

Assignment - 1

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Abstract—This document contains the solution to Exercise 3.23 (a) of Oppenheim.

Problem 1. An LTI system is characterized by the system function

$$H(z) = \frac{1 - \frac{1}{2}z^{-2}}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{4}z^{-1}\right)} \quad (1)$$

Find the impulse response $h(n)$ of the system. **Solution:**

$$H(z) = \frac{1 - \frac{1}{2}z^{-2}}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{4}z^{-1}\right)} \quad (2)$$

On expanding,

$$H(z) = -8 + \frac{-6z^{-1} + 9}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{4}z^{-1}\right)} \quad (3)$$

$$= -8 + \frac{-6}{1 - \frac{1}{2}z^{-1}} + \frac{15}{1 - \frac{1}{4}z^{-1}} \quad (4)$$

We know that

$$a^n u(n) \stackrel{\mathcal{Z}}{\rightleftharpoons} \frac{1}{1 - az^{-1}} \quad (5)$$

and

$$\delta(n) \stackrel{\mathcal{Z}}{\rightleftharpoons} 1 \quad (6)$$

Thus, using (5) and (6) in (4),

$$h(n) = -8\delta(n) + -6\left(-\frac{1}{2}\right)^n u(n) + 15\left(-\frac{1}{4}\right)^n u(n) \quad (7)$$

The plot of $h(n)$ has been generated using the Python code.

https://raw.githubusercontent.com/gowrigovindaraj/EE3900/main/Assignment_0/5_3.py

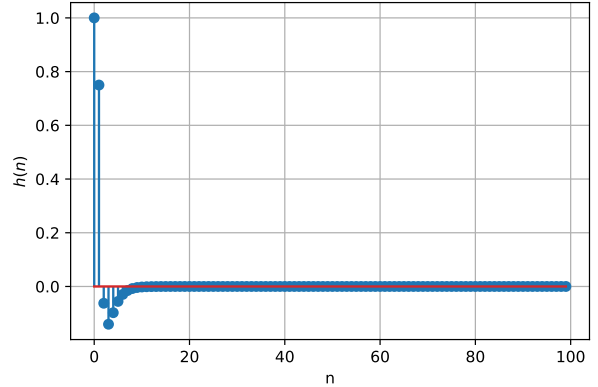


Fig. 1: Plot of $h(n)$ against n