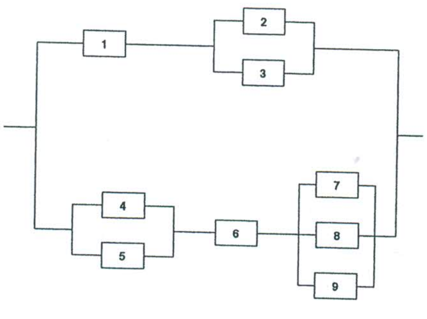
Probability and Stochastic Process (MA20106) Code: Abxw1037x6cFep

Name: Class Test-1

Roll No. Marks:10 Time: 45 mins

Q.No	Question	Answer	Marks
1.	Suppose that $\Omega = \{1,2,3,4,5,6,7,8,9,10\}$ and $P(\{i\}) = \frac{i}{55}$ for $i \in \Omega$. If a number is drawn at random, determine the probability that a prime number is drawn.	$17/35 = 0.309$	1
2.	Suppose that two fair dice will be rolled repeatedly and independently until a total of 6 or a total of 7 appears. Find the probability that a total of 6 is rolled before a total of 7 is rolled.	$\frac{5}{11} = 0.4545$	1
3.	Suppose that a basketball player makes 40% of his three-point shot attempts. If the players shoots 10 three-point shots in a game, and X is the number of three-point shots made. Assuming that the shots are independent, find the mean of random variable X.	4	1
4.	Let the CDF of a random variable X is given by $$F(x) = \begin{cases} \frac{1}{2}e^{kx}, & x < 0 \\ 1 - \frac{1}{2}e^{-kx}, & x \geq 0 \end{cases}$$ when $k = 2$. Find $P\left(\lvert X \rvert \leq \frac{1}{k}\right)$.	$1 - e^{-1} = 0.63$	1
5.	A computer center has three printers A, B and C, which print at different speeds. Programmes are routed to the first available printer. The probability that the programmes are routed to the printers A, B and C are 0.6, 0.3 and 0.1 respectively. Occasinally, a printer will jam and destroy a print out. The probability that printers A, B and C will jam are 0.01, 0.05 and 0.04 respectively. Your programme is destroyed when a printer jams. What is the probability that printer A is involved.	$6/25 = 0.24$	1
6.	Suppose that $P(A) = 0.3$, $P(B) = 0.4$, and A and B are independent events. Determine $P(A^c \cup B^c)$.	$22/25 = 0.88$	1
7.	Let X be a discrete random variable with MGF $M_X(t) = \frac{e^{3t}}{6} + \frac{e^{4t}}{2} + \frac{e^{5t}}{3}$. Determine $\mathrm{Var}(X)$.	$17/36 = 0.4722$	1
8.	An archer can hit the target 2 time if he makes 5 attempts. (a) What is the probability that he can hit the target for the first time on 10 th attempts? (b) What is the probability that he fails 9 times before he hits the target? (c) Suppose it is known that he failed to hit the target in 6 attempts, what is the probability that he can hit the target in next 2 attempts?	(a) 4.03×10^{-3} (b) 4.03×10^{-3} (c) $16/25 = 0.64$	1
9.	In a recent production, 5% of certain electronic components are defective. We need to find 3 non-defective components for our 3 new computers.Components are tested until 3 non-defectives are found. What is the probability that more than 5 components will be tested?	0.0012	1
10.	Consider the system represented in following figure. Let $r_i, i = 1,2,\ldots 9$ denotes the probability of functioning of ith component, and is given by $r_1 = 0.98, r_2 = r_3 = 0.87, r_4 = r_5 = 0.94, r_6 = 0.99, r_7 = r_8 = r_9 = 0.74$. Compute R_s, the probability of functioning of the system.	0.99	1

Any 1 correct 1/2 mark any two full marks



Q1. $P(2,3,5,7) = \frac{2+3+5+7}{55} = \frac{17}{55}.$

Q2. Event A_i that first total of 6 is rolled before a total of 7 is rolled in ith trail.

$$P(A_i) = P(\{\bigcap_{j=1}^{i-1} (6 \cup 7)^c\} \cap 6)$$

$$\overset{\text{indep}}{=} \left(\frac{25}{36}\right)^{i-1} \times \frac{5}{36}$$

$$P(6 \text{ before } 7) = P\left(\bigcup_{i=1}^{\infty} A_i\right) \overset{\text{disjoint}}{=} \sum_{i=1}^{\infty} P(A_i)$$

$$= \sum_{i=1}^{\infty} \left(\frac{25}{36}\right)^{i-1} \times \frac{5}{36} = \frac{5}{36} \times \frac{1}{1-\frac{25}{36}} = \frac{5}{11}$$

Total		# of cases
2	(1,1)	1
3	(1,2) (2,1)	2
4		3
5		4
6		5
7		6
8		5
9		4
10		3
11		2
12	(6,6)	1

Q3. $X \sim Bin(n, p)$, $n=10$, $p = 0.40$

$E(X) = np = 10 \times 0.40 = 4$

For $k>0$,

Q4. $P(-\frac{1}{k} \le X \le \frac{1}{k}) = F(\frac{1}{k}) - F(-\frac{1}{k}) = 1 - \frac{1}{2} e^{-k \times \frac{1}{k}} - \frac{1}{2} e^{k \times (-\frac{1}{k})}$

$$= 1 - \frac{1}{2} e^{-1} - \frac{1}{2} e^{-1} = 1 - e^{-1}$$

Q5. $P(A) = 0.6$, $P(B) = 0.3$, $P(C) = 0.1$

events D jam destroy printout, programme.

$P(D|A) = 0.01$, $P(D|B) = 0.05$, $P(D|C) = 0.04$

$$\therefore P(A|D) = \frac{P(D|A)\,P(A)}{P(D|A)P(A) + P(D|B)\,P(B) + P(D|C)P(C)}$$

$$= \frac{0.01 \times 0.6}{.01 \times .6 + .05 \times .3 + .04 \times .1} = \frac{.006}{.025} = \frac{6}{25}$$

$$= 0.24$$

Q6. $P(A^c \cup B^c) = P(A^c) + P(B^c) - P(A^c \cap B^c)$

$$= 1 - P(A) + 1 - P(B) - P(A^c)\,P(B^c)$$

| A&B are indep $\Rightarrow$ A^c & B^c are indep

$$= 1 - 0.3 + 1 - 0.4 - (1-0.3)(1-0.4)$$

$$= .7 + .6 - .7 \times .6 = 0.88$$

Q7. $E(X) = \frac{d}{dt} m_X(t)\Big|_{t=0} = \left(\frac{e^{3t}}{2} + 2e^{4t} + \frac{5}{3}e^{5t}\right)\Big|_{t=0} = \frac{1}{2} + 2 + \frac{5}{3} = \frac{25}{6}$

$E(X^2) = \frac{d^2}{dt^2} m_X(t)\Big|_{t=0} = \left(\frac{3}{2}e^{3t} + 8e^{4t} + \frac{25}{3}e^{5t}\right)\Big|_{t=0} = \frac{3}{2} + 8 + \frac{25}{3} = \frac{107}{6}$

$Var(X) = E(X^2) - (E(X))^2 = \frac{107}{6} - \left(\frac{25}{6}\right)^2 = \frac{642-625}{36} = \frac{17}{36} = 0.47$

Q.8. $p = 2/5$, $q = 3/5$

(a) $q^9 p = (3/5)^9 \times \frac{2}{5} = 4.03 \times 10^{-3}$

(b) $q^9 p = 4.03 \times 10^{-3}$

(c) $P(X=7 \text{ or } X=8 \mid X>6) = P(X=7 \mid X>6) + P(X=8 \mid X>6)$

$= P(X=1) + P(X=2)$

$= p + qp = p(1+q) = \frac{2}{5}\left(1+\frac{3}{5}\right) = \frac{16}{25}$

Q9. X # of comp. tested for getting r non-defectives

$P(X=x) = \binom{x-1}{r-1} p^r q^{x-r}$; $x = r, r+1, r+2, \ldots$; $p = 0.95$

$q = 0.05$

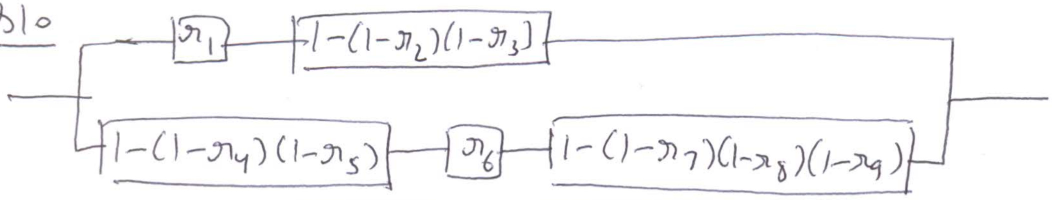
$r = 3$

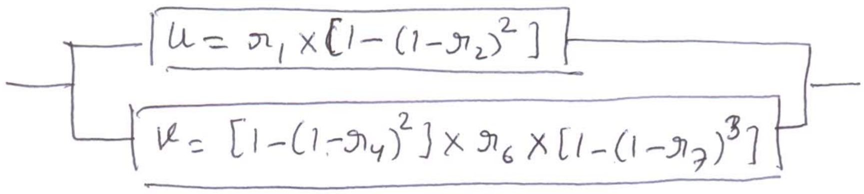
$P(X>5) = 1 - P(X \le 5) = 1 - [P(X=3) + P(X=4) + P(X=5)]$

$= 1 - [p^3 + 3p^2 q + 6p^3q^2] = 1 - p^3[1 + 3q + 6q^2]$

$= 1 - (0.95)^3[1 + 3 \times 0.05 + 6 \times (0.05)^2] = 0.0012$

Q10





$\because r_2 = r_3$
$r_4 = r_5$
$r_7 = r_8 = r_9$

$R_s = 1 - (1-u)(1-v) = 1 - (1 - 0.9634)(1 - 0.9690984)$

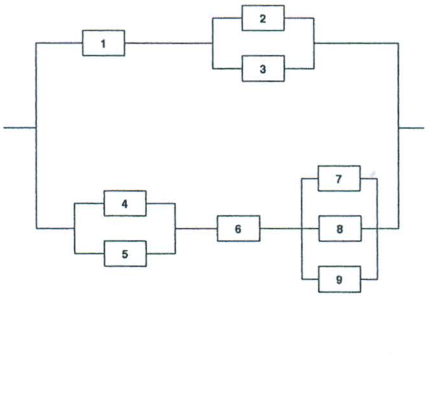
$= 1 - 0.0366 \times 0.0309$

$= 0.9988$

Probability and Stochastic Process (MA20106)	Code: Zqpe2148x6dWer
Name:	Class Test-1
Roll No. Marks:10	Time: 45 mins

Q.No	Question	Answer	Marks
1.	Suppose that $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $P(\{i\}) = \frac{i}{55}$ for $i \in \Omega$. If a number is drawn at random, determine the probability that a even number is drawn.	$6/11 = 0.545$	1
2.	Suppose that two fair dice will be rolled repeatedly and independently until a total of 5 or a total of 7 appears. Find the probability that a total of 5 is rolled before a total of 7 is rolled.	$2/5 = 0.4$	1
3.	Suppose that a basketball player makes 30% of his three-point shot attempts. If the players shoots 10 three-point shots in a game, and X is the number of three-point shots made. Assuming that the shots are independent, find the mean of random variable X.	3	1
4.	Let the CDF of a random variable X is given by $$F(x) = \begin{cases} \frac{1}{2}e^{kx}, & x < 0 \\ 1 - \frac{1}{2}e^{-kx}, & x \geq 0 \end{cases}$$ when $k = 3$. Find $P\left(\lvert X \rvert \leq \frac{1}{k}\right)$.	$1 - e^{-1} = 0.6321$	1
5.	A computer center has three printers A, B and C, which print at different speeds. Programmes are routed to the first available printer. The probability that the programmes are routed to the printers A, B and C are 0.6, 0.3 and 0.1 respectively. Occasinally, a printer will jam and destroy a print out. The probability that printers A, B and C will jam are 0.01, 0.05 and 0.04 respectively. Your programme is destroyed when a printer jams. What is the probability that printer B is involved.	$3/5 = 0.6$	1
6.	Suppose that $P(A) = 0.4$, $P(B) = 0.5$, and A and B are independent events. Determine $P(A^c \cup B^c)$.	$4/5 = 0.8$	1
7.	Let X be a discrete random variable with MGF $M_X(t) = \frac{e^{2t}}{6} + \frac{e^{3t}}{2} + \frac{e^{4t}}{3}$. Determine $\mathrm{Var}(X)$.	$17/36 = 0.4722$	1
8.	An archer can hit the target 1 time if he makes 5 attempts. (a) What is the probability that he can hit the target for the first time on 10 th attempts? (b) What is the probability that he fails 9 times before he hits the target? (c) Suppose it is known that he failed to hit the target in 6 attempts, what is the probability that he can hit the target in next 2 attempts?	(a) 0.026 (b) 0.026 (c) $9/25 = 0.36$	1
9.	In a recent production, 4% of certain electronic components are defective. We need to find 3 non-defective components for our 3 new computers.Components are tested until 3 non-defectives are found. What is the probability that more than 5 components will be tested?	0.0006	1
10.	Consider the system represented in following figure. Let $r_i, i = 1, 2, \ldots 9$ denotes the probability of functioning of ith component, and is given by $r_1 = 0.98, r_2 = r_3 = 0.87, r_4 = r_5 = 0.94, r_6 = 0.99, r_7 = r_8 = r_9 = 0.80$. Compute R_s, the probability of functioning of the system.	0.99	1

any 1 correct 1/2 marks
any 2 correct full marks



Probability and Stochastic Process (MA20106)		Code: Uywe5641y6eTer
Name:		Class Test-1
Roll No.	Marks:10	Time: 45 mins

Q.No	Question	Answer	Marks
1.	Suppose that $\Omega = \{1,2,3,4,5,6,7,8,9,10\}$ and $P(\{i\}) = \frac{i}{55}$ for $i \in \Omega$. If a number is drawn at random, determine the probability that a odd number is drawn.	5/11 =0.4545	1
2.	Suppose that two fair dice will be rolled repeatedly and independently until a total of 4 or a total of 7 appears. Find the probability that a total of 4 is rolled before a total of 7 is rolled.	1/3 =0.33	1
3.	Suppose that a basketball player makes 60% of his three-point shot attempts. If the players shoots 10 three-point shots in a game, and X is the number of three-point shots made. Assuming that the shots are independent, find the mean of random variable X.	6	1
4.	Let the CDF of a random variable X is given by $$F(x) = \begin{cases} \frac{1}{2}e^{kx}, & x < 0 \\ 1 - \frac{1}{2}e^{-kx}, & x \geq 0 \end{cases}$$ when $k = 4$. Find $P\left(\lvert X\rvert \leq \frac{1}{k}\right)$.	$1-e^{-1}$ =0.632	1
5.	A computer center has three printers A, B and C, which print at different speeds. Programmes are routed to the first available printer. The probability that the programmes are routed to the printers A, B and C are 0.6, 0.3 and 0.1 respectively. Occasinally, a printer will jam and destroy a print out. The probability that printers A, B and C will jam are 0.01, 0.05 and 0.04 respectively. Your programme is destroyed when a printer jams. What is the probability that printer C is involved.	4/25 = 0.16	1
6.	Suppose that $P(A) = 0.5$, $P(B) = 0.6$, and A and B are independent events. Determine $P(A^c \cup B^c)$.	0.7	1
7.	Let X be a discrete random variable with MGF $M_X(t) = \frac{e^{4t}}{6} + \frac{e^{5t}}{2} + \frac{e^{6t}}{3}$. Determine $\mathrm{Var}(X)$.	17/36 =0.4722	1
8.	An archer can hit the target 1 time if he makes 3 attempts. (a) What is the probability that he can hit the target for the first time on 10 th attempts? (b) What is the probability that he fails 9 times before he hits the target? (c) Suppose it is known that he failed to hit the target in 6 attempts, what is the probability that he can hit the target in next 2 attempts?	(a) 0.0086 (b) 0.0086 (c) 5/9 = 0.55	1
9.	In a recent production, 6% of certain electronic components are defective. We need to find 3 non-defective components for our 3 new computers.Components are tested until 3 non-defectives are found. What is the probability that more than 5 components will be tested?	0.002	1
10.	Consider the system represented in following figure. Let $r_i, i = 1,2,\ldots 9$ denotes the probability of functioning of ith component, and is given by $r_1 = 0.98, r_2 = r_3 = 0.87, r_4 = r_5 = 0.94, r_6 = 0.99, r_7 = r_8 = r_9 = 0.70$. Compute R_s, the probability of functioning of the system.	0.99	1

any 1 correct 1/2 marks
any two correct full marks