

Gov 50: 25. Inference for Linear Regression

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Roadmap

1. Inference for linear regression
2. Presenting OLS regressions
3. Wrapping up the class

1/ Inference for linear regression

Data

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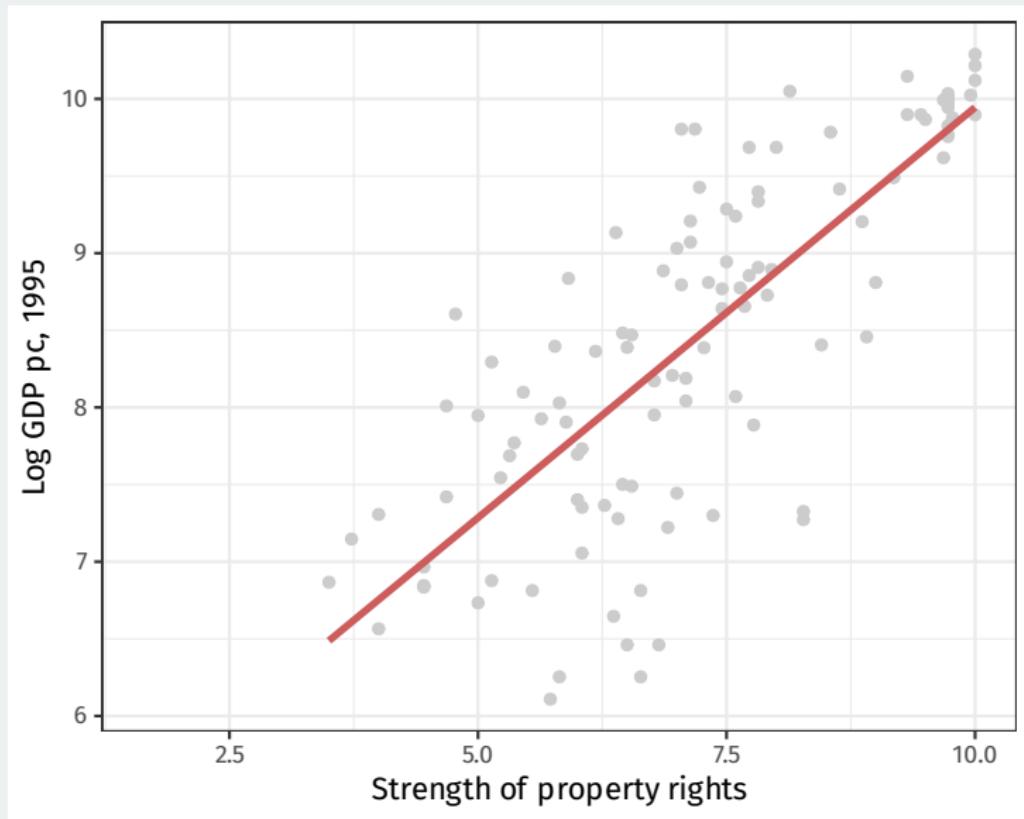
Name	Description
<code>shortnam</code>	three-letter country code
<code>africa</code>	indicator for if the country is in Africa
<code>asia</code>	indicator for if country is in Asia
<code>avexpr</code>	strength of property rights (protection against expropriation)
<code>logpgp95</code>	log GDP per capita

Loading the data

```
library(gov50data)
head(ajr)

## # A tibble: 6 x 15
##   shortnam africa lat_abst malfal94 avexpr logpgp95 logem4
##   <chr>     <dbl>    <dbl>    <dbl>    <dbl>    <dbl>    <dbl>
## 1 AFG        0     0.367  0.00372    NA      NA     4.54
## 2 AGO        1     0.137  0.950     5.36    7.77    5.63
## 3 ARE        0     0.267  0.0123    7.18    9.80    NA
## 4 ARG        0     0.378  0         6.39    9.13    4.23
## 5 ARM        0     0.444  0         NA      7.68    NA
## 6 AUS        0     0.300  0         9.32    9.90    2.15
## # i 8 more variables: asia <dbl>, yellow <dbl>,
## #   baseco <dbl>, leb95 <dbl>, imr95 <dbl>, meantemp <dbl>,
## #   lt100km <dbl>, latabs <dbl>
```

AJR scatterplot



Simple linear regression model

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$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

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 - Represents all unobserved error factors influencing Y_i other than X_i .

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- Get these estimates by the **least squares method**.
- Minimize the **sum of the squared residuals** (SSR):

$$\text{SSR} = \sum_{i=1}^n \hat{\epsilon}_i^2 = \sum_{i=1}^n (Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i)^2$$

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Sample 2: $\{(Y_1, X_1), \dots, (Y_n, X_n)\}$

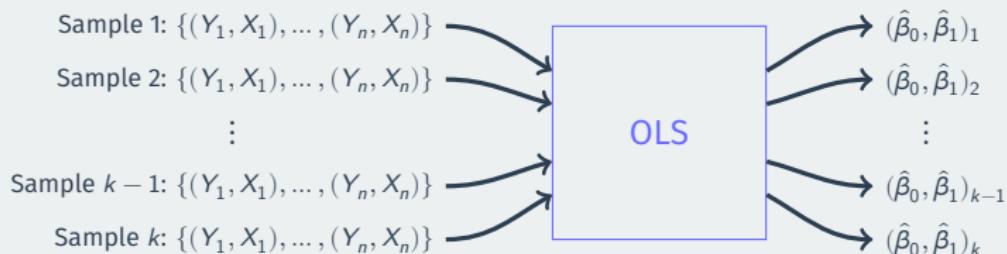


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$(\hat{\beta}_0, \hat{\beta}_1)_2$

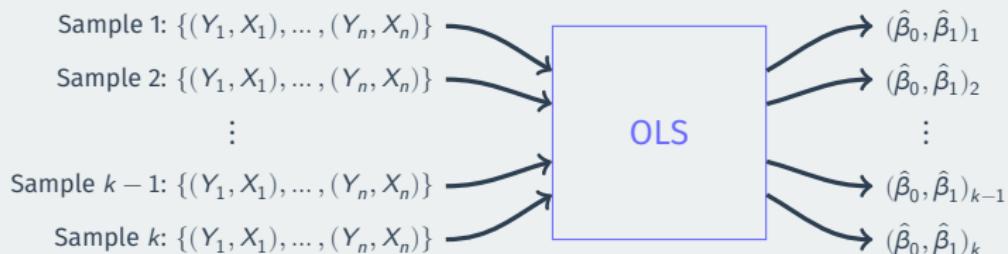
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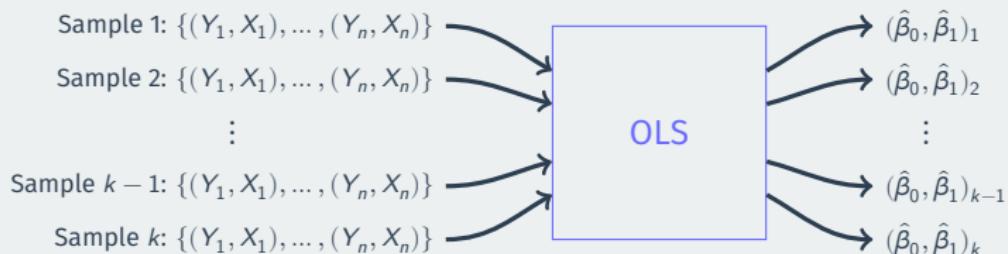
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- \rightsquigarrow sampling distribution with a standard error, etc.

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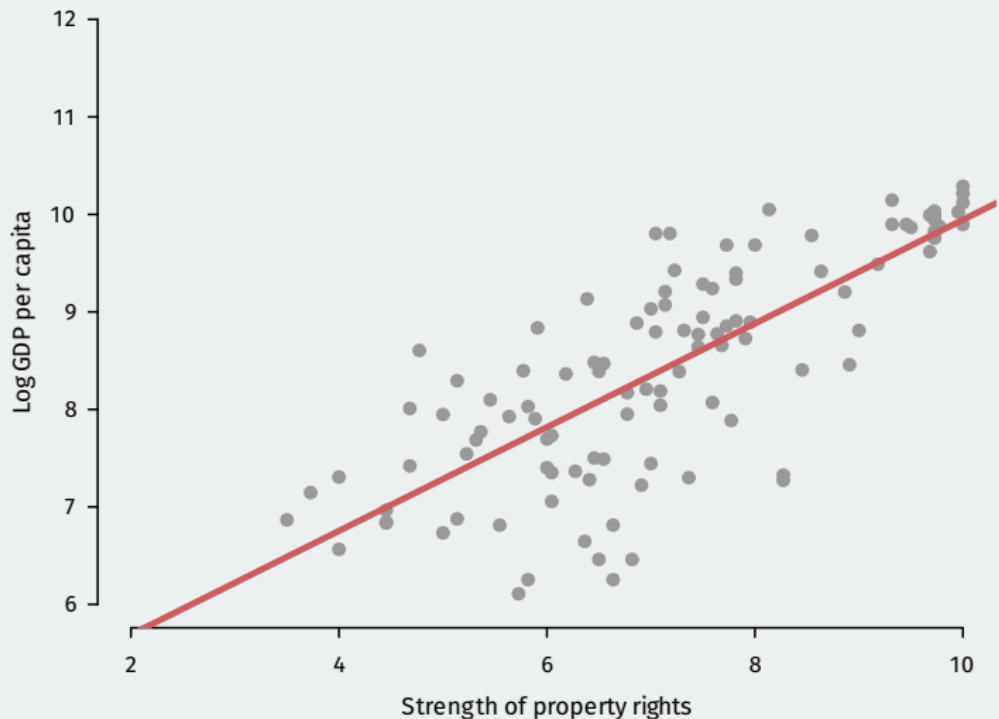
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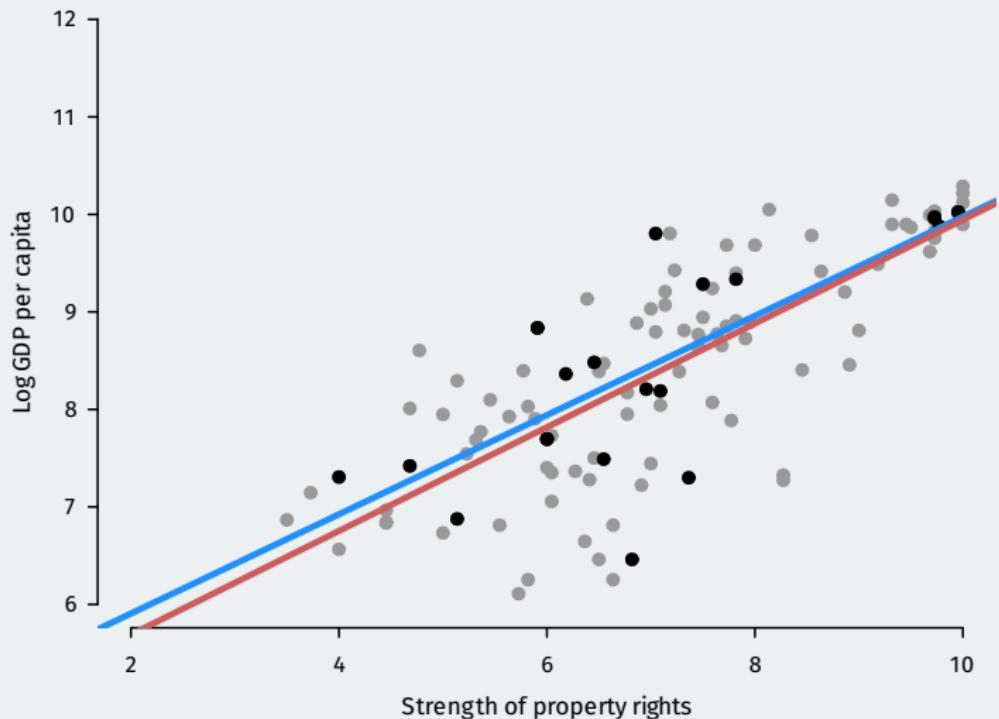
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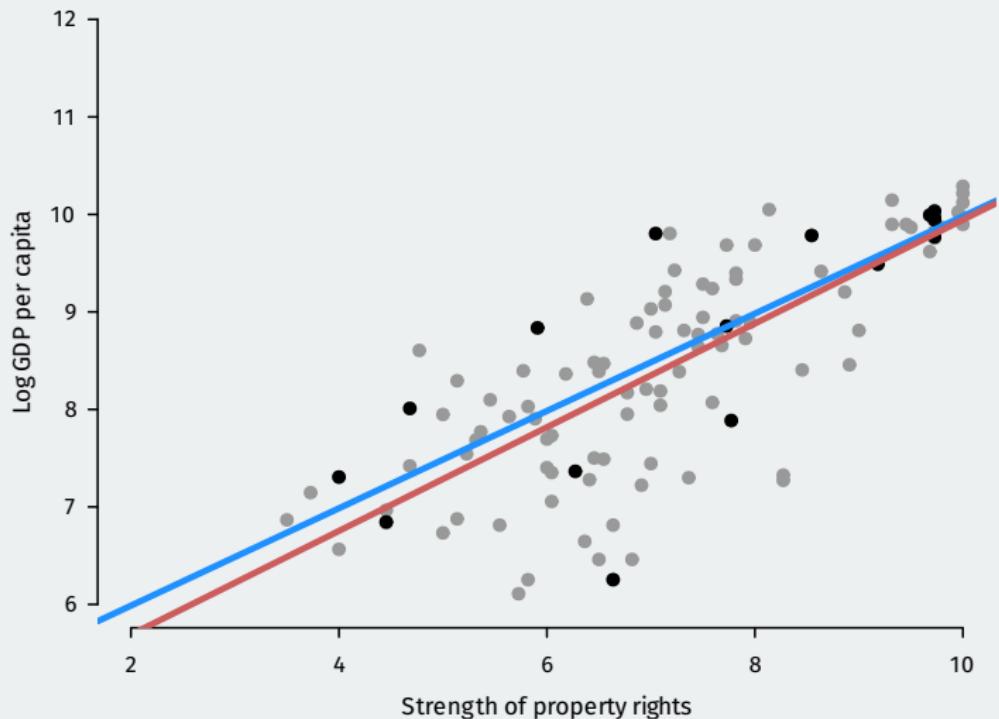
Population regression



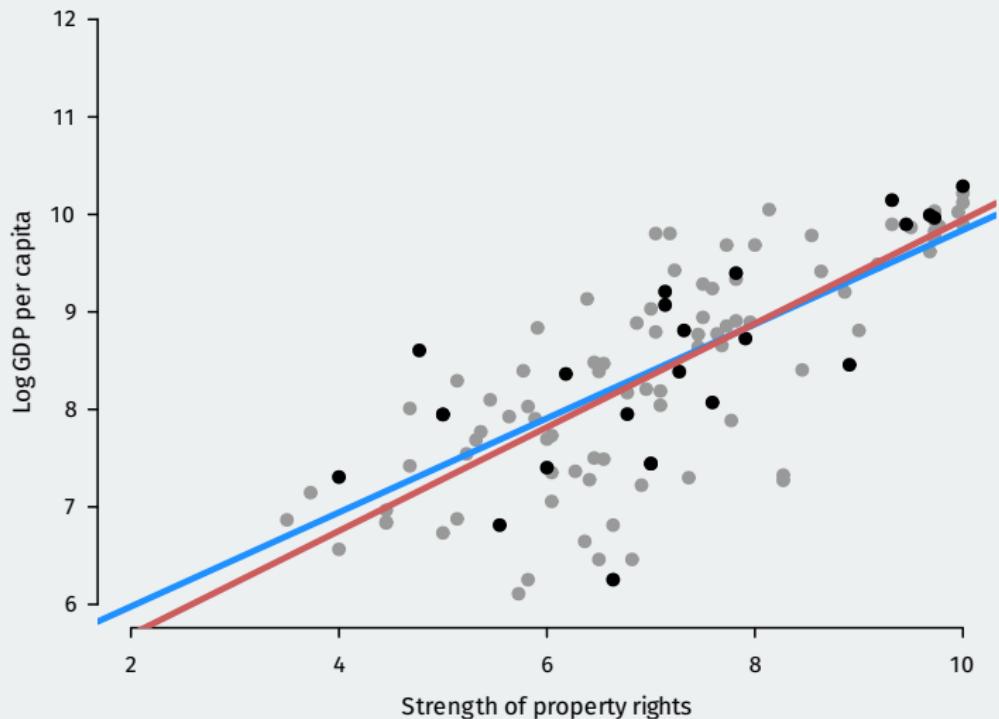
Randomly sample from AJR



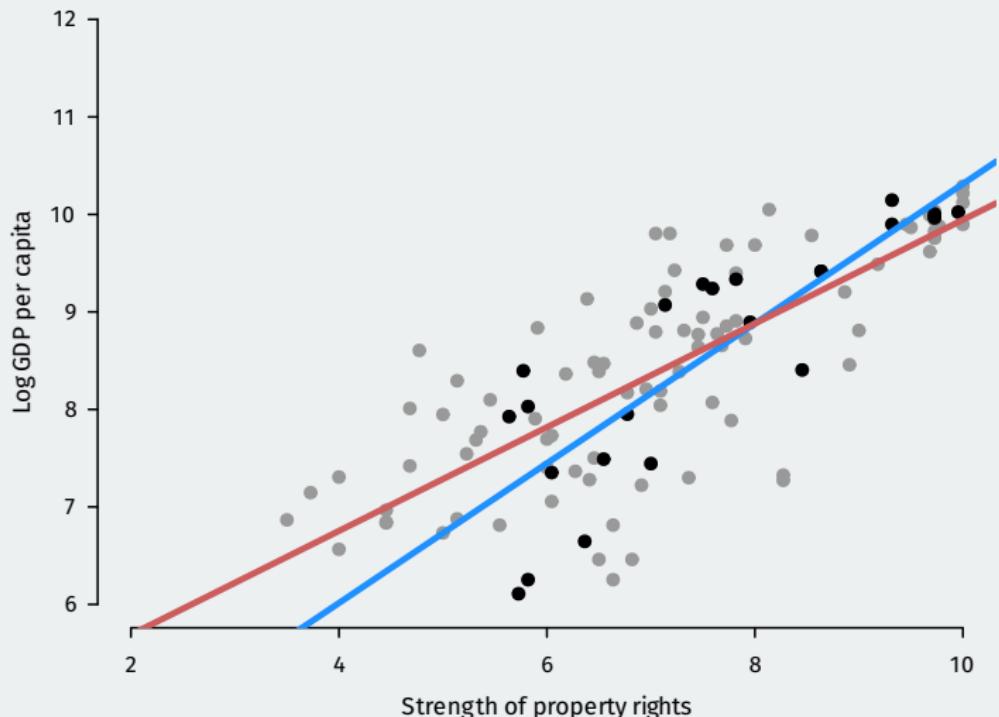
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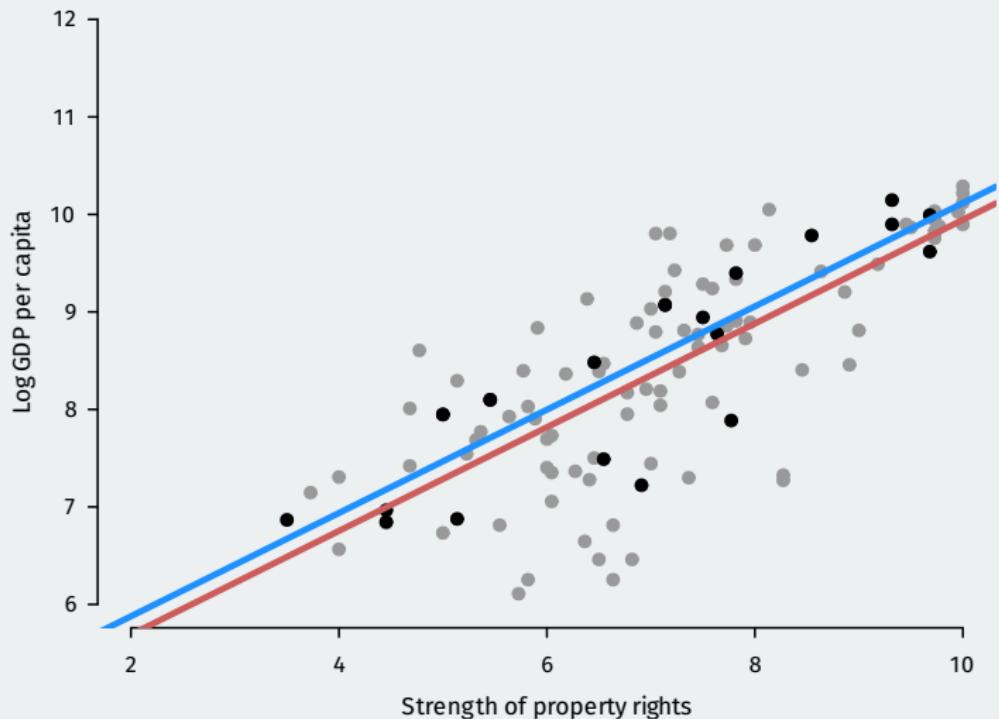
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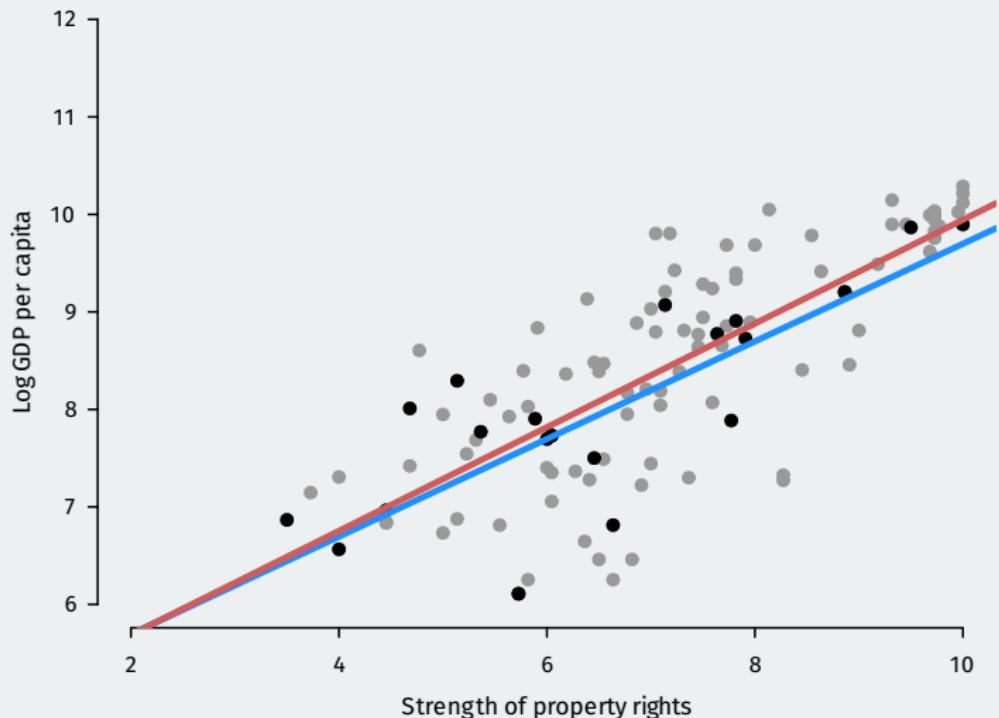
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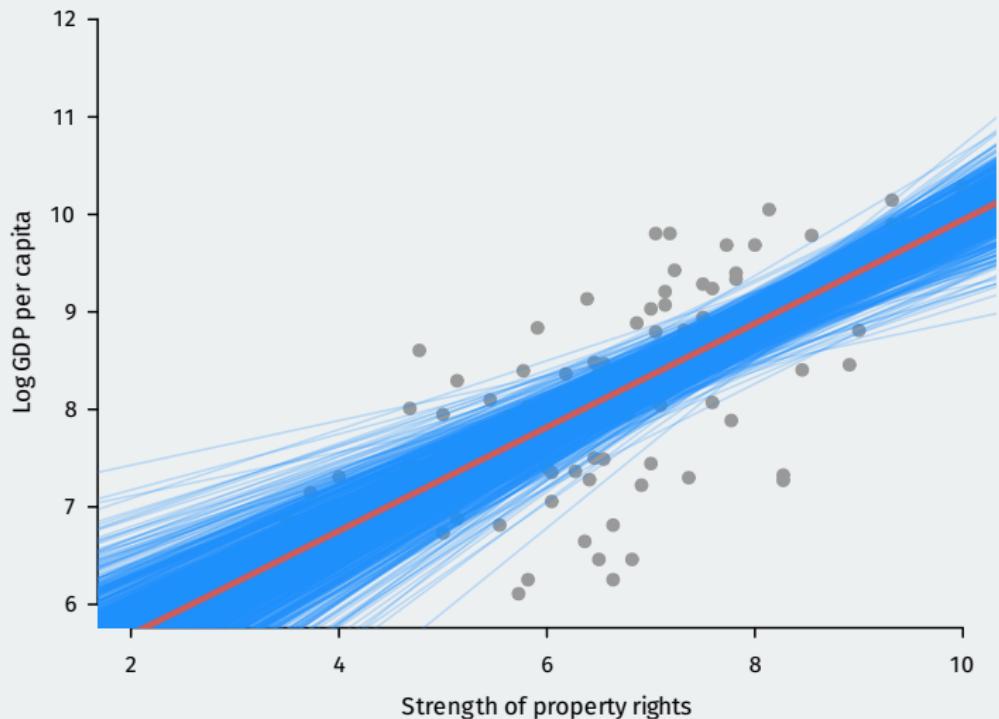
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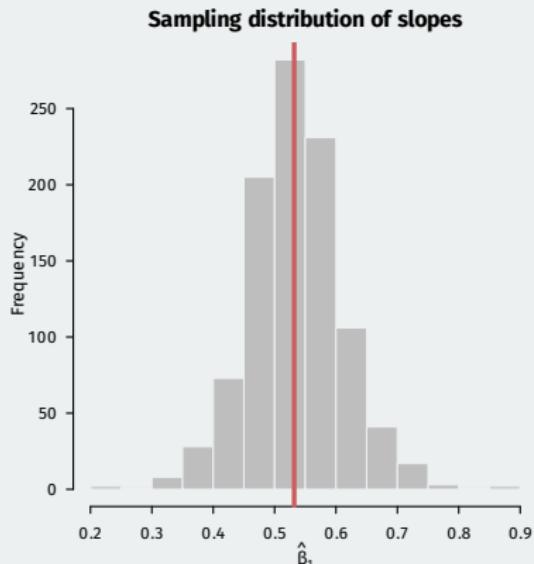
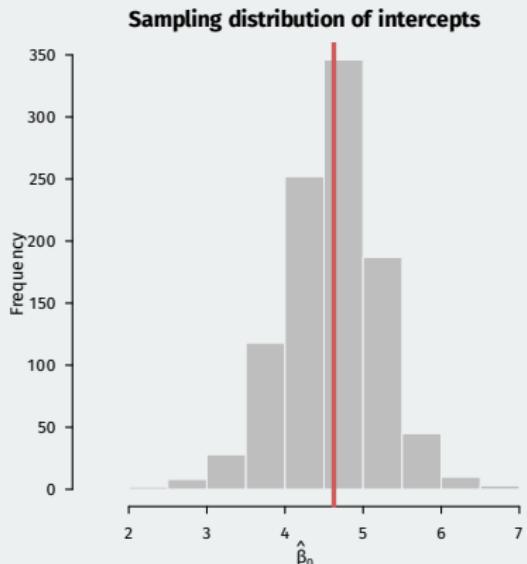


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Sampling distribution of OLS

- Estimated slope and intercept vary between samples, centered on truth.



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 - May not represent a causal effect unless causal assumptions hold.

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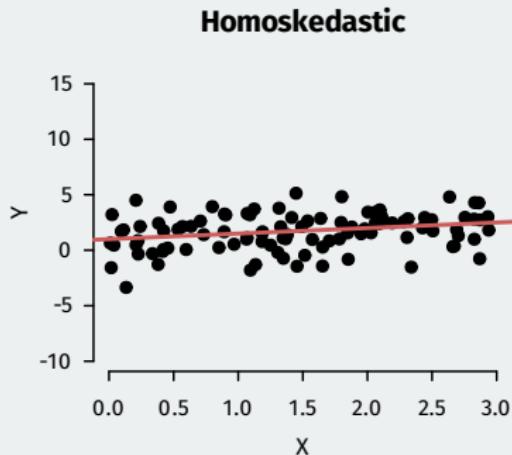
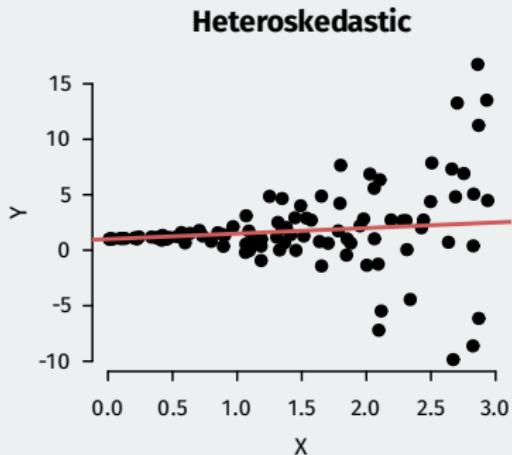
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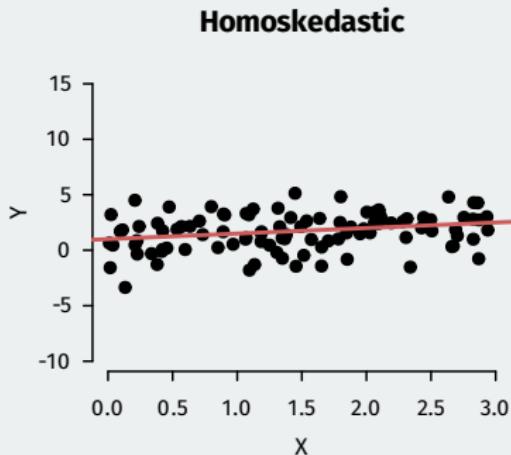
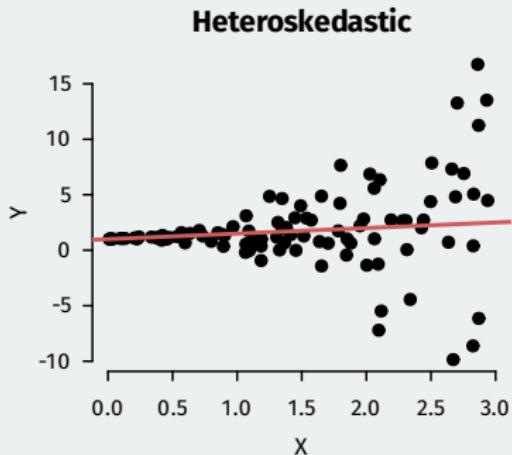
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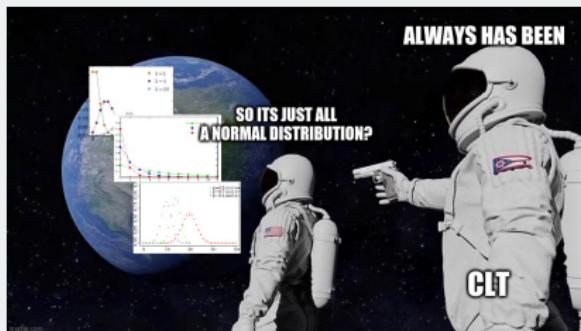
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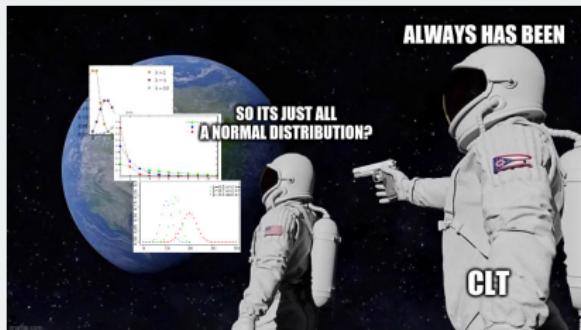
Relatively easy fixes exist, but beyond the scope of this class.

Tests and CIs for regression



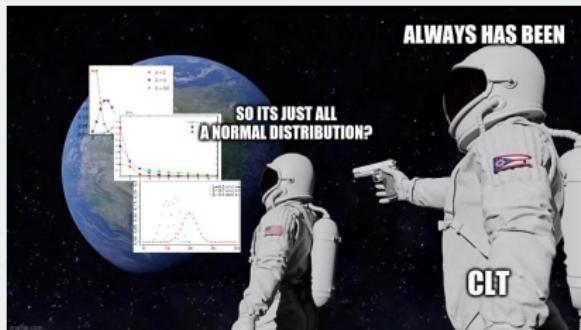
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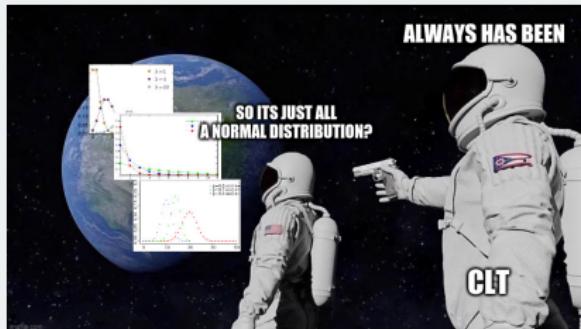
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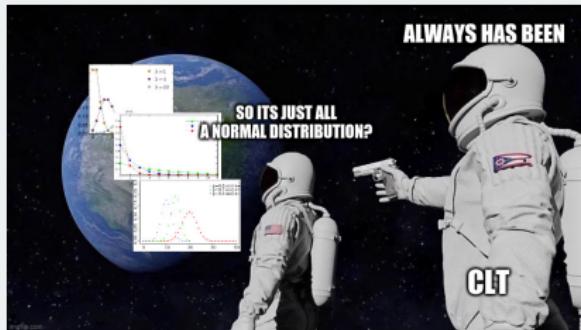
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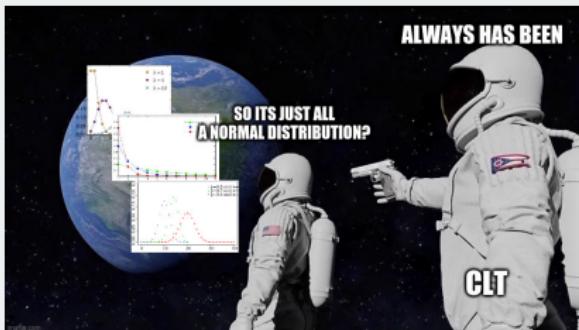
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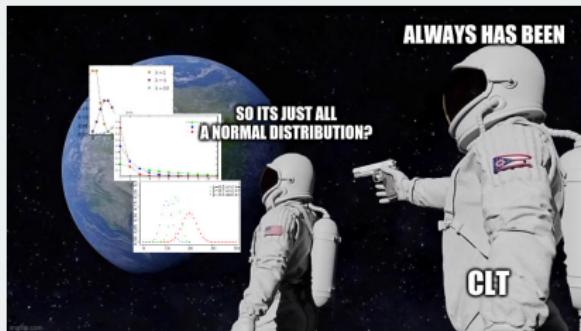
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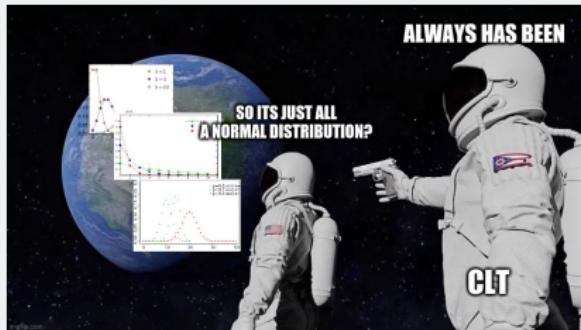
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 - $\hat{\beta}_1$ is **statistically significant** if its p-value from this test is below some threshold (usually 0.05)

```
ajr.reg <- lm(logpgp95 ~ avexpr, data = ajr)
summary(ajr.reg)

##
## Call:
## lm(formula = logpgp95 ~ avexpr, data = ajr)
##
## Residuals:
##    Min      1Q  Median      3Q     Max 
## -1.902 -0.316  0.138  0.422  1.441 
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept)  4.6261    0.3006   15.4   <2e-16 ***
## avexpr       0.5319    0.0406   13.1   <2e-16 ***
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.718 on 109 degrees of freedom
##   (52 observations deleted due to missingness)
## Multiple R-squared:  0.611, Adjusted R-squared:  0.608 
## F-statistic: 171 on 1 and 109 DF,  p-value: <2e-16
```

Using broom with regression

```
library(broom)
tidy(ajr.reg)

## # A tibble: 2 x 5
##   term      estimate std.error statistic p.value
##   <chr>      <dbl>     <dbl>      <dbl>    <dbl>
## 1 (Intercept)  4.63     0.301     15.4 4.28e-29
## 2 avexpr       0.532    0.0406    13.1 4.16e-24
```

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- Interpretation of β_j : an increase in the outcome associated with a one-unit increase in X_{ij} when other variables don't change their values
- Inference:
 - Confidence intervals constructed exactly the same for $\hat{\beta}_j$
 - Hypothesis tests done exactly the same for $\hat{\beta}_j$

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- Inference:
 - Confidence intervals constructed exactly the same for $\hat{\beta}_j$
 - Hypothesis tests done exactly the same for $\hat{\beta}_j$
 - \rightsquigarrow interpret p-values the same as before.

Using knitr::kable to produce tables

```
ajr.multreg <- lm(logpgp95 ~ avexpr + lat_abst + asia + africa, data = ajr)
tidy(ajr.multreg) |>
  knitr::kable(digits = 3)
```

term	estimate	std.error	statistic	p.value
(Intercept)	5.840	0.339	17.239	0.000
avexpr	0.394	0.050	7.843	0.000
lat_abst	0.312	0.444	0.703	0.484
asia	-0.170	0.153	-1.108	0.270
africa	-0.930	0.165	-5.628	0.000

2| Presenting OLS regressions

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- Standard errors, p-values, sample size, and R^2 may be reported as well.

AJR regression table

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TABLE 2—OLS REGRESSIONS

	Whole world (1)	Base sample (2)	Whole world (3)	Whole world (4)	Base sample (5)	Base sample (6)	Whole world (7)	Base sample (8)
Dependent variable is log GDP per capita in 1995								
Dependent variable is log output per worker in 1988								
Average protection against expropriation risk, 1985–1995	0.54 (0.04)	0.52 (0.06)	0.47 (0.06)	0.43 (0.05)	0.47 (0.06)	0.41 (0.06)	0.45 (0.04)	0.46 (0.06)
Latitude			0.89 (0.49)	0.37 (0.51)	1.60 (0.70)	0.92 (0.63)		
Asia dummy				-0.62 (0.19)		-0.60 (0.23)		
Africa dummy				-1.00 (0.15)		-0.90 (0.17)		
“Other” continent dummy				-0.25 (0.20)		-0.04 (0.32)		
<i>R</i> ²	0.62	0.54	0.63	0.73	0.56	0.69	0.55	0.49
Number of observations	110	64	110	110	64	64	108	61

`modelsummary()` to produce tables

We can use `modelsummary()` to produce a table. It takes a list of outputs from `lm` and aligns them in the correct way.

```
modelsummary::modelsummary(list(ajr.reg, ajr.multreg))
```

Output

```
modelsummary::modelsummary(list(ajr.reg, ajr.multreg))
```

	(1)	(2)
(Intercept)	4.626 (0.301)	5.840 (0.339)
avexpr	0.532 (0.041)	0.394 (0.050)
lat_abst		0.312 (0.444)
asia		-0.170 (0.153)
africa		-0.930 (0.165)
Num.Obs.	111	111
R2	0.611	0.713
R2 Adj.	0.608	0.703
AIC	245.4	217.6
BIC	253.5	233.8
Log.Lik.	-119.709	-102.795
RMSE	0.71	0.61

Cleaning up the goodness of fit statistics

```
modelsummary::modelsummary(  
  list(ajr.reg, ajr.multreg),  
  gof_map = c("nobs", "r.squared", "adj.r.squared"))
```

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Cleaning up the variable names

We can also map the variable names to more readable names using the `coef_map` argument. But first, we should do the mapping in a vector. Any term omitted from this vector will be omitted from the table

```
var_labels <- c(  
  "avexpr" = "Avg. Expropriation Risk",  
  "lat_abst" = "Abs. Value of Latitude",  
  "asia" = "Asian country",  
  "africa" = "African country"  
)  
var_labels
```

```
##                               avexpr                  lat_abst  
## "Avg. Expropriation Risk"  "Abs. Value of Latitude"  
##                               asia                   africa  
## "Asian country"            "African country"
```

Nice table

```
modelsummary::modelsummary(  
  list(ajr.reg, ajr.multreg),  
  coef_map = var_labels,  
  gof_map = c("nobs", "r.squared", "adj.r.squared"))
```

	(1)	(2)
Avg. Expropriation Risk	0.532 (0.041)	0.394 (0.050)
Abs. Value of Latitude		0.312 (0.444)
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Num.Obs.	111	111
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3/ Wrapping up the class

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Important takeaways from the course:

1. Data wrangling and data visualizations are really important skills that you now have!
2. Causality is hugely important in the world but difficult to establish.
3. Really important to understand and assess statistical uncertainty when working with data.

I'm really proud of you!



You've come a long way! Hopefully the tools you learned in this course will help you throughout your life and career!

What next?



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- More CS approach to data science: CS109 (Data Science 1)

Thanks!



Fill out your evaluations!