#### PIPG for Rendezvous

Govind Chari

University of Washington

June 2023





#### PIPG Overview

#### PIPG is

- A first order primal-dual conic optimizer
- Easily verifiable
- Factorization free
- Intuitive
- Warmstartable
- Very fast (3-7x faster than ECOS)
- Codable in an afternoon
- Supports infeasibility detection

# Comparison with ECOS

	ECOS	PIPG
Warm Startable	X	✓
Verification Ease		<b>✓</b>
Factorization Free	X	<b>✓</b>
Understandable	X	<b>✓</b>
Implementation Ease	X	<b>✓</b>
Infeasibility Detection	<b>/</b>	<b>✓</b>
Tuning Free	✓	X

### PIPG Algorithm

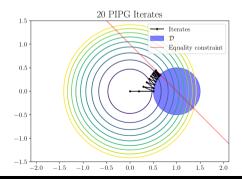
### Optimization Problem

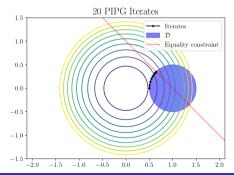
$$\min_{z} \quad \frac{1}{2}z^{\top}Pz + q^{\top}z$$
s.t. 
$$Hz - h = 0$$

$$z \in \mathbb{D}$$

### Algorithm

$$z^{k+1} \leftarrow \Pi_{\mathbb{D}} \left[ z^k - \alpha (Pz^k + q + H^{\top} w^k) \right]$$
$$w^{k+1} \leftarrow w^k + \beta (H(2z^{k+1} - z^k) - h)$$

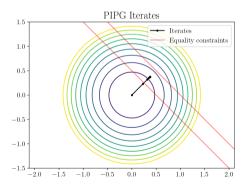


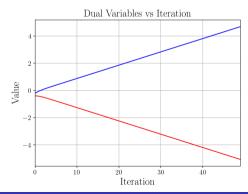


## Infeasibility Detection

#### Primal Infeasibility Criteria:

$$\lim_{k\to\infty}\|w^{k+1}-w^k\|\neq 0$$

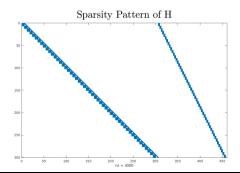


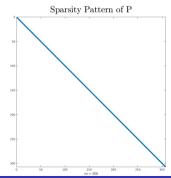


### Structured Sparsity

minimize 
$$x_N^{\top}Q_Nx_N + \sum_{t=1}^{N-1} x_t^{\top}Q_tx_t + u_t^{\top}R_tu_t$$
  
subject to  $x_{t+1} = A_tx_t + B_tu_t$ ,  $x_t \in \mathbb{X}_t$ ,  $u_t \in \mathbb{U}_t$ 

minimize 
$$\frac{1}{2}z^{\top}Pz$$
  
subject to  $Hz - h = 0, z \in \mathbb{D}$ 

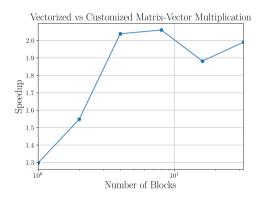




### Customization: Structure Exploitation

$$\begin{bmatrix}
A_1 & & \\ & \ddots & \\ & & A_K
\end{bmatrix}
\underbrace{\begin{bmatrix}
x_1 \\ \vdots \\ x_K
\end{bmatrix}}_{z} = \underbrace{\begin{bmatrix}
A_1 x_1 \\ \vdots \\ A_K x_K
\end{bmatrix}}_{y}$$

We can compute y = Hz subvector by subvector  $(y_i = A_i x_i)$  without constructing H and without using sparse-matrix vector multiplication.



Customization can double structured matrix vector multiplication speed.

### Customization: PIPG

$$w_t \leftarrow v_t + \beta(x_{t+1} - A_t x_t - B_t u_t)$$

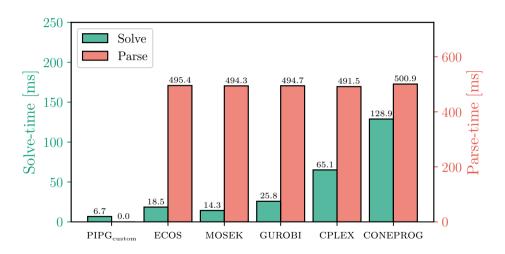
$$u_t \leftarrow \Pi_{\mathbb{U}_t} [u_t - \alpha(R_t u_t - B_t^\top w_{t-1})]$$

$$x_t \leftarrow \Pi_{\mathbb{X}_t} [x_t - \alpha(Q_t x_t + w_{t-1} - A_t^\top w_t)]$$

$$v_t \leftarrow v_t + \beta(x_{t+1} - A_t x_t - B_t u_t)$$

$$w \leftarrow v + \beta(Hz - h)$$
$$z \leftarrow \Pi_{\mathbb{D}}[z - \alpha(Pz + H^{\top}w)]$$
$$v \leftarrow v + \beta(Hz - h)$$

## Parsing

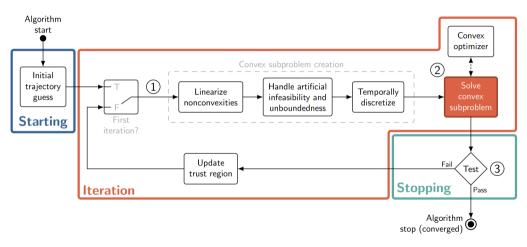


From Elango et al. SciTech 2022

#### Continuous-time Nonconvex Problem

$$\begin{array}{ll} \underset{t_f,\, x(t),\, u(t)}{\text{minimize}} & \int_0^{t_f} \|u(t)\|_2^2 \, dt \\ \\ \text{subject to} & \forall t \in [0,t_f) \\ \hline \text{CW Dynamics} & \dot{x}(t) = f(x(t),u(t)) \\ \hline \text{Max delta-v} & \|u(t)\|_2 \leq u_{\text{max}} \\ \hline \text{Keepout Zone} & \|r(t) - r_c\|_2 \geq \rho \\ \hline \hline \text{Max Speed} & \|v(t)\|_2 \leq v_{\text{max}} \\ \hline \text{Initial Conditions} & r(0) = r_i \\ & v(0) = v_i \\ \hline \hline \text{Terminal Conditions} & r(t_f) = 0_{3\times 1} \\ & v(t_f) = 0_{3\times 1} \\ \hline \end{array}$$

## Sequential Convex Programming (SCP)

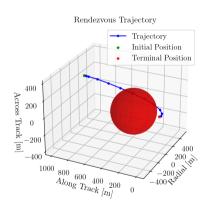


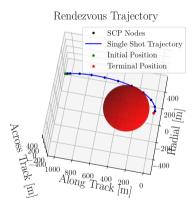
From Malyuta et al. *IEEE CSM 2022* 

## Discrete-time Convex Subproblem

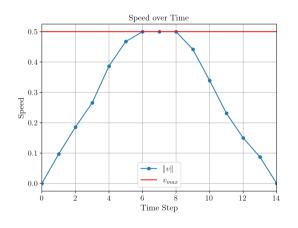
$$\begin{aligned} & \underset{\sigma, x, u}{\text{minimize}} & \sum_{k=1}^{K-1} \|u_k\|_2^2 + w_{tr} \left( \sum_{k=1}^K \|x_k - \hat{x}_k\|_2^2 + \sum_{k=1}^{K-1} [\|u_k - \hat{u}_k\|_2^2 + (\sigma_k - \hat{\sigma}_k)^2] \right) + w_{vc} \|\nu^c\|_1 + w_{vb} \sum_{k=1}^K \nu_k^b \\ & \text{subject to} & \forall k \in [1, K] \\ \hline & \text{Discrete Dynamics} & x_{k+1} = A_k x_k + B_k u_k + S_k \sigma_k + c_k + \nu_k^c \\ \hline & \text{Dilation Constraints} & \sigma_{\min} \leq \sigma_k \leq \sigma_{\max} \\ \hline & \text{Max delta-v} & \|u_k\|_2 \leq u_{\max} \\ \hline & \text{Keepout Zone} & \|\hat{r}_k - r_c\|_2 + \left(\frac{\hat{r}_k - r_c}{\|\hat{r}_k - r_c\|_2}\right)^\top (r_k - \hat{r}_k) + \nu_k^b \geq \rho \\ & \nu_k^b \geq 0 \\ \hline & \text{Max Speed} & \|v_k\|_2 \leq v_{\max} \\ \hline & \text{Initial Conditions} & r(1) = r_i \\ & v(1) = v_i \\ \hline \text{Terminal Conditions} & r(K) = 0_{3\times 1} \\ & v(K) = 0_{3\times 1} \end{aligned}$$

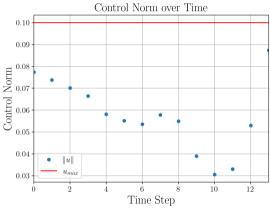
## Trajectory



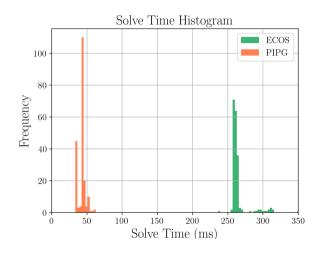


# Trajectory





## Speed Results



	ECOS	PIPG
Mean	261ms	42ms
Median	267ms	44ms
STD	23ms	5ms

PIPG is **6.25**x faster even without customization

200 runs, 3 millisecond bins

## Summary

#### PIPG is

- Easy to understand
- Easy to verify
- Easy to implement
- Faster than ECOS

