

보경, 01 수열의 극한

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1 극한값의 계산

문제 1)

$$\lim_{n \rightarrow \infty} \frac{1}{n} = 0$$

문제 2)

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{5n+4}{2n^2+3n+1} &= \lim_{n \rightarrow \infty} \frac{\frac{5}{n} + \frac{4}{n^2}}{2 + \frac{3}{n} + \frac{1}{n^2}} \\ &= \frac{0+0}{2+0+0} = 0 \end{aligned}$$

문제 3)

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{n^2+5n+4}{2n^2+3n+1} &= \lim_{n \rightarrow \infty} \frac{1 + \frac{5}{n} + \frac{4}{n^2}}{2 + \frac{3}{n} + \frac{1}{n^2}} \\ &= \frac{1+0+0}{2+0+0} = \frac{1}{2} \end{aligned}$$

문제 4)

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{-n^3+n^2+5n+4}{2n^2+3n+1} &= \lim_{n \rightarrow \infty} \frac{-n+1+\frac{5}{n}+\frac{4}{n^2}}{2+\frac{3}{n}+\frac{1}{n^2}} \\ &= -\infty \end{aligned}$$

문제 5)

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{2n+7}{\sqrt{n^2+1}-1} &= \lim_{n \rightarrow \infty} \frac{2+\frac{7}{n}}{\sqrt{1+\frac{1}{n^2}}-\frac{1}{n}} \\ &= \frac{2+0}{\sqrt{1+0}-0} = 2 \end{aligned}$$

문제 6)

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2+4n}-n} &= \lim_{n \rightarrow \infty} \frac{\sqrt{n^2+4n}+n}{4n} \\ &= \lim_{n \rightarrow \infty} \frac{\sqrt{1+\frac{4}{n}}+1}{4} = \frac{1}{2} \end{aligned}$$

문제 7)

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{4^{n+2}}{2^{n+1}-4^n} &= \lim_{n \rightarrow \infty} \frac{16 \times 4^n}{2 \times 2^n - 4^n} \\ &= \lim_{n \rightarrow \infty} \frac{16}{2 \times \left(\frac{1}{2}\right)^n - 1} \\ &= \frac{16}{0-1} = -16 \end{aligned}$$

2 극한의 성질

문제 8)

$\lim_{n \rightarrow \infty} a_n = 2$, $\lim_{n \rightarrow \infty} (a_n - b_n) = 3$ 일 때,

$$\begin{aligned} \lim_{n \rightarrow \infty} b_n &= \lim_{n \rightarrow \infty} (a_n - (a_n - b_n)) \\ &= \lim_{n \rightarrow \infty} a_n - \lim_{n \rightarrow \infty} (a_n - b_n) = 2 - 3 = -1 \end{aligned}$$

문제 9)

$\lim_{n \rightarrow \infty} (n+4)a_n = 4$ 일 때,

$$\begin{aligned} \lim_{n \rightarrow \infty} (2n+5)a_n &= \lim_{n \rightarrow \infty} \left\{ (n+4)a_n \times \frac{2n+5}{n+4} \right\} \\ &= \lim_{n \rightarrow \infty} (n+4)a_n \times \lim_{n \rightarrow \infty} \frac{2n+5}{n+4} \\ &= 4 \times 2 = 8 \end{aligned}$$

문제 10)

방법 1

$b_n = \frac{3a_n-2}{2a_n+1}$ 라고 하면

$$nb_n + 4b_n = 2n + 5$$