$$\Rightarrow \begin{bmatrix} C_1 C_2 & C_2 S_1 & -S_2 \\ -S_1 & C_1 & 0 \\ C_1 S_2 & S_1 S_2 & C_2 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \cdot & -S_5 C_4 \\ 1 & \vdots & S_5 S_4 \\ \vdots & \vdots & C_5 \\ -S_5 C_6 & S_5 S_6 & \end{bmatrix}$$

Since, these rows don't matter for calculation

$$C_1 C_2 r_{13} + r_{23} C_2 S_1 - S_2 r_{33} = -S_5 C_4 \quad -(1)$$

$$-S_1 r_{13} + C_1 r_{23} = S_5 S_4 \quad -(2)$$

$$C_1 S_2 r_{13} + S_1 S_2 r_{23} + C_2 r_{33} = C_5 \quad -(3)$$

$$\boxed{\theta_5 = \cos^{-1}\left(C_1 S_2 r_{13} + S_1 S_2 r_{23} + C_2 r_{33}\right)}$$

Dividing (2) and (1)

$$\tan\theta_4 = \left(\frac{-S_1 r_{13} + C_1 r_{23}}{C_1 C_2 r_{13} + r_{23} C_2 S_1 - S_2 r_{33}}\right)$$

$$\Rightarrow \boxed{\theta_4 = \tan^{-1}\left(\frac{-S_1 r_{13} + C_1 r_{23}}{C_1 C_2 r_{13} + r_{23} C_2 S_1 - S_2 r_{33}}\right)}$$

Also,

$$C_1 S_2 r_{11} + S_1 S_2 r_{21} + C_2 r_{31} = -S_5 C_6$$

$$S_2 C_1 r_{12} + S_1 S_2 r_{22} + C_2 r_{32} = S_5 S_6$$

Dividing the above 2 equations :-

$$\tan\theta_6 = \left(\frac{S_2 C_1 r_{12} + S_1 S_2 r_{22} + C_2 r_{32}}{C_1 S_2 r_{11} + S_1 S_2 r_{21} + C_2 r_{31}}\right)$$

$$\Rightarrow \boxed{\theta_6 = \tan^{-1}\left(\frac{S_2 C_1 r_{12} + S_1 S_2 r_{22} + C_2 r_{32}}{C_1 S_2 r_{11} + S_1 S_2 r_{21} + C_2 r_{31}}\right)}$$

(3) If a body has linear velocity only the velocities can be calculated by the transformation matrix between the two frames

Here $H_1^0 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -3 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

and $v^0(t) = [0 \ -2 \ 4]^T = \begin{bmatrix} 0 \\ -2 \\ 4 \end{bmatrix}$

Since only the Rotation matters

$$V^0 = R_1^0 V^1$$

Taking inverse $\rightarrow$

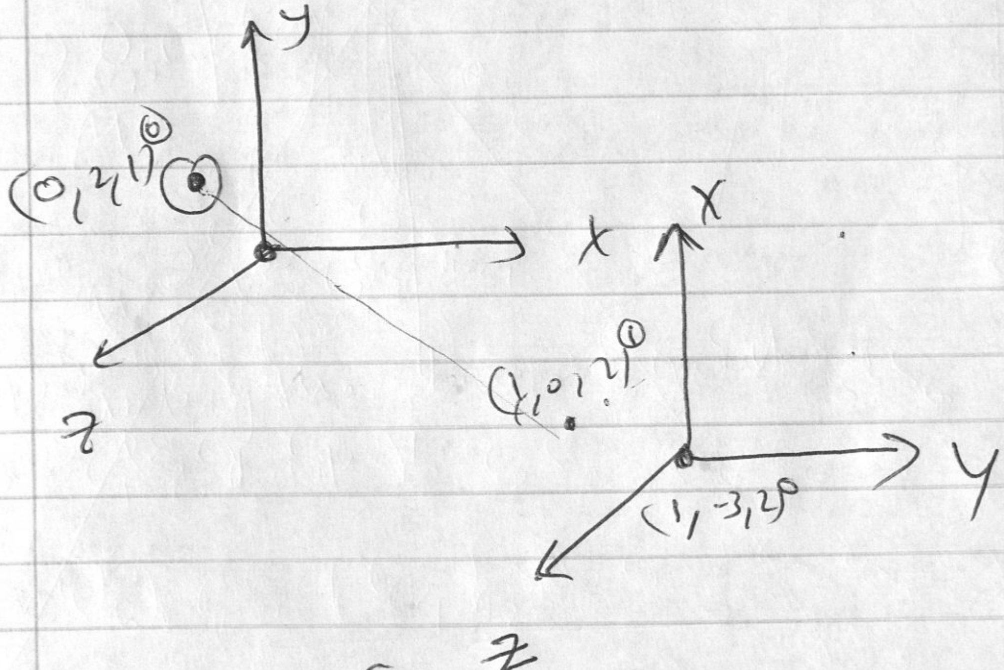
$$(R_1^0)^{-1} V^0 = R_1^0 V^1 (R_1^0)^{-1} = V^1$$

$$\Rightarrow V_1 = (R_1^0)^{-1} V^0 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ -2 \\ 4 \end{bmatrix}$$

$$\Rightarrow V_1 = [4 \ 0 \ -2]^T$$

This is the velocity of the particle relative to the frame 1.

(b) Let A be the base frame.
Let B be the new frame.



old point $(0,2,1)$ about the new point would be $(R_1^0)^T p^0 = p^1$

$\Rightarrow p^1 = (1 \; 0 \; 2)^T$

④ ⓐ

Here,

→ $l_2 > l_3$

→ no limits for the revolute joints so they can move and rotate about freely

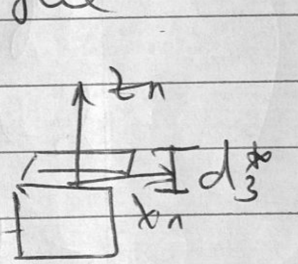
→ $0 \le d_3^* \le d_{max}$

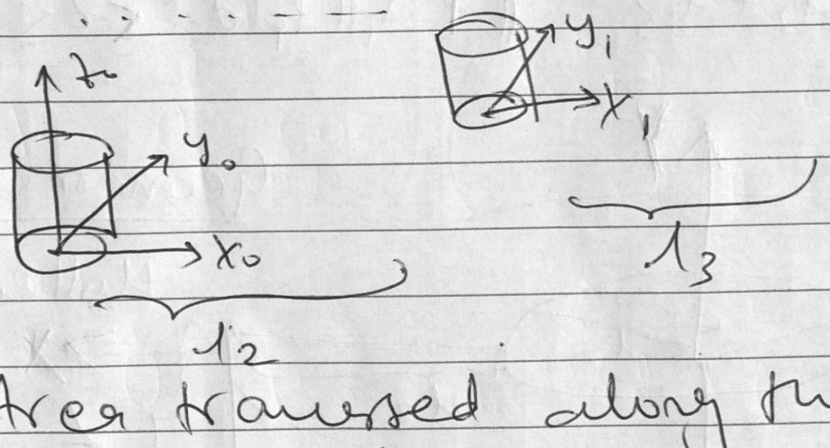
Minimum radius of the workspace would be

$$\underline{l_2 - l_3}$$

and the maximum radius of the workspace would be

$$\underline{\underline{l_2 + l_3}}$$





Area traversed along the XY plane would be.

$$l_2 - l_3 \le \sqrt{x^2 + y^2 + (z - l_1 - l_4 - d_3^*)} \le l_2 + l_3$$

Now along the Z axis one can see that since l_1 and l_4 are constant,

the only difference comes between the translation of the prismatic joints.

Hence

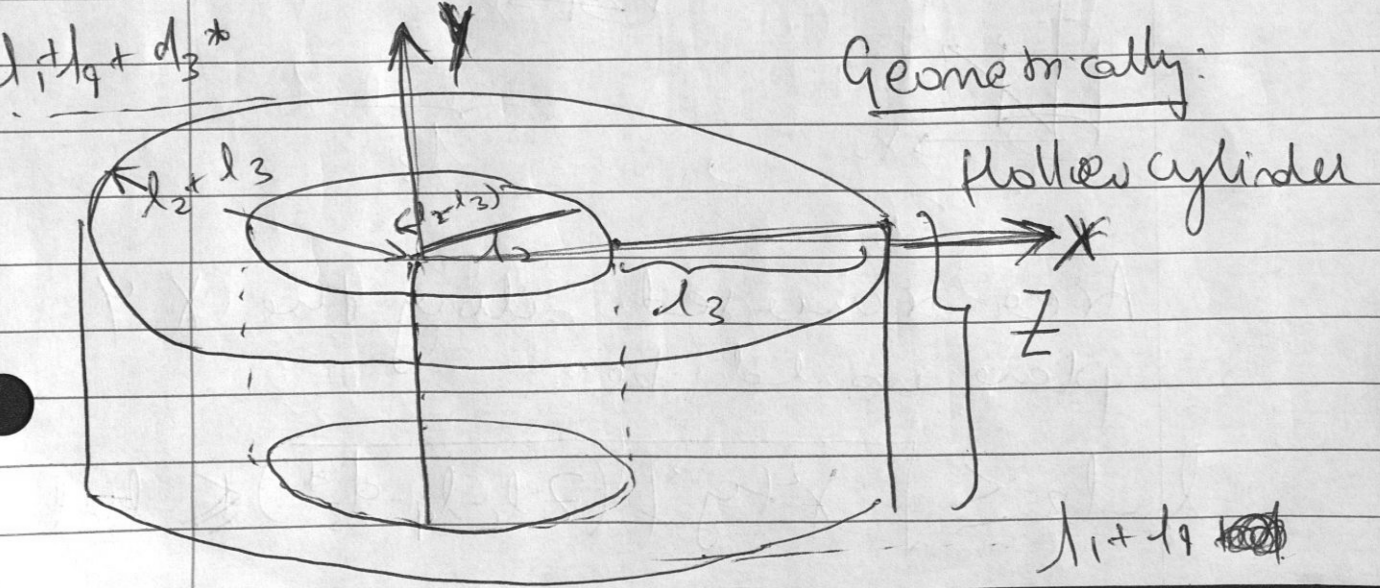
$$l_1 + l_4 + d_3^* \leq Z \leq l_1 + l_4 + d_{3max}.$$

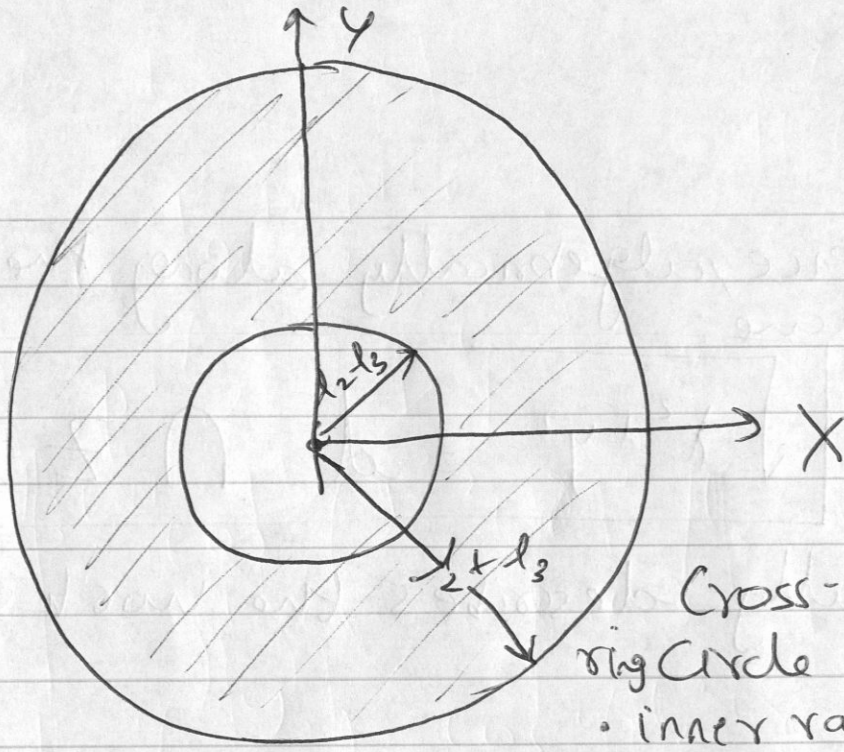
Hence final algebraic representation of the workspace is:-

$$l_2 - l_3 \leq \sqrt{x^2 + y^2 + (z - l_1 - l_4 - d_3^*)^2} \leq l_2 + l_3 \quad \bigcap \quad l_1 + l_4 + d_3^* \leq Z \leq l_1 + l_4 + d_{3max}$$

Geometrically:

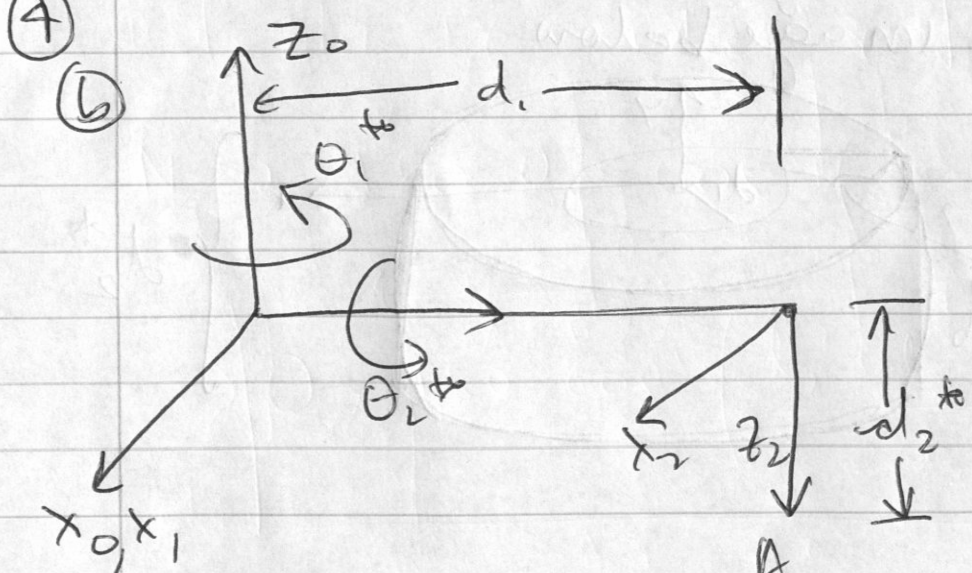
Hollow cylinder





Cross-section:
ring circle with
- inner radius $= l_2 - l_3$
- outer radius $= l_2 + l_3$

(4)
(b)



Since the reachable points are on the X–Y plane, $Z = 0$

θ_1^* can rotate full $360°$.
θ_2^* can rotate full $360°$.

Hence, algebrically along the X-Y plane:

$$\boxed{\sqrt{x^2+y^2} = d \cap Z = 0},$$

fully describes the workspace.

Geometrically we receive a hollow ~~sphered~~ cylinder. Better described in the image below: