Problems:

Problem No 1

Design an overhung crank pin for an engine having the following particulars:

Cylinder diameter	=300mm
Stroke	=500mm
Maximum explosion pressure in the cylinder	=1.8MPa
Engine Speed	=200rpm
Permissible bending stress for pin	=1000MPa
Permissible Bending stress	=85MPa

Given data:

Cylinder diameter	D=300mm
Stroke	L=500mm
Maximum explosion pressure in the cylinder	P_{max}=1.8MPa
Engine Speed	N=200rpm
Permissible bending stress for pin	σ_b=800MPa
Permissible Bearing stress	p_b=85MPa

Solution:

We know that bearing pressure $p_b = \dfrac{F}{(l_p)(d_p)}$ -----------------------------(P1.1)

Where l_p and d_p are length and diameter of the crankpin respectively. p_b is the allowable bearing pressure on the pin, MPa

The length of the crankpin is approximately taken as (0.8 to 1.1) diameter of the crankpin.[Refer page no 50 of the data hand book]

Let us take $l_p=1.1d_p$

We know that gas load $F = \dfrac{\pi}{4} D^2 * p_{max}$

$$F = \frac{\pi}{4} 300^2 * 1.8 = 127234.50N$$

Substituting the values in equation (P1.1) we get

$$85 = \frac{127234.50}{(1.1d_p)(d_p)}$$

Diameter of the crank pin = d_p=36.88mm

Referring the table 3.5a/48, standard diameter of $\mathbf{d_p=40mm}$ is taken. Length of the crankpin $\mathbf{l_p}$=(1.1)(40)=**44mm**

Check:

$$M = \frac{3}{4}(F)(l_p)$$

We know that,

$$\frac{M}{I} = \frac{\sigma_b}{c} \quad \text{……………………………………………………..(1.16/3)}$$

Substituting the values of $c = \frac{d_p}{2}$ and $I = \frac{\pi d_p^4}{64}$ in Equation 1.16 and solving for σ_b we get,

$$(\sigma_b) = \frac{32M}{\pi d_p^3}, \text{ MPa.}$$

$$(\sigma_b) = \frac{32(127234.50)(44)(0.75)}{\pi(40)^3} = 668.25MPa < 800\text{MPa, hence safe.}$$

Problem No 2

A force of 120kN acts tangentially on the crank pin of an overhang crank. The axial distance between the centre of the crankshaft journal and the crank pin is 400mm and the crank is 500mm long. Determine

a) Diameter and length of the crankpin journal.

b) Diameter of the shaft journal

Given that:

Safe bearing pressure	: 5MPa
Bending stress	: 65MPa
Principal stress in the shaft journal	: 65 MPa FEB 2005, [12M] VTU

Given Data:

Referring to Figure 24,

b=400mm and R=500mm

p_b=5MPa, σ_b=65MPa, σ_{max} =65MPa, F=120$(10)^3$N

Solution:

a) We know that, Bearing pressure $p_b = \dfrac{F}{l_p * d_p}$

And assuming ratio of length to diameter of the crank pin as 1.3,

$$5 = \frac{120(10)^3}{1.3(d_p) * d_p}$$

Solving we get, diameter of the crank pin d_p= 135.87mm

Adopting the standard diameter **d_p= 140mm** [T3.4/48]

Minimum length of the crankpin,

$$l_p = \frac{F}{p_b * d_p} = \frac{120(10)^3}{5 * 140} = \mathbf{171.4mm}$$

Check:

$$M = \frac{3}{4}(F)(l_p) \text{ ; We know that, } \frac{M}{I} = \frac{\sigma_u}{c} \text{ ....(1.16/3)}$$

Substituting the values of $c = \dfrac{d_p}{2}$ and $I = \dfrac{\pi d_p^4}{64}$ in Equation 1.16 and solving for σ_b we get,

$$(\sigma_b) = \frac{32M}{\pi d_p^3}\text{, MPa; } (\sigma_b) = \frac{32(120)(10)^3(171.4)(0.75)}{\pi(140)^3} = 57.26MPa < 65MPa \text{ ,hence safe.}$$

b) Bending moment at the shaft journal

M=F(b)=120(10)^3(400)=48(10)^6, N-mm

Twisting moment at the shaft journal,

T=F(R)=120(10)^3(500)=60(10)^6, N-mm

According to maximum normal stress theory,

$$d_s = \left[\frac{16}{\pi\sigma_{max}}\left(M + \sqrt{M^2 + T^2}\right) X \frac{1}{1-K^4}\right]^{\frac{1}{3}} \text{------------------------------(3.5a/42)}$$

Here, because of solid shaft, K=0,

Substituting the values of M, T and σ_{max} in equation 3.5a we get

$$d_S = \left[\frac{16}{\pi(65)}\left((48(10)^6 + \sqrt{(48(10)^6)^2 + (60(10)^6)^2}\right)\right]^{\frac{1}{3}}$$

= 213.85mm

Taking **d_s=220mm** as standard diameter (T3.4/48)

Problem No 3

Determine the maximum normal stress and the maximum shear stress at section A-A for the crank shown in Figure 15 when a load of 10kN is assumed to be concentrated at the center of the crank pin.

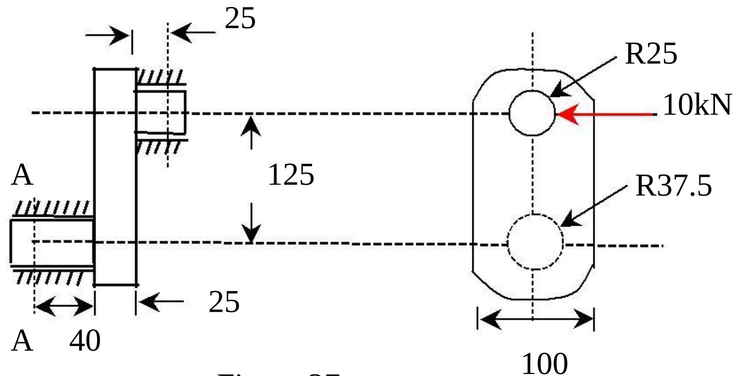

Figure.27

Bending moment $M=10(10)^3(40+25+25)=9(10)^5$, N-mm

Twisting moment $T=10(10)^3(125)=12.5(10)^5$, N-mm

$$\sigma_x = \frac{M(y)}{I} = \frac{9(10)^5(37.5)(64)}{\pi(75)^4} = 21.73MPa$$

$$\tau_{xy} = \frac{T(c)}{J} = \frac{12.5(10)^5(37.5)(32)}{\pi(75)^4} = 15.10MPa$$

Maximum Principal Stress:(σ_1)

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(1.11a/2)$$

$$\sigma_1 = \frac{21.73+0}{2} + \sqrt{\left(\frac{21.72-0}{2}\right)^2 + 15.10^2} = \mathbf{\underline{29.46MPa}}$$

Maximum Shearing Stress:(τ_{max})

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots...(1.12/2)$$

$$\tau_{max} = \pm\sqrt{\left(\frac{21.72-0}{2}\right)^2 + 15.10^2} = \mathbf{\underline{18.60MPa}}$$

Problem No 4

Design a plain carbon steel centre crankshaft for a single acting four stroke single cylinder engine for the following data:
Bore = 400 mm; Stroke = 600 mm; Engine speed = 200 rpm.; Mean effective pressure = 0.5 N/mm^2; Maximum combustion pressure = 2.5 M/mm^2; Weight of flywheel used a pulley = 50 kN; Total belt pull = 6.5 kN.

When the crack has turned through 35^0 from the top dead centre, the pressure on the piston is 1N/mm^2 and the torque on the crank is maximum. The ratio of the connecting rod length to the crank radius is 5. Assume any other date required for the design.

Given Data:

D=400mm, L=600mm or R=300mm, p_{mean}=0.5MPa, p_{max}=2.5MPa, W=50(10)^3 N, T_1+T_2=6.5(10)^3 N, θ=35^0, p_{35}=1MPa, (l/R)=5

Crankshaft is designed for the two positions:

a) Crank is at dead center; b) Angle of maximum twist;

a) Design of the crankshaft when the crank is at the dead center

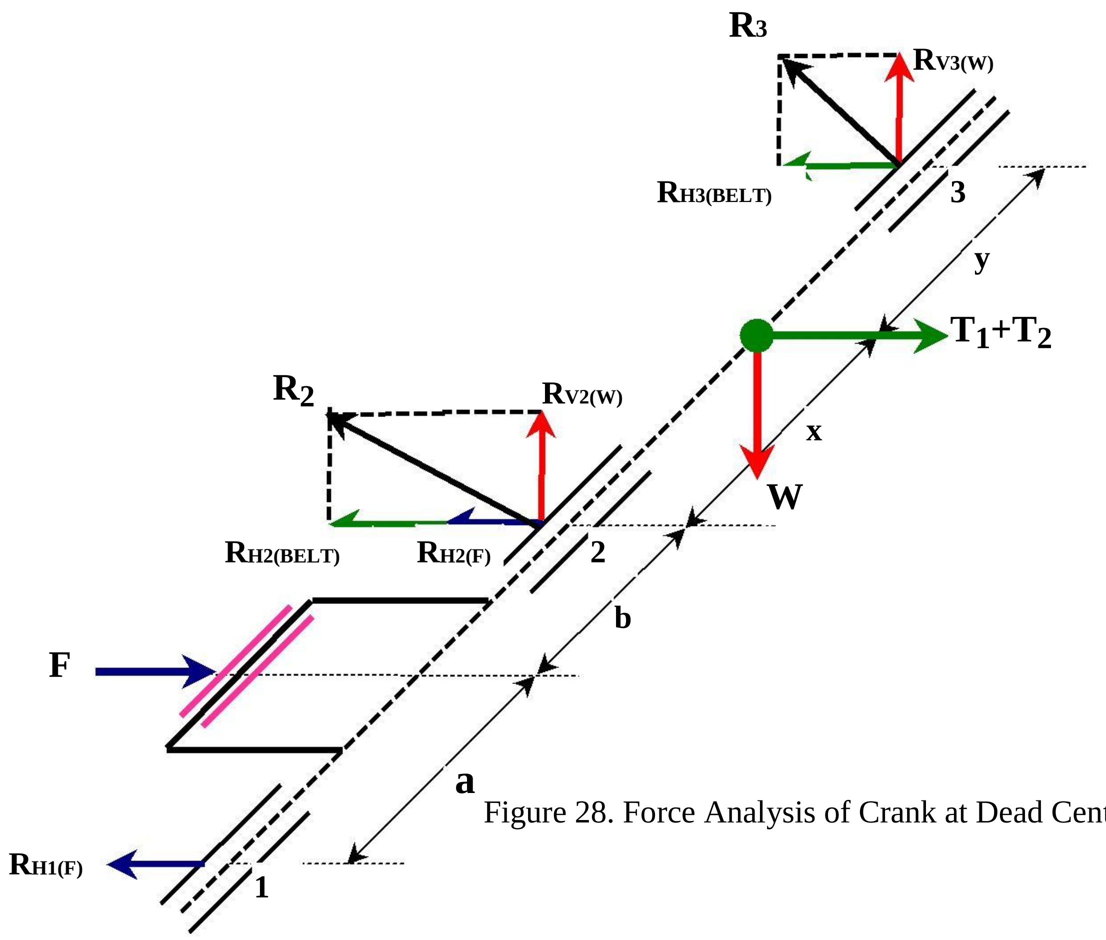

Figure 28. Force Analysis of Crank at Dead Center

Piston Gas load

$$F = \frac{\pi}{4} D^2 * p_{\max} = \frac{\pi}{4} 400^2 * 2.5 = \mathbf{\underline{314.16(10)^3\ N}}$$

Assume that the distance between bearing 1 and 2 is equal to twice the piston diameter (D) and distance a=b.

Therefore $a = b = \frac{2 * D}{2} = \frac{2 * 400}{2} = 400mm$

Due to gas load, there will be two horizontal reactions, $R_{H1(F)}$ at bearing 1, and $R_{H2(F)}$, at bearing 2, so that,

To find the reactions $R_{H1(F)}$ and $R_{H2(F)}$

Since a=b, then, $R_{H1(F)} = R_{H2(F)} = \frac{F}{(2)}, N$

$$R_{H1(F)} = R_{H2(F)} = \frac{314.16(10)^3}{(2)}, N = 157.08(10)^3, N$$

In between bearings 2 and 3, we have two loads

i) Belt pull ($T_1 + T_2$), acting horizontally as shown in Figure 28

ii) Weight of the Flywheel (W), acting vertically as shown in Figure 28

Reactions at bearing 2 and 3 due to Belt Pull,

Due to this there will be two horizontal reactions, $R_{H2(belt)}$ at bearing 2, and $R_{H3(belt)}$ at bearing 3, so that ,

Taking x=y; Its value is computed after calculating the crankpin length.

Since, x=y, then, $R_{H2(belt)} = R_{H3(belt)} = \frac{(T_1 + T_2)}{(2)}$

$$R_{H2(belt)} = R_{H3(belt)} = \frac{(6.5(10)^3)}{(2)} = 3.25(10)^3, N$$

Reactions at bearing 2 and 3 due to Weight of the Flywheel,

Since, x=y, therefore $R_{V2(W)} = R_{V3(W)} = \frac{(W)}{(2)}$

then $R_{V2(W)} = R_{V3(W)} = \frac{(50(10)^3)}{(2)} = 25(10)^3, N$

In this position of the crank, there will be no twisting moment, and the various parts will be designed for bending only.

CRANKPIN:

The bending moment at the centre of the crankpin is,

$M = R_{H1(F)}(a)$, N-mm

$=157.08(10)^3(400) =$

$62832(10)^3$, N-mm

We know that,

$$\frac{M}{I} = \frac{\sigma_b}{c} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.16/3)$$

σ_b=allowable bending stress for the crankpin. It may be assumed as 83MPa. (Refer T3.5b/48)

Substituting the values of $c = \frac{d_p}{2}$ and $I = \frac{\pi d_p^4}{64}$ in Equation 1.16 and solving for M we get,

$$M = \frac{\sigma_b}{c}(I) = \frac{\sigma_b}{\frac{d_p}{2}}\left(\frac{\pi}{64}\right)(d_p^4)$$

$$M = \frac{\pi}{32} d_p^3 (\sigma_b) \text{ , N-mm}$$

$$62832(10)^3 = \frac{\pi}{32} d_p^3 (83)$$

We get d_p=197.56mm.

Standard value of diameter **d_p=200mm** is adopted. (Refer T3.5a/48)

Length of the crankpin (l_p) can be obtained by suitably choosing the value of allowable bearing pressure.

We know that bearing pressure for the given type of engine is between 9.6 MPa to 12.4 MPa. Let us take p_b=10MPa. (Refer T15.11/314)

Bearing pressure, $p_b = \frac{F}{(l_p)(d_p)}$, MPa

Length of the crankpin, $l_p = \frac{F}{(d_p)(p_b)}$, mm $= \frac{314.16(10)^3}{(200)(10)} =$ **157mm**

6.4a.2 Left Hand Crank Web:

The crank web is designed for eccentric loading. There will be two stresses on it, one *direct compressive stress* and the other *bending stress* due to the gas load F.

The thickness h=0.65 d_p + 6.35mm……………………………………(Page No 50)

=0.65(200)+6.35

=136.35mm

Let us take **h=137mm**

The width 'w'may be assumed to be as follows:

$$w = \frac{9}{8} d_p + 12.7,\ mm \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(Page No 50)}$$

$$= \frac{9}{8} 200 + 12.7,\ mm$$

w=237.7mm

Let us take **w=238mm**

Since the empirical relations are used it is advised to check the developed stresses against the given values.

Direct stresses (σ_d)

$$\sigma_d = \frac{R_{1H(F)}}{(w)(h)},\ MPa$$

$$\sigma_d = \frac{157.08(10)^3}{(238)(137)},\ MPa = \mathbf{4.82MPa}$$

Bending stresses: (σ_b)

$$\frac{M}{I} = \frac{\sigma_b}{c}\ ;\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1.16/3)}$$

$$M = R_{1H(F)}\left(a - \frac{l_p}{2} - \frac{h}{2}\right)$$

$$I = \frac{bh^3}{12} \text{ And } c = \frac{h}{2}$$

Substituting the values of M, c and I in bending equation (1.16/3) we get

$$\sigma_b = R_{1H(F)}\left(a - \frac{l_p}{2} - \frac{h}{2}\right)\left(\frac{6}{bh^2}\right),\ MPa$$

$$\sigma_b = 157.08(10)^3\left(400 - \frac{157}{2} - \frac{137}{2}\right)\left(\frac{6}{238(137)^2}\right),\ MPa$$

$$= \mathbf{53.38MPa}$$

Superimposing the direct and bending stresses, we get

Total stress on the crank

web=σ_d+σ_b=4.85+53.38=**58.23MPa<83MPa**, Hence Design is safe.

Right Hand Crank Web:

From the balancing point of view, the dimensions of the right hand crank web **h=137mm** and **w=238mm** are taken equal to the dimensions of the left hand crank web.

Shaft Under the Flywheel: [Diameter of the shaft between bearing 2 and 3]

Length of the bearing, $l_1 = l_2 = l_3 = 2\left(a - \frac{l_p}{2}\right) - h$

$$l_1 = l_2 = l_3 = 2\left(400 - \frac{157}{2}\right) - 137 = \mathbf{369mm}$$

Assuming width of the flywheel as 300mm, we have

x+y=369+300+clearance=369+300+131(to make it round off)

=800mm.

Taking x=y, we have x=y=400mm

Bending moment due to flywheel weight is $M_{FLY} = (R_{V3(W)})(y)$

$=25(10)^3(400)$

$=\mathbf{10(10)^6, N\text{-}mm}$

Bending moment due to the belt pull is $M_{belt} = (R_{H3(BELT)})(y)$

$=3.25(10)^3(400)$

$=\mathbf{1.3(10)^6, N\text{-}mm}$

Since these bending moments act at right angles to each other, the combined bending moment is given by;

$$M_{Total} = \sqrt{M_{FLY}^2 + M_{belt}^2}$$

$$= \sqrt{(10*10^6)^2 + (1.3*10^6)^2}$$

$$\mathbf{=10.08(10)^6, N\text{-}mm}$$

We know that $M_{Total} = \frac{\sigma_b}{c}(I) = \frac{\sigma_b}{\frac{d_W}{2}}\left(\frac{\pi}{64}\right)(d_W^4)$

$$M_{total} = \frac{\pi}{32} d_W^3 (\sigma_b), \text{ N-mm},$$

For plain carbon steel taking σ_b=65MPa

[Ref T1.8/418, taking FOS n=4, Yield stress=196MPa]

$$10.08(10)^6 = \frac{\pi}{32} d_W{}^3 (65)$$

Diameter d_W=116.46mm, Use standard diameter as **<u>d_W=125mm</u>** [Ref. T3/48]

b. Crank at an angle of maximum twisting moment

We know that piston gas load $F_p = \frac{\pi}{4} D^2 * p_{35} = \frac{\pi}{4} 400^2 * 1 =$ **$125.66(10)^3$ N**

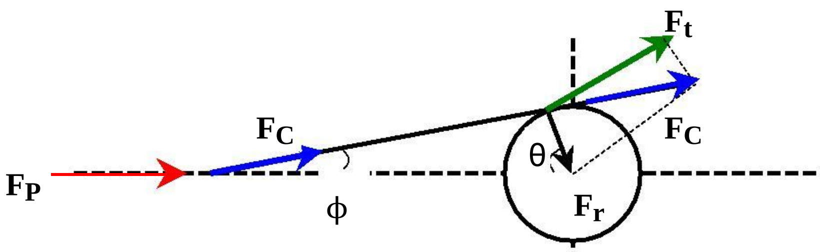

Figure 29. Forces on crank Arm

We know that, $\sin(\varphi) = \frac{\sin(\theta)}{L/R}$ --------------------------<Remember>

Where φ is the angle of inclination of the connecting rod with the line of stroke.

$$\sin(\varphi) = \frac{\sin(35)}{5} = 0.1147$$

Therefore **$\varphi = 6.58^0$**

The force on the connecting Rod or thrust force

$$F_C = \frac{F_P}{\cos(\varphi)} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3.12/45)$$

$$F_C = \frac{125.66(10)^3}{\cos(6.58)} = \mathbf{126.50(10)^3\ N}$$

The tangential force or the rotative effort on the crank

$$Ft = F_C \sin(\varphi + \theta) \quad \ldots\ldots\ldots\ldots\ldots\ldots(3.13/45)$$

$$Ft = 126.50(10)^3 \sin(6.58 + 35) = \mathbf{83.95(10)^3,\ N}$$

The radial force along the crank

$$Fr = F_C \cos(\varphi + \theta) \quad \ldots\ldots\ldots\ldots\ldots\ldots(3.14/45)$$

$$Fr = 126.50(10)^3 \cos(6.58 + 35) = \mathbf{94.63(10)^3,\ N}$$

Tangential force F_t will have two reactions R_{H1FT} and R_{H2FT} at bearing 1 and 2 respectively.

Radial force F_r will have two reactions R_{H1FR} and R_{H2FR} at bearing 1 and 2 respectively.

The reactions at the bearings 2 and 3 due to belt pull (T_1+T_2) and Flywheel W will be same as before.

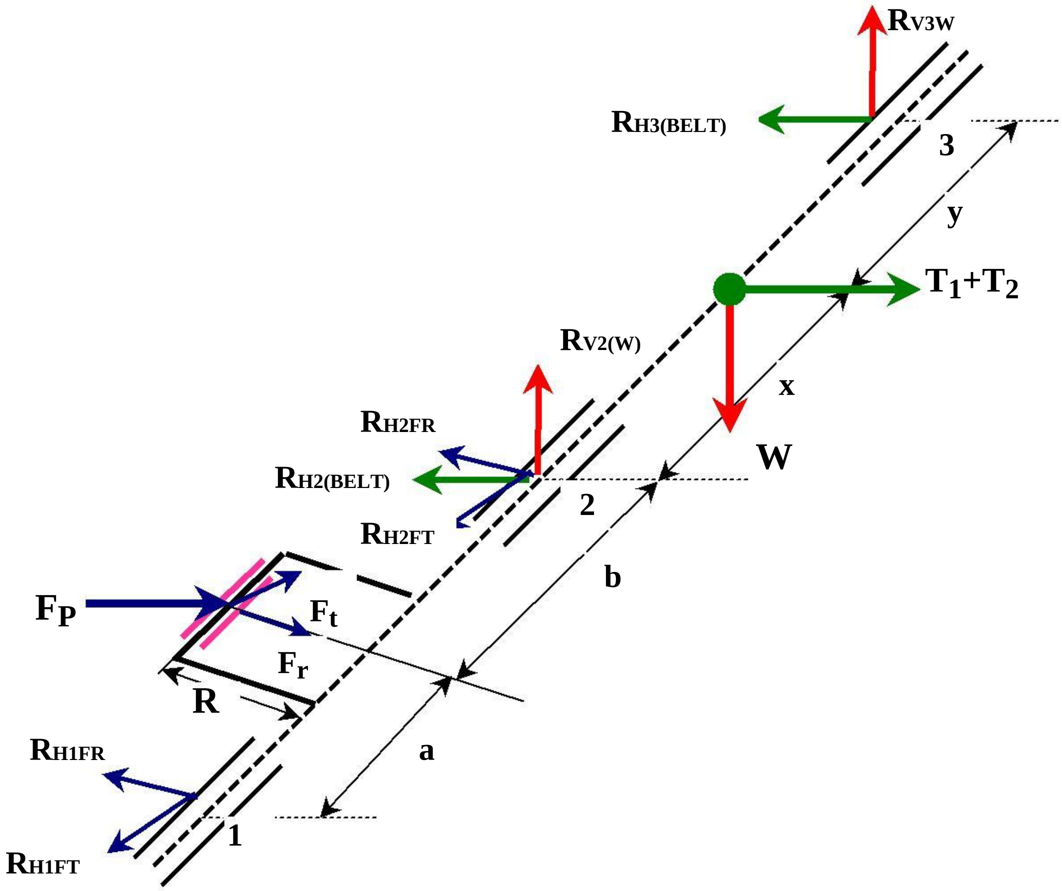

Figure 30. Force Analysis of Crank at angle of maximum twisting Moment

In this position of the crankshaft, the different sections will be subjected to both bending and torsional moments and these must be checked for combined stress. At this point, Shear stress is taken as failure criteria for crankshaft.

The reactions due Radial force (F_r):

$$R_{H1FR} = R_{H2FR} = \frac{F_r}{(2)} = \frac{94.63(10)^3}{2} = \mathbf{\underline{47.315(10)^3, N}}$$

The reactions due tangential force (F_t):

$$R_{H1FT} = R_{H2FT} = \frac{F_t}{(2)} = \frac{83.95(10)^3}{(2)} = \mathbf{\underline{41.975(10)^3, N}}$$

The reactions at the bearings 1 and 2 due to Flywheel weight (W) and resultant belt pull (T_1+T_2) will be as discussed earlier.

Crank pin:

The bending moment at the centre of the crankpin is, $M_b = R_{H1FR}(a)$, N-mm

$M_b = 47.315(10)^3 (400) =$ **$18.926(10)^6$, N-mm**

The Twisting moment is, $T = R_{H1FT}(R)$, N-mm

$T = 41.975(10)^3 (300) =$ **$12.60(10)^6$, N-mm**

Equivalent twisting moment, $T_e = \sqrt{T^2 + M_b^2}$, $N-mm$

$T_e = \sqrt{(12.60 * 10^6)^2 + (18.926 * 10^6)^2}$, $N-mm =$ **$22.737(10)^6$, N-mm**

We know that $T_e = \frac{\pi}{16} d_P^3 (\tau)$, N-mm

$22.737(10)^6 = \frac{\pi}{16} d_P^3 (42)$ *(The value of τ=0.4 to 0.6σ)*

Solving we get, d_p=139.1mm.

Since this value of crankpin is less than the already calculated value of d_p=200mm,(i.e higher among the two).

We shall take **d_p=200mm** and **l_p=157mm**

Shaft under the Flywheel: [Diameter of the shaft between bearing 2 and 3]

The collective bending moment due to flywheel and the belt pull will be the same as earlier.

Bending moment due to flywheel weight is $M_{FLY} = (R_{V3(W)})(y)$

Bending moment due to the belt pull is $M_{belt} = (R_{H3(BELT)})(y)$

Since these bending moments act at right angles to each other, the combined bending moment is given by;

$M_{Total} = \sqrt{M_{FLY}^2 + M_{belt}^2}$, N-mm

$= \sqrt{(10 * 10^6)^2 + (1.3 * 10^6)^2}$

$=$ **$10.08(10)^6$, N-mm**

In addition to this moment there will be a twisting moment because of tangential force F_t. The twisting moment, $T = F_t(R)$, N-mm

$T = 83.95(10)^3 (300) =$ **$25.185(10)^6$, N-mm**

Therefore Equivalent twisting moment,

$T_e = \sqrt{T^2 + M_{Total}^2}$, $N-mm$

$T_e = \sqrt{(25.185(10)^6)^2 + (10.08(10)^6)^2}$, $N-mm =$ **$27.13(10)^6$, N-mm**

We have, $T_e = \frac{\pi}{16} d_W^{\,3} (\tau)$, N-mm,

τ=(0.5 to 0.6)*σ=(0.5 to 0.6)*65=32.5MPa to 39MPa.
Let us take τ=35MPa

$$27.13(10)^6 = \frac{\pi}{16} d_W^{\,3} (35)$$

d_W=157.25mm
Standard value of d_W=160mm is adopted.
Earlier value of d_W is 125mm is less than d_W=160mm. Hence **d_W=160mm**

6.4b.3 Right hand Crank Web:

We have used empirical formulae to obtain the values of crank web dimensions. And also we know that the Right hand Crank Web is severely stressed. In order to find the correctness of the dimensions of the web it is necessary to check the developed stresses against the allowable stresses. This web is subjected to bending stresses in two planes normal to each other, due to radial and tangential components of F_P; to direct compression; and to torsion.

The various dimensions obtained are
w= 238mm; h=137mm; l_p=157mm; d_p=200mm;

The bending moment due to radial component is

$$M_{rad} = R_{H\,2FR}\left(b - \frac{l_p}{2} - \frac{h}{2}\right), \text{ N-mm}$$

$$M_{rad} = 47.315(10)^3\left(400 - \frac{157}{2} - \frac{137}{2}\right) = \mathbf{11.97(10)^6,\ N\text{-}mm}$$

Bending stress in radial direction

$$\sigma_{rad} = M_{rad}\left(\frac{6}{wh^2}\right), MPa$$

$$\sigma_{rad} = 11.97(10)^6\left(\frac{6}{238(137)^2}\right), MPa = \mathbf{16.08MPa}$$

The bending moment due to tangential component is maximum at the juncture of the crank and shaft.

$M_{Tang} = F_t\,(R)$, N-mm (Since here shaft diameter at junction is not considered for calculation. By doing so the bending moment increases and hence the stresses, which leads to safer side.)

$$M_{Tang} = 83.95(10)^3 (300) = \mathbf{25.185(10)^6,\ N\text{-}mm}$$

$$\sigma_{Tang} = M_{Tang} \frac{6}{w^2 h}, \text{ MPa}$$

$$\sigma_{Tang} = 25.185(10)^6\left(\frac{6}{238^2(137)}\right), MPa = \mathbf{\underline{19.47MPa}}$$

The stress due to direct compression, $\sigma_d = \frac{F_r}{2wh}, MPa$

$$\sigma_d = \frac{94.63(10)^3}{2(238)(137)}, MPa = \mathbf{\underline{1.45MPa}}$$

Superimposing the stresses (At the upper left corner to the cross section of the crank) will be equal to

$$\sigma_{total} = \sigma_{rad} + \sigma_{Tang} + \sigma_d, MPa$$

$$\sigma_{total} = 16.08 + 19.47 + 1.45, MPa = \mathbf{\underline{37MPa}}$$

Now the twisting moment, on the arm is

$$T = R_{HIFT}\left(a + \frac{l_p}{2}\right) - F_T\left(\frac{l_p}{2}\right) = R_{H2FT}\left(b - \frac{l_p}{2}\right), N - mm$$

$$T = 41.975(10)^3\left(400 - \frac{157}{2}\right), N - mm = \mathbf{\underline{13.49(10)^6}}$$

We know that,

Shear stress,

$$\tau_{xy} = \frac{T}{J}(c) = \frac{T}{Z}$$

Where Z – polar section modulus, = $\frac{wh^2}{4.5}, mm^3$

$$\tau_{xy} = \frac{T}{J}(c) = \frac{T}{Z} = \frac{13.49(10)^6(4.5)}{238(137)^2} = \mathbf{\underline{13.60MPa}}$$

Therefore maximum combined stress is given by,

Total combined stress,

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \text{………………………………..(1.11a/2)}$$

Here $\sigma_1 = \sigma_{max}$; $\tau_{xy} = \tau$; $\sigma_x = \sigma_{total}$; $\sigma_y = 0$;

$$\sigma_1 = \frac{37 + 0}{2} + \sqrt{\left(\frac{37 - 0}{2}\right)^2 + 13.60^2} = 41.46MPa < 83MPa, \text{Design is safe.}$$

Left hand Crank Web:

This crank web is less severely stressed than the right hand crank since it is not to transmit any power while the right hand crank transmits the power to the flywheel and to the power take off. Hence there is no need to check the left hand crank and its dimensions may be taken as that of the right hand crank.

Crankshaft bearings**:**

The distance between bearing 1 and bearing 2 may be assumed to be equal to twice the cylinder diameter. From the length of the crankpin and the thickness of the arm, the lengths of the bearings can be found out. Bearing 2 is the most heavily loaded, therefore, only this bearing may be checked for the safe bearing pressure.

We know that the total reaction at the bearing 2,

$$R_2 = \frac{F_p}{2} + \frac{W}{2} + \frac{T_1 + T_2}{2}$$

, N here F_P to taken as maximum, i.e. **314.16(10)^3 N** instead of **125.66(10)^3 N**

$$R_2 = \frac{314.16(10)^3}{2} + \frac{50(10)^3}{2} + \frac{6.5(10)^2}{2} = \mathbf{185.33(10)^3,\ N}$$

Therefore bearing pressure $p_b = \frac{R_2}{(L)(d)}$, MPa here d=d_w=160mm, L=369mm

$p_b = \frac{185.33(10)^3}{(369)(160)}$, MPa =3.14MPa<10MPa, hence the design of bearing is safe.

Problem No 5

Design a side or overhung crankshaft for a 250mm X 300 mm gas engine. The weight of the flywheel is 30kN and the explosion pressure is 2.1 MPa. The gas pressure at the maximum torque is 0.9 MPa, when the crank angle is 35^0 from I.D.C. The connecting rod is 4.5 times the crank radius.

Given Data:

D=250mm, L=300mm, or R=150mm, $W=30(10)^3$N, p_{max}=2.1MPa and p_{35}=0.9MPa, l/r=4.5

Material taken: σ_b=allowable bending stress for the crankpin= 83MPa. (Refer T3.5b/48)

Solution:

Crankshaft is designed for the two positions:

a) Crank is at dead center;
b) Angle of maximum twist;

a) Design of the crankshaft when the crank is at the dead center

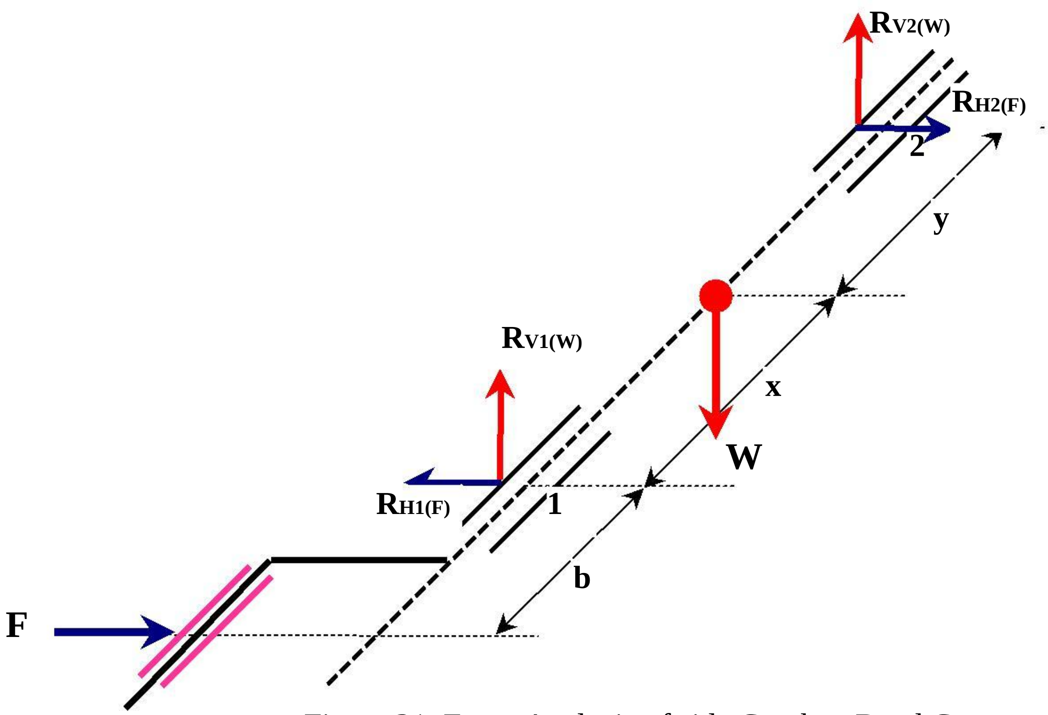

Figure 31. Force Analysis of side Crank at Dead Center

Gas Load, $F = \frac{\pi}{4} D^2 * p_{\max}$

$F = \frac{\pi}{4} 250^2 * 2.1 = \mathbf{103.1(10)^3, N}$

Crankpin:

The dimensions of the crankpin are obtained by considering the crankpin in bearing and then checked for bending stress.

We know that bearing pressure $p_b = \dfrac{F}{(l_p)(d_p)}$

Where l_p and d_p are length and diameter of the crankpin respectively. p_b is the allowable bearing pressure on the pin, MPa

The length of the crankpin is approximately taken as (0.8 to 1.1) diameter of the crankpin.[Refer page no 50 of the data hand book]

Let us take, $l_p=d_p$ And solving for the dimensions of crankpin, we get

$$10 = \frac{103.1(10)^3}{(d_p)(d_p)}$$

Diameter of the crankpin d_p=101.54mm

Standard diameter **d_p= 110mm** is adopted (Refer T3.5a/48) Length of the crankpin **l_p=110mm**

Check:

Bending moment $M = \frac{3}{4}(F)(l_p)$ may be used.

$$M = \frac{3}{4}(103.1(10)^3)(110) = \mathbf{8.51(10)^6, N\text{-}mm}$$

We know that,

$$\frac{M}{I} = \frac{\sigma_b}{c} \quad \text{..........(1.16/3)}$$

Substituting the values of $c = \dfrac{d_p}{2}$ and $I = \dfrac{\pi d_p^4}{64}$ in Equation 1.16 and solving for σ_b we get,

$$(\sigma_b) = \frac{32M}{\pi d_p^3}, \text{ MPa.}$$

$$(\sigma_b) = \frac{32(8.51)(10)^6}{\pi(110)^3} = \mathbf{65.13MPa.<83MPa.}$$

This induced bending stress should be within the permissible limits, Hence design is safe.

Design of bearings:

Let d_1 be the diameter of the bearing 1.

Thickness of web t=h=(0.5 to 0.9) d_p ……………………………………………..(Page No 50) Let us take h=0.6d_p=0.6(110)=**66mm**

Length of the bearing l_1=1.7d_p=1.7(110)=**187mm**

We know that bending moment,

$$M = F\,(0.75 l_p + h + 0.5(l_1)),\ N-mm$$

$$M = 103(10)^3\,[0.75(110) + 66 + 0.5(187)],\ N-mm$$

M=**$25(10)^6$, N-mm**

We know that bending stress, $(\sigma_b) = \dfrac{32M}{\pi d_1^3}$

Assuming bearing material as Phosphor bronze, σ_b=68.65MPa [Refer T15.2/309]

Solving for d_1, we get

$$(68.65) = \frac{32(25(10)^6)}{\pi d_1^3}$$

The diameter of the bearing

d_1=154.72mm Let us take d_1=**155mm**

The bearing dimensions are taken same for bearing 2. i.e **$l_1=l_2$=187mm**

Design of crank web w=Width of the crank web, mm We know that bending moment,

$$M = F\,(0.75 l_p + 0.5(h)),\ N-mm$$

$$M = 103(10)^3\,[0.75(110) + (0.5)66],\ N-mm$$

M=**$11.9(10)^6$, N-mm**

Bending stress $\sigma_b = M \dfrac{6}{w\,h^2}$, MPa

$$\sigma_b = 11.9(10)^6\left(\frac{6}{w(66)^2}\right),\ MPa = \frac{16.39(10)^3}{w},\ MPa$$

The direct Stress, $\sigma_d = \dfrac{F_p}{wh},\ MPa$

$$\sigma_d = \frac{103(10)^3}{(w)(66)},\ MPa = \frac{1.56(10)^3}{w},\ MPa$$

Superimposing the stresses and equating to allowable stress we get

$$\sigma_{all} = \sigma_b + \sigma_d,\ MPa$$

$$83 = \frac{16.39(10)^3}{w} + \frac{1.56(10)^3}{w}$$

The width of crank web **w=216.3mm**

Design of shaft under the flywheel

Let d_S be the Diameter of shaft under the flywheel.
Assuming the width of the flywheel as 250mm

$$\text{Length } (x + y) = 250 + \frac{l_1}{2} + \frac{l_2}{2} + Clearance$$

$$(x + y) = 250 + \frac{187}{2} + \frac{187}{2} + 23 = 460mm$$

$$b = 0.75 l_p + h + \frac{l_1}{2}$$

$$b = 0.75(110) + 66 + \frac{187}{2}$$

=242mm Taking x=y=230mm

Reactions:

Reactions at bearing 1 and 2 due to Weight of the Flywheel (W),

Due to this there will be two Vertical reactions, $R_{v1(W)}$ at bearing1, and $R_{v2(W)}$, at bearing2. Here x=y, then $R_{V1(W)} = R_{V2(W)} = \left(\frac{W}{2}\right)$

$$R_{V1(W)} = R_{V2(W)} = \frac{(30(10)^3)}{(2)} = 15(10)^3, N$$

Reactions at bearing 1 and 2 due to Piston Gas Load(F),

Due to this piston gas load there will be two horizontal reactions, $R_{H1(F)}$ at bearing 1, and $R_{H2(F)}$, at bearing 2.

To find the reactions $R_{H1(F)}$ and $R_{H2(F)}$

$$\sum M_1 = 0, \quad \circlearrowright + \quad \circlearrowleft -$$

Clock Wise direction is taken as positive bending moment and Counter Clockwise as negative bending moment.

$$\rightarrow \quad -F(b) + R_{H2(F)}(x + y) = 0$$

$$R_{H2(F)} = \frac{F(b)}{(x + y)} = \frac{103(10)^3(242)}{(230 + 230)} = 54.2(10)^3, N$$

$\sum F_y = 0$, $\uparrow$, Upward force is taken as positive and downward is taken as negative.

$$\rightarrow \quad R_{H1(F)} - R_{H2(F)} - F = 0$$

$$R_{H1(F)} = F + \frac{F(b)}{(x + y)} = \frac{F(b + x + y)}{(x + y)}, N$$

$$R_{H1(F)} = \frac{103(10)^3(242+230+230)}{(230+230)}, N = 157.2(10)^3, N$$

Since there is no belt tension, therefore the horizontal reactions due to the belt tension are not taken.

In this position of the crank, there will be no twisting moment, and the various parts will be designed for bending only.

Horizontal Bending Moment due to Piston Gas load

$$M_{Gas} = F(b+x) - R_{H1(F)}(x)$$

$$M_{Gas} = 103(10)^3(242+230) - 157.2(10)^3(230) = 12.46(10)^6, N-mm$$

$M_{Belt}=0$;

Therefore total horizontal bending moment is

$M_{HOR}=M_{Gas}+M_{Belt}=12.46(10)^6+0=$**$12.46(10)^6$, N-mm**

Vertical Bending Moment due to ;

Flywheel

$$M_{Vert} = R_{v2(W)}(y)$$

$M_{Vert} = 15(10)^3(230) =$ **$3.45(10)^6$, N-mm**

Resultant Bending Moment

$$M_R = \sqrt{(M_{HOR}^2 + M_{Vert}^2)}$$

$M_R = \sqrt{(12.46*10^6)^2 + (3.45*10^6)^2} =$ **$12.93(10)^6$, N-mm**

We know that,

$$M_R = \left(\frac{\pi}{32}\right)\sigma_b(d_S^3)$$

$$12.93(10)^6 = \left(\frac{\pi}{32}\right)83(d_S^3)$$

The diameter of the shaft under flywheel d_S=116.64mm

Since the diameter of the bearing is 155mm> 116.64mm.

Hence **d_S=155mm** is adopted

b. Crank at an angle of maximum twisting moment

We know that piston gas load $F_p = \frac{\pi}{4} D^2 * p_{35} = \frac{\pi}{4} 250^2 * 0.9 =$ **$44.18(10)^3$ N**

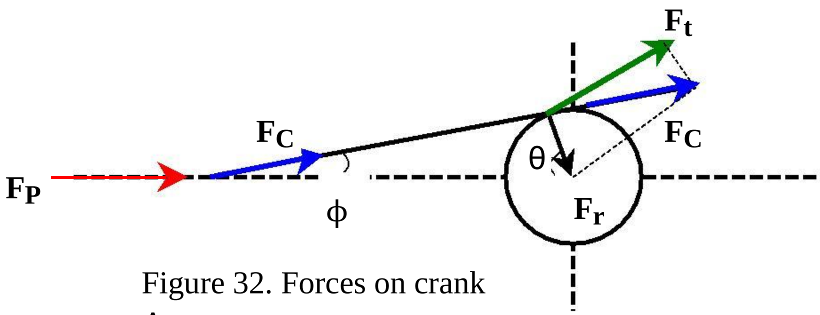

Figure 32. Forces on crank Arm

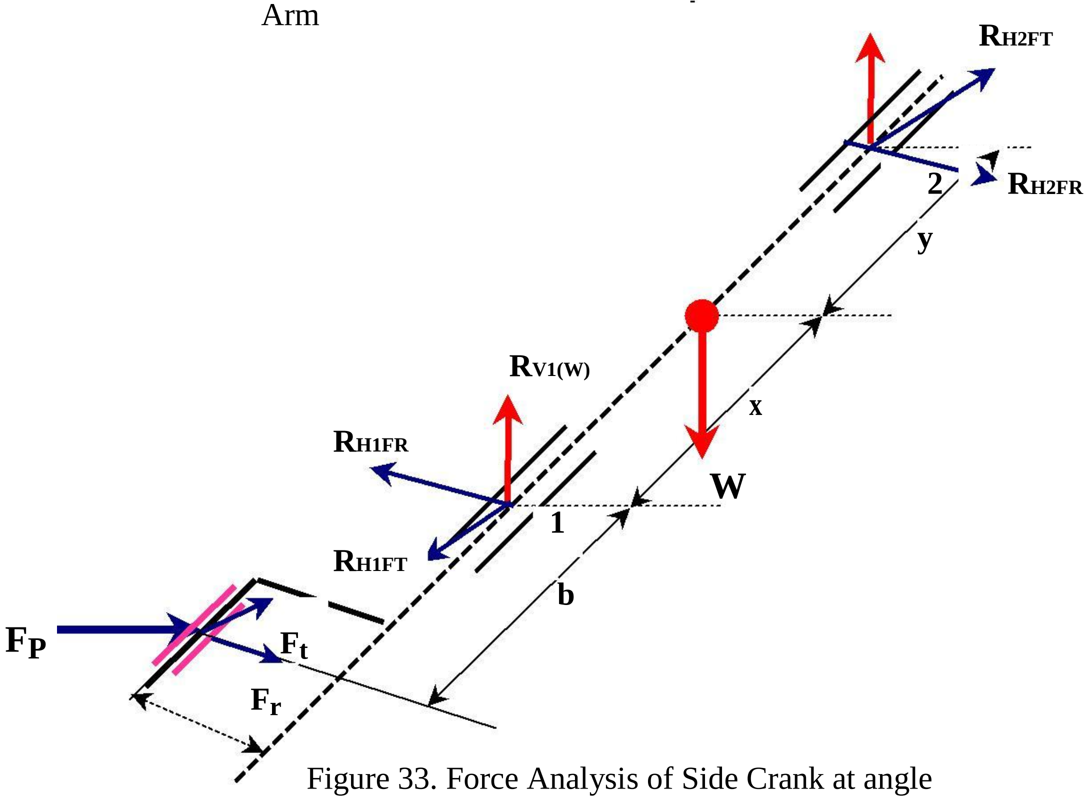

Figure 33. Force Analysis of Side Crank at angle of maximum twisting Moment

We know that, $\sin(\varphi) = \frac{\sin(\theta)}{L/R}$ -------------------------<Remember>

Where φ is the angle of inclination of the connecting rod with the line of stroke.

$$\sin(\varphi) = \frac{\sin(35)}{4.5}$$

$=0.1275$ Therefore **$\varphi = 7.32^0$**

The force on the connecting Rod or thrust force

$$F_C = \frac{F_P}{\cos(\varphi)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.12/45)$$

$$F_C = \frac{44.18(10)^3}{\cos(7.32)} = \underline{\mathbf{44543N}}$$

The tangential force or the rotative effort on the crank

$$Ft = F_C \sin(\varphi + \theta) = \frac{F_P \sin(\varphi + \theta)}{\cos(\varphi)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.13/45)$$

$$Ft = 44543\sin(7.32 + 35) = \underline{\mathbf{29989.50N}}$$

The radial force along the crank

$$Fr = F_C \cos(\varphi + \theta) = \frac{F_P \cos(\varphi + \theta)}{\cos(\varphi)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.14/45)$$

$$Fr = 44543\cos(7.32 + 35) = \underline{\mathbf{32935N}}$$

Tangential force F_t will have two reactions R_{H1FT} and R_{H2FT} at bearing 1 and 2 respectively.

Radial force F_r will have two reactions R_{H1FR} and R_{H2FR} at bearing 1 and 2 respectively.

The reactions at the bearings 1 and 2 due to Flywheel Weight W will be same as before.

In this position of the crankshaft, the different sections will be subjected to both bending and torsional moments and these must be checked for combined stress. At this point, Shear stress is taken as failure criteria for crankshaft.

<u>The reactions due Radial force (F_r):</u> To find the reactions R_{H1FR} and R_{H2FR}

$$\sum M_2 = 0, \quad \circlearrowright + \quad - \circlearrowleft$$

Clock Wise direction is taken as positive bending moment and Counter Clockwise as negative bending moment.

$$\rightarrow \quad -F_r(b + x + y) + R_{H1FR}(x + y) = 0$$

$$R_{H1FR} = \frac{F_r(b + x + y)}{(x + y)}$$

$$R_{H1FR} = \frac{32935\,(242 + 230 + 230)}{(230 + 230)} = \underline{\mathbf{50261.67N}}$$

$\sum F_y = 0$, $-$ $\uparrow$, Upward force is taken as positive and downward is taken as negative.

$$-F_r + R_{H1FR} - R_{H2FR} = 0$$

$$R_{H2FR} = F_r - R_{H1FR} = F_r - \frac{F_r(b+x+y)}{(x+y)}$$

$$R_{H2FR} = \frac{F_r(b)}{(x+y)}$$

$$R_{H2FR} = \frac{32935(242)}{(230+230)} = \mathbf{\underline{17326.67N}}$$

<u>The reactions due tangential force (F_t):</u>

To find the reactions R_{H1FT} and R_{H2FT}

$\sum M_2 = 0$, $+$ $-$

Clock Wise direction is taken as positive bending moment and Counter Clockwise as negative bending moment.

$$\rightarrow \quad -F_T(b+x+y) + R_{H1FT}(x+y) = 0$$

$$R_{H1FT} = \frac{F_T(b+x+y)}{(x+y)}$$

$$R_{H1FT} = \frac{29989.50(242+230+230)}{(230+230)} = \mathbf{\underline{45766.58N}}$$

$\sum F_y = 0$, $-$ $\uparrow$, Upward force is taken as positive and downward is taken as negative.

$$-F_T + R_{H1FT} - R_{H2FT} = 0$$

$$R_{H2FT} = F_T - R_{H1FT} = F_T - \frac{F_T(b+x+y)}{(x+y)}$$

$$R_{H2FT} = \frac{F_T(b)}{(x+y)}$$

$$R_{H2FT} = \frac{29989.50(242)}{(230+230)} = \mathbf{\underline{15777.08N}}$$

The reactions at the bearings 1 and 2 due to Flywheel weight (W) will be same as discussed earlier.

$$R_{V1(W)} = R_{V2(W)} = \frac{(30(10)^3)}{(2)} = 15(10)^3 \text{ , } N$$

Design of crank web

The dimensions are taken same as calculated in crank at dead center.
The same dimensions are checked here for combined stress.
Width of crank web w=216.3mm
Thickness of crank web h=66mm

The most critical section is where the web joins the shaft. This section is subjected to the following stresses:

i) Bending stress due to the tangential force F_T
ii) Bending stress due to the radial force F_r
iii) Direct compressive stress due to radial force F_r and
iv) Shear stress due to the twisting moment of F_T.

Bending stress due to the tangential force F_T

Bending moment due to tangential force, $M_{bT} = F_T\left(R - \frac{d_p}{2}\right)$, N − mm

$M_{bT} = 29989.50\left(150 - \frac{110}{2}\right)$, N − mm = **$2.85(10)^6$, N-mm**

Therefore bending stress due to tangential force $\sigma_{bT} = \frac{6M_{bT}}{hw^2}$

$\sigma_{bT} = \frac{6 * 2.85(10)^6}{66(216.3)^2}$ = **5.54MPa**

Bending stress due to the radial force F_r

Bending moment due to the radial force, $M_{bR} = F_R(0.75 l_p + 0.5h)$

$M_{bR} = 32953(0.75 * 110 + 0.5 * 66)$ = **$3.81(10)^6$ N-mm**

Therefore bending stress due to radial force $\sigma_{bR} = \frac{6M_{bR}}{wh^2}$

$\sigma_{bR} = \frac{6(3.81*10^6)}{(216.3)(66)^2}$ = **24.26MPa**

Direct compressive stress due to radial force F_r

We know that, direct compressive stress, $\sigma_d = \frac{F_R}{wh}$

$\sigma_d = \frac{32953}{(66)(216.3)}$ = **2.31MPa**

Shear stress due to the twisting moment of F_T.

Twisting moment due to the Tangential force, $T = F_T(0.75 l_p + 0.5h)$

$T = 29989.50(0.75(110) + 0.5(66)) =$ **$3.46(10)^6$, N-mm**

Therefore shearing stress due to Tangentiall force $\tau = \frac{T}{I} r = \frac{T}{Z} = \frac{4.5T}{wh^2} = \tau_{xy}$

Where Z-Polar section modulus, $= \frac{wh^2}{4.5}$

$\tau = \frac{4.5(3.46 * 10^6)}{(216..3)(66)^2} = \tau_{xy} =$ **16.53MPa**

Superimposing the stresses we get,

Total compressive stress, $\sigma_C = \sigma_{bT} + \sigma_{bR} + \sigma_d = \sigma_x$

$\sigma_x = 5.54 + 24.26 + 2.31 =$ **32.11MPa**

Now the total or maximum normal and maximum shear stresses are given by,

$$\sigma_{max} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \text{.....................................(1.11b/2)}$$

Here $\sigma_y = 0$;

$$\sigma_{max} = \frac{32.11 + 0}{2} + \sqrt{\left(\frac{32.11 - 0}{2}\right)^2 + 16.53^2} = \mathbf{39.10MPa < 83MPa}$$

Hence the calculated values of dimensions of crank web are safe

Design of Shaft under the flywheel:

Horizontal bending moment acting on the shaft due to piston gas load,

$M_{H1} = F_P(b + x) - \sqrt{(R_{H1FR})^2 + (R_{H1FT})^2} \quad x$

$M_{H1} = 44.18(10)^3(242 + 230) - \sqrt{(50261.67)^2 + (45766.58)^2} * 230$

$=$ **$5.22(10)^6$, N-mm**

Therefore total horizontal bending moment, $M_H = M_{H1} + M_{Hbelt}$

$M_H = 5.22(10)^6 + 0 =$ **$5.22(10)^6$, N-mm**

Vertical bending moment due to flywheel,

$M_{VFLY} = (R_{V2(W)})(y)$

$M_{VFLY} = (15 * 10^3)(230) =$ **$3.45(10)^6$, N-mm**

Since these bending moments act at right angles to each other, the combined bending moment is given by;

$M_{Total} = \sqrt{M_{VFLY}^2 + M_{Ht}^2}$, N-mm

$M_{Total} = \sqrt{\left(3.45 * 10^6\right)^2 + \left(5.22 * 10^6\right)^2}$ = $\underline{\mathbf{6.26(10)^6, N\text{-}mm}}$

In addition to this moment there will be a twisting moment because of tangential force F_t. The twisting moment, $T = F_t (R)$, N-mm

$T = 29989.50(150) = \underline{\mathbf{4.5(10)^6, N\text{-}mm}}$

Therefore Equivalent twisting moment,

$T_e = \sqrt{T^2 + M_{Total}^2}$, $N - mm$

$T_e = \sqrt{(4.5(10)^6)^2 + (6.26(10)^6)^2}$, $N - mm$ = $\underline{\mathbf{7.71(10)^6, N\text{-}mm}}$

We have, $T_e = \frac{\pi}{16} d_S^3 (\tau)$, N-mm,

$7.71(10)^6 = \frac{\pi}{16} d_S^3 (42)$ Here $\tau_{max} = 0.5\sigma_b$

Diameter of the shaft under flywheel $\mathbf{d_S = 97.78mm}$ can be obtained.

Since the diameter of the bearing is 155mm> 97.78mm. Hence $\underline{\mathbf{d_S = 155mm}}$ is adopted

References:

1. Design Data Hand Book, K. Mahadevan and K. Balaveera Reddy, CBS publication, 1989
2. Theory and Problems of Machine Design, Hall, Holowinko, Laughlin, Schaum's Outline Series, 2002.
3. A text Book of Machine Design, P.C.Sharma and D.K.Aggarwal, S K Kataria and Sons, 1993
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5. High Speed Combustion Engines, P M Heldt, Oxford and IBH Publishing Co, 1965
6. Auto Design , R B gupta, Satya Prakashan,2006
7. Automobile Mechanics, N K giri, Khanna Publishers, 2005
8. Automotive Mechanics, Crouse/Anglin, Tata McGraw-Hill, 2003
9. http://www.automotix.net/used-crankshaft-mechanical.html

Questions from Previous University Question Papers.

1. Explain the methods of manufacturing crank shaft? (05M) July 2006. VTU
2. Design a overhung crankshaft for the steam engine to the following specifications:
   Diameter of piston = 400mm
   Stroke of piston = 600mm
   Maximum steam pressure =1.0 N/mm^2 Speed of the engine= 100rpm
   Design shear stress for the crank shaft and crank pin = 3.5 N/mm^2
   Design tensile stress for the crank shaft and key = 6.0 N/mm^2
   The horizontal distance between crank shaft and crank pin=350 mm
   (15M) July 2006.VTU
3. Write a note on balancing of crankshafts. (04M) FEB 2006 VTU
4. Sketch a typical crankshaft used for a four cylinder engine. Indicate clearly the positions of pins & journals and the provision for fabrication. What are the materials used for the crankshaft. (8M) FEB 2006 VTU
5. Design & draw the sketch of an overhung crankpin for an engine having the following particulars.
   Cylinder diameter = 300 mm; Stroke = 500 mm; Maximum explosion pressure in the cylinder = 1.8 N/mm^2
   Engine speed = 200 rpm. Permissible bending stress for pin = 9.81 N / mm^2 &
   Bearing stress = 83.4 N / mm^2 (8M) FEB 2006 VTU
6. Distinguish between i. Center Crankshaft and Overhung Crankshaft. ii Built-up Crankshaft and Integral Crankshaft.
   (6M) Model QP VTU
7. Design a plain carbon steel crankshaft for a 0.40 m by 0.60 m single acting four stroke single Cylinder engine to operate at 200 rev/min. The mean effective pressure is 0.49 MPa and the maximum combustion pressure is 2.625 MPa. At maximum torsional moment, when the crank angle is 36^0, the gas pressure is 0.975 MPa. The ratio of the connecting rod length to the crank radius is 4.8. The flywheel is used as a pulley. The weight of the flywheel is 54.50 KN. And the total belt pull is 6.75 KN. Assume suitable values for the missing data.
   (14M) Model QP VTU

Model Questions

1. Design a plain carbon steel centre crankshaft for a single acting four stroke, single cylinder engine for the following data.

Piston Diameter	250mm
Stroke	400mm
Maximum Combustion Pressure	2.5MPa
Weight of the flywheel	16kN
Total Belt Pull	3kN
Length of the connecting rod	950mm

   When the crank has turned through 30^0 from the top dead center, the pressure on the piston is 1 MPa and the torque on the crank is maximum.
   Any other data required for the design may be assumed.

2. Design aside crank shaft for a 500mmX600mm gas engine. The weight of the flywheel is 80kN and the explosion pressure is 2.5MPa. The gas pressure at maximum torque is 0.9MPa, when the crank angle is 30^0. The connecting rod is 4.5 times the crank radius.
   Any other data required for the design may be assumed.

3. Explain the various types of crank shafts
4. What are the methods and materials used in the manufacture of crankshafts.
5. How a crankshaft is balanced?