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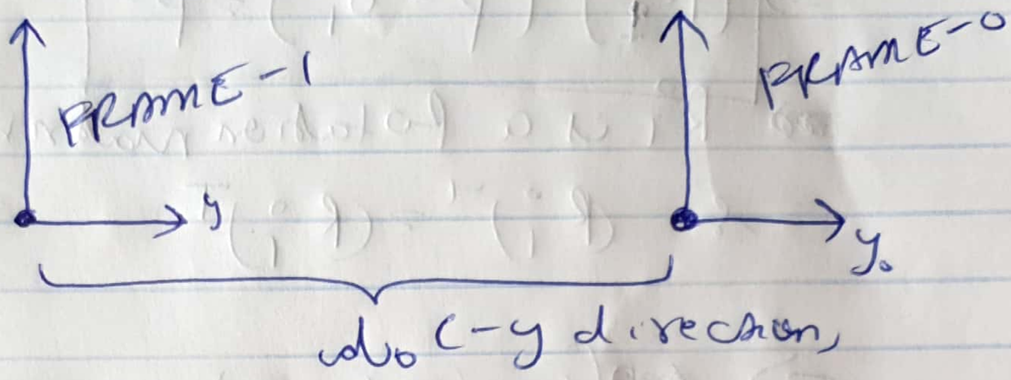
→ GOVIND A.K

→ UID: 116699488

ENPM-662

HW-4

(1)



$$\theta_1^*(t) = (t-2)\frac{\pi}{4} = \left[\frac{\pi t}{4} - \frac{\pi}{2}\right]$$

$$Z_0 = [0 \ 0 \ 1]^T = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$p^0(t) = [t \quad 2t^2 \quad 3t^3+1]^T$$ → position of particle in fixed '0' frame.

(a) $p^0 = R^0_1 p^1 + o^0_1$

Here the Rotation is about the Z axis

Hence the general form of the rotation matrix is given as $\begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = R^0_1$

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We want to find p^1

$$\Rightarrow \boxed{(R_1^0)^{-1}(p^0 - o_1^0) = p^1}$$

R_1^0 is a rotation matrix,

so $(R_1^0)^{-1} = (R_1^0)^T$

$$\Rightarrow \begin{bmatrix} \cos\left(\frac{\pi t}{4} - \frac{\pi}{2}\right) & -\sin\left(\frac{\pi t}{4} - \frac{\pi}{2}\right) & 0 \\ \sin\left(\frac{\pi t}{4} - \frac{\pi}{2}\right) & \cos\left(\frac{\pi t}{4} - \frac{\pi}{2}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix}^T = (R_1^0)^{-1}$$

Writing $\cos\theta$ as $c\theta \rightarrow$

$$\Rightarrow \begin{bmatrix} c\left(\frac{\pi t}{4} - \frac{\pi}{2}\right) & +\sin\left(\frac{\pi t}{4} - \frac{\pi}{2}\right) & 0 \\ -s\left(\frac{\pi t}{4} - \frac{\pi}{2}\right) & \cos\left(\frac{\pi t}{4} - \frac{\pi}{2}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} t \\ 2t^2 \\ 3t^3+1 \end{bmatrix} - \begin{bmatrix} 0 \\ -d \\ 0 \end{bmatrix} \right) = p^1$$

$$\Rightarrow \begin{bmatrix} +s\left(\frac{\pi t}{4}\right) & -c\left(\frac{t\pi}{4}\right) & 0 \\ +c\left(\frac{\pi t}{4}\right) & s\left(\frac{\pi t}{4}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} t \\ 2t^2+d \\ 3t^3+1 \end{bmatrix} = p^1$$

$$\begin{bmatrix} t\cdot S\left(\frac{\pi t}{4}\right) - C\left(\frac{\pi t}{4}\right)\cdot(2t^2+d) \\ + t\cdot C\left(\frac{\pi t}{4}\right) + S\left(\frac{\pi t}{4}\right)\cdot(2t^2+d) \\ 3t^3+1 \end{bmatrix} = p'$$

This is the particles position as a function of time w.r.t frame 1

(iii)

From (i) we know that

$$p' = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 & 0 \\ -\sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} t \\ 2t^2 \\ 3t^3+1 \end{bmatrix}$$

where $\theta_1 = \frac{\pi}{4}t - \frac{\pi}{2}$

Velocity is the differential of p'

$\therefore (p')' =$ velocity.

$$\therefore (p')' = \text{derivative of} \begin{bmatrix} t\cos\theta_1 + 2t^2\sin\theta_1 + 0 \\ -t\sin\theta_1 + 2t^2\cos\theta_1 + 0 \\ 0+0+3t^3+1 \end{bmatrix}$$

$$\therefore (p')' = \begin{bmatrix} t\cos\theta_1 + 2t^2\sin\theta_1 \\ -t\sin\theta_1 + 2t^2\cos\theta_1 \\ 3t^3 + 1 \end{bmatrix} = \text{velocity of the particle.}$$

Let $\cos\theta_1 = c\theta_1$, Let $\sin\theta_1 = c\theta_1$

$$\theta_1 = \frac{\pi}{4}t - \pi/2$$

$$\text{Velocity} = \begin{bmatrix} -ts\theta\left[\frac{\pi}{4}\right] + c\theta + 4ts\theta + 2t^2 c\theta\left(\frac{\pi}{4}\right) \\ + d\,c\theta\left(\frac{\pi}{4}\right) \\ -tc\theta\left(\frac{\pi}{4}\right) + (-s\theta) + 4tc\theta + 2t^2(-s\theta)\left(\frac{\pi}{4}\right) \\ + d(-s\theta)\frac{\pi}{4} \\ 9t^2 + 0 \end{bmatrix}$$

$$\Rightarrow \text{velocity} = \begin{bmatrix} -\frac{t\pi}{4}s\theta + c\theta + 4ts\theta + \frac{t^2\pi}{2}c\theta + \frac{\pi d}{4}c\theta \\ -\frac{\pi t}{4}c\theta - s\theta + 4tc\theta - \frac{\pi t^2}{2}s\theta - \frac{\pi d}{4}s\theta \\ 9t^2 \end{bmatrix}$$

$\Rightarrow$ velocity $= \begin{bmatrix} \left[\frac{t^2\pi}{2} + \frac{t\pi d}{4} + 1\right]\cos\theta + \left[4t - \frac{\pi t}{4}\right]\sin\theta \\ \dfrac{\left[4t - \frac{\pi t}{4}\right]\cos\theta + \left[-1 - \frac{\pi t^2}{4} - \frac{\pi d^2}{4}\right]\sin\theta}{4t^2} \end{bmatrix}$

Now, we know that $\theta = \frac{\pi}{4}t - \frac{\pi}{2}$

$\Rightarrow \cos\theta = \cos\left(\frac{\pi t}{4} - \frac{\pi}{2}\right) = \sin\left(\frac{\pi}{4}t\right)$ $\quad \sin\theta = \sin\left(\frac{\pi}{4}t - \frac{\pi}{2}\right) = -\cos\left(\frac{\pi}{4}t\right)$

Velocity =

$$\begin{bmatrix} \left(\frac{t^2\pi}{2} + \frac{\pi d}{4} + 1\right)\sin\left(\frac{\pi}{4}t\right) + \left(4t - \frac{\pi t}{4}\right)\left(-\cos\frac{\pi}{4}t\right) \\ \dfrac{\left(4t - \frac{\pi t}{4}\right)\sin\left(\frac{\pi t}{4}\right) + \left[-1 - \frac{\pi t^2}{4} - \frac{\pi d^2}{4}\right]\left(-\cos\frac{\pi}{4}t\right)}{4t^2} \end{bmatrix}$$

$\uparrow$

Final velocity of particle in FRAME 1.

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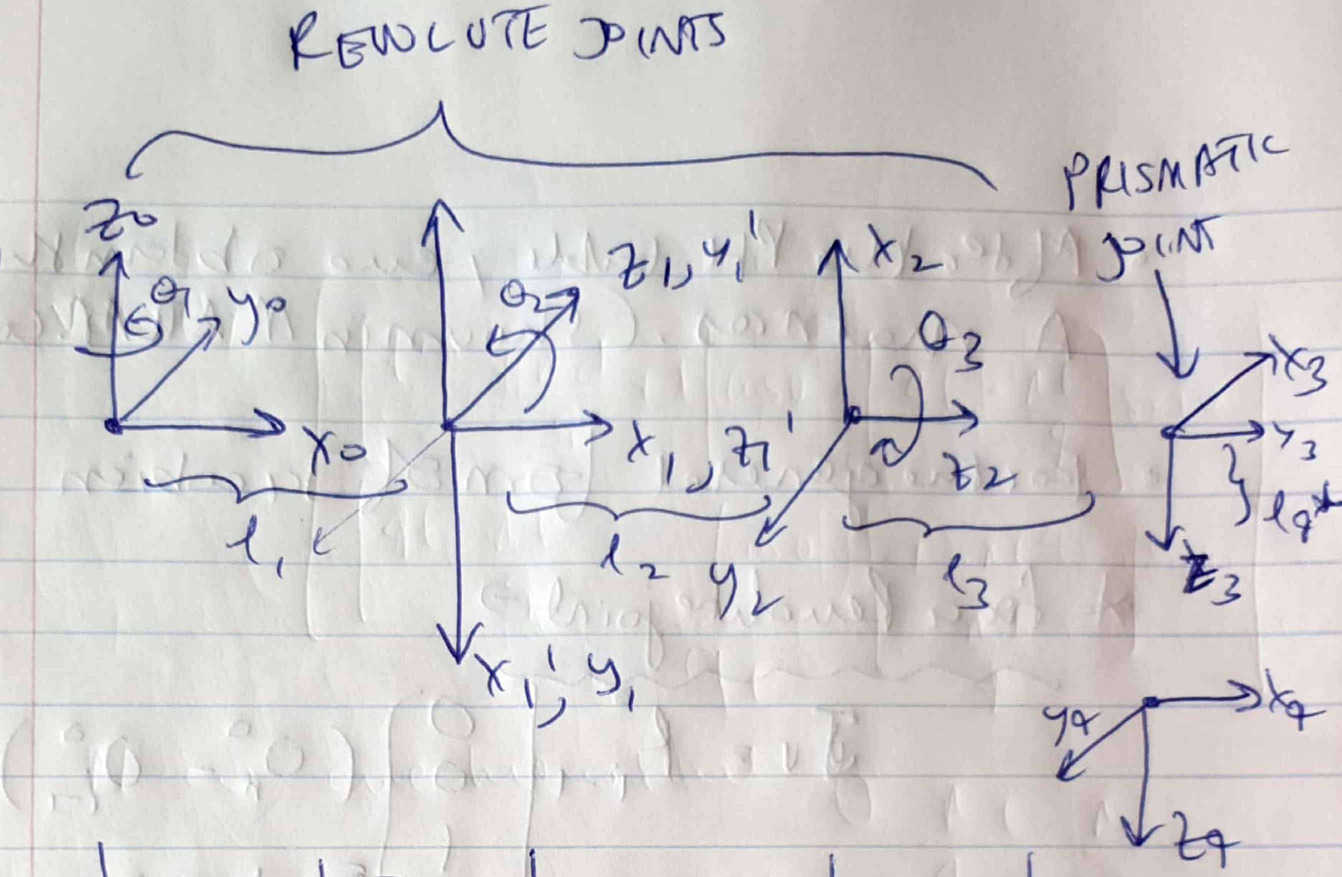

	Z_{n-1} θ	Z_{n-1} d	x_n a	x_n α
$0 \to 1$	θ_1^*	0	l_1	-90
$1 \to 1'$	$\theta_2^* + 90$	0	0	90
$1' \to 2$	0	l_2	0	0
$2 \to 3$	$\theta_3^* + 90$	l_3	0	90
$3 \to 4$	90	l_4^*	0	0

This gives the forward kinematics of the manipulator.

- After the DH table, we obtain the A matrices. (Shown in MATLAB code)

- For the general form of Jacobian

* for a Revolute joint →

$$J_v = R^0_{i-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} (O^0_n - O^0_{i-1})$$

← along z-axis

$$J_\omega = R^0_{i-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

* for Prismatic joint →

$$J_v = R^0_{i-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_\omega = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

We also know → Ans. a

$$J = \begin{bmatrix} z_0(o_1 - o_0) & z_1(o_1 - o_1) & z_2(o_1 - o_2) & z_3 \\ z_0 & z_1 & z_2 & 0 \end{bmatrix}$$

$\theta_1, \theta_2, \theta_n$ are depicted in MATLAB code

Ans: (b)

(c) $l_1 = 3, l_2 = 2, l_3 = 1$

$q = [0, 45, 30, 0.5)$

here $\theta_1 = 0°$ Jacobian =

$\theta_2 = 45°$

$\theta_3 = 30°$

$d_4^* = 0.5$

$$\begin{bmatrix} -0.25 & -2.42 & 0.176 & -0.612 \\ 4.815 & 0 & 0.433 & 0.5 \\ 0 & -1.8157 & 0.176 & -0.612 \\ 0 & 0 & 0.7071 & 0 \\ 0 & 1.00 & 0 & 0 \\ 1.00 & 0 & -0.7071 & 0 \end{bmatrix}$$

Answers on MATLAB code.

Final velocity vector. $\begin{bmatrix} -9.8823 \\ 16.8605 \\ -6.6828 \\ 3.5355 \\ 4.000 \\ -0.5355 \end{bmatrix}$

Z matrices are obtained by multiplying the Rotational matrices with $[0\ 0\ 1]^T$
This is because, the Rotation is about the Z-axis.

Rotation matrices are obtained from the 3x3 matrix part of the homogeneous matrix

The origin is the first 3 elements of the last Column of the homogeneous matrix.

All are depicted clearly in the MATLAB code.

But taking the values of z_0, z_1, z_2, z_3 from the output of MATLAB code, we get

Ans: (b)

$$z_0 = [0 \quad 0 \quad 1]^T$$

$$z_1 = [-S\theta_1 \quad C\theta_1 \quad 0]^T$$

$$z_2 = [(C\theta_1 * S(\theta_2+90)) \quad (S\theta_1 * S(\theta_2+90)) \quad C(\theta_2+90)]^T$$

$$z_3 = \begin{bmatrix} (S\theta_1)(C(\theta_3+90)) + C\theta_1 \cdot C(\theta_2+90) \cdot S(\theta_3+90) \\ (S\theta_1(C(\theta_2+90))S(\theta_3+90)) - (C\theta_1 C(\theta_3+90)) \\ -S(\theta_2+90) * S(\theta_3+90) \end{bmatrix}$$

$\rightarrow$ Expanded in MATLAB publication (attached)