Use the definition with constants to show that $f(n) = n\sqrt{n} - n + 20\lg n$ is $\Theta(n\sqrt{n})$.

Definition: *f(n)* is **Θ***(g(n))* if there exist positive constants c_1, c_2 and n_0 such that:

$c_1\, g(n) \le \boldsymbol{f(n)} \le c_2\, g(n)$ for all $n \ge n_0$.

Here g(n) =

For c_2:

$f(n) = n\sqrt{n} - n + 20\lg(n) \le n\sqrt{n} + 20\lg(n) \le n\sqrt{n} + 20n\sqrt{n} = 21n\sqrt{n}, \forall n \ge 2$ => c_2=21, n_2 = 1

We also want c_1 and n_1: $c_1 n\sqrt{n} \le n\sqrt{n} - n + 20\lg(n), \forall n \ge n_1$. It suffices to show that

$$c_1 n\sqrt{n} \le n\sqrt{n} - n \Rightarrow$$

$$n\sqrt{n} - c_1 n\sqrt{n} - n \ge 0 \Rightarrow$$

$$n[\sqrt{n}(1-c_1) - 1] \ge 0 \Rightarrow$$

$$\sqrt{n}(1-c_1) - 1 \ge 0 \Rightarrow$$

$$\sqrt{n}(1-c_1) \ge 1$$

$(1-c_1)>0$ => $c_1 < 1$. If we pick $c_1 = ½$ we can solve for n =>

$$\sqrt{n}(1-\frac{1}{2}) \ge 1 \Rightarrow \frac{\sqrt{n}}{2} \ge 1 \Rightarrow n \ge 4 \Rightarrow n_1 = 4$$

We want both inequalities to hold at the same time => pick $n_0 = \max(n_1,n_2)=\max(1,4) = 4$.=>$c_1$=1/2, c_2=21, n_0=4.