Deep Learning Seminar: Weekly report

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The document contains some functions which seem to be suitable for our task of analyzing behavior of different optimization algorithms. We can change the parameters of the functions to change the curvature or generate more functions by concatenating reflections.

If these functions are sufficient, please suggest about the task for this week. In not, I will try to collect more functions.

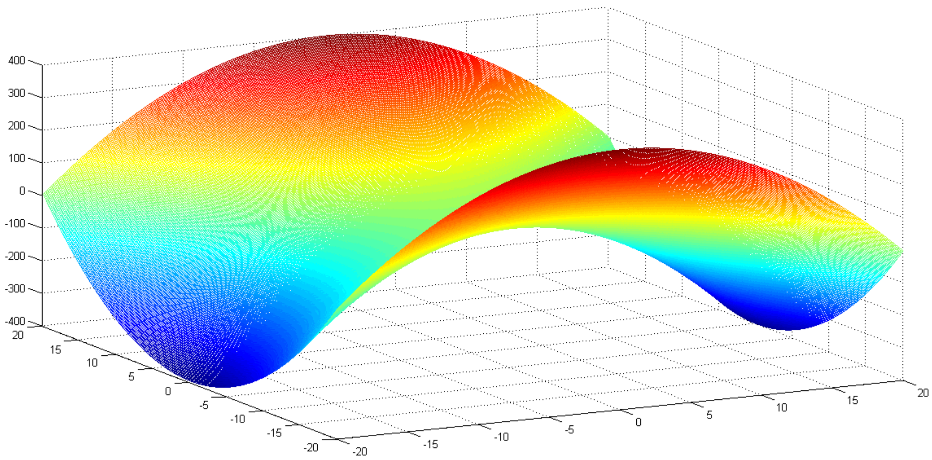

Figure 1: Function: $x^2 - y^2$

Function: $f(x, y) = x^2 - y^2$

Critical Point $= (0, 0)$

Eigenvalues of Hessian at $(0, 0)$ are $2, -2 \implies$ It is a saddle point

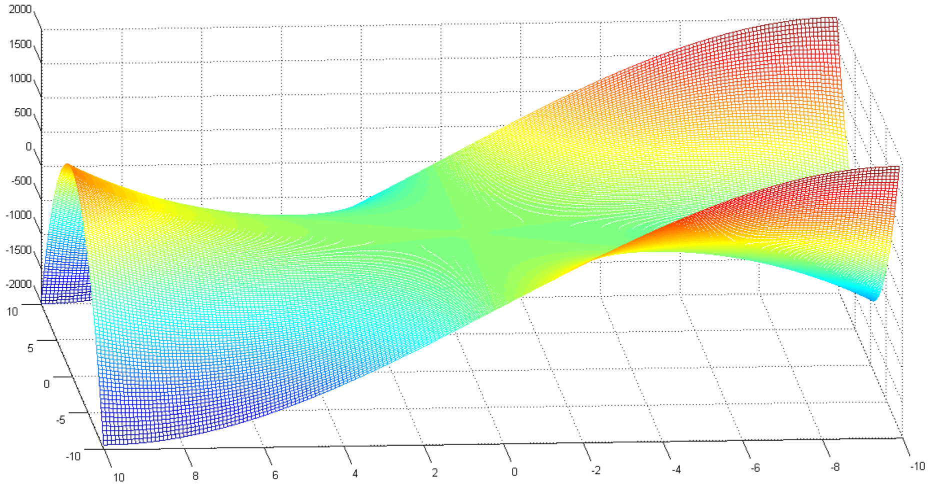

Figure 2: Function: $x^3 - 3xy^2$

Function: $f(x, y) = x^3 - 3xy^2$ (Known as *monkey saddle structure*)

Critical Point $= (0, 0)$

Eigenvalues of Hessian at $(0, 0)$ are $0, 0 \implies$ It is a saddle point.

Function value is 0 for any point $(0, y)$ on the curve. $\implies$ Curvature is zero along that direction. For the direction along $(x, 0)$, the curvature decreases with increase in x, tends to zero as x tends to 0 and then increases with increase in x. (Substituting $y = 0$ in the equation results in $f(x, 0) = x^3$ which has a saddle point at $(0, 0)$)

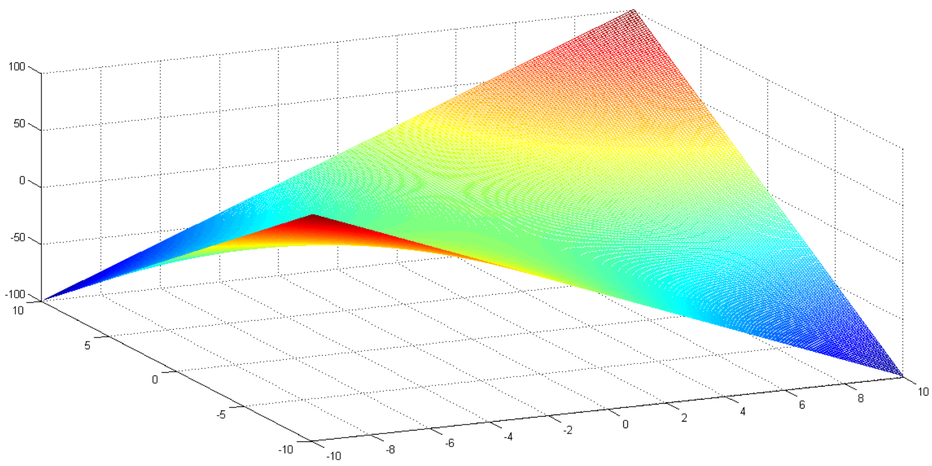

Figure 3: Function: xy

Function: $f(x, y) = xy$

Critical Point $= (0, 0)$

Eigenvalues of Hessian at $(0, 0)$ are $1, -1 \implies$ It is a saddle point.

(x, x) is the direction corresponding to the eigenvalue 1. $\implies$ Function has a positive curvature along this direction. Therefore, the function value increases in this direction, as we move away from $(0, 0)$.

$(x, -x)$ is the direction corresponding to the eigenvalue -1. $\implies$ Function has a negative curvature along this direction. Therefore, the function value decreases in this direction, as we move away from $(0, 0)$.

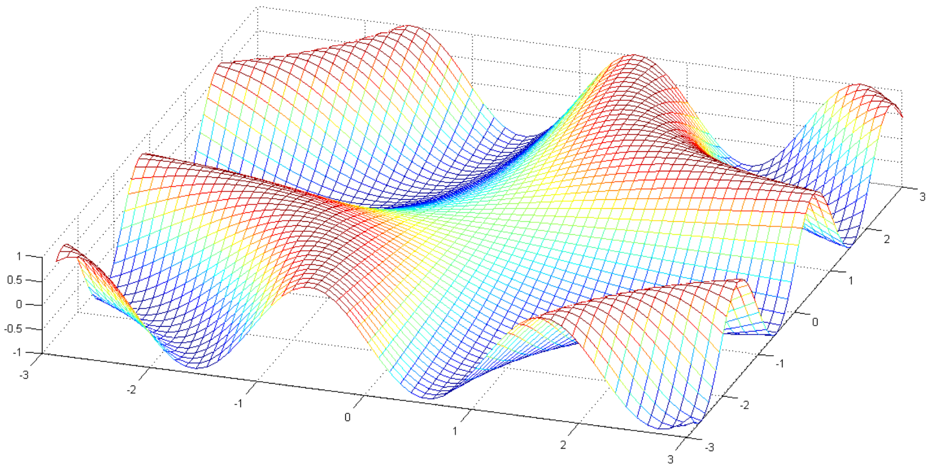

Figure 4: Function: $sin(xy)$

Function: $f(x, y) = sin(xy)$

$$\frac{\partial f}{\partial x} = y * cos(xy)$$
$$\frac{\partial f}{\partial y} = x * cos(xy)$$

There are multiple critical points, $(0, 0)$ is one among them.

Eigenvalues of the Hessian matrix at $(0, 0)$ are 1,-1 $\implies$ It is a saddle point.

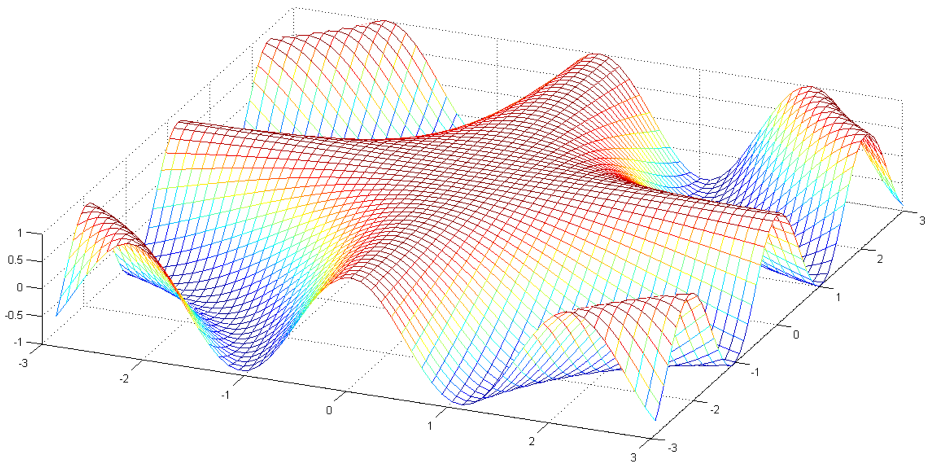

Figure 5: Function: $cos(xy)$

Function: $f(x,y) = cos(xy)$

$$\frac{\partial f}{\partial x} = \text{-}y * sin(xy)$$
$$\frac{\partial f}{\partial y} = \text{-}x * sin(xy)$$

There are multiple critical points, $(0,0)$ is one among them.

Eigenvalues of the Hessian matrix at $(0,0)$ are 0,0 $\implies$ It is a saddle point.

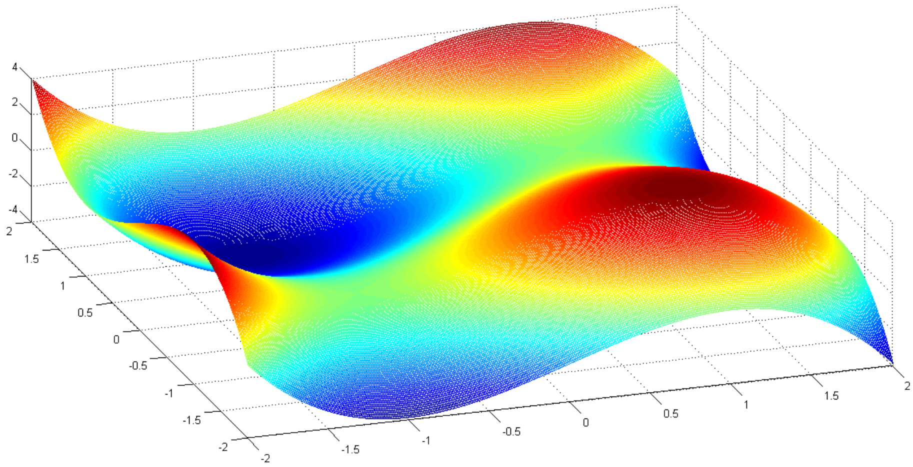

Figure 6: Function: $x^3 - y^3 - 3x + 3y$

Function: $f(x, y) = x^3 - y^3 - 3x + 3y$

Critical points: $(1, 1)$,$(1, -1)$,$(-1, 1)$ and $(-1, -1)$

$(1, 1)$ and $(-1, -1)$ are saddle points,$(-1, 1)$ is a local maximum and $(1, -1)$ is a local minimum.

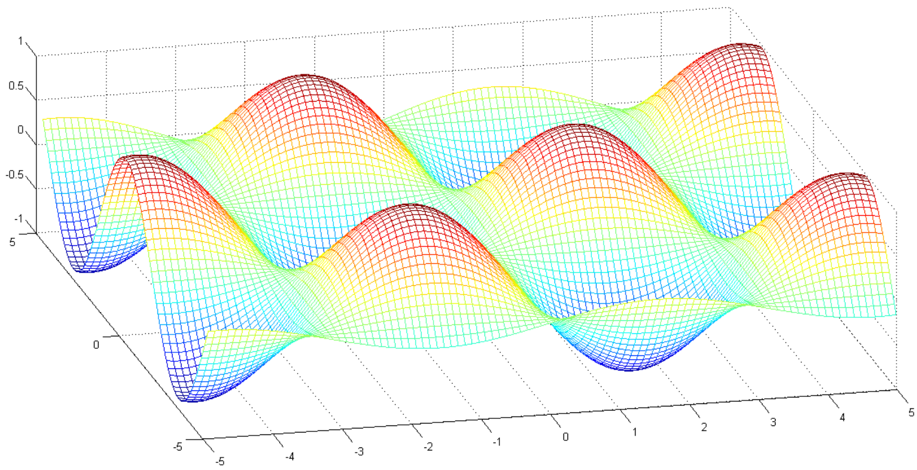

Figure 7: Function: $cos(x) * sin(y)$

Function: $f(x, y) = cos(x) * sin(y)$

$$\frac{\partial f}{\partial x} = \text{-}sin(x) * sin(y)$$

$$\frac{\partial f}{\partial y} = \text{-}cos(x) * cos(y)$$

There are multiple critical points, $(0, \pi/2)$ and $(\pi/2, 0)$ are two among them.

The eigenvalues of the Hessian at $(0, \pi/2)$ are -1,-1. $\implies$ It is a local maximum.
The eigenvalues of the Hessian at $(\pi/2, 0)$ are 1 and -1. $\implies$ It is a saddle point.

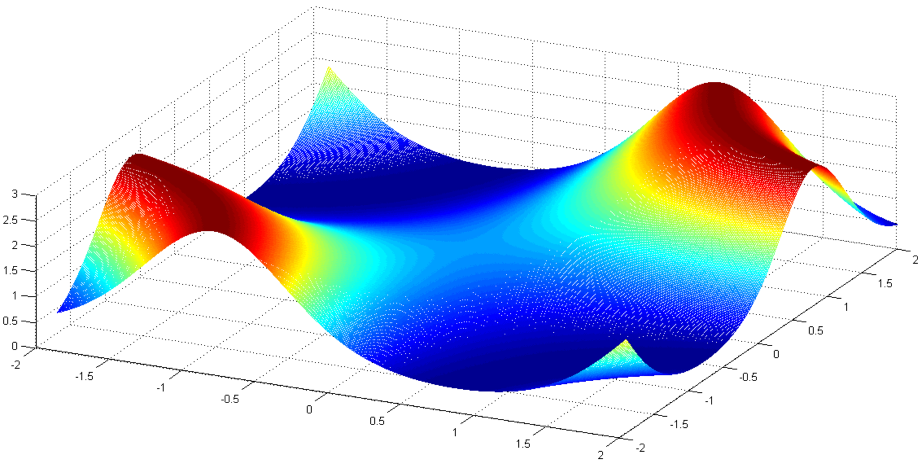

Figure 8: Function: $e^{sin(xy)}$

Function: $f(x, y) = e^{sin(xy)}$

$$\frac{\partial f}{\partial x} = e^{sin(xy)} * cos(xy) * y$$

$$\frac{\partial f}{\partial y} = e^{sin(xy)} * cos(xy) * x$$

There are multiple critical points, $(0, 0)$ is one among them.

Determinant of Hessian at $(0, 0)$ is negative, i.e. sum of eigenvalues is negative. $\implies$ It is a saddle point.

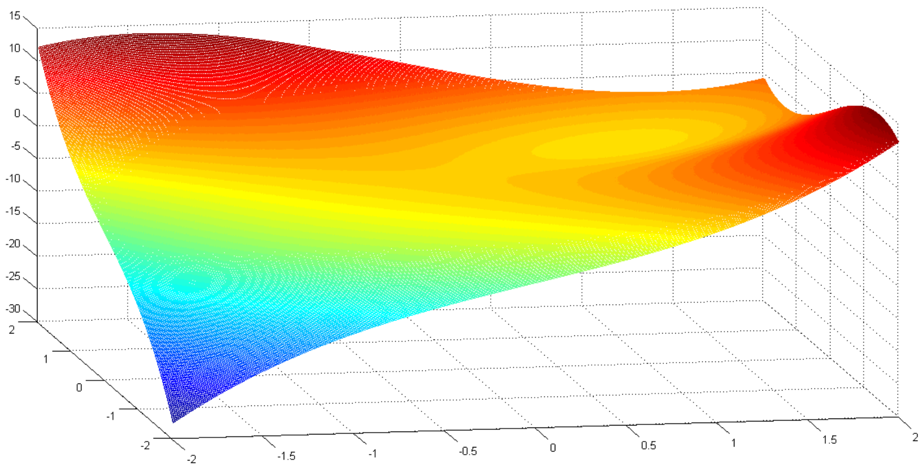

Figure 9: Function: $x^3 + y^3 - 3xy$

Function: $f(x, y) = x^3 + y^3 - 3xy$

Critical points: $(0, 0)$ and $(1, 1)$

Determinant of Hessian at $(0, 0)$ is negative, i.e. sum of eigenvalues is negative. $\implies$ It is a saddle point. $(1, 1)$ is a local minimum.

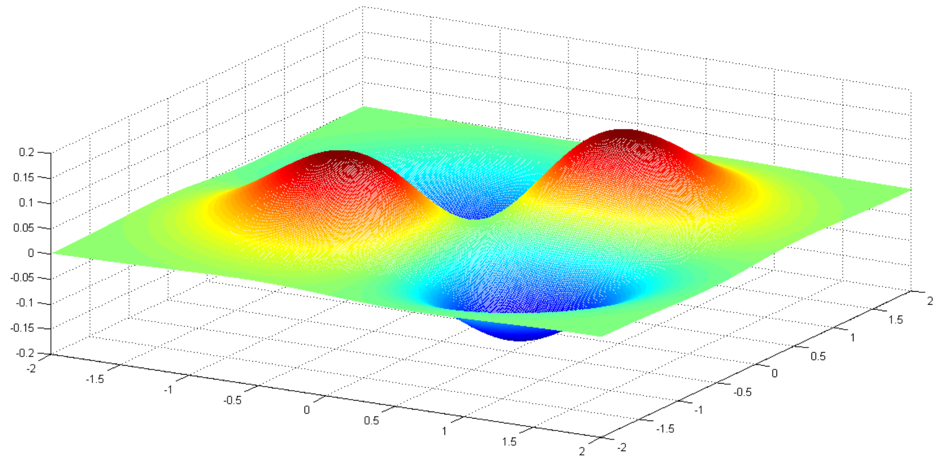

Figure 10: Function: $e^{-(x^2+y^2)} * sin(x) * sin(y)$

Function: $e^{-(x^2+y^2)} * sin(x) * sin(y)$

There are multiple critical points,$(0,0)$ is one among them. There are four more critical points, which can be seen from the figure. To compute them, we have to solve the equations $tan(x) = \frac{1}{2x}$ and $tan(y) = \frac{1}{2y}$ analytically.

$(0,0)$ is a saddle point.(inferred from the figure)

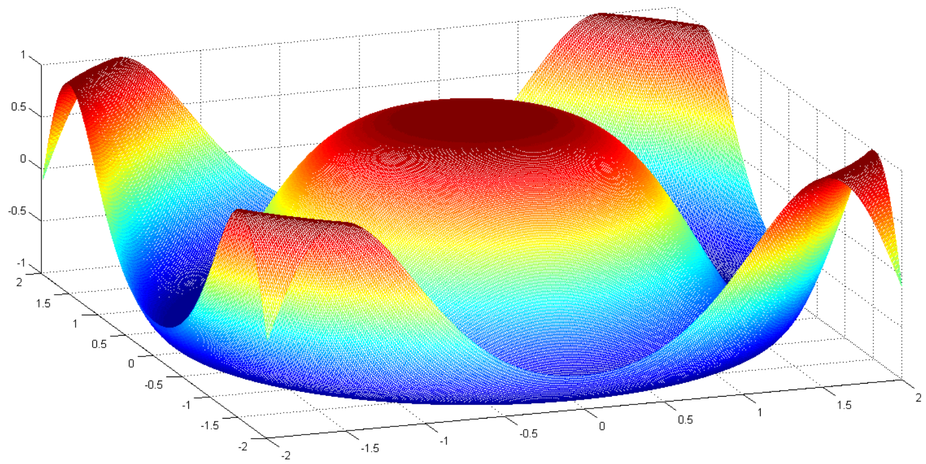

Figure 11: Function: $cos(x^2 + y^2)$

Function: $cos(x^2 + y^2)$

$$\frac{\partial f}{\partial x} = -2x * sin(x^2 + y^2)$$
$$\frac{\partial f}{\partial y} = -2y * sin(x^2 + y^2)$$

There are multiple critical points,$(0, 0)$ is one among them. From the equations, we can infer that any point satisfying $x^2 + y^2 = \pi$ is also a critical point. (Gutter like structure)

Eigenvalues of the Hessian at $(0, 0)$ are 0 and 0. $\implies$ It is a saddle point.

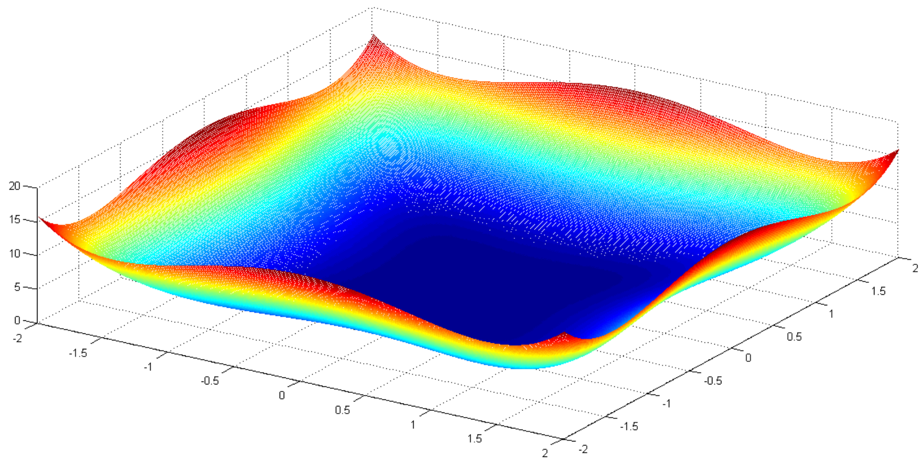

Figure 12: Function: $x^4 + y^4 - x^2y^2$

Function: $x^4 + y^4 - x^2y^2$

Critical Point: $(0, 0)$

Eigenvalues of Hessian at $(0, 0)$ are 0,0. $\implies$ It is a saddle point.