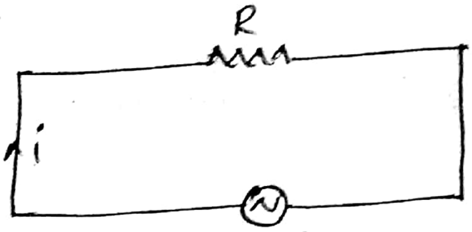
PURE RESISTANCE CIRCUIT (R) : By, ohm's law



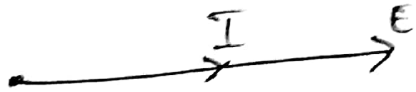
$i = \frac{e}{R}$

$i = \frac{E_m}{R} \sin\omega t = I_m \sin\omega t$

where $I_m = \frac{E_m}{R}$

↳ Comparing $e = E_m \sin\omega t$ & $i = I_m \sin\omega t$, the current is in phase with voltage.

↳ Vector representation,



Instantaneous Power $\Big\}$ $P = ei = E_m \sin\omega t \cdot I_m \sin\omega t = E_m I_m \sin^2\omega t$

$P = \frac{E_m I_m}{2} [1 - \cos 2\omega t]$

$P = \frac{1}{2} E_m I_m - \frac{1}{2} E_m I_m \cos 2\omega t.$

The term $\frac{1}{2} E_m I_m \cos 2\omega t$ is a periodically varying quantity whose frequency is two times the frequency of the applied voltage & its Average value over a period is zero.

$\therefore P = \frac{1}{2} E_m I_m = \frac{E_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$

$\boxed{P = EI}$

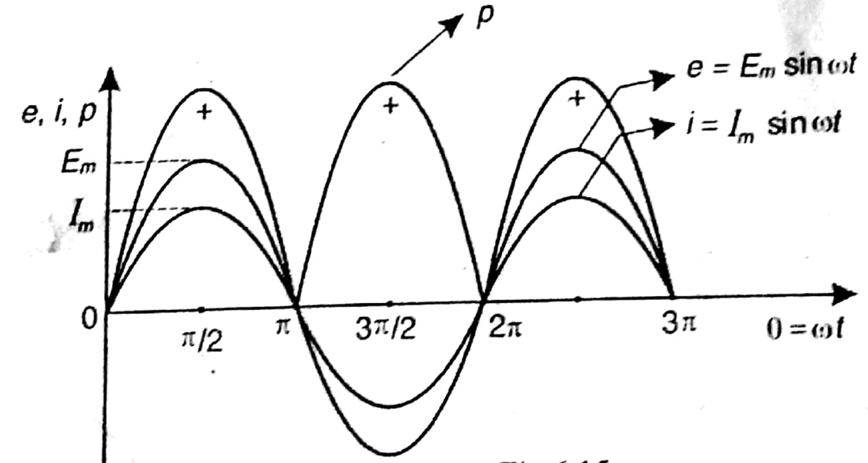


Fig.6.15

FORM FACTOR (K_f) : The form factor of an Alternating quantity represented by a sinusoidal waveform is defined as the ratio of RMS value to its Average value.

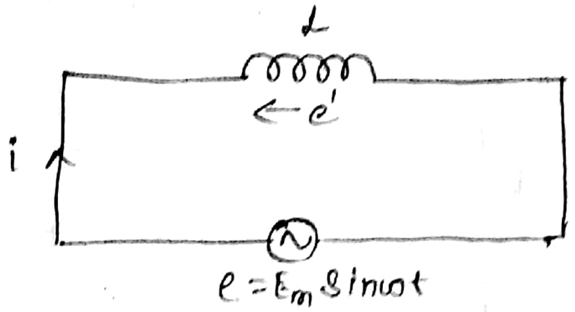
$$\text{Form factor } K_f = \frac{\text{rms value}}{\text{Average value}} = \frac{I_{rms}}{I_{av}}$$

$$K_f = \frac{0.707\, I_m}{0.637\, I_m} = 1.11 \text{ for a sine wave}$$

PEAK FACTOR (K_p) : The Peak factor of an Alternating quantity represented by a sinusoidal waveform is defined as the ratio of its Maximum value to its rms value.

$$\text{Peak factor } K_p = \frac{\text{Maximum value}}{\text{rms value}} = \frac{I_m}{I_{rms}}$$

$$K_p = \frac{I_m}{0.707 I_m} = 1.414, \text{ for a sine wave.}$$



PURE INDUCTANCE CIRCUIT (L) : An Alternating voltage $e = E_m \sin\omega t$ produces an alternating current i, which produces an alternating flux linking the coil, hence an EMF e' is induced in it, which opposes the applied voltage.

$\therefore e = E_m \sin\omega t$

$$e' = -L\frac{di}{dt} = -e \quad \therefore \boxed{e = L\frac{di}{dt}}$$

$$di = \frac{e}{L}dt = \frac{1}{L}E_m \sin\omega t \cdot dt$$

On Integration

$$i = \frac{E_m}{L}\int \sin\omega t \cdot dt$$

$$i = \frac{E_m}{\omega L}(-\cos\omega t)$$

$$i = \frac{E_m}{X_L}\sin(\omega t - \pi/2)$$

$$\boxed{i = I_m \sin(\omega t - \pi/2)}$$

where,

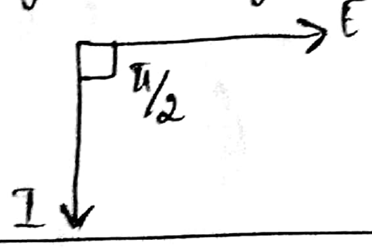
$I_m = \frac{E_m}{X_L}$

$X_L = \omega L = 2\pi f \cdot L$

$X_L \rightarrow$ Inductive reactance in ohms.

$\therefore$ Comparing $e = E_m \sin\omega t$ & $i = I_m \sin(\omega t - \pi/2)$, the current lags the voltage by an angle $\pi/2$.

↳ Vector representation



Instantaneous Power } $P = ei = E_m \sin\omega t \cdot I_m \sin(\omega t - \pi/2)$

$= E_m I_m \sin\omega t \cdot (-\cos 2\omega t)$

$$P = -\frac{1}{2} E_m I_m \sin 2\omega t$$

↳ The equation for 'P' is periodically varying & having a frequency two times the frequency of the applied voltage & whose average value is zero.

Hence the Power Consumed by Pure Inductance is zero ∴ $\underline{\underline{P = 0}}$

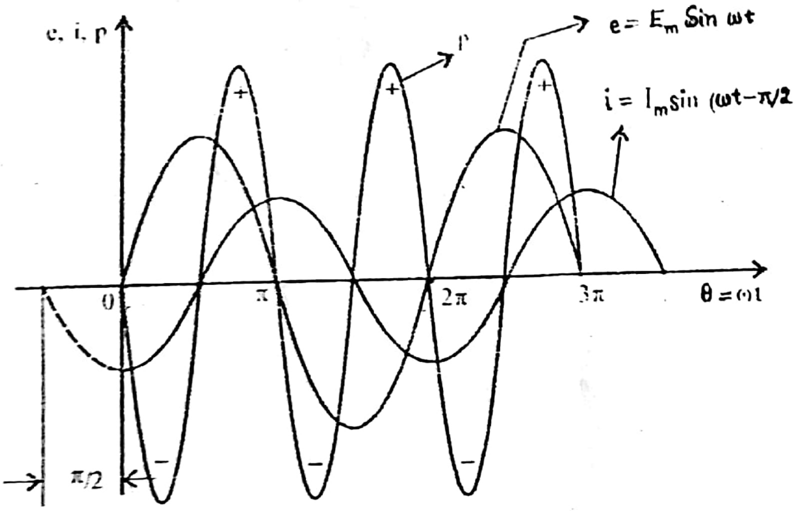
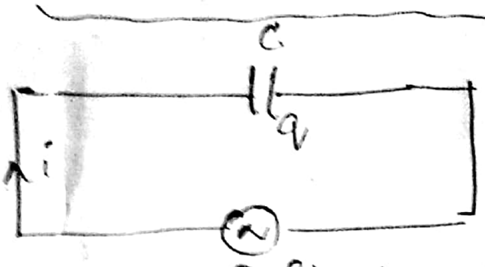


Fig.6.18

PURE CAPACITANCE CIRCUIT (C) : Consider a pure Capacitance 'C' across which an Alternating voltage $e = E_m \sin\omega t$ is applied, due to which an alternating current flows.



$e = E_m \sin\omega t$

$$i = \frac{dq}{dt} = \frac{d[ce]}{dt} = C\frac{d[E_m \sin\omega t]}{dt}$$

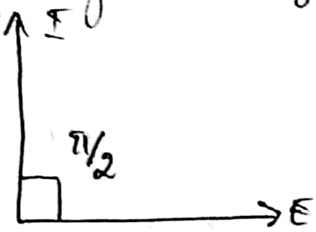
$$= \omega C E_m \cos\omega t = \frac{E_m}{(1/\omega C)} \sin(\omega t + \pi/2) = \frac{E_m \sin(\omega t + \pi/2)}{X_C}$$

$$\boxed{i = I_m \sin(\omega t + \pi/2)}$$ where $I_m = \frac{E_m}{X_C}$ &

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$ → capacitive reactance in ohms.

↳ Comparing $e = E_m \sin\omega t$ & $i = I_m \sin(\omega t + \pi/2)$, Current leads voltage by an angle $\pi/2$.

↳ Vector Representation



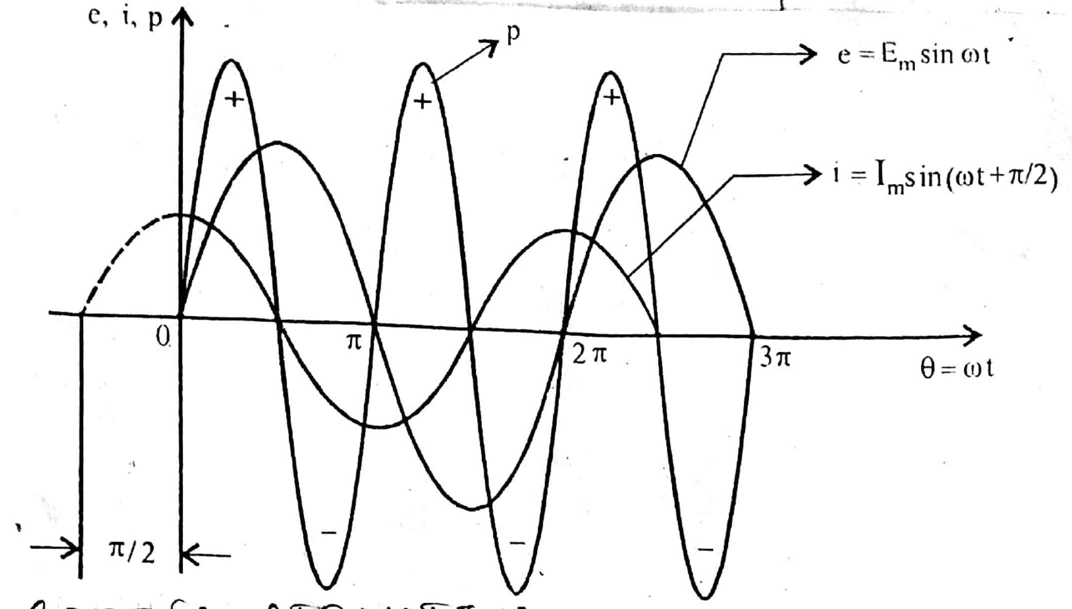
↳ Instantaneous Power $P = ei = E_m \sin\omega t \cdot I_m \sin(\omega t + \pi/2)$

$$= E_m I_m \sin\omega t \cdot \cos\omega t$$

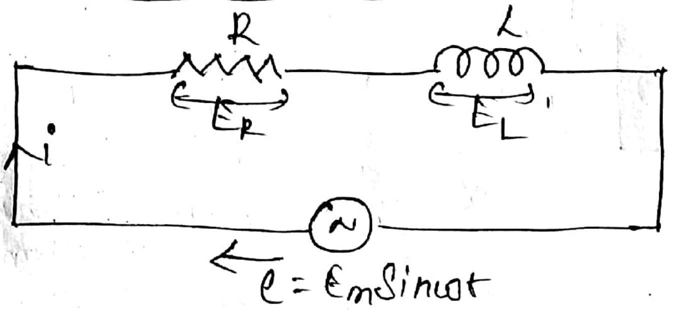
$$P = \frac{1}{2} E_m I_m \sin 2\omega t$$

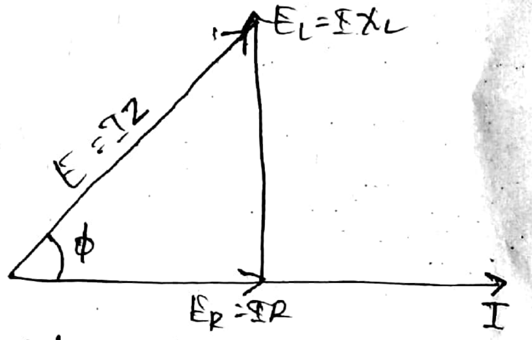
The equation for 'P' is periodically varying & having frequency two times the frequency of the applied voltage & whose Average value is zero.

Hence the Power consumed by pure Capacitance is zero $\therefore P = 0$



RL - SERIES CIRCUIT :-





Vector diagram

↳ From the Vector diagram

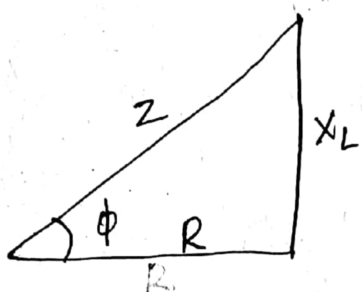
$E_R = IR$ → which is in phase with current.

$E_L = IX_L$ → which leads the current by 90°

$E = IZ$ → The vector sum of E_R & E_L

↳ $I = \frac{E}{Z}$ where $Z = \sqrt{R^2 + X_L^2}$

Z → Impedance of the circuit in Ohms.



Impedance Triangle

↳ Power factor angle } $\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$

↳ Here the current lags the voltage by an angle ϕ i.e if $e = E_m \sin\omega t$ than $i = I_m \sin(\omega t - \phi)$

Instantaneous Power $p = ei = E_m \sin\omega t \cdot I_m \sin(\omega t - \phi)$

$$p = \frac{E_m I_m}{2}\left[\cos\phi - \cos(2\omega t - \phi)\right]$$

$$\therefore\ p = \frac{E_m I_m}{2}\cos\phi - \frac{E_m I_m}{2}\cos(2\omega t - \phi)$$

The second term is a periodically varying quantity whose frequency is two times the frequency of the applied voltage & its average value is zero.

$$\therefore\ P = \frac{1}{2} E_m I_m \cos\phi = \frac{E_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \times \cos\phi$$

$$\therefore\ \boxed{P = EI\cos\phi}$$

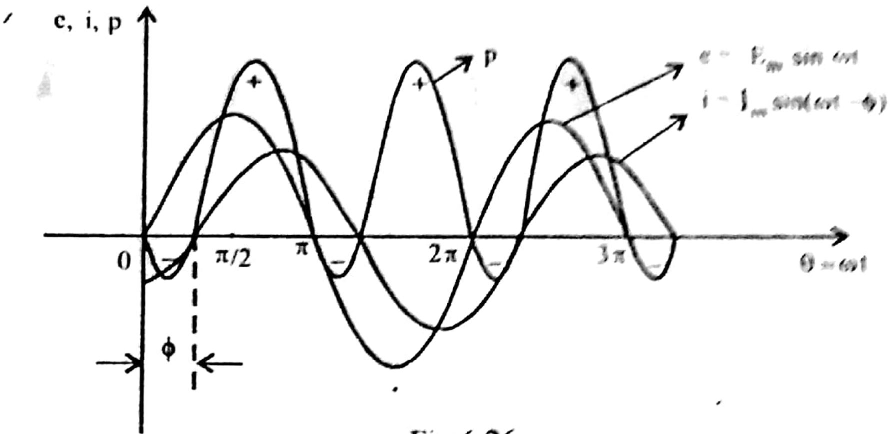
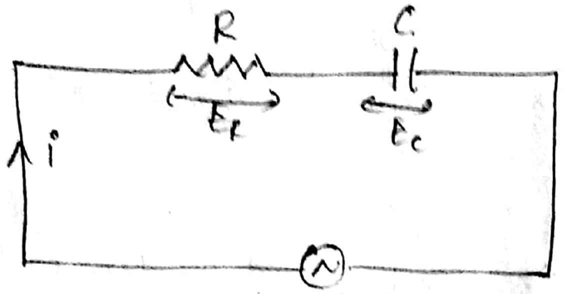
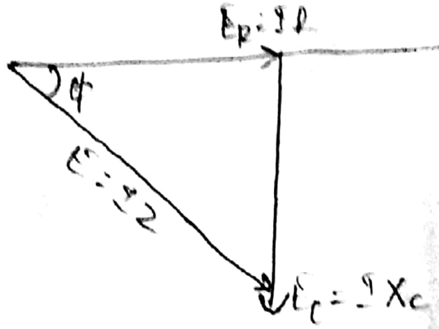


Fig.6.26

RC. SERIES CIRCUIT :



$e = E_m \sin\omega t$



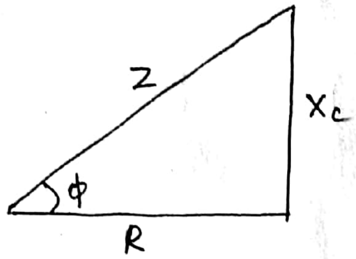
Vector diagram

From the Vector diagram

$E_R = IR$ → which is in phase with Current.

$E_C = IX_C$ → which lags the Current by 90°

$E = IZ$ → The Vector Sum of E_R & E_C.



Impedance Triangle

$I = \dfrac{E}{Z}$ where $Z = \sqrt{R^2 + X_C^2}$

Z → Impedance of the Circuit.

Power factor angle } $\phi = \tan^{-1}\left(\dfrac{X_C}{R}\right)$

↳ Here the Current leads the voltage by an angle 'ϕ' i.e if $e = E_m \sin\omega t$ then $i = I_m \sin(\omega t + \phi)$

↳ Instantaneous Power $P = e\,i = E_m \sin\omega t \cdot I_m \sin(\omega t + \phi)$

$$= \frac{E_m I_m}{2}\left[\cos(-\phi) - \cos(2\omega t + \phi)\right]$$

$$\therefore P = \frac{1}{2} E_m I_m \cos\phi - \frac{1}{2} E_m I_m \cos(2\omega t + \phi)$$

The Second term is a periodically varying quantity, whose frequency is two times the frequency of the applied voltage & its average value is zero.

$$\therefore P = \frac{1}{2} E_m I_m \cos\phi = \frac{E_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \times \cos\phi$$

$$\therefore \boxed{P = EI \cos\phi}$$

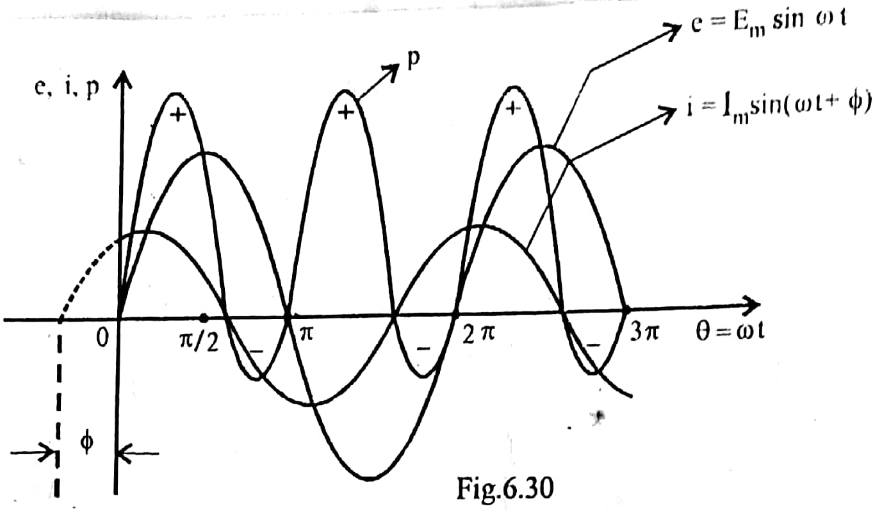
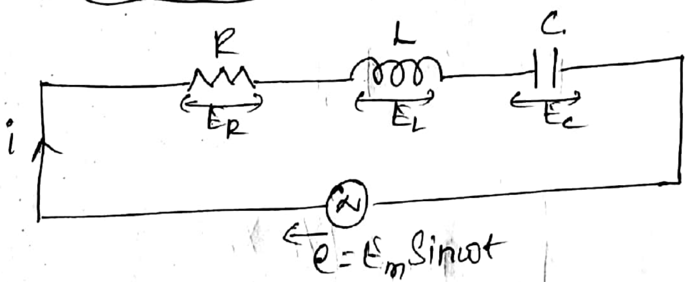


Fig.6.30

RLC - SERIES CIRCUIT :-



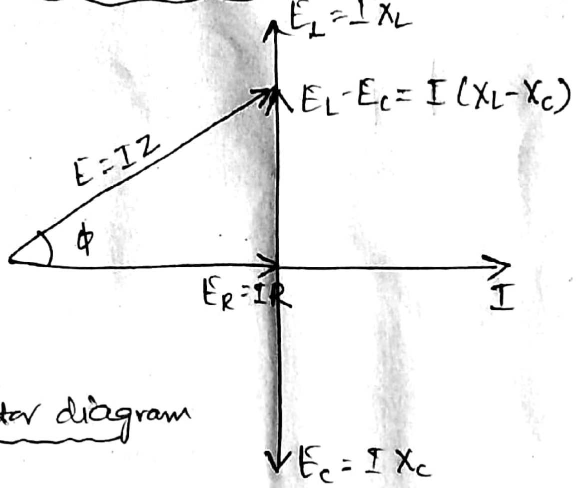
Three Cases are discussed

(i) when $X_L > X_C$

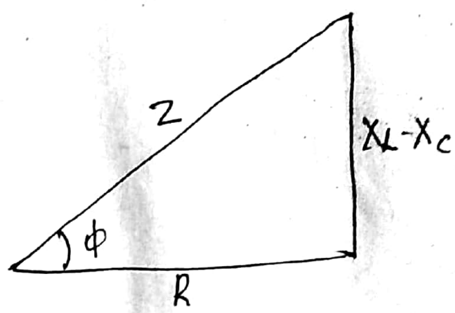
(ii) when $X_L < X_C$

(iii) when $X_L = X_C$

(i) when $X_L > X_C$:



Vector diagram



Impedance Triangle

↳ From the Vector diagram, the current lags the voltage by an angle 'ϕ'

$$I = \frac{E}{Z}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

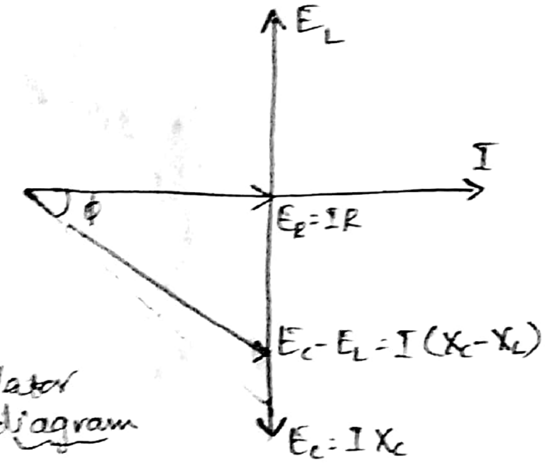
↳ The circuit is similar to an RL-Series circuit, i.e if $e = E_m \sin \omega t$ then $i = I_m \sin(\omega t - \phi)$

Hence $\boxed{P = EI \cos\phi}$

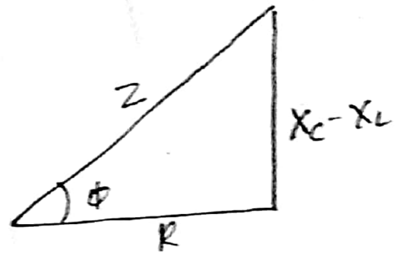
&

Power factor angle } $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

(ii) When $X_L < X_C$:



Vector diagram



Impedance Triangle

↳ From the vector diagram, current leads the voltage by an angle 'ϕ'

$$I = \frac{E}{Z}$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

↳ ∴ The circuit is similar to an RC-Series Circuit, i.e if $e = E_m \sin \omega t$ then $i = I_m \sin(\omega t + \phi)$

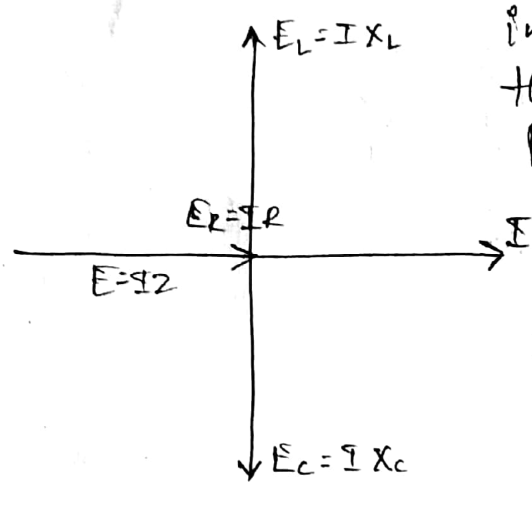
Hence $\boxed{P = EI \cos\phi}$

&

Power factor angle } $\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$

(iii) When $X_L = X_C$:- E_L & E_C get cancelled with each other i.e current is in phase with voltage & the circuit behaves as a pure resistance circuit

$\therefore Z = R$



i.e If $e = E_m \sin\omega t$ then $i = I_m \sin\omega t$

Hence $\boxed{P = EI}$

POWER FACTOR OF A CIRCUIT : Power factor of an A.C circuit is defined in 3-ways.

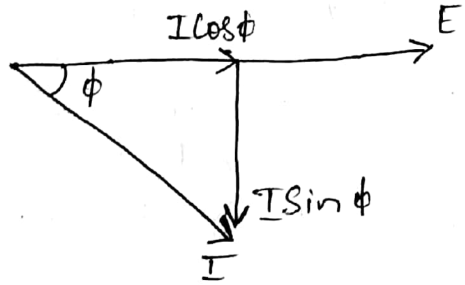
(i) $P.F = \cos\phi$ → Cosine of the angle between voltage & the current.

(ii) $P.F = \frac{R}{Z}$ → Ratio of Resistance to the Impedance of the circuit.

(iii) $P.F = \frac{P}{EI}$ → Ratio of Real power to the Apparent power.

The Maximum value of Power factor is unity.

NOTE :



$I\cos\phi$ → Inphase Component, which contributes to the Real power also known as "Real-component" or "Active Component" or "Wattfull Component"

$I\sin\phi$ → The Quadrature component which does not contribute to Power consumed, also known as "Reactive component" or "Wattless Component"

$P = EI\cos\phi$ → Real Power in watts

$Q = EI\sin\phi$ → Reactive Power in Volt Amps

$S = EI$ → Apparent Power in Volt Amps.

PRACTICAL IMPORTANCE OF POWER FACTOR :

↳ The active power consumed by the load in an A.C Circuit is given by $P = EI\cos\phi$. If the P.F of the load is small, the active power generated by an Alternator & the Active power transmitted & received by the consumer decreases.

↳ If the P.F is small, for transmitting a particular Power, the Current in the Transmission line increases & hence, the Copper losses (I^2R losses) will increase & the efficiency of Transmission decreases.

↳ Due to Low P.F, the current carrying capacity of the conductors has to be increased. Hence large sized conductors have to be used for transmission of Electrical power which involves larger investment.

↳ Hence, for the effective use of supplied energy, the supplying agencies insist on the customers to improve the P.F's of their loads to 0.85 to 0.90 by using static condensors across the load.

↳ The supplying agencies also give some incentive in the tariffs to the consumers for improving the P.F's of the loads.

THREE - PHASE CIRCUITS :

ADVANTAGES OF 3 - ϕ SYSTEMS :

(i) For the same capacity, a three apparatus costs less than a single phase apparatus.

(ii) The size of 3-ϕ apparatus is smaller in size than single phase apparatus of the same capacity & hence requires less material for construction.

(iii) For transmitting same amount of power over the same distance, under the same power loss, the amount of conductor material required is less in case of 3-ϕ system than in case of single phase system.

(iv) Three phase motors produce uniform torque whereas, the torque produced by single phase motors is pulsating.

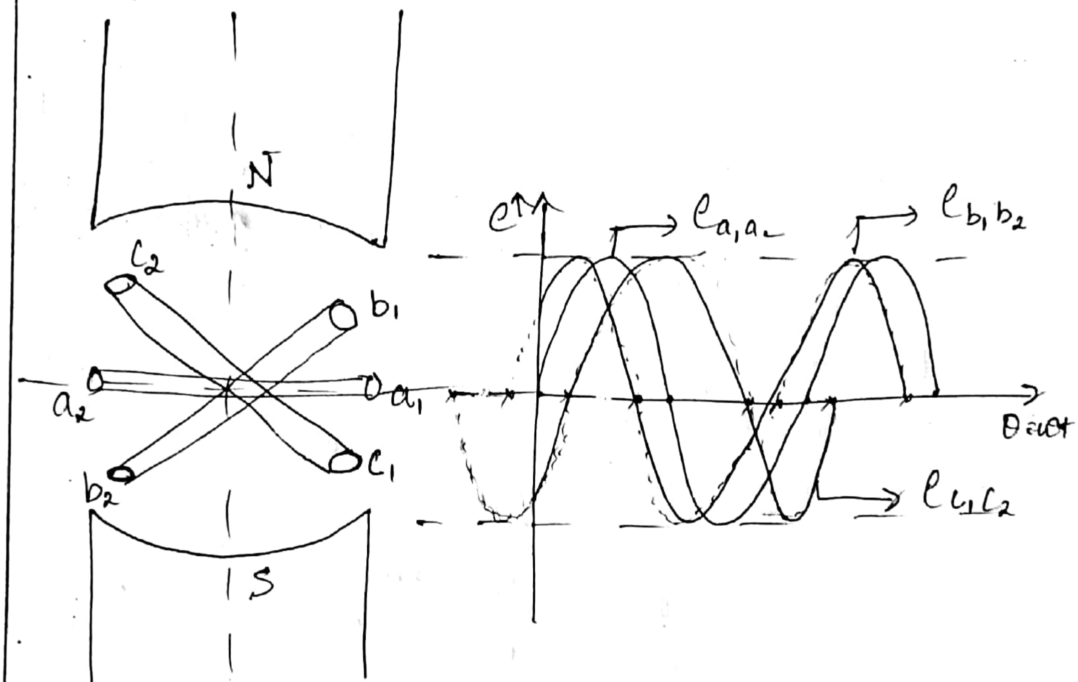
(v) Three phase motors are self starting whereas single phase motors are not self starting.

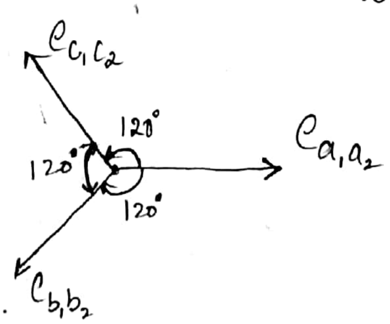
(vi) In case of 3-ϕ system, two different voltages can be obtained [line & phase] whereas only one voltage can be obtained in a single phase system.

(vii) The single phase generators in parallel give rise to harmonics, whereas 3-ϕ generators can be conveniently connected in parallel without giving rise to the generation of harmonics.

GENERATION OF 3-ϕ VOLTAGES :-

- In a 3-ϕ system, there are three equal voltages of same frequency displaced from one another by 120° electrically.
- These voltages are produced by a 3-ϕ generator which has 3-identical windings electrically displaced by 120°.





where,

$$e_{a_1a_2} = E_m \sin \omega t$$
$$e_{b_1b_2} = E_m \sin(\omega t - 120^\circ)$$
$$e_{c_1c_2} = E_m \sin(\omega t - 240^\circ)$$

- When these star connected or delta connected windings are rotated in a magnetic field, an EMF is induced in each of these windings.

↳ These EMFs are of same magnitude & frequency but are displaced by 120° from each other.

Explanation ; when the 3-coils a_1a_2, b_1b_2 & c_1c_2 are rotated under the influence of a magnetic field then three EMFs are induced $e_{a_1a_2}$, $e_{b_1b_2}$ & $e_{c_1c_2}$, which are displaced by 120° with each other.

PHASE SEQUENCE:- The phase sequence of the three phase supply is the order in which maximum values of the three phase voltages occurs

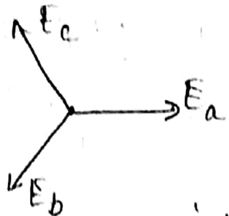


FIG:- Phase Sequence 'abc'

$E_a = E\angle 0°$ V

$E_b = E\angle -120°$ V

$E_c = E\angle +120°$ V

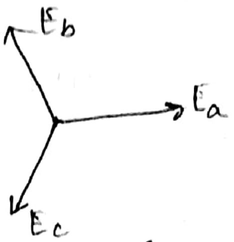


FIG:- Phase Sequence 'acb'.

$E_a = E\angle 0°$ V

$E_c = E\angle 0°$ V

$E_b = E\angle +120°$ V

↳ In the above fig, Three phase voltages occur in the order abc, hence the phase sequence of the supply is abc.

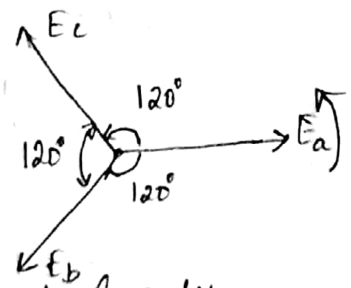
↳ By convention, RYB is considered positive & RBY is negative.

SIGNIFICANCE OF PHASE SEQUENCE :

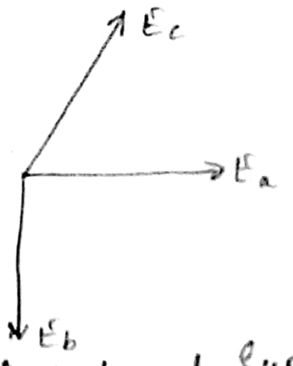
(i) When a three phase supply of a Particular Sequence is given to a Static three phase load, Certain currents flow through the lines & Phases of the load.
If the Phase Sequence is changed, then both magnitude & phase of the currents flowing in the lines & phases of the load will change.

(ii) If the load is a 3-phase Induction motor, when the sequence of the supply is changed, not only the magnitudes & Phases of line current change, but the direction of rotation of the motor changes.

BALANCED THREE PHASE SUPPLY : A three phase supply is said to be Balanced, when all three voltages differ in phase by 120° w.r.t one another.
↳ The three phase supply is said to be Un-balanced, even if one of the above conditions is not satisfied.

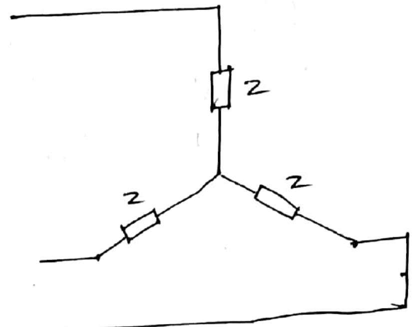


Balanced Supply

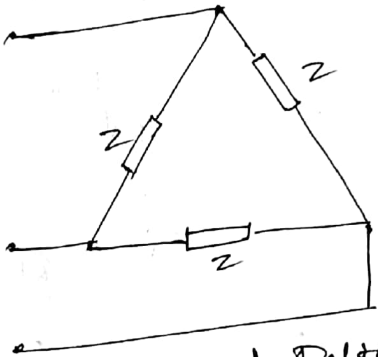


Un-balanced Supply.

BALANCED LOAD : A three phase load is said to be balanced, when impedances of all three phases are exactly the same.



3-ϕ Balanced Star Connected Load.



3-ϕ Balanced Delta Connected Load.

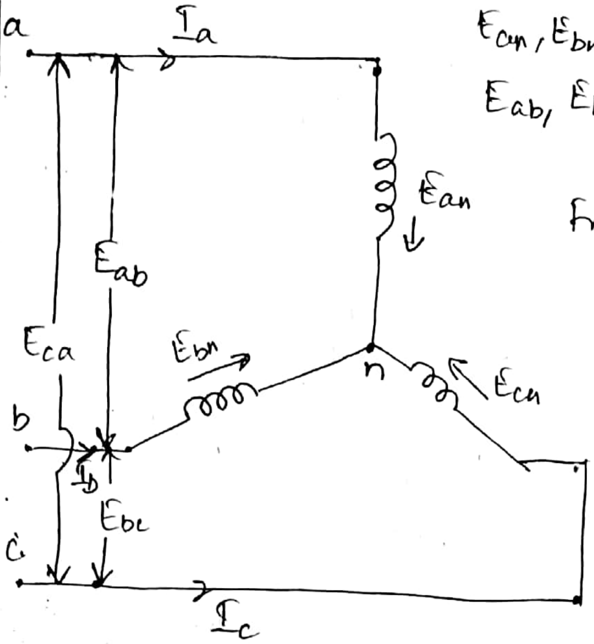
In a three phase Balanced Load, wheather Star or Delta connected the magnitudes of Phase Currents are the same but differ by 120° w.r.t each other, when a Balanced three phase supply is given.

THREE - PHASE CONNECTIONS : There are two types of three phase connections

(i) Star Connection.

(ii). Delta Connection.

(i) STAR CONNECTION (Y)



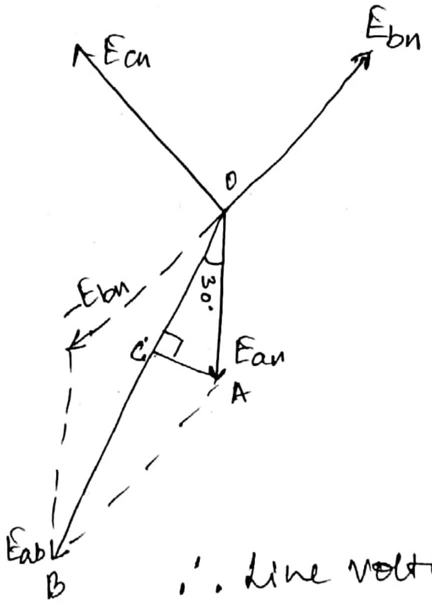
E_{an}, E_{bn} & E_{cn} → Phase Voltages $= E_{ph}$

E_{ab}, E_{bc} & E_{ca} → Line Voltages $= E_L$

From the circuit,

Line Current = Phase Current

$I_L = I_{ph}$



Draw a Perpendicular AC on OB,

From Δ^{le} $A\hat{O}C$,

$$\cos 30° = \frac{OC}{OA} = \frac{OB/2}{OA}$$

$$OB = 2 \times OA \times \cos 30°$$

$$E_{ab} = 2 \times E_{an} \times \sqrt{3}/2$$

$$\boxed{E_L = \sqrt{3}\, E_{ph}}$$

∴ Line voltage = $\sqrt{3}$ × Phase voltage.

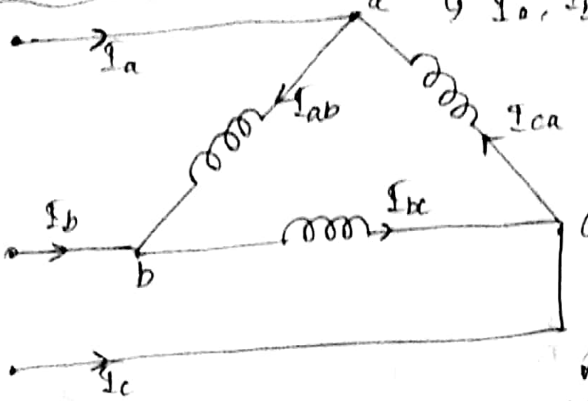
↳ Power consumed by 3-phase circuit

$$P = 3 \times \text{Power in each phase}$$

$$= 3 \times E_{ph} I_{ph} \cos\phi = 3 \times \frac{E_L}{\sqrt{3}} \times I_L \times \cos\phi$$

$$\boxed{P = \sqrt{3}\, E_L I_L \cos\phi}$$ where ϕ = Angle b/w E_{ph} & I_{ph}

DELTA CONNECTION (Δ) :

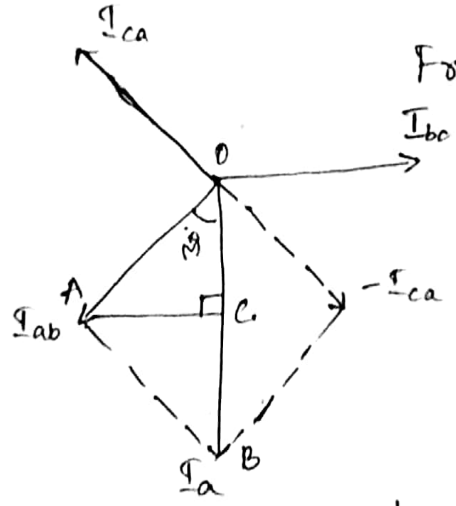


I_a, I_b & I_c → Line Currents } = I_L

I_{ab}, I_{bc} & I_{ca} → Phase Currents } = I_{ph}

Line Voltage } = phase voltage

$E_L = E_{ph}$



From Δ^{le} AOC,

$$\cos 30^\circ = \frac{OC}{OA} = \frac{OB/2}{OA}$$

$$OB = 2 \times OA \times \cos 30^\circ$$

$$I_a = 2 \times I_{ab} \times \frac{\sqrt{3}}{2}$$

$$\boxed{I_L = \sqrt{3}\ I_{ph}}$$

∴ Line Current = Phase Current.

Power Consumed by 3-phase Circuit,

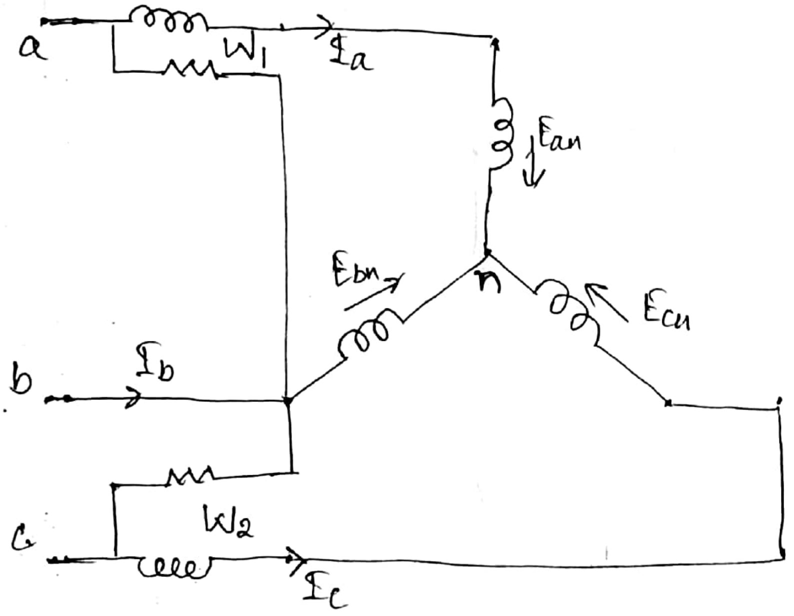
P = 3 × Power in each phase

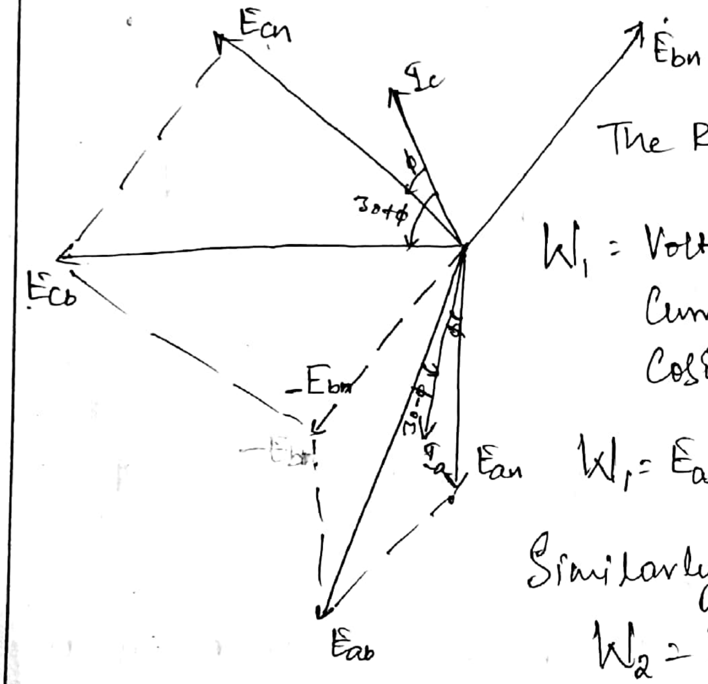
$$= 3 \times E_{ph} I_{ph} \cos\phi = 3 \times E_L \times \frac{I_L}{\sqrt{3}} \times \cos\phi$$

$$\boxed{P = \sqrt{3}\, E_L I_L \cos\phi}$$

where ϕ = Angle b/w E_{ph} & I_{ph}

MEASUREMENT OF POWER IN THREE PHASE CIRCUIT [TWO WATTMETER METHOD] :





The Reading of Wattmeter-1 is,

W_1 = Voltage across potential coil X Current through current coil X Cosine of angle b/n voltage & current.

$$W_1 = E_{ab} I_a \cos\{\angle E_{ab} \;\&\; \angle I_a\}$$

Similarly,

$$W_2 = E_{cb} I_c \cos\{\angle E_{cb} \;\&\; \angle I_c\}$$

Assuming an Inductive load, I_a lags E_{an} by an angle ϕ. Hence the angle b/w E_{ab} & I_a is $(30-\phi)$

$$\therefore W_1 = E_{ab} I_a \cos(30-\phi)$$

$$W_1 = E_L I_L \cos(30-\phi) \quad \ldots\ldots (a)$$

Similarly, I_c lags E_{cn} by an angle ϕ, Hence the angle b/w E_{cb} & I_c is $(30+\phi)$.

$$\therefore W_2 = E_{cb} I_c \cos(30+\phi)$$

$$W_2 = E_L I_L \cos(30+\phi) \quad \ldots\ldots (b)$$

Adding equns. (a) & (b),

$$W_1 + W_2 = E_L I_L \left[\cos(30-\phi) + \cos(30+\phi)\right]$$

$$= E_L I_L \times 2\cos 30 . \cos\phi$$

$$\therefore \boxed{W_1 + W_2 = \sqrt{3} E_L I_L \cos\phi = \text{Three phase Power.}}$$

Expression for Power factor (P.f) :

w.k.t $\quad W_1 = E_L I_L \cos(30-\phi)$

$\quad W_2 = E_L I_L \cos(30+\phi)$

$$W_1 - W_2 = E_L I_L \left[\cos(30-\phi) - \cos(30+\phi)\right]$$

$$= E_L I_L \times 2\sin 30 . \sin\phi$$

$$\therefore W_1 - W_2 = E_L I_L \sin\phi \quad \ldots\ldots (a)$$

w.k.t $\quad W_1 + W_2 = \sqrt{3} E_L I_L \cos\phi \quad \ldots\ldots (b)$

$$\frac{\text{equn}^{n}\,(a)}{\text{equn}^{n}\,(b)} \text{ is } \frac{W_1 - W_2}{W_1 + W_2} = \frac{\operatorname{Tan}\phi}{\sqrt{3}}$$

$$\operatorname{Tan}\phi = \sqrt{3}\left[\frac{W_1 - W_2}{W_1 + W_2}\right]$$

$$\therefore \phi = \operatorname{Tan}^{-1}\left\{\sqrt{3}\left[\frac{W_1 - W_2}{W_1 + W_2}\right]\right\}$$

$$\therefore \boxed{P.f = \cos\phi = \cos\left[\operatorname{Tan}^{-1}\left\{\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)\right\}\right]}$$

EFFECT OF POWER FACTOR (P.f) ON WATTMETER READINGS (W_1 & W_2) :

Case (i) : when P.f = 1 i.e $\phi = 0°$

$W_1 = E_L I_L \cos(30 - \phi) = E_L I_L \cos(30 - 0) = \sqrt{3}/2\, E_L I_L$

$W_2 = E_L I_L \cos(30 + \phi) = E_L I_L \cos(30 + 0) = \sqrt{3}/2\, E_L I_L$

$\therefore$ The two wattmeter readings are positive & equal.

Case (ii) : when P.f = 0.5 i.e $\phi = 60°$

$W_1 = E_L I_L \cos(30 - \phi) = E_L I_L \cos(30 - 60) = \sqrt{3}/2\, E_L I_L$

$W_2 = E_L I_L \cos(30 + \phi) = E_L I_L \cos(30 + 60) = 0$

$\therefore$ One of the wattmeter reads zero.

Case (iii) : when P.f = 0 i.e $\phi = 90°$

$W_1 = E_L I_L \cos(30 - \phi) = E_L I_L \cos(30 - 90) = \frac{1}{2} E_L I_L$

$W_2 = E_L I_L \cos(30 + \phi) = E_L I_L \cos(30 + 90) = -\frac{1}{2} E_L I_L$

$\therefore$ One of the wattmeter reads -ive. The pointer of the wattmeter kicks back & hence readings cannot be taken.