QUIZ 1

Oct 2, 2021
Gowtham Rajeshshekaran (W1607812)
grajeshshekaran@scu.edu

Q) By Mathematical Induction, prove that

$$\sum_{k=1}^{n} (2k-1) = n^2$$

BASIS STEP

The proposition holds true for the initial value (1)

$$(2(1)-1) = 1^2$$

$$1 = 1$$

INDUCTIVE STEP

Let us assume that the inductive hypothesis holds true for an arbitrary value i that is between $1 \ldots i \ldots n$, (ie)

$$\sum_{k=1}^{i} (2k-1) = i^2$$

$$1 + 3 + 5 + \ldots + (2i-1) = i^2 \quad \text{——} \quad (1)$$

We need to prove that it holds true for $(i+1)$, ie

$$\sum_{k=1}^{i+1} (2k-1) = (i+1)^2$$

$$1+3+5+\dots\dots+(2i-1)+(2i+1) = (i+1)^2 \quad \text{—} \ (2)$$

On substituting (1) in (2), we get

$$i^2 + (2i+1) = (i+1)^2 \qquad [\text{FROM } (2), \text{ IND. HYPOTHESIS}]$$

$$i^2 + 2i+1 = (i+1)^2$$

$$= i^2+1+2i \qquad [\text{LHS} = \text{RHS}]$$

$\therefore$ By Mathematical Induction, we proved that

$$\sum_{k=1}^{n} (2k-1) = n^2$$