these numbers in frequent situations involving permutations and combinations, with or without replacement.

Given occurrence of event B, one can compute conditional probability of event A. Unconditional probability of A can be computed from its conditional probabilities by the Law of Total Probability. The Bayes Rule, often used in testing and diagnostics, relates conditional probabilities of A given B and of B given A.

Exercises

2.1. Out of six computer chips, two are defective. If two chips are randomly chosen for testing (without replacement), compute the probability that both of them are defective. List all the outcomes in the sample space.

2.2. Suppose that after 10 years of service, 40% of computers have problems with motherboards (MB), 30% have problems with hard drives (HD), and 15% have problems with both MB and HD. What is the probability that a 10-year old computer still has fully functioning MB and HD?

2.3. A new computer virus can enter the system through e-mail or through the internet. There is a 30% chance of receiving this virus through e-mail. There is a 40% chance of receiving it through the internet. Also, the virus enters the system simultaneously through e-mail and the internet with probability 0.15. What is the probability that the virus does not enter the system at all?

2.4. Among employees of a certain firm, 70% know C/C++, 60% know Fortran, and 50% know both languages. What portion of programmers

(a) does not know Fortran?

(b) does not know Fortran and does not know C/C++?

(c) knows C/C++ but not Fortran?

(d) knows Fortran but not C/C++?

(e) If someone knows Fortran, what is the probability that he/she knows C/C++ too?

(f) If someone knows C/C++, what is the probability that he/she knows Fortran too?

2.5. A computer program is tested by 3 *independent* tests. When there is an error, these tests will discover it with probabilities 0.2, 0.3, and 0.5, respectively. Suppose that the program contains an error. What is the probability that it will be found by at least one test?

2.6. Under good weather conditions, 80% of flights arrive on time. During bad weather, only 30% of flights arrive on time. Tomorrow, the chance of good weather is 60%. What is the probability that your flight will arrive on time?

2.7. A system may become infected by some spyware through the internet or e-mail. Seventy percent of the time the spyware arrives via the internet, thirty percent of the time via e-mail. If it enters via the internet, the system detects it immediately with probability 0.6. If via e-mail, it is detected with probability 0.8. What percentage of times is this spyware detected?

2.8. A shuttle's launch depends on three key devices that may fail independently of each other with probabilities 0.01, 0.02, and 0.02, respectively. If any of the key devices fails, the launch will be postponed. Compute the probability for the shuttle to be launched on time, according to its schedule.

2.9. Successful implementation of a new system is based on three independent modules. Module 1 works properly with probability 0.96. For modules 2 and 3, these probabilities equal 0.95 and 0.90. Compute the probability that at least one of these three modules fails to work properly.

2.10. Three computer viruses arrived as an e-mail attachment. Virus A damages the system with probability 0.4. Independently of it, virus B damages the system with probability 0.5. Independently of A and B, virus C damages the system with probability 0.2. What is the probability that the system gets damaged?

2.11. A computer program is tested by 5 independent tests. If there is an error, these tests will discover it with probabilities 0.1, 0.2, 0.3, 0.4, and 0.5, respectively. Suppose that the program contains an error. What is the probability that it will be found

(a) by at least one test?

(b) by at least two tests?

(c) by all five tests?

2.12. A building is examined by policemen with four dogs that are trained to detect the scent of explosives. If there are explosives in a certain building, and each dog detects them with probability 0.6, independently of other dogs, what is the probability that the explosives will be detected by at least one dog?

2.13. An important module is tested by three independent teams of inspectors. Each team detects a problem in a defective module with probability 0.8. What is the probability that at least one team of inspectors detects a problem in a defective module?

2.14. A spyware is trying to break into a system by guessing its password. It does not give up until it tries 1 million different passwords. What is the probability that it will guess the password and break in if by rules, the password must consist of

(a) 6 different lower-case letters

(b) 6 different letters, some may be upper-case, and it is case-sensitive

(c) any 6 letters, upper- or lower-case, and it is case-sensitive

(d) any 6 characters including letters and digits

2.15. A computer program consists of two blocks written independently by two different programmers. The first block has an error with probability 0.2. The second block has an error with probability 0.3. If the program returns an error, what is the probability that there is an error in both blocks?

2.16. A computer maker receives parts from three suppliers, S1, S2, and S3. Fifty percent come from S1, twenty percent from S2, and thirty percent from S3. Among all the parts supplied by S1, 5% are defective. For S2 and S3, the portion of defective parts is 3% and 6%, respectively.

(a) What portion of all the parts is defective?

(b) A customer complains that a certain part in her recently purchased computer is defective. What is the probability that it was supplied by S1?

2.17. A computer assembling company receives 24% of parts from supplier X, 36% of parts from supplier Y, and the remaining 40% of parts from supplier Z. Five percent of parts supplied by X, ten percent of parts supplied by Y, and six percent of parts supplied by Z are defective. If an assembled computer has a defective part in it, what is the probability that this part was received from supplier Z?

2.18. A problem on a multiple-choice quiz is answered correctly with probability 0.9 if a student is prepared. An unprepared student guesses between 4 possible answers, so the probability of choosing the right answer is 1/4. Seventy-five percent of students prepare for the quiz. If Mr. X gives a correct answer to this problem, what is the chance that he did not prepare for the quiz?

2.19. At a plant, 20% of all the produced parts are subject to a special electronic inspection. It is known that any produced part which was inspected electronically has no defects with probability 0.95. For a part that was not inspected electronically this probability is only 0.7. A customer receives a part and find defects in it. What is the probability that this part went through an electronic inspection?

2.20. All athletes at the Olympic games are tested for performance-enhancing steroid drug use. The imperfect test gives positive results (indicating drug use) for 90% of all steroid-users but also (and incorrectly) for 2% of those who do not use steroids. Suppose that 5% of all registered athletes use steroids. If an athlete is tested negative, what is the probability that he/she uses steroids?

2.21. In the system in Figure 2.7, each component fails with probability 0.3 independently of other components. Compute the system's reliability.

2.22. Three highways connect city A with city B. Two highways connect city B with city C. During a rush hour, each highway is blocked by a traffic accident with probability 0.2, independently of other highways.

(a) Compute the probability that there is at least one open route from A to C.

(b) How will a new highway, also blocked with probability 0.2 independently of other highways, change the probability in (a) if it is built

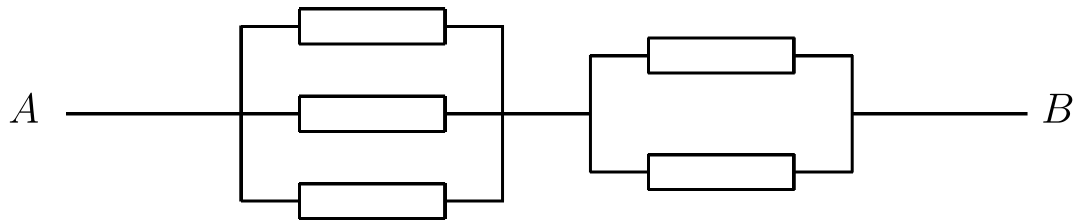

FIGURE 2.7: *Calculate reliability of this system (Exercise 2.21).*

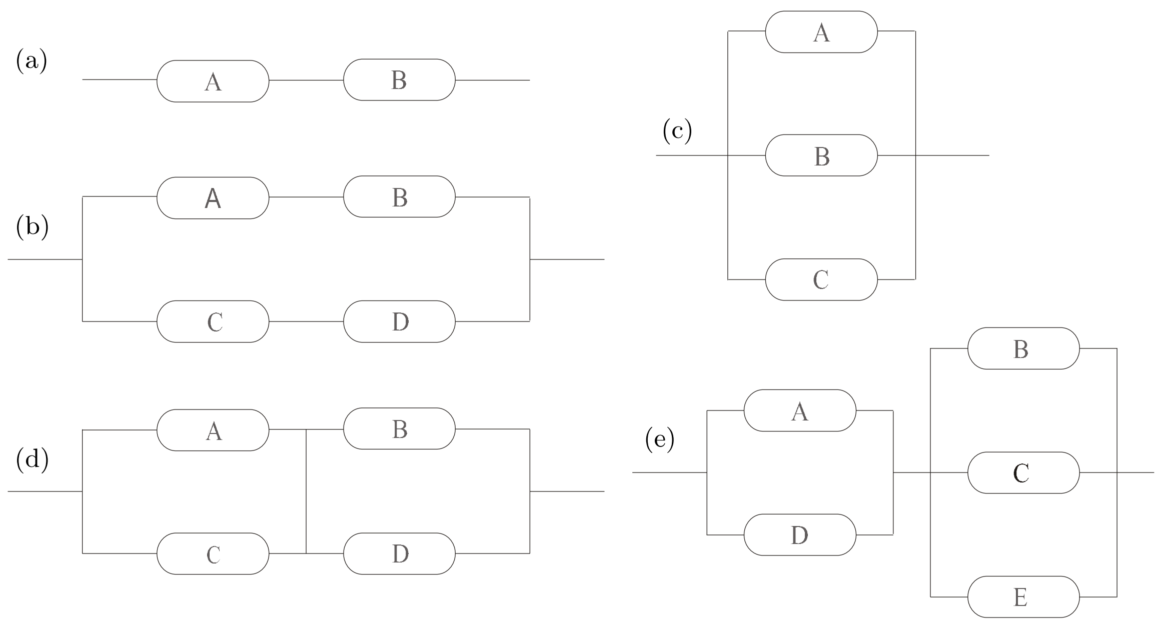

FIGURE 2.8: *Calculate reliability of each system (Exercise 2.23).*

(α) between A and B?

(β) between B and C?

(γ) between A and C?

2.23. Calculate the reliability of each system shown in Figure 2.8, if components A, B, C, D, and E function properly with probabilities 0.9, 0.8, 0.7, 0.6, and 0.5, respectively.

2.24. Among 10 laptop computers, five are good and five have defects. Unaware of this, a customer buys 6 laptops.

(a) What is the probability of exactly 2 defective laptops among them?

(b) Given that *at least* 2 purchased laptops are defective, what is the probability that *exactly* 2 are defective?

2.25. This is known as *the Birthday Problem*.

(a) Consider a class with 30 students. Compute the probability that at least two of them have their birthdays on the same day. (For simplicity, ignore the leap year).

(b) How many students should be in class in order to have this probability above 0.5?

2.26. Two out of six computers in a lab have problems with hard drives. If three computers are selected at random for inspection, what is the probability that none of them has hard drive problems?

2.27. Among eighteen computers in some store, six have defects. Five randomly selected computers are bought for the university lab. Compute the probability that all five computers have no defects.

2.28. A quiz consists of 6 multiple-choice questions. Each question has 4 possible answers. A student is unprepared, and he has no choice but guessing answers completely at random. He passes the quiz if he gets at least 3 questions correctly. What is the probability that he will pass?

2.29. An internet search engine looks for a keyword in 9 databases, searching them in a random order. Only 5 of these databases contain the given keyword. Find the probability that it will be found in at least 2 of the first 4 searched databases.

2.30. Consider the situation described in Example 2.24 on p. 22, but this time let us define the sample space clearly. Suppose that one child is older, and the other is younger, their gender is independent of their age, and the child you meet is one or the other with probabilities 1/2 and 1/2.

(a) List all the outcomes in this sample space. Each outcome should tell the children's gender, which child is older, and which child you have met.

(b) Show that *unconditional* probabilities of outcomes BB, BG, and GB are equal.

(c) Show that *conditional* probabilities of BB, BG, and GB, after you met Leo, are not equal.

(d) Show that the *conditional* probability that Leo has a brother is 1/2.

2.31. Show that events $A, B, C, \ldots$ are disjoint if and only if $\overline{A}, \overline{B}, \overline{C}, \ldots$ are exhaustive.

2.32. Events A and B are independent. Show, intuitively and mathematically, that:

(a) Their complements are also independent.

(b) If they are disjoint, then $\boldsymbol{P}\{A\} = 0$ or $\boldsymbol{P}\{B\} = 0$.

(c) If they are exhaustive, then $\boldsymbol{P}\{A\} = 1$ or $\boldsymbol{P}\{B\} = 1$.

2.33. Derive a computational formula for the probability of a union of N arbitrary events. Assume that probabilities of all individual events and their intersections are given.

2.34. Prove that

$$\overline{E_1 \cap \ldots \cap E_n} = \overline{E}_1 \cup \ldots \cup \overline{E}_n$$

for arbitrary events $E_1, \ldots, E_n$.

2.35. From the "addition" rule of probability, derive the "subtraction" rule:

$$\text{if } B \subset A, \text{ then } \boldsymbol{P}\{A \backslash B\} = \boldsymbol{P}(A) - \boldsymbol{P}(B).$$

2.36. Prove "subadditivity": $\boldsymbol{P}\{E_1 \cup E_2 \cup \ldots\} \leq \sum \boldsymbol{P}\{E_i\}$ for *any* events $E_1, E_2, \ldots \in \mathfrak{M}$.