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11. Obtain an upper bound on the absolute error when we compute $\int_0^6 \sin x^2\,dx$ by means of the composite trapezoid rule using 101 equally spaced points.

Error formula:

$$I - T = -\frac{1}{12}(a-b)h^2 f''(x)$$

$$h = \frac{b-a}{n}$$

$$\therefore\ I - T = -\frac{1}{2}(a-b)\left(\frac{b-a}{h}\right)^2 f''(x)$$

$f(x) = \sin x^2$

$f'(x) = 2x\cos x^2$

$f''(x) = 2\cos x^2 - 4x^2\sin x^2$

$|2\cos x^2 - 4x^2\sin x^2|$

$2\cos x^2 + 4x^2\sin x^2$

$2 + 4x^2(\cos x^2 + \sin x^2)$

$2 + 4x^2(1)$

$2 + 4(6^2)$

$2 + 4(36)$

$2 + 144$

$f''(x) = 146$

$$I-T = -\frac{1}{12}(0-6)\left(\frac{0-6}{100}\right)^2(146)$$

$$= \frac{1}{2}\left(\frac{36\cdot 146}{10000}\right)$$

$$= \frac{5256}{20000}$$

$$\approx 0.2628$$

17. Compute two approximate values for $\int_1^2 dx/x^2$ using $h = \frac{1}{2}$ with the composite trapezoid rule.

$$I = \frac{h}{2}\left[f(x_0) + f(x_n)\right]$$

$x_0 = 1$, $x_1 = 1.5$, $x_2 = 2$

$f(x) = \frac{1}{x^2}$

$f(1) = 1$

$f(1.5) = \frac{1}{1.5^2} = 0.44$

$f(2) = \frac{1}{4} = 0.25$

$h = \frac{2-1}{2} = \frac{1}{2} = 0.5$

$$I = \frac{0.5}{2}\left[1 + 2(0.44) + 0.25\right]$$

$$= \frac{0.5\cdot 2.1388}{2} = 0.5347$$

18. Consider $\int_1^2 dx/x^3$. What is the result of using the composite trapezoid rule with the partition points 1, $\frac{3}{2}$, and 2?

$x_0 = 1$

$x_1 = \frac{3}{2}$

$x_2 = 2$

$f(x) = \frac{1}{x^3}$

$f(1) = \frac{1}{1^3} = 1$

$f\left(\frac{3}{2}\right) = \frac{1}{\left(\frac{3}{2}\right)^3} = \frac{8}{27}$

$f(2) = \frac{1}{2^3} = \frac{1}{8}$

$\int_a^b f(x)\,dx \approx \sum_{i=0}^{n-1} \frac{h}{2}\left(f(x_i) + f(x_{i+1})\right)$

$h_i = x_{i+1} - x_i$

subinterval $[1, \frac{3}{2}]$

$h_1 = \frac{3}{2} - 1 = \frac{1}{2}$

so:

$\frac{1}{2} \cdot \frac{1}{2}\left(f(1) + f(3/2)\right) = \frac{1}{4}\left(1 + \frac{8}{27}\right)$

$= \frac{35}{108}$

For interval $[\frac{3}{2}, 2]$

$h_2 = 2 - \frac{3}{2} = \frac{1}{2}$

$\frac{1}{2} \cdot \frac{1}{2}\left(f(3/2) + f(2)\right) = \frac{1}{4}\left(\frac{8}{27} + \frac{1}{8}\right) =$

$= \frac{91}{864}$

$\frac{35}{108} + \frac{91}{864} = \frac{371}{864} \approx 0.429398$