③ To prove a constraint is non-holonomic we have to prove $\gamma = 0$

in $[\,0 \quad 1 \quad p\sin q_{5} \quad p\cos q_{3} \cos q_{5}\,]\dot{q} = 0$

Here,

$q_1 = 0$

$q_2 = 1$

$q_3 = p\sin q_5$

$q_4 = p\cos q_3$

$q_5 = \cos q_5$

and $\dot{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \end{bmatrix}$

and we use $\dfrac{\partial \gamma(q) a_k(q)}{\partial q_j} = \dfrac{\partial \gamma(q)\, a_j(q)}{\partial q_k}$

$\forall \; j, k = 1, 2, 3 \,--\, n$

The conditions are

j	k
1	2
1	3
1	4
1	5
2	3
2	4
2	5
3	4
3	5
4	5

$$\rightarrow \frac{\partial r a_2}{\partial q_1} = \frac{\partial r a_1}{\partial q_2}$$

$$\Rightarrow \frac{\partial r(1)}{\partial q_1} = \frac{\partial r(0)}{\partial q_2}$$

$$\Rightarrow \boxed{\frac{\partial r}{\partial q_1} = 0} \text{ --- } (1)$$

$$\rightarrow \frac{\partial r a_3}{\partial q_1} = \frac{\partial r a_1}{\partial q_3}$$

$$\Rightarrow r\left(\frac{\partial p \sin q_5}{\partial q_1}\right) + p \sin q_5 \frac{\partial r}{\partial q_1} = 0$$

$$\Rightarrow 0 = 0 \quad \text{(Redundant)}$$

$$\rightarrow \frac{\partial r a_4}{\partial q_1} = \frac{\partial r a_1}{\partial q_4}$$

$$\Rightarrow \frac{\partial r(p \cos q_3)}{\partial q_1} = \frac{\partial r(0)}{\partial q_4} \Rightarrow 0 = 0$$

Redundant

$$\rightarrow \frac{\partial r a_5}{\partial q_1} = \frac{\partial r a_1}{\partial q_5}$$

$$\Rightarrow \frac{r\,\partial(\cos q_5)}{\partial q_1} + \cos q_5 \frac{\partial r}{\partial q_1} = 0$$

$$\Rightarrow 0 \neq 0 \Rightarrow \underline{\underline{\text{Redundant}}}$$

$$\rightarrow \frac{\partial r a_3}{\partial q_2} = \frac{\partial r a_2}{\partial q_3}$$

$$\Rightarrow r(0) + \frac{\partial r}{\partial q_3} = p \sin q_5 \frac{\partial r}{\partial q_2} \text{ to}$$

$$\Rightarrow \boxed{p \sin q_5 \left(\frac{\partial r}{\partial q_2}\right) = \frac{\partial r}{\partial q_3}} \text{ — } (2)$$

$$\rightarrow \frac{\partial r a_4}{\partial q_2} = \frac{\partial r a_2}{\partial q_4}$$

$$\Rightarrow r(0) + \frac{\partial r}{\partial q_4} = p \cos q_3 \frac{\partial r}{\partial q_2}$$

$$\Rightarrow \boxed{p \cos q_3 \frac{\partial r}{\partial q_2} = \frac{\partial r}{\partial q_4}} \text{ — } (3)$$

$$\rightarrow \frac{\partial r a_5}{\partial q_2} = \frac{\partial r a_2}{\partial q_5}$$

$$\Rightarrow \frac{\partial r}{\partial q_5} = 0 + \cos q_5 \frac{\partial r}{\partial q_2}$$

$$\Rightarrow \boxed{\frac{\partial r}{\partial q_5} = \cos q_5 \frac{\partial r}{\partial q_2}} \quad (4)$$

$$\rightarrow \frac{\partial r a_4}{\partial q_3} = \frac{\partial r a_3}{\partial q_4}$$

$$\Rightarrow \rho \cos q_3 \frac{\partial r}{\partial q_3} + \rho(-\sin q_3) = r(0) + \rho \sin q_4 \frac{\partial r}{\partial q_4}$$

$$\Rightarrow \boxed{\sin q_5 \frac{\partial r}{\partial q_4} = \cos q_3 \frac{\partial r}{\partial q_3} - r \sin q_3} \quad (5)$$

$$\rightarrow \frac{\partial r a_5}{\partial q_3} = \frac{\partial r a_3}{\partial q_5}$$

$$= \frac{\partial r(\rho \sin q_5)}{\partial q_5} = \frac{\partial r(\cos q_5)}{\partial q_3}$$

$$\Rightarrow \boxed{\cos q_5 \frac{\partial r}{\partial q_3} = r\rho \cos q_5 + \rho \sin q_5 \frac{\partial r}{\partial q_5}} \quad (6)$$

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$$\Rightarrow \frac{\partial r a_5}{\partial q_{v_4}} = \frac{\partial r a_4}{\partial q_{v_5}}$$

$$\Rightarrow r(0) + \cos q_{v_5} \frac{\partial r}{\partial q_{v_4}} = 0 + \cos q_{v_3} \frac{\partial r}{\partial q_{v_5}}$$

$$\Rightarrow \boxed{\cos q_{v_5} \frac{\partial r}{\partial q_{v_4}} = \cos q_{v_3} \frac{\partial r}{\partial q_{v_5}}} \quad \text{— } (7)$$

Using eqn 2 through (7)

* putting (2) and (3) in eq (5) we get:

$$\sin q_{v_5}\, \rho \cos q_{v_3} \frac{\partial r}{\partial q_{v_2}} = \cos q_{v_3}\, \rho \sin q_{v_3} \frac{\partial r}{\partial q_{v_2}} - r \sin q_{v_3}$$

Cancelling out ⇒

$$\boxed{r \sin q_{v_3} = 0} \quad \text{— } (a)$$

* putting (2) and (4) in eqn (6) we get.

$$\cos q_{v_5}\, \rho \sin q_{v_5} \frac{\partial r}{\partial q_{v_2}} = r\rho \cos q_{v_5} + \rho \sin q_{v_5} \cos q_{v_5} \frac{\partial r}{\partial q_{v_2}}$$

$\Rightarrow \boxed{r p \cos q_5 = 0}$ — (b)

from (7)

$$\cos q_5 - p\cos q_3 \frac{\partial r}{\partial q_2} = \cos q_3 \, p\cos q_5 \frac{\partial r}{\partial q_2}$$

$\Rightarrow$ Redundant

Hence from (a) and (b)

the test function $\boxed{r = 0}$

$\Rightarrow$ The differential constraint

$[0 \quad 1 \quad p\sin q_3 \quad p\cos q_3 \quad \cos q_5]\dot{q}$

in $\mathbb{R}^5$ is NON-HOLONOMIC.

Hence Proved.