

## CSE-571

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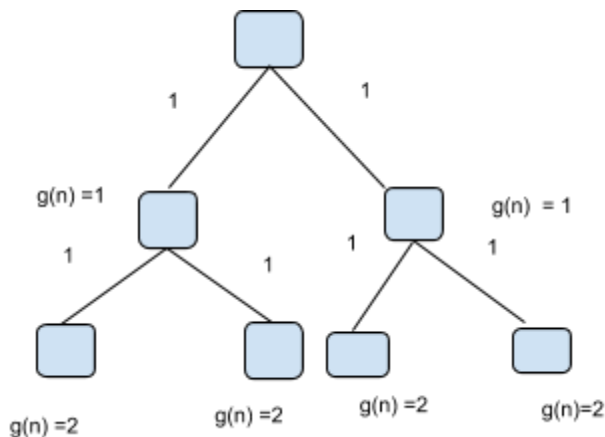
### 1.1

a. Breadth-first Search is a special case of Uniform Cost Search

In uniform cost search, the node with the lowest cost  $g(n)$  in the fringe is expanded first whereas in Breadth First Search, the nodes are expanded at the current level prior to expanding the nodes at a deeper level without considering the lowest cost factor  $g(n)$ .

Breadth First Search will become a special case of Uniform Cost Search

When all the step costs are equal i.e;  $g(n) = \text{depth}(n)$



b. Depth-first Search is a special case of best first tree Search.

In Depth First Search, the nodes are expanded at deeper levels as far as possible before backtracking without considering heuristic cost factor whereas in Best First Search, we explore nodes with low  $h(n)$  heuristic cost.

The Depth-first Search is a special case of Best first tree Search if  $g(n) = - \text{depth}(n)$

c. Uniform Cost Search is a special case of A\* search

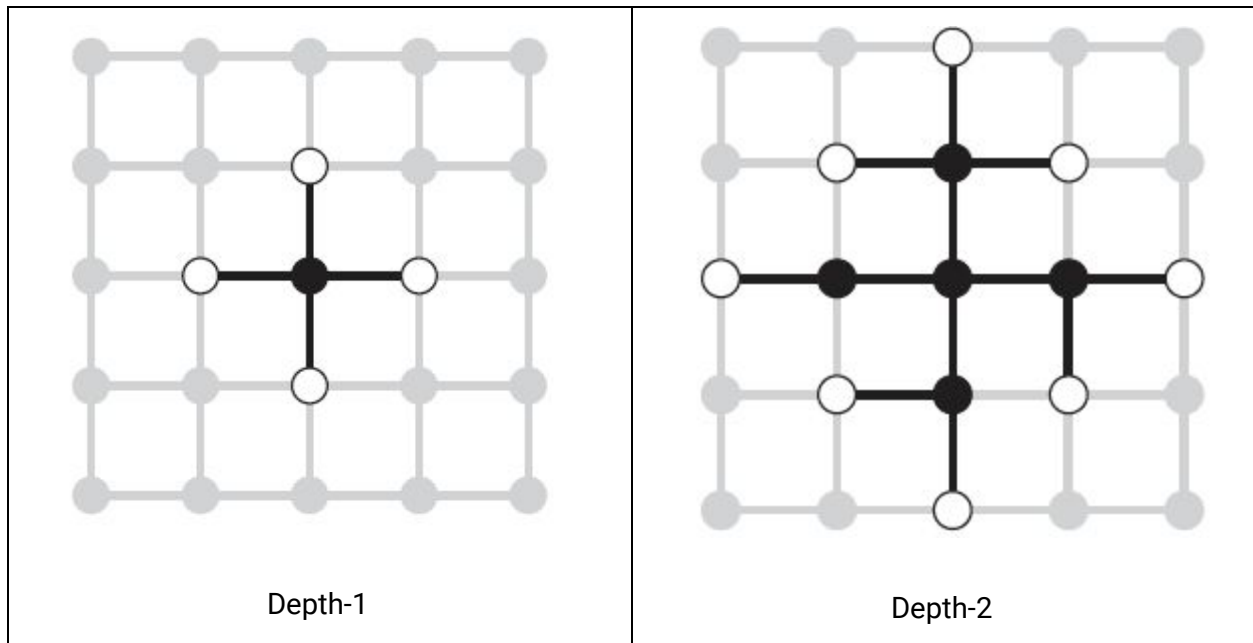
In uniform cost search, the node with the lowest cost  $g(n)$  in the fringe is expanded first whereas in A\* Search, the node with lowest  $f(n)$  in the fringe is expanded first where  $f(n) = g(n) + h(n)$ ,  $g(n)$  = actual cost from start node to node 'n' and  $h(n)$  = estimated cost from node 'n' to goal node. Breadth First Search will become a special case of A\* Search if  $h(n) = 0$  for all nodes

### 1.2

a. What is the branching factor  $b$  in the state space?

The maximum number of locations (left, right, up and bottom) from a given node is 4, Hence the branching factor is **4** or If we consider Stay in any state as one of the choices, the branching factor would become **5**.

**.b.** How many distinct states are there at depth k?



At depth 1 there are 4 nodes.

At depth 2 there are 8 nodes

At depth 3 there are 12 nodes

.....

At depth k there are 4K nodes.

Hence the number of distinct nodes at depth k is **4K**

If loops are considered then it is  $1 + 1*4 + 2*4 + \dots + k*4 = 2k(k + 1) + 1$  nodes

**c.** What is the maximum number of nodes expanded by breadth first tree search without a closed set?

Without a closed set Breadth first tree search does not keep the track of explored nodes, therefore it explores the nodes exponentially every time expands a max of  $5^{x+y}$  nodes for a depth of x+y.

$1 + 5 + 5^2 + 5^3 + \dots + 5^{x+y}$  (Total no of nodes expanded till depth x+y)

$(5^{x+y+1} - 1)/(5-1)$

**$(5^{x+y+1} - 1)/4$  nodes**

**d.** What is the maximum number of nodes expanded by breadth first tree search with a closed set?

If it is a closed set, we won't explore the nodes which are already expanded

At depth 1 there are 4 nodes

At depth 2 there are  $4^2$  nodes

.....

At depth  $x+y$  there are  $4^{x+y}$  nodes

Hence the maximum number of nodes expanded by breadth first tree search with a closed set is  $1 + 4 + 4^2 + 4^3 + \dots + 4^{x+y}$

$$= 1 + 4(1+2+3+ \dots + (x+y))$$

$$= 1 + 4(x+y)(x+y+1)/2$$

$$= 2(x+y)(x+y+1) + 1 \text{ nodes}$$

**e.** What is the maximum number of nodes expanded by breadth first graph search with a closed set?

Breadth First Graph search is as similar as Breadth First Tree search with a closed set.

Therefore the maximum number of nodes expanded by breadth first graph search with the closed set is  **$2(x+y)(x+y+1) + 1$  nodes.**

**f.** Is  $h = |u - x| + |v - y|$  an admissible heuristic for a state at  $(u, v)$ ? Explain.

**Yes,**  $h = |u - x| + |v - y|$ , manhattan distance metric

is an admissible heuristic for the state  $(u,v)$ . If suppose we have to reach the goal state  $(1,1)$  from  $(0,0)$  the actual distance to reach the state is 2  $[(0,0) \rightarrow (0,1) \rightarrow (1,1)]$

Heuristic value =  $|1-0| + |1-0| = 2$ , which is an manhattan distance metric

As the heuristic value  $\leq$  actual distance for all states  $(u, v)$  we can say that the heuristic is admissible.

**g.** How many nodes are expanded by A\* graph search using  $h$ ?

The nodes are expanded by A\* graph search using  $h$  is  $|x| + |y|$  nodes

**h.** Does he remain admissible if some links are removed?

**Yes,** It would remain admissible because if some links are removed then the actual distance would increase. The heuristic cost in this case is still less or equal to actual cost

**i.** Does  $h$  remain admissible if some links are added between nonadjacent states?

**No,**  $h$  won't remain admissible because if some links are added then the actual distance would decrease which can make the actual cost less than heuristic cost.

1.3.

a. Calculate the size of state space function of  $n$ ?

Initially, the 1st vehicle can be kept in any of  $n^2$  locations.

2nd vehicle can be kept in any  $n^2 - 1$  (location of 1st vehicle)

.....

.....

Nth vehicle can be kept in any of  $n^2 - (n-1)$  locations.

Therefore, the state space =  $(n^2) (n^2-1) (n^2-2) \dots (n^2 - (n-1))$

b. Calculate the branching factor as a function of  $n$ .

$5^n$  (for "up, down, left, right, stay put" choices)

c. Suppose that vehicle  $i$  is at  $(x_i, y_i)$  write a nontrivial admissible heuristic  $h_i$  for the number of moves it will require to get to its goal location  $(n-i+1, n)$  assuming no other vehicles are on the grid.

$h_i = |n - i + 1 - x_i| + |n - y_i|$  (manhattan distance heuristic) for a single vehicle

The Manhattan distance heuristic gives the exact cost for a single vehicle to reach the goal.

As the heuristic cost  $\leq$  actual cost we can say that the heuristic is admissible.

d. Which of the following heuristics are admissible for the problem of moving all  $n$  vehicles to their destinations? Explain.

Consider an example with 3X3 dimensions

| Vehicle | StartPosition | GoalPosition | Heuristic cost $ x-u  +  y-v (4,1)$ |
|---------|---------------|--------------|-------------------------------------|
| 1       | (1,1)         | (5,5)        | 7                                   |
| 2       | (2,2)         | (4,5)        | 5                                   |
| 3       | (3,3)         | (3,5)        | 2                                   |

|   |       |       |   |
|---|-------|-------|---|
|   |       |       |   |
| 4 | (4,4) | (2,5) | 3 |

|  |   |   |   |   |
|--|---|---|---|---|
|  |   |   |   |   |
|  |   |   | 4 |   |
|  |   | 3 |   |   |
|  | 2 |   |   |   |
|  | 1 |   |   | 5 |

Vehicle 1 can now reach the goal in 1 step after 3 hops and move right once.  
Vehicle 2 can reach the goal in one step once it moves right and takes 2 hops.  
If vehicle 4 moves bottom, then vehicle 3 can reach the goal in 0 steps.  
Vehicle 4 can reach the goal in 2 steps by moving bottom and right.  
Total Actual Cost= 1 + 1 + 1 + 2 + 0= 5

heuristic cost > actual cost, hence it is not admissible.

(i)  $\sum h_i$ .

The Total heuristic costs of all the vehicles should be less than the total cost of all vehicles to reach the goal positions.  
Total heuristic cost in this case = 7+5+2+3+0 = 17  
Total heuristic cost > actual cost, hence it is **not admissible**.

(ii)  $\max \{h_1, \dots, h_n\}$

To be admissible the max of the vehicle heuristic costs should be less than the total cost of all vehicles to reach the goal positions.  
Max of heuristic costs in this case is  $\max\{7,5,2,3,0\} = 7$   
max of heuristic costs > actual cost, the heuristic is **not Admissible**.

(iii)  $\min \{h_1, \dots, h_n\}$ .

To be admissible the min of the vehicle heuristic costs should be less than the total cost of all vehicles to reach the goal positions.

Min of heuristic costs in this case =  $\{7, 5, 2, 3, 0\} = 0$

min of heuristic costs < actual cost, the heuristic is **admissible**.

#### 1.4

**a.**

**State:** The state is determined by the locations of the farmer, wolf, goat and cabbage.

**Initial state:** [Farmer, Wolf, Goat, Cabbage] are on the left side of the river, we define it as = [LF,LW,LG,LC]

**Actions:** Carry nothing or a single one of his purchases: the wolf, the goat, or the cabbage to the other side of the river with conditions goat and cabbage are not left alone, wolf and goat are not left alone

**Transition model:** Given a state and action, this returns the resulting state.

For example,

|                             | Left side of river | right side of river |
|-----------------------------|--------------------|---------------------|
|                             | [LF,LW,LG,LC]      | [ ]                 |
| Farmer + Goat = right side  | [LW,LC]            | [RF,RG]             |
| Farmer = left side          | [LF,LW,LC]         | [RG]                |
| Farmer + wolf = right side  | [LC]               | [RF,RW,RG]          |
| Farmer + goat = left side   | [LF,LG,LC]         | [RW]                |
| Farmer+cabbage = right side | [ LG ]             | [RF,RW,RC]          |
| Farmer = left side          | [LF,LG]            | [RW,RC]             |
| Farmer + goat = right side  | [ ]                | [RF,RW,RG,RC]       |

Goal test: [Farmer, Wolf, Goat, Cabbage] are on the right side of the river, we define it as [RF,RW,RG,RC]

**Path cost:** Each step costs 1.

**b.** One can obtain a heuristic of an original problem from a relaxed problem. Relaxed problem defined as the original problem without constraints. The optimal cost solution to a relaxed problem is an admissible heuristic cost for the original problem.

Defining a relaxed problem by excluding constraints:

1. Farmer can carry only himself and a single one of his purchases without any further constraints when crossing the river

**heuristic1:** (number of the farmer's purchases on the left \* 2 - 1) (+ 1 if farmer is on the right; + 0 otherwise).

2. The wolf, goat or cabbage can move from one side to another

**heuristic2:** number of the farmer's purchases on the left.

The derived heuristics determine the exact cost for the relaxed problem, it must obey the triangle inequality and therefore it is considered as consistent.