DS 5220

Homework

1. Given $\vec{u} = [2,5/1]^{T}$, $\vec{v} = [3,0,4]^{T}$, $\vec{w} = [1,2,5]^{T}$ for finding $\vec{a} \cdot 3\vec{u} + 4\vec{u}$

-> 3[2] + 4[3] = : Transport - rows will be changed to columns and vice versa

 $\frac{1}{19} = 3\vec{u} + 4\vec{v} = 3\vec{u} + 3\vec{u} + 3\vec{u} + 3\vec{u} + 3\vec{u} + 3\vec{u} + 3$

6. du + 50 - 4w

 $\Rightarrow 2\left(\frac{2}{5}\right) + 5\left(\frac{3}{6}\right) + 4\left(\frac{1}{5}\right) \Rightarrow$

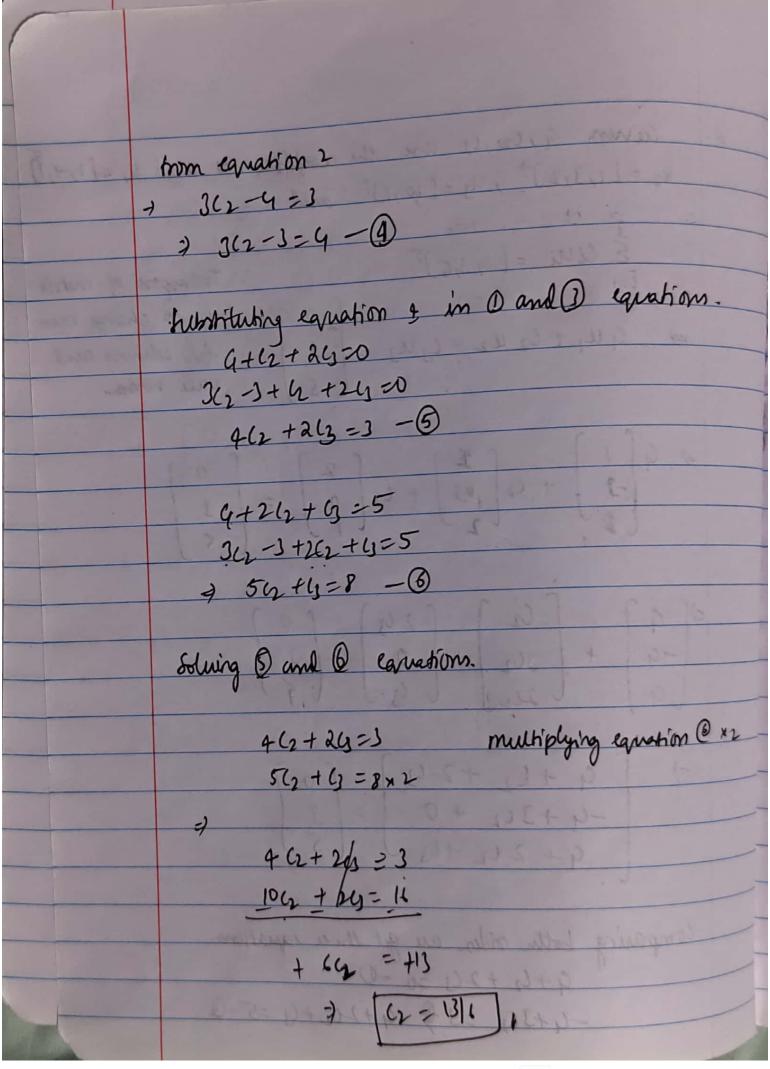
 $\begin{array}{c|c}
3 & 4 \\
10 & + \\
2 & 2
\end{array}$

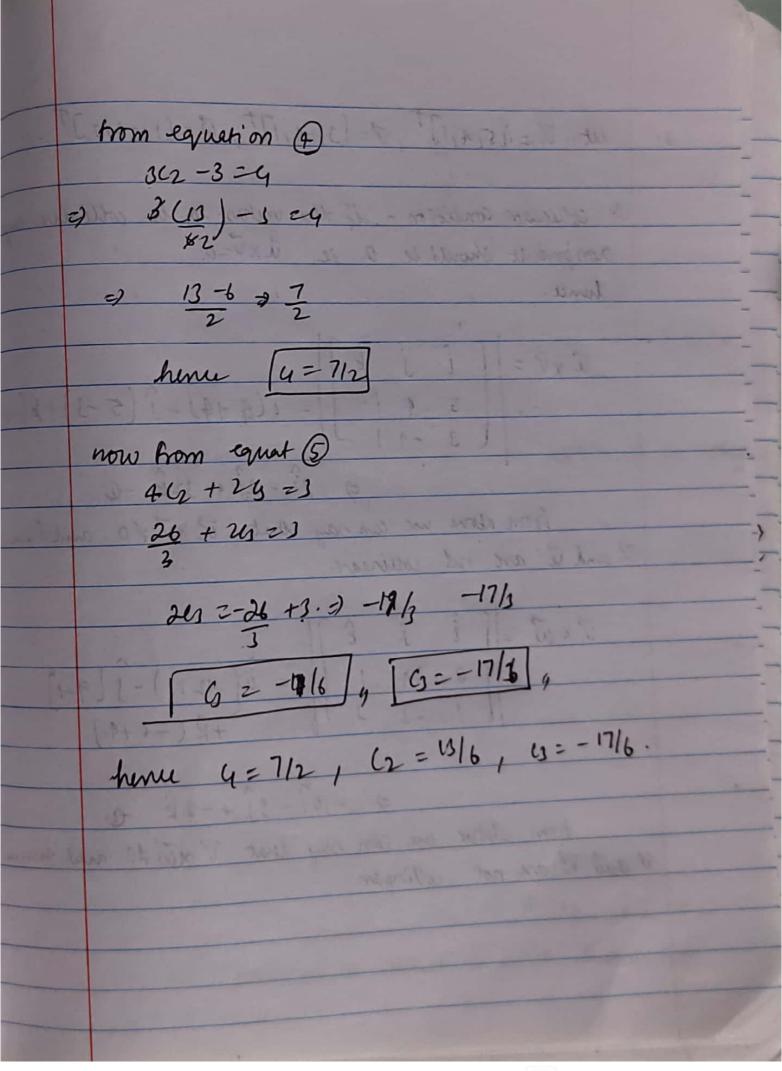
Given 9,(2,1) are the coefficients and 4,2[1,1,1] $42=[1,3,2]^{+}$, $4y=[20;1]^{+}$ and

$$= \begin{cases} 4 \begin{pmatrix} 1 \\ -3 \end{pmatrix} + c_2 \begin{pmatrix} 2 \\ 03 \end{pmatrix} + c_3 \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ 5 \end{pmatrix}$$

$$\begin{cases}
q \\
-q
\end{cases} + \begin{cases}
2y \\
3y \\
4
\end{cases} + \begin{cases}
2y \\
0
\end{cases} = \begin{cases}
0 \\
3
\end{cases}$$

Comparing both vides are get three equations 9 + 62 + 263 = 0-4+362 =3 9 4+262+63 =5-0





ut u=[5,4,1] , v=[3,-4,1] , w=[1,-2,3]

Is collinear condition - if the vertors to be collinear things cross product should be 0. ie il x v=0.

$$\vec{x} \times \vec{v} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 4 & 1 \\ 3 & -4 & 1 \end{bmatrix} = \hat{i} (4 + 4) - \hat{j} (5 - 3) + \hat{k} (-10 - 1)$$

$$= \hat{y} \hat{i} - 2\hat{j} + -32\hat{k} = 0$$

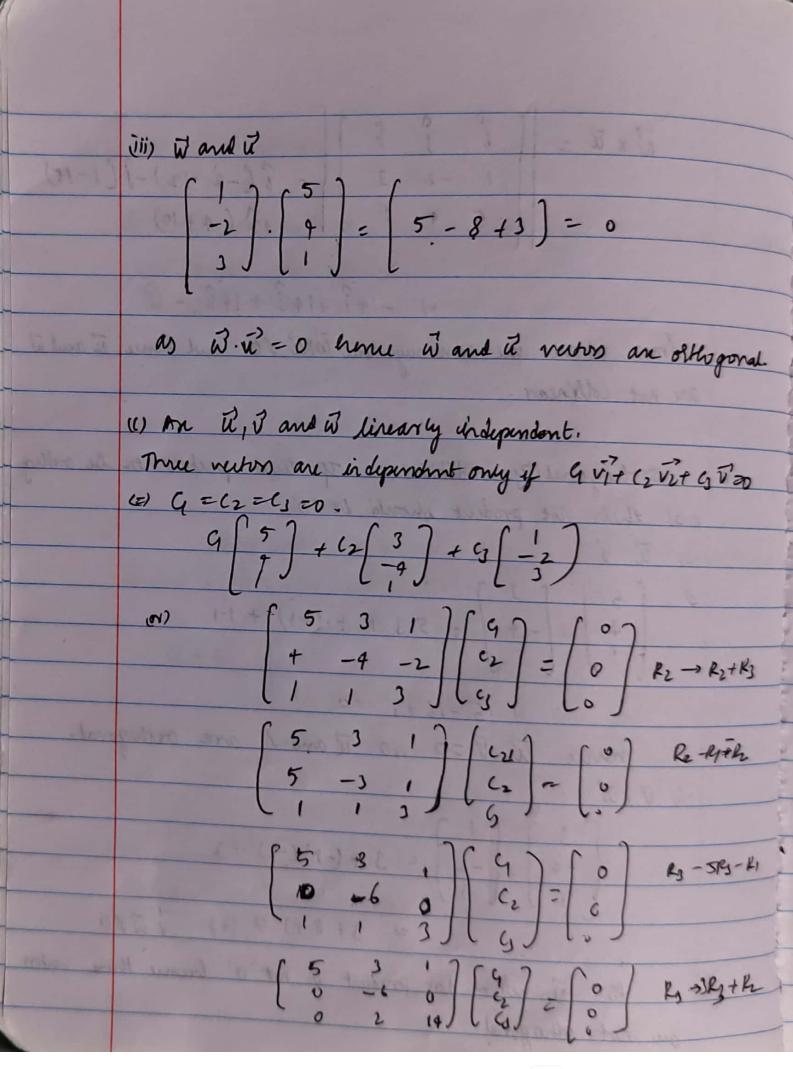
from above we can say that IXXV \$0 and home I and is are not collinear.

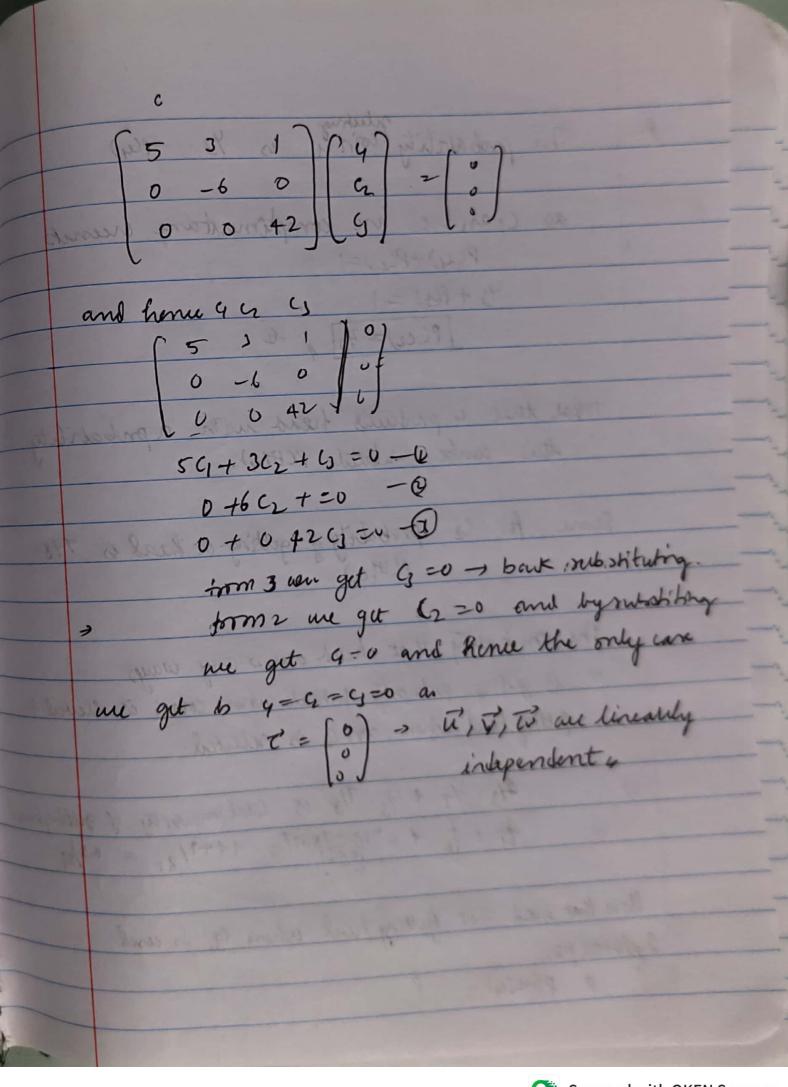
$$\vec{v} \times \vec{\omega} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{E} \\ 3 & -4 & 1 \\ 1 & -2 & 3 \end{bmatrix} = \hat{i} \begin{pmatrix} -12+2 \end{pmatrix} - \hat{j} \begin{pmatrix} 9-1 \\ +\hat{k} \begin{pmatrix} -4+4 \end{pmatrix} \end{pmatrix}$$

= -10î-8j+-2k -

I and I are not collinear.

 $\vec{u} \times \vec{u} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \end{bmatrix} = \hat{i}(-2 - 12) - \hat{j}(1 - 15)$ $= \begin{bmatrix} 5 & 4 & 1 \end{bmatrix} + \hat{k}(4 + 10)$ =) -14î +14î + 14ê - 0 From above we can say that WXW 70 and hence if and is are not collinear. Orthogonality condition is. If two person to be orthig. onal their dot product should be o: $\begin{pmatrix} 5 \\ 4 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix} = 5.3 + 4 \cdot (-4) + 1.1$ henre W.V. 20 no Wand P are orthogonal. $\begin{bmatrix} 3 \\ -4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} = 3 + (-1)(-2) + 3$ As i and is dot product is not o' brence these verbos on not othogonal





W=[1,2,1] T and 0=[3,2,4]T-Le distance = culidian distance. =) 4 (ti, V) = ([ti, -V;]2 $\frac{1}{2} \quad \overline{U} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \quad \overline{V} = \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}$ h= (1-1) + (2-2) + (1-4) 2 = 1 V4+9 = 13y henre 12 distance between it and I is 113. => 3.40 4 distance = 4(1, 7) = É [1, -v,1 $4, 7 \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $7 \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}$ 4 Ly distance = 511-3 + 2-2 + 1-4 = 1-21 +61 + 1-31 henre Wand P = 51

2 = [1,0;5] should be written as linear combonishing of Pi=[1,0,1] , Pi=[1,1,3] , Bi=[2,0,1]

Lo be awitten in linear comprination general from to E Gili hence aniting that way.

$$= \frac{1}{3} \frac{1}{3} \frac{1}{3} + \frac{1}{3} \frac{1}{3} + \frac{1}{3} \frac{1}{3} = \frac{1}{3} = \frac{1}{3} \frac{1}{3} = \frac{1}{3} \frac{1}{3} = \frac{1}{3} \frac{1}{3} = \frac{1}{3} = \frac{1}{3} \frac{1}{3} = \frac{1}{3} \frac{1}{3} = \frac{1}{3} \frac{1}{3} = \frac{1}{3} = \frac{1}{3} \frac{1}{3} = \frac{1}{3}$$

$$= \begin{cases} q + (2 + 2c_3) & = (0) \\ 0 + (2 + 0) & = (0) \\ q + 3(2 - c_3) & = (5) \end{cases}$$

Comparing both miles.

from @ we get (2 to. substituting it in @ and @

hence 9+26=1-4 9+3(0)-4=5-3 => Solving 4 and 5 91+26=1 $3C_{3} = -4C_{3}$ $C_{3} = -4C_{3}$ Substituting 6 in @ 9+24=1 9-8/3=1 4 = 1+8/3 [q=11/3] -0 hence 4=11/3, c2=0, C3=-9/3.

6. Find all $\frac{1}{2}$ K nothat $\tilde{W} = [t_1 K]^T$ and $\tilde{V} = [3] - 4]^T$ are ofthogonal and \tilde{V} has unit length.

for true vectors to be officeronal their dot product is 0. or) should be 0. hence

=> 3t -4k=0 -0

from the injo given its given il has emet lengthe hence distance of is is VE 4.

Vt2+12 =1 S.O.B.S. squaring on Both Lides.

= x2+k2=1 -Q

of from 0 3t-4k to

=> t=411 K -Q

Substituting (1) in (1) =) ((4/s)K) 2+ K2=1

=> 16 K2 + K2 = 1

-) XKL =1 = K= 9/25 | K = ± 3 15 and hence Substituting Kin O t = 418.215 t = 415 -0 t = 418(-(015)) 2 -415 hence we get two pairs of (tik) =

(3/5/4/5) and (-1/5/4)

feny,
$$(n_1y) = \begin{cases} k_{10}, n \in (-4/4) \text{ and } y \in (2/4) \\ 0 \text{ other.} \end{cases}$$

from equation OK=1 + b. Proone (01) disapprone x and y are independent fry (n,y) = frux). fy(y). fn (x) = Sfiny)dy =) } 2/16 dy =) \$ 1/16 dy =) \$ 1/16 } 1 - (4-L) = T1/8]1. fy (4) = John (niy) dn = 1 1/16 dn => 1/16 (4 t+) => 8/16 = 1/2 V me know fry (x14) = f(n). fy (y). so n and y are

) shelpendent. (* Kb). henve e. flyy) = 16 Jet (-4, +) and y (2,+)= fx(x)-fy(4).

P(Y = 3 | X > 0) = fry (x, y) = f fry (n, y) drudy =) J J 1 dn dy =) 3 T 7/16) by P (Y=3 | X = U) = Y+-7 P(x20) = \$ ft 1 dndy 7 1 1 n dy +[1]= 7= Y2 P(463/250) = P(467/x50) = 1/+ PLAZOD

The probability coin & is 43. - 1(4) as complimentary evenus P(4)+P(2)=1 43+P64=1 P(cr)=40 4 6 this canbe whated PCHY. Same for 12 probabily of getting a head is 7/8. The probabity that head orwar any ways is getting probability of head when con a is seleval + getting head when coin is schutch. = 2/3. 1/1 + 2/3. 7/8 is total proposity of gething hear 9 1+ 12 +=+12+7×217 => +++1/8+ = 51/84. Now that orded that getting head when & is und Bystotal pro > P(HVG) = P

243.718 53/84 1 HX 89247 = 49/53 53× 108×84 hence propality. that a head produces and Cz (6) Blad probability from the above we could getting coin 1 and getting head = 1/3.1/7 getting cin 2 and getting head = 25.7/8 1/21 + 7/12 => 53/84. =) 53/84 and it is not equal to 1/2 and hence by this are dispone that probability of getting head its h. and it is P(gukhy head). = 53/84