Line Assignment

Name:A.Gowri Priya Email : gowripriyaappayyagari@gmail.com

1 Problem

In Δ ABC and Δ DEF, AB = DE, AB ∥ DE, BC = EF and BC ∥ EF. Vertices A, B and C are joined to vertices D, E and F respectively (see Figure).
Show that
(i) quadrilateral ABED is a parallelogram
(ii) quadrilateral BEFC is a parallelogram
(iii) AD ∥ CF and AD = CF
(iv) quadrilateral ACFD is a parallelogram
(v) AC = DF
(vi)ΔABC ≅ Δ DEF.

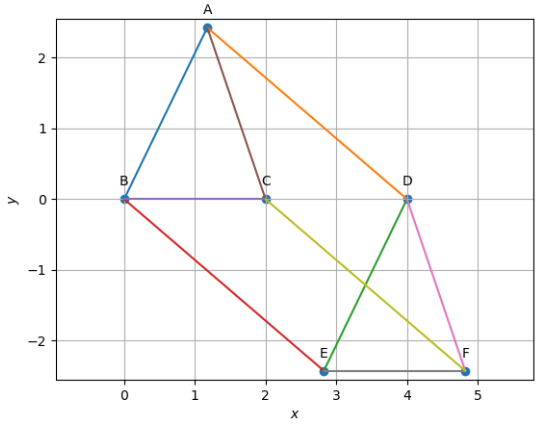

Figure 1: Given Figure

2 Solution

The input parameters for this construction are

Symbol	Value
r1	2
r2	3
θ	$\frac{\pi}{2.5}$

$$\boldsymbol{A} = \begin{pmatrix} \boldsymbol{r1}\cos\boldsymbol{\theta} \\ \boldsymbol{r2}\sin\boldsymbol{\theta} \end{pmatrix}$$

$$\boldsymbol{B} = \begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{0} \end{pmatrix}$$

$$\boldsymbol{D} = \begin{pmatrix} \boldsymbol{4} \\ \boldsymbol{0} \end{pmatrix}$$

$$\boldsymbol{C} = \boldsymbol{B} + \boldsymbol{D}/\boldsymbol{2}$$

$$\boldsymbol{E} = \boldsymbol{B} + \boldsymbol{D} - \boldsymbol{A}$$

$$\boldsymbol{F} = \boldsymbol{E} + \boldsymbol{C} - \boldsymbol{B}$$

Direction vectors

The Direction vectors are

$\boldsymbol{m_1} = \boldsymbol{A} - \boldsymbol{B}$
$\boldsymbol{m_2} = \boldsymbol{B} - \boldsymbol{C}$
$\boldsymbol{m_3} = \boldsymbol{A} - \boldsymbol{C}$
$\boldsymbol{m_4} = \boldsymbol{D} - \boldsymbol{E}$
$\boldsymbol{m_5} = \boldsymbol{E} - \boldsymbol{F}$
$\boldsymbol{m_6} = \boldsymbol{D} - \boldsymbol{F}$
$\boldsymbol{m_7} = \boldsymbol{A} - \boldsymbol{D}$
$\boldsymbol{m_8} = \boldsymbol{C} - \boldsymbol{F}$

To proove

i.Quadrilateral ABED is a parallelogram

Given that AB = DE and AB ∥ DE

Directional vectors $\boldsymbol{m_1}$=$\boldsymbol{m_4}$

i.e $\boldsymbol{A} - \boldsymbol{B} = \boldsymbol{D} - \boldsymbol{E}$

∴ Quadrilateral ABED is a parallelogram.

ii.Quadrilateral BEFC is a parallelogram

Given that BC=EF and BC ∥ EF

Directional vectors $\boldsymbol{m_2}$=$\boldsymbol{m_5}$

i.e $\boldsymbol{B} - \boldsymbol{C} = \boldsymbol{E} - \boldsymbol{F}$

∴ Quadrilateral BEFC is a parallelogram.

iii.AD∥CF and AD=CF

Directional vectors $\boldsymbol{m_7}$=$\boldsymbol{m_8}$

i.e $\boldsymbol{A} - \boldsymbol{D} = \boldsymbol{C} - \boldsymbol{F}$

Since the directional vectors AD and CF are same,AD is parallel to CF.

∴ AD∥CF and AD=CF

iv.Quadrilateral ACFD is a parallelogram

Directional vectors $\boldsymbol{m_3}$=$\boldsymbol{m_6}$

i.e $\boldsymbol{A} - \boldsymbol{C} = \boldsymbol{D} - \boldsymbol{F}$

∴ AC∥DF and AC=DF

∴ACFD is a parallelogram.

v.AC=DF

Directional vectors $\boldsymbol{m_3}$=$\boldsymbol{m_6}$

i.e $\boldsymbol{A - C} = \boldsymbol{D - F}$

∴ AC=DF

vi.ΔABC $\cong$ Δ DEF

$\|\boldsymbol{A - B}\| = \|\boldsymbol{D - E}\|$ and $\|\boldsymbol{B - C}\| = \|\boldsymbol{E - F}\|$ and
$\|\boldsymbol{A - C}\| = \|\boldsymbol{D - F}\|$

∴ By SSS Rule $\Delta ABC \cong \Delta DEF$

3 Execution

*Verify the above proofs in the following code.

`https://github.com/gowripriya-2002/FWC/blob/main/Matrix/line_assignment/code/line.py`