Module-4

Numerical Solution For First Order And First Degree Differential Equation

Taylor's Series Method:-

Step ①:- Write the given differential equation has

$\frac{dy}{dx} = y' = f(x,y)$ to the initial condition $y(x_0) = y_0$.

Step ②:- Find $y'(x_0), y''(x_0), y'''(x_0), \ldots$

Step ③:- Write the Taylor series expansion has

$$y(x) = y(x_0) + \frac{(x-x_0)}{1!} y'(x_0) + \frac{(x-x_0)^2}{2!} y''(x_0) + \frac{(x-x_0)^3}{3!} y'''(x_0) + \ldots$$

and simplify.

① Employ the taylor series method to find y at $x=0.1$ correct to 4-decimal places, given $\frac{dy}{dx} = 2y + 3e^x$, $y(0) = 0$.

Given $\frac{dy}{dx} = y' = 2y + 3e^x$, $y(0) = 0$

$\Rightarrow x_0 = 0, \ y_0 = 0$

$y'(x_0) = 2y_0 + 3e^{x_0} = 2(0) + 3e^0 = 3$

$y''(x) = 2y' + 3e^x$

$\Rightarrow y''(x_0) = 2y'(x_0) + 3e^{x_0}$

$= 2(3) + 3e^0$

$= 9$

$y'''(x) = 2y'' + 3e^x$

$\Rightarrow y'''(x_0) = 2y''(x_0) + 3e^{x_0}$

$= 2(9) + 3$

$= 21$

$y^{iv}(x) = 2y''' + 3e^{x}$

$\Rightarrow y^{iv}(x_0) = 2y'''(x_0) + 3e^{x_0}$

$= 2(21) + 3$

$= 45$

$\therefore$ WKT

$\Rightarrow y(x) = y(x_0) + \frac{(x-x_0)}{1!} y'(x_0) + \frac{(x-x_0)^2}{2!} y''(x_0) + \dots$

$\Rightarrow y(x) = 0 + \frac{x}{1!}(3) + \frac{x^2}{2!}(9) + \frac{x^3}{3!}(21) + \frac{x^4}{4!}(45) + \dots$

$\Rightarrow y(x) = 3 \cdot x + \frac{9}{2}x^2 + \frac{7}{2}x^3 + \frac{45}{4!}x^4 + \dots$

$\Rightarrow y(0.1) = 3(0.1) + \frac{9}{2}(0.1)^2 + \frac{7}{2}(0.1)^3 + \frac{45}{4!}(0.1)^4 + \dots$

$\Rightarrow y(0.1) \cong 0.3487$

② If $y' + y + 2x = 0$, $y(0) = -1$, then find $y(0.1)$ using Taylor's series method.

Given $y' + y + 2x = 0$

$\Rightarrow y' = -(y + 2x)$, $y(0) = -1$

$\Rightarrow x_0 = 0 \quad y_0 = -1$

$y'(x_0) = -(y_0 + 2x_0) = -(-1 + 0) = 1$

$y''(x) = -(y' + 2)$

$\Rightarrow y''(x_0) = -(y'(x_0) + 2)$

$= -(1 + 2)$

$= -3$

$y'''(x) = -(y'')$

$\Rightarrow y'''(x_0) = -(y'''(x_0))$

$= -3$

$y^{iv}(x) = -(y''')$

$\Rightarrow y^{iv}(x_0) = -(y'''(x_0))$

$= 3$

$\therefore$ WKT

$\Rightarrow y(x) = y(x_0) + \frac{(x-x_0)}{1!} y'(x_0) + \frac{(x-x_0)^2}{2!} y''(x_0) + \ldots$

$\Rightarrow y(x) = -1 + \frac{x}{1!}(1) + \frac{x^2}{2!}(-3) + \frac{x^3}{3!}(3) + \frac{x^4}{4!}(-3) + \ldots$

$\Rightarrow y(x) = -1 + x - \frac{3}{2}x^2 + \frac{1}{2}x^3 - \frac{1}{8}x^4 + \ldots$

$\Rightarrow y(0.1) = -1 + (0.1) - \frac{3}{2}(0.1)^2 + \frac{1}{2}(0.1)^3 - \frac{1}{8}(0.1)^4$

$\Rightarrow y(0.1) \cong -0.1945$

(3) If $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$, then find $y(0.1)$ using Taylor's series method.

Given $y' = x^2y - 1 \rightarrow (1)$

$\Rightarrow y' = x^2y - 1$, $y_0 = 1$

$x_0 = 0$

$\therefore y'(x_0) = x_0^2 y_0 - 1 = 0(1) - 1 = -1$

$\therefore (1) \Rightarrow y''(x) = 2xy + x^2y' - 0$

$= 2xy + x^2y'$

$y''(x_0) = 2x_0y_0 + x_0^2 y_0'$

$= 2(0)(1) + 0(-1)$

$= 0$

$$\Rightarrow y'''(x) = 2(1.y + xy') + (2xy' + x^2 y'')$$
$$= 2y + 2xy' + 2xy' + x^2y''$$
$$= 2y + 4xy' + x^2y''$$
$$y'''(x_0) = 2y_0 + 4x_0 y_0' + x_0^2 y_0''$$
$$= 2(1) = 2$$

$\therefore$ WKT

$$y(x) = y(x_0) + \frac{(x-x_0)}{1!} y(x_0) + \frac{(x-x_0)^2}{2!} y''(x_0) + \ldots$$

$$\Rightarrow y(x) = 1 + \frac{x}{1!}(-1) + \frac{x^3}{3!}(2) + \ldots$$

$$\Rightarrow y(x) = 1 - \frac{x}{1!} + \frac{x^3}{3} + \ldots$$

$$\therefore y(0.1) = 1 - \frac{0.1}{1!} + \frac{(0.1)^3}{3} + \ldots$$

$$\Rightarrow y(0.1) = 1 - 0.1 + 0.0003$$
$$\Rightarrow y(0.1) \cong 0.9003$$

④ Employ the Taylor series method to find 'y' at $x = 0.1$ correct to 4 decimal places, given $\frac{dy}{dx} = 3x + y^2$, $y_{(0)} = 1$.

Given $\frac{dy}{dx} = y' = 3x + y^2 \rightarrow ①$ $\quad y_{(0)} = 1$

$x_0 = 0 \qquad y_0 = 1$

$$y'(x_0) = 3x_0 + y_0^2$$
$$= 3(0) + 1^2$$
$$= 1$$

$$y''(x) = 3 + 2yy'$$

$$\Rightarrow y''(x_0) = 3 + 2y_0y_0'$$

$= 3 + 2(1)(1)$

$= 5$

$y'''(x) = 0 + 2[yy'' + (y')^2]$

$= 2[yy'' + (y')^2]$

$\Rightarrow y'''(x_0) = 2[y_0 y_0'' + (y_0')^2]$

$= 2[(1)(5) + 1^2]$

$= 12$

$\therefore$ WKT

$$y(x) = y(x_0) + \frac{(x-x_0)}{1!} y(x_0) + \frac{(x-x_0)^2}{2!} y^2(x_0) + \dots$$

$$\Rightarrow y(x) = 1 + \frac{x}{1!}(1) + \frac{x^2}{2!}(5) + \frac{x^3}{3!}(12) + \dots$$

$$\Rightarrow y(x) = 1 + x + \frac{5}{2}x^2 + 2x^3 + \dots$$

$$\Rightarrow y(0.1) = 1 + (0.1) + \frac{5}{2}(0.1)^2 + 2(0.1)^3 + \dots$$

$$\Rightarrow y(0.1) = 1 + 0.1 + 0.025 + 0.002$$

$$y(0.1) \cong 1.127$$

(6) Solve $\frac{dy}{dx} = e^x - y$, $y(0) = 1$ using taylor series method considering upto the 4^{th} degree terms and find $y(0.1)$.

Given $\frac{dy}{dx} = y' = e^x - y$ $\quad y(0) = 1$

$x_0 = 0 \quad y_0 = 1$

$\Rightarrow y'(x_0) = e^{x_0} - y_0$

$= e^0 - 1$

$= 1 - 1 = 0$

$$\Rightarrow y''(x_0) = e^{x_0} - y_0'$$
$$= 1 - 0$$
$$= 1$$

$$\Rightarrow y'''(x) = e^{x} - y''$$
$$y'''(x_0) = e^{x_0} - y_0''$$
$$= 1 - 1 = 0$$

$$\Rightarrow y^{iv}(x) = e^{x} - y'''$$
$$y^{iv}(x_0) = e^{x_0} - y_0'''$$
$$= 1 - 0$$
$$= 1$$

$\therefore$ WKT

$$y(x) = y(x_0) + \frac{(x-x_0)}{1!} y'(x_0) + \frac{(x-x_0)^2}{2!} y''(x_0) + \dots$$

$$\Rightarrow y(x) = 1 + \frac{x}{1!}(0) + \frac{x^2}{2!}(1) + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(1) + \dots$$

$$\Rightarrow y(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$$

$$\Rightarrow y(0.1) = 1 + \frac{(0.1)^2}{2} + \frac{(0.1)^4}{24} + \dots$$

$$\Rightarrow y(0.1) \cong 1.0050$$

6) Solve $\frac{dy}{dx} = x^3 + y$ $y(1) = 1$ using taylor series method considering upto the 4^{th} degree terms and find $y(1.1)$

Given $\frac{dy}{dx} = y' = x^3 + y$

$y(1) = 1 \Rightarrow x_0 = 1, \ y_0 = 1$

$$\Rightarrow y'(x_0) = x_0^3 + y_0$$

$$= 1 + 1 = 2$$

$$y''(x) = 3x^2 + y'$$

$$\Rightarrow y''(x_0) = 3x_0^2 + y_0'$$

$$= 3(1) + 2$$

$$= 5$$

$$y'''(x) = 6x + y''$$

$$\Rightarrow y'''(x_0) = 6x_0 + y_0''$$

$$= 6(1) + 5$$

$$= 11$$

$$y^{iv}(x) = 6 + y'''$$

$$\Rightarrow y^{iv}(x_0) = 6 + y_0'''$$

$$= 6 + 11$$

$$= 17$$

$\therefore$ WKT

$$y(x) = y(x_0) + \frac{(x-x_0)}{1!} y'(x_0) + \frac{(x-x_0)^2}{2!} y''(x_0) + \cdots$$

$$\Rightarrow y(x) = 1 + \frac{(x-x_0)}{1!}(2) + \frac{(x-1)^2}{2!}(11) + \frac{(x-1)^4}{4!}(17) + \cdots$$

$$\Rightarrow y(1.1) = 1 + \frac{2(0.1)}{1!} + \frac{5}{2}(0.1)^2 + \frac{11}{6}(0.1)^3 + \frac{17}{24}(0.1)^4 + \cdots$$

$$\Rightarrow y(1.1) \cong 1.2269$$

Modified Euler's method :-

Step ① :- consider $\frac{dy}{dx} = f(x,y)$ to the initial condition

$y(x_0) = y_0$ having the step size 'h'.

Step ② :-

To get $y(x_1) = y_1$

I-1: $y(x_1) = y_1^{(1)} = y_0 + hf(x_0, y_0)$

I-2: $y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(1)})]$

I-3: $y_1^{(3)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(2)})]$

I-4: $y_1^{(4)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(3)})]$

Step ③ :-

To find $y(x_2) = y_2$

Consider $y(x_1) = y_1$ has the initial condition

I-1: $y(x_2) = y_2^{(1)} = y_1 + hf[x_1, y_1]$

I-2: $y_2^{(2)} = y_1 + \frac{h}{2}[f(x_1, y_1) + f(x_2, y_2^{(1)})]$

I-3: $y_2^{(3)} = y_1 + \frac{h}{2}[f(x_1, y_1) + f(x_2, y_2^{(2)})]$

① Use modified Euler's method to compute $y_{(0.2)}$. Given $\frac{dy}{dx} - xy^2 = 0$, under the initial condition $y_{(0)} = 2$. Perform three iterations at each step, taking $h = 0.1$.

Sol:- Given $\frac{dy}{dx} - xy^2 = 0$

$$\Rightarrow \frac{dy}{dx} = xy^2 = f(x,y) \rightarrow ①$$

and $y_{(0)} = 2 \Rightarrow x_0 = 0,\ y_0 = 2$

To find $y(x_1) = y_1$

$$\Rightarrow y(0.1) = ?$$

$$\Rightarrow y(x_0 + h) = y(x_1) = y_1^{(1)} = y_0 + hf(x_0, y_0)$$

$$\Rightarrow y_1^{(1)} = 2 + (0.1) f(0, 2)$$

$$\Rightarrow y_1^{(1)} = 2 + 0 = 2$$

$$\Rightarrow y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(1)})]$$

$$y_1^{(2)} = 2 + \frac{0.1}{2}[f(0,2) + f(0.1, 2)]$$

$$y_1^{(2)} = 2 + 0.05[0 + 0.4]$$

$$y_1^{(2)} = 2.02$$

$$\Rightarrow y_1^{(3)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(2)})]$$

$$y_1^{(3)} = 2 + \frac{0.1}{2}[f(0,2) + f(0.1, 2.02)]$$

$$= 2 + 0.05[0 + 0.4080]$$

$$y_1^{(3)} = 2.0204$$

$$\therefore y_{(0.1)} = 2.0204$$

$\Rightarrow x_1 = 0.1 \quad y_1 = 2.0204$

To find $y(x_2) = y_2$

$\Rightarrow y_{(0.2)} = ?$

$\Rightarrow y(x_1+h) = y_{(x_2)} = y_2^{(1)} = y_1 + h f(x_1, y_1)$

$y_2^{(1)} = 2.0204 + (0.1) f(0.1, 2.0204)$

$= 2.0204 + (0.1)(0.4082)$

$y_2^{(1)} = 2.0612$

$\Rightarrow y_2^{(2)} = y_1 + \frac{h}{2}[f(x_1, y_1) + f(x_2, y_2^{(1)})]$

$= 2.0204 + \frac{0.1}{2}[f(0.1, 2.0204) + f(0.2, 2.0612)]$

$= 2.0204 + 0.05[0.4082 + 0.8497]$

$y_2^{(1)} = 2.0833$

$\Rightarrow y_2^{(3)} = y_1 + \frac{h}{2}[f(x_1, y_1) + f(x_2, y_2^{(2)})]$

$= 2.0204 + \frac{0.1}{2}[f(0.1, 2.0204) + f(0.2, 2.0833)]$

$= 2.0204 + 0.05[0.4082 + 0.8680]$

$y_2^{(3)} = 2.0842$

$\therefore y(0.2) \cong 2.0842$

② Use modified Euler's method to compute $y_{(0.1)}$. Given $\frac{dy}{dx} = -xy^2$ under the initial condition $y_{(0)} = 2$. Perform three iterations at each step, taking $h = 0.05$.

Sol: Given $\frac{dy}{dx} = -xy^2 = f(x,y) \rightarrow (1)$

and $y_{(0)} = 2,\ x_0 = 0,\ y_0 = 2$

To find $y(x_1) = y_1 \Rightarrow y_{(0.05)} = ?$

$\Rightarrow y_{(x_0+h)} = y_{(x_1)} = y_1^{(1)} = y_0 + hf(x_0, y_0)$

$= 2 + (0.05) f(0,2)$

$= 2 - 0$

$y_1^{(1)} = 2$

$\Rightarrow y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0,y_0) + f(x_1, y_1^{(1)})]$

$= 2 + \frac{0.05}{2}[f(0,2) + f(0.05, 2)]$

$= 2 + 0.025[0 - 0.2]$

$\Rightarrow y_1^{(2)} = 1.995$

$\Rightarrow y_1^{(3)} = y_0 + \frac{h}{2}[f(x_0,y_0) + f(x_1, y_1^{(2)})]$

$= 2 + \frac{0.05}{2}[f(0,2) + f(0.05, 1.995)]$

$= 2 + 0.025[0 - 0.1990]$

$\Rightarrow y_1^{(3)} = 1.9950$

$\therefore y_{(0.05)} = 1.9950$

$\Rightarrow x_1 = 0.05,\ y_1 = 1.9950$

To find $y(x_2) = y_2 \Rightarrow y_{(0.1)} = ?$

$\Rightarrow y(x_1+h) = y(x_2) = y_2^{(1)} = y_1 + hf(x_1, y_1)$

$$= 1.9950 + (0.05) f(0.05, 1.9950)$$

$$\Rightarrow y_2^{(1)} = 1.9850$$

$$\Rightarrow y_2^{(2)} = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y_2^{(1)})\right]$$

$$= 1.9950 + 0.025\left[f(0.05, 1.9950) + f(0.1, 1.9850)\right]$$

$$= 1.9950 + 0.025\left[-0.19900 - 0.3940\right]$$

$$\Rightarrow y_2^{(2)} = 1.9802$$

$$\Rightarrow y_2^{(3)} = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y_2^{(2)})\right]$$

$$= 1.9950 + 0.025\left[f(0.05, 1.9950) + f(0.1, 1.9802)\right]$$

$$= 1.9950 + 0.025\left[-0.19900 - 0.3921\right]$$

$$\Rightarrow y_2^{(3)} = 1.9802$$

$$\therefore y(0.1) \cong 1.9802$$

(3) Use modified Euler's method to compute $y_{(0.1)}$. Given $\frac{dy}{dx} = 3x + \frac{y}{2}$, $y_{(0)} = 1$. Perform three iterations by taking $h = 0.1$.

Sol: Given $\frac{dy}{dx} = 3x + \frac{y}{2} = f(x,y)$

$$y_{(0)} = 1 \Rightarrow x_0 = 0,\ y_0 = 1,\ h = 0.1$$

$$\therefore y_{(x_0+h)} = y_{(x_1)} = y_1^{(1)} = y_0 + h f(x_0, y_0)$$

$$\Rightarrow y_1^{(1)} = 1 + (0.1) f(0,1)$$

$$= 1 + (0.1)(0.5)$$

$$= 1.05$$

$$\Rightarrow y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

$$= 1 + \frac{0.1}{2}\left[f(0,1) + f(0.1, 1.05)\right]$$

$$= 1.0662$$

$$\Rightarrow y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(2)})\right]$$

$$= 1 + \frac{0.1}{2}\left[f(0,1) + f(0.1, 1.0662)\right]$$

$$= 1 + 0.05\left[0.5 + 0.8331\right]$$

$$= 1.0665$$

$$\therefore y_{(0.1)} \simeq 1.0665$$

(4) Using modified Euler's method to find $y_{(0.1)}$ Given $\frac{dy}{dx} = x^2 - y = f(x,y)$, $y_{(0)} = 1$, $x_0 = 0$, $y_0 = 1$ perform three iterations by taking $h = 0.05$.

Sol:: Given $\frac{dy}{dx} = x^2 - y = f(x,y)$

$y(0) = 1 \Rightarrow x_0 = 0 \quad y_0 = 1. \quad h = 0.05$

To find $y(x_1) = y_1$

$\Rightarrow y(x_0+h) = y(x_1) = y_1^{(1)} = y_0 + hf(x_0, y_0)$

$= 1 + (0.05) f(0,1)$

$= 1 - 0.05$

$= 0.95$

$\Rightarrow y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(1)})]$

$= 1 + \frac{0.05}{2}[f(0,1) + f(0.05, 0.95)]$

$= 1 + \frac{0.05}{2}[-1 - 0.9475]$

$= 0.9513$

$\Rightarrow y_1^{(3)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(2)})]$

$= 1 + \frac{0.05}{2}[f(0,1) + f(0.05, 0.9513)]$

$= 1 + \frac{0.05}{2}[-1 - 0.9488]$

$y_1^{(3)} = 0.9513$

$\therefore y(0.05) = 0.9513$

$x_1 = 0.05 \quad y_1 = 0.9513$

To find $y(x_2) = y_2$

$y(x_1+h) = y_2^{(1)} = y_1 + h(f(x_1, y_1)$

$= 0.9513 + (0.05) f(0.05, 0.9513)$

$= 0.9513 + (0.05)(-0.9488)$

$= 0.90386$

$\Rightarrow y_2^{(2)} = y_1 + \frac{h}{2}[f(x_1, y_1) + f(x_2, y_2^{(1)})]$

$= 0.9513 + \frac{0.05}{2}[f(0.05, 0.9513) + f(0.1, 0.90386)]$

$= 0.9513 + \frac{0.05}{2}[-0.9488 - 0.8938]$

$= 0.9052$

$\Rightarrow y_2^{(3)} = 0.95 + \frac{0.05}{2}[f(0.05, 0.9513) + f(0.1, 0.9052)]$

$= 0.95 + \frac{0.05}{2}[-0.9488 - 0.8932]$

$= 0.9052$

$\therefore y(0.1) \cong 0.9052$

⑤ Solve the differential equation $\frac{dy}{dx} = x\sqrt{y}$ under the initial condition $y(1)=1$ by using modified Euler's method find y at $x=1.4$. Perform three iterations at each step, taking $h=0.2$.

Given: $\frac{dy}{dx} = x\sqrt{y} = f(x,y)$

$y(1)=1 \Rightarrow x_0=1, \; y_0=1, \; h=0.2$

To find $y(x_1)=y_1$

$y(x_0+h) = y(x_1) = y_0 + hf(x_0, y_0)$

$= 1 + (0.2)f(1,1)$

$= 1 + 0.2 \times 1$

$= 1.2$

$y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(1)})]$

$= 1 + \frac{0.2}{2}[f(1,1) + f(1.2, 1.2)]$

$$= 1 + 0.1\,[1 + 1.3145]$$

$$= 1.2314$$

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(2)})\right]$$

$$= 1 + \frac{0.2}{2}\left[f(1,1) + f(1.2, 1.2314)\right]$$

$$= 1 + (0.1)\,[1 + 1.3316]$$

$$= 1.2331$$

$$\therefore y_{(1.2)} \cong 1.2331$$

$$\Rightarrow x_1 = 1.2 \qquad y_1 = 1.2331$$

To find $y(x_2) = y_2$

$$y(x_1 + h) = y_2^{(1)} = y_1 + h\, f(x_1, y_1)$$

$$= 1.2331 + (0.2)\, f(1.2, 1.2331)$$

$$= 1.2331 + (0.2)(1.3325)$$

$$= 1.4996$$

$$y_2^{(2)} = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y_2^{(1)})\right]$$

$$= 1.2331 + \frac{0.2}{2}\left[f(1.2, 1.2331) + f(1.4, 1.4996)\right]$$

$$= 1.2331 + 0.1\,[(1.3325) + (1.7144)]$$

$$= 1.5377$$

$$y_2^{(3)} = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y_2^{(2)})\right]$$

$$= 1.2331 + \frac{0.2}{2}\left[f(1.2, 1.2331) + f(1.4, 1.5377)\right]$$

$$= 1.2331 + 0.1\,[1.3325 + 1.7360]$$

$$= 1.5399$$

$$\therefore y_{(1.4)} \cong 1.5399$$

Runge-Kutta method of 4^{th} order :-

Step ①:- write the given differential equation $\frac{dy}{dx} = f(x,y)$ to the initial condition $y(x_0) = y_0$ having the step size 'h'.

Step ②:- Find $y(x_0+h) = y_0 + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

where $K_1 = hf(x_0, y_0)$

$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$

$K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$

$K_4 = hf(x_0 + h, y_0 + K_3)$

① Use 4^{th}-order Runge-Kutta method to solve $(x+y)\frac{dy}{dx} = 1$, $y(0.4) = 1$, to find $y(0.5)$ taking $h = 0.1$.

Sol:- Given $(x+y)\frac{dy}{dx} = 1$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{(x+y)} = f(x,y)$$

and $y(0.4) = 1 \Rightarrow x_0 = 0.4$, $y_0 = 1$, $h = 0.1$.

$\Rightarrow K_1 = hf(x_0, y_0)$

$= (0.1)f(0.4, 1)$

$= (0.1)(0.7147)$

$K_1 = 0.07142$

$\Rightarrow K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$

$= (0.1)f(0.45, 1.0357)$

$K_2 = 0.0673$

$\Rightarrow K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$

$= (0.1)\, f(0.45, 1.0336)$

$K_3 = 0.0674$

$\Rightarrow K_4 = hf(x_0 + h, y_0 + K_3)$

$= (0.1)\, f(0.5, 1.0674)$

$K_4 = 0.06379$

$\therefore y(x_1) = y_0 + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

$\Rightarrow y_{(0.5)} = 1 + \frac{1}{6}[0.07142 + 2(0.0673) + 2(0.0674) + 0.06379]$

$\Rightarrow y_{(0.5)} = 1 + \frac{1}{6}[0.40461]$

$\Rightarrow y_{(0.5)} \simeq 1.0674$

② Use R-K method of 4^{th} order to solve $\frac{dy}{dx} = 3x + \frac{y}{2}$, $y_{(0)} = 1$ to find $y_{(0.2)}$. Take $h = 0.2$.

Given $\frac{dy}{dx} = 3x + \frac{y}{2} = f(x, y)$

$\Rightarrow K_1 = hf(x_0, y_0)$

$= 0.2\, f(0, 1)$

$K_1 = 0.1$

$\Rightarrow K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$

$= (0.2)\, f\left(0 + \frac{0.2}{2}, 1 + \frac{0.1}{2}\right)$

$= (0.2)\, f(0.1, 1.05)$

$K_2 = 0.165$

$\Rightarrow K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$

$= (0.2) f\left(0 + \frac{0.2}{2}, 1 + \frac{0.165}{2}\right)$

$= (0.2) f(0.1, 1.0825)$

$K_3 = 0.16825$

$\Rightarrow K_4 = hf(x_0 + h, y_0 + K_3)$

$= (0.2) f(0 + 0.2, 1 + 0.16825)$

$= (0.2) f(0.2, 1.16825)$

$K_4 = 0.23682$

$\Rightarrow y_{(0.2)} = 1 + \frac{1}{6}[0.1 + 2 \times 0.165 + 2 \times 0.16825 + 0.23682]$

$y_{(0.2)} \cong 1.16722$

③ Use R-K method to find $y_{(0.2)}$, given $\frac{dy}{dx} = \sqrt{x+y}$, taking $h = 0.2$ initial condition $y_{(0)} = 1$.

Sol: Given $\frac{dy}{dx} = \sqrt{x+y} = f(x,y)$

$h = 0.2$

$y_{(0)} = 1 \Rightarrow x_0 = 0, \; y_0 = 1$

$\Rightarrow K_1 = hf(x_0, y_0)$

$= 0.2 f(0,1)$

$= 0.2$

$\Rightarrow K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$

$= (0.2) f\left(0 + \frac{0.2}{2}, 1 + \frac{0.2}{2}\right)$

$= (0.2) f(0.1, 1.1)$

$= 0.2190$

$\Rightarrow K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$

$= (0.2) f\left(0 + \frac{0.2}{2}, 1 + \frac{0.2190}{2}\right)$

$= (0.2) f(0.1, 1.1095)$

$= 0.2199$

$\Rightarrow K_4 = hf(x_0 + h, y_0 + K_3)$

$= (0.2) f(0 + 0.2, 1 + 0.2199)$

$= (0.2) f(0.2, 1.2199)$

$= 0.2383$

$\Rightarrow y(x_0 + h) = y_0 + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

$= 1 + \frac{1}{6}[0.2 + 2 \times 0.2190 + 2 \times 0.2199 + 0.2383]$

$y_{(0.2)} = 1.21935$

(4) Using R-K method of 4th order find $y_{(0.2)}$ for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}$ $y_{(0)} = 1$ taking $h = 0.2$.

Sol:- Given $\frac{dy}{dx} = \frac{y-x}{y+x} = f(x,y)$

$x_0 = 0, y_0 = 1$

$\Rightarrow K_1 = hf(x_0, y_0)$

$= 0.2 f(0, 1)$

$K_1 = 0.2$

$\Rightarrow K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$

$= (0.2) f\left(0 + \frac{0.2}{2}, 1 + \frac{0.2}{2}\right)$

$= (0.2) f(0.1, 1.1)$

$K_2 = 0.1667$

$\Rightarrow K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$

$= (0.2)\, f\left(0 + \frac{0.2}{2}, 1 + \frac{0.1667}{2}\right)$

$= 0.2\, f(0.1, 1.0833)$

$K_3 = 0.1662$

$\Rightarrow K_4 = hf(x_0 + h, y_0 + K_3)$

$= 0.2\, f(0.2, 1 + 0.1662)$

$= 0.1414$

$\Rightarrow y(x_0+h) = y_0 + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

$= 1 + \frac{1}{6}[0.2 + 0.3334 + 0.3324 + 0.1414]$

$= 1.1679$

⑤ Using R-K method of 4^{th}-orther find $y(0.2)$ for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}$ $y(0) = 1$ taking $h = 0.1$.

Given $\frac{dy}{dx} = \frac{y-x}{y+x} = f(x,y)$

$y(0) = 1, \; x_0 = 0, \; y_0 = 1 \quad h = 0.1$

Stage 1:- $\Rightarrow K_1 = hf(x_0, y_0)$

$= (0.1)\, f(0,1)$

$= 0.1$

$\Rightarrow K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$

$= (0.1)\, f(0.05, 1.05)$

$= 0.091$

$\Rightarrow K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$

$= (0.1)\,f(0.05, 1.0455)$

$= 0.0909$

$\Rightarrow k_4 = hf(x_0+h, y_0+k_3)$

$= (0.1)\,f(0.1, 1.0909)$

$= 0.0832$

$\therefore y(x_0+h) = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$= 1 + \frac{1}{6}(0.1 + 0.182 + 0.1818 + 0.0832)$

$y_{(0.1)} = 1.091167 \cong 1.0912$

Stage - 2 :-

$f(x,y) = \dfrac{y-x}{y+x}$, $x_0 = 0.1$, $y_0 = 1.0912$, $h = 0.1$

$\Rightarrow k_1 = hf(x_0, y_0) = (0.1)\,f(0.1, 1.0912)$

$= 0.0832$

$\Rightarrow k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$= (0.1)\,f(0.15, 1.1328)$

$= 0.0766$

$\Rightarrow k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$= (0.1)\,f(0.15, 1.1295)$

$= 0.07655$

$\Rightarrow k_4 = hf(x_0+h, y_0+k_3)$

$= (0.1)\,f(0.2, 1.16775)$

$= 0.07075$

$\therefore y(x_0+h) = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$$y_{(0.1+0.1)} = 1.0912 + \frac{1}{6}(0.0832 + 0.1532 + 0.1531 + 0.07075)$$

$$\Rightarrow y_{(0.2)} = 1.167908 \cong 1.1679$$

⑥ Solve: $(y^2-x^2)dx = (y^2+x^2)dy$ for $x=0.2$ and 0.4 given that $y=1$ at $x=0$ initially. by applying Runge-Kutta method of order 4. Compute $y(0.2)$ by taking $h=0.2$.

Sol:- Given $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$, $x_0=0$, $y_0=1$, $h=0.2$

Stage ①:- $f(x,y) = \frac{y^2-x^2}{y^2+x^2}$

$$\Rightarrow K_1 = hf(x_0, y_0)$$
$$= (0.2)f(0,1)$$
$$= 0.2$$

$$\Rightarrow K_2 = hf\left(x_0+\frac{h}{2}, y_0+\frac{K_1}{2}\right)$$
$$= (0.2)f(0.1, 1.1)$$
$$= 0.1967$$

$$\Rightarrow K_3 = hf\left(x_0+\frac{h}{2}, y_0+\frac{K_2}{2}\right)$$
$$= (0.2)f(0.1, 1.0984)$$
$$= 0.1967$$

$$\Rightarrow K_4 = hf(x_0+h, y_0+K_3)$$
$$= (0.2)f(0.2, 1.1967)$$
$$= 0.1891$$

$$\therefore y(x_0+h) = y_0 + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$$

$$= 1 + \frac{1}{6}[0.2 + 2 \times 0.1967 + 2 \times 0.1967 + 0.1891]$$

$$y_{(0.2)} = 1.19598 \cong 1.196$$

Stage ②:- $f(x,y) = \frac{y^2 - x^2}{y^2 + x^2}$, $x_0 = 0.2$, $y_0 = 1.196$, $h = 0.2$

$$\Rightarrow K_1 = hf(x_0, y_0) = (0.2) f(0.2, 1.196)$$
$$= 0.1891$$

$$\Rightarrow K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$$
$$= (0.2) f(0.3, 1.29055)$$
$$= 0.1795$$

$$\Rightarrow K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$$
$$= (0.2) f(0.3, 1.28575)$$
$$= 0.1793$$

$$\Rightarrow K_4 = hf(x_0 + h, y_0 + K_3)$$
$$= (0.2) f(0.4, 1.3753)$$
$$= 0.1688$$

$$\therefore y(x_0 + h) = y_0 + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$$

$$y(0.4) = 1.196 + \frac{1}{6}[0.1891 + 2 \times 0.1795 + 2 \times 0.1793 + 0.1688]$$
$$= 1.37525$$

$$y_{(0.4)} \cong 1.3753$$

⑦ Use fourth order Runge-Kutta method to compute $y(1.1)$ given that $\frac{dy}{dx} = xy^{1/3}$, $y(1) = 1$.

Sol:- Given $f(x,y) = xy^{1/3}$, $x_0 = 1$, $y_0 = 1$

Given, $f(x,y) = xy^{1/3}$

$\Rightarrow k_1 = hf(x_0, y_0)$

$= (0.1) f((1)(1)^{1/3})$

$= 0.1$

$\Rightarrow k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$= (0.1) f(1.05, 1.05)$

$= 0.1067$

$\Rightarrow k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$= (0.1) f(1.05, 1.05335)$

$= 0.1068$

$\Rightarrow k_4 = hf(x_0 + h, y_0 + k_3)$

$= (0.1) f(1.1, 1.1068)$

$= 0.1138$

$\therefore y(x_0+h) = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= 1 + \frac{1}{6}[0.1 + 2 \times 0.1067 + 2 \times 0.1068 + 0.1138]$

$y(1.1) = 1.1068$

Milne's, Adams - Bashforth Predictor and Corrector method :-

Step ①:- Consider the given differential equation as $\frac{dy}{dx} = f(x,y)$, to the initial conditions, $y(x_0) = y_0$, $y(x_1) = y_1$ $y(x_2) = y_2$, $y(x_3) = y_3$.

Step ②:- Find $f_0 = f(x_0, y_0)$, $f_1 = f(x_1, y_1)$, $f_2 = f(x_2, y_2)$, $f_3 = f(x_3, y_3)$ and find $y(x_4) = y_4$ using

1. Milne's method:-

$$y_4^{(P)} = y_0 + \frac{4h}{3}[2f_1 - f_2 + 2f_3]$$

$$y_4^{(C)} = y_2 + \frac{h}{3}[f_2 + 4f_3 + f_4^{(P)}] \qquad f_4^{(P)} = f(x_4, y_4^{(P)})$$

2. Adams method:-

$$y_4^{(P)} = y_3 + \frac{h}{24}[55f_3 - 59f_2 + 37f_1 - 9f_0]$$

$$y_4^{(C)} = y_3 + \frac{h}{24}[9f_4^{(P)} + 19f_3 - 5f_2 + f_1] \qquad f_4^{(P)} = f(x_4, y_4^{(P)})$$

① Given $\frac{dy}{dx} = x^2(1+y)$, $y(1)=1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$, $y(1.3) = 1.979$. Evaluate $y(1.4)$ by

1) Milne's Predictor - Corrector method.

2) Adams - Bashforth Predictor - Corrector method.

Sol:- Given $\frac{dy}{dx} = x^2(1+y) = f(x,y)$

$y(1) = 1 \Rightarrow x_0 = 1, \ y_0 = 1$

$y(1.1) = 1.233 \Rightarrow x_1 = 1.1, \ y_1 = 1.233$

$y(1.2) = 1.548 \Rightarrow x_2 = 1.2, \ y_2 = 1.548$

$y(1.3) = 1.979 \Rightarrow x_3 = 1.3, \ y_3 = 1.979$ & $h = 0.1$.

$f_0 = f(x_0, y_0) = f(1,1) = 2$

$f_1 = f(x_1, y_1) = f(1.1, 1.233) = 2.7019$

$f_2 = f(x_2, y_2) = f(1.2, 1.548) = 3.6691$

$f_3 = f(x_3, y_3) = f(1.3, 1.979) = 5.0345$

1) Milne's method:-

$\Rightarrow y_4^{(P)} = y_0 + \frac{4h}{3}[2f_1 - f_2 + 2f_3]$

$= 1 + \frac{4(0.1)}{3}[2(2.7019) - 3.6691 + 2(5.0345)]$

$\Rightarrow y_4^{(P)} = 2.5738$

$\therefore f_4^{(P)} = f(1.4, 2.5738) = 7.0046$

$\therefore y_4^{(C)} = y_2 + \frac{h}{3}[f_2 + 4f_3 + f_4^{(P)}]$

$= 1.548 + \frac{0.1}{3}[3.6691 + 4(5.0345) + 7.0046]$

$\Rightarrow y_4^{(C)} = 2.5750$

$\therefore y(x_4) = y(1.4)$

$= 2.5750$

2) Adams method:-

$y_4^{(P)} = y_3 + \frac{h}{24}[55f_3 - 59f_2 + 37f_1 - 9f_0]$

$= 1.979 + \frac{0.1}{24}[276.8975 - 216.4769 + 99.9703 - 18]$

$\Rightarrow y_4^{(P)} = 2.5722$

$\therefore f_4^{(P)} = f(1.4, 2.5722) = 7.0015$

$\therefore y_4^{(C)} = y_3 + \frac{h}{24}[9f_4^{(P)} + 19f_3 - 5f_2 + f_1]$

$= 1.979 + \frac{0.1}{24}[63.0135 + 95.6555 - 18.3455 + 2.7019]$

$\Rightarrow y_4^{(C)} = 2.5750$

② Given $\frac{dy}{dx} = \frac{1}{x+y}$ $y_{(0)}=2$, $y_{(0.2)} = 2.0933$, $y_{(0.4)} = 2.1755$, $y_{(0.6)} = 2.2493$. Compute y at $x=0.8$ by using

(i) Milne's - Predictor Corrector method.

(ii) Adams - Bashforth Predictor Corrector method.

Sol:- Given. $\frac{dy}{dx} = \frac{1}{x+y} = f(x,y)$

$y_{(0)} = 2 \Rightarrow x_0 = 0,\ y_0 = 2$

$y_{(0.2)} = 2.0933 \Rightarrow x_1 = 0.2,\ y_1 = 2.0933$

$y_{(0.4)} = 2.1755 \Rightarrow x_2 = 0.4,\ y_2 = 2.1755$

$y_{(0.6)} = 2.2493 \Rightarrow x_3 = 0.6,\ y_3 = 2.2493$ and $h = 0.2$

$f_0(x_0, y_0) = f(0,2) = 0.5$

$f_1(x_1, y_1) = f(0.2, 2.0933) = 0.4360$

$f_2(x_2, y_2) = f(0.4, 2.1755) = 0.3882$

$f_3(x_3, y_3) = f(0.6, 2.2493) = 0.3509$

i) Milne's method:-

$\Rightarrow y_4^{(P)} = y_0 + \frac{4h}{3}[2f_1 - f_2 + 2f_3]$

$= 2 + \frac{4(0.2)}{3}(2 \times 0.4360 - 0.3882 + 2 \times 0.3509)$

$\Rightarrow y_4^{(P)} = 2.3162$ $\quad \therefore f_4^{(P)} = f(0.8, 2.3162) = 0.32091$

$\therefore y_4^{(C)} = y_2 + \frac{h}{3}[f_2 + 4f_3 + f_4^{(P)}]$

$= 2.1755 + \frac{0.2}{3}[0.3882 + 4 \times 0.3509 + 0.32091]$

$y_4^{(C)} = 2.3163.$

$$y(x_4) = y(0.8)$$
$$= 2.3163$$

(ii) Adams Method :-

$$y_4^{(P)} = y_3 + \frac{h}{24}[55f_3 - 59f_2 + 37f_1 - 9f_0]$$

$$= 2.2493 + \frac{0.2}{24}[55 \times 0.3509 - 59 \times 0.3882 + 37 \times 0.4360 - 9 \times 0.5]$$

$$y_4^{(P)} = 2.3162$$

$$\therefore f_4^{(P)} = f(0.8, 2.3162) = 0.32090$$

$$\Rightarrow y_4^{(C)} = y_3 + \frac{h}{24}[9f_4^{(P)} + 19f_3 - 5f_2 + f_1]$$

$$= 2.2493 + \frac{0.2}{24}[9 \times 0.32090 + 19 \times 0.3509 - 5 \times 0.3882 + 0.4360]$$

$$y_4^{(C)} = 2.3164$$

$$\therefore y(x_4) = y(0.8)$$
$$= 2.3164$$

③ Apply Milne's and Adam's method to compute $y(0.4)$ given $\frac{dy}{dx} = x + y^2$ with condition

x	y
0.0	1.0000
0.1	1.1000
0.2	1.2310
0.3	1.4020

Sol: Given $\frac{dy}{dx} = x + y^2 = f(x, y)$

$$y(0.0) = 1.0000 \Rightarrow x_0 = 0.0, \; y_0 = 1.0000$$

$$y(0.1) = 1.1000 \Rightarrow x_1 = 0.1, \; y_1 = 1.1000$$

$$y(0.2) = 1.2310 \Rightarrow x_2 = 0.2, \; y_2 = 1.2310$$

$$y(0.3) = 1.4020 \Rightarrow x_3 = 0.3, \; y_3 = 1.4020$$

$$h = 0.1$$

$f_0 = f(x_0, y_0) = f(0.0, 1.0000) = 1$

$f_1 = f(x_1, y_1) = f(0.1, 1.1000) = 1.31$

$f_2 = f(x_2, y_2) = f(0.2, 1.2310) = 1.7154$

$f_3 = f(x_3, y_3) = f(0.3, 1.4020) = 2.2656$

(i) Milne's method:-

$\Rightarrow y_4^{(P)} = y_0 + \frac{4h}{3}[2f_1 - f_2 + 2f_3]$

$= 1.0000 + \frac{4\times 0.1}{3}[2\times 1.31 - 1.7154 + 2\times 2.2656]$

$\Rightarrow y_4^{(P)} = 1.72477$

$\therefore f_4^{(P)} = f(0.4, 1.72477) = 3.37483$

$\Rightarrow y_4^{(C)} = y_2 + \frac{h}{3}[f_2 + 4f_3 + f_4^{(P)}]$

$= 1.2310 + \frac{0.1}{3}[1.7154 + 4\times 2.2656 + 3.37483]$

$\Rightarrow y_4^{(C)} = 1.7027$

(ii) Adams method:-

$\Rightarrow y_4^{(P)} = y_3 + \frac{h}{24}[55f_3 - 59f_2 + 37f_1 - 9f_0]$

$= 1.4020 + \frac{0.1}{24}[55\times 2.2656 - 59\times 1.7154 + 37\times 1.31 - 9\times 1]$

$\Rightarrow y_4^{(P)} = 1.66395$

$\therefore f_4^{(P)} = f(x_4, y_4^{(P)}) = f(0.4, 1.66395) = 3.1687$

$y_4^{(C)} = y_3 + \frac{h}{24}[9f_4^{(P)} + 19f_3 - 5f_2 + f_1]$

$= 1.4020 + \frac{0.1}{24}[9\times 3.1687 + 19\times 2.2656 - 5\times 1.7154 + 1.31]$

$\Rightarrow y_4^{(C)} = 1.6699$

(4) Given $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$, $y_{(1)} = 1$, $y_{(1.1)} = 0.9960$, $y_{(1.2)} = 0.9860$, $y_{(1.3)} = 0.9720$ find $y_{(1.4)}$ using Adam's Bashforth Predictor Corrector method.

Sol: Given $\frac{dy}{dx} = \frac{1}{x^2} - \frac{y}{x}$

$$\Rightarrow \frac{dy}{dx} = \frac{1-xy}{x^2} = f(x,y)$$

$f_0 = f(x_0, y_0) = f(1,1) = 0$

$f_1 = f(x_1, y_1) = f(1.1, 0.9960) = -0.0790$

$f_2 = f(x_2, y_2) = f(1.2, 0.9860) = -0.1272$

$f_3 = f(x_3, y_3) = f(1.3, 0.9720) = -0.15597$

$y_{(1)} = 1 \Rightarrow x_0 = 1, \ y_0 = 1$

$y_{(1.1)} = 0.9960 \Rightarrow x_1 = 1.1, \ y_1 = 0.9960$

$y_{(1.2)} = 0.9860 \Rightarrow x_2 = 1.2, \ y_2 = 0.9860$

$y_{(1.3)} = 0.9720 \Rightarrow x_3 = 1.3, \ y_3 = 0.9720$ & $h = 0.1$

$$\Rightarrow y_4^{(P)} = y_3 + \frac{h}{24}[55f_3 - 59f_2 + 37f_1 - 9f_0]$$

$$= 0.9720 + \frac{h}{24}[55 \times (-0.15597) + 59 \times 0.1272 - 37 \times 0.0790]$$

$$\Rightarrow y_4^{(P)} = 0.95535$$

$$\therefore f_4^{(P)} = f(x_4, y_4^{(P)}) = f(1.4, 0.95535) = -0.17219 \quad -0.172189$$

$$\Rightarrow y_4^{(C)} = y_3 + \frac{h}{24}[9f_4^{(P)} + 19f_3 - 5f_2 + f_1]$$

$$= 0.9720 + \frac{0.1}{24}[9 \times (-0.17219) + 19 \times (-0.15597) - 5 \times (-0.1272) + (-0.0790)]$$

$$y_4^{(C)} = 0.95552$$

(5) Apply Milne's predictor - corrector method to compute $y(2.0)$, given $\frac{dy}{dx} = \frac{1}{2}(x+y)$ and

x	y
0.0	2.0000
0.5	2.6360
1.0	3.5950
1.5	4.9680

Sol:- Given: $\frac{dy}{dx} = \frac{x+y}{2} = f(x,y)$

$y(0.0) = 2.0000,\ x_0 = 0.0,\ y_0 = 2.0000$

$y(0.5) = 2.6360,\ x_1 = 0.5,\ y_1 = 2.6360$

$h = 0.5$

$y(1.0) = 3.5950,\ x_2 = 1.0,\ y_2 = 3.5950$

$y(1.5) = 4.9680,\ x_3 = 1.5,\ y_3 = 4.9680$

$f_0 = f(x_0, y_0) = f(0.0, 2.0000)$
$= 1$

$f_1 = f(x_1, y_1) = f(0.5, 2.6360) = 1.568$

$f_2 = f(x_2, y_2) = f(1.0, 3.5950) = 2.2975$

$f_3 = f(x_3, y_3) = f(1.5, 4.9680) = 3.234$

$\Rightarrow y_4^{(P)} = y_0 + \frac{4h}{3}[2f_1 - f_2 + 2f_3]$

$= 2.0000 + \frac{4\times0.5}{3}[2\times1.568 - 2.2975 + 2\times3.234]$

$\Rightarrow y_4^{(P)} = 6.871$

$\therefore f_4^{(P)} = f(2.0, 6.871) = 4.4355$

$\Rightarrow y_4^{(C)} = y_2 + \frac{h}{3}[f_2 + 4f_3 + f_4^{(P)}]$

$= 3.5950 + \frac{0.5}{3}[2.2975 + 4\times3.234 + 4.4355]$

$y_4^{(C)} = 6.8732$

$\therefore y(x_4) = y(2.0) = 6.8732$