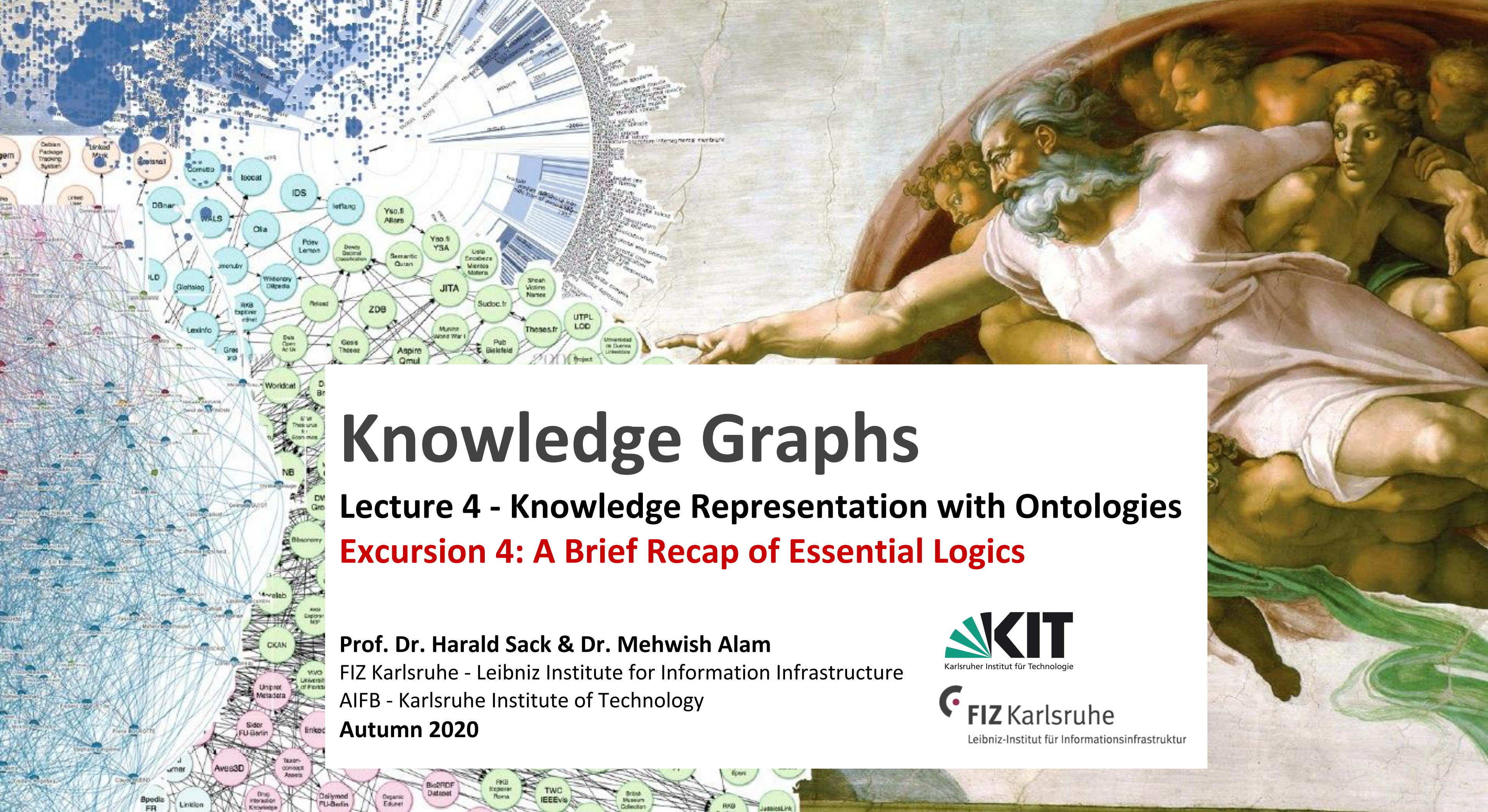


Knowledge Graphs

Lecture 4 - Knowledge Representation with Ontologies

Excursion 4: A Brief Recap of Essential Logics

Prof. Dr. Harald Sack & Dr. Mehwish Alam
FIZ Karlsruhe - Leibniz Institute for Information Infrastructure
AIFB - Karlsruhe Institute of Technology
Autumn 2020

Karlsruher Institut für Technologie

FIZ Karlsruhe
Leibniz-Institut für Informationsinfrastruktur

Knowledge Graphs

Lecture 4: Knowledge Representation with Ontologies

Propositional Logic - PL

- In propositional logic the world consists simply of **facts** and nothing else (**statements** of **assertions**).
- Example for propositional logic assertions and deductions:
  - `If it rains, the road will get wet.`
  - `If the moon is made out of green cheese, then cows can fly.`
  - `If Oliver is in love, then he will be happy.`
- The world consists out of objects and properties that distinguish one object from another.
- Between objects are relations. Some relations are unique, i.e. functions.

Propositional Logic - PL

Syntax:

- **Logical connectives**: Op={ ¬, ∧, ∨, →, ↔, (,) },
- a set of symbols Σ
- with Σ∩Op=∅ and {`true`, `false`}
- **Production rules** for propositional formulae (propositions):
  - all atomic formulas are propositions (*all elements of Σ*)
  - if φ is a proposition, then also ¬φ
  - if φ and ψ are propositions, then also φ∧ψ, φ∨ψ, φ→ψ, φ↔ψ
- **Priority**: ¬ prior to ∧, ∨ prior to →, ↔

connective	name	intentional meaning
¬	negation	“not”
∧	conjunction	“and”
∨	disjunction	“or”
→	implication	“if - then”
↔	equivalence	“if, and only if, then”

Propositional Logic - PL

How to model facts?

Simple Assertion	Modeling
The Moon is made of green cheese	g
It rains	r
The street is getting wet	n

Composed Assertion	Modeling
If it rains, then the street will get wet.	$r \rightarrow n$
If it rains and the street does not get wet, then the moon is made of green cheese.	$(r \land \neg n) \rightarrow g$

Propositional Logic - PL

Model-theoretic Semantics?

- **Interpretation I**:
  Mapping of all atomic propositions to {t,f}.
- If F is a formula and I an interpretation, then I(F) is a truth value computed from F and I via **truth tables**.

I(p)	I(q)	I(¬p)	I(p∨q)	I(p∧q)	I(p→q)	I(p↔q)
f	**f**	t	**f**	**f**	t	t
f	t	t	t	**f**	t	**f**
t	**f**	**f**	t	**f**	**f**	**f**
t	t	**f**	t	t	t	t

Propositional Logic - PL

Model-theoretic Semantics?

- We write $\mathtt{I} \vDash \mathtt{F}$, if $\mathtt{I(F)=t}$,
  and call interpretation $\mathtt{I}$ a **Model** of formula $\mathtt{F}$.
- Rules of Semantics:
  - $\mathtt{I}$ is model of $\neg\varphi$, iff $\mathtt{I}$ is not a model of φ
  - $\mathtt{I}$ is model of $(\varphi \land \psi)$, iff $\mathtt{I}$ is a model of φ AND of ψ
  - ...
- Basic concepts:
  - tautology
  - satisfiable
  - refutable
  - unsatisfiable (contradiction)

Propositional Logic - PL

Model-theoretic Semantics?

- **Decidability**
  - All **true entailments** can be found, and all **false entailments** can be refuted, as long as you spend enough time.
    ⇒ there always exist terminating automatic theorem proofers
- Another useful property:
  - $\{\varphi_1, \ldots, \varphi_n\} \vDash \varphi$ holds, iff $(\varphi_1 \land \ldots \land \varphi_n) \rightarrow \varphi$ is a tautology
  - The decision, if an assertion is an tautology, can be made via truth tables
  - In principle this equals the evaluation of all possible interpretations

semantic entailment/inference

syntactic entailment/inference

First Order Logic - FOL

quantifier	name	intentional meaning
∃	existential quantifier	“it exists”
∀	universal quantifier	“for all”

- **Operators (logical connectives)** as in propositional logic
- **Variables**, e.g., X,Y,Z,…
- **Constants**, e.g., a, b, c, … (i.e. a named object from the domain of discourse)
- **Functions**, e.g., f, g, h, … (incl. arity)
- **Relations / Predicates**,e.g., p, q, r, … (incl. arity)

Example of a FOL formula: $(\forall X)(\exists Y)\ ((p(X) \lor \neg q(f(X),Y)) \rightarrow r(X))$

First Order Logic - FOL

FOL Syntax

- „Correct“ formulation of **Terms** from Variables, Constants and Functions:
  - `f(X), g(a,f(Y)), s(a), .(H,T), x_location(Pixel)`
- „Correct“ formulation of **Atoms** (or **Atomic Formulas**) from Predicates with Terms as arguments
  - `p(f(X)),q(s(a),g(a,f(Y))),add(a,s(a),s(a)), greater_than(x_location(Pixel),128)`
- „Correct“ formulation of (composed) **Formulas** from Atomic Formulas, Operators and Quantifiers:
  - `(∀Pixel)(greater_than(x_location(Pixel),128) → red(Pixel))`
- If in doubt, use brackets!
- All Variables should be quantified!

First Order Logic - FOL

How to model Facts?

- „All kids love ice cream.“
  $\forall X\colon \mathtt{Child}(X) \rightarrow \mathtt{lovesIceCream}(X)$

- „The father of a person is its male parent.“
  $\forall X\ \forall Y\colon \mathtt{isFather}(X,Y) \leftrightarrow (\mathtt{Male}(X) \land \mathtt{isParent}(X,Y))$

- „There are (one or more) interesting lectures.“
  $\exists X\colon \mathtt{Lecture}(X) \land \mathtt{Interesting}(X)$

- „The relation ‚isNeighbor‘ is symmetric.“
  $\forall X\ \forall Y\colon \mathtt{isNeighbor}(X,Y) \rightarrow \mathtt{isNeighbor}(Y,X)$

First Order Logic - FOL

How to model Facts?

- „All kids love ice cream."
  $\forall X\colon \mathrm{Child}(X) \rightarrow \mathrm{lovesIceCream}(X)$
- „The father of a person is its male parent."
  $\forall X\ \forall Y\colon \mathrm{isFather}(X,Y) \leftrightarrow (\mathrm{Male}(X) \wedge \mathrm{isParent}(X,Y))$
- „There are (one or more) interesting lectures."
  $\exists X\colon \mathrm{Lecture}(X) \wedge \mathrm{Interesting}(X)$
- „The relation ‚isNeighbor' is symmetric."
  $\forall X\ \forall Y\colon \mathrm{isNeighbor}(X,Y) \rightarrow \mathrm{isNeighbor}(Y,X)$

First Order Logic - FOL

How to model Facts?

- There is a significant difference:
  - *„All kids love ice cream.“*

$\forall X\texttt{: Child(X)} \rightarrow \texttt{lovesIceCream(X)}$

It is possible that X is not a kid but (nevertheless) loves ice cream

$\forall X\texttt{: Child(X)} \land \texttt{lovesIceCream(X)}$

There are only kids in the universe of discourse and they love ice cream

∀X: Child(X)	∀X: lovesIcecream(x)	∀X: Child(X) → lovesIcecream(X)	∀X: Child(X) ∧ lovesIcecream(X)
0	0	1	0
0	1	1	0
1	0	0	0
1	1	1	1

First Order Logic - FOL

How to model Facts?

- There is a significant difference:
  - *„There are (one or more) interesting lectures.“*

    $\exists X: \mathrm{Lecture}(X) \land \mathrm{Interesting}(X)$ *The things we address here must have both properties*

    $\exists X: \mathrm{Lecture}(X) \rightarrow \mathrm{Interesting}(X)$ *Here X is also qualified, if X is not a lecture*

∃X: Lecture(X)	∃X: Interesting(X)	∃X: Lecture(X) → Interesting(X)	∃X: Lecture(X) ∧ Interesting(X)
0	0	1	0
0	1	1	0
1	0	0	0
1	1	1	1

First Order Logic - FOL

Model-theoretic Semantics

- **Structure**:
  - Definition of a **domain** D.
  - **Constant symbols** are mapped to elements of D.
  - **Function symbols** are mapped to functions in D.
  - **Relation symbols** are mapped to relations over D.

- **Then**:
  - **Terms** will become elements of D.
  - **Relation symbols** with arguments will become `true` or `false`.
  - **Logical connectives** and **quantifiers** are treated likewise.

First Order Logic - FOL

The Penguin Example

Penguins are Black and White.
Some old TV shows are Black and White.
Therefore, some Penguins are Old TV Shows.

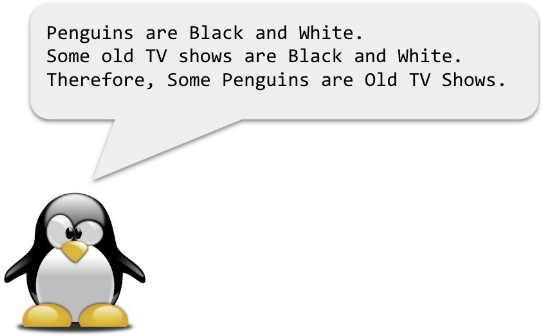
Logic: another thing that Penguins aren't very good at...

First Order Logic - FOL

The Penguin Example

$$
\begin{array}{l}
(\ (\forall X)(\ \mathrm{penguin}(X) \rightarrow \mathrm{blackandwhite}(X)\ ) \\
\land\ (\exists X)(\ \mathrm{oldTVshow}(X)\ \land\ \mathrm{blackandwhite}(X) \\
) \\
)\ \rightarrow\ (\exists X)(\ \mathrm{penguin}(X)\ \land\ \mathrm{oldTVshow}(X)\ )
\end{array}
$$

What is the intentional semantics?



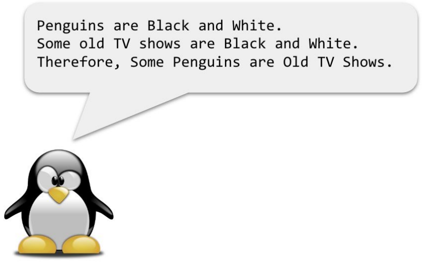
Logic: another thing that Penguins aren't very good at...

First Order Logic - FOL

The Penguin Example

```
( (∀X)( penguin(X) → blackandwhite(X) )
∧ (∃X)( oldTVshow(X) ∧ blackandwhite(X)
)
) → (∃X)( penguin(X) ∧ oldTVshow(X) )
```

- Interpretation I:
  - **Domain**: a set M, containing elements a, b.
  - … no constants or function symbols …
  - We show: the formula is *refutable* (i.e. it is not a tautology)
  - If I(penguin)(a), I(blackandwhite)(a), I(oldTVshow)(b), I(blackandwhite)(b) is true , I(oldTVshow)(a) is false,
  - then the formula with Interpretation I is false, i.e. $\mathrm{I} \nvDash \mathrm{F}$



Logic: another thing that Penguins aren't very good at...

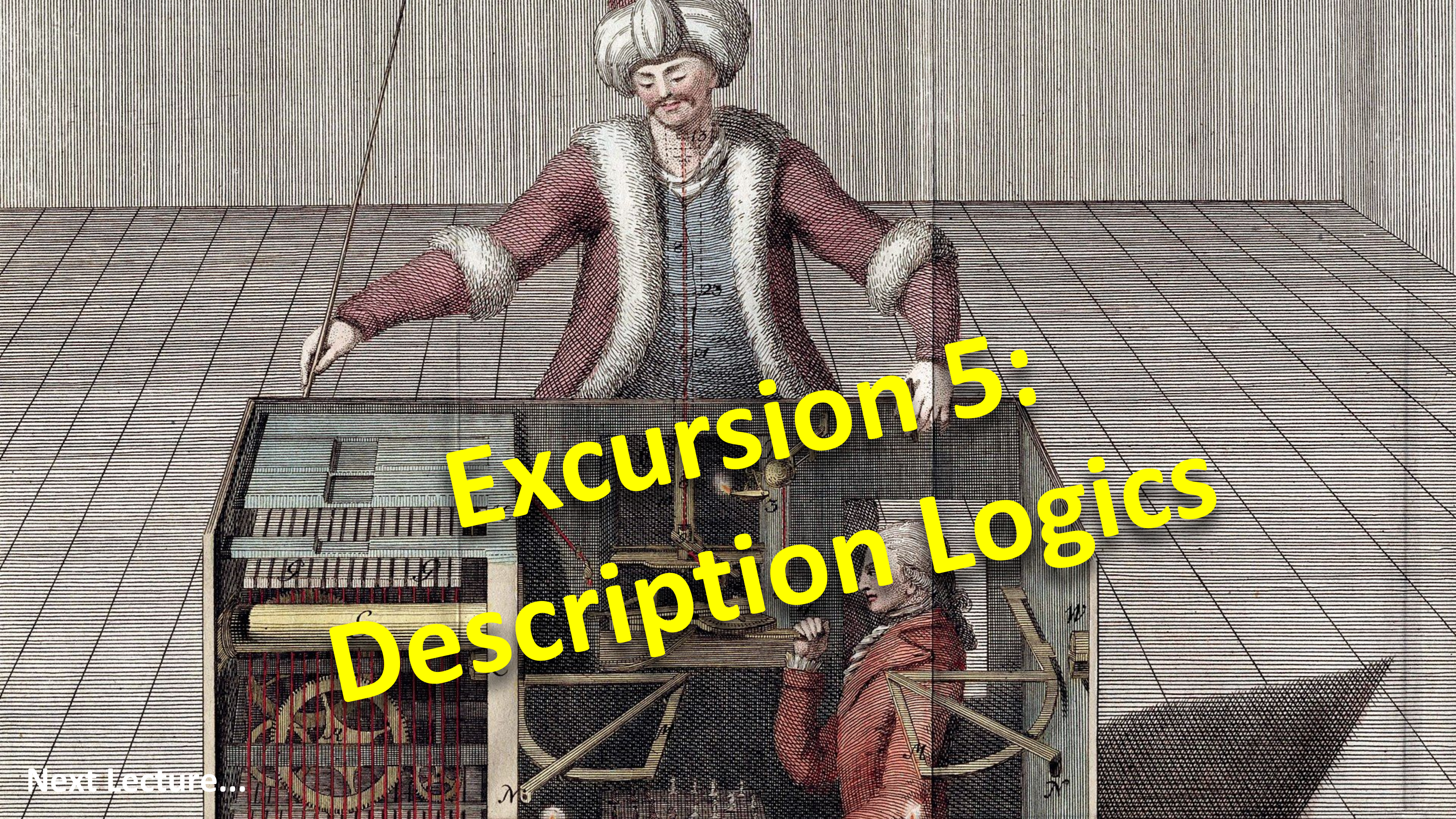
First Order Logic - FOL

Logical Entailment

Karlsruher Institut für Technologie
FIZ Karlsruhe
Leibniz Institute for Information Infrastructure

- A **theory** $\mathtt{T}$ is a set of formulas.
- An interpretation $\mathtt{I}$ is a **model** of $\mathtt{T}$, iff $\mathtt{I} \models \mathtt{G}$ for all formulas $\mathtt{G}$ in $\mathtt{T}$.
- A formula $\mathtt{F}$ is a **logical consequence** of $\mathtt{T}$, iff all models of $\mathtt{T}$ are also models of $\mathtt{F}$.
- Then we write $\mathbf{T} \models \mathbf{F}$.
- Two formulas $\mathtt{F}$, $\mathtt{G}$ are called **logically equivalent**, iff $\{\mathtt{F}\} \models \mathtt{G}$ and $\{\mathtt{G}\} \models \mathtt{F}$.
- Then we write $\mathbf{F} \equiv \mathbf{G}$

Theory ≙ Knowledge Base


Excursion 5:
Description Logics
Next Lecture...

Picture References:

- [1] Penguin, pixabay [Public Domain]
  https://pixabay.com/vectors/penguin-tux-animal-bird-cute-158551/
- [2] Joseph Racknitz, From book that tried to explain the illusions behind the Kempelen chess playing automaton (known as The Turk) after making reconstructions of the device. 1789 [Public Domain]
  https://commons.wikimedia.org/wiki/File:Racknitz_-_The_Turk_3.jpg