1) a) $P(M) = 0.05$

$P(S) = 0.95$

$P(\text{low temp} \mid \text{Maine}) = 1 - 0.2 = 0.8$

$P(\text{low temp} \mid \text{sahara}) = 1 - 0.9 = 0.1$

$$P(\text{Maine} \mid \text{low temp}) = \frac{P(M)\,P(\text{low temp} \mid \text{Maine})}{P(M)\,P(\text{low temp} \mid \text{Maine}) + P(S)\,P(\text{low temp} \mid \text{sahara})}$$

$$= \frac{0.05\,(0.8)}{0.05(0.8) + (0.95)(0.1)} = \underline{0.2963}$$

b) $P(\text{sahara} \mid \text{low temp}) = 1 - 0.2963 = \underline{0.7037}$

$P(\text{Temp}) = P(\text{Maine} \mid \text{low temp}) \times P(\text{low temp} \mid \text{Maine}) +$
$P(\text{sahara} \mid \text{low temp}) \times P(\text{low temp} \mid \text{sahara})$

$= (0.2963 \times 0.8) + (0.7037 \times 0.1)$

$= \underline{0.3074}$

c) ~~P(E)~~ $P(E1, E2, E3) = P(E3 \mid E2, E1) \times P(E2 \times E1)\,P(E3, E2, E1)$

$= P(E3 \mid E2, E1) \times P(E2 \mid E1) \times P(E1)\,P(E3, E2, E1)$

$= P(E3 \mid E1) \times P(E2 \mid E1) \times P(E1)$

$$P(E3 \mid E1) = \frac{P(E1 \mid E3) \times P(E3)}{P(E1)} = 0.639$$

$P(E1) = P(E1 \mid M) \times P(M) + P(E1 \mid \neg M) \times P(\neg M) = 0.135$

$\therefore P(E3, E2, E1) = 0.639 \times 0.135 \times 0.307$

$= \underline{0.2648}$.

(2) 11 Variables.
A has 5 values
B has 7 values

(a) ~~Nos~~ needed to store

$= \cancel{5\times7^{10}} = 5\times7^{10}$

(b) B_j depend on A
B_i is independent

$\Rightarrow$ $7\times10 = 70$ for each B_i
$5\times1 = 5$ for each A.

$\Rightarrow 70\times5 = \underline{350}$

$\therefore$ 350 numbers are stored.

(3) (a) $P(A|B) = \alpha P(A,B)$

$= \alpha[P(A,B,C) + P(A,B,\neg C)]$

$= \alpha[\langle 0.048, 0.012\rangle + \langle 0.196, 0.294\rangle]$

$= \alpha\langle 0.244, 0.306\rangle$

$= \langle 0.44, 0.56\rangle$

(b) $P(A|B,C) = P(A|C) = \alpha P(A,C)$

$= \alpha P[P(A,C,B) + P(A,C,\neg B)]$

$= \alpha[\langle 0.048, 0.012\rangle + \langle 0.192, 0.048\rangle]$

$= \alpha[0.24, 0.06]$

$= \langle 0.8, 0.2\rangle$

(c) $P(A,C|B) = P(A|B) \cdot P(C|B)$

$A(A|B) = \langle 0.44, 0.56 \rangle$

$P(C|B) = \alpha P(C,B) = \alpha [P(C,B,A) + P(C,B,\neg A)]$

$= \alpha [\langle 0.048, 0.196 \rangle + \langle 0.012, 0.2014 \rangle]$

$= \alpha [\langle 0.06, 0.497 \rangle]$

$= \langle 0.109, 0.891 \rangle$

(d) A is conditionally independent of C, given B.

$\because P(A|C,B) = P(A|B)$ (conditional independence property).