

Gov 50: 18. The Bootstrap

Matthew Blackwell

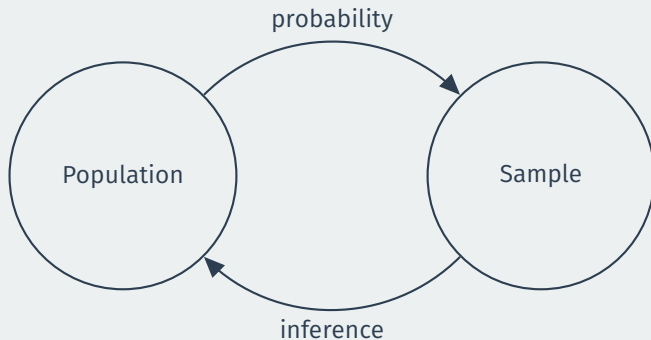
Harvard University

Roadmap

1. Resampling from our sample
2. Confidence intervals
3. Calculating confidence intervals

1/ Resampling from our sample

Where are we?



Can we approximate the **sampling distribution** with our single sample?

American National Election Survey data

Name	Description
state	State of respondent
district	Congressional district of respondent
pid7	Party ID (1=Strong D, 7=Strong R)
pres_vote	Self reported vote in 2020
sci_therm	0-100 therm score for scientists
rural_therm	0-100 therm score for rural Americans
favor_voter_id	1 if respondent thinks voter ID should be required
envir_doing_more	1 if respondent thinks gov't should be doing more about climate change

ANES data

```
library(gov50data)
anes
```

```
## # A tibble: 5,162 x 8
##   state district pid7 pres_vote sci_therm rural_therm
##   <chr>      <dbl> <dbl> <chr>      <dbl>      <dbl>
## 1 ID          2      4 Other        70        60
## 2 VA          2      3 Biden       100       75
## 3 CO          4      4 Trump        60       90
## 4 TX          5      3 Biden        85       85
## 5 WI          6      6 Trump        85       70
## 6 CA         40      2 Biden        50       50
## 7 WI          5      2 Biden       100       70
## 8 OR          4      7 Trump        70       50
## 9 MA          5      3 Biden        80       70
## 10 NV         3      1 Biden        85       40
## # i 5,152 more rows
## # i 2 more variables: favor_voter_id <dbl>,
## #   envir_doing_more <dbl>
```

Sample statistic

What is the average thermometer score for scientists?

```
anes |>  
  summarize(mean(sci_therm))
```

```
## # A tibble: 1 x 1  
##   `mean(sci_therm)`  
##               <dbl>  
## 1               80.6
```

Sample statistic

What is the average thermometer score for scientists?

```
anes |>  
  summarize(mean(sci_therm))
```

```
## # A tibble: 1 x 1  
##   `mean(sci_therm)`  
##               <dbl>  
## 1               80.6
```

What is the sampling distribution of this average? We only have this 1 draw!

Notation review

Population: all US adults.

Population parameter: average feeling thermometer score for scientists among all US adults.

Sample: (complicated) random sample of all US adults.

Sample statistic/point estimate: sample average of thermometer scores.

Notation review

Population: all US adults.

Population parameter: average feeling thermometer score for scientists among all US adults.

Sample: (complicated) random sample of all US adults.

Sample statistic/point estimate: sample average of thermometer scores.

Roughly how far our point estimate is likely to be from the truth?

The bootstrap

Mimic sampling from the population by **resampling** many times from the sample itself.

Bootstrap resampling done **with replacement** (same row can appear more than once)

One bootstrap resample

```
boot_1 <- anes |>
  slice_sample(prop = 1, replace = TRUE)
boot_1
```

```
## # A tibble: 5,162 x 8
##   state district  pid7 pres_vote sci_therm rural_therm
##   <chr>      <dbl> <dbl> <chr>      <dbl>      <dbl>
## 1 KS          4     7 Trump         60         100
## 2 TX          6     1 Biden        100         70
## 3 CA          3     3 Biden        100         50
## 4 TN          4     3 Biden        100        100
## 5 SC          1     7 Trump         85         90
## 6 TN          9     1 Biden        100         40
## 7 GA          7     1 Biden         50          0
## 8 CT          2     7 Trump         15        100
## 9 MD          8     5 Trump         85        100
## 10 IN         8     6 Trump         70         50
## # i 5,152 more rows
## # i 2 more variables: favor_voter_id <dbl>,
## #   enviro_doing_more <dbl>
```

Sample mean in the bootstrap sample

```
boot_1 |>  
  summarize(mean(sci_therm))
```

```
## # A tibble: 1 x 1  
##   `mean(sci_therm)`  
##               <dbl>  
## 1                80.6
```

Many bootstrap samples

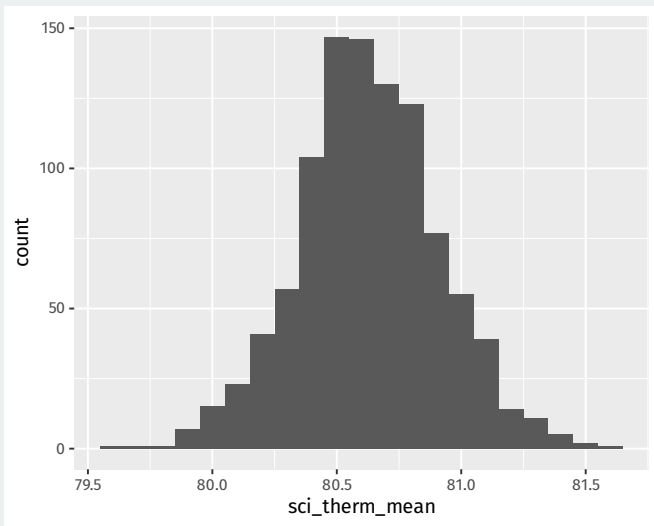
```
library(infer)
bootstrap_dist <- anes |>
  rep_slice_sample(prop = 1, reps = 1000, replace = TRUE) |>
  group_by(replicate) |>
  summarize(sci_therm_mean = mean(sci_therm))
bootstrap_dist
```

Many bootstrap samples

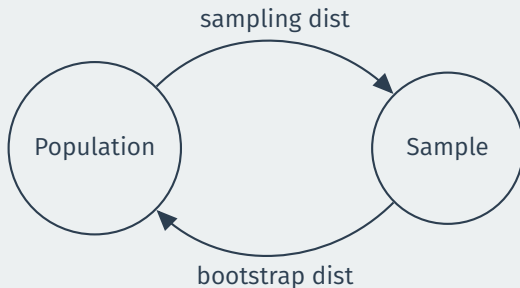
```
## # A tibble: 1,000 x 2
##   replicate sci_therm_mean
##   <int>      <dbl>
## 1         1         80.8
## 2         2         80.1
## 3         3         80.8
## 4         4         80.7
## 5         5         80.6
## 6         6         80.4
## 7         7         80.6
## 8         8         80.7
## 9         9         80.4
## 10        10         80.5
## # i 990 more rows
```

Visualizing the bootstrap distribution

```
bootstrap_dist |>  
  ggplot(aes(x = sci_therm_mean)) + geom_histogram(binwidth = 0.1)
```

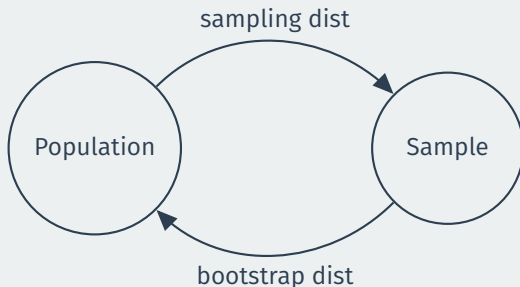


Bootstrap distribution



Bootstrap distribution **approximates** the sampling distribution of the estimator.

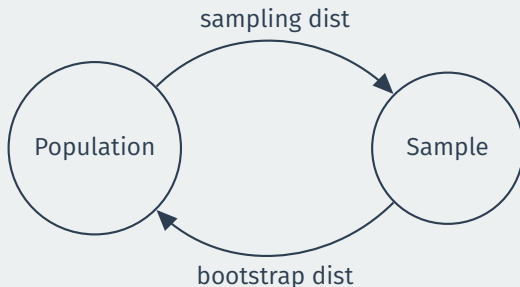
Bootstrap distribution



Bootstrap distribution **approximates** the sampling distribution of the estimator.

Both should have a **similar shape and spread** if sampling from the distribution \approx bootstrap resampling.

Bootstrap distribution



Bootstrap distribution **approximates** the sampling distribution of the estimator.

Both should have a **similar shape and spread** if sampling from the distribution \approx bootstrap resampling.

Approximation gets better as sample gets bigger.

Comparing to the point estimate

Given the sampling, not surprising that bootstrap distribution is centered on the point estimate:

```
bootstrap_dist |>  
  summarize(mean(sci_therm_mean))
```

```
## # A tibble: 1 x 1  
##   `mean(sci_therm_mean)`  
##               <dbl>  
## 1                80.6
```

```
anes |>  
  summarize(mean(sci_therm))
```

```
## # A tibble: 1 x 1  
##   `mean(sci_therm)`  
##               <dbl>  
## 1                80.6
```

2/ Confidence intervals

What is a confidence interval?



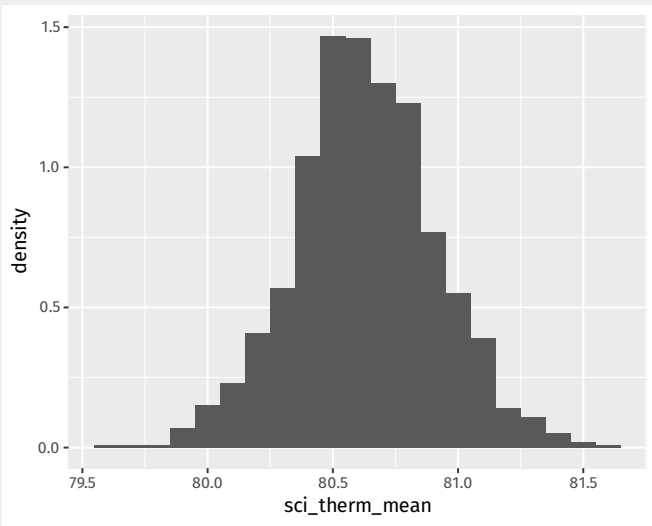
Point estimate: best single guess about the population parameter. Unlikely to be exactly correct.



Confidence interval: a range of plausible values of the population parameter.

Where is most of the bootstrap distribution?

```
bootstrap_dist |>  
  ggplot(aes(x = sci_therm_mean)) +  
  geom_histogram(aes(y= ..density..), binwidth = 0.1)
```



Confidence intervals



- Each sample gives a different CI or toss of the ring.

Confidence intervals



- Each sample gives a different CI or toss of the ring.
- Some samples the ring will contain the target (the CI will contain the truth) other times it won't.

Confidence intervals



- Each sample gives a different CI or toss of the ring.
- Some samples the ring will contain the target (the CI will contain the truth) other times it won't.
 - We don't know if the CI for our sample contains the truth!

Confidence intervals



- Each sample gives a different CI or toss of the ring.
- Some samples the ring will contain the target (the CI will contain the truth) other times it won't.
 - We don't know if the CI for our sample contains the truth!
- **Confidence level:** percent of the time our CI will contain the population parameter.

Confidence intervals



- Each sample gives a different CI or toss of the ring.
- Some samples the ring will contain the target (the CI will contain the truth) other times it won't.
 - We don't know if the CI for our sample contains the truth!
- **Confidence level:** percent of the time our CI will contain the population parameter.
 - Number of ring tosses that will hit the target.

Confidence intervals



- Each sample gives a different CI or toss of the ring.
- Some samples the ring will contain the target (the CI will contain the truth) other times it won't.
 - We don't know if the CI for our sample contains the truth!
- **Confidence level:** percent of the time our CI will contain the population parameter.
 - Number of ring tosses that will hit the target.
 - We get to choose, but typical values are 90%, 95%, and 99%

Confidence intervals as occasional liars

The **confidence level** of a CI determine how often the CI will be wrong.

Confidence intervals as occasional liars

The **confidence level** of a CI determine how often the CI will be wrong.

A 95% confidence interval will:

- Tell you the truth in 95% of repeated samples (contain the population parameter 95% of the time)

Confidence intervals as occasional liars

The **confidence level** of a CI determine how often the CI will be wrong.

A 95% confidence interval will:

- Tell you the truth in 95% of repeated samples (contain the population parameter 95% of the time)
- Lie to you in 5% of repeated sample (not contain the population parameter 5% of the time)

Confidence intervals as occasional liars

The **confidence level** of a CI determine how often the CI will be wrong.

A 95% confidence interval will:

- Tell you the truth in 95% of repeated samples (contain the population parameter 95% of the time)
- Lie to you in 5% of repeated sample (not contain the population parameter 5% of the time)

Confidence intervals as occasional liars

The **confidence level** of a CI determine how often the CI will be wrong.

A 95% confidence interval will:

- Tell you the truth in 95% of repeated samples (contain the population parameter 95% of the time)
- Lie to you in 5% of repeated sample (not contain the population parameter 5% of the time)

Can you tell if your particular confidence interval is telling the truth? No!

Percentile method

Percentile method: find the middle 95% of the bootstrap distribution.

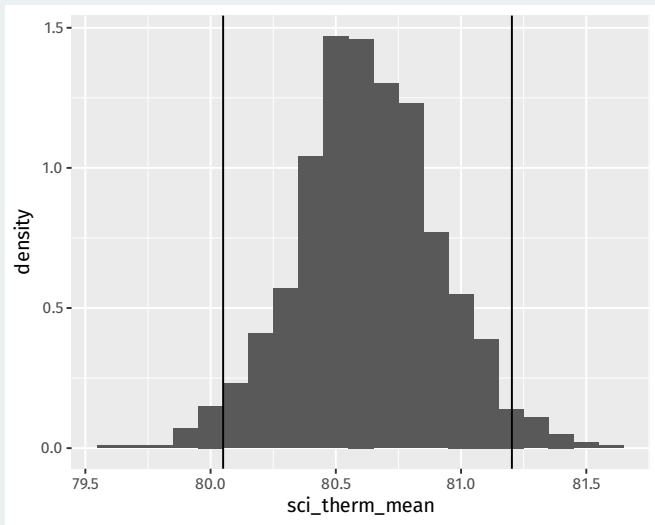
We can do this by finding the points that the 2.5th percentile and the 97.5th percentile.

```
perc_ci95 <- quantile(bootstrap_dist$sci_therm_mean,  
                      probs = c(0.025, 0.975))  
perc_ci95
```

```
## 2.5% 97.5%
```

```
## 80.1 81.2
```

Visualizing the CI



Width of the interval

What happens if we want the CI to be right more often? Will the width of a 99% confidence interval be wider or narrower?

99% confidence interval

For 99% CI we need to find the middle 99% of the bootstrap distribution.

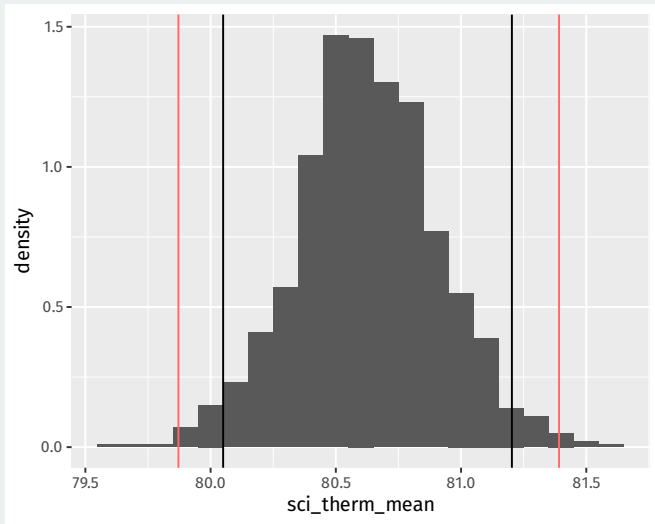
We can do this by finding the points that the 0.5th percentile and the 99.5th percentile.

```
perc_ci99 <- quantile(bootstrap_dist$sci_therm_mean,  
                      probs = c(0.005, 0.995))  
perc_ci99
```

```
## 0.5% 99.5%
```

```
## 79.9 81.4
```

Visualizing the CIs



3/ Calculating confidence intervals

infer package

Possible to use `quantile` to calculate CIs, but `infer` package is a more unified framework for CIs and hypothesis tests.

We'll use a `dplyr`-like approach of chained calls.

Step 1: define an outcome of interest

Start with defining the variable of interest:

```
anes |>  
  specify(response = sci_therm)
```

```
## Response: sci_therm (numeric)  
## # A tibble: 5,162 x 1  
##   sci_therm  
##   <dbl>  
## 1         70  
## 2        100  
## 3         60  
## 4         85  
## 5         85  
## 6         50  
## 7        100  
## 8         70  
## 9         80  
## 10        85  
## # i 5,152 more rows
```

Step 2: generate bootstraps

Next `infer` can generate bootstraps with the `generate()` function (similar to `rep_slice_sample()`):

```
anes |>  
  specify(response = sci_therm) |>  
  generate(reps = 1000, type = "bootstrap")
```

```
## Response: sci_therm (numeric)
## # A tibble: 5,162,000 x 2
## # Groups:   replicate [1,000]
##   replicate sci_therm
##         <int>      <dbl>
##  1           1         50
##  2           1         85
##  3           1         70
##  4           1         50
##  5           1        100
##  6           1         85
##  7           1         60
##  8           1         70
##  9           1         50
## 10          1        100
## # i 5,161,990 more rows
```

Step 3: calculate sample statistics

Use `calculate()` to do the `group_by(replicate)` and summarize commands in one:

```
boot_dist_infer <- anes |>  
  specify(response = sci_therm) |>  
  generate(reps = 1000, type = "bootstrap") |>  
  calculate(stat = "mean")
```

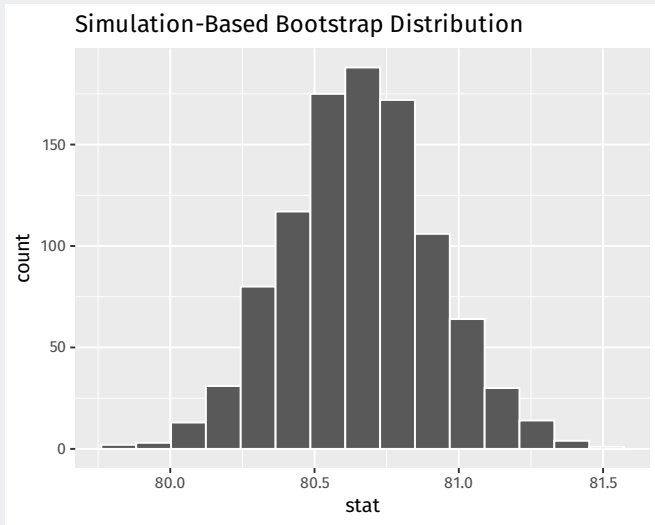
```
boot_dist_infer
```

```
## Response: sci_therm (numeric)
## # A tibble: 1,000 x 2
##   replicate stat
##   <int> <dbl>
## 1         1  80.4
## 2         2  80.6
## 3         3  81.0
## 4         4  80.7
## 5         5  80.3
## 6         6  81.1
## 7         7  80.6
## 8         8  80.6
## 9         9  80.3
## 10        10  80.9
## # i 990 more rows
```

Step 3(b): visualize the bootstrap distribution

`infer` also has a shortcut for plotting called `visualize()`:

```
visualize(boot_dist_infer)
```



Step 4: calculate CIs

Finally we can calculate the CI using the percentile method with `get_confidence_interval()`:

```
perc_ci_95 <- boot_dist_infer |>  
  get_confidence_interval(level = 0.95, type = "percentile")  
perc_ci_95
```

```
## # A tibble: 1 x 2  
##   lower_ci upper_ci  
##   <dbl>    <dbl>  
## 1    80.2    81.2
```


Step 4(b): visualize CIs

```
visualize(boot_dist_infer) +  
  shade_confidence_interval(endpoints = perc_ci_95)
```

