CS 161: Fundamentals of Artificial Intelligence
Fall 2021 – Assignment 5
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1. Consider the following sentences and decide for each whether it is valid, unsatisfiable, or neither. Justify your answer using truth tables.

(a) (Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)

Smoke	Fire	(Smoke ⇒ Fire)	(¬Smoke ⇒ ¬Fire)	(Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)
T	T	**T**	**T**	**T**
F	T	**T**	**F**	**F**
T	F	**F**	**T**	**T**
F	F	**T**	**T**	**T**

(a) **is neither Valid nor Unsatisfiable**. It is satisfiable however

(b) (Smoke ⇒ Fire) ⇒ ((Smoke ∨ Heat) ⇒ Fire)

Smoke	Fire	Heat	(Smoke ⇒ Fire)	((Smoke ∨ Heat) ⇒ Fire)	(Smoke ⇒ Fire) ⇒ ((Smoke ∨ Heat) ⇒ Fire)
T	T	T	**T**	**T**	**T**
F	T	T	**T**	**T**	**T**
T	F	T	**F**	**F**	**T**
F	F	T	**T**	**F**	**F**
T	T	F	**T**	**T**	**T**
F	T	F	**T**	**T**	**T**
T	F	F	**F**	**F**	**T**
F	F	F	**T**	**T**	**T**

(b) **is neither Valid nor Unsatisfiable -** it's simply satisfiable

(c) ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))

Smoke	Fire	Heat	((Smoke ∧ Heat) ⇒ Fire)	(Smoke ⇒ Fire)	(Heat ⇒ Fire)	((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))
T	T	T	**T**	**T**	**T**	**T**
F	T	T	**T**	**T**	**T**	**T**
T	F	T	**F**	**F**	**F**	**T**
F	F	T	**T**	**T**	**F**	**T**
T	T	F	**T**	**T**	**T**	**T**
F	T	F	**T**	**T**	**T**	**T**
T	F	F	**T**	**F**	**T**	**T**
F	F	F	**T**	**T**	**T**	**T**

(c) **is Valid - every possible truth value of all variables results in the expression evaluating to True**

2. Consider the following:

If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is mortal and it is a mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

(a) Represent the above information using a propositional logic knowledge base (a set of sentences in propositional logic). Make sure to clearly define all propositional symbols (variables) first.

Variable Definitions (used in parts a, b, and c):
M - Unicorn is mythical
I - Unicorn is immortal
A - Unicorn is a mammal
H - Unicorn is horned
S - Unicorn is magical

Derived from sentence 1: $(M \Rightarrow I) \land (\neg M \Rightarrow (\neg I \land A))$ -- this can be broken into two sentences
Derived from sentence 2: $(I \lor A) \Rightarrow H$
Derived from sentence 3: $H \Rightarrow S$

Knowledge Base:
$M \Rightarrow I$
$\neg M \Rightarrow (\neg I \land A)$
$(I \lor A) \Rightarrow H$
$H \Rightarrow S$

(b) Convert the knowledge base into CNF.

Converting each sentence into CNF:
Converts conditional $A \Rightarrow B$ into $(\neg A \vee B)$, uses double negation and De Morgans, then distributes disjunctions over conditionals

$M \Rightarrow I$; $(\neg M \vee I)$;

$\neg M \Rightarrow (\neg I \wedge A)$; $\neg\neg M \vee (\neg I \wedge A)$; $M \vee (\neg I \wedge A)$; $(\neg I \vee M) \wedge (M \vee A)$;

$(I \vee A) \Rightarrow H$; $\neg(I \vee A) \vee H$; $(\neg I \wedge \neg A) \vee H$; $(\neg I \vee H) \wedge (\neg A \vee H)$;

$H \Rightarrow S$; $(\neg H \vee S)$;

Individual clauses:
$(\neg M \vee I)$
$(\neg I \vee M)$
$(M \vee A)$
$(\neg I \vee H)$
$(\neg A \vee H)$
$(\neg H \vee S)$

Conjunction of all clauses:
$(\neg M \vee I) \wedge (\neg I \vee M) \wedge (M \vee A) \wedge (\neg I \vee H) \wedge (\neg A \vee H) \wedge (\neg H \vee S)$

(c) (i) Is it possible to derive from the knowledge base that the unicorn is mythical? (ii) How about magical? (iii) Horned? Justify your answers using resolution.

(i) Show: M = True
Assume CD: ~M

~M ^ (~I v M) |= ~I
~M ^ (M v A) |= A
A |= ~~A
~~A ^ (~A v H) |= H
H |= ~~H
~~H ^ (~H v S) |= S

(i cont.) We have assigned truth values to all five variables and not arrived at a contradiction:
M = False
I = False
A = True
H = True
S = True
Thus, it is not possible to derive M=True from our knowledge base.

(ii) Show: S = True
Assume CD: ~S

~S ^ (~H v S) |= ~H
~H ^ (~A v H) |= ~A
~H ^ (~I v H) |= ~I
~I ^ (~M v I) |= ~M
~A ^ (M v A) |= M

~M ^ M is a contradiction! Thus, our original assumption is false and S is true. We have shown that the unicorn is Magical from our knowledge base.

(iii) Show: H = True
Assume CD: ~H

~H ^ (~I v H) |= ~I
~H ^ (~A v H) |= ~A
~A ^ (M v A) |= M
~I ^ (~M v I) |= ~M

~M ^ M is a contradiction! Thus, our original assumption is false which means H is true, and we may conclude that the unicorn is Horned based on our knowledge base.

3. Consider the following:
An oil well may be drilled on Mr. Y's farm in Texas. Based on what has happened to similar farms, we judge the probability of only oil being present to be .5, the probability of only natural gas being present to be .2, and the probability of neither being present to be .3. Oil and gas never occur together. If oil is present, a geological test will give a positive result with probability .9; if natural gas is present, it will give a positive result with probability .3; and if neither are present, the test will be positive with probability .1.
Suppose the test comes back positive. What's the probability that oil is present?

Given data:
Probability Oil is present = $P(O) = .5$
Probability Natural Gas is present = $P(G) = .2$
Probability nothing is present = $P(N) = .3$
Probability of positive test given oil present = $P(T|O) = .9$
Probability of positive test given gas present = $P(T|G) = .3$
Probability of positive test given nothing present = $P(T|N) = .1$

Solve for: Probability of oil present given a positive test, $P(O|T)$

According to Bayes' Theorem,
$P(O|T) = (P(T|O) * P(O)) / P(T)$

To use this we must solve for $P(T)$, the probability of a positive test result occurring.
According to the product rule / using conditioning,
$P(T) = \text{Sum}_i(P(T|i) * P(i))$

so $P(T) = P(T|O) * P(O) + P(T|G) * P(G) + P(T|N) * P(N)$

$P(T) = .9 * .5 + .3 * .2 + .3 * .1$
$P(T) = 0.45 + 0.06 + 0.03 = 0.54$

Returning to Bayes' Theorem,
$P(O|T) = (P(T|O) * P(O)) / P(T)$
$P(O|T) = (.9 * .5) / .54 = 0.8333333\ldots$

Therefore, if a geological test returns a positive result, the probability of oil being present is 83.3%