CSE 2320 - Homework 5 NAME:_____

Total points: 100 Topics: Recurrences , solved with methods: Master Theorem, Tree, Substitution (induction)

Convention: ⌈ ⌉ *means rounded up and* ⌊ ⌋ *means rounded down.*

P1. (**23 points**) Use the tree and table method to compute the **Θ** time complexity for $T(N)=5T(\lfloor N/4 \rfloor)+2N^3$.
Assume T(0) = 1 and T(1) = 1. Fill in the table below and finish the computations outside of it:

Level	Argument/ Problem size	Cost of one node	Nodes per level	Cost of whole level
0				
1				
2				
i				
k= *Leaf level. Write k as a function of N.*				

Total tree cost calculation: ……..

T(N) = Θ(……)
Draw the tree. Show **levels 0,1,2** and the **leaves level**. Show the problem size T(…) as a label next to the node and inside the node show the local cost (cost of one node) as done in class. For the leaf level and level 2 it suffices to show a few nodes.

P2. (23 **points**) Use the tree and table method to compute the **Θ** time complexity for $T(N) = 4T(N-5) + 7$. Assume T(N) = 1 for all 0≤N≤4. Assume N is a convenient value for your computations.

Fill in the table below and finish the computations outside of it:

Level	Argument/ Problem size	Cost of one node	Nodes per level	Cost of whole level
0				
1				
2				
i				
k= *Leaf level. Write k as a function of N.*				

Total tree cost calculation: …….

T(N) = Θ(……)

Draw the tree. Show **levels 0,1,2** and the **leaves level**. Show the problem size T(…) as a label next to the node and inside the node show the local cost (cost of one node) as done in class. For the leaf level and level 2 it suffices to show a few nodes.

P3. (**36 points**) Can you use the Master Theorem to solve the recurrences below? If yes, solve it with this method, if no, show why you cannot use it.

a) $T(N)=5T(\lfloor N/4 \rfloor)+2N^3$. Assume T(0) = 1 and T(1) = 1.

b) $T(N)=4T(\lceil N/4 \rceil)+d$, for some constant d>0. Assume T(0) = 1 and T(1) = 1.

c) $T(N) = 6T(N/6) + 5N$, Assume T(0) = 1 and T(1) = 1.

d) $T(N) = 8T(N/2) + cN^3 lgN$, Assume T(0) = 1, and T(1) = 1 .

P4. (4 **points**) Go to the Wikipedia webpage https://en.wikipedia.org/wiki/Master_theorem_(analysis_of_algorithms) . See section "Inadmissible equations" and list the equation and the reason why it does not satisfy the Master Theorem requirements.

P5. (**14 points**) Show that $T(N)=5T(\lfloor N/4 \rfloor)+2N^3=\Theta(N^3)$ by showing that it is $O(N^3)$ and also $\Omega(N^3)$. Assume T(0) = 1 and T(1) = 1

a) (9 points) Use the induction method to show $O(N^3)$. As done in class, start with the inductive step and then check and refine for enough low values of N until the inductive step can be applied (See lecture from Wed, Oct 11).

b) (5 points) Use just the definition with c and n_0 to show that it is $\Omega(N^3)$. Assume that $T(N)\geq 0$, for all $N\geq 0$. You should not need to use induction. (It is easier without induction.)

Write your answers in a document called **2320_H5.pdf**. It can be hand-written and scanned, but it must be uploaded electronically. Submit just the 2320_H5.pdf.

Remember to include your name at the top.