9.7 (a) $\bar{x} = 37.7$, $n = 100$, $\sigma = 7.2$, 90% confidence inter

$1-\alpha = 0.90 \Rightarrow \alpha = 0.10$

CI : $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$37.7 \pm z_{0.05} \frac{9.2}{\sqrt{100}}$ $\qquad z_{0.05} = 1.645$

$\Rightarrow 37.7 \pm 1.645 \cdot \frac{9.2}{100} = 37.7 \pm 1.5134$

$[36.1866, 39.2134]$ is the confidence interval

(b) 1% significance level.

$H_0 : \mu = 35$
$H_A : \mu > 35$

$$\text{RTT} \begin{cases} \text{reject } H_0 \text{ if } z \geq z_\alpha \\ \text{accept } H_0 \text{ if } z < z_\alpha. \end{cases}$$

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{37.7 - 35}{9.2/\sqrt{100}} = \frac{2.7 \times \sqrt{100}}{9.2} = 2.9347$$

$z_\alpha = z_{0.01} = 2.326$

As $\underset{(z)}{2.9347} > 2.326\ (z_\alpha)$, we reject H_0 & Accept H_A.

It means we have enough significant evidence that mean number of concurrent users is greater than 35.

Scanned by CamScanner

9.8 $\sigma = 5$ min

a) $n = 64$, $\mu = 42$ min, 95% CI

$\alpha = 0.05$

$CI = \bar{X} \pm Z_{\alpha/2} \sigma/\sqrt{n}$

$$CI = 42 \pm Z_{0.05/2} \frac{5}{\sqrt{64}}$$

$$= 42 \pm 1.960 (0.625)$$

$$= 42 \pm 1.225$$

$$= [40.775, 43.225]$$

b) $P(40.775 < X < 43.225)$

$$= P\left(\frac{40.755-40}{5} < Z < \frac{43.225-40}{5}\right)$$

$$= P(0.151 < Z < 0.645)$$

$$= 0.7405 - 0.5600$$

$$= 0.1805$$

9.10 $n = 200$, defective $= 24$

Sample proportion $\hat{p} = 24/200 = 0.12$

a) $\alpha = 0.04$ $Z_{0.02} = 2.054$

$$CI = \hat{p} \pm Z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$= 0.12 \pm (2.054) \sqrt{\frac{(0.12)(0.88)}{200}}$$

$$= 0.12 \pm (2.054) \sqrt{0.0005}$$

$$= 0.12 \pm \cancel{0.0000} \; 0.0459$$

$$= [\cancel{0.0000} \; 0.0741, 0.1659]$$

$$CI = [0.074, 0.165]$$

b) $H_0 : \hat{p} = 0.1$

$H_A : \hat{p} > 0.1$ Disproving claim ~ rejecting H_0

$$Z = \frac{(0.12 - 0.1)}{\sqrt{\frac{(0.12)(0.88)}{200}}}$$

(Reject H_0, $Z \geq Z_\alpha$
accept H_0, $Z < Z_\alpha$)

$$= 0.8704.$$

From table, $Z_{0.04} = 1.75$, ~~Z.0075~~ $Z_{0.15} = 1.04$

$\therefore$ We don't have significance evidence at 4% & 15% to disprove claim

9.11

$H_0 : \hat{p}_1 = \hat{p}_2$

$H_A : \hat{p}_1 < \hat{p}_2$

$\hat{p}_2 = 13/150$
$= 0.087$

$\hat{p}_1 = 0.12$

Pooled sample proportion

$$\hat{p} = \frac{\hat{p}_1 * n_1 + \hat{p}_2 \times n_2}{n_1 + n_2}$$

$$= \frac{0.12 \times 200 + 0.087 \times 150}{200 + 150}$$

$$= 0.1059$$

$$\hat{Var}(\hat{p}_A - \hat{p}_B) = \hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)$$

$$= 0.1059 * (1 - 1.0591) * \left(\frac{1}{200} + \frac{1}{150}\right)$$

$$= 1.104 \times 10^{-3}$$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{Var}}} = \frac{0.12 - 0.087}{0.0332}$$

$$= 0.99$$

$$\text{P-Value} = P(z < 0.99) = 0.8389$$

At 5% Significance level, As Pvalue > significance level, we should not reject Ho. So, At this level, there is no sufficient evidence to conclude that quality of items produced by new supplier is higher.

9.17 $\hat{P}_1 = \frac{45}{100}$ $\hat{P}_2 = \frac{35}{100}$ $\hat{P}_1 - \hat{P}_2 = \frac{10}{100}$

$n = 900$, $(1-\alpha)100 = 95$ $\Rightarrow \alpha = 0.05$

Margin of error of $\hat{P}_1 = z_{0.025}\sqrt{\frac{0.45\times(1-0.45)}{900}}$

$= 1.96\sqrt{\frac{0.45\times 0.55}{900}}$

$= 1.96 \times 0.016 = 0.0325$

Margin of error of $\hat{P}_2 = z_{0.025}\sqrt{\frac{0.35\times(1-0.35)}{900}}$

$= 1.96 \times (0.015) = 0.0311$

Margin of error for $P_1 - P_2$

$z_{0.025}\sqrt{\frac{\hat{P}_1(1-\hat{p})}{n} + \frac{\hat{P}_2(1-\hat{p})}{n}} = 1.96\sqrt{\frac{0.45\times0.55}{900} + \frac{0.35\times0.65}{900}}$

$= 1.96\sqrt{\frac{0.2475 + 0.2275}{900}}$

$= 0.045$

9.18 Before firewall: 56, 47, 49, 37, 38, 60, 50,

n=14 43, 43, 59, 50, 56, 54, 58

After firewall: 53, 21, 32, 49, 45, 38, 44, 33, 32,

n=20 43, 53, 46, 36, 48, 39, 35, 37, 36, 39, 45

$\bar{x}_1 = \frac{700}{14} = 50$ $\bar{x}_2 = \frac{804}{20} = 40.2$

$S_1 = 7.62$ $S_2 = 7.96$

a) ~~t value for 95% CI = 2.1604~~

$$S_p = \sqrt{\frac{13 \times (7.62)^2 + 19\,(7.96)^2}{14+20-2}}$$

$$= \sqrt{61.209} = 7.82$$

$$CI = \bar{x} - \bar{y} \pm t_{\alpha/2}\, S_p \sqrt{\frac{1}{n}+\frac{1}{m}}$$

32 df

$t_{0.025} = 2.037$

$$= (50-40.2) \pm \overset{2.037}{(\cancel{})}(7.82)\left(\sqrt{\frac{1}{14}+\frac{1}{20}}\right)$$

$$= 9.8 \pm \overset{2.037}{(\cancel{})}(7.82)(0.3484)$$

$$= 9.8 \pm 5.5434$$

$$= \cancel{[]}$$

$$= [4.25, 15.35]$$

b) $H_0 : \mu_B = \mu_A$ (Assume equal Variance)

$H_A : \mu_B \neq \mu_A$

$\left(t = \frac{\bar{x} - \bar{y}}{s_p\sqrt{\frac{1}{n}+\frac{1}{m}}} \right)$

Test statistic, $t = \frac{50 - 40.2}{2.7244}$

$= 3.597$

p-value $= P\{t > 3.597\}$ = b/w 0.0005 and 0.001

$\alpha = 0.05$

So, pvalue < α, By rejection rule, it can be concluded that there is evidence to reject H_0 at $\alpha = 0.05$.

$\therefore$ Hence, there is significant difference b/w avg no of instrusion attempts per day before and after change of firewall setting.

(ii) $H_0 : \mu_B = \mu_A$ (Assume unequal variance)

$H_A : \mu_B \neq \mu_A$

$$t = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{s_x^2}{n} + \frac{s_y^2}{m}}} = \frac{50 - 40.2}{\sqrt{\frac{(7.62)^2}{14} + \frac{(7.96)^2}{20}}}$$

$$= \frac{9.8}{2.704}$$

$$\approx 3.62$$

P value $= P\{t > 3.62\}$ = b/w 0.0005 & 0.001

P value < α

It can be concluded that there is evidence to reject H_0 at $\alpha = 0.05$

Hence, there is significance difference b/w avg no of intrusion attempts per day before & after change of firewall setting. , i.e, there is significant reduction

(Considering 2% significance level)

9.20

$\sigma = 5, \quad S = 6.2, \quad n = 40$

$H_0 : \sigma = 5$

$H_A : \sigma \neq 5$

Level of significance = 0.02

Test statistic

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

$$= \frac{(40-1)(6.2)^2}{5^2}$$

$$= \frac{39 \times 38.44}{1499.16} = 59.96$$

$d.f = n-1 = 40-1 = 39.$

Twosided TT

~~P-values~~

Critical Values, $\chi^2_{0.01} = 72.1$

$\chi^2_{0.999} = 17.3$

As χ^2_{obs} lies b/w Critical values.

We accept H_0.

At 2% significance level, there is significant evidence that std deviation is 5 minutes

X - Anthony
y - Eric

9.23 $\bar{X} = 85$ $m = n = 6$

$\bar{Y} = 80$

$S_X = 12.76$

$S_Y = 3.22$

a) Test $H_0 : \mu_X = \mu_Y$

$H_A : \mu_X > \mu_Y$

Test stats :- $H_0 : \sigma_X = \sigma_Y$, $H_A : \sigma_X \neq \sigma_Y$ (To choose method of testing, we compare variances)

$$F_{obs} = \frac{S_X^2}{S_Y^2} = 15.65$$

P value $= 2\min(P\{F \geq F_{obs}\}, P\{F \leq F_{obs}\}) =$ between 0.002 & 0.01

There is significant evidence that $\sigma_X \neq \sigma_Y$, we use Satterwaite approximation.

Test statistic for testing,

$H_0 : \mu_X = \mu_Y$

$H_A : \mu_X > \mu_Y$.

$$t_{obs} = \frac{85 - 80}{\sqrt{\frac{(12.76)^2}{6} + \frac{(3.22)^2}{6}}} = 0.93$$

$$d.f = \frac{\left(\frac{S_x^2}{n} + \frac{S_y^2}{m}\right)^2}{\frac{S_x^4}{n^2(n-1)} + \frac{S_y^4}{m^2(m-1)}}$$

$$= \frac{\left(\frac{12.76^2}{6} + \frac{3.22^2}{6}\right)^2}{\frac{12.76^4}{180} + \frac{3.22^4}{180}} = 5.64.$$

$$P\text{value} = P\{t > t_{obs}\} > 0.10$$

There is no evidence that Anthony is a stronger student, i.e, his avg grade is higher than Eric

b) $H_0: \sigma_x = \sigma_y$

$H_A: \sigma_x > \sigma_y$

$$F_{obs} = 15.65$$

$$P\text{value} = P\{F \geq F_{obs}\} \in (0.01, 0.05)$$

There is a significant evidence that $\sigma_x > \sigma_y$ supporting Eric's claim that he is stable.

$H = -p \log_2 p$

9. First dice – unbiased

Second dice	X	1	2	3	4	5	6
	P(X)	1/9	1/9	1/9	2/9	2/9	2/9
Third dice	X	1	2	3	4	5	6
	P(X)	1/9	1/9	2/9	2/9	2/9	1/9

(i) Avg info (in bits) encoded by each dice

- Fair dice takes $-6 \cdot \frac{1}{6} \log_2 (1/6) = -(-2.584) \sim$ bits

$= 2.584$ bits

- Second dice :

$$H = -\frac{1}{9}\log_2 1/9 - 1/9 \log_2 1/9 - 1/9 \log_2 1/9 - 2/9 \log_2 2/9$$
$$- \frac{2}{9}\log_2 2/9 - \frac{2}{9}\log_2 2/9$$

$$= -\frac{3}{9}[\log_2 1/9] - 3 \times \frac{2}{9}[\log_2 (2/9)]$$

$$= -\frac{3}{9}[-3.1699] - \frac{6}{9}[-2.1699]$$

$$= 1.0566 + 1.4466$$

$$= 2.5032 \text{ bits}$$

- Third dice =

$$H = -\frac{1}{9}\log_2 1/9 - \frac{1}{9}\log_2 1/9 - \frac{2}{9}\log_2 (2/9) - 2/9 \log_2 (2/9)$$
$$- \frac{2}{9}\log_2 (2/9) - \frac{1}{9}\log_2 (1/9)$$

$D(p||q)$

$$= -\sum_x p(x)\log_b\left(p(x)/q(x)\right)$$

$$= -\frac{3}{9}\left[\log_2 1/9\right] - 3\times\frac{2}{9}\left[\log_2 2/9\right]$$

$$= 1.0566 + 1.4466 = 2.5032 \text{ bits.}$$

(ii) Relative Entropy:-

a) Second dice w.r.t first dice

$$= \Big[-\frac{1}{9}\log_2(1/9/1/6) - 1/9\log_2(1/9/1/6) - \frac{1}{9}\log(1/9/1/6)$$

$$- \frac{2}{9}\log_2(2/9/1/6) - \frac{2}{9}\log_2(2/9/1/6) - \frac{2}{9}\log_2(2/9/1/6)\Big]$$

$$= -\frac{3}{9}\log_2(6/9) - \frac{3\times2}{9}\log_2(12/9)$$

$$= -\frac{3}{9}[-0.5849] - \frac{6}{9}[0.4150]$$

$$= 0.1949 - 0.2766 = -0.0817$$

b) Third dice w.r.t First dice

$$= -1/9\log_2(1/9/1/6) - 1/9\log_2\left(\frac{6}{9}\right) - \frac{2}{9}\log_2(12/9)$$

$$- \frac{2}{9}\log_2(2/9) - \frac{2}{9}\log_2(2/9)$$

$$- \frac{1}{9}\log_2(6/9)$$

$$= -\frac{3}{9}\log_2(6/9) - \frac{6}{9}\log_2(12/9)$$

$$= 0.1949 - 0.2766 = -0.0817$$

c) Second dice w.r.t third dice

$$= -\frac{1}{9}\log_2\left(\frac{1/9}{1/9}\right) - \frac{1}{9}\log_2\left(\frac{1/9}{1/9}\right) - \frac{1}{9}\log_2\left(\frac{1/9}{2/9}\right)$$

$$- \frac{2}{9}\log_2\left(\frac{2/9}{2/9}\right) - \frac{2}{9}\log_2\left(\frac{2/9}{2/9}\right) - \frac{2}{9}\log_2\left(\frac{2/9}{1/9}\right)$$

$$= -\frac{1}{9}\log_2(1) - \frac{1}{9}\log_2(1) - \frac{1}{9}\log(1/2)$$

$$- \frac{2}{9}\log_2(1) - \frac{2}{9}\log_2(1) - \frac{2}{9}\log_2(2)$$

$$= 0 - 0 - \frac{1}{9}(-1) - 0 - 0 - \frac{2}{9}$$

$$= -\frac{2}{9} + \frac{1}{9} = -1/9 = -0.111$$