Assignment - 1

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***Abstract*—This document contains the solution to Exercise 3.23 (a) of Oppenheim.**

Problem 1. An LTI system is characterized by the system function

$$H(z) = \frac{1 - \frac{1}{2}z^{-2}}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{4}z^{-1}\right)} \tag{1}$$

Find the impulse response $h(n)$ of the system. **Solution:**

$$H(z) = \frac{1 - \frac{1}{2}z^{-2}}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{4}z^{-1}\right)} \tag{2}$$

On expanding,

$$H(z) = -8 + \frac{-6z^{-1} + 9}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{4}z^{-1}\right)} \tag{3}$$

$$= -8 + \frac{-6}{1 - \frac{1}{2}z^{-1}} + \frac{15}{1 - \frac{1}{4}z^{-1}} \tag{4}$$

We know that

$$a^n u(n) \overset{\mathcal{Z}}{\rightleftharpoons} \frac{1}{1 - az^{-1}} \tag{5}$$

and

$$\delta(n) \overset{\mathcal{Z}}{\rightleftharpoons} 1 \tag{6}$$

Thus, using (5) and (6) in (4),

$$h(n) = -8\delta(n) + -6\left(-\frac{1}{2}\right)^n u(n) + 15\left(-\frac{1}{4}\right)^n u(n) \tag{7}$$

The plot of $h(n)$ has been generated using the Python code.

https://https://raw.githubusercontent.com/gowrigovindaraj/EE3900/main/Assignment_1/opp1.py

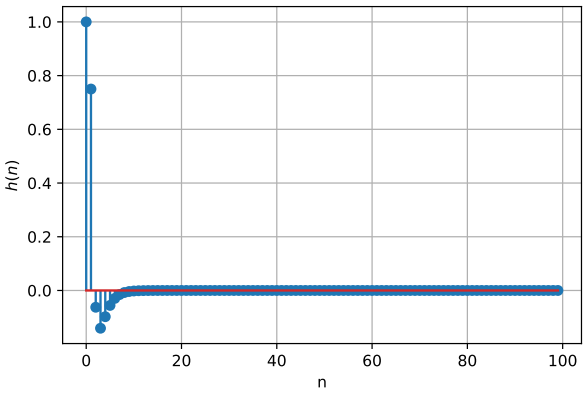

Fig. 1: Plot of $h(n)$ against n