Gov 50: 24. More Inference with Mathematical Models

Matthew Blackwell

Harvard University

Roadmap

1. Confidence intervals for experiments
2. Hypothesis testing with the CLT
3. Two-sample tests

1/ Confidence intervals for experiments

Comparison between groups

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    - Sample size of the control group, $n_C = 1890$

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- Estimated **average treatment effect**

$$\widehat{\text{ATE}} = \overline{Y}_T - \overline{Y}_C = 0.07$$

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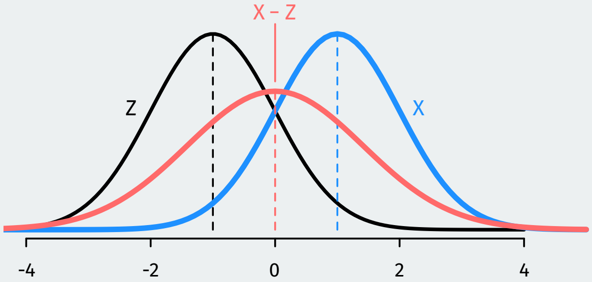
But what is the SE_{diff} in this case?

Spread of a difference in normals

If we take the difference between two independent normal r.v.s, what happens to the spread?



The spread of the difference is **larger** than the spread of the two variables being differenced!

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    - We can construct a 95% CI with $\widehat{\text{ATE}} \pm 1.96 \times SE_{\text{diff}}$

Confidence intervals

But we don't know μ_T or μ_C! Plug in our sample proportions to estimate the SE:

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Range of possibilities taking into account plausible chance errors.

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    5. Use p-value to decide whether to reject the null hypothesis or not

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Sampling distribution, in pictures

population distribution

estimator

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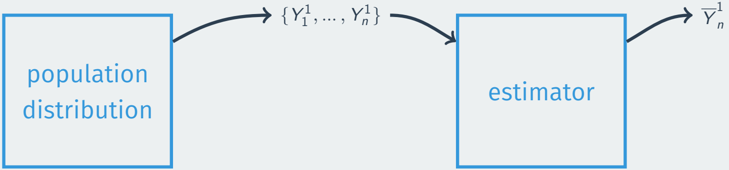

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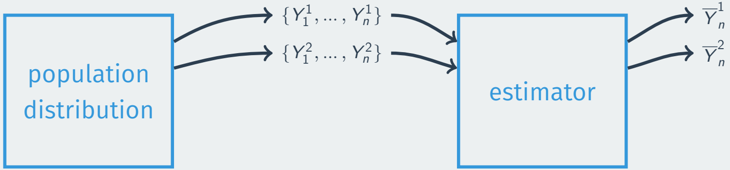

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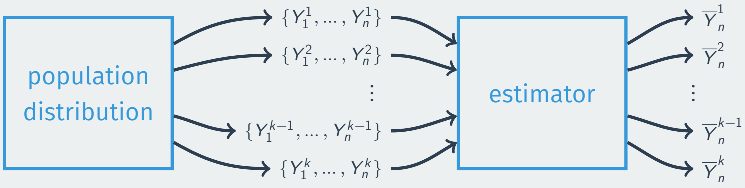

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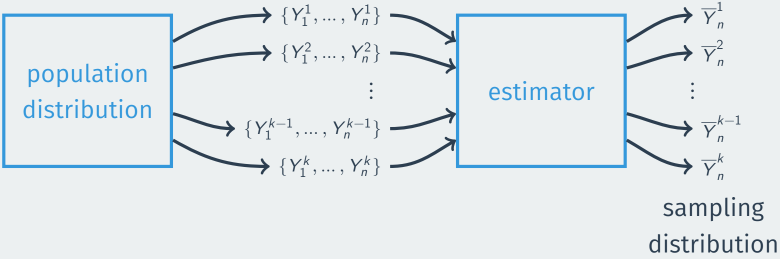

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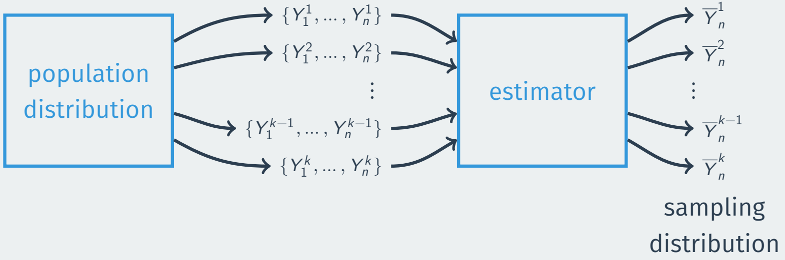

CLT for hypothesis testing

Under the null, we know the distribution of $\overline{Y}$:

$$\overline{Y} \approx N\left(p, \frac{p(1-p)}{n}\right) = N\left(0.5, \frac{0.5 \times 0.5}{812}\right)$$

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Then under the null, know the distribution of the following test statistic:

$$Z = \frac{\overline{Y} - 0.5}{0.5/\sqrt{812}} \approx N(0,1)$$

p-values

What we observe:

$$Z_{\text{obs}} = \frac{\overline{Y} - 0.5}{0.5/\sqrt{812}} = \frac{0.42 - 0.5}{0.5/\sqrt{812}}$$
$$= -\frac{0.08}{0.018} = -4.44$$

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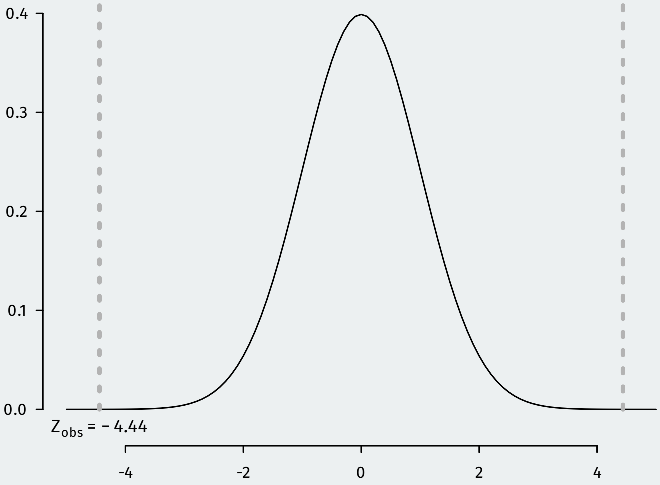
Our observed sample proportion is 4.44 SEs away from 0.5 under the null. What's the probability of being that far away? (**p-value**)

```
pnorm(-4.44, mean = 0, sd = 1) + ## prob being below -4.44
  (1 - pnorm(4.44, mean = 0, sd = 1)) ## prob being above 4.44
```

```
## [1] 0.000009
```


0.4
0.3
0.2
0.1
0.0
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-4
-2
0
2
4

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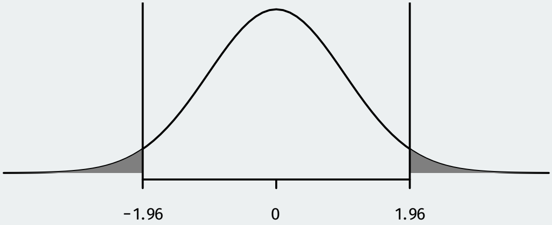
- p-value: probability of a standard normal being bigger than $|Z_{\text{obs}}|$

Rejecting regions



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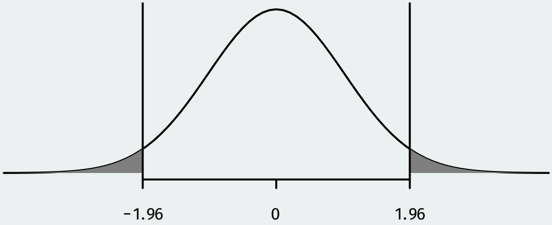

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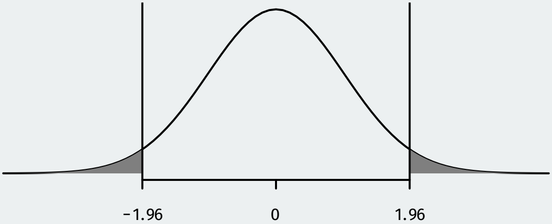

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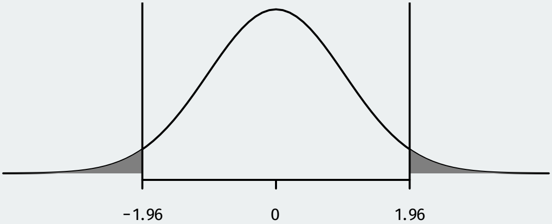

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    - If $Z_{\text{obs}} > 1.96$ or $Z_{\text{obs}} < -1.96$, then the p-value must be below 0.05
    - We can reject if $|Z_{\text{obs}}| > 1.96$

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  - Two-sided alternative: $H_1 : \mu_T - \mu_C \neq 0$
- In words: are the differences in sample means just due to chance?

Difference-in-means review

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- Estimated SE for the difference in means:

$$\widehat{\text{SE}}_{\text{diff}} = \sqrt{\frac{\overline{Y}_T(1-\overline{Y}_T)}{n_T} + \frac{\overline{Y}_C(1-\overline{Y}_C)}{n_C}} = 0.028$$

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Same general form of the test statistic as with one sample mean/proportion:

$$\frac{\text{observed - null guess}}{\text{SE}}$$

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- In large samples, we can replace true SE with an estimate:

$$\widehat{\text{SE}}_{\text{diff}} = \sqrt{\widehat{\text{SE}}_T^2 + \widehat{\text{SE}}_C^2}$$

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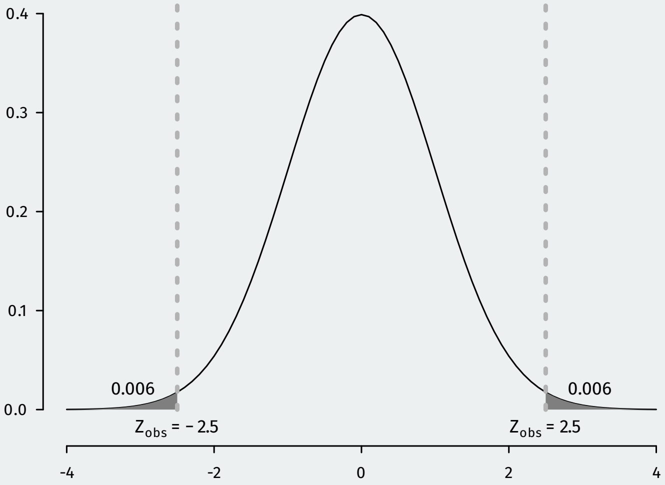
    - Lower p-values $\rightsquigarrow$ stronger evidence against the null.




```r
2 * pnorm(2.5, lower.tail = FALSE)
```

```
## [1] 0.0124
```