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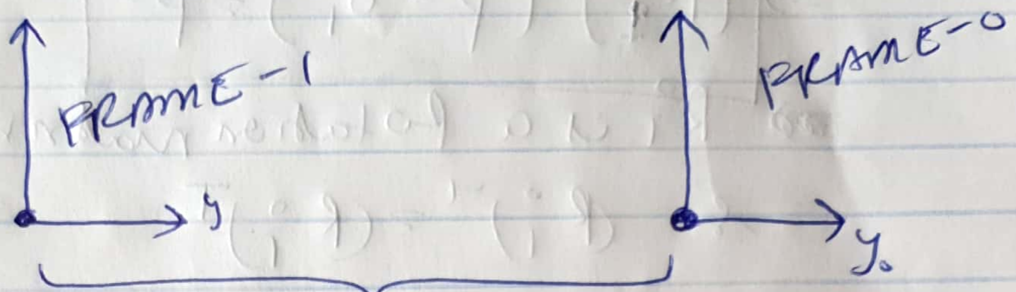
ENPM-662

HW-4

(1)



$$\theta_1^*(t) = (t-2)\frac{\pi}{4} = \left[\frac{\pi t}{4} - \frac{\pi}{2}\right]$$

$$Z_0 = [0 \;\; 0 \;\; 1]^T = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$P^0(t) = [t \quad 2t^2 \quad 3t^3+1]^T$$ → position of particle in fixed '0' frame.

(a) $P^0 = R_1^0 P^1 + O_1^0$

Here the Rotation is about the Z axis

Hence the general form of the rotation matrix is given as $\begin{bmatrix} +C\theta & -S\theta & 0 \\ +S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = R_1^0$

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We want to find p^1

$$\Rightarrow \boxed{(R_1^0)^{-1}(p^0 - o_1^0) = p^1}$$

R_1^0 is a rotation matrix,

so $(R_1^0)^{-1} = (R_1^0)^T$

$$\Rightarrow \begin{bmatrix} \cos\left(\frac{\pi t}{4} - \frac{\pi}{2}\right) & -\sin\left(\frac{\pi t}{4} - \frac{\pi}{2}\right) & 0 \\ \sin\left(\frac{\pi t}{4} - \frac{\pi}{2}\right) & \cos\left(\frac{\pi t}{4} - \frac{\pi}{2}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix}^T = (R_1^0)^{-1}$$

Writing $\cos\theta$ as $c\,\theta \rightarrow$

$$\Rightarrow \begin{bmatrix} c\left(\frac{\pi t}{4} - \frac{\pi}{2}\right) & +\sin\left(\frac{\pi t}{4} - \frac{\pi}{2}\right) & 0 \\ -s\left(\frac{\pi t}{4} - \frac{\pi}{2}\right) & \cos\left(\frac{\pi t}{4} - \frac{\pi}{2}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} t \\ 2t^2 \\ 3t^3+1 \end{bmatrix} - \begin{bmatrix} 0 \\ -d \\ 0 \end{bmatrix} \right) = p^1$$

$$\Rightarrow \begin{bmatrix} +s\left(\frac{\pi t}{4}\right) & -c\left(\frac{t\pi}{4}\right) & 0 \\ +c\left(\frac{\pi t}{4}\right) & s\left(\frac{\pi t}{4}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} t \\ 2t^2+d \\ 3t^3+1 \end{bmatrix} = p^1$$

$$\begin{bmatrix} t\cdot S\left(\frac{\pi t}{4}\right) - C\left(\frac{\pi t}{4}\right)\cdot(2t^2+d) \\ + t\cdot C\left(\frac{\pi t}{4}\right) + S\left(\frac{\pi t}{4}\right)\cdot(2t^2+d) \\ 3t^3+1 \end{bmatrix} = p'$$

This is the particle's position as a function of time w.r.t frame $\underline{\underline{1}}$

(iii)

From (i) we know that

$$p' = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 & 0 \\ -\sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} t \\ 2t^2 \\ 3t^3+1 \end{bmatrix}$$

where $\theta_1 = \frac{\pi}{4}t - \frac{\pi}{2}$

Velocity is the differential of p'

$\therefore (p')' =$ velocity.

$$\therefore (p')' = \text{derivative of} \begin{bmatrix} t\cos\theta_1 + 2t^2\sin\theta_1 + 0 \\ -t\sin\theta_1 + 2t^2\cos\theta_1 + 0 \\ 0+0+3t^3+1 \end{bmatrix}$$

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$$\therefore (p')' = \begin{bmatrix} t\cos\theta_1 + 2t^2 \sin\theta_1 \\ -t\sin\theta_1 + 2t^2\cos\theta_1 \\ 3t^3 + 1 \end{bmatrix} = \text{velocity of the particle}$$

Let $\cos\theta_1 = c\theta_1$, Let $\sin\theta_1 = c\theta_1$

$\theta_1 = \frac{\pi}{4}t - \frac{\pi}{2}$

$$\text{Velocity} = \begin{bmatrix} -ts\theta\left[\frac{\pi}{4}\right] + c\theta + 4ts\theta + 2t^2 c\theta\left(\frac{\pi}{4}\right) + d\,c\theta\frac{\pi}{4} \\ -tc\theta\left(\frac{\pi}{4}\right) + (-s\theta) + 4tc\theta + 2t^2(-s\theta)\left(\frac{\pi}{4}\right) + d(-s\theta)\frac{\pi}{4} \\ 9t^2 + 0 \end{bmatrix}$$

$$\Rightarrow \text{velocity} = \begin{bmatrix} -\frac{t\pi}{4}s\theta + c\theta + 4ts\theta + \frac{t^2\pi}{2}c\theta + \frac{\pi d}{4}c\theta \\ -\frac{\pi t}{4}c\theta - s\theta + 4tc\theta - \frac{\pi t^2}{2}s\theta - \frac{\pi d}{4}s\theta \\ 9t^2 \end{bmatrix}$$

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$$\Rightarrow \text{velocity} = \begin{bmatrix} \left[\frac{t^2\pi}{2} + \frac{t\pi d}{4} + 1\right]\cos\theta + \left[4t - \frac{\pi t}{4}\right]\sin\theta \\ \left[4t - \frac{\pi t}{4}\right]\cos\theta + \left[-1 - \frac{\pi t^2}{4} - \frac{\pi d}{4}\right]\sin\theta \\ 9t^2 \end{bmatrix}$$

Now, we know that $\theta = \frac{\pi}{4}t - \frac{\pi}{2}$

$\Rightarrow \cos\theta = \cos\left(\frac{\pi t}{4} - \frac{\pi}{2}\right) = \sin\left(\frac{\pi}{4}t\right)$ $\quad \sin\theta = \sin\left(\frac{\pi}{4}t - \frac{\pi}{2}\right) = -\cos\left(\frac{\pi}{4}t\right)$

Velocity =

$$\begin{bmatrix} \left(\frac{t^2\pi}{2} + \frac{\pi d}{4} + 1\right)\sin\left(\frac{\pi}{4}t\right) + \left(4t - \frac{\pi t}{4}\right)\left(-\cos\frac{\pi}{4}t\right) \\ \left(4t - \frac{\pi t}{4}\right)\sin\left(\frac{\pi t}{4}\right) + \left[-1 - \frac{\pi t^2}{4} - \frac{\pi d^2}{4}\right]\left(-\cos\frac{\pi}{4}t\right) \\ 9t^2 \end{bmatrix}$$

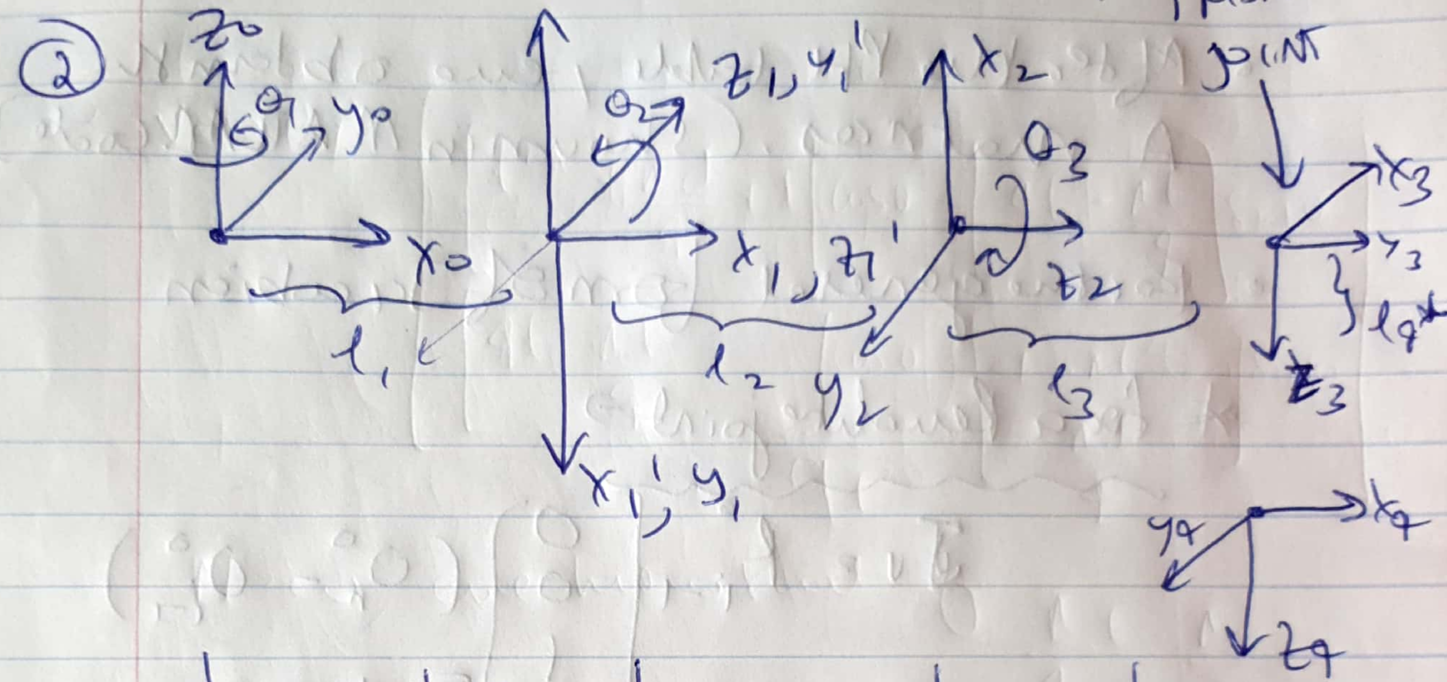
$\uparrow$

Final velocity of particle in FRAME 1.

REVOLUTE JOINTS

PRISMATIC JOINT

(2)



	Z_{n-1}	Z_{n-1}	X_n	X_n
	θ	d	a	α
$0 \to 1$	θ_1^*	0	l_1	-90
$1 \to 1'$	$\theta_2^* + 90$	0	0	90
$1' \to 2$	0	l_2	0	0
$2 \to 3$	$\theta_3^* + 90$	l_3	0	90
$3 \to 4$	90	l_4^*	0	0

This gives the forward kinematics of the manipulator.

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- After the PH table, we obtain the A matrices. (Shown in MATLAB code)

- For the general form of Jacobian

* for a Revolute joint →

$$J_v = R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}(O_n^0 - O_{i-1}^0)$$

← along z-axis

$$J_\omega = R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$$

* for Prismatic joint →

$$J_v = R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$J_\omega = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

→ Ans. a

We also know

$$J = \begin{bmatrix} z_0(o_1-o_0) & z_1(o_1-o_1) & z_2(o_1-o_2) & z_3 \\ z_0 & z_1 & z_2 & 0 \end{bmatrix}$$

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$\theta_1, \theta_2, \theta_n$ are depicted in MATLAB code

↑

Ans: (b)

(c) $l_1 = 3, l_2 = 2, l_3 = 1$

$q = [0, 45, 30, 0.5]$

here $\theta_1 = 0°$

$\theta_2 = 45°$

$\theta_3 = 30°$

$d_4^* = 0.5$

Jacobian =

$$\begin{bmatrix} -0.25 & -2.42 & 0.176 & -0.612 \\ 4.815 & 0 & 0.433 & 0.5 \\ 0 & -1.8157 & 0.176 & -0.612 \\ 0 & 0 & 0.7071 & 0 \\ 0 & 1.00 & 0 & 0 \\ 1.00 & 0 & -0.7071 & 0 \end{bmatrix}$$

Answers on MATLAB code.

Final velocity vector:

$$\begin{bmatrix} -9.8823 \\ 16.8605 \\ -6.6828 \\ 3.5355 \\ 4.000 \\ -0.5355 \end{bmatrix}$$

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Z matrices are obtained by multiplying the Rotational matrices with $[0\ 0\ 1]^T$. This is because, the Rotation is about the Z-axis.

Rotation matrices are obtained from the 3x3 matrix part of the homogeneous matrix

The origin is the first 3 elements of the last Column of the homogeneous matrix.

All are depicted clearly in the MATLAB code.

But taking the values of z_0, z_1, z_2, z_3 from the output of MATLAB code, we get

Ans: (b)

$$z_0 = [0 \quad 0 \quad 1]^T$$

$$z_1 = [-S\theta_1 \quad C\theta_1 \quad 0]^T$$

$$z_2 = [(C\theta_1 * S(\theta_2+90)) \quad (S\theta_1 * S(\theta_2+90)) \quad C(\theta_2+90)]^T$$

$$z_3 = \begin{bmatrix} (S\theta_1)(C(\theta_3+90)) + C\theta_1 \cdot C(\theta_2+90) \cdot S(\theta_3+90) \\ (S\theta_1(C(\theta_2+90))S(\theta_3+90)) - (C\theta_1 S(C(\theta_3+90)) \\ - S(\theta_2+90) * S(\theta_3+90) \end{bmatrix}$$

→ Expanded in MATLAB publication (attached)

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```
%MATLAB CODE ASSIGNMENT 4 ENPM662
%ANSWER 2 (a) 2 (b) 2 (c)

%%2(a) generic jacobian form written by hand
clc
clear all
%writing the code for A matrices
%%declaring the variables and symbols for the matrix multiplication
syms theta1 theta2 theta3 l1 l2 l3 l4 %%declaring all the unknown
 values as symbols

%%declaring the values of the a variables for translation along Xn
a2=0;
a3=0;
a4=0;
a5=0;
%%declaring the values of the alpha variables for rotation about Xn
alpha1=-90;
alpha2=90;
alpha3=0;
alpha4=90;
alpha5=0;
%%declaring the value of theta for frame 3 to 4 transformation
theta5=90;

%%The general form of the matrices are obtained by multiplying the
%%following

%%first A matrix A_1
Rz_theta1=[cosd(theta1) -sind(theta1) 0 0;sind(theta1) cosd(theta1) 0
 0;0 0 1 0;0 0 0 1];
Rx_alpha1=[1 0 0 0;0 cosd(alpha1) -sind(alpha1) 0;0 sind(alpha1)
 cosd(alpha1) 0;0 0 0 1];
Tz_d1=[1 0 0 0;0 1 0 0;0 0 1 (0);0 0 0 1];
Tx_a1=[1 0 0 l1;0 1 0 0;0 0 1 0;0 0 0 1];

A_1=Rz_theta1*Tz_d1*Tx_a1*Rx_alpha1

%%second A matrix A_2
Rz_theta2=[cosd(theta2+90) -sind(theta2+90) 0 0;sind(theta2+90)
 cosd(theta2+90) 0 0;0 0 1 0;0 0 0 1];
Rx_alpha2=[1 0 0 0;0 cosd(alpha2) -sind(alpha2) 0;0 sind(alpha2)
 cosd(alpha2) 0;0 0 0 1];
Tz_d2=[1 0 0 0;0 1 0 0;0 0 1 (0);0 0 0 1];
Tx_a2=[1 0 0 a2;0 1 0 0;0 0 1 0;0 0 0 1];

A_2=Rz_theta2*Tz_d2*Tx_a2*Rx_alpha2

%%third A matrix A_3
Rz_theta3=[cosd(0) -sind(0) 0 0;sind(0) cosd(0) 0 0;0 0 1 0;0 0 0 1];
Rx_alpha3=[1 0 0 0;0 cosd(alpha3) -sind(alpha3) 0;0 sind(alpha3)
 cosd(alpha3) 0;0 0 0 1];
```

```
Tz_d3=[1 0 0 0;0 1 0 0;0 0 1 (l2);0 0 0 1];
Tx_a3=[1 0 0 a3;0 1 0 0;0 0 1 0;0 0 0 1];

A_3=Rz_theta3*Tz_d3*Tx_a3*Rx_alpha3

%%fourth A matrix A_4
Rz_theta4=[cosd(theta3+90) -sind(theta3+90) 0 0;sind(theta3+90)
 cosd(theta3+90) 0 0;0 0 1 0;0 0 0 1];
Rx_alpha4=[1 0 0 0;0 cosd(alpha4) -sind(alpha4) 0;0 sind(alpha4)
 cosd(alpha4) 0;0 0 0 1];
Tz_d4=[1 0 0 0;0 1 0 0;0 0 1 (l3);0 0 0 1];
Tx_a4=[1 0 0 a4;0 1 0 0;0 0 1 0;0 0 0 1];

A_4=Rz_theta4*Tz_d4*Tx_a4*Rx_alpha4

%%fifth A matriX A_5
Rz_theta5=[cosd(theta5) -sind(theta5) 0 0;sind(theta5) cosd(theta5) 0
 0;0 0 1 0;0 0 0 1];
Rx_alpha5=[1 0 0 0;0 cosd(alpha5) -sind(alpha5) 0;0 sind(alpha5)
 cosd(alpha5) 0;0 0 0 1];
Tz_d5=[1 0 0 0;0 1 0 0;0 0 1 (l4);0 0 0 1];
Tx_a5=[1 0 0 a5;0 1 0 0;0 0 1 0;0 0 0 1];

A_5=Rz_theta5*Tz_d5*Tx_a5*Rx_alpha5

%%FINDING HOMOGENEOUS, ROTATION MATRICES AND ORIGINS FOR JACOBIAN
 CALCULATION
%%JOINT 1
H_1_0=A_1 %%Homogeneous Matrix for Joint 1
R_1_0=H_1_0(1:3,1:3) %%Rotation matrix for Joint 1
O_1_0=H_1_0(1:3,4); %%Origin for matrix 1
%%JOINT 2
H_2_0=A_1*A_2*A_3 %%Homogeneous Matrix for Joint 2
R_2_0=H_2_0(1:3,1:3) %%Rotation matrix for Joint 2
O_2_0=H_2_0(1:3,4); %%Origin for matrix 2
%%JOINT 3
H_3_0=A_1*A_2*A_3*A_4 %%Homogeneous Matrix for Joint 3
R_3_0=H_3_0(1:3,1:3)  %%Rotation matrix for Joint 3
O_3_0=H_3_0(1:3,4); %%Origin for matrix 3
%%JOINT 4
H_4_0=A_1*A_2*A_3*A_4*A_5  %%Homogeneous Matrix for Joint 4
R_4_0=H_4_0(1:3,1:3) %%Rotation matrix for Joint 4
O_4_0=H_4_0(1:3,4); %%Origin for matrix 4

fprintf("Answer B: ")
fprintf("The o matrices are given here :  ")
O_1_0=H_1_0(1:3,4)
O_2_0=H_2_0(1:3,4)
O_3_0=H_3_0(1:3,4)
O_4_0=H_4_0(1:3,4)
fprintf("The z matrices are given below : ")
z1=R_1_0*[0 0 1]' %%z1 matrix for jacobian calculation
z2=R_2_0*[0 0 1]' %%z2 matrix for jacobian calculation
z3=R_3_0*[0 0 1]' %%z3 matrix for jacobian calculation
```

```
fprintf("Answer C: ")
%%substituting the values of l given in the question
l1=3;
l2=2;
l3=1;
%%substituting the values  of theta given in the question
theta1=0;
theta2=45;
theta3=30;
l4=0.5;
%%substituting the values of a
a2=0;
a3=0;
a4=0;
a5=0;
%%substituting the values of alpha
alpha1=-90;
alpha2=90;
alpha3=0;
alpha4=90;
alpha5=0;
%%substituting the values of theta
theta5=90;

%%The general form of the matrices are obtained by multiplying the
%%following

%%first A matrix
Rz_theta1=[cosd(theta1) -sind(theta1) 0 0;sind(theta1) cosd(theta1) 0
 0;0 0 1 0;0 0 0 1];
Rx_alpha1=[1 0 0 0;0 cosd(alpha1) -sind(alpha1) 0;0 sind(alpha1)
 cosd(alpha1) 0;0 0 0 1];
Tz_d1=[1 0 0 0;0 1 0 0;0 0 1 (0);0 0 0 1];
Tx_a1=[1 0 0 l1;0 1 0 0;0 0 1 0;0 0 0 1];

%%second A matrix
Rz_theta2=[cosd(theta2+90) -sind(theta2+90) 0 0;sind(theta2+90)
 cosd(theta2+90) 0 0;0 0 1 0;0 0 0 1];
Rx_alpha2=[1 0 0 0;0 cosd(alpha2) -sind(alpha2) 0;0 sind(alpha2)
 cosd(alpha2) 0;0 0 0 1];
Tz_d2=[1 0 0 0;0 1 0 0;0 0 1 (0);0 0 0 1];
Tx_a2=[1 0 0 a2;0 1 0 0;0 0 1 0;0 0 0 1];

%%third A matrix
Rz_theta3=[cosd(0) -sind(0) 0 0;sind(0) cosd(0) 0 0;0 0 1 0;0 0 0 1];
Rx_alpha3=[1 0 0 0;0 cosd(alpha3) -sind(alpha3) 0;0 sind(alpha3)
 cosd(alpha3) 0;0 0 0 1];
Tz_d3=[1 0 0 0;0 1 0 0;0 0 1 (l2);0 0 0 1];
Tx_a3=[1 0 0 a3;0 1 0 0;0 0 1 0;0 0 0 1];

%%fourth A matrix
Rz_theta4=[cosd(theta3+90) -sind(theta3+90) 0 0;sind(theta3+90)
 cosd(theta3+90) 0 0;0 0 1 0;0 0 0 1];
```

```
Rx_alpha4=[1 0 0 0;0 cosd(alpha4) -sind(alpha4) 0;0 sind(alpha4)
 cosd(alpha4) 0;0 0 0 1];
Tz_d4=[1 0 0 0;0 1 0 0;0 0 1 (l3);0 0 0 1];
Tx_a4=[1 0 0 a4;0 1 0 0;0 0 1 0;0 0 0 1];

%%fifth A matrix
Rz_theta5=[cosd(theta5) -sind(theta5) 0 0;sind(theta5) cosd(theta5) 0
 0;0 0 1 0;0 0 0 1];
Rx_alpha5=[1 0 0 0;0 cosd(alpha5) -sind(alpha5) 0;0 sind(alpha5)
 cosd(alpha5) 0;0 0 0 1];
Tz_d5=[1 0 0 0;0 1 0 0;0 0 1 (l4);0 0 0 1];
Tx_a5=[1 0 0 a5;0 1 0 0;0 0 1 0;0 0 0 1];

%%Matrix multiplicaion
A_1=Rz_theta1*Tz_d1*Tx_a1*Rx_alpha1
A_2=Rz_theta2*Tz_d2*Tx_a2*Rx_alpha2
A_3=Rz_theta3*Tz_d3*Tx_a3*Rx_alpha3
A_4=Rz_theta4*Tz_d4*Tx_a4*Rx_alpha4
A_5=Rz_theta5*Tz_d5*Tx_a5*Rx_alpha5
%%Multipltying to get the T matrix we get

%%REVOLUTE MATRICES FOR JACOBIAN CALCULATION
%%JOINT 1
O_0_0=[0 0 0]'
H_1_0=A_1
R_1_0=H_1_0(1:3,1:3)
O_1_0=H_1_0(1:3,4)
%%JOINT 2
H_2_0=A_1*A_2*A_3
R_2_0=H_2_0(1:3,1:3)
O_2_0=H_2_0(1:3,4)
%%JOINT 3
H_3_0=A_1*A_2*A_3*A_4
R_3_0=H_3_0(1:3,1:3)
O_3_0=H_3_0(1:3,4)
%%JOINT 4
H_4_0=A_1*A_2*A_3*A_4*A_5
R_4_0=H_4_0(1:3,1:3)
O_4_0=H_4_0(1:3,4)

z0=eye(3)*[0 0 1]'
z1=R_1_0*[0 0 1]'
z2=R_2_0*[0 0 1]'
z3=R_3_0*[0 0 1]'

%%taking cross products of the vectors and calculating the Jacobian
 Matrix
q_dot=[3,4,5,0.5]'%given in the question
Jacobian=[cross(z0,O_4_0-O_0_0) cross(z1,O_4_0-O_1_0) cross(z2,O_4_0-
O_2_0) z3;z0 z1 z2 [0 0 0]']
velocity_vectors=Jacobian*q_dot
```

A_1 =

```
[ cos((pi*theta1)/180),  0, -sin((pi*theta1)/180),
 l1*cos((pi*theta1)/180)]
[ sin((pi*theta1)/180),  0,  cos((pi*theta1)/180),
 l1*sin((pi*theta1)/180)]
[                    0, -1,                     0,
    0]
[                    0,  0,                     0,
    1]


A_2 =

[ cos((pi*(theta2 + 90))/180), 0,  sin((pi*(theta2 + 90))/180), 0]
[ sin((pi*(theta2 + 90))/180), 0, -cos((pi*(theta2 + 90))/180), 0]
[                           0, 1,                            0, 0]
[                           0, 0,                            0, 1]


A_3 =

[ 1, 0, 0,  0]
[ 0, 1, 0,  0]
[ 0, 0, 1, l2]
[ 0, 0, 0,  1]


A_4 =

[ cos((pi*(theta3 + 90))/180), 0,  sin((pi*(theta3 + 90))/180),  0]
[ sin((pi*(theta3 + 90))/180), 0, -cos((pi*(theta3 + 90))/180),  0]
[                           0, 1,                            0, l3]
[                           0, 0,                            0,  1]


A_5 =

[ 0, -1, 0,  0]
[ 1,  0, 0,  0]
[ 0,  0, 1, l4]
[ 0,  0, 0,  1]


H_1_0 =

[ cos((pi*theta1)/180),  0, -sin((pi*theta1)/180),
 l1*cos((pi*theta1)/180)]
[ sin((pi*theta1)/180),  0,  cos((pi*theta1)/180),
 l1*sin((pi*theta1)/180)]
[                    0, -1,                     0,
    0]
[                    0,  0,                     0,
    1]
```

```
R_1_0 =

[ cos((pi*theta1)/180),  0, -sin((pi*theta1)/180)]
[ sin((pi*theta1)/180),  0,  cos((pi*theta1)/180)]
[                    0, -1,                     0]


H_2_0 =

[ cos((pi*theta1)/180)*cos((pi*(theta2 + 90))/180), -
sin((pi*theta1)/180), cos((pi*theta1)/180)*sin((pi*(theta2
 + 90))/180), l1*cos((pi*theta1)/180) +
 l2*cos((pi*theta1)/180)*sin((pi*(theta2 + 90))/180)]
[ sin((pi*theta1)/180)*cos((pi*(theta2 + 90))/180),
 cos((pi*theta1)/180), sin((pi*theta1)/180)*sin((pi*(theta2
 + 90))/180), l1*sin((pi*theta1)/180) +
 l2*sin((pi*theta1)/180)*sin((pi*(theta2 + 90))/180)]
[                      -sin((pi*(theta2 + 90))/180),
  0,                     cos((pi*(theta2 + 90))/180),
                               l2*cos((pi*(theta2 + 90))/180)]
[                                                 0,
  0,                                                 0,
                                                            1]


R_2_0 =

[ cos((pi*theta1)/180)*cos((pi*(theta2 + 90))/180), -
sin((pi*theta1)/180), cos((pi*theta1)/180)*sin((pi*(theta2 +
 90))/180)]
[ sin((pi*theta1)/180)*cos((pi*(theta2 + 90))/180),
 cos((pi*theta1)/180), sin((pi*theta1)/180)*sin((pi*(theta2 +
 90))/180)]
[                      -sin((pi*(theta2 + 90))/180),
  0,                     cos((pi*(theta2 + 90))/180)]


H_3_0 =

[ cos((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*cos((pi*(theta3
 + 90))/180) - sin((pi*theta1)/180)*sin((pi*(theta3
 + 90))/180), cos((pi*theta1)/180)*sin((pi*(theta2
 + 90))/180), sin((pi*theta1)/180)*cos((pi*(theta3 +
 90))/180) + cos((pi*theta1)/180)*cos((pi*(theta2 +
 90))/180)*sin((pi*(theta3 + 90))/180), l1*cos((pi*theta1)/180)
 + l2*cos((pi*theta1)/180)*sin((pi*(theta2 + 90))/180) +
 l3*cos((pi*theta1)/180)*sin((pi*(theta2 + 90))/180)]
[ cos((pi*theta1)/180)*sin((pi*(theta3 + 90))/180) +
 sin((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*cos((pi*(theta3
 + 90))/180), sin((pi*theta1)/180)*sin((pi*(theta2 + 90))/180),
 sin((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*sin((pi*(theta3
 + 90))/180) - cos((pi*theta1)/180)*cos((pi*(theta3 + 90))/180),
```

```
 l1*sin((pi*theta1)/180) + l2*sin((pi*theta1)/180)*sin((pi*(theta2 +
 90))/180) + l3*sin((pi*theta1)/180)*sin((pi*(theta2 + 90))/180)]
[
      -cos((pi*(theta3 + 90))/180)*sin((pi*(theta2 + 90))/180),
                  cos((pi*(theta2 + 90))/180),
                                                    -sin((pi*(theta2 +
 90))/180)*sin((pi*(theta3 + 90))/180),
                                        l2*cos((pi*(theta2 + 90))/180) +
 l3*cos((pi*(theta2 + 90))/180)]
[
                                                            0,
                                        0,

                             0,

                     1]


R_3_0 =

[ cos((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*cos((pi*(theta3
 + 90))/180) - sin((pi*theta1)/180)*sin((pi*(theta3 +
 90))/180), cos((pi*theta1)/180)*sin((pi*(theta2 + 90))/180),
 sin((pi*theta1)/180)*cos((pi*(theta3 + 90))/180) +
 cos((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*sin((pi*(theta3 +
 90))/180)]
[ cos((pi*theta1)/180)*sin((pi*(theta3 + 90))/180) +
 sin((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*cos((pi*(theta3
 + 90))/180), sin((pi*theta1)/180)*sin((pi*(theta2 + 90))/180),
 sin((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*sin((pi*(theta3 +
 90))/180) - cos((pi*theta1)/180)*cos((pi*(theta3 + 90))/180)]
[
      -cos((pi*(theta3 + 90))/180)*sin((pi*(theta2 + 90))/180),
                  cos((pi*(theta2 + 90))/180),
                                                    -sin((pi*(theta2 +
 90))/180)*sin((pi*(theta3 + 90))/180)]


H_4_0 =

[ cos((pi*theta1)/180)*sin((pi*(theta2 + 90))/180),
  sin((pi*theta1)/180)*sin((pi*(theta3 + 90))/180) -
 cos((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*cos((pi*(theta3
 + 90))/180), sin((pi*theta1)/180)*cos((pi*(theta3 +
 90))/180) + cos((pi*theta1)/180)*cos((pi*(theta2 +
 90))/180)*sin((pi*(theta3 + 90))/180), l1*cos((pi*theta1)/180)
 + l4*(sin((pi*theta1)/180)*cos((pi*(theta3 + 90))/180) +
 cos((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*sin((pi*(theta3 +
 90))/180)) + l2*cos((pi*theta1)/180)*sin((pi*(theta2 + 90))/180) +
 l3*cos((pi*theta1)/180)*sin((pi*(theta2 + 90))/180)]
[ sin((pi*theta1)/180)*sin((pi*(theta2 + 90))/180),
 - cos((pi*theta1)/180)*sin((pi*(theta3 + 90))/180) -
 sin((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*cos((pi*(theta3
 + 90))/180), sin((pi*theta1)/180)*cos((pi*(theta2
```

+ 90))/180)*sin((pi*(theta3 + 90))/180) -
cos((pi*theta1)/180)*cos((pi*(theta3 + 90))/180),
l1*sin((pi*theta1)/180) - l4*(cos((pi*theta1)/180)*cos((pi*(theta3
+ 90))/180) - sin((pi*theta1)/180)*cos((pi*(theta2
+ 90))/180)*sin((pi*(theta3 + 90))/180)) +
l2*sin((pi*theta1)/180)*sin((pi*(theta2 + 90))/180) +
l3*sin((pi*theta1)/180)*sin((pi*(theta2 + 90))/180)]
[cos((pi*(theta2 + 90))/180),

cos((pi*(theta3 + 90))/180)*sin((pi*(theta2 + 90))/180),
-
sin((pi*(theta2 + 90))/180)*sin((pi*(theta3 + 90))/180),

l2*cos((pi*(theta2 + 90))/180) + l3*cos((pi*(theta2 + 90))/180) -
l4*sin((pi*(theta2 + 90))/180)*sin((pi*(theta3 + 90))/180)]
[0,

0,

0,

1]

R_4_0 =

[cos((pi*theta1)/180)*sin((pi*(theta2 + 90))/180),
sin((pi*theta1)/180)*sin((pi*(theta3 + 90))/180) -
cos((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*cos((pi*(theta3
+ 90))/180), sin((pi*theta1)/180)*cos((pi*(theta3 + 90))/180) +
cos((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*sin((pi*(theta3 +
90))/180)]
[sin((pi*theta1)/180)*sin((pi*(theta2 + 90))/180),
- cos((pi*theta1)/180)*sin((pi*(theta3 + 90))/180) -
sin((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*cos((pi*(theta3
+ 90))/180), sin((pi*theta1)/180)*cos((pi*(theta2
+ 90))/180)*sin((pi*(theta3 + 90))/180) -
cos((pi*theta1)/180)*cos((pi*(theta3 + 90))/180)]
[cos((pi*(theta2 + 90))/180),

cos((pi*(theta3 + 90))/180)*sin((pi*(theta2 + 90))/180),
-
sin((pi*(theta2 + 90))/180)*sin((pi*(theta3 + 90))/180)]

Answer B: The o matrices are given here :
O_1_0 =

l1*cos((pi*theta1)/180)
l1*sin((pi*theta1)/180)
0

```
O_2_0 =

 l1*cos((pi*theta1)/180) + l2*cos((pi*theta1)/180)*sin((pi*(theta2 +
 90))/180)
 l1*sin((pi*theta1)/180) + l2*sin((pi*theta1)/180)*sin((pi*(theta2 +
 90))/180)
                                                  l2*cos((pi*(theta2 +
 90))/180)


O_3_0 =

 l1*cos((pi*theta1)/180) + l2*cos((pi*theta1)/180)*sin((pi*(theta2 +
 90))/180) + l3*cos((pi*theta1)/180)*sin((pi*(theta2 + 90))/180)
 l1*sin((pi*theta1)/180) + l2*sin((pi*theta1)/180)*sin((pi*(theta2 +
 90))/180) + l3*sin((pi*theta1)/180)*sin((pi*(theta2 + 90))/180)

 l2*cos((pi*(theta2 + 90))/180) + l3*cos((pi*(theta2 + 90))/180)


O_4_0 =

 l1*cos((pi*theta1)/180) + l4*(sin((pi*theta1)/180)*cos((pi*(theta3
 + 90))/180) + cos((pi*theta1)/180)*cos((pi*(theta2
 + 90))/180)*sin((pi*(theta3 + 90))/180)) +
 l2*cos((pi*theta1)/180)*sin((pi*(theta2 + 90))/180) +
 l3*cos((pi*theta1)/180)*sin((pi*(theta2 + 90))/180)
 l1*sin((pi*theta1)/180) - l4*(cos((pi*theta1)/180)*cos((pi*(theta3
 + 90))/180) - sin((pi*theta1)/180)*cos((pi*(theta2
 + 90))/180)*sin((pi*(theta3 + 90))/180)) +
 l2*sin((pi*theta1)/180)*sin((pi*(theta2 + 90))/180) +
 l3*sin((pi*theta1)/180)*sin((pi*(theta2 + 90))/180)


    l2*cos((pi*(theta2 + 90))/180) + l3*cos((pi*(theta2 + 90))/180) -
 l4*sin((pi*(theta2 + 90))/180)*sin((pi*(theta3 + 90))/180)

The z matrices are given below :
z1 =

 -sin((pi*theta1)/180)
  cos((pi*theta1)/180)
                     0


z2 =

 cos((pi*theta1)/180)*sin((pi*(theta2 + 90))/180)
 sin((pi*theta1)/180)*sin((pi*(theta2 + 90))/180)
                      cos((pi*(theta2 + 90))/180)


z3 =
```

```
 sin((pi*theta1)/180)*cos((pi*(theta3 + 90))/180) +
 cos((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*sin((pi*(theta3 +
 90))/180)
 sin((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*sin((pi*(theta3 +
 90))/180) - cos((pi*theta1)/180)*cos((pi*(theta3 + 90))/180)

  -sin((pi*(theta2 + 90))/180)*sin((pi*(theta3 + 90))/180)

Answer C:
A_1 =

     1     0     0     3
     0     0     1     0
     0    -1     0     0
     0     0     0     1


A_2 =

   -0.7071         0    0.7071         0
    0.7071         0    0.7071         0
         0    1.0000         0         0
         0         0         0    1.0000


A_3 =

     1     0     0     0
     0     1     0     0
     0     0     1     2
     0     0     0     1


A_4 =

   -0.5000         0    0.8660         0
    0.8660         0    0.5000         0
         0    1.0000         0    1.0000
         0         0         0    1.0000


A_5 =

         0   -1.0000         0         0
    1.0000         0         0         0
         0         0    1.0000    0.5000
         0         0         0    1.0000


O_0_0 =

     0
     0
```

```
     0


H_1_0 =

     1     0     0     3
     0     0     1     0
     0    -1     0     0
     0     0     0     1


R_1_0 =

     1     0     0
     0     0     1
     0    -1     0


O_1_0 =

     3
     0
     0


H_2_0 =

   -0.7071         0    0.7071    4.4142
         0    1.0000         0         0
   -0.7071         0   -0.7071   -1.4142
         0         0         0    1.0000


R_2_0 =

   -0.7071         0    0.7071
         0    1.0000         0
   -0.7071         0   -0.7071


O_2_0 =

    4.4142
         0
   -1.4142


H_3_0 =

    0.3536    0.7071   -0.6124    5.1213
    0.8660         0    0.5000         0
    0.3536   -0.7071   -0.6124   -2.1213
         0         0         0    1.0000
```

```
R_3_0 =

    0.3536    0.7071   -0.6124
    0.8660         0    0.5000
    0.3536   -0.7071   -0.6124


O_3_0 =

    5.1213
         0
   -2.1213


H_4_0 =

    0.7071   -0.3536   -0.6124    4.8151
         0   -0.8660    0.5000    0.2500
   -0.7071   -0.3536   -0.6124   -2.4275
         0         0         0    1.0000


R_4_0 =

    0.7071   -0.3536   -0.6124
         0   -0.8660    0.5000
   -0.7071   -0.3536   -0.6124


O_4_0 =

    4.8151
    0.2500
   -2.4275


z0 =

     0
     0
     1


z1 =

     0
     1
     0


z2 =

    0.7071
```

```
         0
   -0.7071


z3 =

   -0.6124
    0.5000
   -0.6124


q_dot =

    3.0000
    4.0000
    5.0000
    0.5000


Jacobian =

   -0.2500   -2.4275    0.1768   -0.6124
    4.8151         0    0.4330    0.5000
         0   -1.8151    0.1768   -0.6124
         0         0    0.7071         0
         0    1.0000         0         0
    1.0000         0   -0.7071         0


velocity_vectors =

   -9.8823
   16.8605
   -6.6828
    3.5355
    4.0000
   -0.5355
```

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(3)

$$J = \begin{vmatrix} -S_1(a_2C_2 + a_3C_{23}) & -C_1(a_2S_2 + a_3S_{23}) & a_3C_1S_{23} \\ C_1(a_2C_2 + a_3C_{23}) & -S_1(a_2S_2 + a_3S_{23}) & a_3S_1S_{23} \\ 0 & a_2C_2 + a_3C_{23} & -a_3C_{23} \\ 0 & S_1 & -S_1 \\ 0 & -C_1 & C_1 \\ 1 & 0 & 0 \end{vmatrix}$$

$$J = \begin{bmatrix} J_v \\ J_w \end{bmatrix}$$

Finding det of $J_w \Rightarrow \begin{vmatrix} 0 & S_1 & -S_1 \\ 0 & -C_1 & C_1 \\ 1 & 0 & 0 \end{vmatrix}$

$\Rightarrow$ Det $= 0 - S_1(C\phi) - S_1 C0$

$= 0$

$\therefore$ this size Singularity decomposition case

$\Rightarrow$ Det of J_v should also be equal to zero.

$$\Rightarrow \begin{vmatrix} -S_1(a_2C_2 + a_3C_{23}) & -C_1(a_2S_2 + a_3S_{23}) & a_3C_1S_{23} \\ C_1(a_2C_2 + a_3C_{23}) & -S_1(a_2S_2 + a_3S_{23}) & a_3S_1S_{23} \\ 0 & a_2C_2 + a_3C_{23} & -a_3C_{23} \end{vmatrix} = 0$$

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$$\Rightarrow -\begin{vmatrix} 0 & a_2c_2+a_3c_{23} & -a_3c_{23} \\ c_1(a_2c_2+a_3c_{23}) & -s_1(a_2s_2+a_3s_{23}) & a_3s_1s_{23} \\ -s_1(a_2c_2+a_3c_{23}) & -c_1(a_2s_2+a_3s_{23}) & a_3c_1s_{23} \end{vmatrix}$$

$$\Rightarrow -\Big[0 - (a_2c_2+a_3c_{23})\big[(a_2c_2+a_3c_{23})\big[a_3c_1^2s_{23} - a_3s_1^2s_{23}\big]\big]$$
$$- a_3c_{23}\big[-c_1^2\big[(a_2c_2+a_3c_{23})(a_2s_2+a_3s_{23})\big]$$
$$- s_1^2\big[(a_2s_2+a_3s_{23})(a_2c_2+a_3c_{23})\big]\big]\Big]$$

$$\Rightarrow -\Big[-(a_2c_2+a_3c_{23})\,[a_2c_2+a_3c_{23}]\,[a_3s_{23}]$$
$$+a_3c_{23}\,[a_2c_2+a_3c_{23}]\,[a_2s_2+a_3s_{23}]\Big]$$

$$\Rightarrow -(a_2c_2+a_3c_{23})a_3\big[(a_2c_2+a_3c_{23})s_{23}$$
$$-(a_2s_2+a_3s_{23})c_{23}\big]$$

$$\Rightarrow -(a_2c_2+a_3c_{23})\;a_3\big[a_2c_2s_{23}+\cancel{a_3c_{23}s_{23}}$$
$$-a_2s_2c_{23}-\cancel{a_3s_{23}c_{23}}\big]$$

$$\Rightarrow -a_3a_2[a_2c_2+a_3c_{23}]\,[s_{23}c_2 - c_{23}s_2]$$

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a_3 and a_2 are constants.
Hence, they don't matter.

$$\Rightarrow \underbrace{[a_2 C_2 + a_3 C_{23}]}[\underbrace{S_{23} C_2 - C_{23} S_2}_{\sin[\underbrace{(\theta_2+\theta_3)-\theta_2}_{\text{format}}]}) = 0$$

$a_2 C_2 + a_3 C_{23}$

$\Rightarrow a_2 \cos(\theta_2) + a_3 \cos(\theta_2 + \theta_3)$

$\Rightarrow \theta_2 + \theta_3 - \theta_2 = 0$

$\Rightarrow \boxed{\theta_3 = 0}$

Case: 1

if $\theta_3 = 0$

$\Rightarrow \theta_2$ can be any value.

if $\theta_3 \neq 0$

$\cancel{\theta_2 = -\theta_3}$ $\cos\theta_2 = 0$

$\Rightarrow$

$\Rightarrow \theta_2 = \left(\frac{\pi}{2}\right)$

$\therefore$ either $\theta_3 = 0$ or $\theta_2 = \frac{\pi}{2}$

between 0 and 2π

Hence:
θ_1 doesn't matter.

$\left.\begin{array}{l} \theta_3 = 0, \pi, 2\pi \\ \theta_2 = \frac{\pi}{2}, \frac{3\pi}{2} \end{array}\right] \rightarrow$ FINAL Values.

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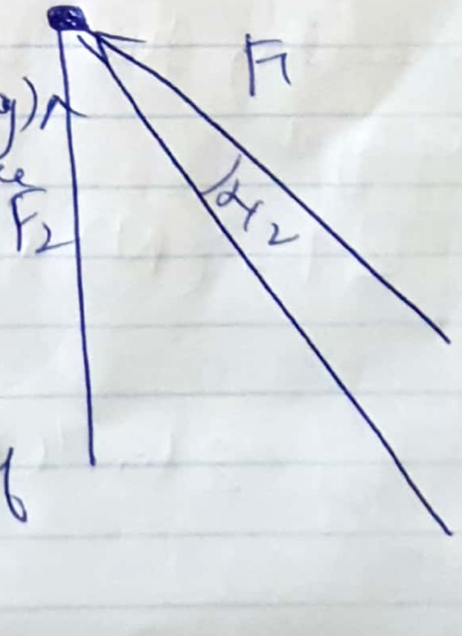
④ Let the 2 normal forces be F_1 and F_2.
The radius of the ball is 'r'.
The span measure between the 2 forces is 's'.

The forces F_1 and F_2 act along the centre of the ball.
The angle between the 2 forces are α.
The arc span between α is 's'

Let us assume.
$F_1 = F_2$ (for palming)
Let F_N = Normal force
Let F_t = Tangential force
Let N = Coefficient of forcition



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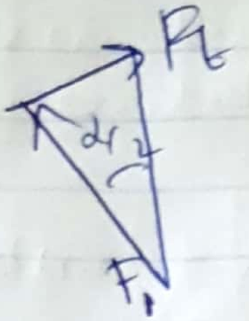
$\mu F_N = F_t$ (by properties of mechanics.)



$$\tan\left(\frac{\alpha}{2}\right) = \frac{1}{(F_N/F_t)}$$

$$\Rightarrow \tan \alpha/2 = (F_t/F_N)$$

But $F_t = \mu F_N \Rightarrow \tan(\alpha/2) = \mu$

$\alpha = 2\tan^{-1}(\mu)$

Relation between α and s.

$$\frac{\alpha}{360} = \frac{s}{2\pi r} \quad \left\{ \begin{array}{l} s = \text{arc length} \\ r = r \text{ radius} \end{array} \right\}$$

$$\Rightarrow \alpha = \frac{360 s}{2\pi r} \quad [\text{in degrees}]$$

But $\alpha = 2\tan^{-1}(\mu)$

Converting the radians to degrees we get

$$2\tan^{-1}(\mu) \times \frac{180}{\pi} = \frac{360 \times s}{2\pi r}$$

$$\Rightarrow 2\tan^{-1}(\mu) = \frac{s}{r}$$

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$$2\tan^{-1}(\mu) = \frac{s}{r}$$

$$\tan^{-1}(\mu) = \frac{s}{2r}$$

$$\boxed{\mu = \tan\left(\frac{s}{2r}\right)}$$

Coefficient of friction, μ is minimum when $\frac{s}{2r}$ is minimum, which happens when $s = 0$

$\therefore$ 2 ~~parts~~ vectors of forces should coincide for μ to be minimum.

Further the arc length between the 2 forces increase, the coefficient of friction proportionally increases.

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