7T1: Stochastic Model

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Stochastic signals

- Described by the laws of probability; mean, variance, probability distributions

- Autocorrelation

$$Z_{xx}[k] = \sum_{n=0}^{n=N-1} x[n]\,x[n+k] \qquad k = -N+1, \ldots, N-1$$

- Power spectral density

$$Xp[k] = \lim_{N \to \infty} |X[k]|^2$$

where $X[k] = \sum_{n=0}^{N-1} x[n]\,e^{-j2\pi kn/N} \qquad k = 0, \ldots, N-1$

Stochastic model

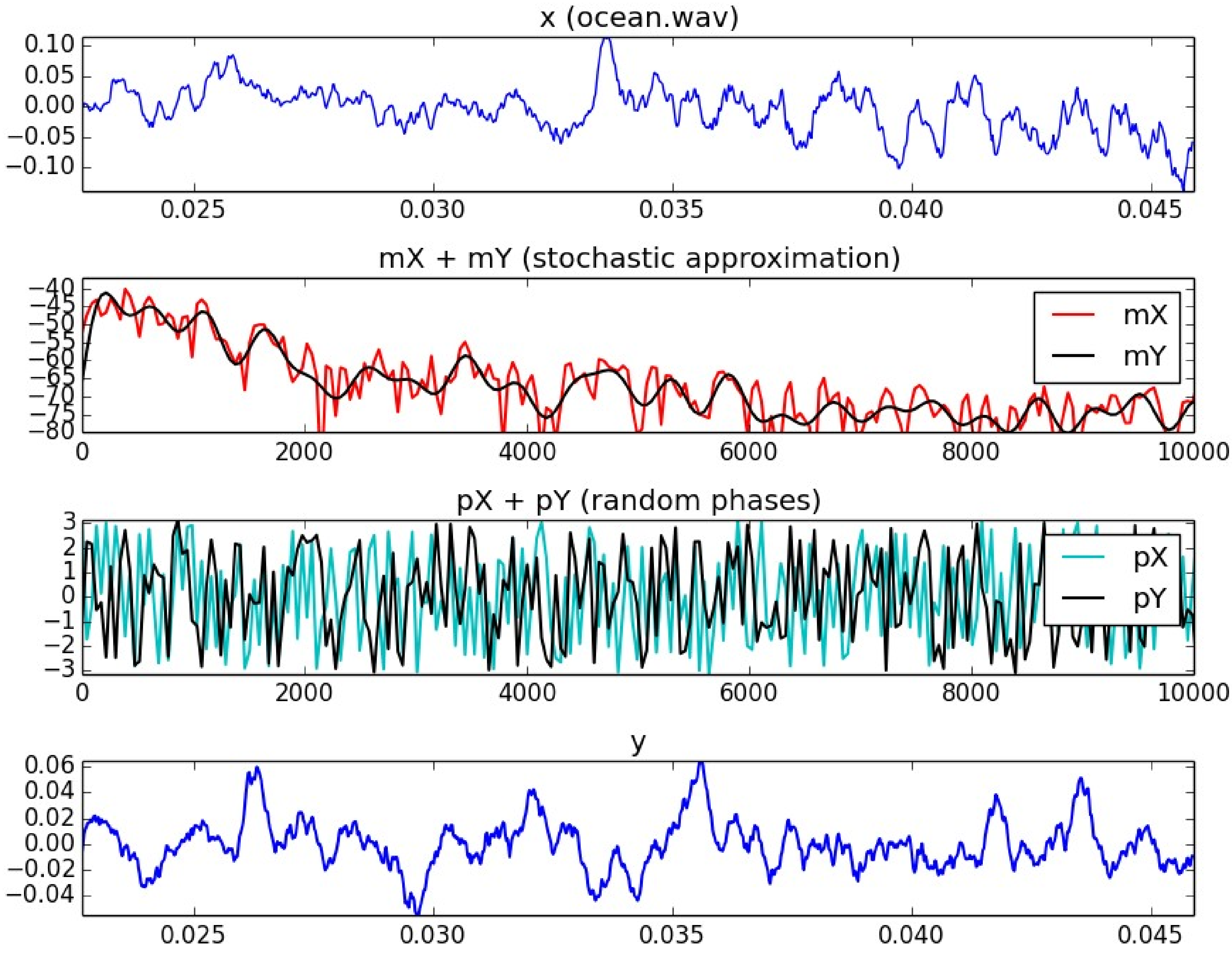
$$yst[n] = \sum_{k=0}^{N-1} u[k]h[n-k]$$

$u[n]$: white noise
$h[n]$: impulse response of filter approximating input signal $x[n]$

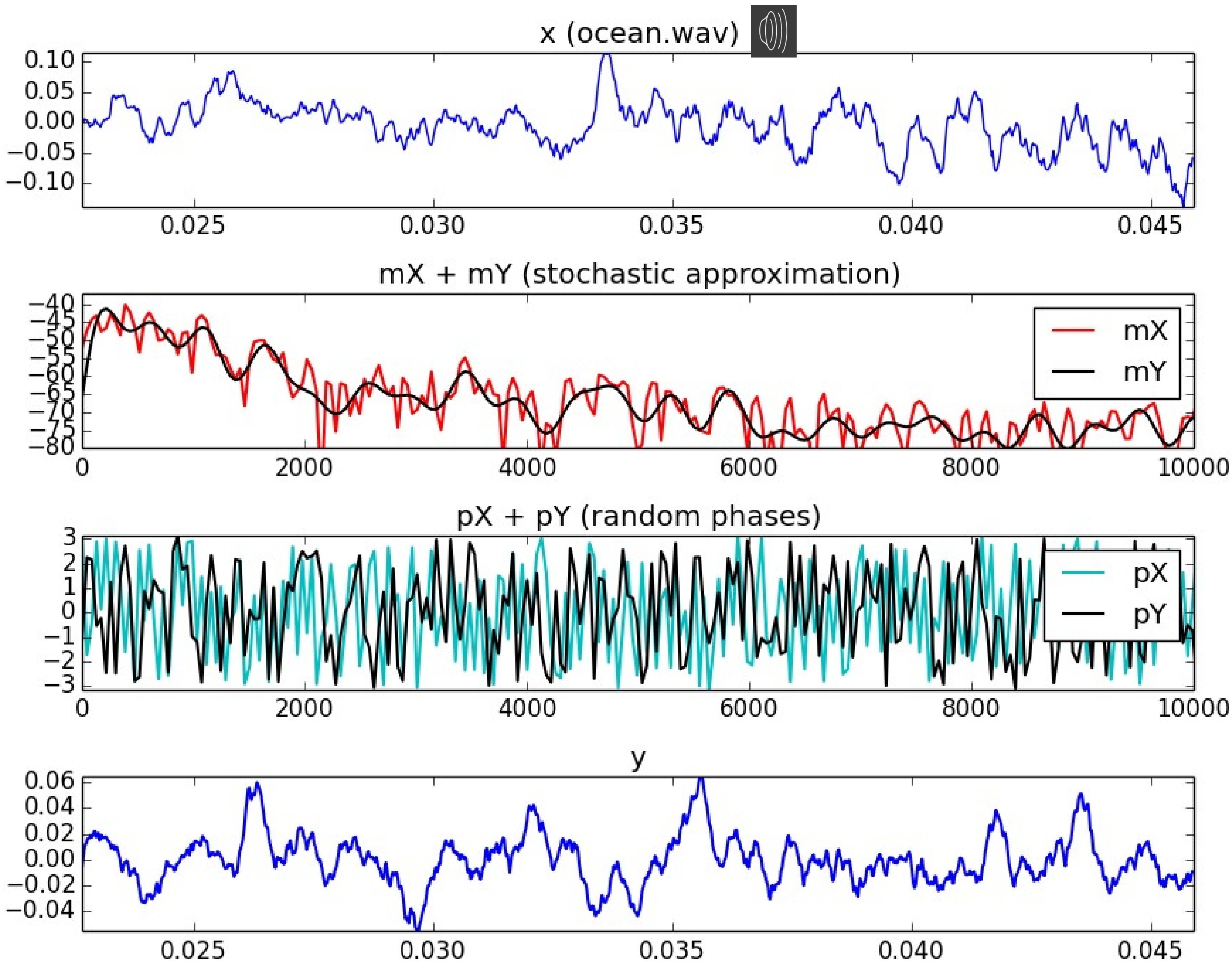
Spectral view:

$$Yst_l[k] = |H_l[k]||U[k]|e^{j(\measuredangle H[k] + \measuredangle U[k])} = |\tilde{X}_l[k]|e^{j\measuredangle U[k]}$$

$|\tilde{X}[k]|$: approximation of magnitude spectrum of input signal $x[n]$
$\measuredangle U[k]$: spectral phases of noise signal
l: frame number



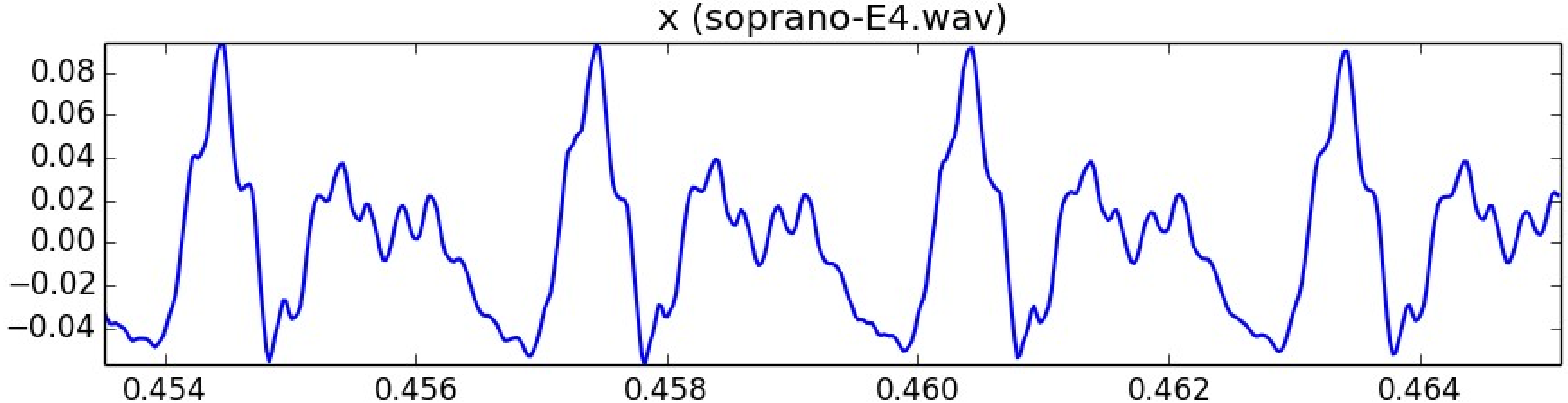
x (ocean.wav)
mX + mY (stochastic approximation)
mX
mY
pX + pY (random phases)
pX
pY
y

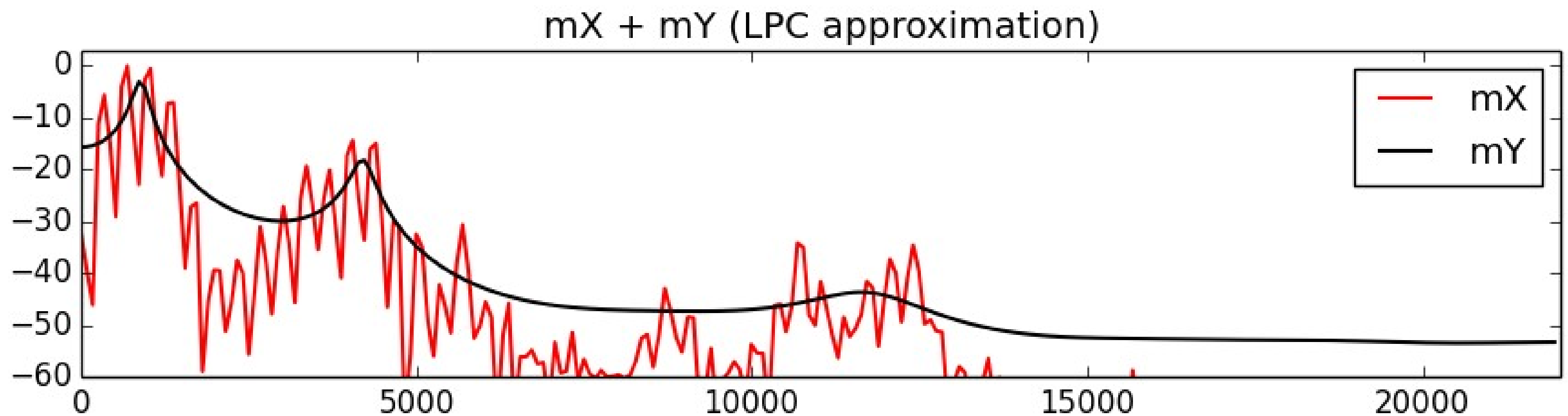


x (ocean.wav)
0.10
0.05
0.00
−0.05
−0.10
0.025
0.030
0.035
0.040
0.045
mX + mY (stochastic approximation)
−40
−45
−50
−55
−60
−65
−70
−75
−80
0
2000
4000
6000
8000
10000
mX
mY
pX + pY (random phases)
3
2
1
0
−1
−2
−3
0
2000
4000
6000
8000
10000
pX
pY
y
0.06
0.04
0.02
0.00
−0.02
−0.04
0.025
0.030
0.035
0.040
0.045

LPC approximation

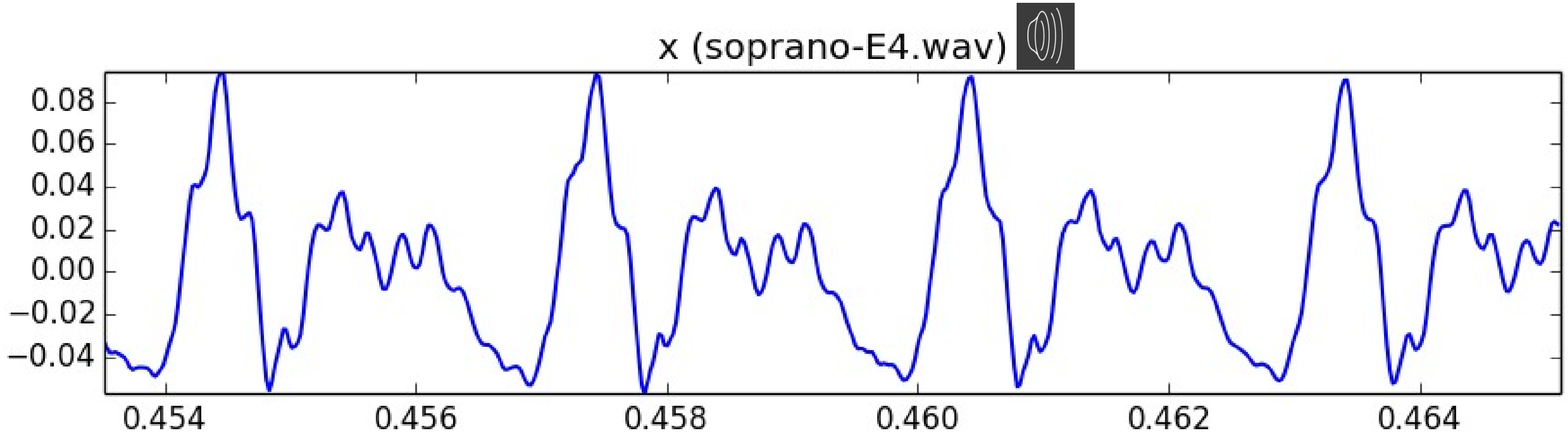
$$\hat{x}[n] = \sum_{k=1}^{K} a_k x[n-k] \qquad Error = \sum_{n=-\infty}^{\infty} \left(x[n] + \sum_{k=1}^{K} a_k x[n-k] \right)^2$$

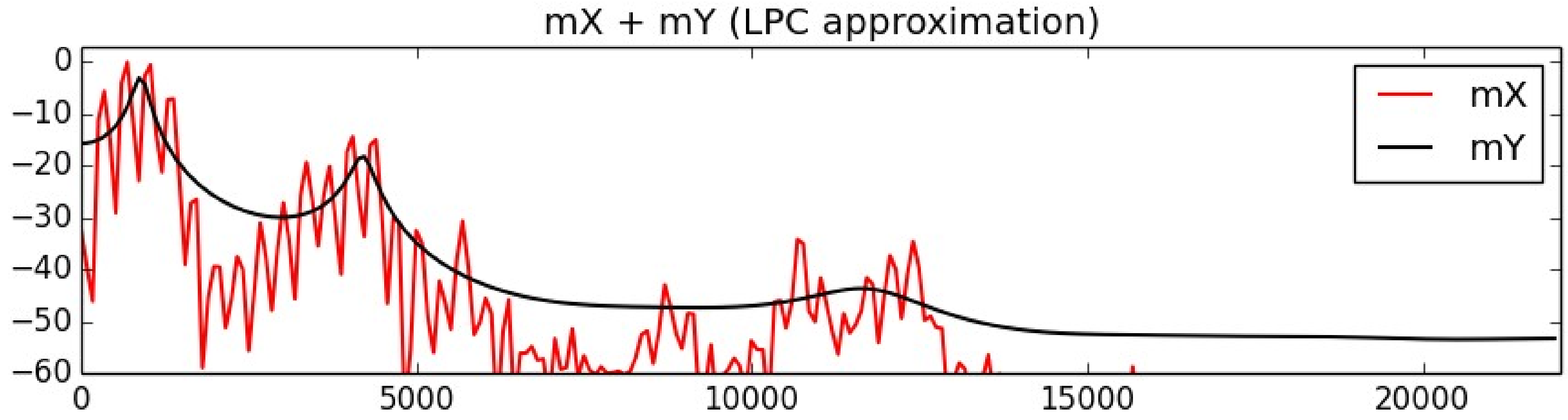




LPC approximation

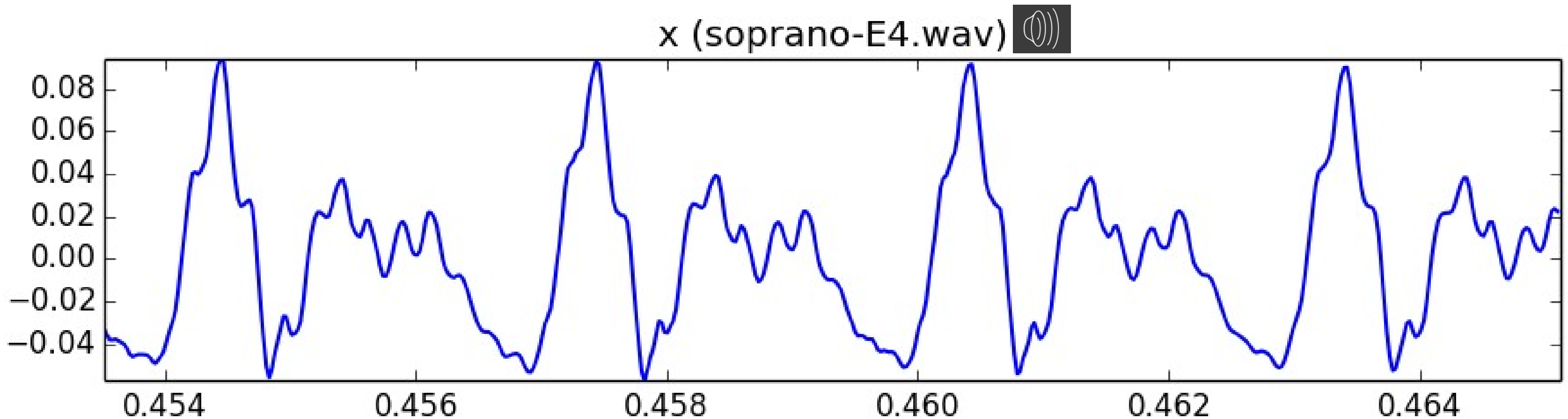
$$\hat{x}[n] = \sum_{k=1}^{K} a_k x[n-k] \qquad Error = \sum_{n=-\infty}^{\infty} \left(x[n] + \sum_{k=1}^{K} a_k x[n-k] \right)^2$$

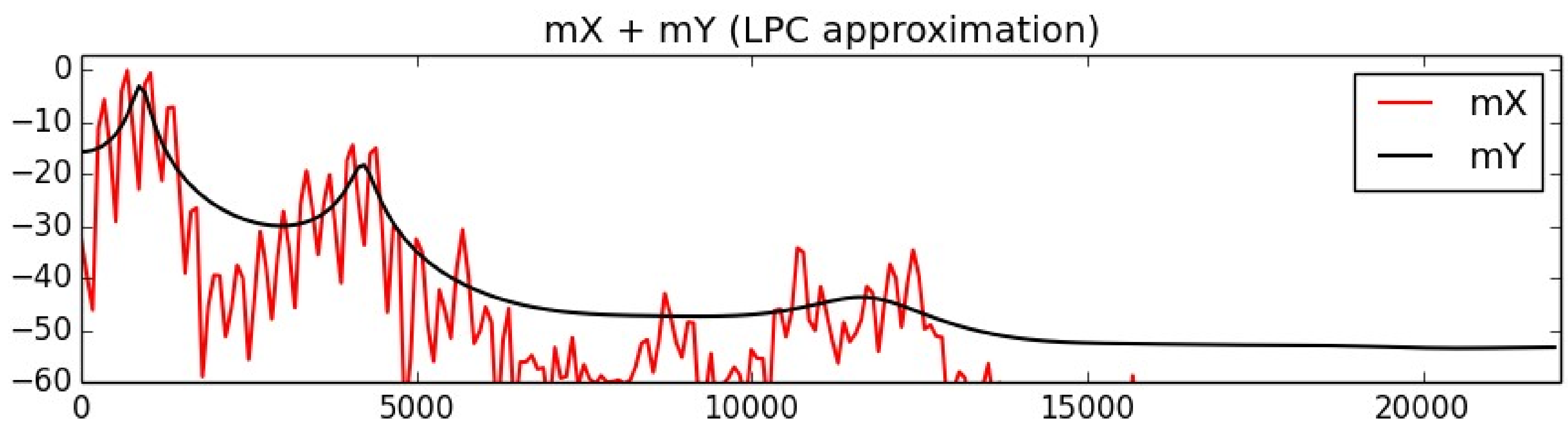




LPC approximation

$$\hat{x}[n]=\sum_{k=1}^{K} a_k\, x[n-k] \qquad Error=\sum_{n=-\infty}^{\infty}\left(x[n]+\sum_{k=1}^{K} a_k\, x[n-k]\right)^2$$

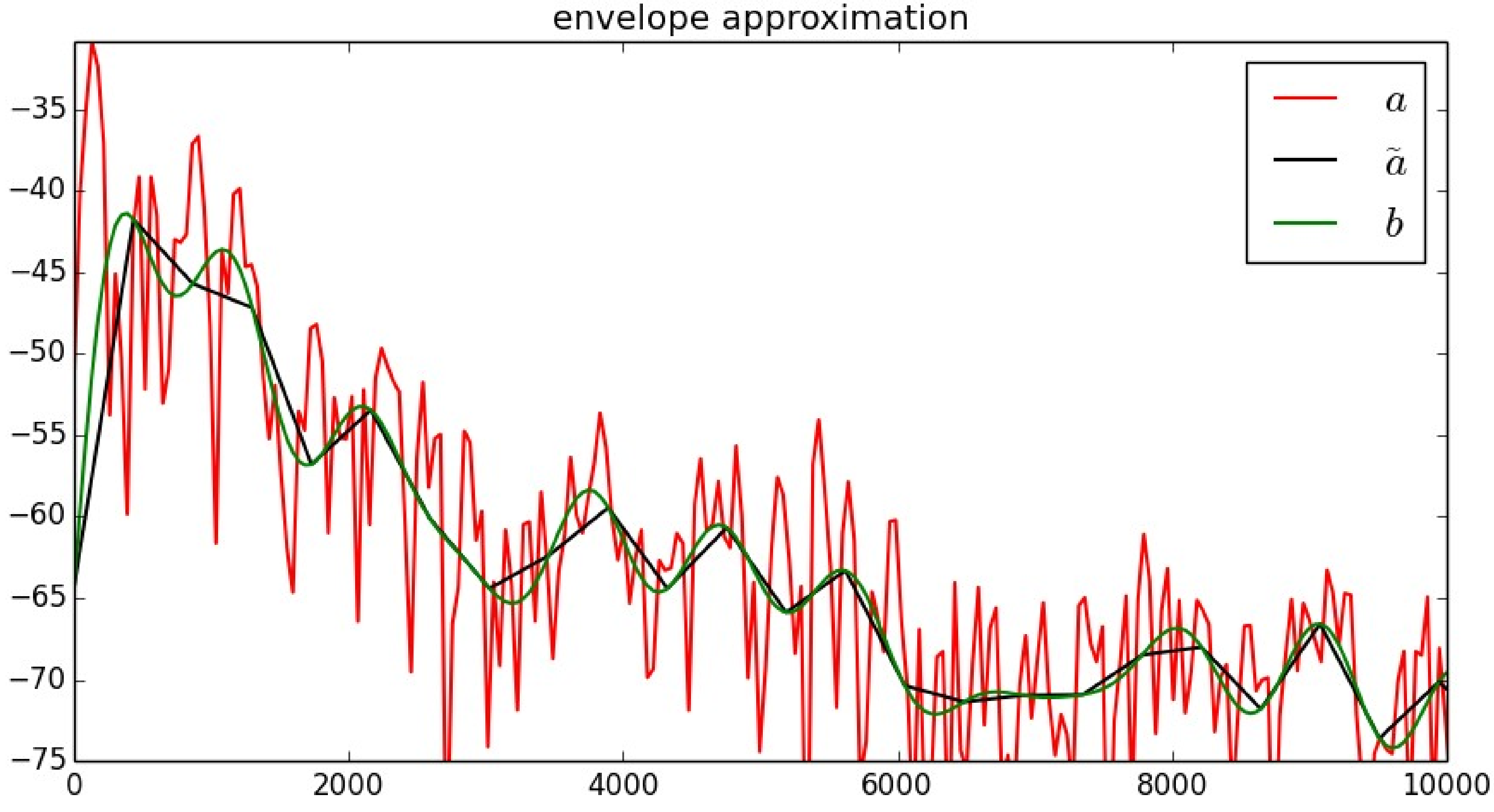




Envelope approximation

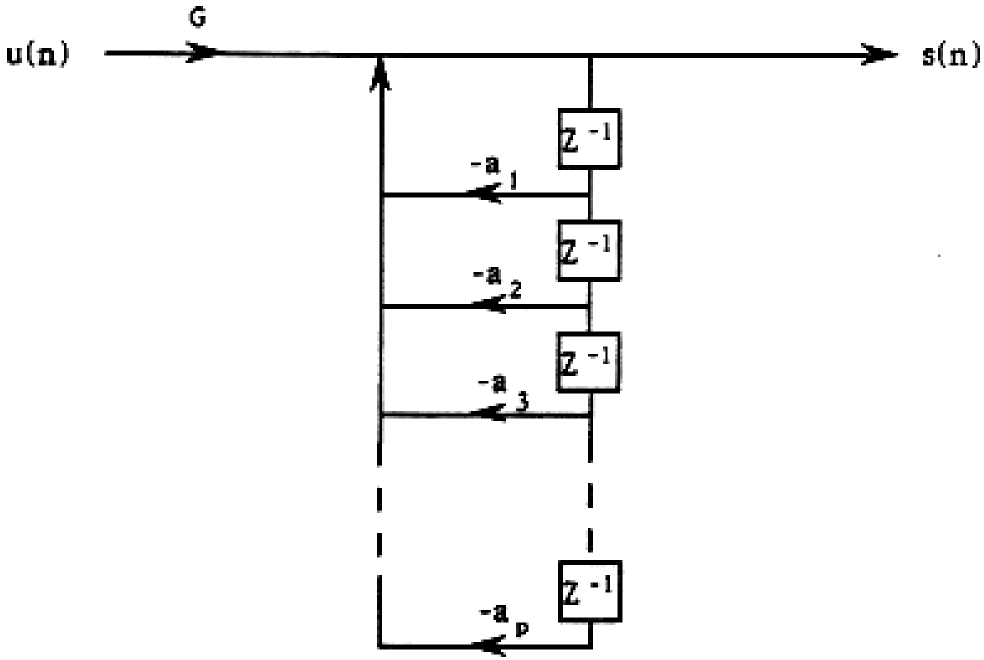
$$\widetilde{a}[k] = IDFT(LP(DFT(a[k])))$$
LP : low-pass filter

$$b[k] = IDFT(ZP(DFT(\widetilde{a}[k])))$$
ZP : zero-padding

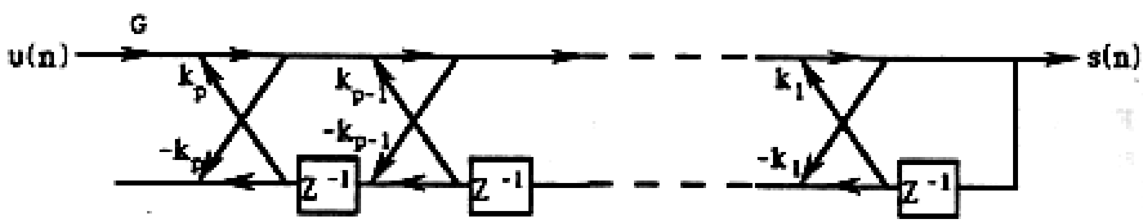


Stochastic synthesis using LPC

$$yst[n]=\sum_{k=1}^{K} a_k u[n-k],\quad a_k:\text{filter coefficients}\,;\, u[n]:\text{white noise}$$



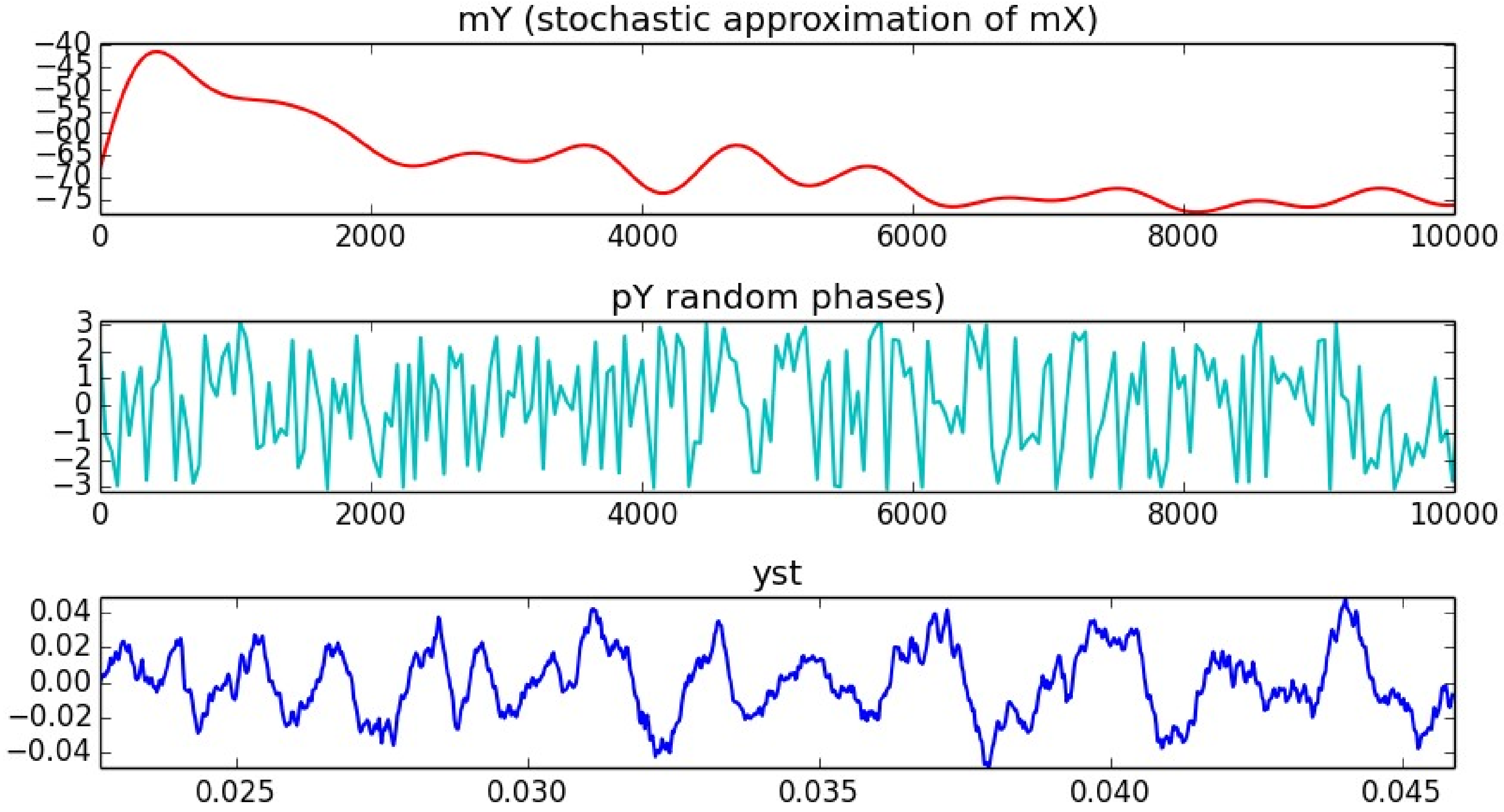
direct form structure



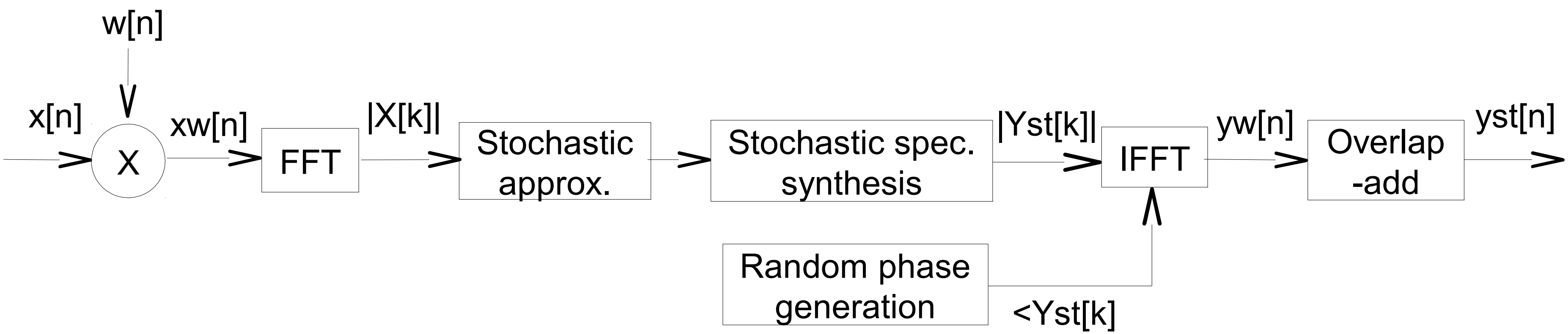
lattice structure

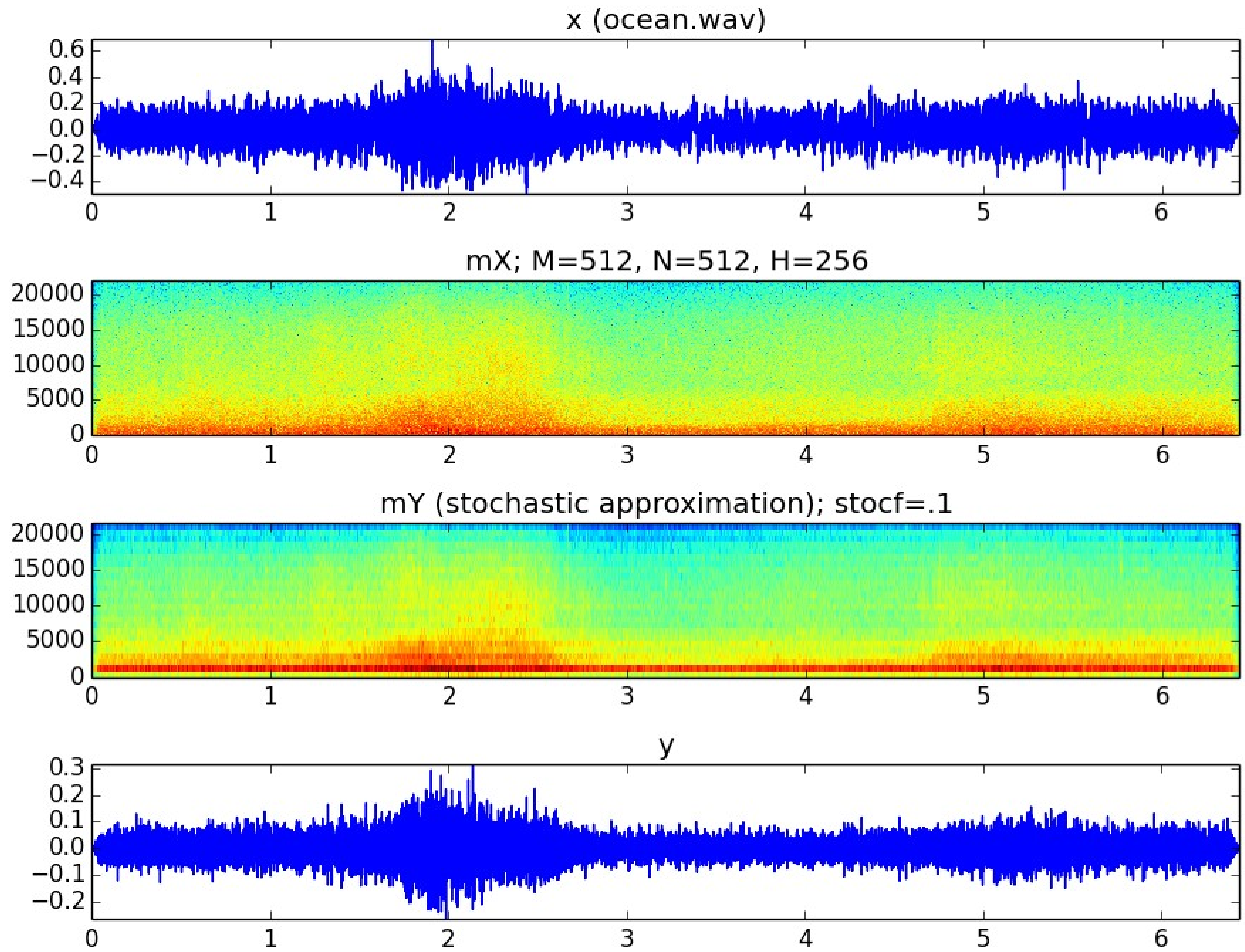
Stochastic synthesis using envelopes

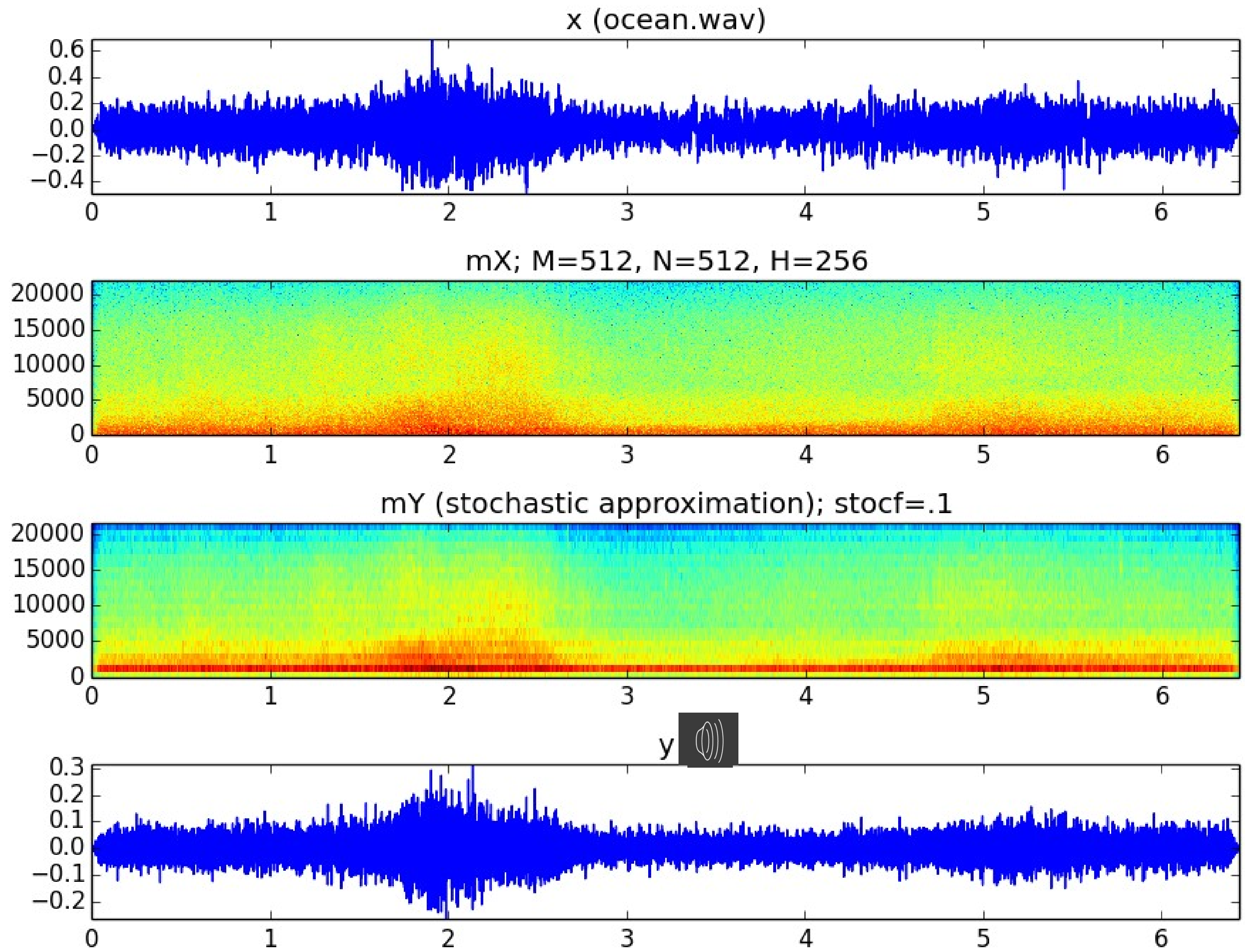
$$yst[n] = IDFT\left(|\tilde{X}[k]| e^{j \sphericalangle U[k]}\right)$$



Stochastic model system




x (ocean.wav)
mX; M=512, N=512, H=256
mY (stochastic approximation); stocf=.1
y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mX; M=512, N=512, H=256
mY (stochastic approximation); stocf=.1
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References and credits

- More information in:
  - http://en.wikipedia.org/wiki/Statistical_signal_processing
  - http://en.wikipedia.org/wiki/Stochastic_process
  - http://en.wikipedia.org/wiki/Linear_predictive_coding
- Sounds: http://www.freesound.org/people/xserra/packs/13038/
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