AI1103:Assignment 2

Gowri Govindaraj, EP20BTECH11007

Download all python codes from

https://github.com/gowrigovindaraj/AI/Assignment2/codes

and latex-tikz codes from

https://github.com/gowrigovindaraj/AI/Assignment2.tex

1 GATE CS 2020 Question 45

For $n > 2$, let $a \in \{0,1\}^n$ be a non-zero vector. Suppose that x is chosen uniformly at random from $\{0,1\}^n$. Then, the probability that $\sum_{i=1}^{n} a_i x_i$ is an odd number is _____ ?

2 Solution

Consider

$$\mathbf{a} = \begin{pmatrix} a_1 & a_2 & \dots & a_n \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{x} = \begin{pmatrix} x_1 & x_2 & \dots & x_n \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{ax} = \begin{pmatrix} a_1x_1 & a_2x_2 & \dots & a_nx_n \end{pmatrix} \tag{2.0.3}$$

Each element of both the vectors can take the values 0 or 1. Therefore $\sum_{i=1}^{n} a_i x_i$ can take values between 0 and n.

Total number of cases=2^n

The favourable cases are which the value of $\sum_{i=1}^{n} a_i x_i$ are odd, i.e., $1, 3, \dots$

TABLE 0: Values

$\sum_{i=1}^{n} a_i x_i$	0	1	2	...	n
number of cases	${}^nC_0$	${}^nC_1$	${}^nC_2$	...	${}^nC_n$

$$\text{P}\left[\sum_{i=1}^{n} a_i x_i \text{ is odd}\right]$$

$$= \frac{{}^nC_1 + {}^nC_3 + \dots}{2^n}$$

$$= \frac{2^{n-1}}{2^n} \qquad \{{}^nC_1 + {}^nC_3 + \dots = 2^{n-1}\}$$

$$= \frac{1}{2}$$

$$= 0.5$$