Analyzing the Performances of a Compliant 3R Planar Robot using the ESP Control

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Outline

- **Problem**: Avoid oscillatory behaviours due to nonlinear elastic transmissions of a robotic system. The plant is underactuated due to compliant elements.

- **Objective**: Move to an equivalent dynamical model without gravity and with a link-side damping dynamics through feedback.

Our presentation is organized as follows:

- Theoretical introduction of ESP and ESP+ control schemes
- Dynamical model used for simulations
- Computational results obtained from simulations
- Conclusions

Coordinates Transformation

- We impose the equivalence

$$\psi(\theta - q) = \psi(\eta - \tilde{q}) + n(t, \tilde{q}, \dot{\tilde{q}})$$

where η is estimated using MATLAB nonlinear solver `fsolve`

- Differentiating the above equation w.r.t. time we can isolate $\dot{\eta}$

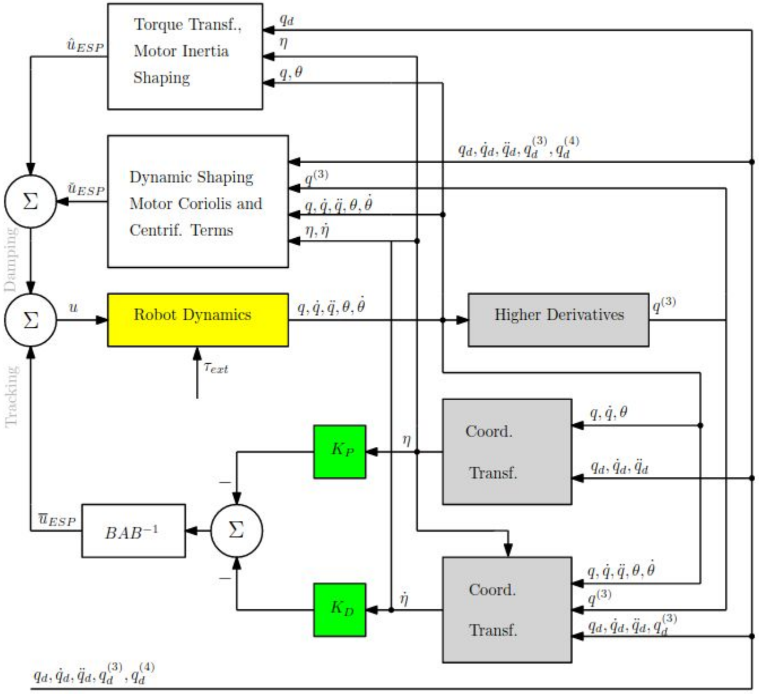
$$\kappa(\theta - q)(\dot{\theta} - \dot{q}) = \kappa(\eta - \tilde{q})(\dot{\eta} - \dot{\tilde{q}}) + \dot{n}(t, \tilde{q}, \dot{\tilde{q}})$$

- Then the transformated motor dynamics becomes

$$BA\ddot{\eta} + B\dot{A}\dot{\eta} + B\dot{a} + \psi(\theta - q) = u$$

$$\Rightarrow \quad A = \kappa^{-1}(\theta - q)\kappa(\eta - \tilde{q})$$

ESP control scheme



$$BA\ddot{\eta} + B\dot{A}\dot{\eta} + B\dot{a} + \psi(\theta - q) = u$$

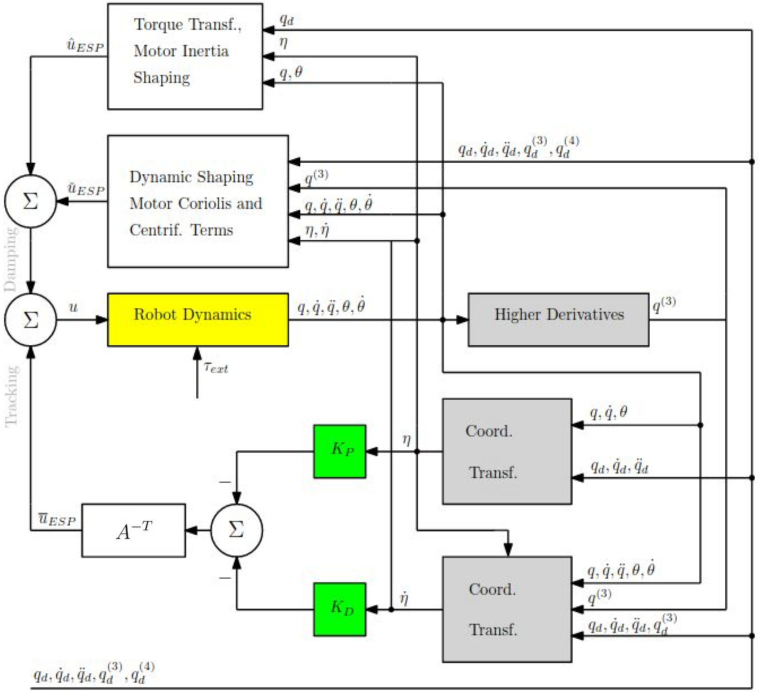
Imposition of a Damping Behaviour

$$\check{u}_{ESP} = B(\dot{A}\dot{\eta} + \dot{a})$$
$$\hat{u}_{ESP} = \psi(\theta - q) - BAB^{-1}\psi(\eta - \tilde{q})$$
$$\bar{u}_{ESP} = -BAB^{-1}(K_D\dot{\eta} + K_P\eta)$$

PD in η for Motion Tracking

$$B\ddot{\eta} + \psi(\eta - \tilde{q}) = -K_D\dot{\eta} - K_P\eta$$

ESP+ control scheme



$$B_\eta \ddot{\eta} + S_\eta \dot{\eta} + A^T \psi(\theta - q) + A^T B \dot{a} = A^T u$$

Imposition of a Damping Behaviour

$$\check{u}_{ESP+} = B\dot{a}$$
$$\hat{u}_{ESP+} = \psi(\theta - q) - A^{-T}\psi(\eta - \tilde{q})$$
$$\bar{u}_{ESP+} = -A^{-T}(K_D \dot{\eta} + K_P \eta)$$

PD in η for Motion Tracking

$$B_\eta \ddot{\eta} + S_\eta \dot{\eta} + \psi(\eta - \tilde{q}) = -K_D \dot{\eta} - K_P \eta$$

Real Implementation

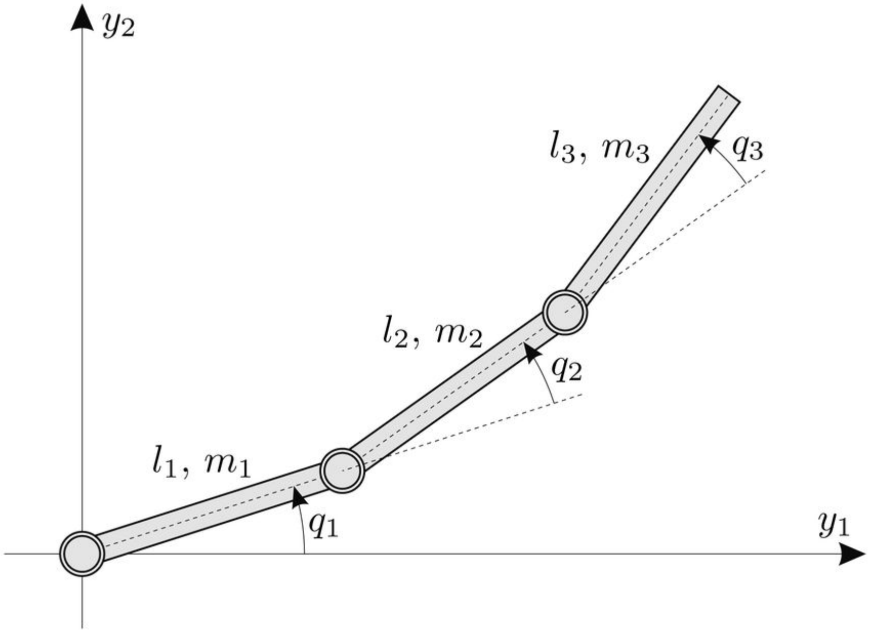
- Implementation on DLR Hand Arm System

- Variable Stiffness Actuators
  - FSJ
  - BAVS

- Regulation problem

- Comparison with a PD Control

Simulation Setting

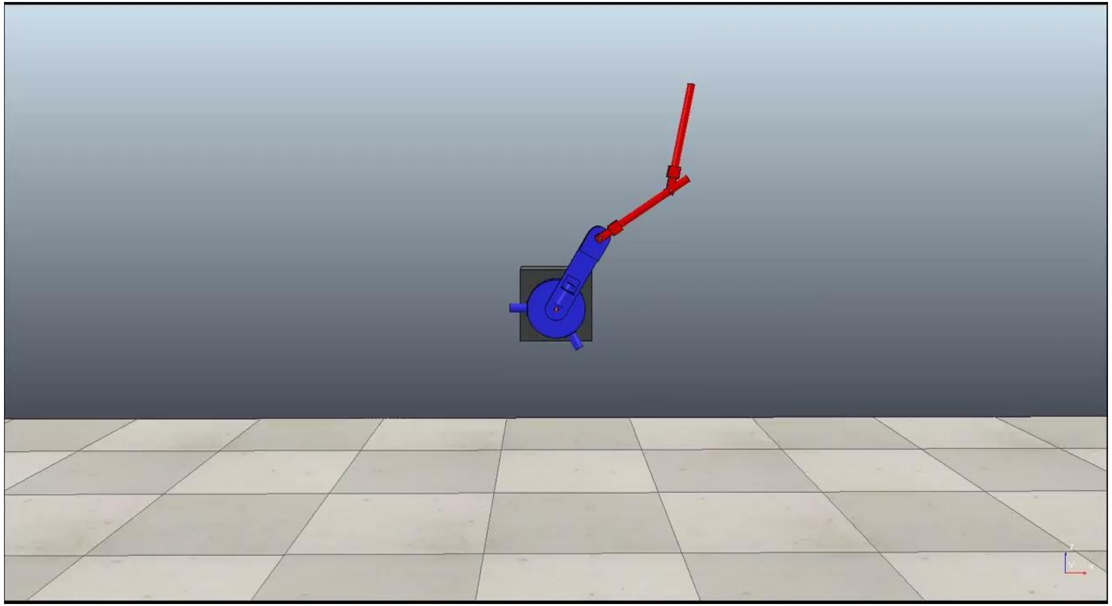


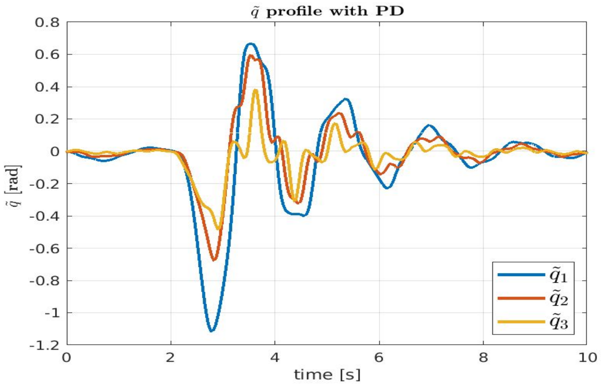
- Use of the Euler-Lagrange formalism
- The angular speed of the motors is related just to a rotation around their spinning axis

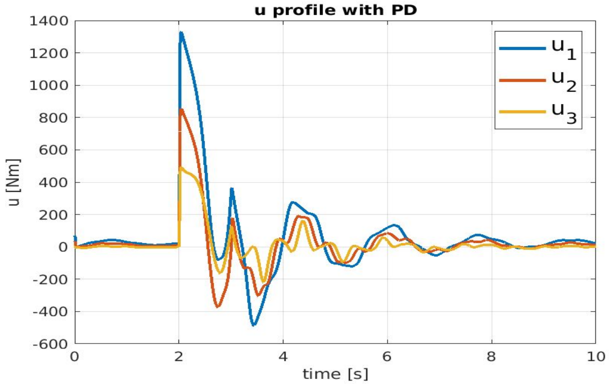
$$M(q)\ddot{q} + S(q,\dot{q})\dot{q} + g(q) - \psi(\theta - q) = \tau_{ext}$$
$$B\ddot{\theta} + \psi(\theta - q) = u$$

$$\Rightarrow \quad \psi(\theta - q) = k[(\theta - q) + (\theta - q)^3]$$

Generic Pose - PD control

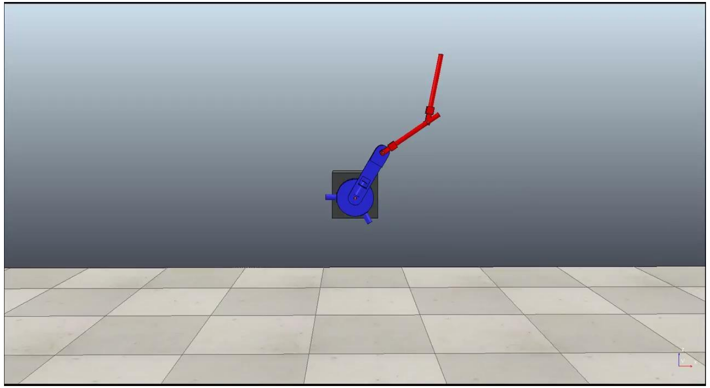




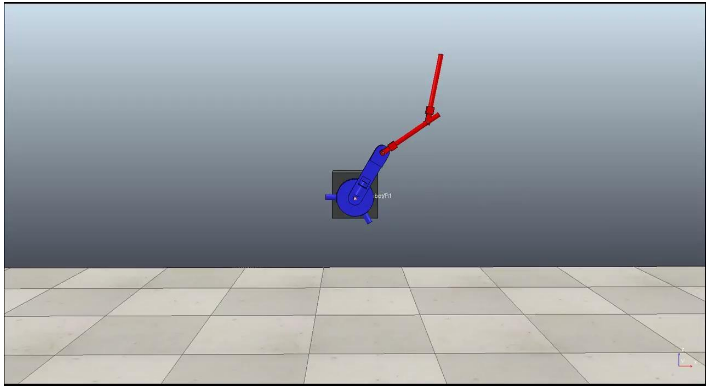


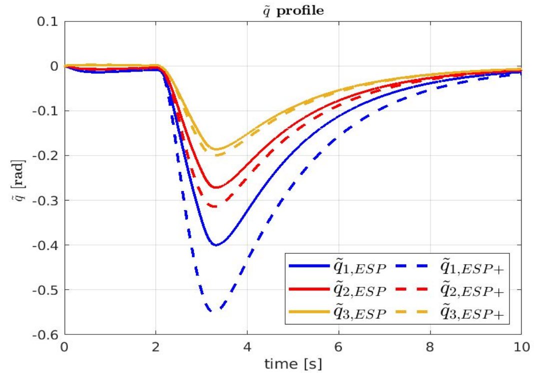
Generic Pose - ESP vs ESP+ control

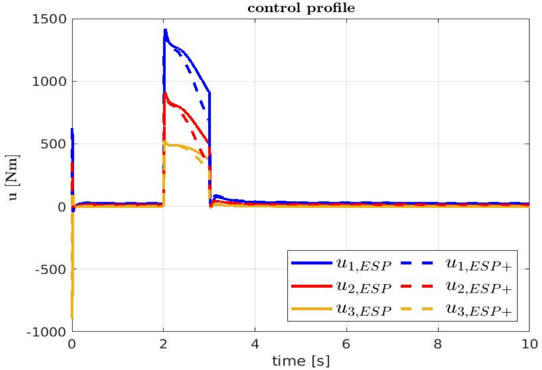
ESP →



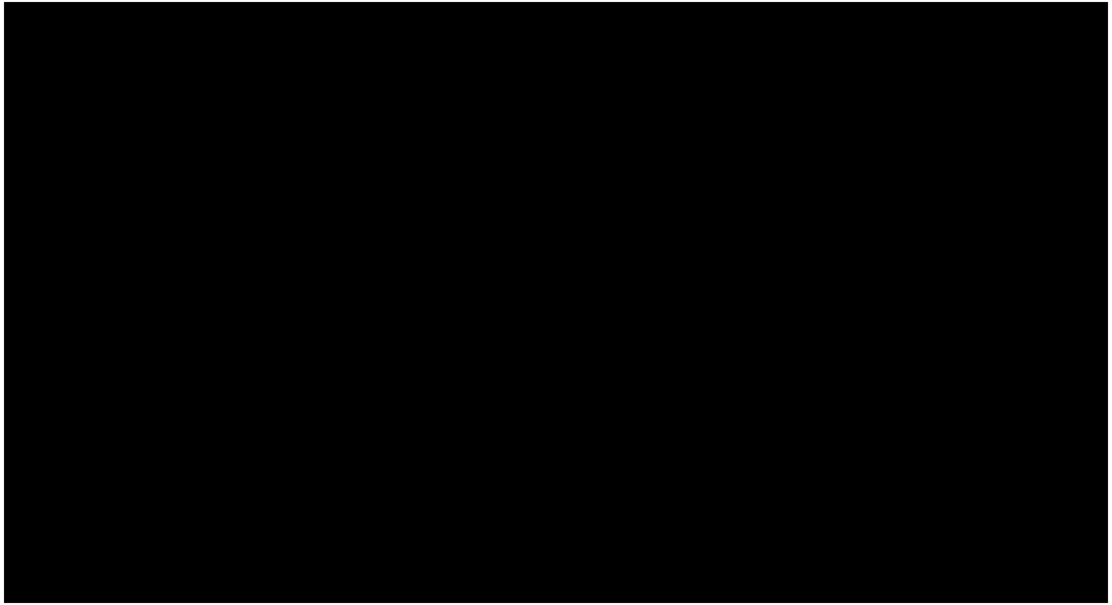
ESP+ →

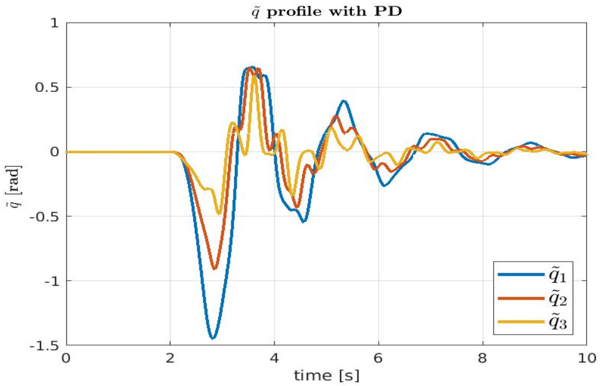


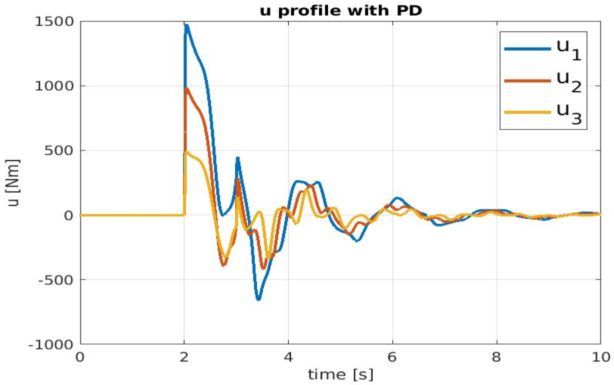




Stretched Arm - PD control

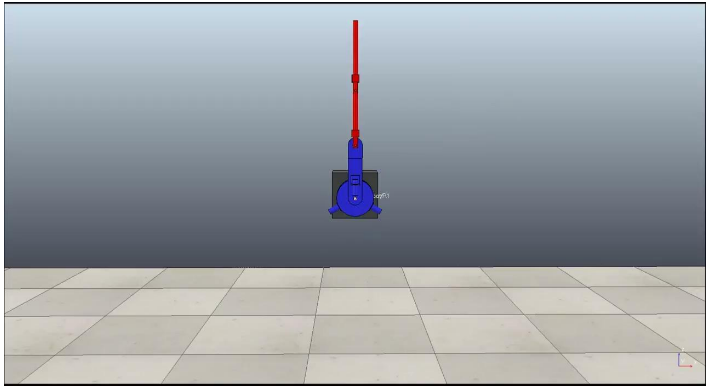




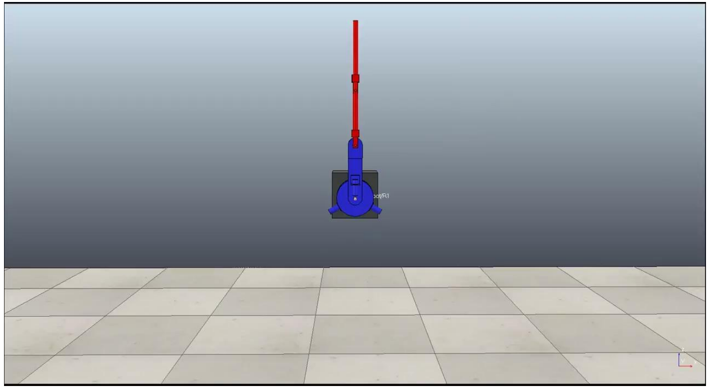


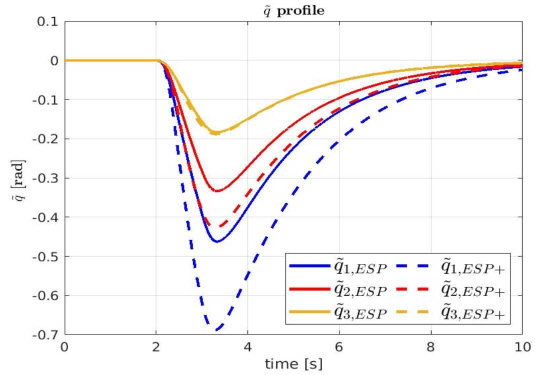
Stretched Arm - ESP vs ESP+ control

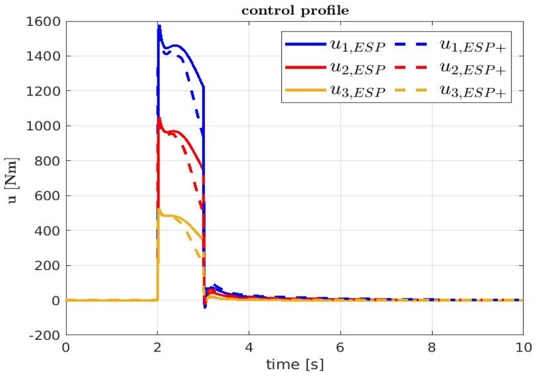
ESP



ESP+







Shaping the Motor Inertia

- **Goal:** shaping B imposing a desired stiffness to the transmissions
    - Choose linear stiffness for better understanding the effects

$$\psi(\theta - q) = k_1(\theta - q)$$
$$\psi(\eta - \tilde{q}) = k_2(\eta - \tilde{q})$$

    - This choice simplifies the equations of the controls
        - matrix $A = k_1^{-1} \cdot k_2$ become constant ⇨ its derivative is 0
        - η can be computed analytically
    - ESP+ shapes B through matrix A that is user-dependent

$$B_\eta = A^T B A$$

Shaping the Motor Inertia

- The natural frequency of a spring is $\omega = Bk^{-1}$
- **Remark:** $B_\eta = A^T B A \quad A = k_2 k_1^{-1}$
- Imposing a different stiffness to the transmission the new frequency reads as

$$\omega_\eta = B_\eta k_2^{-1} = \frac{Bk_2^2}{k_1^2 k_2} = \frac{Bk_2}{k_1^2} = \frac{B}{k_1}\frac{k_2}{k_1} = \omega \frac{k_2}{k_1}$$

- This implies that if
  - $k_2 < k_1 \Rightarrow \omega_\eta < \omega$
  - $k_2 > k_1 \Rightarrow \omega_\eta > \omega$

Shaping the Motor Inertia

- **Nonlinear Stiffness**

$$\check{u}_{ESP} = B(\dot{A}\dot{\eta} + \dot{a})$$
$$\hat{u}_{ESP} = \psi(\theta - q) - BAB^{-1}\psi(\eta - \tilde{q})$$
$$\bar{u}_{ESP} = -BAB^{-1}(K_D\dot{\eta} + K_P\eta)$$

$$\check{u}_{ESP+} = B\dot{a}$$
$$\hat{u}_{ESP+} = \psi(\theta - q) - A^{-T}\psi(\eta - \tilde{q})$$
$$\bar{u}_{ESP+} = -A^{-T}(K_D\dot{\eta} + K_P\eta)$$

$\Rightarrow$

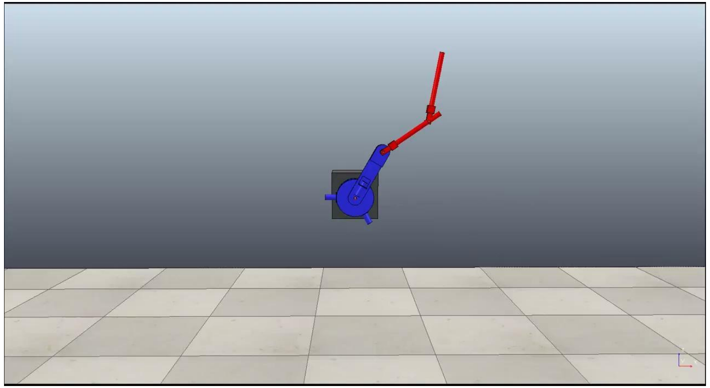
- **Linear Stiffness**

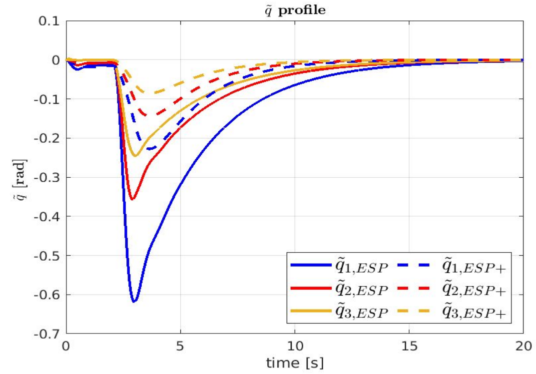
$$\check{u}_{ESP} = B\dot{a}$$
$$\hat{u}_{ESP} = k_1(\theta - q) - BAB^{-1}k_2(\eta - \tilde{q})$$
$$\bar{u}_{ESP} = -BAB^{-1}(K_D\dot{\eta} + K_P\eta)$$

$$\check{u}_{ESP+} = B\dot{a}$$
$$\hat{u}_{ESP+} = k_1(\theta - q) - A^{T}k_2(\eta - \tilde{q})$$
$$\bar{u}_{ESP+} = -A^{-T}(K_D\dot{\eta} + K_P\eta)$$

Generic Pose - ESP vs ESP+ control

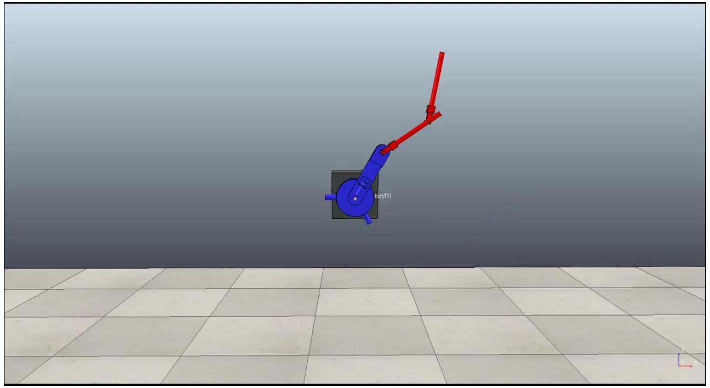
ESP

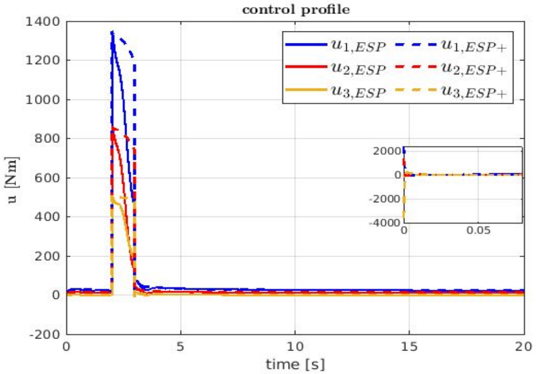




k_2=250

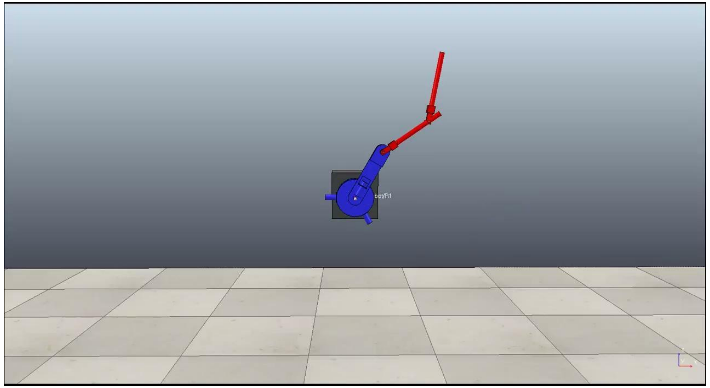
ESP+





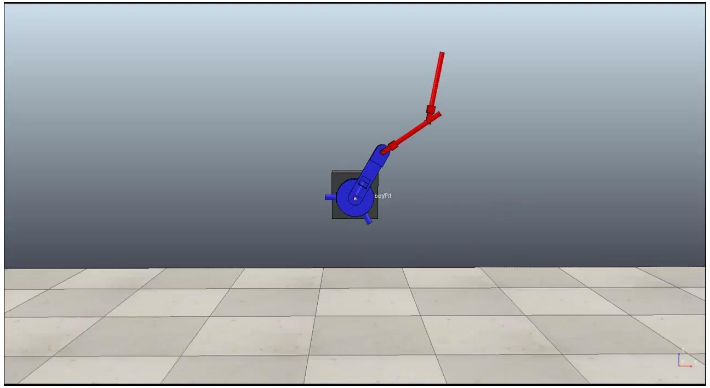
Generic Pose - ESP vs ESP+ control

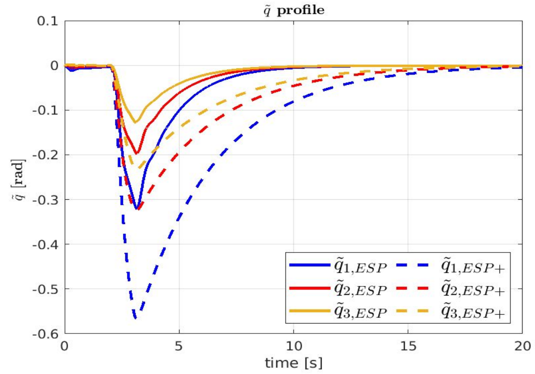
ESP →

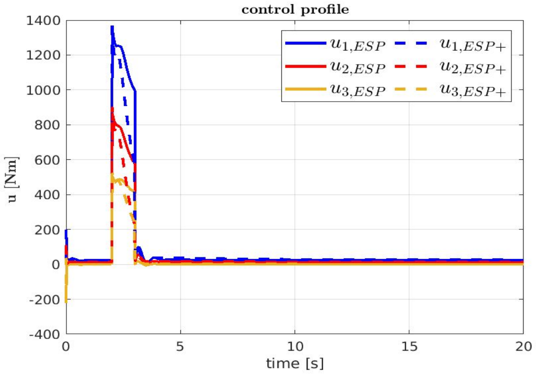


k_2=1000

ESP+ →







Conclusions

We have simulated a 3R planar robot with compliant transmissions that lies on a vertical plane analyzing:

- the ESP and ESP+ control schemes
- a simple PD controller

We have found that:

- The control scheme works even for the worst case scenario
- Matrix D is crucial for the behaviours of ESP and ESP+ control schemes
    - Best choice may be implement D configuration-dependent
- Oscillations in the PD framework are completely removed with ESP/ESP+
- In the linear stiffness case we are able to shape the motor inertia using ESP+

References

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