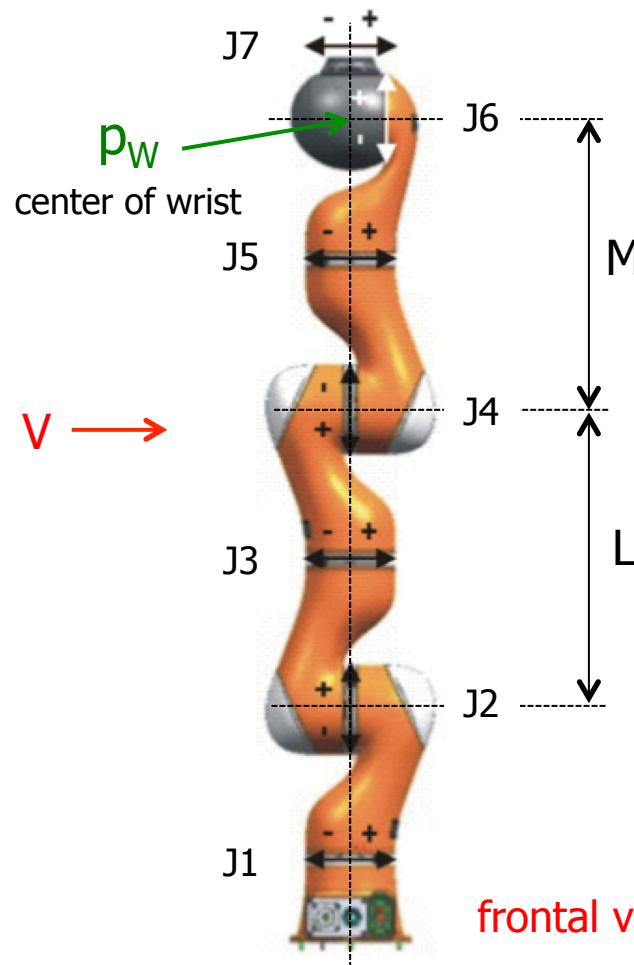
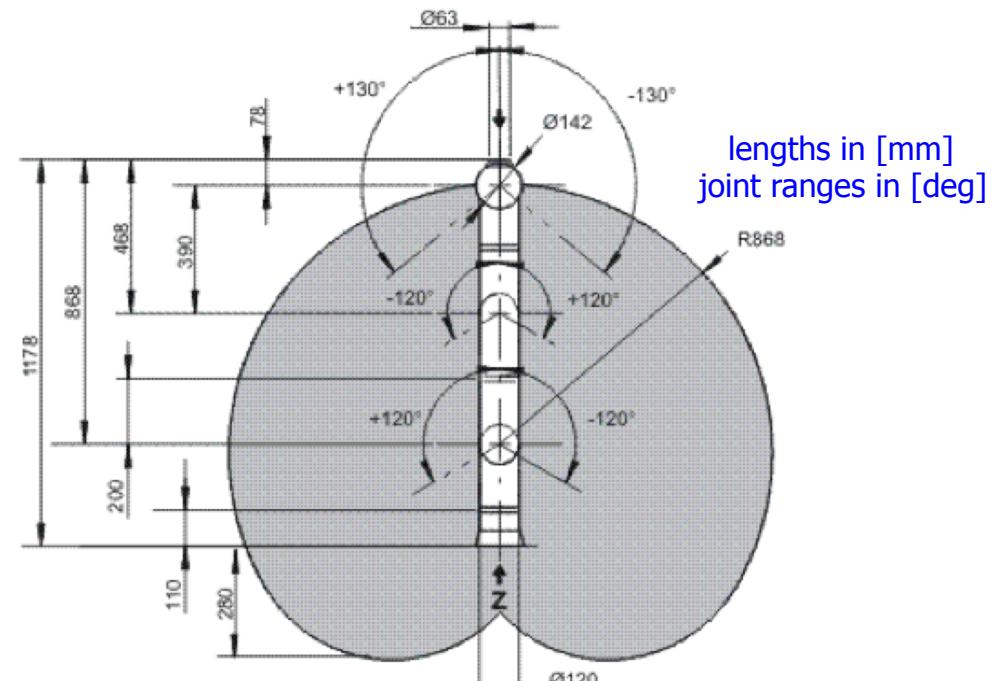


KUKA LWR-IV

- 7R manipulator: spherical shoulder (3R), elbow joint (1R), spherical wrist (3R)



- the robot is shown in its **zero configuration ($q=0$)**
- positive directions of joint rotations are indicated



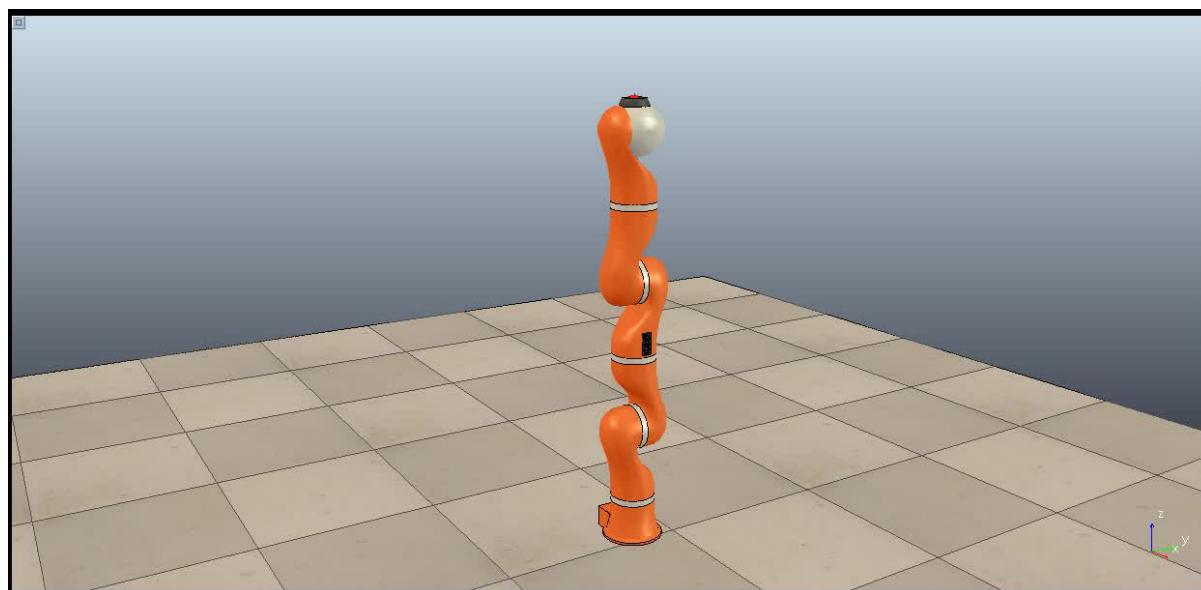
KUKA LWR-IV in motion



↑
side view from
observer in V

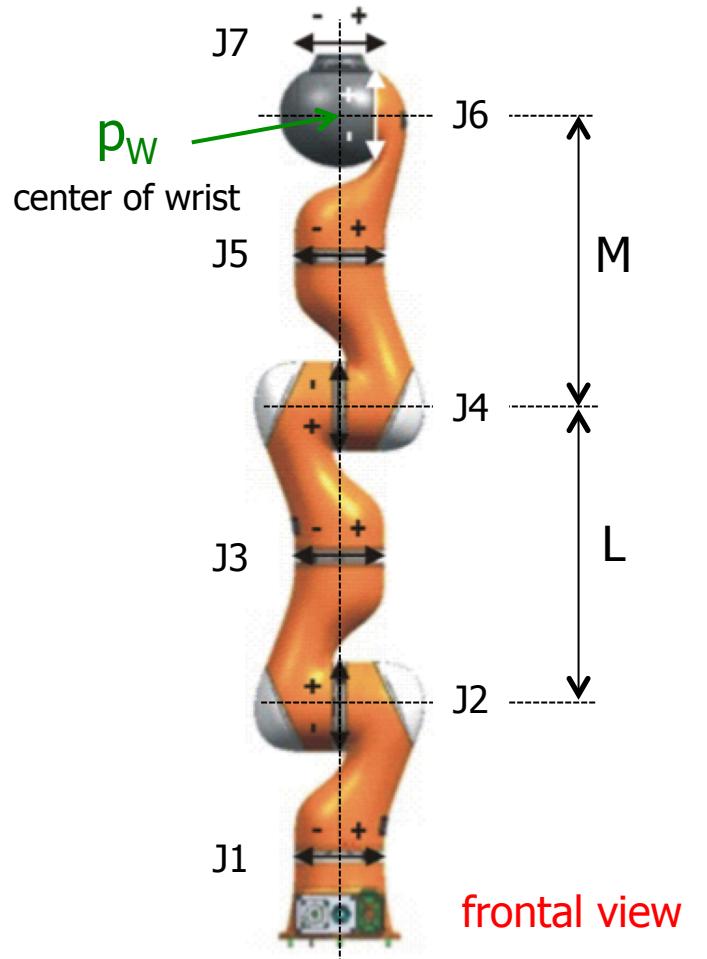


↗
videos at DIAG
Robotics Lab



VREP video

KUKA LWR-IV



- determine
 - frames and table of D-H parameters
 - be consistent with positive rotations indicated by KUKA
 - only the two kinematic lengths L and M should be needed
 - homogeneous transformation matrices
 - direct kinematics of the center of wrist p_w in **symbolic** form
 - **numerically**, in the configuration
 $q = (0, \pi/2, \pi/2, -\pi/2, 0, \pi/2, 0)$ [rad]
the position 0p_d in frame 0 of a **tool point** P_d whose coordinates in frame 7 are given by
 ${}^7p_d = (0, 0.05, 0.1)$ [m]

Assignment of D-H frames

steps!

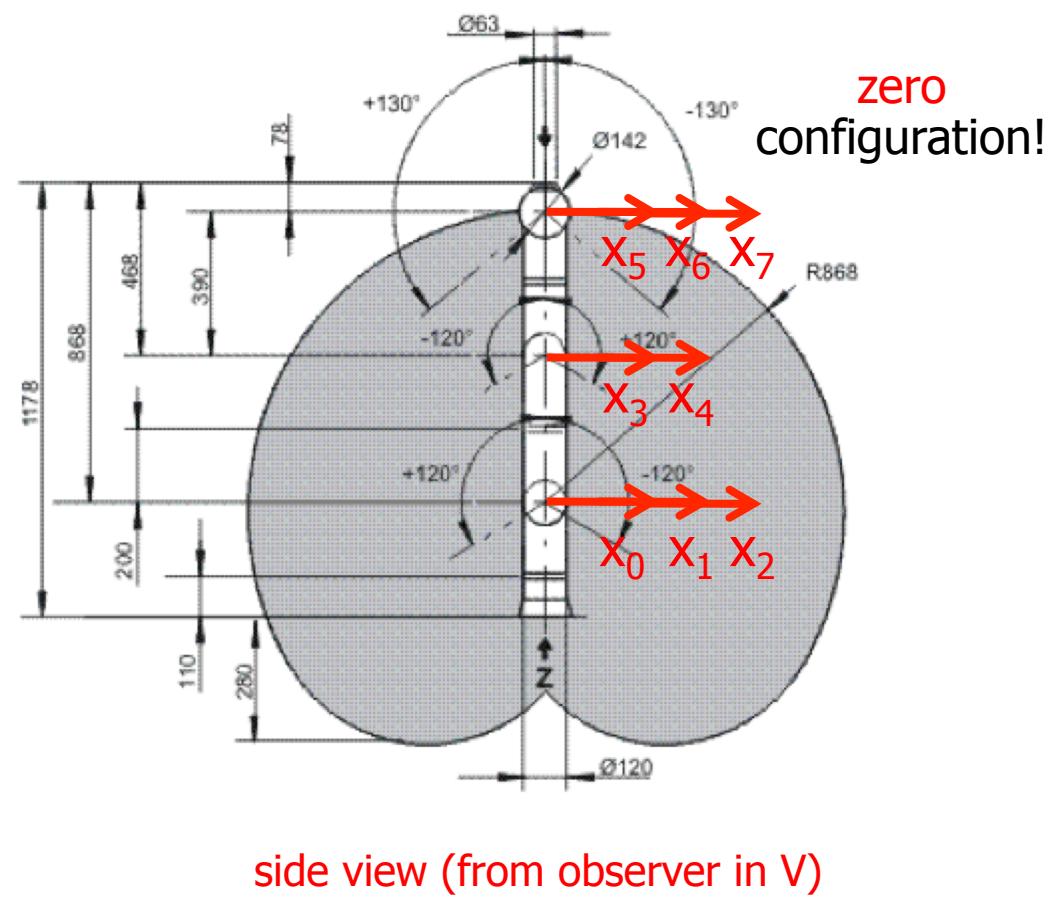
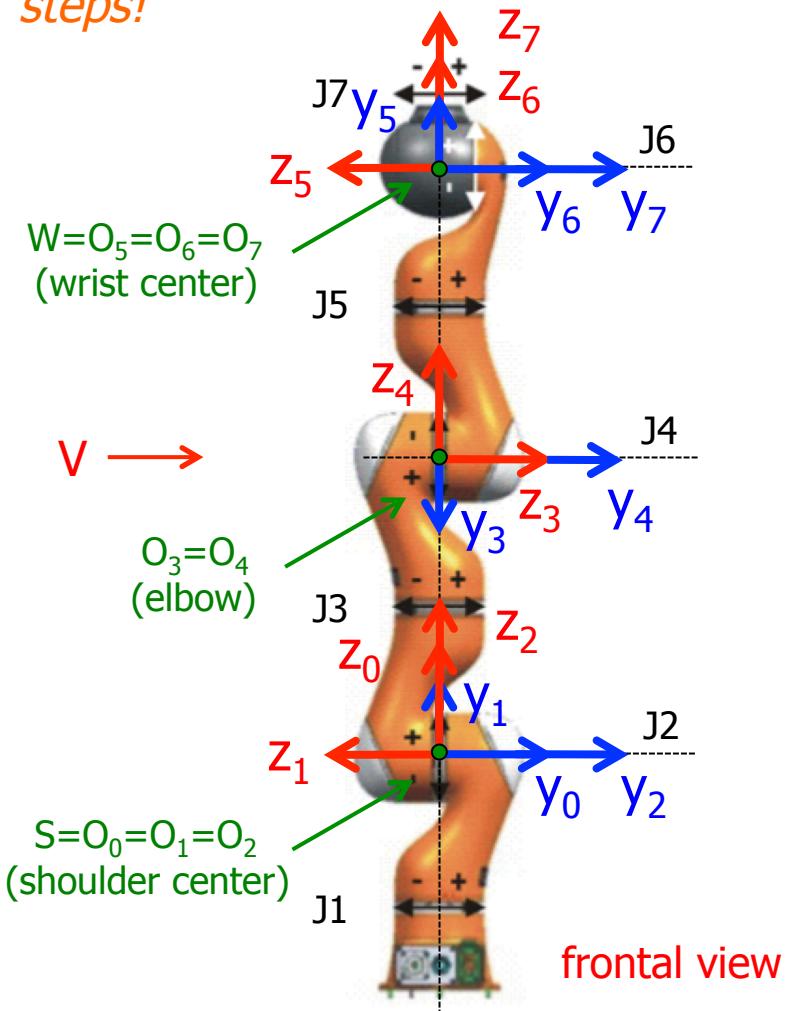
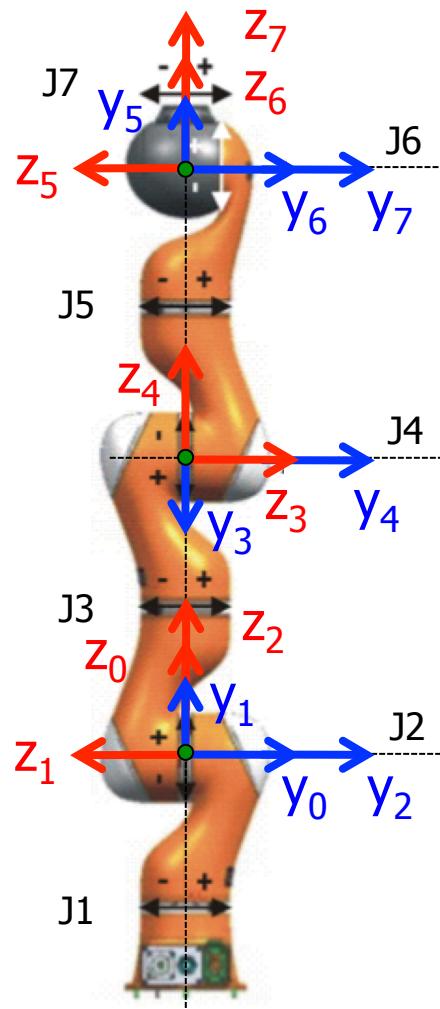


Table of D-H parameters



i	α_i	d_i	a_i	θ_i
1	$\pi/2$	0	0	$q_1=0$
2	$-\pi/2$	0	0	$q_2=0$
3	$-\pi/2$	L	0	$q_3=0$
4	$\pi/2$	0	0	$q_4=0$
5	$\pi/2$	M	0	$q_5=0$
6	$-\pi/2$	0	0	$q_6=0$
7	0	0	0	$q_7=0$

in the shown
configuration

with

$$d_3 = L = 0.40, \quad d_5 = M = 0.39 \quad [\text{m}]$$

D-H homogeneous matrices

output from Matlab (symbolic) program

```
A1 =  
  
[ cos(q1), 0, sin(q1), 0]  
[ sin(q1), 0, -cos(q1), 0]  
[ 0, 1, 0, 0]  
[ 0, 0, 0, 1]  
  
A2 =  
  
[ cos(q2), 0, -sin(q2), 0]  
[ sin(q2), 0, cos(q2), 0]  
[ 0, -1, 0, 0]  
[ 0, 0, 0, 1]  
  
A3 =  
  
[ cos(q3), 0, -sin(q3), 0]  
[ sin(q3), 0, cos(q3), 0]  
[ 0, -1, 0, L]  
[ 0, 0, 0, 1]  
  
A4 =  
  
[ cos(q4), 0, sin(q4), 0]  
[ sin(q4), 0, -cos(q4), 0]  
[ 0, 1, 0, 0]  
[ 0, 0, 0, 1]
```

```
A5 =  
  
[ cos(q5), 0, sin(q5), 0]  
[ sin(q5), 0, -cos(q5), 0]  
[ 0, 1, 0, M]  
[ 0, 0, 0, 1]  
  
A6 =  
  
[ cos(q6), 0, -sin(q6), 0]  
[ sin(q6), 0, cos(q6), 0]  
[ 0, -1, 0, 0]  
[ 0, 0, 0, 1]  
  
A7 =  
  
[ cos(q7), -sin(q7), 0, 0]  
[ sin(q7), cos(q7), 0, 0]  
[ 0, 0, 1, 0]  
[ 0, 0, 0, 1]
```

Direct kinematics of the wrist center

output from Matlab (**symbolic**) program

$$\begin{aligned} p_{W,hom} &= A_1(q_1)A_2(q_2)A_3(q_3)A_4(q_4) \underbrace{A_5(q_5)A_6(q_6)A_7(q_7)}_{\text{these can be replaced (by inspection) with } [0 \ 0 \ M \ 1]^T} \cdot [0 \ 0 \ 0 \ 1]^T \\ &= A_1(q_1)A_2(q_2)A_3(q_3)A_4(q_4) \cdot [0 \ 0 \ M \ 1]^T \\ &\quad \underbrace{\qquad\qquad\qquad}_{\text{faster (symbolic) recursive position transformations}} \quad \underbrace{\qquad\qquad\qquad}_{\text{(matrix} \cdot \text{vector products, in homogenous coordinates)}} \\ &\quad \text{the last three (spherical)} \\ &\quad \text{joints do not move W} \end{aligned}$$

```
pW =  
- cos(q1)*(sin(q2)*(L + M*cos(q4)) - M*cos(q2)*cos(q3)*sin(q4)) - M*sin(q1)*sin(q3)*sin(q4)  
M*cos(q1)*sin(q3)*sin(q4) - sin(q1)*(sin(q2)*(L + M*cos(q4)) - M*cos(q2)*cos(q3)*sin(q4))  
cos(q2)*(L + M*cos(q4)) + M*cos(q3)*sin(q2)*sin(q4)
```

Tool position evaluation

output from Matlab (numerical) program

- in the given configuration $q = (0, \pi/2, \pi/2, -\pi/2, 0, \pi/2, 0)$ [rad]
for the tool point P_d of coordinates (in frame 7) ${}^7p_d = (0, 0.05, 0.1)$ [m]

$${}^0p_{d,hom} = A_1(q_1)A_2(q_2)A_3(q_3)A_4(q_4)A_5(q_5)A_6(q_6)A_7(q_7) {}^7p_{d,hom}$$

etc.

again, (numerical) recursive position transformations (matrix \cdot vector products)

```
Anum{1}=subs(A1,{q1},{0});  
Anum{2}=subs(A2,{q2},{pi/2});  
Anum{3}=subs(A3,{q3,L},{pi/2,0.4}); ← numerical evaluation  
Anum{4}=subs(A4,{q4},{-pi/2});  
Anum{5}=subs(A5,{q5,M},{0,0.39});  
Anum{6}=subs(A6,{q6},{pi/2});  
Anum{7}=subs(A7,{q7},{0});  
  
pd = [0 0.05 0.1 1]';  
  
for i=N:-1:1  
    pd = Anum{i}*pd;  
end  
  
pd= pd(1:3)
```

of symbolic quantities



```
pd =  
-0.3000  
-0.3900  
-0.0500
```

7p_d in homogeneous coordinates

