Game Theory-1

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1 Introduction

1.1 Static Game/Normal Form Game

A Strategic game is a model of interactive decision making in which each decision makes chooses his plan of actions once and for all these choices are made simultaneously.

Formal Setting of the Game

- Finite set θ of players
- For each player i, a set(non-empty) S_i of actions
- For each player i, a preference relation $\succsim_i$ on $S = \bigtimes_{j\in\theta} S_j$

If the set S_i of actions of every player i is finite then the game is *finite*.

Usually it the case that the players preferences are not naturally defined over action profiles but over their consequences. To circumvent this we introduce a set C of consequences, a function $g : A \Rightarrow C$ that associates consequences with action profiles, and a profile $(\succsim_i^\star)$ of preference relations over C.

Then the preference relation $\succsim_i^\star$ of each player i in the strategic game is defined as follows:

$$\text{a} \succsim_i^\star \text{b if and only if } g(a) \succsim_i^\star g(b)$$

The Preference relation $\succsim_i$ of player i can be represented by a payoff function. $u_i : S \Rightarrow \mathbb{R}$. We refer tp values of such a function as payoffs.

In such a case we denote the game as $\langle \theta, \{S_i\}_{i=1}^{\theta}, \{u_i)\}_{i=1}^{\theta} \rangle$

Prisoner's Dilemma

Two suspects in a crime are put into separate cells. If they both confess, each will be sentenced to three years in prison. If only one of them confesses, he will be freed and used as a witness against the other, who will receive a sentence of four years. If neither confesses, they will both be convicted of a minor offense and spend one year in prison. Choosing a convenient payoff representation for the preferences.

Let's represent the above game in the rigorous form,

- $\theta = \{1, 2\}$
- $S_1 = S_2 = \{\text{Confess(C)}, \text{Don't Confess(DC)}\}$

- Strategy profile, $S = S_1 \times S_2 = \{(DC, C), (C, DC), (C, C), (DC, DC)\}$

$$
\begin{aligned}
U_i(\mathrm{DC}, \mathrm{DC}) &= -2 \quad \forall i \in \{1, 2\} \\
U_i(\mathrm{C}, \mathrm{C}) &= -5 \quad \forall i \in \{1, 2\} \\
U_1(\mathrm{C}, \mathrm{DC}) &= -1 \\
U_1(\mathrm{DC}, \mathrm{C}) &= -10 \\
U_2(\mathrm{C}, \mathrm{DC}) &= -10 \\
U_2(\mathrm{DC}, \mathrm{C}) &= -1
\end{aligned}
$$

Here we have the tabular representation of the game,

		Player 1	
		C	DC
Player 2	C	(-2,-2)	(-10,-1)
	DC	(-1,-10)	(-5,-5)

This is a game in which there are gains from cooperation—the best outcome for the players is that neither confesses—but each player has an incentive to be a "free rider". Whatever one player does, the other prefers Confess to Don't Confess, so that the game has a unique Nash equilibrium (Confess, Confess).