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2. If we use the secant method on $f(x) = x^3 - 2x + 2$ starting with $x_0 = 0$ and $x_1 = 1$, what is x_2?

$$x_{n+1} = x_n - \left(\frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}\right) \cdot f(x_n)$$

$$x_2 = x_1 - \left(\frac{x_1 - x_0}{f(x_1) - f(x_0)}\right) f(x_1)$$

$$= 1 - \left(\frac{1-0}{1-2}\right)(1) = 2$$

$$f(1) = 1^3 - (2 \cdot 1) + 2 = 1$$

$$f(0) = 2$$

11. Show that if the iterates in Newton's method converge to a point r for which $f'(r) \neq 0$, then $f(r) = 0$. Establish the same assertion for the secant method.
Hint: In the latter, the Mean-Value Theorem of Differential Calculus is useful. This is the case $n = 0$ in Taylor's Theorem.

Newton Method:

$$x_{n+1} = x_n - \frac{f(x)}{f'(x_n)}$$

if the sequence converges to r then:

$\lim_{n \to \infty} x_n = r$ then:

$$r = r - \frac{f(r)}{f'(r)} \quad \text{then:} \quad \frac{f(r)}{f'(r)} = 0$$

Since $f'(r) \neq 0$, therefore $f(r) = 0$

Therefoer r is roof of $f(x)$

Now secant method:

$$x_{n+1} = x_n - \left(\frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}\right) f(x_n)$$

if $x_n \to r$, then $x_{n-1} \to r$

then we have:

$$\frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}} = f'(\xi)$$

for:

$$\frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \approx \frac{1}{f'(\xi)}$$

by substituting the formula we have:

$$x_{n+1} = x_n - f(x_n) \cdot \frac{1}{f'(\xi)}$$

by taking the limit:

$$r = r - \frac{f(r)}{f'(r)}$$

$$f(r) = 0$$

$\therefore$ r is the roof of $f(x)$

13. Test the following sequences for different types of convergence (i.e., linear, superlinear, or quadratic), where $n = 1, 2, 3, \ldots$.

[a]**a.** $x_n = n^{-2}$ **b.** $x_n = 2^{-n}$ [a]**c.** $x_n = 2^{-2^n}$
d. $x_n = 2^{-a_n}$ with $a_0 = a_1 = 1$ and $a_{n+1} = a_n + a_{n-1}$ for $n \geqq 2$

$$x_n = 2^{-n}$$

First we take the limit:

$\lim_{n \to \infty} 2^{-n} = 0$, this shows that the sequence converges.

Now we need to check convergence rate:

$$\lim_{n \to \infty} \frac{|x_{n+1} - L|}{|x_n - L|^{\alpha}} = C$$

$$\frac{x_{n+1}}{x_n} = \frac{2^{-(n+1)}}{2^{-n}}$$

$$= 2^{-n-1} \cdot 2^{n} = 2^{-1} = \frac{1}{2}$$

Since the ratio is $\frac{1}{2}$, constant, the convergence is <u>Linear</u>