

$$\Rightarrow \begin{pmatrix} C_1 C_2 & C_2 S_1 & -S_2 \\ -S_1 & C_1 & 0 \\ C_1 S_2 & S_1 S_2 & C_2 \end{pmatrix} \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

$$= \begin{pmatrix} 0 & & -S_5 C_4 \\ 1 & 1 & S_5 S_4 \\ -S_5 C_6 & S_5 S_6 & C_5 \end{pmatrix}$$

Since C_1, C_2 ~~their values don't matter for calculation~~
 ans don't matter

$$C_1 C_2 r_{13} + r_{23} C_2 S_1 - S_2 r_{33} = -S_5 C_4 \quad \textcircled{1}$$

$$-S_1 r_{13} + C_1 r_{23} = -S_5 S_4 \quad \textcircled{2}$$

$$C_1 S_2 r_{13} + S_1 S_2 r_{23} + C_2 r_{33} = C_5 \quad \textcircled{3}$$

$$\boxed{\Theta_5 = \cos^{-1}(C_1 S_2 r_{13} + S_1 S_2 r_{23} + C_2 r_{33})}$$

Dividing $\textcircled{2}$ and $\textcircled{1}$

$$\tan \Theta_9 = \left(\frac{-S_1 r_{13} + C_1 r_{23}}{C_1 C_2 r_{13} + r_{23} C_2 S_1 - S_2 r_{33}} \right)$$

$$\Rightarrow \theta_4 = \tan^{-1} \left(\frac{-\varsigma_1 r_{13} + c_1 r_{23}}{c_1 c_2 r_{13} + r_{23} (c_2 s_1 - \varsigma_2 r_{33})} \right)$$

Also,

$$c_1 \varsigma_2 r_{11} + \varsigma_1 \varsigma_2 r_{21} + c_2 r_{31} = -\varsigma_5 \quad \text{---}$$

$$\varsigma_2 c_1 r_{12} + \varsigma_1 c_2 r_{22} + c_2 r_{32} = \varsigma_5 \varsigma_6$$

Dividing the above 2 equations :-

$$\tan(\theta_6) = \frac{\varsigma_2 c_1 r_{12} + \varsigma_1 \varsigma_2 r_{22} + c_2 r_{32}}{c_1 \varsigma_2 r_{11} + \varsigma_1 \varsigma_2 r_{21} + c_2 r_{31}}$$

$$\Rightarrow \theta_6 = \tan^{-1} \left(\frac{\varsigma_2 c_1 r_{12} + \varsigma_1 \varsigma_2 r_{22} + c_2 r_{32}}{c_1 \varsigma_2 r_{11} + \varsigma_1 \varsigma_2 r_{21} + c_2 r_{31}} \right)$$

③ If a body has linear velocity only
the velocities can be calculated
by the transformation matrix between
the two frames

$$\text{Here } H_1 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -3 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{and } V^o(t) = \begin{bmatrix} 0 & -2 & 4 \end{bmatrix}^T = \begin{bmatrix} 0 \\ -2 \\ 4 \end{bmatrix}$$

Since only the rotation matters.

$$V^o = R_1^o V'$$

Taking inverse →

$$(R_1^o)^{-1} V^o = R_1^o V' (R_1^o)^{-1} = V'$$

$$\Rightarrow V_1 = (R_1^o)^{-1} V^o = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ -2 \\ 4 \end{bmatrix}$$

$$\Rightarrow V_1 = [4 \ 0 \ -2]^T$$