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ENPM 662

HW-6

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(1)

(a) $J = \begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 4 & 3 & 1 \\ 2 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 2 & 3 & 0 \\ 1 & 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} J_v \\ J_w \end{bmatrix}$

$\tau_1 = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 5 \end{bmatrix} \quad \tau_2 = \begin{bmatrix} 2 \\ 5 \\ 6 \\ 5 \end{bmatrix}$

The pseudoinverse of this can only be used to determine the endeffector forces iff the condition

$\text{rank}(J) = \text{rank}[J \mid \tau_i]$ be obeyed

Hence, we have to test for this

Testing for τ_1

$\text{rank}(J_w^T) = \text{rank} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 3 & 0 \\ 1 & 0 & 0 & 5 \end{bmatrix}^T$

$= \text{rank} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} = 3 = \text{full rank}$

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Now:

$$\text{Rank}\left(J_\omega^T \mid \tau_1\right) = \text{Rank}\begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 2 & 0 & 4 \\ 0 & 3 & 0 & 6 \\ 0 & 0 & 5 & 5 \end{bmatrix}$$

Checking the MATLAB code we see that this Rank = 3

Hence the pseudo inverse can be used to determine the end-effector forces.

testing for τ_2

$\text{Rank } J_\omega^T = 3$ (as we have seen before)

$$\text{Ran}\left(J_\omega^T \mid \tau_2\right) = \text{Rank}\begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 2 & 0 & 5 \\ 0 & 3 & 0 & 6 \\ 0 & 0 & 5 & 5 \end{bmatrix} = 4$$

(Check MATLAB code)

$\therefore 3 \neq 4$

Hence, the pseudo inverse method for calculating the end effector torques might yield out wrong values

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(b) $\tau = J^T F \rightarrow$ Relation between end-effector forces and torques

We have to find F

Now J_ω^T has a dimension of 4×3 and τ has dimensions of 4×1

Hence,

$F_1 \rightarrow$ Force for τ_1

$F_2 \rightarrow$ Force for τ_2

$$\boxed{J_\omega \tau = J_\omega J_\omega^T F_1}$$

Now to get F, we have to get rid of $J_\omega J_\omega^T$ on the RHS

Hence,

$$\boxed{(J_\omega J_\omega^T)^{-1} J_\omega \tau_1 = (J_\omega J_\omega^T)^{-1} (J_\omega J_\omega^T) F_1}$$

Now $J_\omega = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 3 & 0 \\ 1 & 0 & 0 & 5 \end{bmatrix}$ $\quad \tau_1 = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 5 \end{bmatrix}$

$$J_\omega^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

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Now, $(J_\omega J_\omega^T)^{-1}(J_\omega J_\omega^T) = I$

Doing the calculations on MATLAB →

$\therefore F_1 = (J_\omega J_\omega^T)^{-1} J_\omega \tau_1$

$$= \left[\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 3 & 0 \\ 1 & 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}\right]^{-1} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 3 & 0 \\ 1 & 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & -0.20 \\ 0 & 0.1538 & 0.23 & 0 \\ 0 & 0 & 0 & 0.2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \longrightarrow F_1 \text{ for } \tau_1$$

Now for τ_2 →

$$\tau_2 = \begin{bmatrix} 2 \\ 5 \\ 6 \\ 5 \end{bmatrix}$$

$F_2 = (J_\omega J_\omega^T)^{-1} J_\omega \tau_2$

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$$\Rightarrow F_2 = \begin{bmatrix} 1 & 0 & 0 & -0.2 \\ 0 & 0.1538 & 0.23 & 0 \\ 0 & 0 & 0 & 0.2 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 6 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2.1538 \\ 1.0000 \end{bmatrix}$$

ⓒ Now to verify our theory from part ⓐ,

we do the following

$\tau_1 = J_w^T F_1$ should yield the same $\tau_1 = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 5 \end{bmatrix}$

and doing MATLAB multiplication, we find that

$$\tau_1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$ Hence, Verified.

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Similarly

$$T_2 = J_w^T F_2$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1.000 \\ 2.1538 \\ 1.000 \end{bmatrix}$$

$$t_2 = \begin{bmatrix} 2.00 \\ 4.30 \\ 6.4615 \\ 5.000 \end{bmatrix}$$ Hence this is NOT THE SAME as

initial T_2 of $\begin{bmatrix} 2 \\ 5 \\ 6 \\ 5 \end{bmatrix}$

<u>Hence verified.</u>

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```
%%HW 6
%%Q1
%%a subsection
clc
clear all
Jw = [1 0 0 0;0 2 3 0;1 0 0 5]
t1 = [2 4 6 5]'
t2 = [2 5 6 5]'
%%testing for the first torque value
r1=rank(Jw)
r2=rank(cat(2,Jw',t1))
r1==r2 %%indicating the equality between r1 and r2
%%testing for the first torque value
r3=rank(Jw)
r4=rank(cat(2,Jw',t2))
r3==r4 %%indicating the equality between r3 and r4

%%bsubsection
F1=(inv(Jw*Jw')*Jw)*t1
F2=(inv(Jw*Jw')*Jw)*t2
%%getting back the values of tau
newt1 =Jw'*F1 %%tau1
newt2 =Jw'*F2 %%tau2
```

```
Jw =

     1     0     0     0
     0     2     3     0
     1     0     0     5


t1 =

     2
     4
     6
     5


t2 =

     2
     5
     6
     5


r1 =

     3
```

```
r2 =

     3


ans =

  logical

   1


r3 =

     3


r4 =

     4


ans =

  logical

   0


F1 =

     1
     2
     1


F2 =

    1.0000
    2.1538
    1.0000


newt1 =

     2
     4
     6
     5


newt2 =
```

```
    2.0000
    4.3077
    6.4615
    5.0000
```

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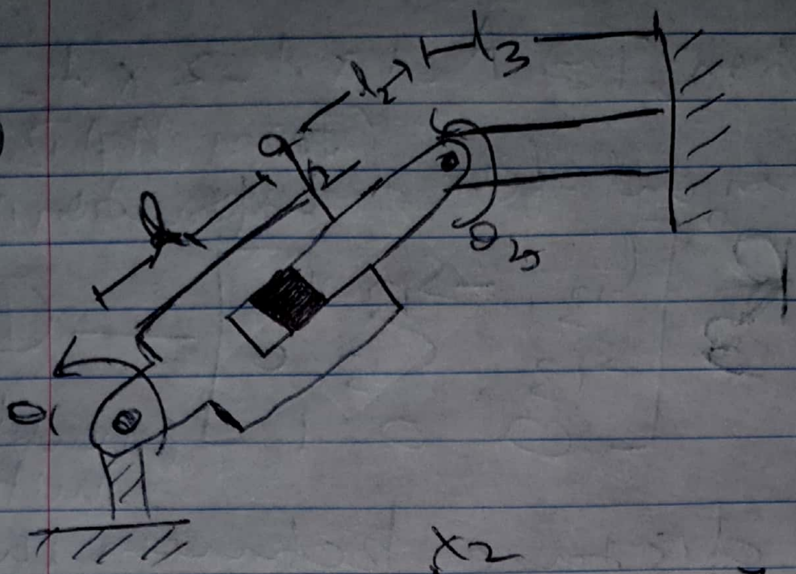
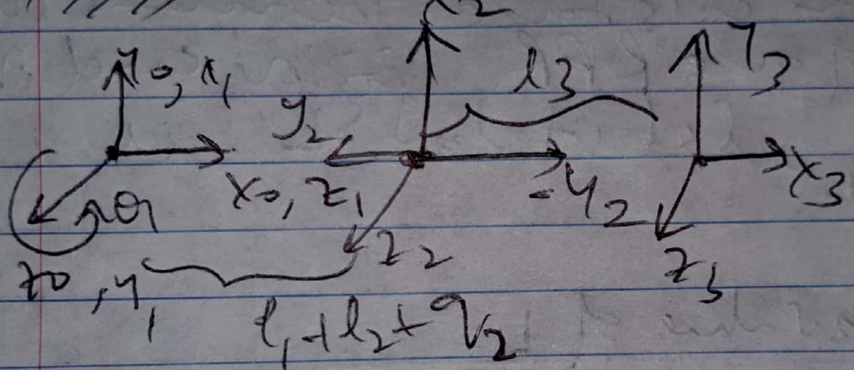
②





Writing DH parameters we get,

	θ	d	a	α
$0 \to 1$	θ_1+90	0	0	90
$1 \to 2$	0	$l_1+l_2+q_2^*$	0	-90
$2 \to 3$	θ_3-90	0	l_3	0

The transformation matrix of this can be seen on MATLAB

This can be written as:

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Transformation, T = A_1 * A_2 x A_3
(from MATLAB)

Taking the Rotation out of the transformation matrix:

$$\Rightarrow R_3^0 \begin{bmatrix} C_{13} & -S_{13} & 0 \\ S_{13} & C_{13} & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{---} (1)$$

(manually multiplying)

Now the distance between the base of the RPR joint and the wall, let it be 'L' is a Constant

$\Rightarrow$ This value of L =

$$\boxed{(l_1 + q_2 + l_2)\cos\theta_1 + l_3 = 0} \text{---} (2)$$

This is the constraint to make sure that the link is always normal to the wall

Now from eqn (1)

$$\begin{bmatrix} \cos(\theta_1 + \theta_3) \\ \sin(\theta_1 + \theta_3) \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

This HAS to be true because the

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link is parallel to the ground.

$\therefore \cos(\theta_1+\theta_3) = 1$

$\Rightarrow \theta_1+\theta_3 = 0$ or π or 2π ...

$= \underline{\underline{n\pi}}$ —— (3)

Now the question says that, q_2 has a limited travel.

Let this be between 0 and q_{max}

$$\boxed{\therefore 0 \le q_2 \le q_{max}} \text{ —— (4)}$$

Now, looking at all the constraints

$\boxed{\text{CONSTRAINT-1}} \rightarrow$

$$(l_1+q_2+l_2)\cos\theta_1 + l_3 = 0$$

if we look at eq (4) we know that the motion of this is limited from

$$(l_1+l_2)\cos\theta_1 + l_3 \le (l_1+q_2+l_2)\cos\theta_1 + l_3$$

$$\le (l_1+q_{max}+l_2)\cos\theta_1 + l_3$$

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Hence, this is a <u>HOLONOMIC</u>

constraint

Now,

$\boxed{\text{CONSTRAINT 2}}$ →

$$\cos(\theta_1 + \theta_3) = 1$$

$$\Rightarrow \theta_1 + \theta_3 = 0^\circ \text{ or } (n\pi),$$
$$n \in 0, 1, 2 \ldots n$$

This is a indicative of the parallel nature of the ~~joint~~ link at all times, hence, this is also a <u>HOLONOMIC</u>

constraint

$\boxed{\text{CONSTRAINT 3}}$ →

$$0 \leq q_2 \leq l_{max}$$

also limits the motion, hence

these constraints are <u>HOLONOMIC IN NATURE</u>

```
clc
clear all
%%code for
%%answer 2 HW 6
syms l1 l2 l3 q2 theta1 theta3
alpha1 = 90;
alpha2 = -90;
alpha3 = 0;

%%matrix 1
Rz_theta1=[cosd(theta1+90) -sind(theta1+90) 0 0;sind(theta1+90)
 cosd(theta1+90) 0 0;0 0 1 0;0 0 0 1];
Rx_alpha1=[1 0 0 0;0 cosd(alpha1) -sind(alpha1) 0;0 sind(alpha1)
 cosd(alpha1) 0;0 0 0 1];
Tz_d1=[1 0 0 0;0 1 0 0;0 0 1 l1;0 0 0 1];
Tx_a1=[1 0 0 0;0 1 0 0;0 0 1 0;0 0 0 1];

%%matrix 2
Rz_theta2=[cosd(0) -sind(0) 0 0;sind(0) cosd(0) 0 0;0 0 1 0;0 0 0 1];
Rx_alpha2=[1 0 0 0;0 cosd(alpha2) -sind(alpha2) 0;0 sind(alpha2)
 cosd(alpha2) 0;0 0 0 1];
Tz_d2=[1 0 0 0;0 1 0 0;0 0 1 (l1 + l2 +q2);0 0 0 1];
Tx_a2=[1 0 0 0;0 1 0 0;0 0 1 0;0 0 0 1];

%matrix 3
Rz_theta3=[cosd(theta3-90) -sind(theta3-90) 0 0;sind(theta3-90)
 cosd(theta3-90) 0 0;0 0 1 0;0 0 0 1];
Rx_alpha3=[1 0 0 0;0 cosd(alpha3) -sind(alpha3) 0;0 sind(alpha3)
 cosd(alpha3) 0;0 0 0 1];
Tz_d3=[1 0 0 0;0 1 0 0;0 0 1 0;0 0 0 1];
Tx_a3=[1 0 0 l3;0 1 0 0;0 0 1 0;0 0 0 1];

%%multiplying to get homogeneous matrices
A_1=Rz_theta1*Tz_d1*Tx_a1*Rx_alpha1
A_2=Rz_theta2*Tz_d2*Tx_a2*Rx_alpha2
A_3=Rz_theta3*Tz_d3*Tx_a3*Rx_alpha3

transformation = A_1*A_2*A_3
```

```
A_1 =

[ cos((pi*(theta1 + 90))/180), 0,  sin((pi*(theta1 + 90))/180),  0]
[ sin((pi*(theta1 + 90))/180), 0, -cos((pi*(theta1 + 90))/180),  0]
[                           0, 1,                            0, l1]
[                           0, 0,                            0,  1]


A_2 =

[ 1,  0, 0,            0]
[ 0,  0, 1,            0]
```

```
[ 0, -1, 0, l1 + l2 + q2]
[ 0,  0, 0,            1]


A_3 =

[ cos((pi*(theta3 - 90))/180), -sin((pi*(theta3 - 90))/180), 0,
 l3*cos((pi*(theta3 - 90))/180)]
[ sin((pi*(theta3 - 90))/180),  cos((pi*(theta3 - 90))/180), 0,
 l3*sin((pi*(theta3 - 90))/180)]
[                           0,                            0, 1,
                              0]
[                           0,                            0, 0,
                              1]


transformation =

[ cos((pi*(theta1 + 90))/180)*cos((pi*(theta3 - 90))/180) -
 sin((pi*(theta1 + 90))/180)*sin((pi*(theta3 - 90))/180), -
 cos((pi*(theta1 + 90))/180)*sin((pi*(theta3 - 90))/180) -
 cos((pi*(theta3 - 90))/180)*sin((pi*(theta1 + 90))/180), 0,
 sin((pi*(theta1 + 90))/180)*(l1 + l2 + q2) + l3*cos((pi*(theta1
 + 90))/180)*cos((pi*(theta3 - 90))/180) - l3*sin((pi*(theta1 +
 90))/180)*sin((pi*(theta3 - 90))/180)]
[ cos((pi*(theta1 + 90))/180)*sin((pi*(theta3 - 90))/180) +
 cos((pi*(theta3 - 90))/180)*sin((pi*(theta1 + 90))/180),
  cos((pi*(theta1 + 90))/180)*cos((pi*(theta3 - 90))/180) -
 sin((pi*(theta1 + 90))/180)*sin((pi*(theta3 - 90))/180), 0,
 l3*cos((pi*(theta1 + 90))/180)*sin((pi*(theta3 - 90))/180) -
 cos((pi*(theta1 + 90))/180)*(l1 + l2 + q2) + l3*cos((pi*(theta3 -
 90))/180)*sin((pi*(theta1 + 90))/180)]
[
                                                                0,

                     0, 1,

                                                                      l1]
[
                                                                0,

                     0, 0,

                                                                       1]
```

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③ To prove a constraint is non-holonomic we have to prove $\gamma = 0$

in $[\,0 \quad 1 \quad p\sin q_5 \quad p\cos q_3 \cos q_5\,]\dot{q} = 0$

Here,

$a_1 = 0$

$a_2 = 1$

$a_3 = p\sin q_5$

$a_4 = p\cos q_3$

$a_5 = \cos q_5$

and $\dot{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \end{bmatrix}$

and we use $\dfrac{\partial \gamma(q) a_k(q)}{\partial q_j} = \dfrac{\partial \gamma(q)\, a_j(q)}{\partial q_k}$

$\forall\; j, k = 1, 2, 3 \ldots n$

The conditions are

j	k
1	2
1	3
1	4
1	5
2	3
2	4
2	5
3	4
3	5
4	5

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$$\rightarrow \frac{\partial r_{q_2}}{\partial q_1} = \frac{\partial r_{q_1}}{\partial q_2}$$

$$\Rightarrow \frac{\partial r(1)}{\partial q_1} = \frac{\partial r(0)}{\partial q_2}$$

$$\Rightarrow \boxed{\frac{\partial r}{\partial q_1} = 0} \text{ — } (1)$$

$$\rightarrow \frac{\partial r_{q_3}}{\partial q_1} = \frac{\partial r_{q_1}}{\partial q_3}$$

$$\Rightarrow r\left(\frac{\partial p \sin q_5}{\partial q_1}\right) + p \sin q_5 \frac{\partial r}{\partial q_1} = 0$$

$$\Rightarrow 0 = 0 \quad (\text{Redundant})$$

$$\rightarrow \frac{\partial r_{q_4}}{\partial q_1} = \frac{\partial r_{q_1}}{\partial q_4}$$

$$\Rightarrow \frac{\partial(p\cos q_3)}{\partial q_1} = \frac{\partial r(0)}{\partial q_4} \Rightarrow 0 = 0$$

Redundant

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$$\Rightarrow \frac{\partial r a_5}{\partial q_1} = \frac{\partial r a_1}{\partial q_5}$$

$$\Rightarrow \frac{r\,\partial(\cos q_5)}{\partial q_1} + \cos q_5 \frac{\partial r}{\partial q_1} = 0$$

$\Rightarrow 0 + 0 = 0$ <u>Redundant</u>

$$\Rightarrow \frac{\partial r a_3}{\partial q_2} = \frac{\partial r a_2}{\partial q_3}$$

$$\Rightarrow r(0) + \frac{\partial r}{\partial q_3} = p \sin q_5 \frac{\partial r}{\partial q_2} + 0$$

$$\Rightarrow \boxed{p \sin q_5 \left(\frac{\partial r}{\partial q_2}\right) = \frac{\partial r}{\partial q_3}} \quad (2)$$

$$\Rightarrow \frac{\partial r a_4}{\partial q_2} = \frac{\partial r a_2}{\partial q_4}$$

$$\Rightarrow r(0) + \frac{\partial r}{\partial q_4} = p \cos q_3 \frac{\partial r}{\partial q_2}$$

$$\Rightarrow \boxed{p \cos q_3 \frac{\partial r}{\partial q_2} = \frac{\partial r}{\partial q_4}} \quad (3)$$

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$$\rightarrow \frac{\partial r a_5}{\partial q_2} = \frac{\partial r a_2}{\partial q_5}$$

$$\Rightarrow \frac{\partial r}{\partial q_5} = 0 + \cos q_5 \frac{\partial r}{\partial q_2}$$

$$\Rightarrow \boxed{\frac{\partial r}{\partial q_5} = \cos q_5 \frac{\partial r}{\partial q_2}} \quad (4)$$

$$\rightarrow \frac{\partial r a_4}{\partial q_3} = \frac{\partial r a_3}{\partial q_4}$$

$$\Rightarrow p \cos q_3 \frac{\partial r}{\partial q_3} + p(-\sin q_3) = r(0) + p \sin q_5 \frac{\partial r}{\partial q_4}$$

$$\Rightarrow \boxed{\sin q_5 \frac{\partial r}{\partial q_4} = \cos q_3 \frac{\partial r}{\partial q_3} - r \sin q_3} \quad (5)$$

$$\rightarrow \frac{\partial r a_5}{\partial q_3} = \frac{\partial r a_3}{\partial q_5}$$

$$= \frac{\partial r(p \sin q_5)}{\partial q_5} = \frac{\partial r(\cos q_5)}{\partial q_3}$$

$$\Rightarrow \boxed{\cos q_5 \frac{\partial r}{\partial q_3} = r p \cos q_5 + p \sin q_5 \frac{\partial r}{\partial q_5}} \quad (6)$$

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$$\Rightarrow \frac{\partial r a_5}{\partial q_{v_4}} = \frac{\partial r a_4}{\partial q_{v_5}}$$

$$\Rightarrow r(0) + \cos q_{v_5} \frac{\partial r}{\partial q_{v_4}} = 0 + \cos q_{v_3} \frac{\partial r}{\partial q_{v_5}}$$

$$\Rightarrow \boxed{\cos q_{v_5} \frac{\partial r}{\partial q_{v_4}} = \cos q_{v_3} \frac{\partial r}{\partial q_{v_5}}} \quad (7)$$

Using eqn 2 through (7)

* putting (2) and (3) in eq (1) we get:

$$\sin q_{v_5}\, \rho \cos q_{v_3} \frac{\partial r}{\partial q_{v_2}} = \cos q_{v_3}\, \rho \sin q_{v_3} \frac{\partial r}{\partial q_{v_2}} - r \sin q_{v_3}$$

Cancelling out $\Rightarrow$

$$\boxed{r \sin q_{v_3} = 0} \quad (a)$$

* putting (2) and (4) in eqn (6) we get.

$$\cos q_{v_5}\, \rho \sin q_{v_5} \frac{\partial r}{\partial q_{v_2}} = r \rho \cos q_{v_5} + \rho \sin q_{v_5} \cos q_{v_5} \frac{\partial r}{\partial q_{v_2}}$$

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$$\Rightarrow \boxed{r p \cos q_5 = 0} \quad \text{(6)}$$

from (7)

$$\cos q_5 - p\cos q_3 \frac{\partial r}{\partial q_2} = \cos q_3 \, p\cos q_5 \frac{\partial r}{\partial q_2}$$

$\Rightarrow$ Redundant

Hence from (9) and (6)

the test function $\boxed{r = 0}$

$\Rightarrow$ The differential constraint

$$[0 \quad 1 \quad p\sin q_3 \quad p\cos q_3 \quad \cos q_5]\dot{q}$$

in R^5 is NON-HOLONOMIC.

Hence Proved.

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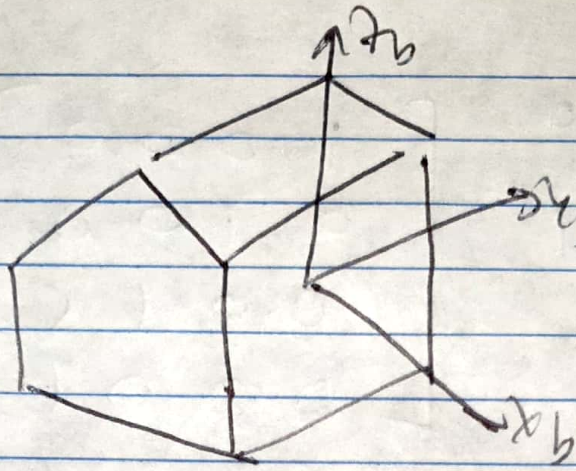
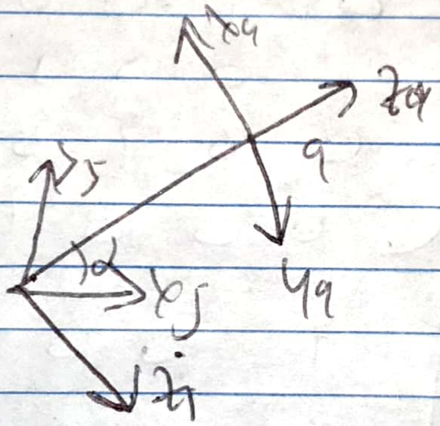
4





DH table

link	θ	a	d	α
1	$\alpha + 90$	0	0	90
	0	5	0	0

For T_j^b

θ	d	a	α
0	5	0	0
0	0	-5	0
90	0	25	0
0	0	0	95
0	$\frac{S}{S_2}$	0	0

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$$A^{0}_{1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

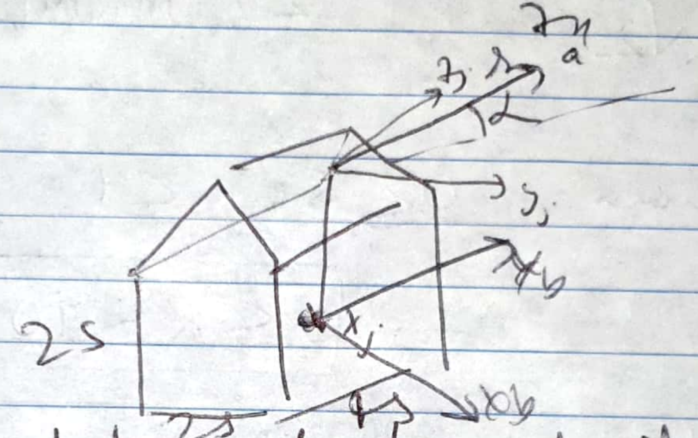
$$A^{2}_{3} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A^{1}_{2} = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A^{7}_{8} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

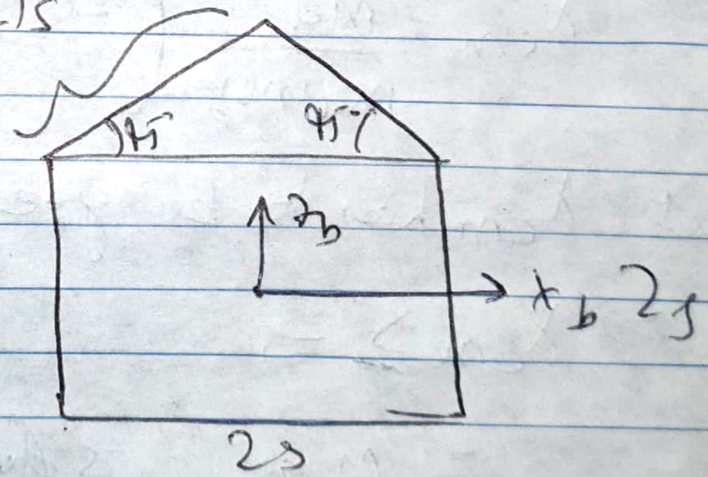
Continuing, $T = \begin{bmatrix} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{5}{2} \\ 1 & 0 & 0 & 25 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{35}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$

looking at it from the top



and then looking at it from the side

$\sqrt{s^2+s^2} = (\sqrt{2})s$



we know, $r = d_s$

$$O_q^b = \begin{bmatrix} -\frac{s}{2} - \frac{d_s}{\sqrt{2}}\sin\alpha \\ 2s + d_s\cos\alpha \\ \frac{d_s}{\sqrt{2}}\sin\alpha + \frac{3s}{2} \end{bmatrix}$$

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$$X_{cm} = \left(-\frac{ds}{\sqrt{2}}\sin\alpha - \frac{s}{2}\right)\frac{m_a}{m_a+m_b} \quad \text{— (1)}$$

We know $0 \le \sin\alpha \le 1$

$$\Rightarrow \boxed{0 \le \alpha \le 90^\circ}$$

Substituting in eqn (1)

$$X_{cm} = \frac{m_a}{m_a+m_b}\left(\frac{-s - ds\sqrt{2}}{2}\right)$$

But X_{cm} has to be greater than $-s$

$$\Rightarrow X_{cm} \ge -s$$

$$\Rightarrow \frac{m_a}{m_a+m_b}\left(-s\left(\frac{d\sqrt{2}+1}{2}\right)\right) \ge -s$$

$$\Rightarrow \frac{m_a}{m_a+m_b} \ge \left(\frac{2}{d\sqrt{2}+1}\right)$$

taking maximum →

$$\frac{d\sqrt{2}H}{2} \geq \frac{m_a + m_b}{m_a}$$

Now, from the basic equations of the center of mass of a rigid body

$$X_{cm} = \sum_{i=1}^{n} \frac{m_i x_i}{\sum_{i=1}^{n} m_i}$$

$\Rightarrow$ Here $X_{cm} = \dfrac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$

But $x_b = x_2 = 0$

$$\Rightarrow \boxed{X_{cm} = \frac{m_a x_a}{m_a + m_b}}$$

Similarly

$$\boxed{Y_{cm} = \frac{m_a y_a}{m_a + m_b}}$$

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Now, Rotation is 45° about the y axis

$$O_a^b = \begin{bmatrix} -\frac{r}{\sqrt{2}} \sin\alpha - \frac{s}{2} \\ 2s + r\cos\alpha \\ \frac{r}{\sqrt{2}} \sin\alpha + \frac{3s}{2} \end{bmatrix}$$

going back to the initial equation of the center of mass

$$X_{cm} = \frac{M_a x_a}{M_a + M_b}$$

$$= \frac{M_a}{M_a + M_b}\left(-\frac{s}{2} (\sqrt{2} d + 1) \right)$$

But $X_{cm} \geq -s$

$$\Rightarrow \frac{M_a}{M_a + M_b}\left(-\frac{s}{2} (\sqrt{2} d + 1) \right) \geq -s$$

$$\Rightarrow \frac{(d\sqrt{2} + 1)}{2} \geq \frac{M_a + M_b}{M_a}$$

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$$\Rightarrow \frac{d\sqrt{2}+1}{2} \geq 1 + \frac{m_b}{m_a}$$

$$\Rightarrow \frac{d\sqrt{2}+1}{2} - 1 \geq \frac{m_b}{m_a}$$

$$\Rightarrow \boxed{\frac{m_b}{m_a} \leq \frac{d\sqrt{2}-1}{2}}$$

also, $\boxed{Y_{cm} = \frac{M_a}{M_a+m_b}(d\cos\alpha + 2s)}$

For the maximum values of Y_{cm}

$0 \leq \cos\alpha \leq 1$

$\Rightarrow$ ~~90 ≥ α ≥ 0~~ $0 \leq \alpha \leq 90$

$\therefore \boxed{\alpha = 0}$ $\left[\because \cos 0 = 1\right]$

Substituting in Y_{CM}

$$Y_{CM} = \frac{M_a}{M_a+m_b}(d + 2s)$$

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But $y_{cm} \leq 2s$

$$\Rightarrow \frac{d+2s}{2s} \leq \frac{m_a+m_b}{m_a}$$

$$\Rightarrow \frac{d}{2s} + \cancel{1} \leq \cancel{1} + \frac{m_b}{m_a}$$

$$\Rightarrow \boxed{\frac{m_b}{m_a} \geq \frac{d}{2s}}$$

$\therefore$ The conditions are:

$$\boxed{\frac{d}{2s} \leq \frac{m_b}{m_a} \leq \frac{d\sqrt{2}-1}{2}}$$

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