데이터 과학

L03: Regression

Kookmin University

지도학습 Supervised Learning

훈련 데이터(Training Data)로부터 하나의 함수를 유추해내기 위한 기계 학습(Machine Learning)의 한 방법

Training Data

```
[1.2,  3.8, -1.4, …,  4.1] →  1.1
[3.2, -1.2, -0.2, …,  2.1] →  2.7
[2.8, -1.4, -0.3, …,  2.3] →  2.8
[1.2,  3.4, -1.5, …,  4.2] →  0.9
[4.2,  2.1,  2.8, …, -0.5] → -0.1
                    …
[3.2,  2.2,  2.2, …, -0.4] → -0.2
```

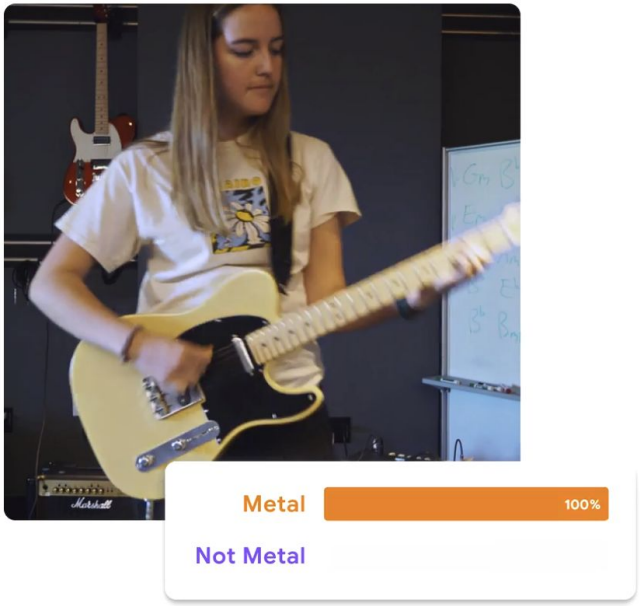
Test

```
[1.3,  3.2, -1.5, …,  4.1] → ?
```

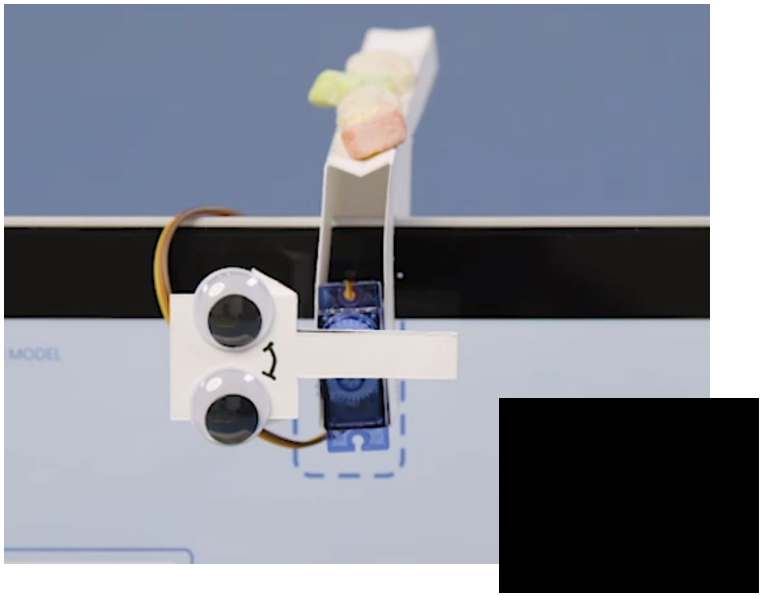
지도학습 예제: Teachable Machine

https://teachablemachine.withgoogle.com/

누구나 쉽게 **지도학습**을 할 수 있도록 구글에서 만든 온라인 서비스
이미지, 소리, 자세 등을 입력으로 받아서 분류작업Classification 수행



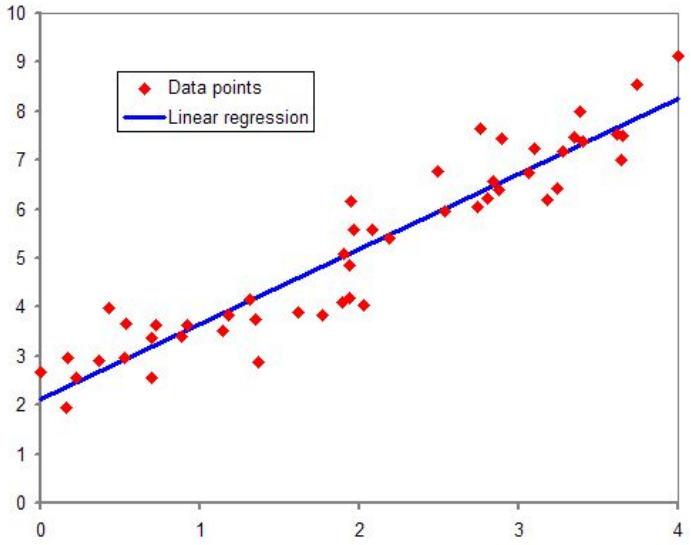
https://experiments.withgoogle.com/tiny-sorter/view



회귀분석 (Regression)

관찰된 연속형 변수들에 대해 **두 변수 사이의 모형**을 구한뒤 **적합도를 측정**해 내는 분석 방법 (출처: 위키피디아)

선형회귀분석 (Linear Regression): 종속 변수 y와 한 개 이상의 **독립 변수 (또는 설명 변수) X**와의 **선형 상관 관계를 모델링**하는 회귀분석 기법 (출처: 위키피디아)



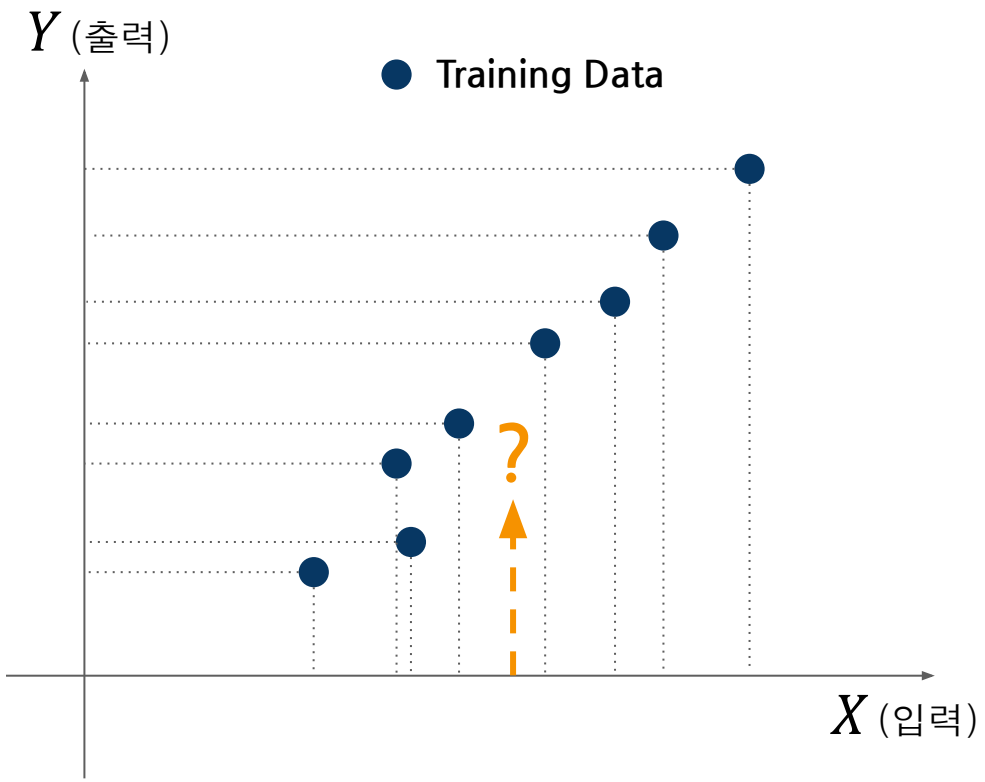
Linear Regression

Training Data

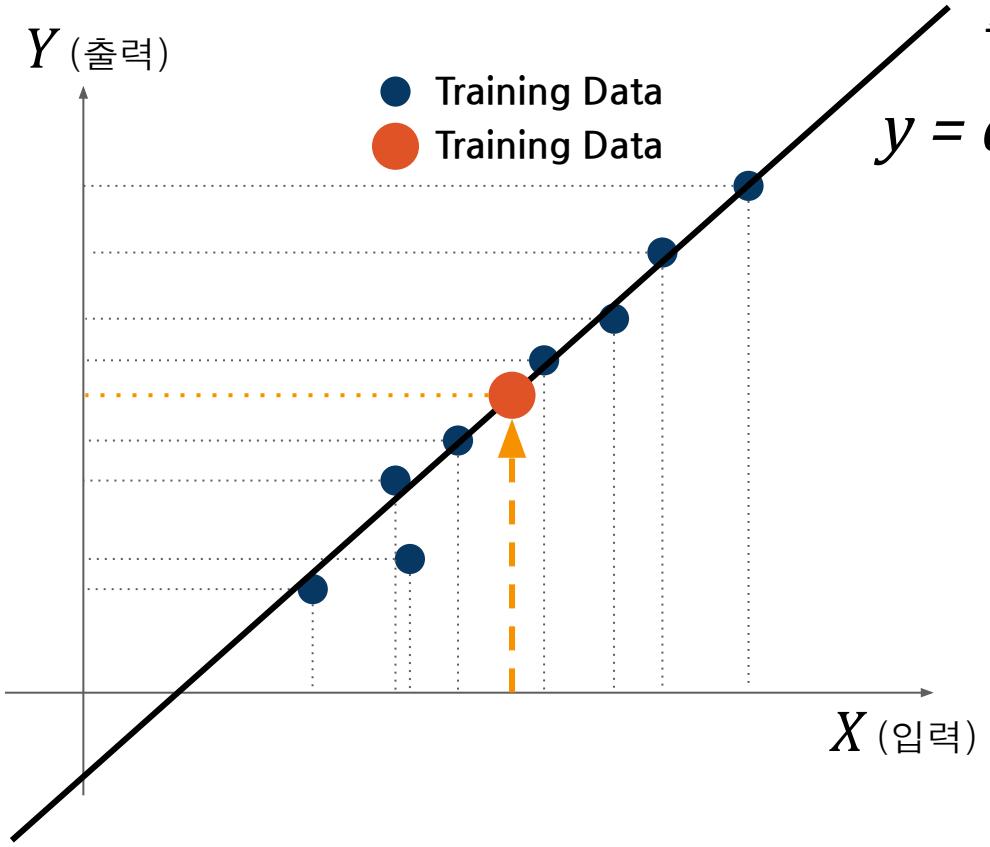
```
[1.2] →  1.1
[3.2] →  2.7
[2.8] →  2.8
[1.2] →  0.9
[4.2] → -0.1
   …
[3.2] → -0.2
```

Test

```
[1.3] →  ?
```



Linear Regression



이 **선**만 구할 수 있으면,
어떤 **입력**이 들어와도
출력을 **예측**할 수 있다!

$y = ax+c$

선을 구한다
= a 값과 c 값을 구한다

어떻게 구하지..?

Linear Regression

입력이 복잡한 지도학습…

Training Data

```
[1.2,  3.8, -1.4, …,  4.1] →  1.1
[3.2, -1.2, -0.2, …,  2.1] →  2.7
[2.8, -1.4, -0.3, …,  2.3] →  2.8
[1.2,  3.4, -1.5, …,  4.2] →  0.9
[4.2,  2.1,  2.8, …, -0.5] → -0.1
                …
[3.2,  2.2,  2.2, …, -0.4] → -0.2
```

Test

```
[1.3,  3.2, -1.5, …,  4.1] → ?
```

쉬운 설명을 위해…
단순한 예제.

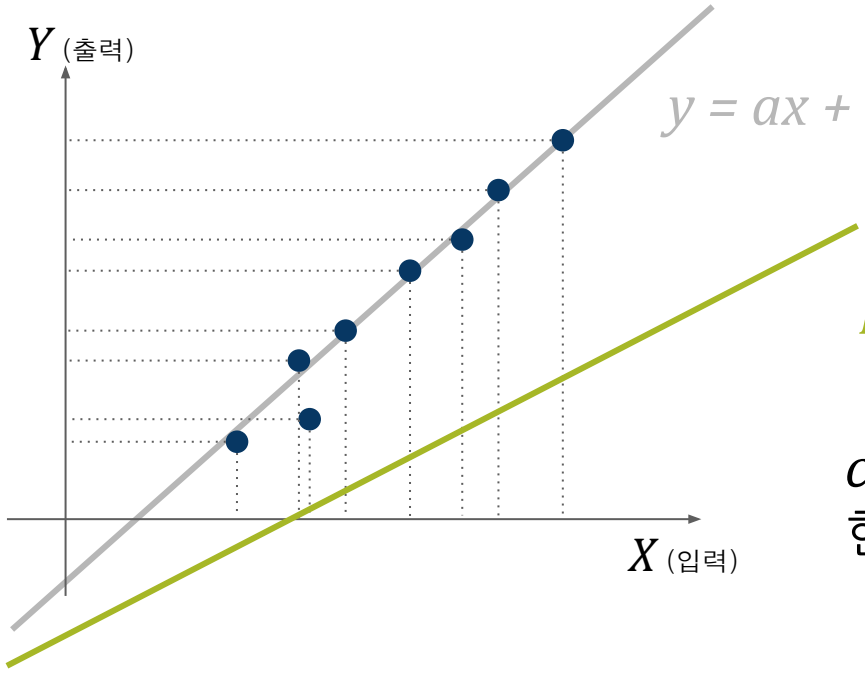
Training Data

```
[1.2] →  1.1
[3.2] →  2.7
[2.8] →  2.8
[1.2] →  0.9
[4.2] → -0.1
   …
[3.2] → -0.2
```

Test

```
[1.3] → ?
```

가설과 비용 Hypothesis and Cost function



가설Hypothesis

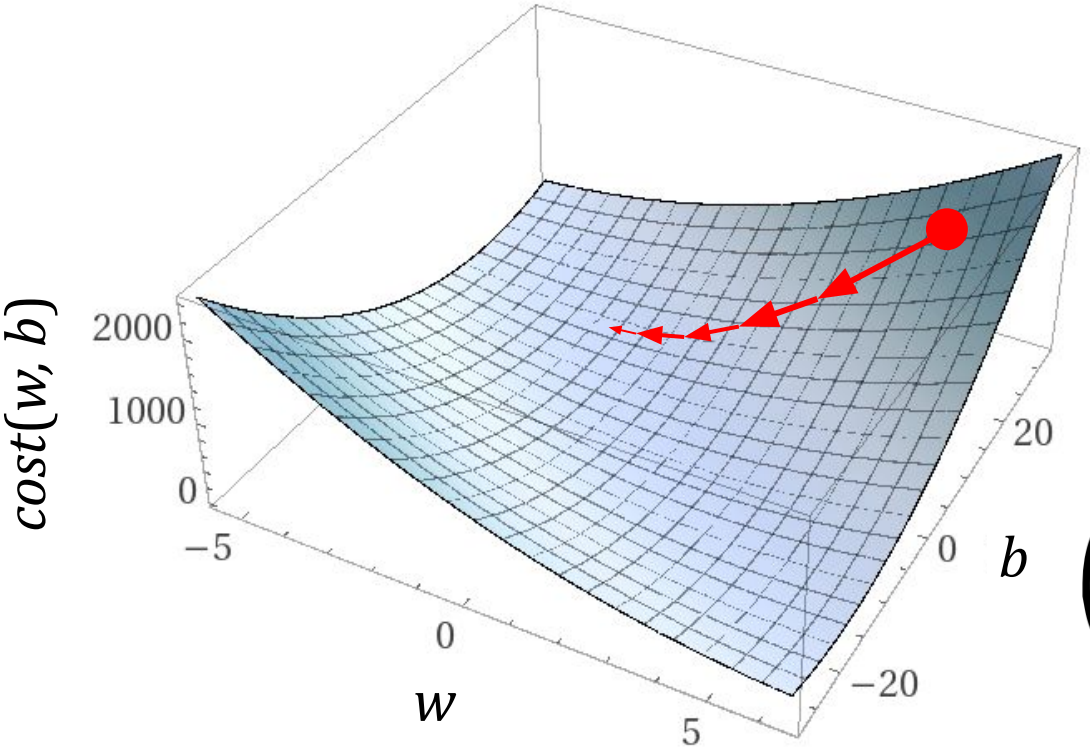
$H(x) = wx + b$

$cost(w,b)$:
현재 가설은 얼마나 잘못되었는가?

경사 하강법 Gradient Descent

우리의 목표: cost를 최소화 하자! = cost를 최소로 만드는 w, b 값을 찾자!

$$\arg\min_{w,b} cost(w,b)$$



경사 따라 내려가야하는데,
경사는 어떻게 구하지?
⇒ **편미분** 이용

경사:

$$\left(\frac{\partial cost(w,b)}{\partial w}, \frac{\partial cost(w,b)}{\partial b} \right)$$

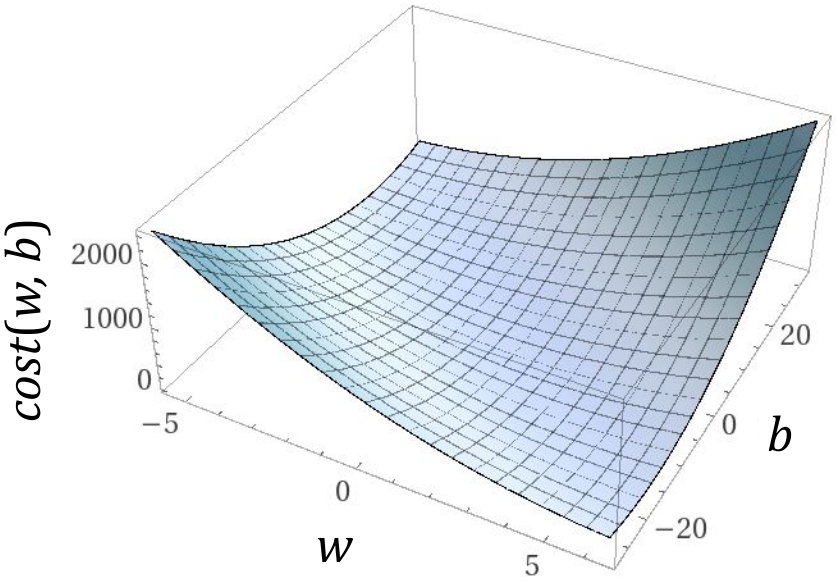
경사 하강법 Gradient Descent

경사: $\left(\frac{\partial cost(w,b)}{\partial w}, \frac{\partial cost(w,b)}{\partial b} \right)$

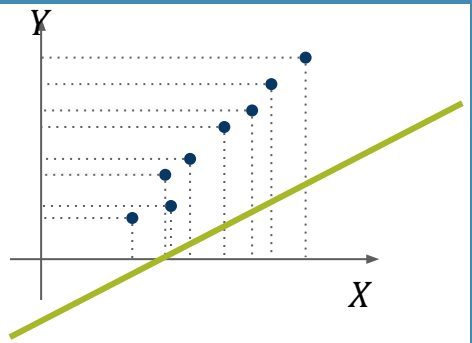
업데이트: Learning Rate

$$w = w - \alpha \frac{\partial cost(w,b)}{\partial w}$$

$$b = b - \alpha \frac{\partial cost(w,b)}{\partial b}$$



Linear Regression (2)



입력이 조금 더 복잡할 때?

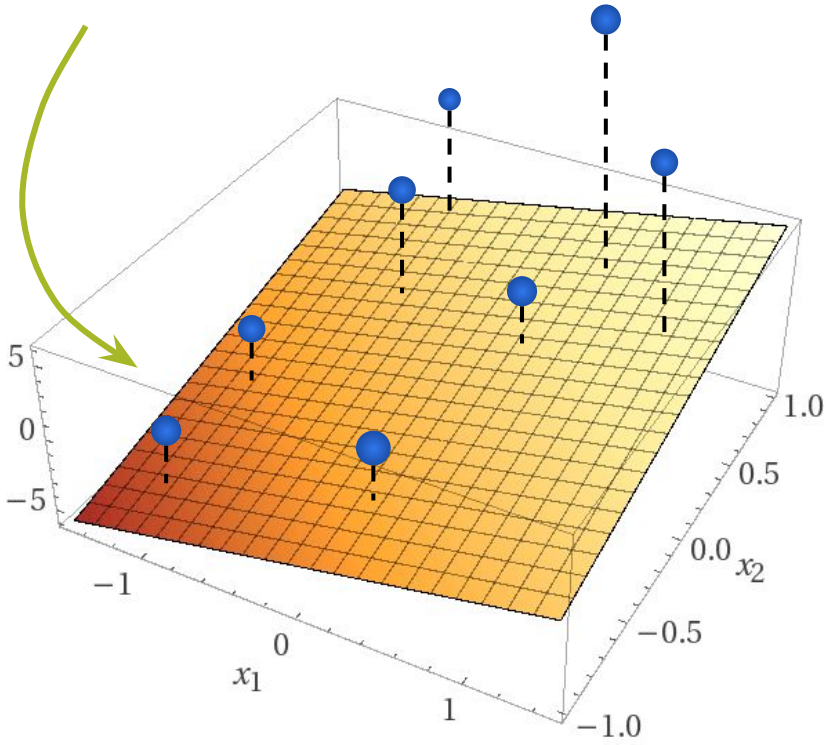
Training Data

```
[1.2,  3.8] →  1.1
[3.2, -1.2] →  2.7
[2.8, -1.4] →  2.8
[1.2,  3.4] →  0.9
[4.2,  2.1] → -0.1
          …
[3.2,  2.2] → -0.2
```

Test

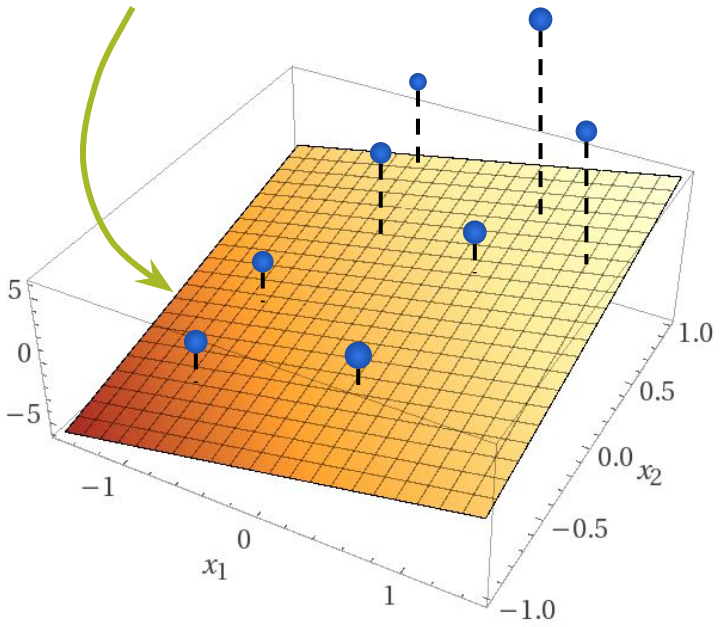
```
[1.3,  3.2] → ?
```

$$H(x_1, x_2) = w_1 x_1 + w_2 x_2 + b$$



가설과 비용 (2) Hypothesis and Cost function (2)

$H(x_1, x_2) = w_1x_1 + w_2x_2 + b$



$$\mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$H(x_1, x_2) = H(\mathbf{x}) = \mathbf{w}^\mathsf{T}\mathbf{x} + b$$

$$cost(\mathbf{w}, b) = \frac{1}{n}\sum_{i=0}^{n}\left(y_i - H(\mathbf{x}_i)\right)^2$$

경사 하강법 (2) Gradient Descent (2)

우리의 목표: cost를 최소화 하자! = cost를 최소로 만드는 w, b 값을 찾자!

$$\arg\min_{\mathbf{w},b} cost(\mathbf{w}, b)$$

업데이트:

$$w_1 = w_1 - \alpha \frac{\partial cost(\mathbf{w},b)}{\partial w_1}$$

$$w_2 = w_2 - \alpha \frac{\partial cost(\mathbf{w},b)}{\partial w_2}$$

$$b = b - \alpha \frac{\partial cost(\mathbf{w},b)}{\partial b}$$

$$\mathbf{w} = \mathbf{w} - \alpha \frac{\partial cost(\mathbf{w},b)}{\partial \mathbf{w}}$$

Learning Rate

경사 (Gradient)

Linear Regression (3)

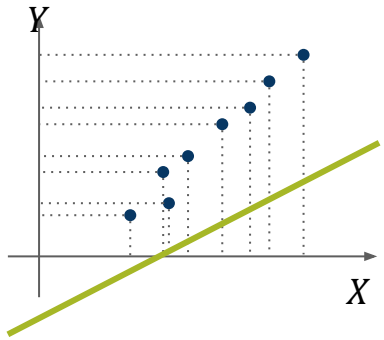
입력이 조금 더 더 복잡할 때?

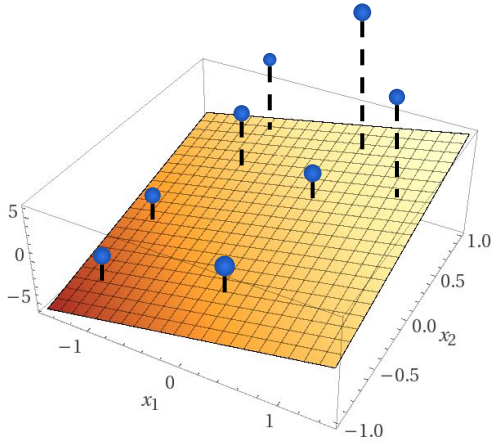
Training Data

```
[1.2,  3.8, -1.4, …,  4.1] →  1.1
[3.2, -1.2, -0.2, …,  2.1] →  2.7
[2.8, -1.4, -0.3, …,  2.3] →  2.8
[1.2,  3.4, -1.5, …,  4.2] →  0.9
[4.2,  2.1,  2.8, …, -0.5] → -0.1
                   …
[3.2,  2.2,  2.2, …, -0.4] → -0.2
```

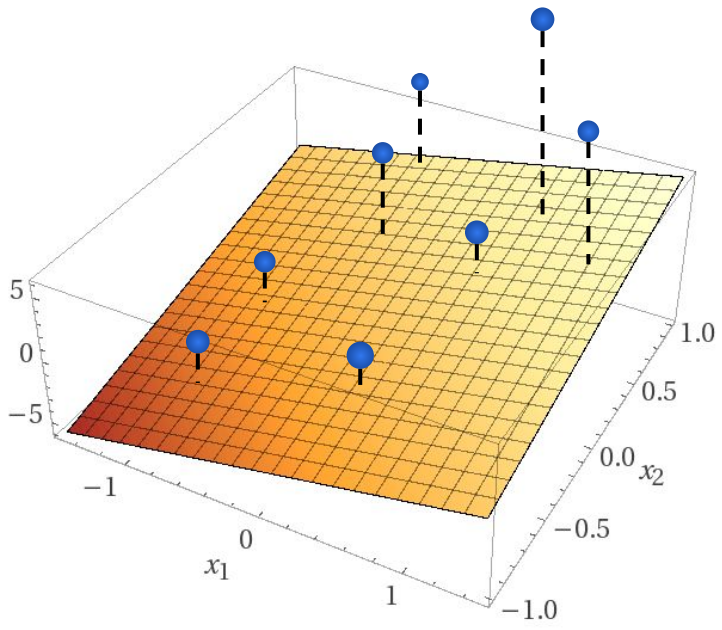
Test

```
[1.3,  3.2, -1.5, …,  4.1] → ?
```





가설과 비용 (3) Hypothesis and Cost function (3)



$$\mathbf{w} = \begin{bmatrix} w_1 \\ \vdots \\ w_m \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix}$$

$$H(\mathbf{x}) = \mathbf{w}^{\mathsf{T}}\mathbf{x} + b$$

$$cost(\mathbf{w}, b) = \frac{1}{n}\sum_{i=0}^{n}\left(y_i - H(\mathbf{x}_i)\right)^2$$

경사 하강법 (3) Gradient Descent (3)

우리의 목표: cost를 최소화 하자! = cost를 최소로 만드는 w, b 값을 찾자!

$$\arg\min_{\mathbf{w},b} cost(\mathbf{w}, b)$$

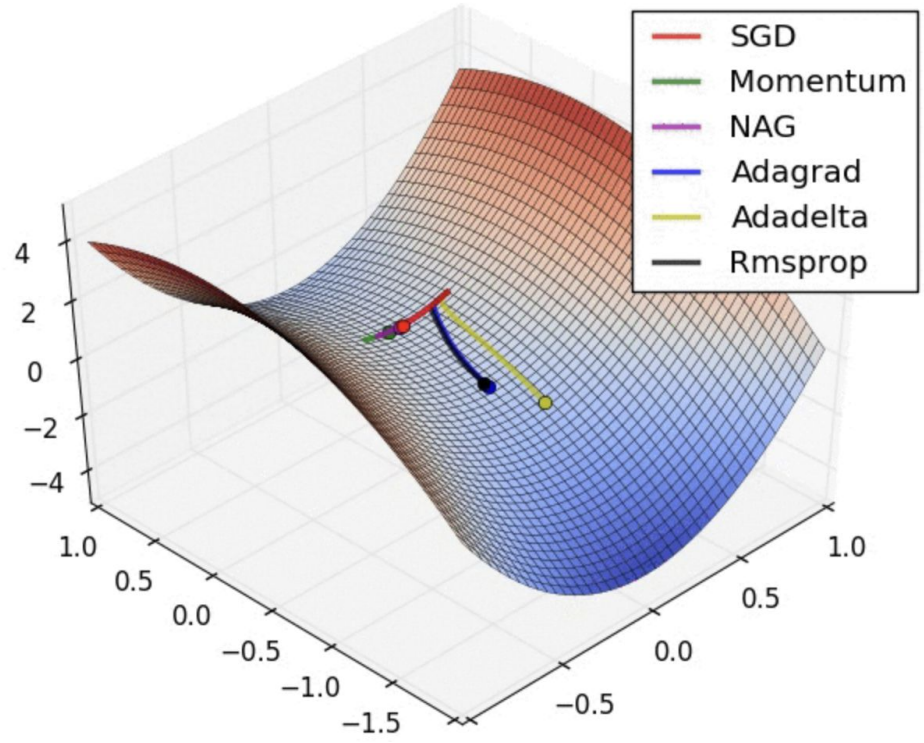
업데이트:

$$\mathbf{w} = \mathbf{w} - \alpha \frac{\partial cost(\mathbf{w},b)}{\partial \mathbf{w}}$$

$$b = b - \alpha \frac{\partial cost(\mathbf{w},b)}{\partial b}$$

그밖의 학습 방법들

경사하강법은 최적해를 찾기위한 한 가지 방법일 뿐, 다양한 업데이트 방법이 존재: [링크] Gradient Descent Optimization Algorithms 정리

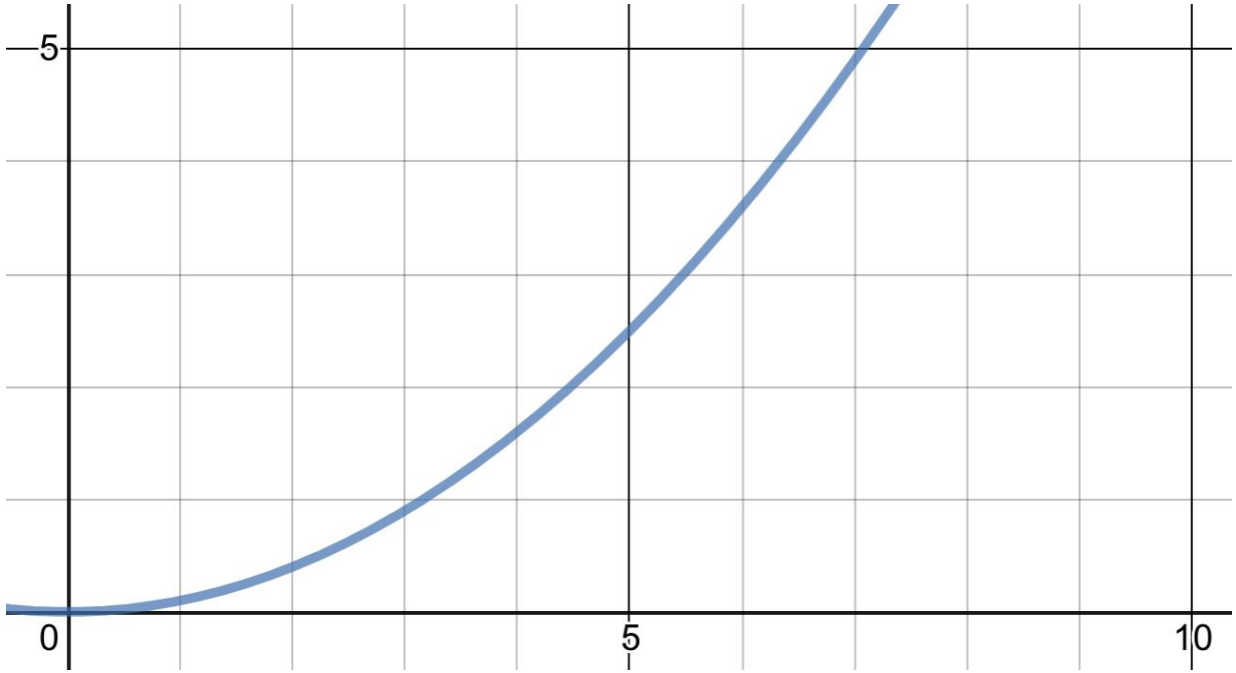


부록: 미분

평균변화율

x가 a에서 b로 증가하는 동안, f(x)가 평균적으로 증가한 양은?

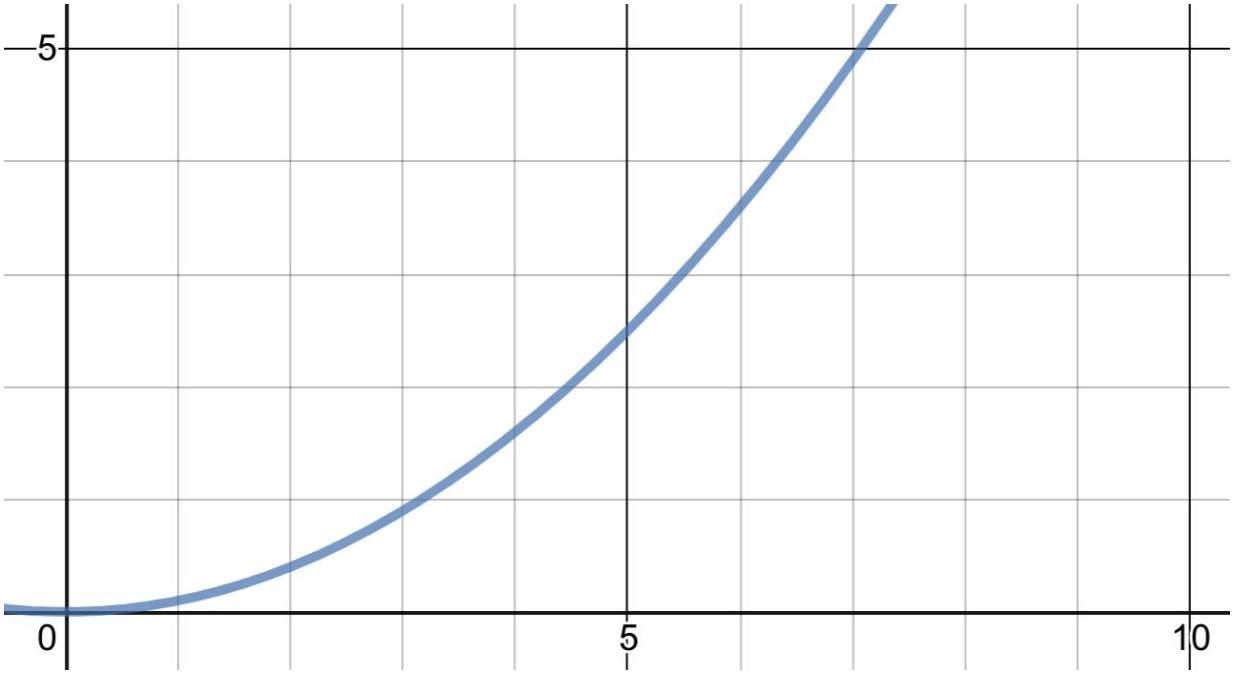
$$\frac{f(b)-f(a)}{b-a} = \frac{f(a+\Delta x)-f(a)}{\Delta x}$$



순간변화율 = 미분계수

x가 a일 때, f(x)의 증가율은? = x가 a일 때, f(x)의 미분계수는?

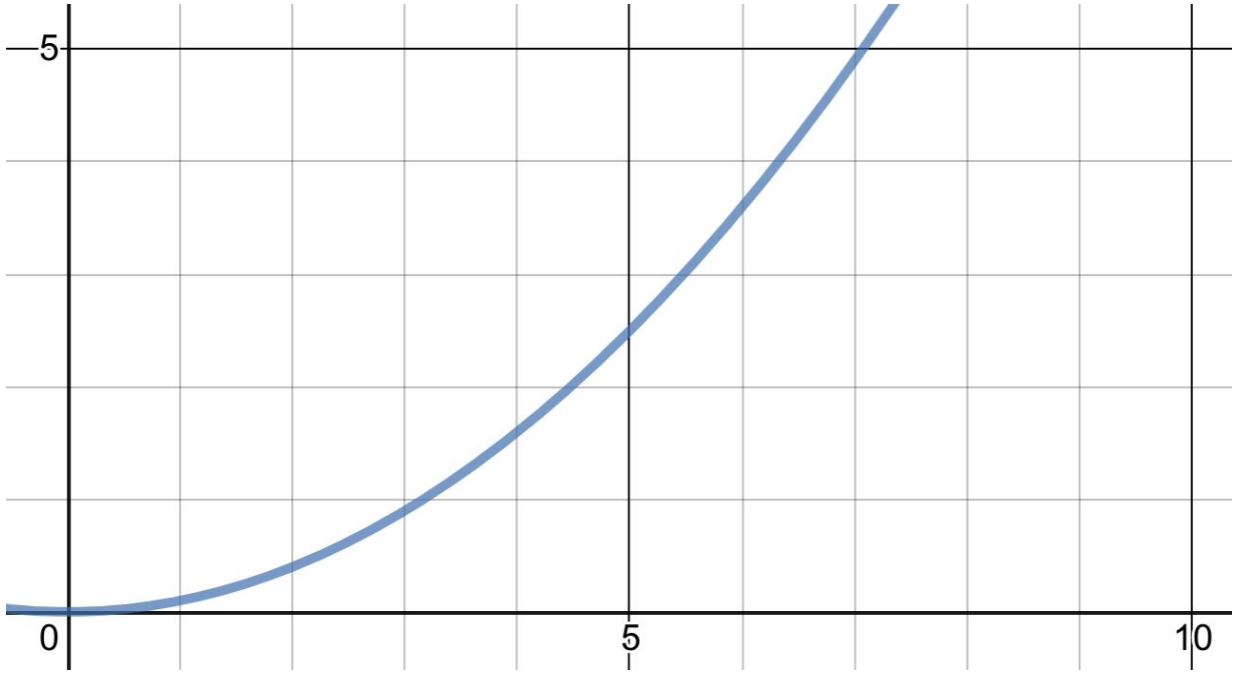
$$\lim_{\Delta x \to 0} \frac{f(a+\Delta x) - f(a)}{\Delta x}$$



도함수

항상 limit 값을 계산해야하나...? 도함수 f'(x)를 정의하자! ~ 미분한다..는 듯

$$f'(x) = \frac{d}{dx} f(x) = \lim_{\Delta x \to 0} \frac{f(a + \Delta x) - f(a)}{\Delta x}$$

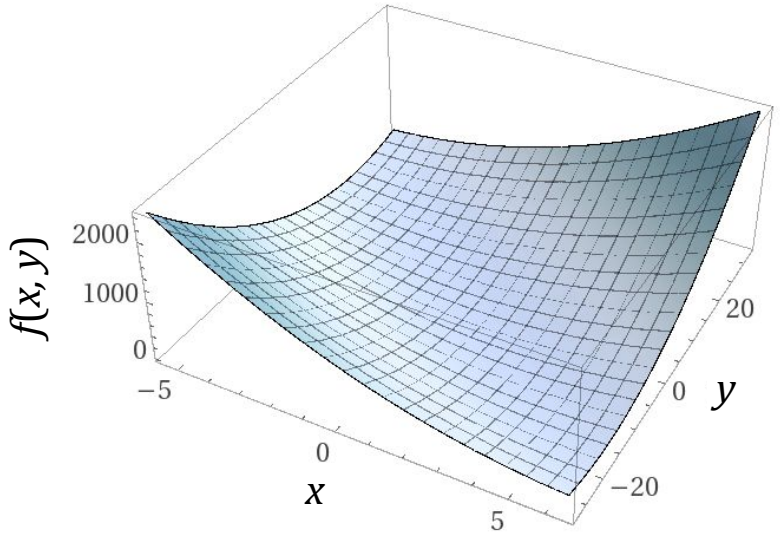


편미분

미지수가 2개일 때는 어떻게 미분을 하나...? 각각 나눠서.

$$\frac{\partial}{\partial x} f(x,y) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$$

$$\frac{\partial}{\partial y} f(x,y) = \lim_{\Delta y \to 0} \frac{f(x, y+\Delta y) - f(x,y)}{\Delta y}$$



Questions?