

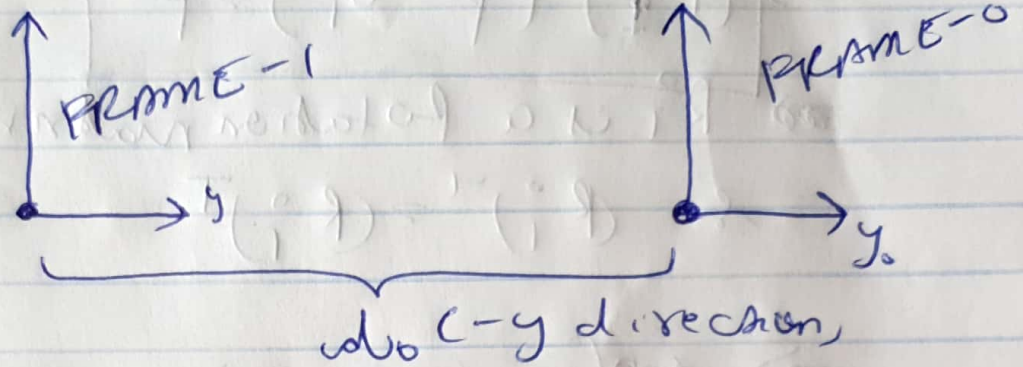
October 8th 19

→ GOVIND A.K
→ UID: 116699488

ENPM-662

HW-4

①



$$\theta_1^*(t) = (t - 2) \frac{\pi}{4} = \left[\frac{\pi t}{4} - \frac{\pi}{2} \right]$$

$$Z_0 = [0 \ 0 \ 1]^T = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$P^0(t) = [t \ 2t^2 \ 3t^3 + 1]^T \rightarrow \text{position of particle in fixed '0' frame.}$$

② $P^0 = R_1 P^1 + O_1$

Here the Rotation is about the Z axis

Hence the general form of the rotation matrix is given as:

$$\begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = R_1$$

We want to find p'

$$\Rightarrow (R_i^0)^{-1} (p^0 - o_i) = p'$$

R_i^0 is a rotation matrix,
so $(R_i^0)^{-1} = (R_i^0)^T$

$$\Rightarrow \begin{bmatrix} \cos\left(\frac{\pi t}{4} - \frac{\pi}{2}\right) & -\sin\left(\frac{\pi t}{4} - \frac{\pi}{2}\right) & 0 \\ \sin\left(\frac{\pi t}{4} - \frac{\pi}{2}\right) & \cos\left(\frac{\pi t}{4} - \frac{\pi}{2}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix}^T = (R_i^0)^{-1}$$

Writing $\cos \theta$ as $(\theta \rightarrow$

$$\Rightarrow \begin{bmatrix} \cos\left(\frac{\pi t}{4} - \frac{\pi}{2}\right) & +\sin\left(\frac{\pi t}{4} - \frac{\pi}{2}\right) & 0 \\ -\sin\left(\frac{\pi t}{4} - \frac{\pi}{2}\right) & \cos\left(\frac{\pi t}{4} - \frac{\pi}{2}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} t \\ 2t^2 \\ 3t^3 + 1 \end{bmatrix} - \begin{bmatrix} 0 \\ -d \\ 0 \end{bmatrix} = p'$$

$$\Rightarrow \begin{bmatrix} +\sin\left(\frac{\pi t}{4}\right) & -\cos\left(\frac{\pi t}{4}\right) & 0 \\ +\cos\left(\frac{\pi t}{4}\right) & \sin\left(\frac{\pi t}{4}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} t \\ 2t^2 + d \\ 3t^3 + 1 \end{bmatrix} = p'$$

$$\begin{bmatrix} t \cdot \sin\left(\frac{\pi t}{4}\right) + \cos\left(\frac{\pi t}{4}\right) \cdot (2t^2 + d) \\ + t \cdot \cos\left(\frac{\pi t}{4}\right) + \sin\left(\frac{\pi t}{4}\right) \cdot (2t^2 + d) \\ 3t^3 + 1 \end{bmatrix} = p'$$

This is the position as a function of time w.r.t frame

(ii)

From (i), we know that

$$p' = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 & 0 \\ -\sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} t \\ 2t^2 \\ 3t^3 + 1 \end{bmatrix}$$

$$\text{where } \theta_1 = \frac{\pi}{4} t - \frac{\pi}{2}$$

Velocity is the differential of p'

$$\therefore (p')' = \text{velocity}$$

$$\therefore (p')' = \text{derivative of } \begin{bmatrix} t \cos\theta_1 + 2t^2 \sin\theta_1 & t \sin\theta_1 + 2t^2 \cos\theta_1 & 0 \text{ or } 3t^3 + 1 \end{bmatrix}$$

$$\therefore (p')' = \begin{bmatrix} t \cos \theta_1 + 2t^2 \sin \theta_1 \\ -t \sin \theta_1 + 2t^2 \cos \theta_1 \\ 3t^3 \end{bmatrix} = \text{velocity of the particle.}$$

Let $\cos \theta_1 = c\theta$, Let $\sin \theta_1 = s\theta$
 $\theta_1 = \frac{\pi}{4} t - \frac{\pi}{2}$

$$\text{Velocity} = \begin{bmatrix} -t \sin \left[\frac{\pi}{4} \right] + c\theta + 4t s\theta + 2t^2 \left(c\left[\frac{\pi}{4} \right] \right) + d \left(c\left[\frac{\pi}{4} \right] \right) \\ -t c\left(\frac{\pi}{4} \right) + (-s\theta) + 4t c\theta + 2t^2 (-s\theta) + d(-s\theta) \\ 9t^2 + 0 \end{bmatrix}$$

$$\Rightarrow \text{velocity} = \begin{bmatrix} -\frac{t\pi}{4} s\theta + c\theta + 4t s\theta + \frac{t^2\pi}{2} c\theta + \frac{\pi d}{4} c\theta \\ -\frac{\pi t}{4} c\theta - s\theta + 4t c\theta - \frac{\pi t^2}{2} s\theta - \frac{\pi d}{4} s\theta \\ 9t^2 \end{bmatrix}$$

$$\Rightarrow \text{velocity} = \begin{bmatrix} \left[\frac{t^2 \pi}{2} + \frac{\pi d}{4} + 1 \right] \cos \theta + \left[4t - \frac{\pi t}{4} \right] \sin \theta \\ \left[4t - \frac{\pi t}{4} \right] \cos \theta + \left[-1 - \frac{\pi t^2}{4} \right] \sin \theta \\ q t^2 \quad \frac{-\pi d^2}{4} \end{bmatrix}$$

Now, we know that $\theta = \frac{\pi t}{4} - \frac{\pi}{2}$

$$\Rightarrow \cos \theta = \cos \left(\frac{\pi t}{4} - \frac{\pi}{2} \right) = \sin \left(\frac{\pi t}{4} \right) \quad \sin \theta = \sin \left(\frac{\pi t}{4} - \frac{\pi}{2} \right) = -\cos \left(\frac{\pi t}{4} \right)$$

Velocity =

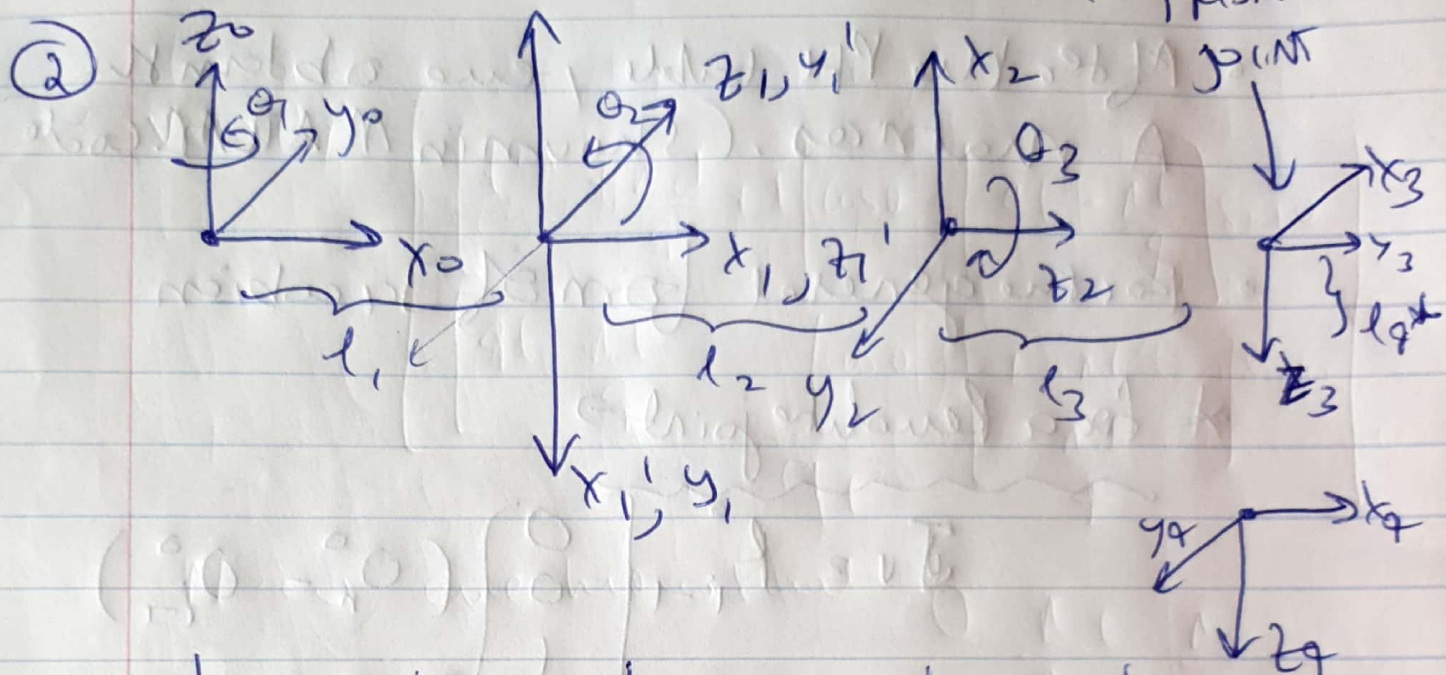
$$\begin{bmatrix} \left[\frac{t^2 \pi}{2} + \frac{\pi d}{4} + 1 \right] \sin \left(\frac{\pi t}{4} \right) + \left(4t - \frac{\pi t}{4} \right) (-\cos \frac{\pi t}{4}) \\ \left(4t - \frac{\pi t}{4} \right) \sin \left(\frac{\pi t}{4} \right) + \left[-1 - \frac{\pi t^2}{4} \right] (-\cos \frac{\pi t}{4}) \\ q t^2 \quad \frac{-\pi d^2}{4} \end{bmatrix}$$



Final velocity of particle in frame E_1 .

REVOLUTE JOINTS

PRISMATIC JOINT



	z_{n-1}	z_{n-1}	x_n	x_n
	θ	d	a	α
$0 \rightarrow 1$	θ_1^*	0	l_1	-90
$1 \rightarrow 1'$	$\theta_2^* + 90$	0	0	90
$1' \rightarrow 2$	0	l_2	0	0
$2 \rightarrow 3$	$\theta_3^* + 90$	l_3	0	90
$3 \rightarrow 4$	90	l_4^*	0	0

This gives the forward kinematics of the manipulator.

• After the PH table, we obtain the A matrices. (shown in MATLAB code)

• For the general form of Jacobian

* for a Revolute joint \rightarrow

$$J_v = R_{i-1}^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} (0_n^0 - 0_{i-1}^0) \quad \leftarrow \text{along } z\text{-axis}$$

$$J_w = R_{i-1}^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

* for Prismatic joint \rightarrow

$$J_v = R_{i-1}^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_w = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

We also know

\rightarrow Ans. (a)

$$J = \begin{bmatrix} z_0(0_1 - 0_0) & z_1(0_1 - 0_0) & z_2(0_1 - 0_0) & z_3 \\ z_0 & z_1 & z_2 & 0 \end{bmatrix}$$

$\theta_1, \theta_2, \theta_3$ are depicted in MATLAB code

Ans: (b)

(c) $l_1 = 3, l_2 = 2, l_3 = 1$

$q = [0, 45, 30, 0.5]$

here $\theta_1 = 0^\circ$

$\theta_2 = 45^\circ$

$\theta_3 = 30^\circ$

$\dot{\theta}_4 = 20.5$

Jacobian =

$$\begin{bmatrix} 0.25 & -2.42 & 0.176 & -0.612 \\ 4.815 & 0 & 0.433 & 0.5 \\ 0 & -1.817 & 0.176 & -0.612 \\ 0 & 0 & 0.2071 & 0 \\ 0 & 1.00 & 0 & 0 \\ 1.00 & 0 & -0.2071 & 0 \end{bmatrix}$$

Answers on MATLAB Code

Final velocity vector: $\begin{bmatrix} -9.8823 \\ 16.8605 \\ -6.6828 \\ 3.5355 \\ 4.000 \\ -0.5215 \end{bmatrix}$

Z matrices are obtained by multiplying the Rotational matrices with $[0 \ 0 \ 1]^T$. This is because, the rotation is about the Z-axis.

Rotation matrices are obtained from the 3×3 matrix part of the homogeneous matrix.

The origin is the first 3 elements of the last column of the homogeneous matrix.

All are depicted clearly in the MATLAB code.

But taking the values of z_0, z_1, z_2, z_3 from the output of MATLAB code, we get

Ans. (b)

$$\begin{aligned} z_0 &= [0 \ 0 \ 1]^T \\ z_1 &= [-\sin \theta_1 \ 0 \ 0]^T \\ z_2 &= [(\cos \theta_1 \sin(\theta_2 + 90)) \ (\sin \theta_1 \sin(\theta_2 + 90)) \ (\cos(\theta_2 + 90))]^T \\ z_3 &= \begin{bmatrix} (\sin \theta_1 (\cos(\theta_3 + 90)) + (\cos \theta_1 (\theta_2 + 90) \cdot \sin(\theta_3 + 90)) \\ (\sin \theta_1 (\cos(\theta_2 + 90) \sin(\theta_3 + 90)) - (\cos \theta_1 \sin(\theta_3 + 90)) \\ -\sin(\theta_2 + 90) \times \sin(\theta_3 + 90) \end{bmatrix} \end{aligned}$$

→ Expanded in MATLAB publication (attached)

```

%MATLAB CODE ASSIGNMENT 4 ENPM662
%ANSWER 2 (a) 2 (b) 2 (c)

%2(a) generic jacobian form written by hand
clc
clear all
%writing the code for A matrices
%%declaring the variables and symbols for the matrix multiplication
syms theta1 theta2 theta3 l1 l2 l3 l4 %%declaring all the unknown
    values as symbols

%%declaring the values of the a variables for translation along Xn
a2=0;
a3=0;
a4=0;
a5=0;
%%declaring the values of the alpha variables for rotation about Xn
alpha1=-90;
alpha2=90;
alpha3=0;
alpha4=90;
alpha5=0;
%%declaring the value of theta for frame 3 to 4 transformation
theta5=90;

%%The general form of the matrices are obtained by multiplying the
%%following

%%first A matrix A_1
Rz_theta1=[cosd(theta1) -sind(theta1) 0 0;sind(theta1) cosd(theta1) 0
    0;0 0 1 0;0 0 0 1];
Rx_alpha1=[1 0 0 0;0 cosd(alpha1) -sind(alpha1) 0;0 sind(alpha1)
    cosd(alpha1) 0;0 0 0 1];
Tz_d1=[1 0 0 0;0 1 0 0;0 0 1 (0);0 0 0 1];
Tx_a1=[1 0 0 l1;0 1 0 0;0 0 1 0;0 0 0 1];

A_1=Rz_theta1*Tz_d1*Tx_a1*Rx_alpha1

%%second A matrix A_2
Rz_theta2=[cosd(theta2+90) -sind(theta2+90) 0 0;sind(theta2+90)
    cosd(theta2+90) 0 0;0 0 1 0;0 0 0 1];
Rx_alpha2=[1 0 0 0;0 cosd(alpha2) -sind(alpha2) 0;0 sind(alpha2)
    cosd(alpha2) 0;0 0 0 1];
Tz_d2=[1 0 0 0;0 1 0 0;0 0 1 (0);0 0 0 1];
Tx_a2=[1 0 0 a2;0 1 0 0;0 0 1 0;0 0 0 1];

A_2=Rz_theta2*Tz_d2*Tx_a2*Rx_alpha2

%%third A matrix A_3
Rz_theta3=[cosd(0) -sind(0) 0 0;sind(0) cosd(0) 0 0;0 0 1 0;0 0 0 1];
Rx_alpha3=[1 0 0 0;0 cosd(alpha3) -sind(alpha3) 0;0 sind(alpha3)
    cosd(alpha3) 0;0 0 0 1];

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Tz_d3=[1 0 0 0;0 1 0 0;0 0 1 (l2);0 0 0 1];
Tx_a3=[1 0 0 a3;0 1 0 0;0 0 1 0;0 0 0 1];

A_3=Rz_theta3*Tz_d3*Tx_a3*Rx_alpha3

%%fourth A matrix A_4
Rz_theta4=[cosd(theta3+90) -sind(theta3+90) 0 0;sind(theta3+90)
cosd(theta3+90) 0 0;0 0 1 0;0 0 0 1];
Rx_alpha4=[1 0 0 0;0 cosd(alpha4) -sind(alpha4) 0;0 sind(alpha4)
cosd(alpha4) 0;0 0 0 1];
Tz_d4=[1 0 0 0;0 1 0 0;0 0 1 (l3);0 0 0 1];
Tx_a4=[1 0 0 a4;0 1 0 0;0 0 1 0;0 0 0 1];

A_4=Rz_theta4*Tz_d4*Tx_a4*Rx_alpha4

%%fifth A matrix A_5
Rz_theta5=[cosd(theta5) -sind(theta5) 0 0;sind(theta5) cosd(theta5) 0
0;0 0 1 0;0 0 0 1];
Rx_alpha5=[1 0 0 0;0 cosd(alpha5) -sind(alpha5) 0;0 sind(alpha5)
cosd(alpha5) 0;0 0 0 1];
Tz_d5=[1 0 0 0;0 1 0 0;0 0 1 (l4);0 0 0 1];
Tx_a5=[1 0 0 a5;0 1 0 0;0 0 1 0;0 0 0 1];

A_5=Rz_theta5*Tz_d5*Tx_a5*Rx_alpha5

%%FINDING HOMOGENEOUS, ROTATION MATRICES AND ORIGINS FOR JACOBIAN
CALCULATION
%%JOINT 1
H_1_0=A_1 %%Homogeneous Matrix for Joint 1
R_1_0=H_1_0(1:3,1:3) %%Rotation matrix for Joint 1
O_1_0=H_1_0(1:3,4); %%Origin for matrix 1
%%JOINT 2
H_2_0=A_1*A_2*A_3 %%Homogeneous Matrix for Joint 2
R_2_0=H_2_0(1:3,1:3) %%Rotation matrix for Joint 2
O_2_0=H_2_0(1:3,4); %%Origin for matrix 2
%%JOINT 3
H_3_0=A_1*A_2*A_3*A_4 %%Homogeneous Matrix for Joint 3
R_3_0=H_3_0(1:3,1:3) %%Rotation matrix for Joint 3
O_3_0=H_3_0(1:3,4); %%Origin for matrix 3
%%JOINT 4
H_4_0=A_1*A_2*A_3*A_4*A_5 %%Homogeneous Matrix for Joint 4
R_4_0=H_4_0(1:3,1:3) %%Rotation matrix for Joint 4
O_4_0=H_4_0(1:3,4); %%Origin for matrix 4

fprintf("Answer B: ")
fprintf("The o matrices are given here : ")
O_1_0=H_1_0(1:3,4)
O_2_0=H_2_0(1:3,4)
O_3_0=H_3_0(1:3,4)
O_4_0=H_4_0(1:3,4)
fprintf("The z matrices are given below : ")
z1=R_1_0*[0 0 1]' %%z1 matrix for jacobian calculation
z2=R_2_0*[0 0 1]' %%z2 matrix for jacobian calculation
z3=R_3_0*[0 0 1]' %%z3 matrix for jacobian calculation

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fprintf("Answer C: ")
%%substituting the values of l given in the question
l1=3;
l2=2;
l3=1;
%%substituting the values of theta given in the question
theta1=0;
theta2=45;
theta3=30;
l4=0.5;
%%substituting the values of a
a2=0;
a3=0;
a4=0;
a5=0;
%%substituting the values of alpha
alpha1=-90;
alpha2=90;
alpha3=0;
alpha4=90;
alpha5=0;
%%substituting the values of theta
theta5=90;

%%The general form of the matrices are obtained by multiplying the
%%following

%%first A matrix
Rz_theta1=[cosd(theta1) -sind(theta1) 0 0;sind(theta1) cosd(theta1) 0
0;0 0 1 0;0 0 0 1];
Rx_alpha1=[1 0 0 0;0 cosd(alpha1) -sind(alpha1) 0;0 sind(alpha1)
cosd(alpha1) 0;0 0 0 1];
Tz_d1=[1 0 0 0;0 1 0 0;0 0 1 (0);0 0 0 1];
Tx_a1=[1 0 0 l1;0 1 0 0;0 0 1 0;0 0 0 1];

%%second A matrix
Rz_theta2=[cosd(theta2+90) -sind(theta2+90) 0 0;sind(theta2+90)
cosd(theta2+90) 0 0;0 0 1 0;0 0 0 1];
Rx_alpha2=[1 0 0 0;0 cosd(alpha2) -sind(alpha2) 0;0 sind(alpha2)
cosd(alpha2) 0;0 0 0 1];
Tz_d2=[1 0 0 0;0 1 0 0;0 0 1 (0);0 0 0 1];
Tx_a2=[1 0 0 a2;0 1 0 0;0 0 1 0;0 0 0 1];

%%third A matrix
Rz_theta3=[cosd(0) -sind(0) 0 0;sind(0) cosd(0) 0 0;0 0 1 0;0 0 0 1];
Rx_alpha3=[1 0 0 0;0 cosd(alpha3) -sind(alpha3) 0;0 sind(alpha3)
cosd(alpha3) 0;0 0 0 1];
Tz_d3=[1 0 0 0;0 1 0 0;0 0 1 (l2);0 0 0 1];
Tx_a3=[1 0 0 a3;0 1 0 0;0 0 1 0;0 0 0 1];

%%fourth A matrix
Rz_theta4=[cosd(theta3+90) -sind(theta3+90) 0 0;sind(theta3+90)
cosd(theta3+90) 0 0;0 0 1 0;0 0 0 1];

```

```

Rx_alpha4=[1 0 0 0;0 cosd(alpha4) -sind(alpha4) 0;0 sind(alpha4)
cosd(alpha4) 0;0 0 0 1];
Tz_d4=[1 0 0 0;0 1 0 0;0 0 1 (l3);0 0 0 1];
Tx_a4=[1 0 0 a4;0 1 0 0;0 0 1 0;0 0 0 1];

%%fifth A matrix
Rz_theta5=[cosd(theta5) -sind(theta5) 0 0;sind(theta5) cosd(theta5) 0
0;0 0 1 0;0 0 0 1];
Rx_alpha5=[1 0 0 0;0 cosd(alpha5) -sind(alpha5) 0;0 sind(alpha5)
cosd(alpha5) 0;0 0 0 1];
Tz_d5=[1 0 0 0;0 1 0 0;0 0 1 (l4);0 0 0 1];
Tx_a5=[1 0 0 a5;0 1 0 0;0 0 1 0;0 0 0 1];

%%Matrix multiplicaion
A_1=Rz_theta1*Tz_d1*Tx_a1*Rx_alpha1
A_2=Rz_theta2*Tz_d2*Tx_a2*Rx_alpha2
A_3=Rz_theta3*Tz_d3*Tx_a3*Rx_alpha3
A_4=Rz_theta4*Tz_d4*Tx_a4*Rx_alpha4
A_5=Rz_theta5*Tz_d5*Tx_a5*Rx_alpha5
%%Multipltying to get the T matrix we get

%%REVOLUTE MATRICES FOR JACOBIAN CALCULATION
%%JOINT 1
O_0_0=[0 0 0]';
H_1_0=A_1
R_1_0=H_1_0(1:3,1:3)
O_1_0=H_1_0(1:3,4)
%%JOINT 2
H_2_0=A_1*A_2*A_3
R_2_0=H_2_0(1:3,1:3)
O_2_0=H_2_0(1:3,4)
%%JOINT 3
H_3_0=A_1*A_2*A_3*A_4
R_3_0=H_3_0(1:3,1:3)
O_3_0=H_3_0(1:3,4)
%%JOINT 4
H_4_0=A_1*A_2*A_3*A_4*A_5
R_4_0=H_4_0(1:3,1:3)
O_4_0=H_4_0(1:3,4)

z0=eye(3)*[0 0 1]';
z1=R_1_0*[0 0 1]';
z2=R_2_0*[0 0 1]';
z3=R_3_0*[0 0 1]';

%%taking cross products of the vectors and calculating the Jacobian
Matrix
q_dot=[3,4,5,0.5]';%given in the question
Jacobian=[cross(z0,O_4_0-O_0_0) cross(z1,O_4_0-O_1_0) cross(z2,O_4_0-
O_2_0) z3;z0 z1 z2 [0 0 0]']
velocity_vectors=Jacobian*q_dot

A_1 =

```

```

[ cos((pi*theta1)/180), 0, -sin((pi*theta1)/180),
  11*cos((pi*theta1)/180)]
[ sin((pi*theta1)/180), 0, cos((pi*theta1)/180),
  11*sin((pi*theta1)/180)]
[          0, -1,          0,
  0]
[          0, 0,          0,
  1]

```

A_2 =

```

[ cos((pi*(theta2 + 90))/180), 0, sin((pi*(theta2 + 90))/180), 0]
[ sin((pi*(theta2 + 90))/180), 0, -cos((pi*(theta2 + 90))/180), 0]
[          0, 1,          0, 0]
[          0, 0,          0, 1]

```

A_3 =

```

[ 1, 0, 0, 0]
[ 0, 1, 0, 0]
[ 0, 0, 1, 12]
[ 0, 0, 0, 1]

```

A_4 =

```

[ cos((pi*(theta3 + 90))/180), 0, sin((pi*(theta3 + 90))/180), 0]
[ sin((pi*(theta3 + 90))/180), 0, -cos((pi*(theta3 + 90))/180), 0]
[          0, 1,          0, 13]
[          0, 0,          0, 1]

```

A_5 =

```

[ 0, -1, 0, 0]
[ 1, 0, 0, 0]
[ 0, 0, 1, 14]
[ 0, 0, 0, 1]

```

H_1_0 =

```

[ cos((pi*theta1)/180), 0, -sin((pi*theta1)/180),
  11*cos((pi*theta1)/180)]
[ sin((pi*theta1)/180), 0, cos((pi*theta1)/180),
  11*sin((pi*theta1)/180)]
[          0, -1,          0,
  0]
[          0, 0,          0,
  1]

```

R_1_0 =

```
[ cos((pi*theta1)/180),  0, -sin((pi*theta1)/180)]  
[ sin((pi*theta1)/180),  0,  cos((pi*theta1)/180)]  
[                          0, -1,                          0]
```

H_2_0 =

```
[ cos((pi*theta1)/180)*cos((pi*(theta2 + 90))/180), -  
sin((pi*theta1)/180), cos((pi*theta1)/180)*sin((pi*(theta2  
+ 90))/180), l1*cos((pi*theta1)/180) +  
l2*cos((pi*theta1)/180)*sin((pi*(theta2 + 90))/180)]  
[ sin((pi*theta1)/180)*cos((pi*(theta2 + 90))/180),  
cos((pi*theta1)/180), sin((pi*theta1)/180)*sin((pi*(theta2  
+ 90))/180), l1*sin((pi*theta1)/180) +  
l2*sin((pi*theta1)/180)*sin((pi*(theta2 + 90))/180)]  
[                          -sin((pi*(theta2 + 90))/180),  
0,                          cos((pi*(theta2 + 90))/180),  
                          l2*cos((pi*(theta2 + 90))/180)]  
[                          0,  
0,                          0,  
                          0,  
                          1]
```

R_2_0 =

```
[ cos((pi*theta1)/180)*cos((pi*(theta2 + 90))/180), -  
sin((pi*theta1)/180), cos((pi*theta1)/180)*sin((pi*(theta2 +  
90))/180)]  
[ sin((pi*theta1)/180)*cos((pi*(theta2 + 90))/180),  
cos((pi*theta1)/180), sin((pi*theta1)/180)*sin((pi*(theta2 +  
90))/180)]  
[                          -sin((pi*(theta2 + 90))/180),  
0,                          cos((pi*(theta2 + 90))/180)]
```

H_3_0 =

```
[ cos((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*cos((pi*(theta3  
+ 90))/180) - sin((pi*theta1)/180)*sin((pi*(theta3  
+ 90))/180), cos((pi*theta1)/180)*sin((pi*(theta2  
+ 90))/180), sin((pi*theta1)/180)*cos((pi*(theta3 +  
90))/180) + cos((pi*theta1)/180)*cos((pi*(theta2 +  
90))/180)*sin((pi*(theta3 + 90))/180), l1*cos((pi*theta1)/180)  
+ l2*cos((pi*theta1)/180)*sin((pi*(theta2 + 90))/180) +  
l3*cos((pi*theta1)/180)*sin((pi*(theta2 + 90))/180)]  
[ cos((pi*theta1)/180)*sin((pi*(theta3 + 90))/180) +  
sin((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*cos((pi*(theta3  
+ 90))/180), sin((pi*theta1)/180)*sin((pi*(theta2 + 90))/180),  
sin((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*sin((pi*(theta3  
+ 90))/180) - cos((pi*theta1)/180)*cos((pi*(theta3 + 90))/180),
```

```

11*sin((pi*theta1)/180) + 12*sin((pi*theta1)/180)*sin((pi*(theta2 +
90))/180) + 13*sin((pi*theta1)/180)*sin((pi*(theta2 + 90))/180)]
[
    -cos((pi*(theta3 + 90))/180)*sin((pi*(theta2 + 90))/180),
        cos((pi*(theta2 + 90))/180),
                                -sin((pi*(theta2 +
90))/180)*sin((pi*(theta3 + 90))/180),
                                    12*cos((pi*(theta2 + 90))/180) +
13*cos((pi*(theta2 + 90))/180)]
[
                                0,
                                0,
                                0,
                                1]
1]

```

R_3_0 =

```

[ cos((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*cos((pi*(theta3
+ 90))/180) - sin((pi*theta1)/180)*sin((pi*(theta3 +
90))/180), cos((pi*theta1)/180)*sin((pi*(theta2 + 90))/180),
sin((pi*theta1)/180)*cos((pi*(theta3 + 90))/180) +
cos((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*sin((pi*(theta3 +
90))/180)]
[ cos((pi*theta1)/180)*sin((pi*(theta3 + 90))/180) +
sin((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*cos((pi*(theta3
+ 90))/180), sin((pi*theta1)/180)*sin((pi*(theta2 + 90))/180),
sin((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*sin((pi*(theta3 +
90))/180) - cos((pi*theta1)/180)*cos((pi*(theta3 + 90))/180)]
[
    -cos((pi*(theta3 + 90))/180)*sin((pi*(theta2 + 90))/180),
        cos((pi*(theta2 + 90))/180),
                                -sin((pi*(theta2 +
90))/180)*sin((pi*(theta3 + 90))/180)]

```

H_4_0 =

```

[ cos((pi*theta1)/180)*sin((pi*(theta2 + 90))/180),
    sin((pi*theta1)/180)*sin((pi*(theta3 + 90))/180) -
cos((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*cos((pi*(theta3
+ 90))/180), sin((pi*theta1)/180)*cos((pi*(theta3 +
90))/180) + cos((pi*theta1)/180)*cos((pi*(theta2 +
90))/180)*sin((pi*(theta3 + 90))/180), 11*cos((pi*theta1)/180)
+ 14*(sin((pi*theta1)/180)*cos((pi*(theta3 + 90))/180) +
cos((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*sin((pi*(theta3 +
90))/180)) + 12*cos((pi*theta1)/180)*sin((pi*(theta2 + 90))/180) +
13*cos((pi*theta1)/180)*sin((pi*(theta2 + 90))/180)]
[ sin((pi*theta1)/180)*sin((pi*(theta2 + 90))/180),
    - cos((pi*theta1)/180)*sin((pi*(theta3 + 90))/180) -
sin((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*cos((pi*(theta3
+ 90))/180), sin((pi*theta1)/180)*cos((pi*(theta2

```

```

+ 90))/180)*sin((pi*(theta3 + 90))/180) -
cos((pi*theta1)/180)*cos((pi*(theta3 + 90))/180),
11*sin((pi*theta1)/180) - 14*(cos((pi*theta1)/180)*cos((pi*(theta3
+ 90))/180) - sin((pi*theta1)/180)*cos((pi*(theta2
+ 90))/180)*sin((pi*(theta3 + 90))/180)) +
12*sin((pi*theta1)/180)*sin((pi*(theta2 + 90))/180) +
13*sin((pi*theta1)/180)*sin((pi*(theta2 + 90))/180)]
[
cos((pi*(theta2 + 90))/180),

cos((pi*(theta3 + 90))/180)*sin((pi*(theta2 + 90))/180),
-
sin((pi*(theta2 + 90))/180)*sin((pi*(theta3 + 90))/180),

12*cos((pi*(theta2 + 90))/180) + 13*cos((pi*(theta2 + 90))/180) -
14*sin((pi*(theta2 + 90))/180)*sin((pi*(theta3 + 90))/180)]
[
0,

0,

0,

1]

```

R_4_0 =

```

[ cos((pi*theta1)/180)*sin((pi*(theta2 + 90))/180),
sin((pi*theta1)/180)*sin((pi*(theta3 + 90))/180) -
cos((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*cos((pi*(theta3
+ 90))/180), sin((pi*theta1)/180)*cos((pi*(theta3 + 90))/180) +
cos((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*sin((pi*(theta3 +
90))/180)]
[ sin((pi*theta1)/180)*sin((pi*(theta2 + 90))/180),
- cos((pi*theta1)/180)*sin((pi*(theta3 + 90))/180) -
sin((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*cos((pi*(theta3
+ 90))/180), sin((pi*theta1)/180)*cos((pi*(theta2
+ 90))/180)*sin((pi*(theta3 + 90))/180) -
cos((pi*theta1)/180)*cos((pi*(theta3 + 90))/180)]
[
cos((pi*(theta2 + 90))/180),

cos((pi*(theta3 + 90))/180)*sin((pi*(theta2 + 90))/180),
-
sin((pi*(theta2 + 90))/180)*sin((pi*(theta3 + 90))/180)]

```

Answer B: The o matrices are given here :

O_1_0 =

```

11*cos((pi*theta1)/180)
11*sin((pi*theta1)/180)
0

```

$O_{2_0} =$

$$\begin{aligned} & l1 \cdot \cos((\pi \cdot \theta_1)/180) + l2 \cdot \cos((\pi \cdot \theta_1)/180) \cdot \sin((\pi \cdot (\theta_2 + 90))/180) \\ & l1 \cdot \sin((\pi \cdot \theta_1)/180) + l2 \cdot \sin((\pi \cdot \theta_1)/180) \cdot \sin((\pi \cdot (\theta_2 + 90))/180) \\ & \qquad \qquad \qquad l2 \cdot \cos((\pi \cdot (\theta_2 + 90))/180) \end{aligned}$$

$O_{3_0} =$

$$\begin{aligned} & l1 \cdot \cos((\pi \cdot \theta_1)/180) + l2 \cdot \cos((\pi \cdot \theta_1)/180) \cdot \sin((\pi \cdot (\theta_2 + 90))/180) + l3 \cdot \cos((\pi \cdot \theta_1)/180) \cdot \sin((\pi \cdot (\theta_2 + 90))/180) \\ & l1 \cdot \sin((\pi \cdot \theta_1)/180) + l2 \cdot \sin((\pi \cdot \theta_1)/180) \cdot \sin((\pi \cdot (\theta_2 + 90))/180) + l3 \cdot \sin((\pi \cdot \theta_1)/180) \cdot \sin((\pi \cdot (\theta_2 + 90))/180) \\ & l2 \cdot \cos((\pi \cdot (\theta_2 + 90))/180) + l3 \cdot \cos((\pi \cdot (\theta_2 + 90))/180) \end{aligned}$$

$O_{4_0} =$

$$\begin{aligned} & l1 \cdot \cos((\pi \cdot \theta_1)/180) + l4 \cdot (\sin((\pi \cdot \theta_1)/180) \cdot \cos((\pi \cdot (\theta_3 + 90))/180) + \cos((\pi \cdot \theta_1)/180) \cdot \cos((\pi \cdot (\theta_2 + 90))/180) \cdot \sin((\pi \cdot (\theta_3 + 90))/180)) + \\ & l2 \cdot \cos((\pi \cdot \theta_1)/180) \cdot \sin((\pi \cdot (\theta_2 + 90))/180) + \\ & l3 \cdot \cos((\pi \cdot \theta_1)/180) \cdot \sin((\pi \cdot (\theta_2 + 90))/180) \\ & l1 \cdot \sin((\pi \cdot \theta_1)/180) - l4 \cdot (\cos((\pi \cdot \theta_1)/180) \cdot \cos((\pi \cdot (\theta_3 + 90))/180) - \sin((\pi \cdot \theta_1)/180) \cdot \cos((\pi \cdot (\theta_2 + 90))/180) \cdot \sin((\pi \cdot (\theta_3 + 90))/180)) + \\ & l2 \cdot \sin((\pi \cdot \theta_1)/180) \cdot \sin((\pi \cdot (\theta_2 + 90))/180) + \\ & l3 \cdot \sin((\pi \cdot \theta_1)/180) \cdot \sin((\pi \cdot (\theta_2 + 90))/180) \\ & l2 \cdot \cos((\pi \cdot (\theta_2 + 90))/180) + l3 \cdot \cos((\pi \cdot (\theta_2 + 90))/180) - \\ & l4 \cdot \sin((\pi \cdot (\theta_2 + 90))/180) \cdot \sin((\pi \cdot (\theta_3 + 90))/180) \end{aligned}$$

The z matrices are given below :

$z1 =$

$$\begin{aligned} & -\sin((\pi \cdot \theta_1)/180) \\ & \cos((\pi \cdot \theta_1)/180) \\ & \qquad \qquad \qquad 0 \end{aligned}$$

$z2 =$

$$\begin{aligned} & \cos((\pi \cdot \theta_1)/180) \cdot \sin((\pi \cdot (\theta_2 + 90))/180) \\ & \sin((\pi \cdot \theta_1)/180) \cdot \sin((\pi \cdot (\theta_2 + 90))/180) \\ & \qquad \qquad \qquad \cos((\pi \cdot (\theta_2 + 90))/180) \end{aligned}$$

$z3 =$

```

sin((pi*theta1)/180)*cos((pi*(theta3 + 90))/180) +
cos((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*sin((pi*(theta3 +
90))/180)
sin((pi*theta1)/180)*cos((pi*(theta2 + 90))/180)*sin((pi*(theta3 +
90))/180) - cos((pi*theta1)/180)*cos((pi*(theta3 + 90))/180)

-sin((pi*(theta2 + 90))/180)*sin((pi*(theta3 + 90))/180)

```

Answer C:

A_1 =

```

1      0      0      3
0      0      1      0
0     -1      0      0
0      0      0      1

```

A_2 =

```

-0.7071      0      0.7071      0
 0.7071      0      0.7071      0
      0     1.0000      0      0
      0      0      0     1.0000

```

A_3 =

```

1      0      0      0
0      1      0      0
0      0      1      2
0      0      0      1

```

A_4 =

```

-0.5000      0      0.8660      0
 0.8660      0      0.5000      0
      0     1.0000      0     1.0000
      0      0      0     1.0000

```

A_5 =

```

      0     -1.0000      0      0
 1.0000      0      0      0
      0      0     1.0000     0.5000
      0      0      0     1.0000

```

O_0_0 =

```

0
0

```

0

$H_{1_0} =$

1	0	0	3
0	0	1	0
0	-1	0	0
0	0	0	1

$R_{1_0} =$

1	0	0
0	0	1
0	-1	0

$O_{1_0} =$

3
0
0

$H_{2_0} =$

-0.7071	0	0.7071	4.4142
0	1.0000	0	0
-0.7071	0	-0.7071	-1.4142
0	0	0	1.0000

$R_{2_0} =$

-0.7071	0	0.7071
0	1.0000	0
-0.7071	0	-0.7071

$O_{2_0} =$

4.4142
0
-1.4142

$H_{3_0} =$

0.3536	0.7071	-0.6124	5.1213
0.8660	0	0.5000	0
0.3536	-0.7071	-0.6124	-2.1213
0	0	0	1.0000

$R_{3_0} =$

0.3536	0.7071	-0.6124
0.8660	0	0.5000
0.3536	-0.7071	-0.6124

$O_{3_0} =$

5.1213
0
-2.1213

$H_{4_0} =$

0.7071	-0.3536	-0.6124	4.8151
0	-0.8660	0.5000	0.2500
-0.7071	-0.3536	-0.6124	-2.4275
0	0	0	1.0000

$R_{4_0} =$

0.7071	-0.3536	-0.6124
0	-0.8660	0.5000
-0.7071	-0.3536	-0.6124

$O_{4_0} =$

4.8151
0.2500
-2.4275

$z0 =$

0
0
1

$z1 =$

0
1
0

$z2 =$

0.7071

```

    0
    -0.7071

z3 =

    -0.6124
     0.5000
    -0.6124

q_dot =

    3.0000
    4.0000
    5.0000
    0.5000

Jacobian =

    -0.2500    -2.4275     0.1768    -0.6124
     4.8151         0     0.4330     0.5000
         0    -1.8151     0.1768    -0.6124
         0         0     0.7071         0
         0     1.0000         0         0
     1.0000         0    -0.7071         0

velocity_vectors =

    -9.8823
    16.8605
    -6.6828
     3.5355
     4.0000
    -0.5355
```

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