

## 2.4 Signed and Unsigned Numbers

정수 (integer)

# Base- $n$ Numbers ( $n$ 진수)

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- $i$ 번째 자리수가  $d$ 이면  $d \times \text{Base}^i$  를 의미 ( $i$ 는 0부터 시작)
- $d$  의 범위는  $0 \sim \text{Base} - 1$

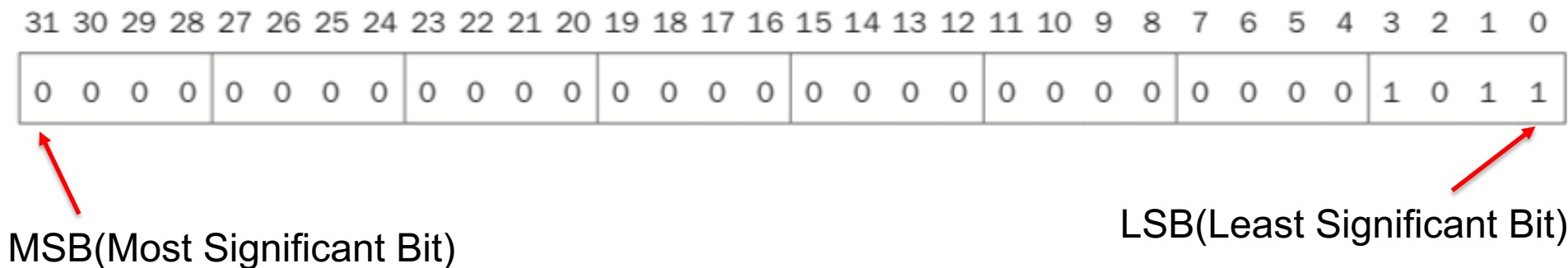
$$138_{10} = 1 \times 10^2 + 3 \times 10^1 + 8 \times 10^0 \quad \text{when } n = 10$$

## Base-2 Numbers = Binary Numbers (2진수) when $n=2$

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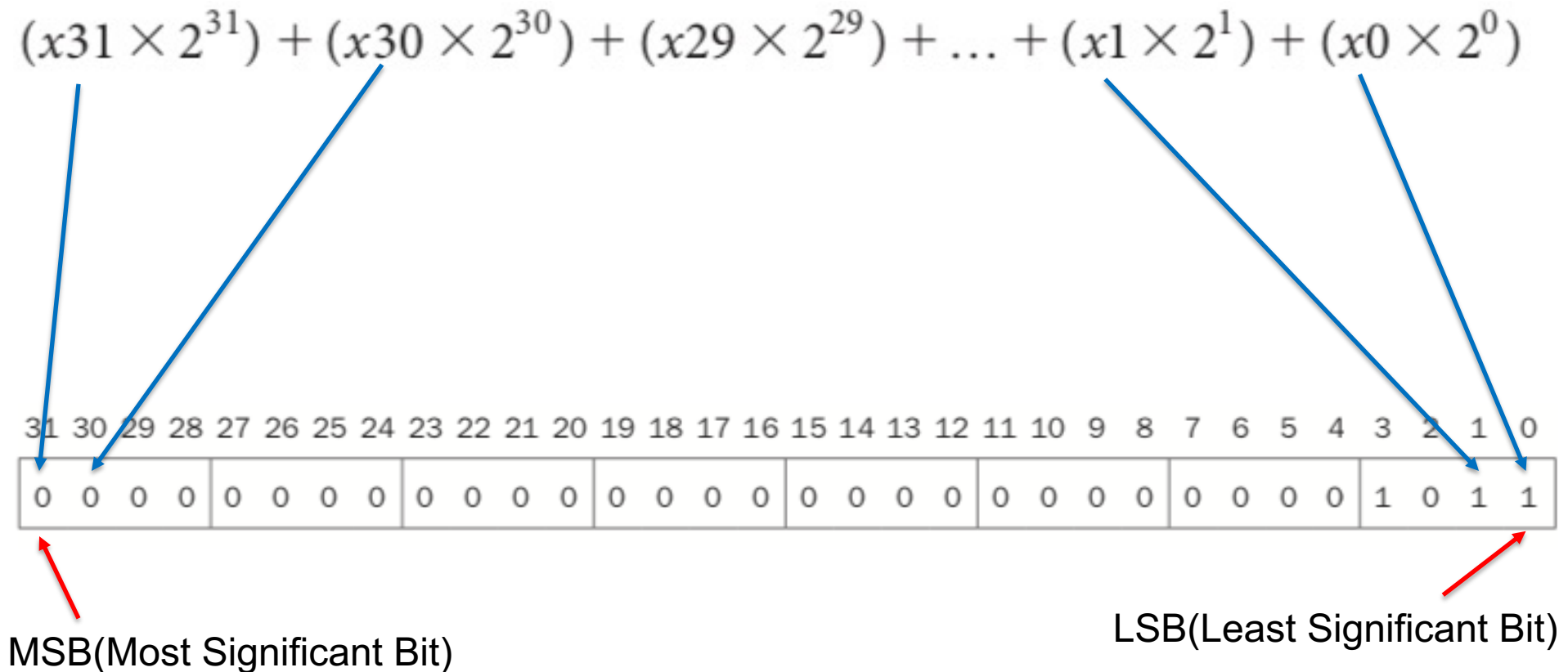
- $i$ 번째 자리수가  $d_i$ 이면  $d_i \times 2^i$ 를 의미 ( $i$ 는 0부터 시작)
- $d_i$ 의 범위는 0 ~ 1

$$\begin{aligned} 1011_2 &= 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\ &= 8 + 0 + 2 + 1 = 11_{10} \end{aligned}$$



# Unsigned Binary Numbers

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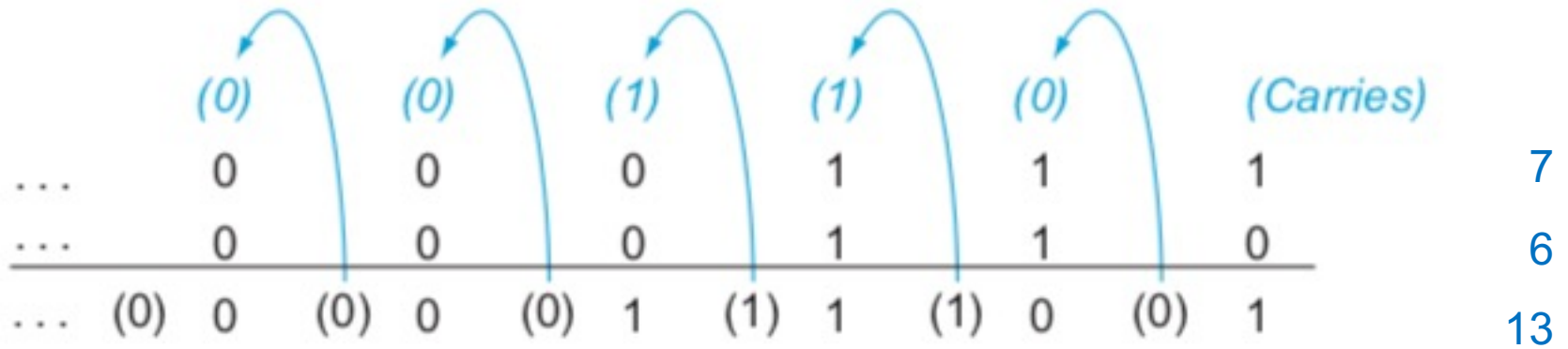
# unsigned 32-bit 2진수

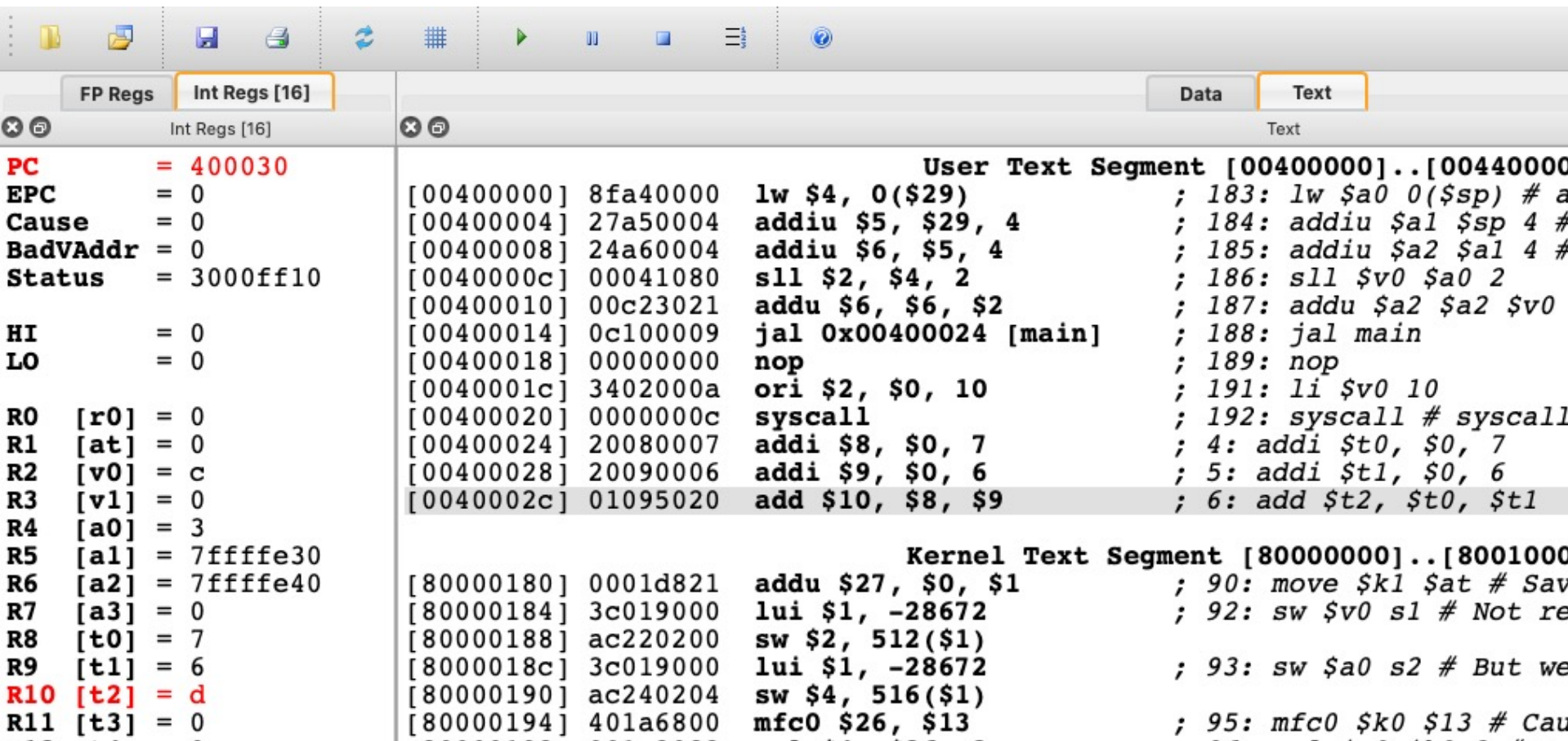
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0000	0000	0000	0000	0000	0000	0000	0000	$_{\text{two}}$	=	$0_{\text{ten}}$
0000	0000	0000	0000	0000	0000	0000	0001	$_{\text{two}}$	=	$1_{\text{ten}}$
0000	0000	0000	0000	0000	0000	0000	0010	$_{\text{two}}$	=	$2_{\text{ten}}$
...									...	
1111	1111	1111	1111	1111	1111	1111	1101	$_{\text{two}}$	=	$4,294,967,293_{\text{ten}}$
1111	1111	1111	1111	1111	1111	1111	1110	$_{\text{two}}$	=	$4,294,967,294_{\text{ten}}$
1111	1111	1111	1111	1111	1111	1111	1111	$_{\text{two}}$	=	$4,294,967,295_{\text{ten}}$

# unsigned binary number 의 덧셈 : 7+6

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## 2.4 Signed and Unsigned Numbers

Category	Instruction	Example	Meaning	Comments
Arithmetic	add	add \$s1,\$s2,\$s3	$\$s1 = \$s2 + \$s3$	Three operands; overflow detected
	subtract	sub \$s1,\$s2,\$s3	$\$s1 = \$s2 - \$s3$	Three operands; overflow detected
	add immediate	addi \$s1,\$s2,100	$\$s1 = \$s2 + 100$	+ constant; overflow detected
	add unsigned	addu \$s1,\$s2,\$s3	$\$s1 = \$s2 + \$s3$	Three operands; overflow undetected
	subtract unsigned	subu \$s1,\$s2,\$s3	$\$s1 = \$s2 - \$s3$	Three operands; overflow undetected
	add immediate unsigned	addiu \$s1,\$s2,100	$\$s1 = \$s2 + 100$	+ constant; overflow undetected
	move from coprocessor register	mfc0 \$s1,\$epc	$\$s1 = \$epc$	Copy Exception PC + special regs
	multiply	mult \$s2,\$s3	Hi, Lo = $\$s2 \times \$s3$	64-bit signed product in Hi, Lo
	multiply unsigned	multu \$s2,\$s3	Hi, Lo = $\$s2 \times \$s3$	64-bit unsigned product in Hi, Lo
	divide	div \$s2,\$s3	Lo = $\$s2 / \$s3$ , Hi = $\$s2 \bmod \$s3$	Lo = quotient, Hi = remainder
	divide unsigned	divu \$s2,\$s3	Lo = $\$s2 / \$s3$ , Hi = $\$s2 \bmod \$s3$	Unsigned quotient and remainder
	move from Hi	mfhi \$s1	$\$s1 = \text{Hi}$	Used to get copy of Hi
	move from Lo	mflo \$s1	$\$s1 = \text{Lo}$	Used to get copy of Lo

$\$t0 = \$t1 - 1$  은 MIPS instruction 으로 어떻게?

there is no 'subi' instruction.



---

```
addi $t0, $t1, -1
```

# MIPS source file

---

```
.text
```

```
.globl main
```

```
main:
```

```
    addi $t1, $0, -2
```

```
    addi $t0, $t1, -1
```

FP Regs		Int Regs [16]		Data		Text	
FP Regs		Int Regs [16]		Data		Text	
PC	= 0			<b>User Text Segment [00400000]..[00440000]</b>			
EPC	= 0	[00400000]	8fa40000	lw \$4, 0(\$29)		; 183: lw \$a0 0(\$sp) # argc	
Cause	= 0	[00400004]	27a50004	addiu \$5, \$29, 4		; 184: addiu \$a1 \$sp 4 # argv	
BadVAddr	= 0	[00400008]	24a60004	addiu \$6, \$5, 4		; 185: addiu \$a2 \$a1 4 # envp	
Status	= 3000ff10	[0040000c]	00041080	sll \$2, \$4, 2		; 186: sll \$v0 \$a0 2	
		[00400010]	00c23021	addu \$6, \$6, \$2		; 187: addu \$a2 \$a2 \$v0	
HI	= 0	[00400014]	0c100009	jal 0x00400024 [main]		; 188: jal main	
LO	= 0	[00400018]	00000000	nop		; 189: nop	
		[0040001c]	3402000a	ori \$2, \$0, 10		; 191: li \$v0 10	
R0 [r0]	= 0	[00400020]	0000000c	syscall		; 192: syscall # syscall 10 (ex	
R1 [at]	= 0	[00400024]	2009ffffe	addi \$9, \$0, -2		; 4: addi \$t1, \$0, -2	
R2 [v0]	= 0	[00400028]	2128ffff	addi \$8, \$9, -1		; 5: addi \$t0, \$t1, -1	
R3 [v1]	= 0			<b>Kernel Text Segment [80000000]..[80010000]</b>			
R4 [a0]	= 0	[80000180]	0001d821	addu \$27, \$0, \$1		; 90: move \$k1 \$at # Save \$at	
R5 [a1]	= 0	[80000184]	3c019000	lui \$1, -28672		; 92: sw \$v0 s1 # Not re-entran	
R6 [a2]	= 7ffffe48	[80000188]	ac220200	sw \$2, 512(\$1)			
R7 [a3]	= 0	[8000018c]	3c019000	lui \$1, -28672		; 93: sw \$a0 s2 # But we need t	
R8 [t0]	= 0	[80000190]	ac240204	sw \$4, 516(\$1)			
R9 [t1]	= 0	[80000194]	401a6800	mfc0 \$26, \$13		; 95: mfc0 \$k0 \$13 # Cause regi	
R10 [t2]	= 0	[80000198]	001a2082	srl \$4, \$26, 2		; 96: srl \$a0 \$k0 2 # Extract E	
R11 [t3]	= 0	[8000019c]	3084001f	andi \$4, \$4, 31		; 97: andi \$a0 \$a0 0x1f	
R12 [t4]	= 0	[800001a0]	34020004	ori \$2, \$0, 4		; 101: li \$v0 4 # syscall 4 (pr	
R13 [t5]	= 0	[800001a4]	3c049000	lui \$4, -28672 [ m1 ]		; 102: la \$a0 m1	

The screenshot displays the Immunity Debugger interface. On the left, the 'CPU Registers' window is open, showing the state of various registers. The 'PC' (Program Counter) is highlighted in red and set to 400028. The 'R9' register, labeled as 't1', is also highlighted in red and set to ffffffff. The 'Cause' register is 0, 'BadVAddr' is 0, and 'Status' is 3000fff0. Other registers like 'HI', 'LO', 'R0', 'R1', 'R2', 'R3', 'R4', 'R5', 'R6', 'R7', 'R10', 'R11', 'R12', 'R13', and 'R14' are also shown with their respective values.

The main window displays the disassembly of the 'User Text Segment' starting at address 00400000. The assembly code is as follows:

```

[00400000] 8fa40000  lw $4, 0($29)
[00400004] 27a50004  addiu $5, $29, 4
[00400008] 24a60004  addiu $6, $5, 4
[0040000c] 00041080  sll $2, $4, 2
[00400010] 00c23021  addu $6, $6, $2
[00400014] 0c100009  jal 0x00400024 [main]
[00400018] 00000000  nop
[0040001c] 3402000a  ori $2, $0, 10
[00400020] 0000000c  syscall
[00400024] 2009ffff  addi $9, $0, -2
[00400028] 2128ffff  addi $8, $9, -1

```

The instruction 'addi \$8, \$9, -1' at address 00400028 is highlighted in blue. Below this, the 'Kernel Text Segment' starting at address 80000000 is visible, showing instructions like 'addu \$27, \$0, \$1', 'lui \$1, -28672', 'sw \$2, 512(\$1)', 'lui \$1, -28672', 'sw \$4, 516(\$1)', 'mfc0 \$26, \$13', 'srl \$4, \$26, 2', 'andi \$4, \$4, 31', 'ori \$2, \$0, 4', and 'lui \$4, -28672 [ m1 ]'.

# How to represent negative numbers

---

```
addi $t0, $t1, -1
```

# unsigned 32-bit 2진수

---

0000	0000	0000	0000	0000	0000	0000	0000	$_{\text{two}}$	=	$0_{\text{ten}}$
0000	0000	0000	0000	0000	0000	0000	0001	$_{\text{two}}$	=	$1_{\text{ten}}$
0000	0000	0000	0000	0000	0000	0000	0010	$_{\text{two}}$	=	$2_{\text{ten}}$
...										...
1111	1111	1111	1111	1111	1111	1111	1101	$_{\text{two}}$	=	$4,294,967,293_{\text{ten}}$
1111	1111	1111	1111	1111	1111	1111	1110	$_{\text{two}}$	=	$4,294,967,294_{\text{ten}}$
1111	1111	1111	1111	1111	1111	1111	1111	$_{\text{two}}$	=	$4,294,967,295_{\text{ten}}$

# signed 32-bit 2진수 in 2's complement representation

sign bit

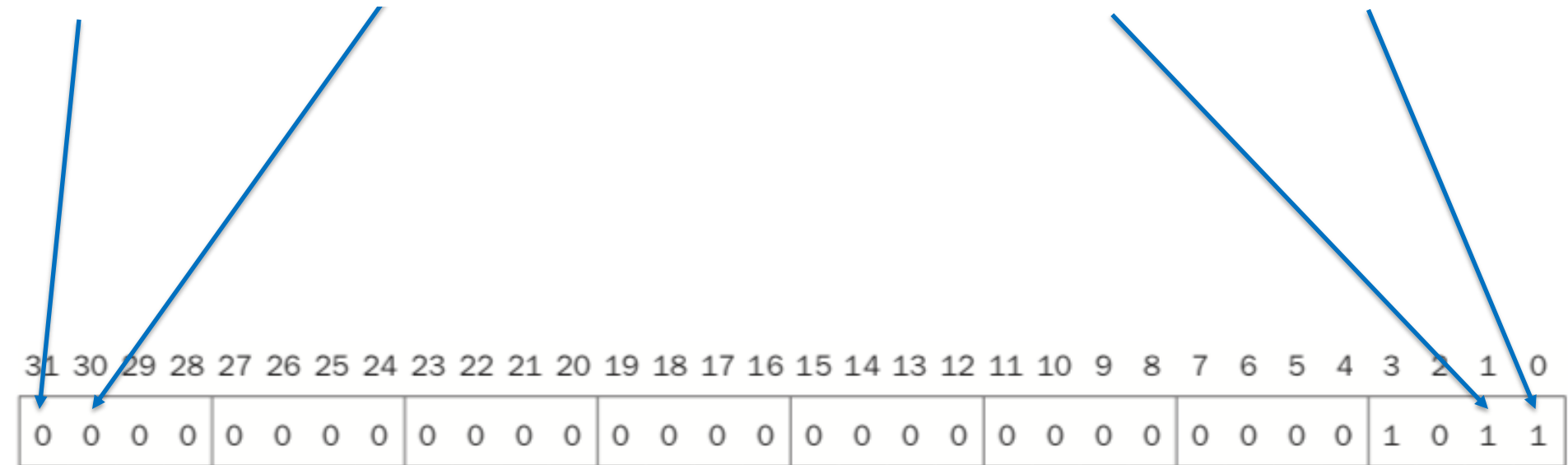


0000	0000	0000	0000	0000	0000	0000	0000	$_{\text{two}}$	$= 0_{\text{ten}}$
0000	0000	0000	0000	0000	0000	0000	0001	$_{\text{two}}$	$= 1_{\text{ten}}$
0000	0000	0000	0000	0000	0000	0000	0010	$_{\text{two}}$	$= 2_{\text{ten}}$
...									...
0111	1111	1111	1111	1111	1111	1111	1101	$_{\text{two}}$	$= 2,147,483,645_{\text{ten}}$
0111	1111	1111	1111	1111	1111	1111	1110	$_{\text{two}}$	$= 2,147,483,646_{\text{ten}}$
0111	1111	1111	1111	1111	1111	1111	1111	$_{\text{two}}$	$= 2,147,483,647_{\text{ten}}$
1000	0000	0000	0000	0000	0000	0000	0000	$_{\text{two}}$	$= -2,147,483,648_{\text{ten}}$
1000	0000	0000	0000	0000	0000	0000	0001	$_{\text{two}}$	$= -2,147,483,647_{\text{ten}}$
1000	0000	0000	0000	0000	0000	0000	0010	$_{\text{two}}$	$= -2,147,483,646_{\text{ten}}$
...									...
1111	1111	1111	1111	1111	1111	1111	1101	$_{\text{two}}$	$= -3_{\text{ten}}$
1111	1111	1111	1111	1111	1111	1111	1110	$_{\text{two}}$	$= -2_{\text{ten}}$
1111	1111	1111	1111	1111	1111	1111	1111	$_{\text{two}}$	$= -1_{\text{ten}}$

# Signed Binary Numbers

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$$(x_{31} \times -2^{31}) + (x_{30} \times 2^{30}) + (x_{29} \times 2^{29}) + \dots + (x_1 \times 2^1) + (x_0 \times 2^0)$$



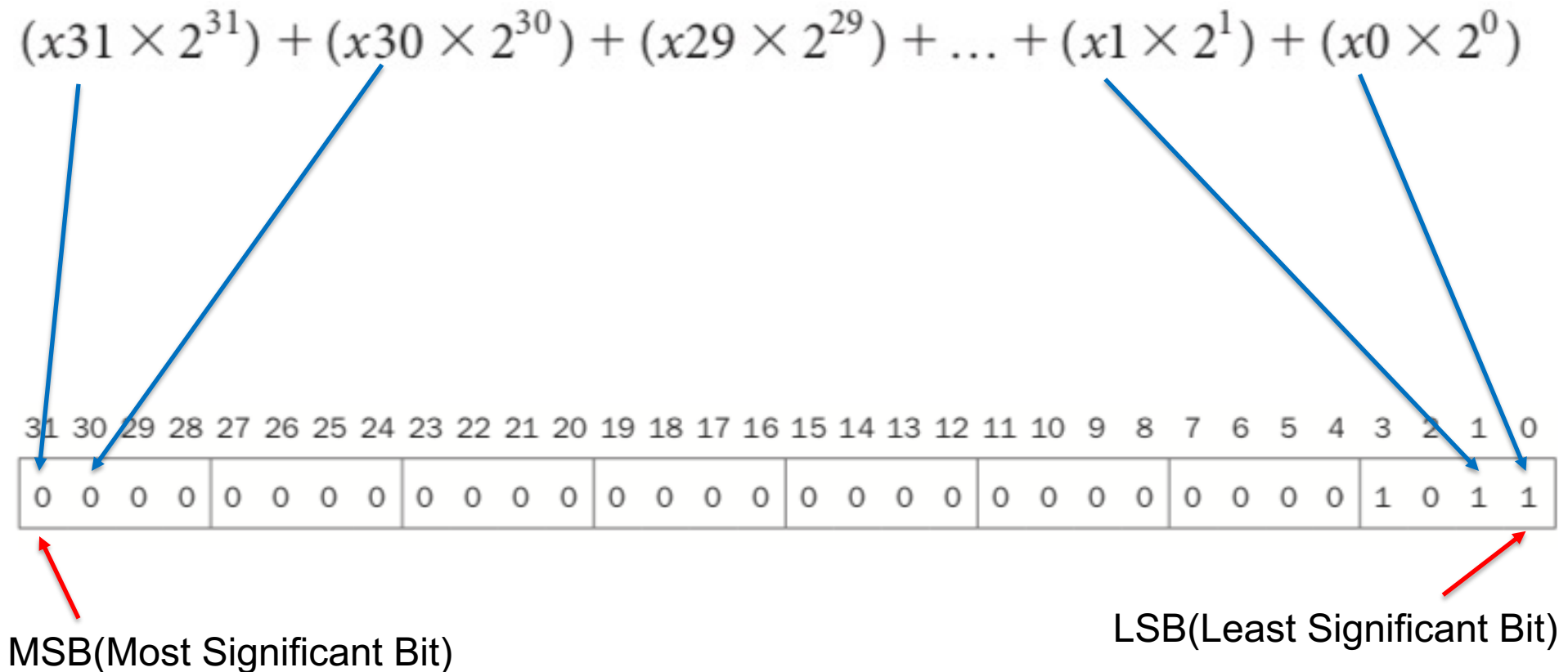
MSB(Most Significant Bit)

LSB(Least Significant Bit)



# Unsigned Binary Numbers

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# Signed Binary Number (2의 보수 표현)

---

What is the decimal value of this 32-bit two's complement number?

1111 1111 1111 1111 1111 1111 1111 1100<sub>two</sub>


Substituting the number's bit values into the formula above:

$$(1 \times -2^{31}) + (1 \times 2^{30}) + (1 \times 2^{29}) + \dots + (1 \times 2^2) + (0 \times 2^1) + (0 \times 2^0) \\ = -2^{31} + 2^{30} + 2^{29} + \dots + 2^2 + 0 + 0$$

$$= -2^{31} + \sum_{i=2}^{30} 2^i = -2^{31} + 4(2^{29} - 1)$$

$$= -2^{31} + 2^{31} - 4$$

$$= -4$$

$$\sum_{k=1}^n a_k = \frac{a(1 - r^n)}{1 - r}$$


**\*\* 2진수를 10진수로 바꾸는 것은 사람이 이해하기 위한  
것이지 컴퓨터는 10진수로 바꾸어 볼 필요가 없습니다.**

## signed 32-bit 2진수 in 2's complement representation

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-2 를 표현하려면 +2 의 2의 보수를 만든다.

1) +2 를 2진수로 표현한다.  $\rightarrow a$

$$2_{\text{ten}} = 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0010_{\text{two}}$$

2) a 의 1의 보수를 만든다.  $\rightarrow b$

$$1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1101_{\text{two}}$$

3) b 에 1을 더한다.  $\rightarrow c$  는 -2

$$\begin{array}{r} 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1101_{\text{two}} \\ + \phantom{1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ } 1_{\text{two}} \\ \hline = 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1110_{\text{two}} \\ = -2_{\text{ten}} \end{array}$$

## signed 32-bit 2진수 in 2's complement representation

---

$a = 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111_2$

은 십진수로 얼마인가?

1) 음수이면  $a$ 의 1의 보수를 만든다  $\rightarrow b$

$b = 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000$

2)  $b$ 에 1을 더한다.  $\rightarrow c$ 가  $a$ 의 절대값이다.

$c = 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0001$

3)  $a$ 는 십진수로  $-c$

그러므로  $a$ 는 십진수로  $-1$

\*) 양수이면?

$d = 0111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111_2$

$$= -0 \times 2^{31} + \sum_{i=0}^{30} 2^i = 2^{31} - 1 = 2147483647$$

# Register 값들을 hex 로 보여줌

The screenshot shows a debugger interface with two main panes. The left pane displays the state of various registers, and the right pane displays assembly code for two segments: User Text Segment and Kernel Text Segment.

**Register Values:**

Register	Value
PC	400030
EPC	0
Cause	0
BadVAddr	0
Status	3000ff10
HI	0
LO	0
R0 [r0]	0
R1 [at]	0
R2 [v0]	c
R3 [v1]	0
R4 [a0]	3
R5 [a1]	7ffffe30
R6 [a2]	7ffffe40
R7 [a3]	0
R8 [t0]	fffffffe
R9 [t1]	fffffff
R10 [t2]	fffffff
R11 [t3]	0

**Assembly Code:**

**User Text Segment [00400000]..[00400000]**

Address	Hex	Assembly	Comment
[00400000]	8fa40000	lw \$4, 0(\$29)	; 183: lw \$a0 0(\$sp) # argc
[00400004]	27a50004	addiu \$5, \$29, 4	; 184: addiu \$a1 \$sp 4 # argv
[00400008]	24a60004	addiu \$6, \$5, 4	; 185: addiu \$a2 \$a1 4 # envp
[0040000c]	00041080	sll \$2, \$4, 2	; 186: sll \$v0 \$a0 2
[00400010]	00c23021	addu \$6, \$6, \$2	; 187: addu \$a2 \$a2 \$v0
[00400014]	0c100009	jal 0x00400024 [main]	; 188: jal main
[00400018]	00000000	nop	; 189: nop
[0040001c]	3402000a	ori \$2, \$0, 10	; 191: li \$v0 10
[00400020]	0000000c	syscall	; 192: syscall # syscall 10 (
[00400024]	2008ffffe	addi \$8, \$0, -2	; 4: addi \$t0, \$0, -2
[00400028]	2009fffff	addi \$9, \$0, -1	; 5: addi \$t1, \$0, 0xffffffff
[0040002c]	200afffff	addi \$10, \$0, -1	; 6: addi \$t2, \$0, -1

**Kernel Text Segment [80000000]..[80010000]**

Address	Hex	Assembly	Comment
[80000180]	0001d821	addu \$27, \$0, \$1	; 90: move \$k1 \$at # Save \$at
[80000184]	3c019000	lui \$1, -28672	; 92: sw \$v0 s1 # Not re-entra
[80000188]	ac220200	sw \$2, 512(\$1)	
[8000018c]	3c019000	lui \$1, -28672	; 93: sw \$a0 s2 # But we need
[80000190]	ac240204	sw \$4, 516(\$1)	
[80000194]	401a6800	mfc0 \$26, \$13	; 95: mfc0 \$k0 \$13 # Cause re

# Register 값들을 decimal 로 보여줌

The screenshot shows a debugger interface with two main panes. The left pane displays the state of various registers, and the right pane displays assembly code for two different segments.

**Register Values (Left Pane):**

Register	Value
PC	4194352
EPC	0
Cause	0
BadVAddr	0
Status	805371664
HI	0
LO	0
R0 [r0]	0
R1 [at]	0
R2 [v0]	12
R3 [v1]	0
R4 [a0]	3
R5 [a1]	2147483184
R6 [a2]	2147483200
R7 [a3]	0
R8 [t0]	-2
R9 [t1]	-1
R10 [t2]	-1
R11 [t3]	0

**Assembly Code (Right Pane):**

**User Text Segment [00400000]..[00440000]**

Address	Hex	Assembly	Comment
[00400000]	8fa40000	lw \$4, 0(\$29)	; 183: lw \$a0 0(\$sp) # argc
[00400004]	27a50004	addiu \$5, \$29, 4	; 184: addiu \$a1 \$sp 4 # argv
[00400008]	24a60004	addiu \$6, \$5, 4	; 185: addiu \$a2 \$a1 4 # envp
[0040000c]	00041080	sll \$2, \$4, 2	; 186: sll \$v0 \$a0 2
[00400010]	00c23021	addu \$6, \$6, \$2	; 187: addu \$a2 \$a2 \$v0
[00400014]	0c100009	jal 0x00400024 [main]	; 188: jal main
[00400018]	00000000	nop	; 189: nop
[0040001c]	3402000a	ori \$2, \$0, 10	; 191: li \$v0 10
[00400020]	0000000c	syscall	; 192: syscall # syscall 10 (
[00400024]	2008ffffe	addi \$8, \$0, -2	; 4: addi \$t0, \$0, -2
[00400028]	2009fffff	addi \$9, \$0, -1	; 5: addi \$t1, \$0, 0xffffffff
[0040002c]	200afffff	addi \$10, \$0, -1	; 6: addi \$t2, \$0, -1

**Kernel Text Segment [80000000]..[80010000]**

Address	Hex	Assembly	Comment
[80000180]	0001d821	addu \$27, \$0, \$1	; 90: move \$k1 \$at # Save \$at
[80000184]	3c019000	lui \$1, -28672	; 92: sw \$v0 s1 # Not re-entra
[80000188]	ac220200	sw \$2, 512(\$1)	
[8000018c]	3c019000	lui \$1, -28672	; 93: sw \$a0 s2 # But we need
[80000190]	ac240204	sw \$4, 516(\$1)	
[80000194]	401a6800	mfc0 \$26, \$13	; 95: mfc0 \$k0 \$13 # Cause re

# Register 값들을 binary 로 보여줌

The screenshot shows a debugger interface with two main panels. The left panel, titled 'Int Regs [2]', displays the values of various registers in binary. The right panel, titled 'Text', displays assembly code for the 'User Text Segment' and 'Kernel Text Segment'.

**Int Regs [2] Panel:**

- PC = 10000000000000000110000
- EPC = 0
- Cause = 0
- BadVAddr = 0
- Status = 1100000000000001111111100010000
- HI = 0
- LO = 0
- R0 [r0] = 0
- R1 [at] = 0
- R2 [v0] = 1100
- R3 [v1] = 0
- R4 [a0] = 11
- R5 [a1] = 11111111111111111111000110000
- R6 [a2] = 111111111111111111111001000000
- R7 [a3] = 0
- R8 [t0] = 11111111111111111111111111111110
- R9 [t1] = 11111111111111111111111111111111
- R10 [t2] = 11111111111111111111111111111111
- R11 [t3] = 0

**Text Panel:**

**User Text Segment [00400000]..[00400000]**

- [00400000] 8fa40000 lw \$4, 0(\$29) ; 183: lw
- [00400004] 27a50004 addiu \$5, \$29, 4 ; 184: ad
- [00400008] 24a60004 addiu \$6, \$5, 4 ; 185: ad
- [0040000c] 00041080 sll \$2, \$4, 2 ; 186: sl
- [00400010] 00c23021 addu \$6, \$6, \$2 ; 187: ad
- [00400014] 0c100009 jal 0x00400024 [main] ; 188: ja
- [00400018] 00000000 nop ; 189: no
- [0040001c] 3402000a ori \$2, \$0, 10 ; 191: li
- [00400020] 0000000c syscall ; 192: sy
- [00400024] 2008ffff addi \$8, \$0, -2 ; 4: addi
- [00400028] 2009ffff addi \$9, \$0, -1 ; 5: addi
- [0040002c] 200affff addi \$10, \$0, -1 ; 6: addi

**Kernel Text Segment [80000000]..[80000000]**

- [80000180] 0001d821 addu \$27, \$0, \$1 ; 90: mov
- [80000184] 3c019000 lui \$1, -28672 ; 92: sw
- [80000188] ac220200 sw \$2, 512(\$1)
- [8000018c] 3c019000 lui \$1, -28672 ; 93: sw

and we can't trust \$sp  
use these registers