Differentiation Review

Support Workshop A

04-2-2014

Topics: Squeeze theorem, implicit differentiation, logarithmic differentiation, l'Hôpital's rule.

Practice exercises:

1. **The Squeeze Theorem.** If $\ell(x) \leq f(x) \leq u(x)$ when x is near a (except possibly at a) and
$$\lim_{x\to a} \ell(x) = L = \lim_{x\to a} u(x),$$
then
$$\lim_{x\to a} \ell(x) \leq \lim_{x\to a} f(x) \leq \lim_{x\to a} u(x),$$
and the limit of $f(x)$ is *squeezed*
$$L \leq \lim_{x\to a} f(x) \leq L,$$
thus $\lim_{x\to a} f(x) = L$.
   (a) $\lim_{x\to 0} x \sin\left(\frac{1}{2x}\right)$.
   (b) $\lim_{x\to 0} \sqrt{x^3 + x^2} \cos\left(\frac{\pi}{x}\right)$.
   (c) $\lim_{x\to \infty} e^{-x} \cos\left(\frac{3}{\pi x}\right)$.

2. Calculate y' using implicit differentiation.
   (a) $x^2 - 2xy + y^3 = 1$.
   (b) $\sin x + \cos y = \sin xy$.
   (c) $\sqrt{x + y} = 1 + x^2 y$.
   (d) $\frac{y}{x+y} = \ln y$.

3. Calculate y' using logarithmic differentiation.
   (a) $f(x) = x^{2x}$.
   (b) $f(x) = (\sin x)^{x^2}$.
   (c) $f(x) = x^{x^x}$.
   (d) $f(x) = \sqrt{\frac{x^2+1}{x^3+1}}$.

4. Use L'Hôpital's rule to compute the following limits. Make sure to verify that L'Hôpital's rule applies.
   (a) $\lim_{x\to 0} \frac{e^x - 1}{x}$.
   (b) $\lim_{x\to (\pi/2)^+} \frac{\cos x}{1 - \sin x}$.
   (c) $\lim_{x\to \infty} \frac{\ln x}{x^3 + 5x + 2}$.
   (d) $\lim_{x\to \infty} x e^{-x}$.
   (e) $\lim_{x\to \infty} (x - \ln x)$.
   (f) $\lim_{x\to \infty} \left(1 + \frac{1}{x}\right)^x$.