26th Nov '19

ENPM 662

HW-6

Govind AK
116699488

① (a)

$$J = \begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 4 & 3 & 1 \\ 2 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 2 & 3 & 0 \\ 1 & 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} J_v \\ J_w \end{bmatrix}$$

$$\tau_1 = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 5 \end{bmatrix} \qquad \tau_2 = \begin{bmatrix} 2 \\ 5 \\ 6 \\ 5 \end{bmatrix}$$

The pseudoinverse of this can only be used to determine the endeffector forces iff the condition

$$\text{rank}(J) = \text{rank}[J \mid \tau_i] \text{ be obeyed}$$

Hence, we have to test for this

Testing for τ_1

$$\text{rank}(J_w^T) = \text{rank} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 3 & 0 \\ 1 & 0 & 0 & 5 \end{bmatrix}^T$$

$$= \text{rank} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} = 3 = \text{full rank}$$

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Now:

$$\text{Rank}\left(J_\omega^T | \tau_1\right) = \text{Rank}\begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 2 & 0 & 4 \\ 0 & 3 & 0 & 6 \\ 0 & 0 & 5 & 5 \end{bmatrix}$$

Checking the MATLAB code we see that this Rank = 3

Hence the pseudo inverse can be used to determine the end-effector forces.

testing for τ_2

Rank $J_\omega^T = 3$ (as we have seen before)

$$\text{Ran}\left(J_\omega^T | \tau_2\right) = \text{Rank}\begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 2 & 0 & 5 \\ 0 & 3 & 0 & 6 \\ 0 & 0 & 5 & 5 \end{bmatrix} = 4$$

(Check MATLAB code)

$\therefore 3 \neq 4$

Hence, the pseudoinverse method for calculating the end effector torques might yield out wrong values

(b) $\tau = J^T F$ → Relation between end-effector forces and torques

We have to find F

Now J_w^T has a dimension of 4×3 and τ has dimensions of 4×1

Hence,

$F_1 \to$ Force for τ_1

$F_2 \to$ Force for τ_2

$$\boxed{J_w \tau = J_w J_w^T F_1}$$

Now to get F, we have to get rid of $J_w J_w^T$ on the RHS

Hence,

$$\boxed{(J_w J_w^T)^{-1} J_w \tau_1 = (J_w J_w^T)^{-1} (J_w J_w^T) F_1}$$

Now $J_w = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 3 & 0 \\ 1 & 0 & 0 & 5 \end{bmatrix}$ $\quad \tau_1 = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 5 \end{bmatrix}$

$$J_w^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

Now, $(J_\omega J_\omega^T)^{-1}(J_\omega J_\omega^T) = I$

Doing the calculations on MATLAB →

$\therefore F_1 = (J_\omega J_\omega^T)^{-1} J_\omega \tau_1$

$$= \left[\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 3 & 0 \\ 1 & 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}\right]^{-1}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 3 & 0 \\ 1 & 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \\ 6 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & -0.20 \\ 0 & 0.1538 & 0.23 & 0 \\ 0 & 0 & 0 & 0.2 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \\ 6 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \longrightarrow F_1 \text{ for } \tau_1$$

Now for τ_2 →

$$\tau_2 = \begin{bmatrix} 2 \\ 5 \\ 6 \\ 5 \end{bmatrix}$$

$F_2 = (J_\omega J_\omega^T)^{-1} J_\omega \tau_2$

$$\Rightarrow F_2 = \begin{bmatrix} 1 & 0 & 0 & -0.2 \\ 0 & 0.1538 & 0.23 & 0 \\ 0 & 0 & 0 & 0.2 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 6 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2.1538 \\ 1.0000 \end{bmatrix}$$

(c) Now to verify our theory from part (a),

we do the following

$\tau_1 = J_w^T F_1$ should yield the same $\tau_1 = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 5 \end{bmatrix}$

and doing MATLAB multiplication, we find that

$$\tau_1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$= \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ Hence, Verified.

Similarly

$$T_2 = J_w^T F_2$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1.000 \\ 2.1538 \\ 1.000 \end{bmatrix}$$

$$t_2 = \begin{bmatrix} 2.00 \\ 4.30 \\ 6.4615 \\ 5.000 \end{bmatrix}$$ Hence this is NOT THE SAME as

initial T_2 of $\begin{bmatrix} 2 \\ 5 \\ 6 \\ 5 \end{bmatrix}$

<u>Hence verified.</u>