

List of topics

- About feasibility algorithm in its current form: bound and proof for single arm, bound and proof for multiple arms, patterns observed in multiple arm.
- About modified algorithm by building the sequence from the back to combat earlier drawbacks.
- Perfect algorithm for single arm, potential expansion into multiple arms.

Current feasibility algorithm

Single arm:

- Explain intuition
- Lower bound = $1 - \lfloor (T-1)/2 \rfloor / T$
- Asymptotically towards approximation ratio of 50%.
- Is initial assumption correct? Can't think of any worse scenarios.

Current feasibility algorithm

Multiple arms:

- Explain intuition: Multiple identical arms with sharp spikes in reward
- Algorithm fails to see better alternatives. Alternative algorithm can be a solution.
- Lower bound = $1/k$
- If initial assumptions for worst case scenario are correct, then this is a tight bound. Unable to think of any worse scenarios.
- The approximation ratio will never reach so low for normal bitonic functions.

Current feasibility algorithm

Single arm:

- For prior mentioned special functions which peak at single point.
- Let there be k functions and let them all peak at $\tau=T$.
- Clear patterns emerge. Can't say how relevant they are.
- Approximation ratio = 100% till $t=T+1$.
- Periodic minimas in graph with a time period of T .
- First minima is seen at $t=T+k$. Value = $1/k$.
- From $t=T+1$ to $T=T+k$, value decays predictably. For $t=T+i$, value= $100/i$.
- Number of consecutive minimas also follows a pattern.
- For large T , value seems to stabilize.

Modified Algorithm

- Employ similar feasibility concept, but start building sequences from the back.
- More high feasibility entries can be covered using this strategy.
- Solves problem of all maxima occurring at the same value of τ . (Clashing entries).
- Previous entries are not blindly allocated like before.
- Need to work out finer details, will do so over the next 2 days.
- Have not yet thought about proving its efficiency mathematically.

Perfect algorithm for one arm

- Advantage with only one arm: when we play the arm, it resets to initial state.
- Thus, use dynamic programming to solve. Will also need some notion of feasibility here.
- Define feasibility as $\text{reward}(t) * [T/(t+1)]$
- Find optimum for $T=1$. Then for $T=2$ and so on.
- For $T=T$, find element with highest feasibility. Hit it as many times as possible. Fill up the remaining slots using previously stored information.

Potential expansion

- For multiple arms, we can find the best way to fill a sequence of length T with each arm individually. If we find the N best ways with each arm, we will have KN sequences.
- Challenge: To find a compatible set of sequences that can be merged to maximize reward.
- Of the order N^K .
- If computational limit = L , take $N = 10^{(\log(L)/K)}$

Other queries:

- Will the functions be defined until $\text{Tau} = T-1$?
- Do we want to find a measure for average performance along with worst performance?
- I have not been able to incorporate the bitonic function assumption in the algorithm or proof. Is there any way I can do so?
- How do we hope to apply such an algorithm?
- How likely are the worst case scenarios?
- Should I start working on trying to implement the problem variations? Will they be eventually incorporated anyway?