Assignment - 2

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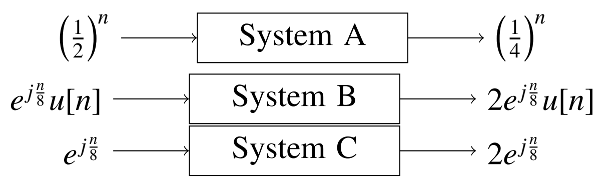
***Abstract*—This document contains the solution to Exercise 2.27 of Oppenheim.**

Problem 1. Three systems A, B and C have the inputs and outputs as indicated in Fig. Determine whether each system could be LTI. If your answer is yes, specify whether there could be more than one LTI system with the given input-output pair. Explain your answer.



Solution:

- Consider System A

$$x[n] = \left(\frac{1}{2}\right)^n \tag{1}$$

For an LTI system, the output will be a linearly scalable multiple of the input. As here, $y[n] = \left(\frac{1}{4}\right)^n$ is not linearly scalable by a complex constant to $x[n]$, the system is not an LTI.

- Consider system B

$$x[n] = e^{j\frac{n}{8}}u[n] \tag{2}$$

The Fourier Transform of x[n] is

$$X(e^{j\omega}) = \sum_{-\infty}^{\infty} e^{j\frac{n}{8}}u[n]e^{j\omega n} \tag{3}$$

$$= \sum_{n=0}^{\infty} e^{-j(\omega-\frac{1}{8})} \tag{4}$$

$$= \frac{1}{1-e^{-j(\omega-\frac{1}{8})}} \tag{5}$$

Given the output $y[n] = 2x[n]$
$\therefore$

$$Y(e^{j\omega}) = \frac{2}{1-e^{-j(\omega-\frac{1}{8})}} \tag{6}$$

$\therefore$ the frequence response of the system is

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} \tag{7}$$

$$= 2 \tag{8}$$

System B is a unique Linear Amplifier and is an LTI.

- Consider System C
We observe that the given output is a linear scalable of the input,

$$y[n] = 2x[n] \tag{9}$$

$\implies$ the system is LTI.
The system is constrained by

$$H(e^{j\omega})\big|_{\omega=\frac{1}{8}} = 2 \tag{10}$$

This is not sufficient to satisfy the condition of a unique system.