Module - 5

Numerical methods

Divided differences :- Suppose $y=f(x)$ be a function in the Variable of x. Let the set of Values of y are Value of y are $y_0 . y_1 . y_2$ ---- y_n corresponding to the Value of an arguments $x_0 . x_1 . x_2 . x_3$ ---- x_n the divided difference are classified into 2 types in the Equal length of Interval

1. forward divided difference
2. Backward divided difference

Forward divided differences :- the Symbol Δ (delta) is called the forward divided differences Operator and $\Delta^1, \Delta^2, \Delta^3$ ---- are called The 1^{st}, 2nd, 3rd order divided forward differences

x	$y=f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$
x_0	y_0			
		$y_1 - y_0 = \Delta y_0$		
x_1	y_1		$\Delta y_1 - \Delta y_0 = \Delta^2 y_0$	
				$\Delta^2 y_1 - \Delta^2 y_0 = \Delta^3 y_0$
x_2	y_2	$y_2 - y_1 = \Delta y_1$		
			$\Delta y_2 - \Delta y_1 = \Delta^2 y_1$	
x_3	y_3	$y_3 - y_2 = \Delta y_2$		$\Delta^2 y_2 - \Delta^2 y_1 = \Delta^3 y_1$
			$\Delta y_3 - \Delta y_2 = \Delta^2 y_2$	
x_4	y_4	$y_4 - y_3 = \Delta y_3$		

$\Delta^4 y$ $\quad \Delta^3 y_1 - \Delta^3 y_0 = \Delta^4 y_0$

Here $\Delta y_0, \Delta^2 y_0, \Delta^3 y_0, \Delta^4 y_0$ is called leading forward divided differences

Backward differences :- the Symbol dell (∇) is called the Backward divided differences Operator. $\nabla, \nabla^2, \nabla^3$ ---- are called the 1st, 2nd, 3rd ---- Order backward divided differences

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
x_0	y_0				
		$y_1 - y_0 = \nabla y_1$			
x_1	y_1		$\Delta y_2 - \nabla y_1 = \nabla^2 y_2$		
		$y_2 - y_1 = \nabla y_2$		$\nabla^2 y_3 - \nabla^2 y_2 = \nabla^3 y_3$	
x_2	y_2		$\nabla y_3 - \nabla y_2 = \nabla^2 y_3$		$\nabla^3 y_4 - \nabla^3 y_3 = \nabla^4 y_4$
		$y_3 - y_2 = \nabla y_3$		$\nabla^2 y_4 - \nabla^2 y_3 = \nabla^3 y_4$	
x_3	y_3		$\nabla y_4 - \nabla y_3 = \nabla^2 y_4$		
		$y_4 - y_3 = \nabla y_4$			
x_4	y_4				

Here $\nabla y_n, \nabla^2 y_n, \nabla^3 y_n$ --- are called the leading Backward divided differences

Interpolation and Extrapolation

the Evaluation of y in the given range x_0 to x_n is called in interpolation and Outside of x_0 to x_n is called Extrapolation

Interpolation formula

i) $y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0$
$+ \text{------}$ when $p = \frac{x - x_0}{h}$ is called the newton's forward interpolation formula

ii) $y(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_n$

---- where $p = \frac{x - x_n}{h}$ is called the newton's Backward Interpolation formula

1 The population of a town is given by the table

year	1951	1961	1971	1981	1991
population	19.6	39.65	58.81	72.21	94.61

Using newtons forward and Backward interpolation formula. Calculate the Increase in population from the year 1955 to 1985

year	population (y)	I DD	II DD	III DD	IV DD
$x_0 = 1951$	$y_0 = 19.6$				
		20.05			
$x_1 = 1961$	$y_1 = 39.65$		-0.89		
		19.16		-4.87	
$x_2 = 1971$	$y_2 = 58.81$		-5.76		19.63
		13.4		-14.76	
$x_3 = 1981$	$y_3 = 72.21$		9		
		22.4			
$x_4 = 1991$	$y_4 = 94.61$				

To find $y(1955)$

Since 1955 is near to $x_0 = 1951$

$\Rightarrow p = \frac{x - x_0}{h}$

$\Rightarrow p = \frac{1955 - 1951}{10}$

$\Rightarrow p = \frac{4}{10}$

$\Rightarrow p = 0.4$

$\therefore$ By the newtons forward intercolation formula

$$\Rightarrow y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0$$

$$\Rightarrow y(1955) = 19.6 + (0.4)(20.05) + \frac{(0.4)(-0.6)(-0.84)}{2} + \frac{(0.4)(0.6)(-1.6)(-4.8}{6}$$

$$+ \frac{(0.4)(-0.6)(-1.6)(-2.6)(19.63)}{24}$$

$$\Rightarrow y(1955) = 19.6 + 8.02 + 0.1008 - 0.31168 - 0.8166$$

$$\Rightarrow y(1955) = 26.60$$

<u>To find y(1985)</u>

Since 1985 is near to $x_0 = 1951$

$$p = \frac{x - x_0}{h} = \frac{1985 - 1991}{10} = \frac{-6}{10} = -0.6$$

$\therefore$ The newtons Backward intercolation formula

$$\Rightarrow y(x) = y_4 + p\nabla y_4 + \frac{p(p+1)}{2!}\nabla^2 y_4 + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_4 +$$

$$\frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_4$$

$$\Rightarrow y(1985) = 94.61 + (-0.6)(22.4) + \frac{(-0.6)(0.4)(9)}{2} +$$

$$\frac{(-0.6)(0.4)(1.4)(14.76)}{6} + \frac{(-1.6)(0.4)(1.4)(2.4)(19.6)}{24}$$

$$\Rightarrow y(1985) = 94.61 - 13.44 - 1.88 - 0.826 - 0.659$$

$$\Rightarrow y(1985) = 78.605$$

x.y from population between 1955 to 1985

$$\Longrightarrow 78.605 - 26.60$$

$$\Longrightarrow 52.005 \,//$$

2) Use an appropriate interpolation formula to compute $g(42)$ using the following data

x	40	50	60	70	80	90
y	184	204	226	250	276	304

x	y	I DD	II DD	III DD	IV DD	V
40	184					
		20				
50	204		2			
		22		0		
60	226		2		0	
		24		0		0
70	250		2		0	
		26		0		
80	276		2			
		28				
90	304					

To find $y(42)$

$y(42)$ is near to $x_0 = 40$

$$p = \frac{x - x_0}{h} = \frac{42 - 40}{10} = \frac{2}{10} = 0.2$$

By newtone forward interpolation formula

$$\Rightarrow y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 - - - -$$

$$\Rightarrow y(42) = 184 + (0.2)(20) + \frac{(0.2)(0.8)(2)}{(2)}$$

$$\Rightarrow y(42) = 184 + 4 - 0.16$$

$$\Rightarrow y(42) = 187.84$$

3 Use an appropriate interpolation formula to compute $y(82)$ and $y(92)$ for the data

x	80	85	90	95	100
y	5026	5674	6362	7088	7854

x	y	I DD	II DD	III DD	IV DD
80	5026				
		648			
85	5674		40		
		688		−2	
90	6362		38		4
		726		2	
95	7088		40		
		766			
100	7854				

i) To find $y(82)$

Since 82 is near to $x_0 = 80$

$$p = \frac{x - x_0}{h} = \frac{82-80}{5} = \frac{2}{5} = 0.4$$

By the newtons forward interpolation formula

$$\Rightarrow y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots$$

$$\Rightarrow y(82) = 5026 + (0.4)(648) + \frac{(0.4)(-0.6)(40)}{2} + \frac{(0.4)(-0.6)(-1.6)(-2)}{6} + \frac{4 \times 0.4(-0.6)(-1.6)(-2.6)}{24}$$

$$\Rightarrow y(82) = 5026 + 259.2 - 4.8 - 0.128 - 0.1664$$

$$\Rightarrow y(82) = 5280.0056$$

ii) To find $y(92)$

Since 92 is near to $x_4 = 90$

$$p = \frac{x - x_0}{n} = \frac{92-100}{5} = \frac{-8}{5} = -1.6$$

By the newtons Backward Interpolation formula

$$\Rightarrow y(x) = y_4 + p\nabla y_4 + \frac{p(p+1)}{2!}\nabla^2 y_4 + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_4 + \text{------}$$

$$\Rightarrow y(92) = 7854 + (-16)(766) + \frac{(-1.6)(-0.6)(40)}{2} + \frac{(-1.6)(-0.6)(0.4)(2)}{6}$$

$$+ \frac{(-1.6)(-0.6)(0.4)(1.4)(4)}{24}$$

$$\Rightarrow y(92) = 7854 - 1225 + 19.2 + 0.128 + 0.0896$$

$$\Rightarrow y(92) = 6647.8176$$

$\boxed{4}$ Using find $f(12.5)$ for The following data (NBIF)

x	10	11	12	13
y	22	24	28	34

x	y	I DD	II DD	III DD
10	22			
		2		
11	24		2	
		4		0
12	28		2	
		6		
13	34			

To find $y(12.5)$

$$p = \frac{x - x_4}{h} = \frac{-13 + 12.5}{1} = -0.5$$

$$y(x) = y_4 + p\Delta y_4 + \frac{p(p+1)}{2!}\Delta^2 y_4 + \text{-----}$$

$$y(12.5) = (34) + (-0.5)(6) + \frac{(-0.5)(0.5)}{2} 2 + \text{---}$$

$\Rightarrow y(12.5) = 34 - 3 - 0.25$

$\Rightarrow y(12.5) = \underline{\underline{30.75}}$

5. ~~formula~~ ~~following~~ table and Estimate the number of Students who obtained marks between 40 and 45

Marks	30-40	40-50	50-60	60-70	70-80
No of Students	31	42	51	35	31

The number of Student who obtained $<40 = 31$, $<50 = 73$, $<60 = 124$, $<70 = 154$, $<80 = 190$

X	y	I DD	II DD	III DD	IV DD
40	31				
		42			
50	73		9		
		51		−25	
60	124		−16		37
		35		12	
70	159		−4		
		51			
80	190				

x	$y=f(x)$	I DD
x_0	$f(x_0)$	$\frac{f(x_1)-f(x_0)}{x_1-x_0} = f(x_0,x_1)$
x_1	$f(x_1)$	$\frac{f(x_2)-f(x_1)}{x_2-x_1} = f(x_1,x_2)$
x_2	$f(x_2)$	$\frac{f(x_3)-f(x_2)}{x_3-x_1} = f(x_2,x_3)$
x_3	$f(x_3)$	

$$\frac{f(x_1,x_3)-f(x_0,x_3)}{x_3-x_0} = f(x_0, x_1, x_2)$$

$$\frac{f(x_2,x_3)-f(x_1,x_2)}{x_3-x_1} = f(x_1,x_2,x_3)$$

$$\frac{f(x_1,x_2,x_3)-f(x_0,x_1,x_2)}{x_3-x_0} = f(x_0,x_1,x_2,x_3)$$

$$f(x) = f(x_0) + (x-x_0)f(x_0-x_1) + (x-x_0)(x-x_1)f(x_0 x_1 x_2) + (x-x_0)(x-x_1)(x_1-x_2)\, f(x_0,x_1,x_2,x_3)$$

6 Using newtons interpolation formula to construct the polynomial by the following data

x	2	4	5	6	8	10
$f(x)$	10	96	196	350	868	1746

x	$f(x)$	I DD	II DD	III DD	IV DD	V DD
2	10					
		43				
4	96		19			
		100		2		
5	196		27		0	
		154		2		0
6	350		35		0	
		259		2		
8	868		45			
		439				
10	1746					

To find $y(45)$

Since 45 is near to $x_0 = 40$

$$p = \frac{x - x_0}{h} = \frac{45-40}{10} = 0.5$$

By the newton's forward interpolation formula

$$\Rightarrow y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}$$

$$\Delta^4 y_0 + \text{---------}$$

$$\Rightarrow y(45) = 31 + (0.5)(42) + \frac{(0.5)(-0.5)(9)}{2!} + \frac{(0.5)(-0.5)(-1.5)(-25)}{6}$$

$$+ \frac{(0.5)(-0.5)(-1.5)(-2.5)(37)}{24}$$

$$\Rightarrow y(45) = 31 + 21 - 1.125 - 1.5625 - 1.4453$$

$$\Rightarrow y(45) = 47.687 \simeq 48$$

The Number of students who obtained less than 45 marks = 48

The Number of students who obtained between 40 and 45 marks = 48 - 31 = 17

Divided difference for Unequal intervals

Newton's divided differences formula for Unequal intervals :-

Suppose $y = f(x)$ be a function in x and $f(x_0)$, $f(x_1)$, $f(x_2)$, $f(x_3)$ ---- be the values of $f(x)$ corresponding to the values of x_0, x_1, x_2, x_3 ---- with unequal intervals

7] find the Interpolating polynomial

X	0	1	2	3	4	5
$f(x)$	3	2	7	24	59	118

x	$f(x)$	I DD	II DD	III DD	IV DD	V
0	3					
		-1				
			3			
1	2					
				1		
		5				
					0	
2	7		6			
						0
		17				
				1		
					0	
3	24		9			
		35				
				1		
4	59		12			
		59				
5	118					

$$f(x) = f(x_0) + (x-x_0) f(x_0 - x_1) + (x-x_0)(x-x_1) + f(x_0, x_1, x_2) +$$

$$f(x) = 3 + (-1)(x-0) + 3(x-0)(x-1) + 1(x-0)(x-1)(x-2)$$

$$f(x) = 3 - x + 3x^2 - 3x + x^3 - 3x^2 + 2x$$

$$f(x) = x^3 - 2x + 3$$

when $x = 6$

$$f(6) = (6)^3 - (2 \times 6) + 3$$

$$f(6) = \underline{\underline{207}}$$

8. Find the number of men getting wages below Rs 3.5 from the following data

Wages in Rs	0-10	10-20	20-30	30-40
frequency	9	30	35	42

The number of men who getting less than 10 = 9

$< 20 = 30$

$< 30 = 35$

$< 40 = 42$

x	y	I DD	II DD	III DD
10	9			
		30		
20	39		5	
		35		2
30	74		7	
		42		
40	116			

35 is near to $x_3 = 40$

By newton Backward Integrated factor

$$p = \frac{x - x_3}{h} = \frac{35-40}{10} = -0.5$$

$$y(x) = y_3 + p\nabla y_3 + \frac{p(p+1)}{2!}\nabla^2 y_3 + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_3$$

$$y(35) = 116 + (-0.5)42 + \frac{(-0.5)(0.5)7}{2} + \frac{2(-0.5)(0.5)(1.5)}{6}$$

$$y(35) = 116 - 21 - 0.875 - 0.125$$

$$y(35) = \underline{\underline{94}}$$

9] Determine the Interplating formula construct the polynomial for the following data Hence find

x	3	7	9	10
f(x)	168	120	72	63

Soln

x	f(x)	$f(x_0,x_1)$	$f(x_0,x_1,x_2)$	$f(x_0,x_1,x_2,x_3)$
3	168			
		−12		
7	120		−2	
		−24		1
9	72		5	
		−9		
10	63			

$\Rightarrow f(x) = 168 - 12(x-3) - 2(x-3)(x-7) + 1(x-3)(x-7)(x-9)$

$\Rightarrow f(x) = 168 - 12x + 36 - 2x^2 + 20x - 42 + x^3 - 16x + 63x$
$-3x^2 + 48x - 189$

$\Rightarrow f(x) = x^3 - 21x^2 + 119x - 27$

$\Rightarrow f(8) = 8^3 - 21(8^2) + 119(8) - 27$

$\Rightarrow f(8) = 93$

10] from the table of half year premium for policy measuring of different states Estimate the premium for policy measuring on the age of (46)

age	45	50	55	60	65
premium	114.84	96.16	83.32	74.48	68.48

age (x)	premium	I DD	II DD	III DD	IV DD
45	114.84				
		−18.68			
50	96.16		5.84		
		−12.84		−1.84	
55	83.32		4		0.68
		−8.84		−1.16	
60	74.48		2.84		
		−6			
65	68.48				

To find f(46) it is near to $x_0 = 45$

$$p = \frac{x - x_0}{h} = \frac{46-45}{5} = 0.2$$

By newton forward integrated factor

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0$$

$$y(46) = 114.84 + (0.2 \times -18.68) + \frac{(0.2)(-0.8)}{2} \times 5.84 + \frac{(0.2)(-0.8)(-1.8)(-1.84)}{6} + \frac{(0.2)(-0.8)(-1.8)(-2.8)}{24} \times 0.68$$

$$y(46) = 114.84 - 3.76 - 0.4672 - 0.08832 - 0.022846$$

$$y(46) = 110.5256$$

Lachranges interpolation formula for Unequal Intervals

Suppose $y_0, y_1, y_2, y_3 ---- y_n$ be the set of values of $y=f(x)$ corresponding to $x_0, x_1, x_2 ---- x_n$ then the Lachranges interpolation formula is followe as

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)-----(x-x_n)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)----(x_0-x_n)} y_0 +$$

$$\frac{(x-x_0)(x-x_2)(x-x_3)---(x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)--(x_1-x_n)} y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)---(x-x_n)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)----(x_2-x_n)} y_2$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_n-x_0)(x_n-x_1)(x_n-x_2)} ----- \frac{(x-x_{n-1})}{(x_n-x_{n-1})} y_n$$

Illy the Inverse legranges interpolation formula can be

$$x = g(x) = \frac{(y-y_1)(y-y_2)(y-y_3)-----(y-y_n)}{(y_0-y_1)(y_0-y_2)(y_0-y_3)----(y_0-y_n)} x_0 +$$

$$\frac{(y-y_0)(y-y_2)(y-y_3)-----(y_1-y_n)}{(y_1-y_0)(y_1-y_2)(y_1-y_3----(y_1-y_n)} x_1 +$$

$$\frac{(y-y_0)(y-y_1)(y-y_2)-----(y-y_n)}{(y_2-y_0)(y_2-y_1)(y_2-y_3)----(y_2-y_n)} x_2 +$$

$$\frac{(y-y_0)(y-y_1)(y-y_2)-----(y-y_{n-1})}{(y_n-y_0)(y_n-y_1)(y_n-y_3)----(y_n-y_{n-1})} x_n$$

1] Use the legranches interpolation formula to find y at $x=0$

x	5	6	9	11
y	12	13	14	16

Given :-

x	y
$x_0 = 5$	$y_0 = 12$
$x_1 = 6$	$y_1 = 13$
$x_2 = 9$	$y_2 = 14$
$x_3 = 11$	$y_3 = 16$

To find $y(x) = y(10)$

$$y(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$y(10) = \frac{(10-6)(10-9)(10-11)}{(5-6)(5-9)(5-11)}(12) + \frac{(10-5)(10-0)(10-11)}{(6-5)(6-9)(6-11)}(13)$$

$$+ \frac{(10-5)(10-6)(10-11)}{(9-5)(9-6)(9-11)}(14) + \frac{(10-5)(10-6)(10-9)}{(11-5)(11-6)(11-9)}(16)$$

$$y(10) = \frac{4(1)(-1)(12)}{(-1)(-4)(-6)} + \frac{5(1)(-1)(13)}{(1)(-3)(-5)} + \frac{(5)(4)(-1)(14)}{(4)(3)(-1)} + \frac{(5)(4)(1)(16)}{(6)(5)(2)}$$

$$y(10) = 2 - 4.33 + 11.66 + 5.33$$

$$y(10) = 14.66$$

12 By the polynomial $f(x)$ using langrange's formula from the following data

x	0	1	2	5
y	2	3	12	147

Soln :-

x	y
$x_0 = 0$	$y_0 = 2$
$x_1 = 1$	$y_1 = 3$
$x_2 = 2$	$y_2 = 12$
$x_3 = 5$	$y_3 = 147$

$$f(x) = \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 +$$
$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$f(x) = \frac{(x-1)(x-2)(x-5)}{(0-1)(0-2)(0-5)}(2) + \frac{(x-0)(x-2)(x-5)}{(1-0)(1-2)(1-5)}(3) +$$
$$\frac{(x-0)(x-1)(x-5)}{(2-0)(2-1)(2-5)}(12) + \frac{(x-0)(x-1)(x-2)}{(5-0)(5-1)(5-2)}(147)$$

$$f(x) = \frac{2(x-1)(x-2)(x-5)}{(-1)(-2)(-5)} + \frac{3(x-0)(x-2)(x-5)}{(1)(-1)(-4)} + \frac{12(x)(x-1)(x-5)}{2(1)(-4)}$$
$$+ \frac{147(x-0)(x-1)(x-2)}{5(4)(3)}$$

$$f(x) = \frac{1}{5}(x-1)(x-2)(x-5) + \frac{3}{4}(x-2)(x-5)(x-0) - 2x(x-1)(x-5)$$
$$+ \frac{49}{20}x(x-1)(x-2)$$

$$f(x) = \frac{1}{20}\Big| -4(x-1)(x^2-7x+10) + 15x(x^2-7x+10) - 40x(x^2-(x-5)$$
$$+ 49x(x^2-3x+2) \Big|$$

$$f(x) = \frac{1}{20}\Big| -4x^3+32x^2-68x+40+15x^3-105x^2+150x-40x^3+240x^2$$
$$-200x+44x^3-147x^2-98x \Big|$$

$$f(x) = \frac{1}{20}\Big| 20x^3+20x^2-20x+40 \Big|$$

$$f(x) = x^3+x^2-x+2 //$$

Numerical integration

Suppose $y=f(x)$ be a function and let $y_0 \cdot y_1 \cdot y_2 --- y_n$ are the Set of Values Corresponding to the partition of

$P=[a=x_0,\ x_0+h=x_1,\ x_1+h=x_2,\ x_2+h=x_3+ ------ +x_{n-1}+h=x_n=b]$

with the Equal length of the Cartesion $h=\frac{b-a}{n}$ where

n is the number of Equal Strips [or $n+1$ ordinate] we can Evaluate $\int_a^b f(x)\,dx$ by the following method

i) Simpsons' $\frac{1}{3}$rd rule

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[(y_0+y_n)+2(y_2+y_4+y_6 ---- y_{n-2}) + 4(y_1+y_3+y_5 + ----- + y_{n-1})\right]$$

ii) Simpson's $\frac{3}{8}$th rule

$$\int_a^b f(x)\,dx = \frac{3h}{8}\left[(y_0+y_n)+2(y_3+y_6+y_9+---y_{n-3})+ 3(y_1+y_2+y_4+y_5 --- -1)\right]$$

iii) weedles rules for $n=6$

$$\int_a^b f(x)\,dx = \frac{3h}{10}\left[y_0+5y_1+y_2+6y_3+y_4+5y_5+y_6\right]$$

13] Evaluate $\int_0^1 \frac{1}{(1+x^2)}dx$ by taking Equal strips and Hence diduse an approximate Value of radian π by following method

i] Simpson's $\frac{1}{3}$ rd rule

ii] Simpson's $\frac{3}{8}$ rule

iii] weddle's rule

Soln :- Let $I = \int_0^1 \frac{1}{1+x^2} dx$

$a=0, b=1, n=6, \quad y = \frac{1}{1+x^2}$

$h = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$

$P = \{a = x_0 = 0, x_1 = \frac{1}{6}, x_2 = \frac{2}{6}, x_3 = \frac{3}{6}, x_4 = \frac{4}{6}, x_5 = \frac{5}{6}$
$x_6 = 1 = b\}$

x	$y = \frac{1}{1+x^2}$
$x_0 = 0$	$y_0 = \frac{1}{1+0} = 1$
$x_1 = 1/6$	$y_1 = \frac{1}{1+36} = 0.9730$
$x_2 = 1/3$	$y_2 = \frac{1}{1+\frac{1}{9}} = 0.9$
$x_3 = 1/2$	$y_3 = \frac{1}{1+\frac{1}{4}} = 0.8$
$x_4 = 2/3$	$y_4 = \frac{1}{1+\frac{4}{9}} = 0.6923$
$x_5 = 5/6$	$y_5 = \frac{1}{1+\frac{25}{36}} = 0.5902$
$x_6 = 1$	$y_6 = \frac{1}{1+1^2} = 0.5$

i) Simpson's $\frac{1}{3}$rd rule

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[(y_0 - y_6) + 2(y_2+y_4) + 4(y_1+y_3+y_5)\right]$$

$$\Rightarrow \int_0^1 \frac{1}{1+x^2} dx = \frac{1/6}{3}\left[(1+0.5) + 2(0.9+0.6923) + 4(0.97304 + 0.8 + 0.5902)\right]$$

$$\Rightarrow \int_0^1 \frac{1}{1+x^2} dx = \frac{1}{18}\left[1.5 + 3.1846 + 9.4528\right]$$

$$\Rightarrow \int_0^1 \frac{1}{1+x^2}dx = 0.7854$$

$$\Rightarrow \tan^{-1}(x)\Big|_0^1 = 0.7854$$

$$\Rightarrow \tan^{-1}(1) - \tan^{-1}(0) = 0.7854$$

$$\Rightarrow \frac{\pi}{4} = 0.7854$$

$$\Rightarrow \pi = 0.7854 \times 4$$

$$\Rightarrow \pi \simeq \underline{\underline{3.1416}}$$

ii) Simpson's $\frac{3}{8}^{Th}$ rule

$$\int_a^b f(x)dx = \frac{3h}{8}\left[(y_0+y_6)+2(y_3)+3(y_1+y_2+y_3+y_4+y_5)\right]$$

$$\Rightarrow \int_a^b \frac{1}{1+x^2}dx = \frac{3(1/6)}{8}\left[(1+0.5)+2(0.8)+3(0.973+0.9+0.6923+0.5402)\right]$$

$$\Rightarrow \frac{1}{16}\left[1.5+1.6+9.4665\right]$$

$$\Rightarrow \tan^{-1}(x)\Big|_0^1 = 0.7854$$

$$\Rightarrow \tan^{-1}(1) - \tan^{-1}(0) = 0.7854$$

$$\Rightarrow \frac{\pi}{4} = 0.7854$$

$$\Rightarrow \pi = 0.7854 \times 4$$

$$\Rightarrow \pi \simeq 3.1416$$

iii) weddle rule

$$\int_a^b f(x)dx = \frac{3h}{10}\left[y_0+5y_1+y_2+6y_3+y_4+5y_5+y_0\right]$$

$$\Rightarrow \int_0^1 \frac{1}{1+x^2}dx = \frac{3(1/6)}{10}\left[1+4.865+0.9+4.8+0.6923+2.95\right]$$

$$\Rightarrow \int_0^1 \frac{1}{1+x^2}dx = \frac{1}{20}\left[15.7083\right]$$

$\Rightarrow \tan^{-1}(1) - \tan^{-1}(0) = 0.7854$

$\Rightarrow \frac{\pi}{4} = 0.7854$

$\Rightarrow \pi \simeq 3.1416$

14 Evaluate $\int_0^1 \frac{1}{1+x} dx$ taking 7 ordinates and Hence deduce the value of $\log_e^2$ by using
i) Simpsons

Soln :- Let $a=0$. $b=1$, $n=6$

$h = \frac{b-a}{6} = \frac{1-0}{6} = \frac{1}{6}$

$p = (a = x_0 = 0,\ x_1 = 1/6,\ x_2 = 2/6,\ x_3 = 3/6,\ x_4 = 4/6,\ x_5 = 5/6$
$x_6 = 6/6)$

x	$y = \frac{1}{1+x}$
$x_0 = 0$	$y_0 = 1$
$x_1 = 1/6$	$y_1 = 0.8571$
$x_2 = 1/3$	$y_2 = 0.73$
$x_3 = 1/2$	$y_3 = 0.667$
$x_4 = 2/3$	$y_4 = 0.6$
$x_5 = 5/6$	$y_5 = 0.545$
$x_6 = 1$	$y_6 = 0.5$

i) Simpsons $\frac{1}{3}$rd rule

$$\int_a^b f(x)dx = \frac{h}{3}\left[(y_0+y_6) + 2(y_2+y_4) + 4(y_1+y_2+y_5)\right]$$

$$= \left(\frac{1/6}{3}\right)\left[(1+0.5) + 2(0.75+0.6) + 4(0.857 + 0.669 + 0.5455)\right]$$

$$= \frac{1}{18}\left[1.5 + 2.7 + 8.2784\right]$$

$$\log_e(x+1)\Big|_0^1 = 0.6932$$

$$\log_2^e = 0.6932$$

15 Find the approximate value of $\int_0^{\pi/2} \sqrt{\cos\theta}\, d\theta$ by Simpson's $\frac{1}{3}$rd rule by dividing $[0, \frac{\pi}{2}]$ into 6 equal parts

Given :- $y(\theta) = \sqrt{\cos\theta}$

$a = 0,\ b = \frac{\pi}{2},\ n = 6$

$h = \frac{b-a}{n} = \frac{\frac{\pi}{2} - 0}{6} = \frac{\pi}{12} = 15^\circ (0.2619)$

$P = \{a = \theta_0 = 0,\ \theta_1 = 15^\circ,\ \theta_2 = 30^\circ,\ \theta_3 = 45^\circ,\ \theta_4 = 60^\circ,\ \theta_5 = 75^\circ,\ \theta_6 = 90^\circ = b\}$

θ	$y = \sqrt{\cos\theta}$
$\theta = 0$	$y = \sqrt{\cos 0} = 1 = y_0$
$\theta_1 = 15$	$y = \sqrt{\cos 15} = 0.982 = y_1$
$\theta_2 = 30^\circ$	$y = \sqrt{\cos 30} = 0.9306 = y_2$
$\theta_3 = 45^\circ$	$y = \sqrt{\cos 45} = 0.8109 = y_3$
$\theta_4 = 60^\circ$	$y = \sqrt{\cos 60} = 0.7071 = y_4$
$\theta_5 = 75^\circ$	$y = \sqrt{\cos 75} = 0.508 = y_5$
$\theta_6 = 90^\circ$	$y = \sqrt{\cos 90} = 0 = y_6$

By the Simpsons $\frac{1}{3}$rd rule
$n = 6$

$$\Rightarrow \int_a^b f(x)\,dx = \frac{h}{3}\left[(y_0 + y_6) + 2(y_2 + y_5) + 4(y_1 + y_3 + y_5)\right]$$

$$\Rightarrow \int_0^{\pi/2} \sqrt{\cos\theta}\,d\theta = \frac{0.2619}{3}\left[(1+0) + 2(0.9306 + 0.7071) + 4(0.9828 + 0.8409 + 0.5087)\right]$$

$$\Rightarrow \int_0^{\pi/2} \sqrt{\cos\theta}\,d\theta = 1.1777$$

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10) Evaluate $\int_0^1 \frac{x}{1+x^2}\,dx$ taking 7 ordinates using weddle rule Hence find $\log_e 2$

Given :- $y = \frac{x}{1+x^2}$

$a=0,\ b=1,\ n=6$

$h = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$

$P = \{a = x_0 = 0,\ x_1 = \frac{1}{6},\ x_2 = \frac{1}{3},\ x_3 = \frac{1}{2},\ x_4 = \frac{2}{3},\ x_5 = \frac{5}{6},\ x_6 = 1 = b\}$

x	$\frac{x}{1+x^2}$
0	$0 = y_0$
1/6	$0.1621 = y_1$
1/3	$0.30 = y_2$
1/2	$0.4 = y_3$
2/3	$0.4615 = y_4$
5/6	$0.4915 = y_5$
1	$0.5 = y_6$

By The weddle rule

$$\int_0^b f(x)\,dx = \frac{3h}{10}\left(y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6\right)$$

$$\Rightarrow \int_0^1 \frac{x}{1+x^2}\,dx = \frac{3(1/6)}{10}\left[0 + 0.8105 + 0.3 + 2.4 + 0.4615 + 2.4590 + 0.5\right]$$

$$\Rightarrow \int_0^1 \frac{x}{1+x^2}\,dx = 0.3466$$

$$\Rightarrow \frac{1}{2}\int_0^1 \frac{2x}{1+x^2}\,dx = 0.3466$$

$$\log(1+x^2)\Big|_0^1 = 2\times 0.3466$$

$$\log(2)-\log(1) = 0.6932$$

$$\log_e^2 = 0.6932$$

7 Use simpson's $\frac{1}{3}$rd rule 7 ordinates to Evaluate $\int_2^8 \frac{1}{\log_e x}dx$

Given :- $y = \frac{1}{\log_{10} x}$

$$a=2,\ b=8,\ n=6$$

$$h = \frac{b-a}{n} = \frac{8-2}{6} = 1$$

$$P = \{a = x_0 = 2,\ x_1 = 3,\ x_2 = 4,\ x_3 = 5,\ x_4 = 6,\ x_5 = 7,\ x_6 = 8 = b\}$$

x	$y = \frac{1}{\log_{10} x}$
2	$y_0 = 3.3219$
3	$y_1 = 2.0460$
4	$y_2 = 1.6609$
5	$y_3 = 1.4306$
6	$y_4 = 1.2851$
7	$y_5 = 1.1833$
8	$y_6 = 1.1073$

By simpson's 1/3rd rule

$$n=6$$

$$\Rightarrow \int_a^b f(x)dx = \frac{h}{3}\left[(y_0+y_0)+2(y_2+y_4)+4(y_1+y_3+y_5)\right]$$

$$\Rightarrow \int_2^8 \frac{1}{\log_{10} x}dx = \frac{1}{3}\left[(3.3219+1.1093)+2(1.6609+1.285) + 4(2.096+1.4306+1.1833)\right]$$

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$$\Rightarrow \frac{1}{3}\left(4.4292 + 5.6920 + 8.9596\right)$$

$$\int_2^8 \frac{1}{\log_{10}^x}\,dx = 9.6936$$

1B Use Simpson's $\frac{1}{3}$rd rule to find $\int_0^{0.6} e^{-x^2}dx$ by taking 6 Subintervals

Given :- $y = e^{-x^2}$

$a=0,\quad b=0.6,\quad n=6$

$$h = \frac{b-a}{n} = \frac{0.6-0}{6} = 0.1$$

$p = \{x_0 = 0,\ x_1 = 0,\ x_2 = 0.2,\ x_3 = 0.3,\ x_4 = 0.4,\ x_5 = 0.5,\ x_6 = 0.6\}$

x	e^{-x^2}
0	$e^{0.1} = 1 \Rightarrow y_0$
0.1	$e^{-0.01} = 0.99 \Rightarrow y_1$
0.2	$e^{-0.04} = 0.9607 \Rightarrow y_2$
0.3	$e^{-0.09} = 0.9137 \Rightarrow y_3$
0.4	$e^{-0.16} = 0.8521 \Rightarrow y_4$
0.5	$e^{-0.25} = 0.7788 \Rightarrow y_5$
0.6	$e^{-0.36} = 0.697 \Rightarrow y_6$

Numerical Soln for trancidental Equation

An Eqn which involves algebraic logarithmic Exponential Trignometry function is called the trancidental Eqn

Regular falsi method (or) falsi position method

Step 1:- Write the given trancidental Eqn in the form of $f(x)=0$

Step 2:- Choose x_0 and x_1 nearest to the real root for which $f(x_0)<0$, $f(x_0)>0$

Step 3:-

Let the real root $x_2 = \dfrac{x_1 f(x_0) - x_0 f(x_1)}{f(x_0) - f(x_1)}$

Suppose $f(x_2)<0$. We say that the root lies b/w x_2 and x_1 and as follows

Step 4:-

$$x_3 = \frac{x_2 f(x_1) - x_1 f(x_2)}{f(x_1) - f(x_2)}$$

If $f(x_3)>0$ we say that the roots lies b/w x_2 and x_3 and follows the same

Step 5:-

Continue the same procedure until $f(x_n)$ is 0 (or) approximatly zero

[19] find the real root of the equation $xe^x - 3 = 0$ by regula falsi method correct to 3 decimal places

Soln $xe^x - 3 = 0$

$\Longrightarrow f(x) = 0$

$f(x) = xe^x - 3$

Let $x_0 = 0$, $x_1 = 1.1$

$f(x_0) = f(1) = 1 \cdot e^1 - 3 = -0.2817 < 0$

$f(x_1) = f(1.1) = 1.1e^{1.1} - 3 = 0.30467$

the roots lies between x_0 and x_1

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = \frac{(1)(0.3046) - (1.1)(-0.2817)}{0.3046 + 0.2817}$$

$\Rightarrow x_2 = 1.0480$

$\therefore f(x_2) = (1.048) e^{1.048} - 3 = -0.011 < 0$

the roots lies b/w x_2 and x_1

$$x_3 = \frac{x_2 f(x_1) - x_1 f(x_2)}{f(x_1) - f(x_2)}$$

$$x_3 = \frac{(1.048)(0.3046) - (1.1)(-0.011)}{0.3046 + 0.011}$$

$\Rightarrow x_3 = 1.0497$

$\therefore f(x_3) = (1.0497) e^{1.044} - 3 = -0.0012 < 0$

$$x_4 = \frac{x_3 f(x_1) - x_1 f(x_3)}{f(x_1) - f(x_3)}$$

$$x_4 = \frac{(1.0497)(0.3046) - (1.1)(-0.0012)}{0.3046 + 0.0012}$$

$\Rightarrow x_4 = 1.0498$

$f(x_4) = (1.049 + e^{1.0498} - 3) = 0.0006 \approx 0$

the real root is $x = 1.0498$

20 Find the real root of Eqn $xe^x - 2 = 0$ by regular-false method

Soln :-

Given :- $xe^x - 2 = 0$

$$f(x) = 0$$

$$\therefore f(x) = xe^x - 2$$

Let $x_0 = 0.8$, $x_1 = 0.9$

$$\Rightarrow f(x_0) = f(0.8) = -0.2195 < 0$$

$$f(x_1) = f(0.9) = 0.2136 > 0$$

$$\Rightarrow x_2 = \frac{(0.8)(0.2136) - (0.9)(-0.2195)}{0.2136 + 0.2195} = 0.8506$$

$$\therefore f(x_2) = f(0.8536) = -0.0087 < 0$$

$$\Rightarrow x_3 = \frac{x_2 f(x_1) - x_1 f(x_2)}{f(x_1) - f(x_2)}$$

$$x_3 = \frac{(0.8506)(0.2136) - (0.9)(-0.0087)}{0.2136 + 0.0087}$$

$$x_3 = 0.08525$$

$$\therefore f(x_3) = f(0.8525) = -0.0004 < 0$$

$$\Rightarrow x_4 = \frac{x_3 f(x_1) - x_1 f(x_3)}{f(x_1) - f(x_3)}$$

$$x_4 = \frac{(0.8525)(0.2136) - (0.9)(-0.0004)}{0.2136 + 0.0004}$$

$$x_4 = 0.08526$$

$$\therefore f(x_4) = f(0.8526) = -0.00002 < 0$$

$\therefore \; f(x_4) = f(0.8526) = -0.00002 < 0$

$$\boxed{\text{The real root of the Eqn is } x = 0.8526}$$

21 Use the regula falsi method to obtain a root of the Eqn $2x - \log_{10}x = 7$ which lies b/w 3.5 and 4 carry out 2 itteration

Soln:- $2x - \log_{10}x = 7$

$\Rightarrow 2x - \log_{10}x - 7 = 0$

$f(x) = 0$

$f(x) = 2x - \log_{10}x - 7$

and $x_0 = 3.5$ and $x_1 = 4$

$f(x_0) = f(3.5) = 2(3.5) - \log_{10}(3.5) - 7 = -0.5441 < 0$

$f(x_1) = f(4) = 2(4) - \log_{10}(4) - 7 = 0.397 > 0$

the roots are lies b/w x_0 and x_1

$$x_2 = \frac{x_1 f(x_0) - x_0 f(x_1)}{f(x_0) - f(x_1)}$$

$$x_2 = \frac{4(-0.544) - (3.5)(0.3970)}{-0.5441 - 0.397}$$

$x_2 = 3.788$

$f(x_2) = 2(3.7882) - \log_{10}(3.788) - 7 = 0.0009 < 0$

the roots lies b/w x_2 and x_1

$$x_3 = \frac{x_2 f(x_1) - x_1 f(x_2)}{f(x_1) - f(x_2)}$$

$$x_3 = \frac{(3.788)(0.3979) - (4)(-0.0009)}{0.3978 + 0.0009}$$

$x_3 = 3.7892$

$f(x_3) = 2(3.7892) - \log_{10}(3.7892) - 7$

$f(x_3) = -0.0001 \approx 0$

$\boxed{\text{The roots lies b/w } x = 3.7892}$

22) find the real roots of $x\log_{10}x - 1.2 = 0$ Correct to 3 decimal places lying in the interval $(2,3)$ using regula falsi method

Soln :- Given :- $x\log_{10}x - 1.2 = 0$

$f(x) = 0$

$f(x) = x\log_{10}x - 1.2$

Let $x_0 = 2.7$, $x_1 = 2.8$

$\Rightarrow f(x_0) = f(2.7) = -0.0353 < 0$

$f(x_1) = f(2.8) = 0.0520 > 0$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$\Rightarrow x_2 = \frac{(2.8)(0.0520) - (2.9)(-0.0353)}{0.0520 + 0.0353} = 2.7404$$

$f(x_2) = f(2.7404) = -0.0002 < 0$

$$x_3 = \frac{x_2 f(x_1) - x_1 f(x_2)}{f(x_1) - f(x_2)}$$

$$\Rightarrow x_3 = \frac{(2.7404)(0.05201) - (2.9)(-0.0002)}{0.0520 + 0.0002}$$

$x_3 = 2.7406$

$f(x_3) = f(2.7406) = -0.0000 2 < 0$

$\boxed{\text{The real root of the Eqn } x = 2.7406}$

Q3 Find the 4^{th} root of 12 by using regula-falsi method

Soln :- Let $x = \sqrt[4]{12}$

$\Rightarrow x^4 = 12$

$\Rightarrow x^4 - 12 = 0$

$\Rightarrow f(x) = 0$

$f(x) = x^4 - 12$

Let $x_0 = 1.8$, $x_1 = 1.9$

$\therefore f(x_0) = -1.5024 < 0$

$f(x_1) = 1.0321 > 0$

$\therefore$ the root lies between x_0 and x_1

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$\Rightarrow x_2 = \frac{(1.8)(1.0321) - (1.9)(-1.5024)}{1.0321 + 1.5024}$$

$\Rightarrow x_2 = 1.8592$

$\therefore f(x_2) = -0.0517 < 0$

$$x_3 = \frac{x_2 f(x_1) - x_1 f(x_2)}{f(x_1) - f(x_2)}$$

$$\Rightarrow x_3 = \frac{(1.8592)(1.0321) - (1.9)(-0.0517)}{1.0321 + 0.0517}$$

$\Rightarrow x_3 = 1.8611$

$\therefore f(x_3) = 0.0028 < 0$

$$x_4 = \frac{x_3 f(x_1) - x_1 f(x_3)}{f(x_1) - f(x_3)}$$

$$= \frac{(1.8611)(1.0321) - (1.9)(-0.0028)}{1.0321 + 0.0028}$$

$x_4 = 1.8612$

$f(x_4) = -0.0001 \approx 0$

$x = 1.8612$ is real root

$\therefore \sqrt[4]{12} = 1.8612$

24 Using regula falsi method to find the real root for the Equation $x^3 - 2x - 5 = 0$

Soln :-

$x^3 - 2x - 5 = 0$

$\Rightarrow f(x) = 0$

$f(x) = x^3 - 2x - 5 = 0$

Let $x_0 = 2 \Rightarrow f(x_0) = -1 < 0$

$x_1 = 2.1 \Rightarrow f(x_1) = 0.061 > 0$

$$\therefore x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$\Rightarrow x_2 = \frac{(2)(0.06) - (2.1)(-1)}{0.061 + 1}$$

$\Rightarrow x_2 = 2.0947$

$f(x_2) = -0.0039 < 0$

$$\therefore x_3 = \frac{x_2 f(x_1) - x_1 f(x_2)}{f(x_1) - f(x_2)}$$

$$\Rightarrow x_3 = \frac{(2.0942)(0.061) - (2.1)(-0.0039)}{0.061 + 0.0039}$$

$\Rightarrow x_3 = 2.0947$

$f(x_3) = -0.0005 < 0$

$$\therefore x_4 = \frac{(2.0945)(0.061) - (2.1)(-0.0005)}{0.061 + 0.0005}$$

$\Rightarrow x_4 = 2.0945$

$f(x_4) = -0.0005 \approx 0$

$\therefore x = 2.0945$ is real root

Newton Raphson Method

Step 1 :- Write the given transcedental Eqn in the form of $f(x)=0$ and find $f'(x)$

Step 2 :- Choose x_0. for which $f(x_0) < 0$

Step 3 :- Use of formula $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ for $n = 0, 1, 2$ ----

for the real root and continue the Same Until to get the Value of x has to be Equal

25 Find $\sqrt[3]{37}$ by newton Raphson method

Soln :-

Let $x = \sqrt[3]{37}$

$\Rightarrow x^3 = 37$

$\Rightarrow x^3 - 37 = 0$

$\Rightarrow f(x) = 0$

$\therefore f(x) = x^3 - 37 \Rightarrow f'(x) = 3x^2$

Let $x_0 = 3.3$

$\Rightarrow f(x_0) = -1.063 \;,\; f'(x_0) = 32.6700$

$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$\Rightarrow 3.3 - \frac{(-1.063)}{32.67}$

$\Rightarrow 3.3325$

$\Rightarrow f(x_1) = -0.0092 \;,\; f'(x_1) = 33.3166$

$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$\Rightarrow 3.3325 - \frac{(-0.0092)}{33.3166}$

$\Rightarrow 3.3327$

$f(x_2) = 0.0159 \qquad f'(x_2) = 33.3206$

$$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$\Rightarrow 3.3327 - \frac{(0.0159)}{33.3206}$$

$$\Rightarrow 3.3322$$

$$x = \sqrt[3]{27}$$

$$x = 3.3322$$

26 find a real root of Eqn $x\sin x + \cos x = 0$ near to $x = \pi$ radians correct to 4 decimal places by newtons draphson method

Soln :-

$$x\sin x + \cos x = 0$$

$$\Rightarrow f(x) = 0$$

$$\therefore f(x) = x\sin x + \cos x$$

$$\Rightarrow f'(x) = x\cos x + \sin x - \sin x$$

$$f'(x) = x\cos x$$

and $x_0 = \pi$

$$\Rightarrow f(x_0) = \pi\sin\pi + \cos\pi = -1$$

$$f'(x_0) = \pi\cos\pi = -\pi$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$\Rightarrow \pi - \frac{(-1)}{(-\pi)}$$

$$\Rightarrow \pi - \frac{1}{\pi}$$

$$\Rightarrow \frac{22}{7} - \frac{7}{22}$$

$$\Rightarrow x_1 \Rightarrow 2.8246$$

$\therefore\ x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \Rightarrow 2.8246 - \frac{(-0.0697)}{(-2.6838)} = 2.7986$

$\therefore\ x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \Rightarrow 2.7986 - \frac{(-0.00056)}{(-2.6355)} = 2.7983$

$\therefore\ x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} \Rightarrow 2.7983 - \frac{(0.00022)}{(-2.6350)} = 2.7983$

$x = 2.7983$ is a real root

27 Using newton raphson method find the real root of Equation $3x = \cos x + 1$ [Use radian mode]

Soln :-

given :- $3x = \cos x + 1$

$\Rightarrow 3x - \cos x - 1 = 0$

$\Rightarrow f(x) = 0$

$f(x) = 3x - \cos x - 1$

$\Rightarrow f'(x) = 3 + \sin x$

Let $x_0 = 0.5$

$\Rightarrow f(x_0) = 0.3715 < 0$

$f'(x_0) = 3.4794$

$\therefore\ x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.5 - \frac{(-0.3735)}{3.4794} = 0.6084$

$\therefore\ x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.6084 - \frac{0.00463}{3.5715} = 0.6071$

$\therefore\ x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.6071 - \frac{(-0.0000058)}{3.5704} = 0.6071$

$x = 0.6071$ is real root

28 Find the real root of $x\log_{10}^{x} = 1.2$ using newton raphson method near to 2.5

Soln :- Given :- $x\log_{10}^{x} = 1.2$

$$\Rightarrow x\log_{10}^{x} - 1.2 = 0$$

$$\Rightarrow \frac{x\log_e^x}{\log_e^{10}} - 1.2 = 0$$

$$\Rightarrow (0.4343)\, x\log x - 1.2 = 0$$

$$\Rightarrow f(x) = 0$$

$$\therefore f(x) = (0.4343)x\log x - 1.2$$

$$f'(x) = (0.4343)\left[x \cdot \frac{1}{x} + (1)\log x\right] - 0$$

$$\Rightarrow f'(x) = (0.4343)(1 + \log x)$$

and $x_0 = 2.5$

$$f(x_0) = -0.2051$$

$$f'(x_0) = 0.8322$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \Rightarrow 2.5 - \frac{(-0.2051)}{(0.8322)} = 2.7465$$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \Rightarrow 2.7465 - \frac{(0.0051)}{(0.8730)} = 2.7406$$

$$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \Rightarrow 2.7406 - \frac{(0.00002)}{(0.8722)} = 2.7406$$

$\boxed{\text{The real root is } 2.7406}$

29 Using newton raphson method find the real root that lies near $x = 4.5$ of the Equation $\tan x = x$ correct to 4 decimal places [take x as a radian]

Soln Given, $\tan x = x$

$$f(x) = x - \tan x$$

$$f'(x) = 1 - \sec^2 x$$

$f'(x) = -(\sec^2 x - 1)$

$f'(x) = -\tan^2 x$

and $x_0 = 4.5$

$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \Rightarrow 4.5 - \frac{(-0.1373)}{(-21.5048)} = 4.4936$

$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \Rightarrow 4.4936 - \frac{(-0.0039)}{(-20.227)} = 4.4934$

$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \Rightarrow 4.493 - \frac{(0.0002)}{(-20.1869)} = 4.4934$

$\boxed{\text{The real root is } x = 4.4934}$

30] find the real root of the Eqn $xe^x - \cos x = 0$ Correct to 3 decimal places by using newton Rapson method near To the root $x = 0.5$

Soln :-

$xe^x - \cos x = 0$

$\Rightarrow f(x) = 0$

$\therefore f(x) = xe^x - \cos x$

$f'(x) = 1 \cdot e^x + xe^x + \sin x$

$\Rightarrow f'(x) = e^x(1+x) + \sin x$

and $x_0 = 0.5$

$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \Rightarrow 0.5 - \frac{(-0.0532)}{2.9525} = 0.5180$

$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \Rightarrow 0.5180 - \frac{(0.0007)}{(3.0434)} = 0.5177$

$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \Rightarrow 0.5177 - \frac{(-0.0001)}{(3.0418)} = 0.5177$

$\boxed{\text{the real root is } x = 0.5177}$