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ENPM 662
Hw-6

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①

$$② \quad J = \begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 4 & 3 & 1 \\ 2 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 2 & 3 & 0 \\ 1 & 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} JV \\ JW \end{bmatrix}$$

$$\tau_1 = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 5 \end{bmatrix} \quad \tau_2 = \begin{bmatrix} 2 \\ 5 \\ 6 \\ 5 \end{bmatrix}$$

The pseudoinverse of this can only be used to determine the ~~real~~ effective forces if the condition

$$\text{rank}(J) = \text{rank}[J | \tau_i] \text{ be obeyed}$$

hence we have to test for this

Testing for τ_1

$$\text{rank}(J^T) = \text{rank} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 3 & 0 \\ 1 & 0 & 0 & 5 \end{bmatrix}^T$$

$$= \text{rank} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} = 3 - \text{full rank}$$

Now:

$$\text{Rank } (J_w^T | I_1) = \text{Rank} \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 2 & 0 & 4 \\ 0 & 3 & 0 & 6 \\ 0 & 0 & 5 & 5 \end{bmatrix}$$

Checking the MATLAB code
we see that this Rank ≥ 3

Hence the pseudo inverse can be used
to determine the end-effector forces.

Testing for I_2

Rank $J_w^T = 3$ (as we have seen before)

Rank $(J_w^T | I_2)$

$$= \text{Rank} \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 2 & 0 & 5 \\ 0 & 3 & 0 & 6 \\ 0 & 0 & 5 & 5 \end{bmatrix} \geq 4$$

(Check

MATLAB
code)

$$\therefore 3 \neq 4$$

Hence, the pseudoinverse method
for calculating the end effector torques
might yield out wrong values

⑥

$T = J^T F \rightarrow$ Relation between
end-effector forces and
torques

We have to find F

Now J_w^T has a dimension of 4×3
and T has dimensions of 4×1
Hence,

~~$J_w^T T = F$~~ \rightarrow $F_1 \rightarrow$ Force for T_1 ,
 $F_2 \rightarrow$ Force for T_2

$$\boxed{J_w^T T = J_w J_w^T F}$$

Now to get F , we have to get
rid of $J_w J_w^T$ on the RHS

Hence,

$$\boxed{(J_w J_w^T)^{-1} J_w T_1 = (J_w J_w^T)(J_w J_w^T)^{-1} F_1}$$

Now $J_w = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 3 & 0 \\ 1 & 0 & 0 & 5 \end{bmatrix}$

$$T_1 = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 5 \end{bmatrix}$$

$$J_w^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$N_{\text{deg}} (J_w J_w^T)^{-1} (J_w J_w^T) = 1$$

Doing the calculations on MATLAB \rightarrow

$$\therefore F_1 = (J_w J_w^T)^{-1} J_w T_1$$

$$= \begin{bmatrix} [1 & 0 & 0 & 0] & [1 & 0 & 1] \\ [0 & 2 & 3 & 0] & [0 & 2 & 0] \\ [1 & 0 & 0 & 5] & [0 & 3 & 0] \\ & [0 & 0 & 5] \end{bmatrix}^{-1} \begin{bmatrix} [0 & 0 & 0] \\ [0 & 2 & 3 & 0] \\ [1 & 0 & 0 & 5] \\ [2 & 4 & 6 & 5] \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 6 & 0 - 0.2 & 0 \\ 0 & 0.1538 & 0.23 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \rightarrow F_1 \text{ for } T_1$$

Now for $T_2 \rightarrow$

$$T_2 = \begin{bmatrix} 2 \\ 5 \\ 6 \\ 5 \end{bmatrix}$$

$$F_2 = (J_w J_w^T)^{-1} J_w T_2$$

$$\Rightarrow F_2 = \begin{bmatrix} 1 & 6 & 0 & -0.2 \\ 0 & 0.1538 & 0.23 & 0 \\ 0 & 0 & 0 & 0.2 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 6 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2.1538 \\ 1.000 \end{bmatrix}$$

② Now to verify our theory from part (a),

We do the following.

$$E_1 = Jw^T T_1 \text{ should yield the same } T_1 = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 5 \end{bmatrix}$$

and doing MATLAB multiplication, we find that

$$T_1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix} \text{ Hence. Verified.}$$

Similarly

$$T_2 = \int \omega^T F_2$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1.000 \\ 2.1538 \\ 1.000 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} 2.00 \\ 4.30 \\ 6.4615 \\ 5.000 \end{bmatrix}$$

Hence this is
NOT THE
SAME as

initial T_2 of $\begin{bmatrix} 2 \\ 5 \\ 6 \\ 5 \end{bmatrix}$

Hence Verified.

```

%%HW 6
%%Q1
%%a subsection
clc
clear all
Jw = [1 0 0 0;0 2 3 0;1 0 0 5]
t1 = [2 4 6 5]'
t2 = [2 5 6 5]'
%%testing for the first torque value
r1=rank(Jw)
r2=rank(cat(2,Jw',t1))
r1==r2 %%indicating the equality between r1 and r2
%%testing for the first torque value
r3=rank(Jw)
r4=rank(cat(2,Jw',t2))
r3==r4 %%indicating the equality between r3 and r4

%%bsubsection
F1=(inv(Jw'*Jw')*Jw)*t1
F2=(inv(Jw'*Jw')*Jw)*t2
%%getting back the values of tau
newt1 =Jw'*F1 %%taul
newt2 =Jw'*F2 %%tau2

```

Jw =

1	0	0	0
0	2	3	0
1	0	0	5

t1 =

2
4
6
5

t2 =

2
5
6
5

r1 =

3

r2 =

3

ans =

logical

1

r3 =

3

r4 =

4

ans =

logical

0

F1 =

1

2

1

F2 =

1.0000

2.1538

1.0000

newt1 =

2

4

6

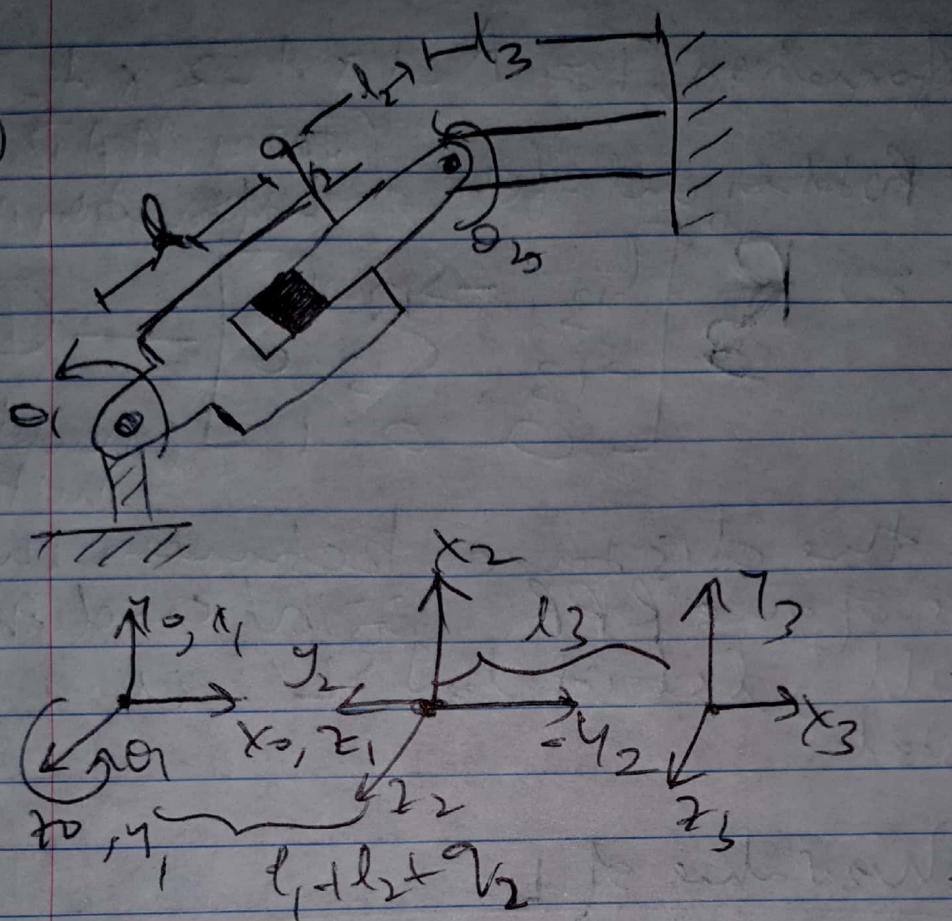
5

newt2 =

*2.0000
4.3077
6.4615
5.0000*

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②



Writing DH Parameters we get,

	θ	d	a	α
$0 \rightarrow 1$	$\theta_1 + 90^\circ$	0	0	q_0
$1 \rightarrow 2$	0	$l_1 + l_2 + q_2$	0	$-q_0$
$2 \rightarrow 3$	$\theta_3 - 90^\circ$	0	l_3	0

The transformation matrix of this can be seen on MATLAB

This can be written as:

Transformation, $T = A_{-1} \times A_{-2} \times A_{-3}$
 (from MATLAB)

Taking the rotation out of the transformation matrix:

$$\Rightarrow R_3 \begin{bmatrix} C_{13} & -S_{13} & 0 \\ S_{13} & C_{13} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{(mentally multiplying)} \quad \textcircled{1}$$

Now the distance between the base of the RPR joint and the wall, let it be ' L' is a constant

\Rightarrow This value of $L =$

$$[(l_1 + q_1 + l_2) \cos \theta_1 + l_3] = 0 \quad \text{--- } \textcircled{2}$$

This is the constraint to make

sure that the link is always normal to the wall

Now from eqn $\textcircled{1}$

$$\begin{bmatrix} \cos(\theta_1 + \theta_3) \\ \sin(\theta_1 + \theta_3) \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

This HAS to be true because the

link is parallel to the ground.

$$\therefore \cos(\theta_1 + \theta_3) = 1$$

$$\Rightarrow \theta_1 + \theta_3 = 0 \text{ or } \pi \text{ or } 2\pi \dots \\ = \underline{\underline{n\pi}} \quad \textcircled{3}$$

Now the question says that,
 q_2 has a limited bound.

let this be between 0 and q_{\max}

$$\boxed{0 \leq q_2 \leq q_{\max}} \quad \textcircled{4}$$

Now, looking at all the constraints

CONSTRAINT - 1 \rightarrow

$$(l_1 + q_2 l_2) \cos \theta_1 + l_3 = 0$$

if we look at eq ④ we know that
the motion of this is limited from

$$(l_1 + l_2) \cos \theta_1 + l_3 \leq (l_1 + q_{\max} l_2) \cos \theta_1 + l_3 \\ \leq (l_1 + q_{\max} l_2) \cos \theta_1 + l_3$$

Hence, this is a holonomic
constraint

Now,

CONSTRAINT 2 →

$$\cos(\theta_1 + \theta_2) = 1$$

$$\Rightarrow \theta_1 + \theta_2 = 0^\circ \text{ or } (n\pi), \\ n \in 0, 1, 2, \dots, n$$

This is a indicative of the parallel
nature of the joint link at all times,
hence, this is also a holonomic
constraint

CONSTRAINT 3 →

$0 \leq \varphi_2 \leq l_{\max}$
also limits the motion, hence

these constraints are
holonomic in nature

```

clc
clear all
%code for
%%answer 2 HW 6
syms l1 l2 l3 q2 thetal theta3
alphal = 90;
alpha2 = -90;
alpha3 = 0;

%%matrix 1
Rz_thetal=[cosd(thetal+90) -sind(thetal+90) 0 0;sind(thetal+90)
            cosd(thetal+90) 0 0;0 0 1 0;0 0 0 1];
Rx_alpha1=[1 0 0 0;0 cosd(alpha1) -sind(alpha1) 0;0 sind(alpha1)
            cosd(alpha1) 0;0 0 0 1];
Tz_d1=[1 0 0 0;0 1 0 0;0 0 1 11;0 0 0 1];
Tx_a1=[1 0 0 0;0 1 0 0;0 0 1 0;0 0 0 1];

%%matrix 2
Rz_theta2=[cosd(0) -sind(0) 0 0;sind(0) cosd(0) 0 0;0 0 1 0;0 0 0 1];
Rx_alpha2=[1 0 0 0;0 cosd(alpha2) -sind(alpha2) 0;0 sind(alpha2)
            cosd(alpha2) 0;0 0 0 1];
Tz_d2=[1 0 0 0;0 1 0 0;0 0 1 (l1 + l2 +q2);0 0 0 1];
Tx_a2=[1 0 0 0;0 1 0 0;0 0 1 0;0 0 0 1];

%%matrix 3
Rz_theta3=[cosd(theta3-90) -sind(theta3-90) 0 0;sind(theta3-90)
            cosd(theta3-90) 0 0;0 0 1 0;0 0 0 1];
Rx_alpha3=[1 0 0 0;0 cosd(alpha3) -sind(alpha3) 0;0 sind(alpha3)
            cosd(alpha3) 0;0 0 0 1];
Tz_d3=[1 0 0 0;0 1 0 0;0 0 1 0;0 0 0 1];
Tx_a3=[1 0 0 13;0 1 0 0;0 0 1 0;0 0 0 1];

%%multiplying to get homogeneous matrices
A_1=Rz_thetal*Tz_d1*Tx_a1*Rx_alpha1
A_2=Rz_theta2*Tz_d2*Tx_a2*Rx_alpha2
A_3=Rz_theta3*Tz_d3*Tx_a3*Rx_alpha3

transformation = A_1*A_2*A_3

A_1 =

```

$$\begin{bmatrix}
\cos((\pi * (\text{thetal} + 90)) / 180), & 0, & \sin((\pi * (\text{thetal} + 90)) / 180), & 0 \\
\sin((\pi * (\text{thetal} + 90)) / 180), & 0, & -\cos((\pi * (\text{thetal} + 90)) / 180), & 0 \\
0, & 1, & 0, & 11 \\
0, & 0, & 0, & 1
\end{bmatrix}$$


```

A_2 =

```

$$\begin{bmatrix}
1, & 0, & 0, & 0 \\
0, & 0, & 1, & 0
\end{bmatrix}$$

```

[ 0, -1, 0, 11 + 12 + q2]
[ 0, 0, 0, 1]

A_3 =

[ cos((pi*(theta3 - 90))/180), -sin((pi*(theta3 - 90))/180), 0,
  13*cos((pi*(theta3 - 90))/180)]
[ sin((pi*(theta3 - 90))/180), cos((pi*(theta3 - 90))/180), 0,
  13*sin((pi*(theta3 - 90))/180)]
[ 0, 0, 0, 1,
  0]
[ 0, 0, 1]

transformation =

[ cos((pi*(theta1 + 90))/180)*cos((pi*(theta3 - 90))/180) -
  sin((pi*(theta1 + 90))/180)*sin((pi*(theta3 - 90))/180), -
  cos((pi*(theta1 + 90))/180)*sin((pi*(theta3 - 90))/180) -
  cos((pi*(theta3 - 90))/180)*sin((pi*(theta1 + 90))/180), 0,
  sin((pi*(theta1 + 90))/180)*(11 + 12 + q2) + 13*cos((pi*(theta1 +
  90))/180)*cos((pi*(theta3 - 90))/180) - 13*sin((pi*(theta1 +
  90))/180)*sin((pi*(theta3 - 90))/180)]
[ cos((pi*(theta1 + 90))/180)*sin((pi*(theta3 - 90))/180) +
  cos((pi*(theta3 - 90))/180)*sin((pi*(theta1 + 90))/180),
  cos((pi*(theta1 + 90))/180)*cos((pi*(theta3 - 90))/180) -
  sin((pi*(theta1 + 90))/180)*sin((pi*(theta3 - 90))/180), 0,
  13*cos((pi*(theta1 + 90))/180)*sin((pi*(theta3 - 90))/180) -
  cos((pi*(theta1 + 90))/180)*(11 + 12 + q2) + 13*cos((pi*(theta3 -
  90))/180)*sin((pi*(theta1 + 90))/180)]
[ 0, 0, 11]
[ 0, 0, 1]

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③ To prove a constraint is non-holonomic we have to prove $\dot{V} = 0$

$$\text{in } [0 \ 1 \ p \sin q_5 - p \cos q_3 \cos q_5]_{q=0}$$

Here,

$$q_1 = 0$$

$$q_2 = 1$$

$$q_3 = p \sin q_5 \quad \text{and} \quad q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \end{bmatrix}$$

$$q_4 = p \cos q_3$$

$$q_5 = \cos q_5$$

$$\text{and we use } \frac{\partial V(q) q_h(q)}{\partial q_j} = \frac{\partial V(q)}{\partial q_j} a_j(q)$$

$$\forall j, h = 1, 2, 3, \dots, n$$

The conditions are:

j	h
1	2
1	3
1	4
1	5
2	3
2	4
2	5
3	4
3	5
4	5

$$\rightarrow \frac{\partial r_{q_2}}{\partial q_1} = \frac{\partial r_{q_1}}{\partial q_2}$$

$$\Rightarrow \frac{\partial r(0)}{\partial q_1} = \frac{\partial r(0)}{\partial q_2}$$

$$\Rightarrow \left[\frac{\partial r}{\partial q_i} = 0 \right] \quad \text{--- (1)}$$

$$\rightarrow \frac{\partial r_{q_3}}{\partial q_1} = \frac{\partial r_{q_1}}{\partial q_3}$$

$$\Rightarrow r(\frac{\partial p_{\sin q_3}}{\partial q_1}) + p_{\sin q_3} \frac{\partial r}{\partial q_1} = 0$$

$\Rightarrow 0 = 0$ (Redundant)

$$\rightarrow \frac{\partial r_{q_4}}{\partial q_1} = \frac{\partial r_{q_1}}{\partial q_4}$$

$$\Rightarrow \frac{\partial r(p_{\cos q_3})}{\partial q_1} = \frac{\partial r(0)}{\partial q_4} \Rightarrow 0 = 0$$

Redundant

$$\rightarrow \frac{\partial r_{q_5}}{\partial q_1} = \frac{\partial r_{q_1}}{\partial q_5}$$

$$\Rightarrow r \frac{\partial (\cos q_5)}{\partial q_1} + \cos q_5 \frac{\partial r}{\partial q_1} = 0$$

$\Rightarrow 0 \text{ to } \underline{\text{Redundant}}$

$$\rightarrow \frac{\partial r_{q_3}}{\partial q_2} = \frac{\partial r_{q_2}}{\partial q_3}$$

$$\Rightarrow r(0) + \frac{\partial r}{\partial q_3} = p \sin q_5 \frac{\partial r}{\partial q_2} \text{ to}$$

$$\Rightarrow \boxed{p \sin q_5 \left(\frac{\partial r}{\partial q_2} \right) = \frac{\partial r}{\partial q_3}} \quad ②$$

$$\rightarrow \frac{\partial r_{q_4}}{\partial q_2} = \frac{\partial r_{q_2}}{\partial q_4}$$

$$\Rightarrow r(0) + \frac{\partial r}{\partial q_4} = p \cos q_3 \frac{\partial r}{\partial q_2}$$

$$\Rightarrow \boxed{p \cos q_3 \frac{\partial r}{\partial q_2} = \frac{\partial r}{\partial q_4}} \quad ③$$

$$\rightarrow \frac{\partial r \alpha_5}{\partial \gamma_{V_2}} \rightarrow \frac{\partial r \alpha_2}{\partial \gamma_{V_5}}$$

$$\Rightarrow \frac{\partial \gamma}{\partial \gamma_{V_5}} = 0 + \cos \gamma_{V_5} \frac{\partial r}{\partial \gamma_{V_2}}$$

$$\Rightarrow \boxed{\frac{\partial r}{\partial \gamma_{V_5}} = \cos \gamma_{V_5} \frac{\partial r}{\partial \gamma_{V_2}}} \quad (4)$$

$$\rightarrow \frac{\partial r \alpha_3}{\partial \gamma_{V_3}} \rightarrow \frac{\partial r \alpha_3}{\partial \gamma_{V_F}}$$

$$\Rightarrow \cancel{\rho \cos \gamma_{V_3} \frac{\partial r}{\partial \gamma_{V_3}}} + \rho (-\sin \gamma_{V_3}) = r(0)$$

$$\Rightarrow \boxed{\sin \gamma_{V_5} \frac{\partial r}{\partial \gamma_{V_3}} = \cos \gamma_{V_3} \frac{\partial r}{\partial \gamma_{V_3}} - r \sin \gamma_{V_3} + \rho \sin \gamma_{V_F} \frac{\partial r}{\partial \gamma_{V_F}}} \quad (4)$$

$$\rightarrow \frac{\partial r \alpha_5}{\partial \gamma_{V_5}} \rightarrow \frac{\partial r \alpha_3}{\partial \gamma_{V_F}}$$

$$= \frac{\partial r (\rho \sin \gamma_{V_F})}{\partial \gamma_{V_5}} = \frac{\partial r (\cos \gamma_{V_5})}{\partial \gamma_{V_3}}$$

$$\Rightarrow \boxed{\cos \gamma_{V_5} \frac{\partial r}{\partial \gamma_{V_3}} = \rho \cos \gamma_{V_F} + \rho \sin \gamma_{V_F} \frac{\partial r}{\partial \gamma_{V_F}}} \quad (6)$$

$$\rightarrow \frac{\partial r_{q_5}}{\partial q_4} = \frac{\partial r_{q_3}}{\partial q_5}$$

$$\rightarrow r(0) + \cos q_5 \frac{\partial r}{\partial q_4} = 0 + \cos q_3 \frac{\partial r}{\partial q_5}$$

$$\Rightarrow \boxed{\cos q_5 \frac{\partial r}{\partial q_4} = \cos q_3 \frac{\partial r}{\partial q_5}}$$

(7)

Using eqn 2 through (7)

* Putting (2) and (3) in eq (7) we get:

$$\sin q_5 \rho \cos q_3 \frac{\partial r}{\partial q_2} = \cos q_3 \rho \sin q_3 \frac{\partial r}{\partial q_2}$$

- $\rho \sin q_3$

Cancelling out \rightarrow

$$\boxed{\rho \sin q_3 = 0} - (a)$$

* putting (2) and (4) in eq (6) we get:

$$\cos q_5 \rho \sin q_3 \frac{\partial r}{\partial q_2} = \rho \cos q_1 + \rho \sin q_5 \cos q_1 \frac{\partial r}{\partial q_1}$$

$$\Rightarrow \boxed{N_p \cos \gamma_5 = 0} \quad (b)$$

from (7)

$$\cos \gamma_5 - p \cos \gamma_3 \frac{\partial r}{\partial \gamma_5} = \cos \gamma_3 p \cos \gamma_5 \frac{\partial r}{\partial \gamma_2}$$

\Rightarrow Redundant

Hence from (a) and (b)

the test function $\boxed{r=0}$

\Rightarrow The differential constraint

$$[0 \ 1 \ p \sin \gamma_5 \ p \cos \gamma_3 \ \cos \gamma_5]^\top$$

in \mathbb{R}^5 is NON-HOLONOMIC.

Hence Powell.