

# 시계열 회귀분석(Time Series Regression - 3)

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이것은 Time Series Regression - Part 3 강의에 대한 노트이다.

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$$y_t = TR_t + SN_t + \varepsilon_t$$

$y_t$  = the value of the time series in period t

$TR_t$  = the trend in time period t

$SN_t$  = the seasonal factor in time period t

$\varepsilon_t$  = the error term(irregular factor) in time period t

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위의 식에서 보듯이, 많은 경우에 시계열은 추세(trend)와 계절성(seasonality), 그리고 오차로 이루어져 있다. 이번 강의에서는 seasonal variation을 모델링하는 네 가지 모델에 대해서 다루는 것 같다. 그것들은

- binary variable models
- trigonometric models
- growth curve models
- first-order autoregressive process

이다. 이 중에서 앞의 세 개는 고전적인 방법이고 마지막 방법이 앞으로 계속 사용할 방법인 것 같다.

이번 정리부터는 유튜브 캡쳐를 나열한 후 정리하려고 하고, 강의 내용 자체에만 충실히 적어보려고 한다.

## 1 Binary Variable Models

### Modeling Seasonal Variations Using Binary Variables

The seasonal factor expressed using binary variables is  
 $SN_t = \beta_1 x_{s1,t} + \beta_2 x_{s2,t} + \dots + \beta_{L-1} x_{s(L-1),t}$

where  $x_{s1,t}, x_{s2,t}, \dots, x_{s(L-1),t}$  are binary variables that are defined as follows:

$$x_{s1,t} = \begin{cases} 1 & \text{if time period } t \text{ is season 1} \\ 0 & \text{otherwise} \end{cases}$$

$$x_{s2,t} = \begin{cases} 1 & \text{if time period } t \text{ is season 2} \\ 0 & \text{otherwise} \end{cases}$$

$$\vdots$$

$$x_{s(L-1),t} = \begin{cases} 1 & \text{if time period } t \text{ is season } (L-1) \\ 0 & \text{otherwise} \end{cases}$$

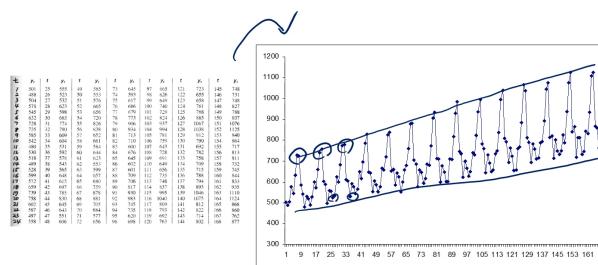
### Example - Binary Variable Models

| t  | y <sub>t</sub> | t  | y <sub>t</sub> | t   | y <sub>t</sub> | t   | y <sub>t</sub> | t   | y <sub>t</sub> | t    | y <sub>t</sub> | t    | y <sub>t</sub> |
|----|----------------|----|----------------|-----|----------------|-----|----------------|-----|----------------|------|----------------|------|----------------|
| 1  | 50             | 25 | 535            | 39  | 235            | 75  | 645            | 97  | 655            | 121  | 723            | 145  | 731            |
| 2  | 488            | 26 | 523            | 53  | 533            | 74  | 593            | 98  | 586            | 122  | 655            | 146  | 731            |
| 3  | 504            | 27 | 532            | 51  | 576            | 75  | 617            | 99  | 649            | 123  | 658            | 147  | 748            |
| 4  | 578            | 28 | 623            | 52  | 665            | 76  | 686            | 100 | 740            | 124  | 761            | 147  | 827            |
| 5  | 545            | 29 | 598            | 53  | 656            | 77  | 679            | 101 | 729            | 125  | 768            | 149  | 788            |
| 6  | 632            | 30 | 683            | 54  | 720            | 78  | 773            | 102 | 824            | 126  | 885            | 150  | 937            |
| 7  | 728            | 31 | 774            | 826 | 79             | 903 | 103            | 963 | 127            | 1027 | 151            | 1079 | 160            |
| 8  | 725            | 32 | 800            | 56  | 838            | 80  | 834            | 104 | 904            | 128  | 1038           | 152  | 1256           |
| 9  | 585            | 33 | 609            | 57  | 652            | 81  | 713            | 105 | 781            | 129  | 812            | 153  | 840            |
| 10 | 542            | 34 | 694            | 58  | 661            | 82  | 710            | 106 | 759            | 130  | 790            | 154  | 864            |
| 11 | 480            | 35 | 531            | 59  | 584            | 83  | 600            | 107 | 643            | 131  | 692            | 155  | 717            |
| 12 | 530            | 36 | 592            | 60  | 644            | 84  | 676            | 108 | 728            | 132  | 782            | 156  | 813            |
| 13 | 570            | 37 | 570            | 61  | 627            | 85  | 676            | 109 | 691            | 133  | 758            | 157  | 811            |
| 14 | 489            | 38 | 543            | 62  | 553            | 86  | 672            | 110 | 649            | 141  | 709            | 158  | 732            |
| 15 | 528            | 39 | 565            | 63  | 599            | 87  | 601            | 111 | 656            | 135  | 715            | 159  | 745            |
| 16 | 599            | 40 | 648            | 64  | 657            | 88  | 709            | 112 | 735            | 136  | 788            | 160  | 844            |
| 17 | 572            | 41 | 615            | 65  | 680            | 89  | 706            | 113 | 745            | 137  | 794            | 161  | 833            |
| 18 | 659            | 42 | 697            | 66  | 759            | 90  | 817            | 114 | 837            | 138  | 893            | 162  | 935            |
| 19 | 739            | 43 | 785            | 67  | 878            | 91  | 901            | 115 | 995            | 139  | 1046           | 163  | 1110           |
| 20 | 758            | 44 | 839            | 68  | 889            | 92  | 938            | 116 | 949            | 140  | 1053           | 164  | 1154           |
| 21 | 662            | 45 | 645            | 69  | 705            | 93  | 745            | 117 | 809            | 141  | 812            | 165  | 868            |
| 22 | 587            | 46 | 643            | 70  | 684            | 94  | 735            | 118 | 793            | 142  | 822            | 166  | 860            |
| 23 | 497            | 47 | 551            | 71  | 577            | 95  | 620            | 119 | 692            | 143  | 714            | 167  | 762            |
| 24 | 558            | 48 | 606            | 72  | 656            | 96  | 698            | 120 | 763            | 144  | 802            | 168  | 877            |

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### Seasonal Variations

- A linear trend and increasing seasonal variations



### Example - Binary Variable Models

$$y_t^* = TR_t + SN_t + \varepsilon_t$$

$$= \beta_0 + \beta_1 t + \beta_2 M_1 + \beta_3 M_2 + \dots + \beta_{12} M_{12} + \varepsilon_t$$

where  $y_t^* = \ln y_t$  and  $M_1, M_2, \dots, M_{12}$  are seasonal binary variables.

|     | $X_1$<br>( $t \in M_1$ ) | $X_2$<br>( $M_2$ ) | $X_3$<br>( $M_3$ ) | $X_4$<br>( $M_4$ ) | $\dots$ | $X_{12}$<br>( $M_{12}$ ) | $\bar{Y}$ |
|-----|--------------------------|--------------------|--------------------|--------------------|---------|--------------------------|-----------|
| 1   | 1                        | 1                  | 0                  | 0                  | ...     | 0                        | 50.1      |
| 2   | 2                        | 0                  | 1                  | 0                  | ...     | 0                        | 48.8      |
| 3   | 3                        | 0                  | 0                  | 1                  | ...     | 0                        | 50.4      |
| 4   | 4                        | 0                  | 0                  | 0                  | ...     | 0                        | 50.7      |
| ... | ...                      | ...                | ...                | ...                | ...     | ...                      | ...       |
| 168 | 168                      | 0                  | 0                  | 0                  | ...     | 0                        | 877       |

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## Example - Binary Variable Models

$$y_t = TR_t + SN_t + \varepsilon_t$$

$$y_t = \beta_0 + \beta_1 t + \underbrace{\beta_2 M_1 + \beta_3 M_2 + \cdots + \beta_{12} M_{11}}_{TR_t} + \underbrace{\beta_{13} M_{12}}_{SN_t} + \varepsilon_t$$

$$SN_t = \beta_{s1} x_{s1,t} + \beta_{s2} x_{s2,t} + \cdots + \beta_{s(L-1)} x_{s(L-1),t}$$

$M_1 \quad M_2 \quad \dots \quad M_{12}$

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## Example - Binary Variable Models

$$\tilde{y}_t^* = TR_t + SN_t + \varepsilon_t$$

$$= \beta_0 + \beta_1 t + \beta_2 M_1 + \beta_3 M_2 + \cdots + \beta_{12} M_{11} + \varepsilon_t$$

where  $\tilde{y}_t^* = \ln y_t$  and  $M_1, M_2, \dots, M_{11}$  are seasonal binary variables.

$M_{12}$

- Here we have arbitrarily set the seasonal parameter for season 12 (the last month, December) equal to zero. Thus, the other 11 seasonal parameters are defined with respect to December.
- For example,  $\beta_2$  is the difference between the level of time series in January and the level of the time series in December.
- A positive  $\beta_2$  implies that the level of the time series in January can be expected to be greater than the level in December.

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## Example - Binary Variable Models

| Variable        | DF | Parameter Estimates |                |         |         |
|-----------------|----|---------------------|----------------|---------|---------|
|                 |    | Estimate            | Standard Error | t Value | Pr >  t |
| Intercept       | 1  | 6.28756             | 0.00643        | 978.26  | <.0001  |
| Time            | 1  | 0.00223             | 0.00003379     | 60.65   | <.0001  |
| 1               | 1  | -0.04161            | 0.00801        | -4.93   | <.0001  |
| 2               | 1  | -0.11208            | 0.00801        | -13.98  | <.0001  |
| 3               | 1  | -0.08446            | 0.00801        | -10.54  | <.0001  |
| 4               | 1  | 0.03985             | 0.00801        | 4.97    | <.0001  |
| 5               | 1  | 0.02050             | 0.00801        | 2.55    | 0.0119  |
| 6               | 1  | 0.14691             | 0.00801        | 18.94   | <.0001  |
| 7               | 1  | 0.28992             | 0.00801        | 36.09   | <.0001  |
| 8               | 1  | 0.31119             | 0.00801        | 38.86   | <.0001  |
| 9               | 1  | 0.05539             | 0.00801        | 6.99    | <.0001  |
| 10              | 1  | 0.03600             | 0.00801        | 4.94    | <.0001  |
| 11              | 1  | -0.11222            | 0.00801        | -14.01  | <.0001  |
| Durbin-Watson D |    | 1.190               |                |         |         |

$\tilde{y}_{169}^* = b_0 + b_1(169) + b_2(1) + \dots$   
 $= 6.28756 + .00223(169) + (-0.04161)(1)$   
 $= 6.7065$   
 $\hat{y}_{169} = e^{6.7065} = 817.70$

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seasonal variation을 다루기 위한 여러 모델들 중 첫번째로 다루는 모델은 binary variable model이다. 예시로 주어지고 있는 데이터셋은 호텔의 투숙된 객실 수에 대한 데이터이다. 시간  $t$ 의 단위는 ‘월’로 주어져있고, 총 7년의 데이터가 있으므로  $t \in 1, 2, \dots, 168$ 이다. 독립변수는  $t$ , 종속변수는  $y$ 로 이루어져 있다. 다시 말해서, 몇 번째 달( $t$ )에 몇 개의 객실들( $y$ )이 투숙되어 있는지 하는 단변수 회귀 (univariate regression) 문제이다.

그런데 세 번째 슬라이드에서 보듯 계절성이 뚜렷이 드러나고 있으므로, 예측모델  $f_\beta$ 을 설정할 때 추세(trend,  $TR_t$ ) 말고도 계절성(seasonal variation,  $SN_t$ )도 고려할 것이다. 즉

$$f_\beta(t) = TR_t + SN_t \quad (1)$$

이고

$$y_t = f_\beta(t) + \varepsilon_t$$

이다. 추세는 일차함수(affine function)로 나타낼 것이어서

$$TR_t = \beta_0 + \beta_1 t$$

로 표현할 것이고, 계절성은 각각의 계절에 대하여 상수회귀(no trend, constant regression)를 진행한다. 식으로 표현하면

$$SN_t = \begin{cases} \beta_2 & (t = 12n + 1) \\ \beta_3 & (t = 12n + 2) \\ \vdots & (\text{단, } n = 0, 1, 2, \dots, 6) \\ \beta_{11} & (t = 12n + 10) \\ 0 & (t = 12n + 11) \end{cases}$$

이다. 이것을 표현하기 위해서 강의에서는  $M_1, M_2, \dots, M_{11}$ 을 사용하고 있는데, 이건 각 월에 대한 characteristic(indicator) function으로 이해하면 될 것 같다. 여하튼, 식 (1)을 다시 정리하면

$$f_\beta(t) = \begin{cases} \beta_0 + \beta_2 + \beta_1 t & (t = 12n + 1) \\ \beta_0 + \beta_3 + \beta_1 t & (t = 12n + 2) \\ \vdots & \\ \beta_0 + \beta_{11} + \beta_1 t & (t = 12n + 10) \\ \beta_0 + \beta_1 t & (t = 12n + 11) \end{cases} \quad (\text{단, } n = 0, 1, 2, \dots, 6) \quad (2)$$

이 된다. 그런 의미에서  $M_{12}$ 는 굳이 사용하지 않았다. 각 월(1월 ~ 11월)에 대한 정보를 담고 있는 항은  $\beta_2, \dots, \beta_{11}$ 이다. 그리고 12월에 대한 정보를 담고 있는 항은 없다. 하지만 문제가 되지 않는다.

다시 말해서, 1월부터 11월까지의 월들에 각각 어떤 기본값을 부여할 지에 대해서는 매개변수  $\beta_2, \dots, \beta_{11}$ 로 조정하면 된다. 하지만,  $y$ 절편에 해당하는  $\beta_0$ 를 이미 설정해놓았으므로, 12월에는  $\beta_0$ 라는 기본값을 부여받게 되는 것이다.

그런데, 어차피 이렇게 할거면, trend를 설정할 때,  $y$ 절편이 없는 일차함수로 잡은 다음  $M_1, M_2, \dots, M_{12}$ 를 설정하는 게 깔끔해보인다. 하지만 그렇게 하지 않고, 지금과 같은 방법을 취하는 것이 아마도 통계 방면에서의 관습인 게 아닐까 싶기도 하다.

이렇게 parametric model  $f_\beta$ 를 설정했다. 그 다음으로 하는 것은 기존의 회귀분석(ordinary regression analysis)을 진행하는 것이다. 강의에서는 45분 32초쯤에 ‘일반적인 최소제곱법(LSE, least square estimation ; OLS ordinary least square, ordinary least squares)’을 사용하는 것이다. 표에서 관측치가 주어져있었다.

$$\begin{aligned} \text{관측치} &= \{(t, y_t) : t = 1, 2, \dots, 168\} \\ &= \{(1, 501), (2, 488), \dots, (168, 877)\} \end{aligned}$$

이걸 가지고 MSE를 계산하면

$$\text{MSE} = \frac{1}{168} \sum_{t=1}^{168} (y_t - f_\beta(t))^2$$

이 된다.  $MSE$ 를  $\beta_0, \beta_1, \dots, \beta_{11}$ 로 편미분한 것을 0으로 두면 미지수가 12개이고 식이 12개인 연립방정식이 나오는데, 그 연립방정식을 풀어 근을  $\beta_0 = \hat{\beta}_0, \beta_1 = \hat{\beta}_1, \dots, \beta_{11} = \hat{\beta}_{11}$ <sup>1</sup>  $\hat{\beta}_i$ 들을 가지고 최적의 함수  $\hat{f}$ 를 찾을 수 있다.

이 계산들은 보통은 컴퓨터를 통해, 몇개의 명령어를 입력하여 계산하는 것 같고, 마지막 캡쳐의 표에 이  $\hat{\beta}_i$ 들의 값이 적혀있는 것으로 보인다.

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<sup>1</sup> $\beta = \hat{\beta}$

## 2 Trigonometric Models

### Trigonometric Models

$$\text{Model 1: } y_t = \beta_0 + \beta_1 t + \beta_2 \sin\left(\frac{2\pi t}{L}\right) + \beta_3 \cos\left(\frac{2\pi t}{L}\right) + \varepsilon_t$$

$$\text{Model 2: } y_t = \beta_0 + \beta_1 t + \beta_2 \sin\left(\frac{2\pi t}{L}\right) + \beta_3 \cos\left(\frac{2\pi t}{L}\right) + \beta_4 \sin\left(\frac{4\pi t}{L}\right) + \beta_5 \cos\left(\frac{4\pi t}{L}\right) + \varepsilon_t$$

- These models assume a linear trend, but they can be altered to handle other trends.
- The first model (model 1) is useful for modeling a very regular seasonal time series that exhibits constant seasonal variations.
- The second model (model 2) is useful for modeling more complicated seasonal patterns.

### Example-Trigonometric Models

$$\text{Model 1: } y_t = \beta_0 + \beta_1 t + \beta_2 \sin\left(\frac{2\pi t}{L}\right) + \beta_3 \cos\left(\frac{2\pi t}{L}\right) + \varepsilon_t$$

$\beta_3$

### 3 Growth Curve Models

#### Growth Curve Models

- Useful for the models, not linear in the parameters.

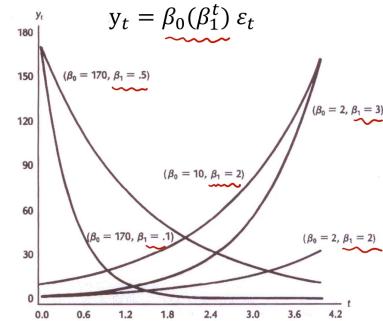
$$\log y_t = \beta_0 + \beta_1 t + \varepsilon_t$$

- Transform a nonlinear model to one that is linear in parameter.

$$\log(y_t) = \log(\beta_0) + t \log(\beta_1) + \log(\varepsilon_t)$$

$$\begin{aligned} \log(AB) &= \log(A) + \log(B) \\ \log(A^r) &= r \log(A) \end{aligned}$$

#### Growth Curve Models



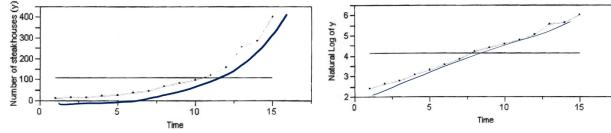
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#### Growth Curve Models - Example

- Number of branches of Western Steakhouses for 15 years.

| Year (t) | $y_t$ | $\ln y_t$ | Year (t) | $y_t$ | $\ln y_t$ |
|----------|-------|-----------|----------|-------|-----------|
| 1        | 11    | 2.398     | 9        | 82    | 4.407     |
| 2        | 14    | 2.639     | 10       | 99    | 4.595     |
| 3        | 16    | 2.773     | 11       | 119   | 4.779     |
| 4        | 22    | 3.091     | 12       | 156   | 5.050     |
| 5        | 28    | 3.332     | 13       | 257   | 5.549     |
| 6        | 36    | 3.584     | 14       | 284   | 5.649     |
| 7        | 46    | 3.829     | 15       | 403   | 5.999     |
| 8        | 67    | 4.205     |          |       |           |



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#### Growth Curve Models - Example

| Ln y = 2.07 + 0.257Year |              |                   |       |       |  |
|-------------------------|--------------|-------------------|-------|-------|--|
| Predictor               | Coeff        | SE Coef           | T     | P     |  |
| Constant                | 2.07012      | 0.04103           | 50.45 | 0.000 |  |
| Year                    | 0.25688      | 0.00451           | 56.92 | 0.000 |  |
| S = 0.07552             | R-Sq = 99.6% | R-Sq(adj) = 99.6% |       |       |  |

| New Obs | Fit    | SE Fit | 95.0% CI         | 95.0% PI         |
|---------|--------|--------|------------------|------------------|
| 1       | 6.1802 | 0.0410 | (6.0916, 6.2684) | (5.9945, 6.3659) |

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The point prediction of  $\ln y_{16}$ , where  $y_{16}$  is the number of steakhouses that will be in operation in period 16, is

$$\ln \hat{y}_{16} = \hat{\beta}_0 + \hat{\beta}_1 t = 2.07012 + 0.25688(16) = 6.1802$$

Thus a point prediction of  $y_{16}$  is

$$\hat{y}_{16} = e^{6.1802} = 483.09$$

The 95% prediction interval for  $\ln y_{16}$  is

$$[5.9945, 6.3659]$$

And thus a 95% prediction interval for  $y_{16}$  is

$$[e^{5.9945}, e^{6.3659}] = [401.22, 581.67]$$

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## 4 First-Order Autoregressive Process

### Time Series Regression with Autocorrelation

- How to model a time series that possesses a first-order autocorrelated error structure.
- If we ignore autocorrelated error terms, we will pay a penalty in terms of wider prediction interval.
- By taking autocorrelation into account, we can obtain more precise prediction intervals.
- We can check this using the residual plots or the Durbin-Watson test.

$$Y = \beta_0 + \beta_1 X + \varepsilon_t$$

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| Example |      |        |         |          |        |          |        |
|---------|------|--------|---------|----------|--------|----------|--------|
| year    | tfr  | partic | degrees | fconvict | ftheft | mconvict | mtheft |
| 1 1931  | 3200 | 234    | 12.4    | 77.1     | NA     | 778.7    | NA     |
| 2 1932  | 3084 | 234    | 12.9    | 92.9     | NA     | 745.7    | NA     |
| 3 1933  | 2864 | 235    | 13.9    | 98.3     | NA     | 768.3    | NA     |
| 4 1934  | 2803 | 237    | 13.6    | 88.1     | NA     | 733.6    | NA     |
| 5 1935  | 2755 | 238    | 13.2    | 79.4     | 20.4   | 765.7    | 247.1  |
| 6 1936  | 2696 | 240    | 13.2    | 91.0     | 22.1   | 816.5    | 254.9  |
| ...     |      |        |         |          |        |          |        |
| 37 1967 | 2586 | 339    | 80.4    | 115.2    | 70.6   | 781.1    | 272.0  |
| 38 1968 | 2441 | 338    | 90.4    | 122.9    | 73.0   | 849.7    | 274.7  |

- year, 1931–1968.
- tfr, the total fertility rate, births per 1000 women.
- partic, women's labor-force participation rate, per 1000.
- degrees, women's post-secondary degree rate, per 10,000.
- fconvict, women's indicable-offense conviction rate, per 100,000.
- ftheft, women's theft conviction rate, per 100,000.
- mconvict, men's indicable-offense conviction rate, per 100,000.
- mtheft, theft conviction rate, per 100,000.

### First-Order Autoregressive Process

$$\varepsilon_t = \phi_1 \varepsilon_{t-1} + a_t$$

$a_t$  is assumed to be an error term with mean 0 that satisfies the constant variance, independence, and normality assumption.

- $\phi_1$  is the correlation coefficient between error terms separated by one time period.
- If  $\phi_1 > 0$ , this indicates that the error terms are positively autocorrelated. That means a positive error term  $\varepsilon_t$  tends to produce another positive error term  $\varepsilon_{t-1}$ .
- If  $\phi_1 < 0$ , the error terms are negatively autocorrelated. In this case, a positive error term  $\varepsilon_{t-1}$  tends to produce a negative error term  $\varepsilon_t$  and vice versa.

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### Example

#### • Ordinary multiple regression result

```
> mod.ols <- lm(fconvict ~ tfr + partic + degrees + mconvict, data=Hartnagel)
> summary(mod.ols)
```

Coefficients:

|             | Estimate  | Std. Error | t value | Pr(> t ) |
|-------------|-----------|------------|---------|----------|
| (Intercept) | 127.64000 | 59.95704   | 2.13    | 0.041    |
| tfr         | -0.04657  | 0.00803    | -5.80   | 1.8e-06  |
| partic      | 0.25342   | 0.11513    | 2.20    | 0.035    |
| degrees     | -0.21205  | 0.21145    | -1.00   | 0.323    |
| mconvict    | 0.05910   | 0.04515    | 1.31    | 0.200    |

Residual standard error: 19.2 on 33 degrees of freedom  
Multiple R-Squared: 0.695, Adjusted R-squared: 0.658  
F-statistic: 18.8 on 4 and 33 DF, p-value: 3.91e-008

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### Example

#### • Time series regression with second-order autoregressive process

Correlation Structure: ARMA(2,0)  
Formula: ~1  
Parameter estimate(s):  
Phi1 Phi2  
1.0683 -0.5507

Coefficients:

|             | Value | Std. Error | t-value | p-value |
|-------------|-------|------------|---------|---------|
| (Intercept) | 83.34 | 59.47      | 1.401   | 0.1704  |
| tfr         | -0.04 | 0.01       | -4.309  | 0.0001  |
| partic      | 0.29  | 0.11       | 2.568   | 0.0150  |
| degrees     | -0.21 | 0.21       | -1.016  | 0.3171  |
| mconvict    | 0.08  | 0.04       | 2.162   | 0.0380  |

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#### ① Ordinary regression analysis

|             | Estimate  | Std. Error | t value | Pr(> t ) |
|-------------|-----------|------------|---------|----------|
| (Intercept) | 127.64000 | 59.95704   | 2.13    | 0.041    |
| tfr         | -0.04657  | 0.00803    | -5.80   | 1.8e-06  |
| partic      | 0.25342   | 0.11513    | 2.20    | 0.035    |
| degrees     | -0.21205  | 0.21145    | -1.00   | 0.323    |
| mconvict    | 0.05910   | 0.04515    | 1.31    | 0.200    |

#### ② Time series regression analysis

|             | Value | Std. Error | t-value | p-value |
|-------------|-------|------------|---------|---------|
| (Intercept) | 83.34 | 59.47      | 1.401   | 0.1704  |
| tfr         | -0.04 | 0.01       | -4.309  | 0.0001  |
| partic      | 0.29  | 0.11       | 2.568   | 0.0150  |
| degrees     | -0.21 | 0.21       | -1.016  | 0.3171  |
| mconvict    | 0.08  | 0.04       | 2.162   | 0.0380  |

In our example, ML estimate of the regression parameters under the AR(2) error-correlation model are not terribly different from the OLS estimates.

The coefficient for mconvict is statistically significant.

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