Exercises

3.1. A computer virus is trying to corrupt two files. The first file will be corrupted with probability 0.4. Independently of it, the second file will be corrupted with probability 0.3.

(a) Compute the probability mass function (pmf) of X, the number of corrupted files.

(b) Draw a graph of its cumulative distribution function (cdf).

3.2. Every day, the number of network blackouts has a distribution (probability mass function)

x	0	1	2
$P(x)$	0.7	0.2	0.1

A small internet trading company estimates that each network blackout results in a \$500 loss. Compute expectation and variance of this company's daily loss due to blackouts.

3.3. There is one error in one of five blocks of a program. To find the error, we test three randomly selected blocks. Let X be the number of errors in these three blocks. Compute $\mathbf{E}(X)$ and $\mathrm{Var}(X)$.

3.4. Tossing a fair die is an experiment that can result in any integer number from 1 to 6 with equal probabilities. Let X be the number of dots on the top face of a die. Compute $\mathbf{E}(X)$ and $\mathrm{Var}(X)$.

3.5. A software package consists of 12 programs, five of which must be upgraded. If 4 programs are randomly chosen for testing,

(a) What is the probability that at least two of them must be upgraded?

(b) What is the expected number of programs, out of the chosen four, that must be upgraded?

3.6. A computer program contains one error. In order to find the error, we split the program into 6 blocks and test two of them, selected at random. Let X be the number of errors in these blocks. Compute $\mathbf{E}(X)$.

3.7. The number of home runs scored by a certain team in one baseball game is a random variable with the distribution

x	0	1	2
$P(x)$	0.4	0.4	0.2

The team plays 2 games. The number of home runs scored in one game is independent of the number of home runs in the other game. Let Y be the *total* number of home runs. Find $\mathbf{E}(Y)$ and $\mathrm{Var}(Y)$.

3.8. A computer user tries to recall her password. She knows it can be one of 4 possible passwords. She tries her passwords until she finds the right one. Let X be the number of wrong passwords she uses before she finds the right one. Find $\mathbf{E}(X)$ and $\mathrm{Var}(X)$.

3.9. It takes an average of 40 seconds to download a certain file, with a standard deviation of 5 seconds. The actual distribution of the download time is unknown. Using Chebyshev's inequality, what can be said about the probability of spending more than 1 minute for this download?

3.10. Every day, the number of traffic accidents has the probability mass function

x	0	1	2	more than 2
$P(x)$	0.6	0.2	0.2	0

independently of other days. What is the probability that there are more accidents on Friday than on Thursday?

3.11. Two dice are tossed. Let X be *the smaller* number of points. Let Y be *the larger* number of points. If both dice show the same number, say, z points, then $X = Y = z$.

(a) Find the joint probability mass function of (X, Y).

(b) Are X and Y independent? Explain.

(c) Find the probability mass function of X.

(d) If $X = 2$, what is the probability that $Y = 5$?

3.12. Two random variables, X and Y, have the joint distribution $P(x, y)$,

$P(x,y)$		x	
		0	1
y	0	0.5	0.2
	1	0.2	0.1

(a) Are X and Y independent? Explain.

(b) Are $(X + Y)$ and $(X - Y)$ independent? Explain.

3.13. Two random variables X and Y have the joint distribution, $P(0, 0) = 0.2$, $P(0, 2) = 0.3$, $P(1, 1) = 0.1$, $P(2, 0) = 0.3$, $P(2, 2) = 0.1$, and $P(x, y) = 0$ for all other pairs (x, y).

(a) Find the probability mass function of $Z = X + Y$.

(b) Find the probability mass function of $U = X - Y$.

(c) Find the probability mass function of $V = XY$.

3.14. An internet service provider charges its customers for the time of the internet use rounding it up to the nearest hour. The joint distribution of the used time (X, hours) and the charge per hour (Y, cents) is given in the table below.

$P(x,y)$		$x=1$	$x=2$	$x=3$	$x=4$
y	1	0	0.06	0.06	0.10
	2	0.10	0.10	0.04	0.04
	3	0.40	0.10	0	0

Each customer is charged $Z = X \cdot Y$ cents, which is the number of hours multiplied by the price of each hour. Find the distribution of Z.

3.15. Let X and Y be the number of hardware failures in two computer labs in a given month. The joint distribution of X and Y is given in the table below.

$P(x,y)$		$x=0$	$x=1$	$x=2$
y	0	0.52	0.20	0.04
	1	0.14	0.02	0.01
	2	0.06	0.01	0

(a) Compute the probability of at least one hardware failure.

(b) From the given distribution, are X and Y independent? Why or why not?

3.16. The number of hardware failures, X, and the number of software failures, Y, on any day in a small computer lab have the joint distribution $P(x, y)$, where $P(0, 0) = 0.6$, $P(0, 1) = 0.1$, $P(1, 0) = 0.1$, $P(1, 1) = 0.2$. Based on this information,

(a) Are X and Y (hardware and software failures) independent?

(b) Compute $\mathbf{E}(X + Y)$, i.e., the expected total number of failures during 1 day.

3.17. Shares of company A are sold at \$10 per share. Shares of company B are sold at \$50 per share. According to a market analyst, 1 share of each company can either gain \$1, with probability 0.5, or lose \$1, with probability 0.5, independently of the other company. Which of the following portfolios has the lowest risk:

(a) 100 shares of A

(b) 50 shares of A + 10 shares of B

(c) 40 shares of A + 12 shares of B

3.18. Shares of company A cost \$10 per share and give a profit of X%. Independently of A, shares of company B cost \$50 per share and give a profit of Y%. Deciding how to invest \$1,000, Mr. X chooses between 3 portfolios:

(a) 100 shares of A,

(b) 50 shares of A and 10 shares of B,

(c) 20 shares of B.

The distribution of X is given by probabilities:

$$P\{X=-3\}=0.3, P\{X=0\}=0.2, P\{X=3\}=0.5.$$

The distribution of Y is given by probabilities:

$$P\{Y=-3\}=0.4, P\{Y=3\}=0.6.$$

Compute expectations and variances of the total dollar profit generated by portfolios (a), (b), and (c). What is the least risky portfolio? What is the most risky portfolio?

3.19. A and B are two competing companies. An investor decides whether to buy

(a) 100 shares of A, or

(b) 100 shares of B, or

(c) 50 shares of A and 50 shares of B.

A profit made on 1 share of A is a random variable X with the distribution $P(X=2)=P(X=-2)=0.5$.

A profit made on 1 share of B is a random variable Y with the distribution $P(Y=4)=0.2, P(Y=-1)=0.8$.

If X and Y are independent, compute the expected value and variance of the total profit for strategies (a), (b), and (c).

3.20. A quality control engineer tests the quality of produced computers. Suppose that 5% of computers have defects, and defects occur independently of each other.

(a) Find the probability of exactly 3 defective computers in a shipment of twenty.

(b) Find the probability that the engineer has to test at least 5 computers in order to find 2 defective ones.

3.21. A lab network consisting of 20 computers was attacked by a computer virus. This virus enters each computer with probability 0.4, independently of other computers. Find the probability that it entered at least 10 computers.

3.22. Five percent of computer parts produced by a certain supplier are defective. What is the probability that a sample of 16 parts contains more than 3 defective ones?

3.23. Every day, a lecture may be canceled due to inclement weather with probability 0.05. Class cancelations on different days are independent.

(a) There are 15 classes left this semester. Compute the probability that at least 4 of them get canceled.

(b) Compute the probability that the tenth class this semester is the third class that gets canceled.

3.24. An internet search engine looks for a certain keyword in a sequence of independent web sites. It is believed that 20% of the sites contain this keyword.

(a) Compute the probability that at least 5 of the first 10 sites contain the given keyword.

(b) Compute the probability that the search engine had to visit at least 5 sites in order to find the first occurrence of a keyword.

3.25. About ten percent of users do not close Windows properly. Suppose that Windows is installed in a public library that is used by random people in a random order.

(a) On the average, how many users of this computer *do not* close Windows properly before someone *does* close it properly?

(b) What is the probability that exactly 8 of the next 10 users will close Windows properly?

3.26. After a computer virus entered the system, a computer manager checks the condition of all important files. She knows that each file has probability 0.2 to be damaged by the virus, independently of other files.

(a) Compute the probability that at least 5 of the first 20 files are damaged.

(b) Compute the probability that the manager has to check at least 6 files in order to find 3 undamaged files.

3.27. Messages arrive at an electronic message center at random times, with an average of 9 messages per hour.

(a) What is the probability of receiving *at least* five messages during the next hour?

(b) What is the probability of receiving *exactly* five messages during the next hour?

3.28. The number of received electronic messages has Poisson distribution with some parameter λ. Using Chebyshev inequality, show that the probability of receiving more than 4λ messages does not exceed $1/(9\lambda)$.

3.29. An insurance company divides its customers into 2 groups. Twenty percent of customers are in the high-risk group, and eighty percent are in the low-risk group. The high-risk customers make an average of 1 accident per year while the low-risk customers make an average of 0.1 accidents per year. Eric had no accidents last year. What is the probability that he is a high-risk driver?

3.30. Eric from Exercise 3.29 continues driving. After three years, he still has no traffic accidents. Now, what is the conditional probability that he is a high-risk driver?

3.31. Before the computer is assembled, its vital component (motherboard) goes through a special inspection. Only 80% of components pass this inspection.

(a) What is the probability that at least 18 of the next 20 components pass inspection?

(b) On the average, how many components should be inspected until a component that passes inspection is found?

3.32. On the average, 1 computer in 800 crashes during a severe thunderstorm. A certain company had 4,000 working computers when the area was hit by a severe thunderstorm.

(a) Compute the probability that less than 10 computers crashed.

(b) Compute the probability that exactly 10 computers crashed.

You may want to use a suitable approximation.

3.33. The number of computer shutdowns during any month has a Poisson distribution, averaging 0.25 shutdowns per month.

(a) What is the probability of at least 3 computer shutdowns during the next year?

(b) During the next year, what is the probability of at least 3 months (out of 12) with exactly 1 computer shutdown in each?

3.34. A dangerous computer virus attacks a folder consisting of 250 files. Files are affected by the virus independently of one another. Each file is affected with the probability 0.032. What is the probability that more than 7 files are affected by this virus?

3.35. In some city, the probability of a thunderstorm on any day is 0.6. During a thunderstorm, the number of traffic accidents has Poisson distribution with parameter 10. Otherwise, the number of traffic accidents has Poisson distribution with parameter 4. If there were 7 accidents yesterday, what is the probability that there was a thunderstorm?

3.36. An interactive system consists of ten terminals that are connected to the central computer. At any time, each terminal is ready to transmit a message with probability 0.7, independently of other terminals. Find the probability that exactly 6 terminals are ready to transmit at 8 o'clock.

3.37. Network breakdowns are unexpected rare events that occur every 3 weeks, on the average. Compute the probability of more than 4 breakdowns during a 21-week period.

3.38. Simplifying expressions, derive from the definitions of variance and covariance that

(a) $\mathrm{Var}(X) = \mathbf{E}(X^2) - \mathbf{E}^2(X)$;

(b) $\mathrm{Cov}(X,Y) = \mathbf{E}(XY) - \mathbf{E}(X)\,\mathbf{E}(Y)$.

3.39. Show that

$$\begin{aligned}&\mathrm{Cov}(aX + bY + c, dZ + eW + f)\\ &\quad = \; ad\,\mathrm{Cov}(X,Z) + ae\,\mathrm{Cov}(X,W) + bd\,\mathrm{Cov}(Y,Z) + be\,\mathrm{Cov}(Y,W)\end{aligned}$$

for any random variables X, Y, Z, W, and any non-random numbers a, b, c, d, e, f.