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SECTION 002

Homework 2

Task 1 a) Is $2^{n+1} = O(2^n)$?

Let $f(n) = 2^{n+1}$ $g(n) = 2^n$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{2^{n+1}}{2^n}$$

$$= \lim_{n\to\infty} \frac{2^n \times 2}{2^n}$$

$$= \lim_{n\to\infty} 2$$

$$= 2 = \text{constant} = c$$

From Big-oh limit theorem,

$$f(n) = O(g(n)) \text{ if } \lim_{n\to\infty} \frac{f(n)}{g(n)} = 0 \text{ or } c$$

$$\text{Here, } \lim \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{2^{n+1}}{2^n} = 2 = c$$

Hence, $\boxed{2^{n+1} = O(2^n) \Rightarrow \text{True}}$

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Task 1 b) Q8. $2^{2n} = O(2^n)$

Let $f(n) = 2^{2n}$ $\quad g(n) = 2^n$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{2^{2n}}{2^n}$$

$$= \lim_{n\to\infty} \frac{2^n \times 2^n}{2^n} = \lim_{n\to\infty} 2^n = \infty \neq C$$

By Big oh limit theorem

$$f(n) = O(g(n)) \text{ if } \lim_{n\to\infty} \frac{f(n)}{g(n)} = 0 \text{ or } C$$

But here $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} 2^n \neq 0 \text{ or } C$

Hence, $\boxed{2^{2n} \neq O(2^n)}$

Task 1 a) LET $f(n) = (4/9)^0 + (4/9)^1 + \dots (4/9)^n$

For $0 < x < 1$ $\quad \sum_{k=0}^{n} x^k = \frac{1 - x^{n+1}}{1-x}$

$$\sum_{k=0}^{n} (4/9)^k = \frac{1 - (4/9)^{n+1}}{1 - 4/9}$$

$$= \frac{1 - \left[(4/9)(4/9)^n\right]}{\frac{9-4}{9}}$$

$$= \frac{1 - \left[(4/9)(4/9)^n\right]}{\frac{5}{9}}$$

$$= \frac{9}{5}\left[1 - (4/9)(4/9)^n\right]$$

$$= \frac{9 - 9(4/9)(4/9)^n}{5}$$

$$= \frac{9 - 4(4/9)^n}{5} \Rightarrow \text{some constant for any value of } n$$

We know that,

$$\sum_{k=0}^{n} x^k \le \sum_{k=0}^{\infty} x^k = \frac{1}{1-x} \Rightarrow \text{Equation } (1)$$

From equation (1)

$$\sum_{k=0}^{n} (4/9)^k \le \sum_{k=0}^{\infty} (4/9)^k = \frac{1}{1-4/9} = \frac{1}{(5/9)} = \underline{\frac{9}{5}} = \text{constant}$$

θ of any constant is 1. Hence, θ of $f(n)$ is $\boxed{\theta(1)}$

$$\boxed{\theta[f(n)] = \theta(1)}$$

Because constant is independent of n

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Task 2b) $f(n) = n\lg n - 15n + 14\sqrt{n}$

$c_1 g(n) \leq f(n) \leq c_2 g(n) \quad \forall n \geq n_0$

$f(n) \leq c_2 g(n) \quad \forall n \geq n_2$

$n\lg n - 15n + 14\sqrt{n} \leq c_2(n\lg n)$

$n\lg n - 15n + 14\sqrt{n} \leq n\lg n + 14\sqrt{n}$

when $n \geq 1$

$\leq n\lg n + 14 n\lg n$

$\leq \underline{\underline{15 n\lg n}}$

$\boxed{c_2 = 15, \ n_2 = 1}$

$c_1 g(n) \leq f(n) \quad \forall n \geq n_1$

$c_1(n\lg n) \leq n\lg n - 15n + 14\sqrt{n}$

$n\lg n - 15n \leq n\lg n - 15n + 14\sqrt{n}$

$c_1 [n\lg n] \leq n\lg n - 15n$

$0 \leq n\lg n - 15n - c_1 n\lg n$

$0 \leq n\lg n \left[1 - \frac{15}{\lg n} - c_1\right]$

$0 \leq 1 - \frac{15}{\lg n} - c_1$

$\frac{15}{\lg n} \leq 1 - c_1$

$15 \leq (1-c_1)\lg n$

$(1-c_1)$ must be high & $\lg n$ must be low for $(1-c_1)\lg n$ to be greater than equal to 15

LET $c_1 = 0.01$, $(1-c_1)\lg n \geq 15$

$(1-0.01)\lg n \geq 15$

$0.99 \lg n \geq 15$

$\lg n \geq 15/0.99$

$2^{\lg n} \geq 2^{[15/0.99]}$

$\boxed{n \geq 2^{16}}$ approximately.

$\boxed{c_1 = 0.01, \ c_2 = 15, \ n_0 = 2^{16}}$