Govher Sapardurdyyeva

6. Find the polynomial p of least degree that takes these values: $p(0) = 2$, $p(2) = 4$, $p(3) = -4$, $p(5) = 82$. Use divided differences to get the correct polynomial. It is *not* necessary to write the polynomial in the standard form $a_0 + a_1x + a_2x^2 + \cdots$.

$$f(x_0, x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$f(x_0, x_1, x_2) = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$$

$$P(x) = 2 + 1(x-0) + (-3)(x-0)(x-2) + 4(x-0)(x-2)(x-3)$$

$$\boxed{P(x) = 4x^3 - 23x^2 + 31x + 2}$$

7. Complete the following divided-difference tables, and use them to obtain polynomials of degree 3 that interpolate the function values indicated:

b.

x	$f[\ ]$	$f[\ ,\ ]$	$f[\ ,\ ,\ ]$	$f[\ ,\ ,\ ,\ ]$
-1	2			
1	-4			
3	46			
		53.5		
4	99.5			

$$f(x_0, x_1) = \frac{-4-2}{1+1} = \frac{-6}{2} = 3$$

$$f(x_1, x_2) = \frac{46-(-4)}{3-1} = \frac{50}{2} = 25$$

$$f(x_2, x_3) = \frac{99.5-46}{4-3} = \frac{535}{1} = 53.5$$

$$f[x_0, x_1, x_2] = \frac{25-(-3)}{3-(-1)} = \frac{28}{4} = 7$$

$$f[x_1, x_2, x_3] = \frac{53.5-25}{4-1} = \frac{28.5}{3} = 9.5$$

$$f[x_0, x_1, x_2, x_3] = \frac{9.5-7}{4-(-1)} = \frac{2.5}{5} = 0.5$$

$$P(x) = 2 - 3(x+1) + 7(x+1)(x-1) + 0.5(x+1)(x-1)(x-3)$$

$$\boxed{P(x) = 0.5x^3 + 5.5x^2 - 3.5x - 6.5}$$

10. a. Construct Newton's interpolation polynomial for the data shown.

x	0	2	3	4
y	7	11	28	63

$$f[x_0, x_1] = \frac{11-7}{2-0} = \frac{4}{2} = 2$$

$$f[x_1, x_2] = \frac{28-11}{3-2} = 17$$

$$f[x_2, x_3] = \frac{63-28}{4-3} = 35$$

$$f[x_0, x_1, x_2] = \frac{17-2}{3-0} = 5$$

$$f[x_1, x_2, x_3] = \frac{35-17}{4-2} = 9$$

$$f[x_0, x_1, x_2, x_3] = \frac{9-5}{4-0} = 1$$

$$P(x) = 7 + 2(x-0) + 5(x-0)(x-2) + 1(x-0)(x-2)(x-3)$$

$$\boxed{P(x) = x^3 - 2x + 7}$$