CSE 2320 Homework 2 Solution

Task 1

Give the answer for the two problems below and **justify your answer using the limit theorem and computing the actual limit**. Saying: "The limit is x because this term dominates the other" is not a valid proof. To clarify, you can use the 'dominant term' argument when you talk about polynomial terms part of a finite summation (i.e. n^d).

a) is $2^{n+1} = O(2^n)$?

Yes. $\lim \frac{2^n}{2^{n+1}} = \lim \frac{1}{2} = \frac{1}{2}$ is a constant.

b) is $2^{2n} = O(2^n)$?

No. $\lim \frac{2^{2n}}{2^n} = \lim 2^n = \infty$ => f(n) grows faster than g(n) as n approaches infinity.

Task 2

a) (5 points) Let f(n) = $(4/9)^0 + (4/9)^1 + (4/9)^2 + \ldots + (4/9)^n$. Find Θ for f(n).

This is a summation of a geometric series where x=(4/9)<1 =>

$\sum_{i=1}^{n} (\frac{4}{9})^i < \sum_{i=1}^{\infty} (\frac{4}{9})^i = \frac{1}{1-\frac{4}{9}} = \frac{9}{5} = \Theta(1)$ because it does not depend on n.

b) (10 points) Use the definition with constants to show that $f(n) = nlg(n) - 15n + 14\sqrt{n}$ is $\Theta(nlg(n))$.

For c_2 and n_2: $f(n) = nlg(n) - 15n + 14\sqrt{n} \leq nlg(n) + 14\sqrt{n} \leq 15nlg(n), \forall n_2 \geq 2$=> c_2=15, n_2 = 2

We also want c_1 and n_1: $c_1 nlg(n) \leq nlg(n) - 15n + 14\sqrt{n}\ \forall n \geq n_1$. It suffices to show that:

$$c_1 nlg(n) \leq nlg(n) - 15n \Leftrightarrow$$
$$n[(1-c1)\lg(n) - 15] \geq 0 \Leftrightarrow$$
$$[(1-c1)\lg(n) - 15] \geq 0$$

We need $(1-c_1)\lg(n)>0$ => need: $(1-c_1)>0$ => $c_1 < 1$. If we pick $c_1 = ½$ we can solve for n:

$\left(1-\frac{1}{2}\right)\lg(n) - 15 \geq 0 \Leftrightarrow \frac{1}{2} lg(n) \geq 12 \Leftrightarrow \lg(n) \geq 30 \Leftrightarrow n \geq 2^{30} \Rightarrow n_1 = 2^{30}$

We want both inequalities to hold at the same time => pick $n_0 = \max(n_1,n_2)=\max(2,2^{30}) = 2^{30}$.=>$c_1$=1/2, c_2=15, $n_0=2^{30}$.