Method of Moments Estimation

Moments

DEFINITION 9.1

The k-th **population moment** is defined as

$$\mu_k = \mathbf{E}(X^k).$$

The k-th **sample moment**

$$m_k = \frac{1}{n}\sum_{i=1}^{n} X_i^k$$

estimates μ_k from a sample $(X_1, \ldots, X_n)$.

The first sample moment is the sample mean $\bar{X}$.

DEFINITION 9.2

For $k \geq 2$, the k-th **population central moment** is defined as

$$\mu_k' = \mathbf{E}(X - \mu_1)^k.$$

The k-th **sample central moment**

$$m_k' = \frac{1}{n}\sum_{i=1}^{n} (X_i - \bar{X})^k$$

estimates μ_k from a sample $(X_1, \ldots, X_n)$.

For the given distribution, find the required moments (or central moments) in terms of the parameters and equate them to the sample moments (or sample central moments) and solve for the parameters.

Method of Maximum Likelihood estimation

DEFINITION 9.3

Maximum likelihood estimator is the parameter value that maximizes the likelihood of the observed sample. For a discrete distribution, we maximize the joint pmf of data $P(X_1, \ldots, X_n)$. For a continuous distribution, we maximize the joint density $f(X_1, \ldots, X_n)$.

To find parameters θ that maximize likelihood

$$\frac{\partial}{\partial \theta} P(\boldsymbol{X}) = 0$$

It might be easier to find the parameter θ that maximized log-likelihood

$$\frac{\partial}{\partial \theta} \ln P(\boldsymbol{X}) = 0$$