Gov 50: 21. More Hypothesis testing

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Roadmap

1. Hypothesis testing using infer
2. Two-sample tests
3. Two-sample permutation tests with infer

1/ Hypothesis testing using infer

Statistical hypothesis testing

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    4. Use this distribution to calculate the **p-value**.

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    3. Derive the **reference distribution** of the test statistic under the null.
    4. Use this distribution to calculate the **p-value**.
    5. Use p-value to decide whether to reject the null hypothesis or not

GSS data from `infer`

```r
library(infer)
gss
```

```
## # A tibble: 500 x 11
##     year   age sex    college   partyid hompop hours income
##    <dbl> <dbl> <fct>  <fct>     <fct>    <dbl> <dbl> <ord>
##  1  2014    36 male   degree    ind          3    50 $25000~
##  2  1994    34 female no degree rep          4    31 $20000~
##  3  1998    24 male   degree    ind          1    40 $25000~
##  4  1996    42 male   no degree ind          4    40 $25000~
##  5  1994    31 male   degree    rep          2    40 $25000~
##  6  1996    32 female no degree rep          4    53 $25000~
##  7  1990    48 female no degree dem          2    32 $25000~
##  8  2016    36 female degree    ind          1    20 $25000~
##  9  2000    30 female degree    rep          5    40 $25000~
## 10  1998    33 female no degree dem          2    40 $15000~
## # i 490 more rows
## # i 3 more variables: class <fct>, finrela <fct>,
## #   weight <dbl>
```

What is the average hours worked?

`dplyr` way:

```
gss |>
  summarize(mean(hours))
```

```
## # A tibble: 1 x 1
##   `mean(hours)`
##           <dbl>
## 1          41.4
```

`infer` way:

```
observed_mean <- gss |>
  specify(response = hours) |>
  calculate(stat = "mean")
observed_mean
```

```
## Response: hours (numeric)
## # A tibble: 1 x 1
##    stat
##   <dbl>
## 1  41.4
```

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How do we perform this test using infer? The **bootstrap!**

Specifying the hypotheses

```r
gss |>
  specify(response = hours) |>
  hypothesize(null = "point", mu = 40)
```

```
## Response: hours (numeric)
## Null Hypothesis: point
## # A tibble: 500 x 1
##    hours
##    <dbl>
##  1    50
##  2    31
##  3    40
##  4    40
##  5    40
##  6    53
##  7    32
##  8    20
##  9    40
## 10    40
## # i 490 more rows
```

Generating the null distribution

We can use the bootstrap to determine how much variation there will be around 40 in the null distribution.

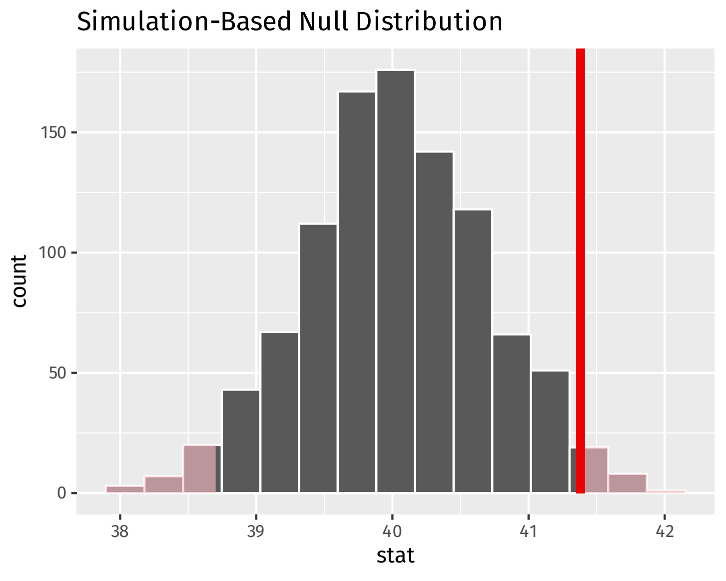
```r
null_dist <- gss |>
  specify(response = hours) |>
  hypothesize(null = "point", mu = 40) |>
  generate(reps = 1000, type = "bootstrap") |>
  calculate(stat = "mean")
null_dist
```

```
## Response: hours (numeric)
## Null Hypothesis: point
## # A tibble: 1,000 x 2
##    replicate  stat
##        <int> <dbl>
##  1         1  39.7
##  2         2  40.3
##  3         3  39.5
##  4         4  39.8
##  5         5  40.1
##  6         6  40.8
##  7         7  40.3
```

Visualizing the p-value

We can visualize our bootstrapped null distribution and the p-value as a shaded region:

```
null_dist |>
  visualize() +
  shade_p_value(observed_mean,
                direction = "two-sided")
```


Simulation-Based Null Distribution
count
150
100
50
0
38
39
40
41
42
stat

2/ Two-sample tests

Social pressure experiment

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  - Neighbors: naming-and-shaming social pressure mailer.

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- Outcome: whether household members voted or not.

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  - Civic Duty: mailer saying voting is your civic duty.
  - Hawthorne: a “we’re watching you” message.
  - Neighbors: naming-and-shaming social pressure mailer.
- Outcome: whether household members voted or not.
- We’ll focus on Neighbors vs Control
- Randomized implies samples are **independent**

Neighbors mailer

Dear Registered Voter:

WHAT IF YOUR NEIGHBORS KNEW WHETHER YOU VOTED?

Why do so many people fail to vote? We've been talking about the problem for years, but it only seems to get worse. This year, we're taking a new approach. We're sending this mailing to you and your neighbors to publicize who does and does not vote.

The chart shows the names of some of your neighbors, showing which have voted in the past. After the August 8 election, we intend to mail an updated chart. You and your neighbors will all know who voted and who did not.

DO YOUR CIVIC DUTY — VOTE!

MAPLE DR	Aug 04	Nov 04	Aug 06
9995 JOSEPH JAMES SMITH	Voted	Voted	______
9995 JENNIFER KAY SMITH		Voted	______
9997 RICHARD B JACKSON		Voted	______
9999 KATHY MARIE JACKSON		Voted	______

Social pressure data

```r
data(social, package = "qss")
social <- as_tibble(social)
social
```

```
## # A tibble: 305,866 x 6
##    sex   yearofbirth primary2004 messages primary2006 hhsize
##    <chr>       <int>       <int> <chr>          <int>  <int>
##  1 male         1941           0 Civic D~           0      2
##  2 fema~        1947           0 Civic D~           0      2
##  3 male         1951           0 Hawthor~           1      3
##  4 fema~        1950           0 Hawthor~           1      3
##  5 fema~        1982           0 Hawthor~           1      3
##  6 male         1981           0 Control            0      3
##  7 fema~        1959           0 Control            1      3
##  8 male         1956           0 Control            1      3
##  9 fema~        1968           0 Control            0      2
## 10 male         1967           0 Control            0      2
## # i 305,856 more rows
```

Two-sample hypotheses

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- Goal: learn about the population difference in means

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  - μ_T: Turnout rate in the population if everyone received treatment.
  - μ_C: Turnout rate in the population if everyone received control.
- Goal: learn about the population difference in means
- Usual null hypothesis: no difference in population means (ATE = 0)
  - Null: $H_0 : \mu_T - \mu_C = 0$

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- Parameter: **population ATE** $\mu_T - \mu_C$
    - μ_T: Turnout rate in the population if everyone received treatment.
    - μ_C: Turnout rate in the population if everyone received control.
- Goal: learn about the population difference in means
- Usual null hypothesis: no difference in population means (ATE = 0)
    - Null: $H_0 : \mu_T - \mu_C = 0$
    - Two-sided alternative: $H_1 : \mu_T - \mu_C \neq 0$

Two-sample hypotheses

- Parameter: **population ATE** $\mu_T - \mu_C$
    - μ_T: Turnout rate in the population if everyone received treatment.
    - μ_C: Turnout rate in the population if everyone received control.
- Goal: learn about the population difference in means
- Usual null hypothesis: no difference in population means (ATE = 0)
    - Null: $H_0 : \mu_T - \mu_C = 0$
    - Two-sided alternative: $H_1 : \mu_T - \mu_C \neq 0$
- In words: are the differences in sample means just due to chance?

Permutation test

How do we generate draws of the difference in means under the null?

$H_0 : \mu_T - \mu_C = 0$

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If the voting distribution is the same in the treatment and control groups, we could randomly swap who is labelled as treated and who is labelled as control and it shouldn't matter.

Permutation test

How do we generate draws of the difference in means under the null?
$H_0 : \mu_T - \mu_C = 0$

If the voting distribution is the same in the treatment and control groups, we could randomly swap who is labelled as treated and who is labelled as control and it shouldn't matter.

Permutation test: generate the null distribution by permuting the group labels and see the resulting distribution of differences in proportions

Permuting the labels

```r
social <- social |>
  filter(messages %in% c("Neighbors", "Control"))

social |>
  mutate(messages_permute = sample(messages)) |>
  select(primary2006, messages, messages_permute)
```

```
## # A tibble: 229,444 x 3
##    primary2006 messages messages_permute
##          <int> <chr>    <chr>
##  1           0 Control  Control
##  2           1 Control  Control
##  3           1 Control  Neighbors
##  4           0 Control  Control
##  5           0 Control  Control
##  6           1 Control  Neighbors
##  7           0 Control  Control
##  8           1 Control  Control
##  9           1 Control  Control
## 10           1 Control  Control
## # i 229,434 more rows
```

3/ Two-sample permutation tests with infer

Calculating the difference in proportion

`infer` functions with binary outcomes work best with factor variables:

```
social <- social |>
  mutate(turnout = if_else(primary2006 == 1, "Voted", "Didn't Vote"))

est_ate <- social |>
  specify(turnout ~ messages, success = "Voted") |>
  calculate(stat = "diff in props", order = c("Neighbors", "Control"))
est_ate
```

```
## Response: turnout (factor)
## Explanatory: messages (factor)
## # A tibble: 1 x 1
##     stat
##    <dbl>
## 1 0.0813
```

Specifying the relationship of interest

infer functions with binary outcomes work best with factor variables:

```r
social |>
  specify(turnout ~ messages, success = "Voted")
```

```
## Response: turnout (factor)
## Explanatory: messages (factor)
## # A tibble: 229,444 x 2
##    turnout    messages
##    <fct>      <fct>
##  1 Didn't Vote Control
##  2 Voted      Control
##  3 Voted      Control
##  4 Didn't Vote Control
##  5 Didn't Vote Control
##  6 Voted      Control
##  7 Didn't Vote Control
##  8 Voted      Control
##  9 Voted      Control
## 10 Voted      Control
## # i 229,434 more rows
```

Setting the hypotheses

The null for these two-sample tests is called `"independence"` for the `infer` package because the assumption is that the two variables are statistically independent.

```
social |>
  specify(turnout ~ messages, success = "Voted") |>
  hypothesize(null = "independence")
```

```
## Response: turnout (factor)
## Explanatory: messages (factor)
## Null Hypothesis: independence
## # A tibble: 229,444 x 2
##    turnout     messages
##    <fct>       <fct>
##  1 Didn't Vote Control
##  2 Voted       Control
##  3 Voted       Control
##  4 Didn't Vote Control
##  5 Didn't Vote Control
##  6 Voted       Control
##  7 Didn't Vote Control
##  8 Voted       Control
```

Generating the permutations

We can tell `infer` to do our permutation test by using the argument `type = "permute"` to `generate()`:

```
social |>
  specify(turnout ~ messages, success = "Voted") |>
  hypothesize(null = "independence") |>
  generate(reps = 1000, type = "permute")
```

```
## Response: turnout (factor)
## Explanatory: messages (factor)
## Null Hypothesis: independence
## # A tibble: 229,444,000 x 3
## # Groups:   replicate [1,000]
##    turnout     messages replicate
##    <fct>       <fct>        <int>
##  1 Voted       Control          1
##  2 Didn't Vote Control          1
##  3 Voted       Control          1
##  4 Didn't Vote Control          1
##  5 Didn't Vote Control          1
##  6 Voted       Control          1
##  7 Voted       Control          1
```

Calculating the diff in proportions in each sample

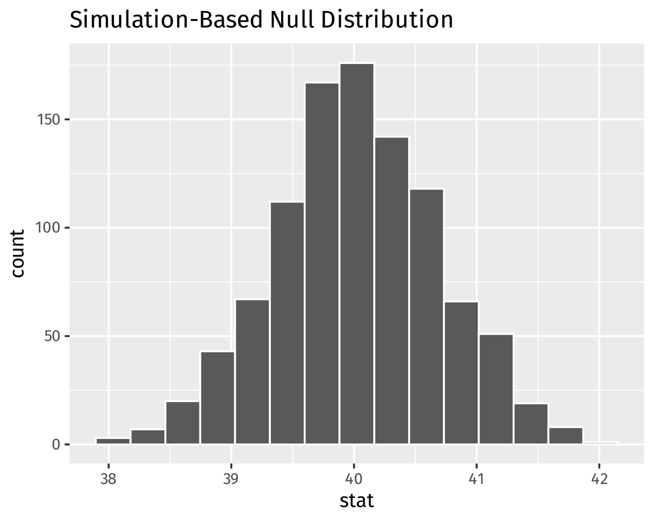
```r
null_dist <- social |>
  specify(turnout ~ messages, success = "Voted") |>
  hypothesize(null = "independence") |>
  generate(reps = 1000, type = "permute") |>
  calculate(stat = "diff in props", order = c("Neighbors", "Control"))
```

```r
null_dist
```

```
## Response: hours (numeric)
## Null Hypothesis: point
## # A tibble: 1,000 x 2
##    replicate  stat
##        <int> <dbl>
##  1         1  39.7
##  2         2  40.3
##  3         3  39.5
##  4         4  39.8
##  5         5  40.1
##  6         6  40.8
##  7         7  40.3
##  8         8  39.8
##  9         9  40.1
## 10        10  39.2
## # i 990 more rows
```

Visualizing

```r
null_dist |>
  visualize()
```



Calculating p-values

```r
ate_pval <- null_dist |>
  get_p_value(obs_stat = est_ate, direction = "both")
ate_pval
```

```
## # A tibble: 1 x 1
##   p_value
##     <dbl>
## 1       0
```

Visualizing p-values

```
null_dist |>
  visualize() +
  shade_p_value(obs_stat = est_ate, direction = "both")
```