# Homework - 1

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2.4) $P(C) = 0.7$, $P(F) = 0.6$, $P(C \cap F) = 0.5 = P(F \cap C)$

a) does not know fortran

$P(\bar{F}) = 1 - P(F)$

$= 1 - 0.6$

$\boxed{P(\bar{F}) = 0.4}$

b) does not know fortran and does not know c/c++

$P(\bar{F} \cap \bar{C}) = 1 - P(\overline{\bar{F} \cap \bar{C}}) = 1 - P(\bar{\bar{F}} \cup \bar{\bar{C}}) = 1 - P(F \cup C)$

$P(F \cup C) = P(F) + P(C) - P(F \cap C)$

$= 0.6 + 0.7 - 0.5 = 0.8$

$P(\bar{F} \cap \bar{C}) = 1 - 0.8 = 0.2$ $\boxed{P(\bar{F} \cap \bar{C}) = 0.2}$

c) knows c/c++ but not fortran

$P(C - F) = P(C) - P(C \cap F)$

$= 0.7 - 0.5 = 0.2$

$\boxed{P(C - F) = 0.2}$

d) knows fortran but not c/c++

$P(F - C) = P(F) - P(F \cap C)$

$= 0.6 - 0.5 = 0.1$

$\boxed{P(F - C) = 0.1}$

e) If someone knows fortran, what is the probability that he/she knows c/c++ too?

$P(C|F) = \dfrac{P(C \cap F)}{P(F)} = \dfrac{0.5}{0.6} = 0.8333$

$\boxed{P(C|F) = 0.8333}$

b) If someone knows c/c++, what is the probability that he/she knows fortran too?

$$P(F|C) = \frac{P(F\cap C)}{P(C)} = \frac{0.5}{0.7} = 0.7143$$

$$\boxed{P(F|C) = 0.7143}$$

2.9) P(Module 1 works) = P(M1) = 0.96

P(Module 2 works) = P(M2) = 0.95

P(Module 3 works) = P(M3) = 0.90

P(at least one of three modules fails to work) = 1 - P(all modules work)

$= 1 - P(M1 \cap M2 \cap M3)$

$= 1 - P(M1) \cdot P(M2) \cdot P(M3)$

$= 1 - (0.96)(0.95)(0.90)$

$= 1 - 0.8208$

$= \underline{\underline{0.1792}}$

2.16) $P(S1) = 0.5$, $P(S2) = 0.2$, $P(S3) = 0.3$

$P(D|S1) = 0.05$, $P(D|S2) = 0.03$, $P(D|S3) = 0.06$

a) What portion of all the parts is defective? $P(D) = ?$

$$P(D) = P(D|S1)\,P(S1) + P(D|S2)\,P(S2) + P(D|S3)\,P(S3)$$

$$= (0.05 \times 0.5) + (0.03 \times 0.2) + (0.06 \times 0.3)$$

$$= 0.025 + 0.006 + 0.018$$

$$\boxed{P(D) = 0.049}$$

b) $P(S1|D) = ?$

$$P(S1|D) = \frac{P(D|S1)\cdot P(S1)}{P(D)} = \frac{0.05 \times 0.5}{0.049} = 0.5102$$

$$\boxed{P(S1|D) = 0.5102}$$

2.19) $P(I) = 0.2$ , $P(\bar{I}) = 1 - P(I) = 1 - 0.2 = 0.8$

$P(\bar{D}|I) = 0.95$ , $P(\bar{D}|\bar{I}) = 0.7$ , $P(I|D) = ?$

$$P(I|D) = \frac{P(D|I)\,P(I)}{P(D|I)\,P(I) + P(D|\bar{I})\,P(\bar{I})}$$

$P(D|I) = 1 - P(\bar{D}|I) = 1 - 0.95 = 0.05$

$P(D|\bar{I}) = 1 - P(\bar{D}|\bar{I}) = 1 - 0.7 = 0.3$

$$P(I|D) = \frac{0.05 \times 0.2}{(0.05 \times 0.2) + (0.3 \times 0.8)} = \frac{0.1}{0.01 + 0.24} = 0.04$$

$\boxed{P(I|D) = 0.04}$

2.21) P( each component failing ) = 0.3

P( system reliability ) = P ( first 3 components working ) × P ( second 2 components working )

P( first 3 components working ) = 1 − P ( first 3 components failing )
$= 1 - (0.3 \times 0.3 \times 0.3)$
$= 0.973$

P( second 2 components working ) = 1 − P( second 2 components failing )
$= 1 - (0.3 \times 0.3)$
$= 0.91$

P( system reliability ) = $0.973 \times 0.91 = \underline{0.8854}$

2.26) No. of computers have problems with hard drive = 2

Total computers = 6

No. of computers don't have problems with hard drive = 6 - 2 = 4

No. of ways 3 computers selected at random = $6C_3 = \frac{6!}{3!\,3!} = 20$

Probability (None of them have hard drive problems) = $\frac{4}{6C_3}$

P(None have problems) = $\frac{4}{20} = 0.2$

3.7)

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $P(x)$ | 0.4 | 0.4 | 0.2 |

The team plays 2 games $x_1$ and $x_2$

$y = x_1 + x_2 = 2 + 2 = 4$

$P(y=0) = P(x_1=0) \times P(x_2=0)$

$= 0.4 \times 0.4 = 0.16$

$P(y=1) = P(x_1=0) \times P(x_2=1) + P(x_1=1)\,P(x_2=0)$

$= (0.4 \times 0.4) + (0.4 \times 0.4) = 0.32$

$P(y=2) = P(x_1=0)\,P(x_2=2) + P(x_1=1)\,P(x_2=1) + P(x_1=2)\,P(x_2=0)$

$= (0.4 \times 0.2) + (0.4 \times 0.4) + (0.2 \times 0.4) = 0.32$

$P(y=3) = P(x_1=1)\,P(x_2=2) + P(x_1=2)\,P(x_2=1)$

$= (0.4 \times 0.2) + (0.2 \times 0.4) = 0.16$

$P(y=4) = P(x_1=2)\,P(x_2=2)$

$= 0.2 \times 0.2 = 0.04$

$E(y) = \sum_y y\,P(y)$

$Var(y) = E(y^2) - [E(y)]^2$

$E(y) = \sum y P(y) = 0(0.16) + 1(0.32) + 2(0.32) + 3(0.16) + 4(0.04)$

$\boxed{E(y) = 1.6}$

$Var(y) = E(y^2) - [E(y)]^2$

$E(y^2) = \sum y^2 P(y) = 0^2(0.16) + 1^2(0.32) + 2^2(0.32) + 3^2(0.16) + 4^2(0.04)$
$= 3.68$

$Var(y) = 3.68 - (1.6)^2 = 1.12$

$\boxed{Var(y) = 1.12}$

3.18) A => \$10 per share. , Profit $= X\% = \frac{x}{100}$

B => \$50 per share, Profit $= Y\% = \frac{y}{100}$

Investment = \$1000

a) ~~100 shares of A = 100A = Portfolio 1 a~~

| X | -3 | 0 | 3 |
|---|---|---|---|
| P(x) | 0.3 | 0.2 | 0.5 |

| Y | -3 | 0 | 3 |
|---|---|---|---|
| P(y) | 0.4 | 0 | 0.6 |

$E(x) = \sum x P(x) = (-3 \times 0.3) + (0 \times 0.2) + (3 \times 0.5) = 0.6$

$Var(x) = \sum x^2 P(x) - \sum x P(x) = E(x^2) - [E(x)]^2$

$Var(x) = [(9 \times 0.3) + (0^2 \times 0.2) + (9 \times 0.5)] - (0.6)^2$

$Var(x) = 6.84$

$E(y) = \sum y P(y) = (-3 \times 0.4) + (0 \times 0) + (3 \times 0.6) = 0.6$

$Var(y) = E(y^2) - [E(y)]^2$
$= [(9 \times 0.4) + 0 + (9 \times 0.6)] - 0.6^2$

$Var(y) = 8.64$

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a) Portfolio 1 $P_1 = (100 \text{ shares} \times \$10)\frac{X}{100} = 10X$

b) Portfolio 2 $P_2 = (50 \text{ shares} \times \$10)\frac{X}{100} + (10 \text{ shares} \times \$50)\frac{Y}{100}$

$$= 5X + 5Y$$

c) Portfolio 3 $P_3 = (20 \times \$50)\frac{Y}{100} = 10Y$

$$\boxed{\begin{array}{l} E(aX) = aE(X) \\ Var.(aX) = a^2 Var(X) \end{array}}$$

$E(P_1) = 10E(X) = 10 \times 0.6 = \underline{6}$

$Var(P_1) = 10^2 Var(X) = 10^2 \times 6.84 = \underline{684}$

$E(P_2) = E(5X+5Y) = 5E(X) + 5E(Y) = (5 \times 0.6) + (5 \times 0.6)$

$E(P_2) = \underline{6}$

$Var(P_2) = Var(5X+5Y) = 5^2 Var(X) + 5^2 Var(Y) = 25(6.84) + 25(8.64)$

$Var(P_2) = \underline{387}$

$E(P_3) = E(10Y) = 10E(Y) = 10 \times 0.6 = 6$

$Var(P_3) = Var(10Y) = 10^2 Var(Y) = 10^2 \times 8.64 = \underline{864}$

To find least and most risky portfolio:

Our goal should be to maximize expectation and minimize variance

Here Expectation of 3 portfolios are same, so we consider only variance.

Maximum variance $= 864 = Var.(P_3)$ = most risky portfolio.

Minimum variance $= 387 = Var(P_2)$ = least risky portfolio

Most risky portfolio is c) 20 shares of B

Least risky portfolio is b) 50 shares of A and 10 shares of B

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3.22) $p = 0.05$ , $n = 16$ , $x > 3$

$x$ - no. of successes $> 3$ } Using Binomial distribution
$n$ - no. of trials $= 16$ } $P(x > 3)$ can be determined

$P(X > x) = 1 - F(x)$

$P(X > 3) = 1 - F(3)$

$= 1 - 0.993 = \underline{\underline{0.007}}$

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3.26) $p = 0.2$

a) $x$ - success - at least 5 of the first 20 files damaged.
$n$ - no. of trials $= 20$ . Using Binomial distribution

$P(X \geq 5) = 1 - P(X \leq 4)$

$= 1 - F(4)$

$= 1 - 0.6296$

$= 0.3704$

b) P ( check at least 6 files in order to find 3 undamaged files )
We can check using negative binomial distribution

$X$ - no. of files to be checked

$P(X \geq 6) =$ P( at least 6 files to be checked)

= P (5 files are not sufficient)

= P (there are fewer than 3 undamaged files)

$= P(Y \leq 2)$

$Y$ - no. of success } We can use binomial distribution
$n$ - no. of trials $= 6$ } $p = 0.2$

$P(y \leq 2) = F(2) = \underline{\underline{0.0579}}$

3.27) Using Poisson distribution with $\lambda = 9$

X - no. of messages

a) $P(X \geq 5) = 1 - P(X \leq 4)$

$= 1 - 0.055 = \underline{\underline{0.945}}$

b) $P(X = 5) = P(X \leq 5) - P(X \leq 4)$

$= 0.116 - 0.055$

$= \underline{\underline{0.061}}$

4.1) $f(x) = \begin{cases} \frac{k}{x^4} & \text{for } x \geq 1 \\ 0 & \text{for } x > 1 \end{cases}$

k can be found from the condition $\int f(x)\,dx = 1$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{1}^{\infty} \frac{k}{x^4}\,dx = \left[-\frac{k}{3x^3}\right]_1^{\infty} = \left[\frac{-k}{3(\infty)^3} - \frac{-k}{3(1)^3}\right] = 1$$

$$\left[0 + \frac{k}{3}\right] = 1 \Rightarrow \boxed{k = 3}$$

Integrating the density we get cdf

$$F(x) = \int_{-\infty}^{x} f(y)\,dy = \int_{1}^{x} \frac{3}{y^4}\,dy = \left[-\frac{1}{y^3}\right]_1^{x} = \left[\frac{-1}{x^3} - \frac{-1}{1^3}\right]$$

$F(x) = 1 - \frac{1}{x^3}$ for $x \geq 1$

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$$P(X>2) = 1 - P(X \le 2)$$
$$= 1 - F(2)$$
$$= 1 - \left[1 - \frac{1}{2^3}\right] = \frac{1}{8} = 0.125$$

$$P(X>2) = \underline{0.125}$$

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A.4) $f(x) = \begin{cases} k - x/50 & \text{for } 0 < x < 10 \text{ years} \\ 0 & \text{for all other } x \end{cases}$

a) To find k we use condition $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_0^{10} \left(k - \frac{x}{50}\right) dx$$
$$= \left[kx - \frac{x^2}{2\times 50}\right]_0^{10} = \left[k\times 10 - \frac{10^2}{2\times 50}\right] - \left[k\times 0 - \frac{0^2}{2\times 50}\right]$$

$$\int_{-\infty}^{\infty} f(x)\,dx = 10k - 1 \Rightarrow \text{equating this to } 1$$

$$10k - 1 = 1 \Rightarrow 10k = 2 \Rightarrow k = \frac{2}{10} = \frac{1}{5} = 0.2$$

$$\boxed{k = 0.2}$$

b) $P(X<5) = \int_0^5 \left(0.2 - \frac{x}{50}\right) dx$

$$= \left[0.2x - \frac{x^2}{2\times 50}\right]_0^5$$
$$= \left[0.2\times 5 - \frac{5^2}{2\times 50}\right] = \left[1 - \frac{25}{100}\right] = 1 - 0.25 = 0.75$$

$$\boxed{P(X<5) = 0.75}$$

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c) $E(X) = \int_{-\infty}^{\infty} x f(x)\, dx$

$= \int_0^{10} x\left(0.2 - \frac{x}{50}\right) dx$

$= \int_0^{10} \left(0.2x - \frac{x^2}{50}\right) dx$

$= \left[\frac{0.2x^2}{2} - \frac{x^3}{3\times 50}\right]_0^{10}$

$= \left[\frac{0.2\times 10^2}{2} - \frac{10^3}{3\times 50}\right]$

$= \left[10 - 6.6666\right] = 3.3333$

$\boxed{E(X) = 3.3333 \text{ years}}$

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A.7) $E(X) = 12 \Rightarrow$ Exponential distribution

$E(X) = \frac{1}{\lambda} = 12$

$\lambda = \frac{1}{12}\ s^{-1}$

Job in the printer is 3rd in line following a gamma distribution with $\alpha = 3$, $\lambda = \frac{1}{12}\ s^{-1}$, $t = 1$ minute $= 60$ seconds

$P(T \leq t) = P(X \geq \alpha)$ By Gamma - Poisson formula Poisson$(\lambda t)$ variable X

$P(T \leq 60) = P(X \geq 3)$

$P(X \geq 3) = 1 - P(X < 3) = 1 - P(X \leq 2) = 1 - F(2) = \underline{0.875}$

4.30) $P(L1) = 0.8$ , $P(L2) = 0.2$ ,

Line I has Gamma connection with $\alpha = 3$, $\lambda = 2 \text{ min}^{-1}$

$P(T > 0.5) = ?$

Probability ($T >$ 30 seconds given Line 1) = ?

30 seconds $= \frac{30}{60} = 0.5$ min

$P(T > 0.5 | L1) = ?$

Using Gamma - Poisson formula

$P(T > t) = P(X < \alpha)$ , Poisson $(\lambda t) \Rightarrow$ Poisson $(2 \times 0.5) \Rightarrow$ Poisson(1)

$$P(T > 0.5 | L1) = P(X < 3)$$
$$= P(X \leq 2)$$
$$= F(2)$$
$$= 0.92$$

Line II has Uniform $(a, b)$ connection with $a = 20$ sec , $b = 50$ sec

$a = \frac{20}{60}$ min $= 0.3333$ min

$b = \frac{50}{60} = 0.8333$ min

$P(T > 0.5 | L2) = ?$

Using Uniform property

$$P(T > t) = \int_{t}^{tth} \frac{1}{b-a} \, dt$$

$$P(T > 0.5 | L2) = \int_{20/60}^{0.5} \frac{1}{\frac{50}{60} - \frac{20}{60}} \, dt$$

$$= 2 \Big[ t \Big]_{1/3}^{0.5}$$

$$= 2 \left[ 0.5 - \frac{1}{3} \right]$$

$$= \frac{1}{3}$$

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$$P(T > 0.5) = P(T > 0.5 \mid L1)\, P(L1) + P(T > 0.5 \mid L2)\, P(L2)$$

$$= 0.92 \times 0.8 + \frac{1}{3} \times 0.2$$

$$= \underline{\underline{0.8027}}$$