Project-8
Design of a Crankshaft

National Institute of Technology, Rourkela

Machine Element Design Practice-II

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ME-1(Group 6)

Problem Statement and Solution

Problem : Design a crankshaft for a four stoke petrol engine with the following given specifications :-

1. Brake Power ($\mathrm{B_p}$) = 8 kW
2. Rated Speed (N) = 950 RPM
3. Mean Effective Pressure ($\mathrm{P_m}$) = 0.8 MPa
4. Mechanical Efficiency (η_m) = 80%

Assume Maximum Pressure is 10 times the mean effective pressure

★Nomenclature :-

$\mathrm{B_p}$ = Brake Power, kW

$\mathrm{P_m}$ = Mean Effective Pressure, MPa

D = Inner Diameter of Cylinder, mm

l = Length of Bearing, mm

w = Width of Crank Web, mm

$P_{\max}$ = Maximum Pressure, MPa

p_b = Bearing Pressure, MPa

P_1, P_2 = Tensions in Belt, N

d_c = Diameter of Crank Pin, mm

fs = Factor of Safety

p' = Gas Pressure for Maximum Torque, MPa

M_t = Torsional Moment, Nmm

$(\sigma_c)_t$ = Total Compressive Stress, MPa

σ_b = Bending Stress, MPa

ϕ = Inclination of Connecting Rod, °

c = Distance Between Bearings 2 & 3, mm

P_t = Tangential Component of force on pin, N

d_{s1} = Diameter of Shaft at Juncture, mm

l_2 = Length of Bearing 2, mm

N = Rated Speed, RPM

η_m = Mechanical Efficiency

L = Length of Connecting Rod, mm

t = Thickness of Crank Web, mm

r = Crank Radius, mm

P_p = Gas Pressure, MPa

W = Weight of Flywheel, N

R = Reaction Force, N

l_c = Length of Crank Pin, mm

d_s = Shaft Diameter Under Flywheel, mm

M_b = Bending Moment, Nmm

σ_c = Compressive Stress, mm

$(\sigma_c)_{\max}$ = Maximum Compressive Stress, MPa

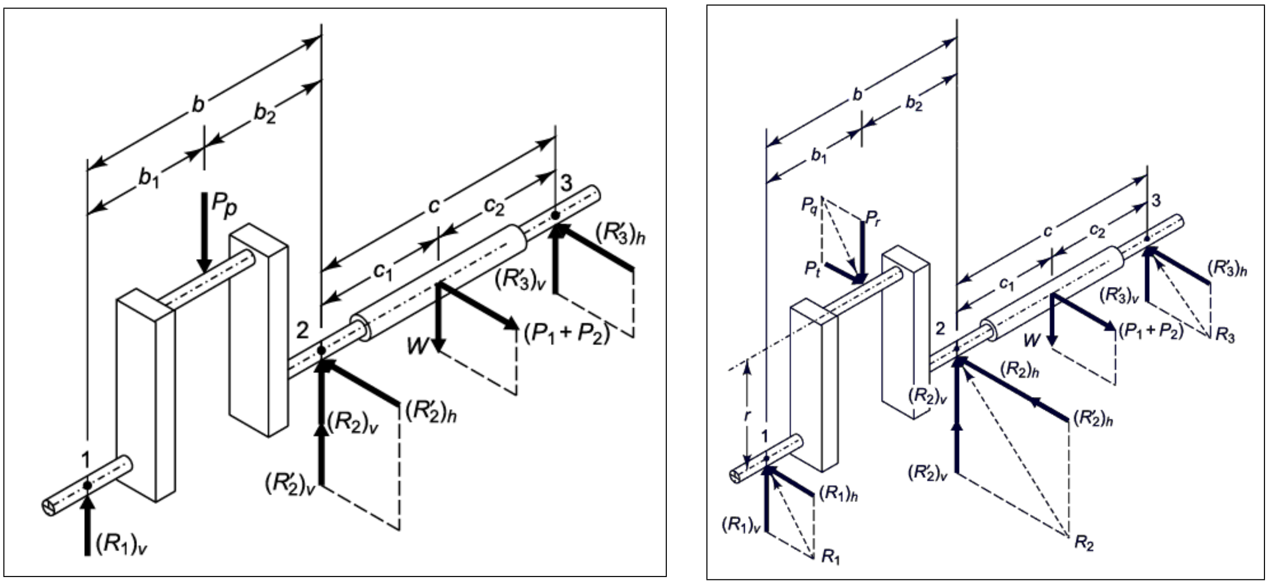
θ = Inclination of Crank, °

b = Distance Between Bearings 1 & 2, mm

P_q = Thrust on Connecting Rod, N

P_r = Radial Component of force on pin, N

τ = Allowable Shear Stress, MPa



(a) Top Dead Centre Position

(b) Angle of Maximum Torque Position

Figure 1: Crankshaft

★Material Selected and Design Data :-

Sl no.	Component	Material	Design Data
1	Crank Web	Plain Carbon Steel	$\sigma_c = 70$ MPa
2	Crank Pin	Plain Carbon Steel	$\sigma_c = 70$ MPa
3	Shaft	Plain Carbon Steel	$\sigma_c = 70$ MPa, $\tau = 45$ MPa

★Assumed Data[1] :-

$$\theta = 25^\circ \qquad L/r = 4$$
$$p' = 6 \text{ MPa} \qquad W = 1 \text{ kN}$$
$$l_c/d_c = 1 \qquad P_1 + P_2 = 2 \text{ kN}$$
$$b_1 = b_2 \qquad c_1 = c_2$$
$$c = 300 \text{ mm} \qquad p_b = 10 \text{ MPa}$$
$$l_2/d_{s1} = 1 \qquad b = 2D$$

Case I : Centre Crankshaft at Top Dead Centre Position :-

★Calculation of Bearing Reactions :-

The bearing reactions[2] are given by :-

$$(R_1)_v = \frac{P_p \times b_2}{b} \qquad (R_2)_v = \frac{P_p \times b_1}{b} \tag{1.1}$$

$$\left(R_3'\right)_v = \frac{W \times c_1}{c} \qquad \left(R_2'\right)_v = \frac{W \times c_2}{c} \tag{1.2}$$

$$\left(R_3'\right)_h = \frac{(P_1 + P_2) \times c_1}{c} \qquad \left(R_2'\right)_h = \frac{(P_1 + P_2) \times c_2}{c} \tag{1.3}$$

$$P_p = \left(\frac{\pi D^2}{4}\right) P_{\max} \tag{1.4}$$

So, we get the reactions as :-

$$\begin{aligned} (R_1)_v = (R_2)_v &= 31415.93 \text{ N} \\ \left(R_3'\right)_v = \left(R_2'\right)_v &= 500 \text{ N} \\ \left(R_3'\right)_h = \left(R_2'\right)_h &= 1000 \text{ N} \end{aligned}$$

★Design of Crank Pin :-

The crank pin diameter[2] can be found by :-

$$(R_1)_v b_1 = \left(\frac{\pi d_c^3}{32}\right) \sigma_b \tag{1.5}$$

So, the dimensions of the crank pin are, $d_c = 80$ mm and $l_c = 80$ mm.

The bearing pressure is given by :-

$$p_b = \frac{P_p}{l_c d_c} \tag{1.6}$$

We get the bearing pressure acting on the crank pin as, $p_b = 9.81$ MPa which is less the allowed bearing pressure of 10 MPa hence our design is safe.

★Design of Left Hand Crank Web :-

The dimensions of the crank web[2] are given by using the following empirical relationships :-

$$t = 0.7 d_c \qquad w = 1.14 d_c \tag{1.7}$$

So we get the crank web dimensions as, $t = 56$ mm and $w = 92$ mm.

The direct compressive stress and the bending stress on the crank web[2] is given by :-

$$\sigma_c = \frac{(R_1)_v}{wt} \qquad \sigma_b = \frac{6\,(R_1)_v}{wt^2}\left[b_1 - \frac{l_c}{2} - \frac{t}{2}\right] \tag{1.8}$$

So, we get the stresses as, $\sigma_c = 6.098$ MPa and $\sigma_b = 20.906$ MPa with their sum being the total compressive stress, $(\sigma_c)_t = 27$ MPa which is less than the allowable bending stress of 70 MPa which means our design is safe.

★Design of Right Hand Crank Web :-

The right hand and left-hand webs should be identical from balancing considerations. Therefore, the thickness and width of the right-hand crank web are made equal to that of the left-hand crank web. .

★Design of Shaft Under Flywheel :-

The shaft diameter[2] is given by :-

$$M_b = \frac{\pi d_s^3}{32}\sigma_b \qquad M_b = \sqrt{\left[\left(R_3'\right)_v c_2\right]^2 + \left[\left(R_3'\right)_h c_2\right]^2} \tag{1.9}$$

So, we get the shaft diameter as, $d_s = 30$ mm.

Case II : Centre Crankshaft at Angle of Maximum Torque :-

★Components of Force on Crank Pin :-

The components of force acting on the crank pin[2] namely the radial and the tangential force are given by :-

$$P_r = P_q \sin(\theta + \phi) \qquad P_t = P_q \cos(\theta + \phi) \tag{1.10}$$

$$P_q = \frac{P_p}{\cos\phi} \qquad \sin\phi = \frac{\sin\theta}{(L/r)} \tag{1.11}$$

$$P_p = \left(\frac{\pi D^2}{4}\right) p' \tag{1.12}$$

So we get, $\phi = 6.064°$, $P_q = 47389.13$ N, $P_t = 24452.56$ N and $P_r = 40593.12$ N.

★Calculation of Bearing Reactions :-

The bearing reactions[2] are given by :-

$$(R_2)_h = \frac{P_t \times b_1}{b} \qquad (R_1)_h = \frac{P_t \times b_2}{b} \tag{1.13}$$

$$(R_2)_v = \frac{P_r \times b_1}{b} \qquad (R_1)_h = \frac{P_r \times b_2}{b} \tag{1.14}$$

$$\left(R_3'\right)_v = \frac{W \times c_1}{c} \qquad \left(R_2'\right)_v = \frac{W \times c_2}{c} \tag{1.15}$$

$$\left(R_3'\right)_h = \frac{(P_1 + P_2) \times c_1}{c} \qquad \left(R_2'\right)_h = \frac{(P_1 + P_2) \times c_2}{c} \tag{1.16}$$

So, we get the bearing reactions as,

$$\begin{aligned}
(R_2)_h = (R_1)_h &= 12226.28 \text{ N} \\
(R_2)_v = (R_1)_h &= 20296.56 \text{ N} \\
\left(R_3'\right)_v = \left(R_2'\right)_v &= 500 \text{ N} \\
\left(R_3'\right)_h = \left(R_2'\right)_h &= 1000 \text{ N}
\end{aligned}$$

★Design of Crank Pin :-

The crank pin diameter can be found by :-

$$d_c^3 = \frac{16}{\pi\tau}\sqrt{(M_b)^2 + (M_t)^2} \tag{1.17}$$

$$M_b = (R_1)_v \times b_1 \qquad M_t = (R_1)_h \times r \tag{1.18}$$

So, we get the crank pin diameter as, $d_c = 65$ mm. But since as calculated in the previous case it is more, the first will be the deciding criterion and hence, $d_c = l_c = 80$ mm.

★Design of Shaft Under Flywheel :-

The shaft diameter will be given by :-

$$d_s^3 = \frac{16}{\pi\tau}\sqrt{(M_b)^2 + (M_t)^2} \tag{1.19}$$

$$M_b = \sqrt{\left[\left(R_3'\right)_v\right]^2 + \left[\left(R_3'\right)_h\right]^2} \times c_2 \qquad M_t = P_t \times r \tag{1.20}$$

So, we get the shaft diameter as, $d_s = 65$ mm. Since in the previous the diameter was obtained as 30 mm which is less, the deciding criterion will be second one and hence, $d_s = 65$ mm.

★Design of Shaft at the Juncture of the Right Hand Crank Web :-

The shaft at the juncture of the right hand crank web will be given by :-

$$d_{s1}^3 = \frac{16}{\pi\tau}\sqrt{(M_b)^2 + (M_t)^2} \tag{1.21}$$

$$M_b = \sqrt{[(M_b)_v]^2 + [(M_b)_h]^2} \qquad M_t = P_t \times r \tag{1.22}$$

$$(M_b)_v = (R_1)_v\left[b_1 + \frac{l_c}{2} + \frac{t}{2}\right] - P_r\left[\frac{l_c}{2} + \frac{t}{2}\right] \quad (M_b)_v = (R_1)_h\left[b_1 + \frac{l_c}{2} + \frac{t}{2}\right] - P_t\left[\frac{l_c}{2} + \frac{t}{2}\right] \tag{1.23}$$

So, we get the shaft at the juncture of the right hand crank web diameter and length as, $d_{s1} = 70$ mm and $l_2 = 70$ mm.

★Design of Right Hand Crank Web :-

The stresses acting on the web[2] are given by :-

$$(R_2)_v\left[b_2 - \frac{l_c}{2} - \frac{t}{2}\right] = (\sigma_b)_r\left[\frac{1}{6}wt^2\right] \tag{1.24}$$

$$P_t\left[r - \frac{d_{s1}}{2}\right] = (\sigma_b)_t\left[\frac{1}{6}tw^2\right] \tag{1.25}$$

$$(\sigma_c)_d = \frac{P_r}{2wt} \tag{1.26}$$

So, we get the stresses as, $(\sigma_b)_r = 13.50$ MPa, $(\sigma_b)_t = 20.89$ MPa and $(\sigma_c)_d = 3.94$ MPa. The total compressive stress is the sum of these three stresses which comes out to be, $(\sigma_c)_t = 38.33$ MPa.

The torsional stress and the maximum compressive stress is given by :-

$$(R_2)_h\left[b_2 - \frac{l_c}{2}\right] = \tau\left[\frac{1}{4.5}wt^2\right] \tag{1.27}$$

$$(\sigma_c)_{\max} = \frac{\sigma_c}{2} + \frac{1}{2}\sqrt{\sigma_c^2 + 4\tau^2} \tag{1.28}$$

So, we get the stresses as, $\tau = 11.44$ MPa and $(\sigma_c)_{\max} = 41.48$ MPa which is less than our allowable compressive stress of 70 MPa and hence our design is safe.

★Design of Left Hand Crank Web :-

The left-hand crank web is not severely stressed to the extent of the right-hand crank web. Therefore, it is not necessary to check the stresses in the left-hand crank web. The thickness and width of the left-hand crank web are made equal to that of the right-hand crank web from balancing consideration.

★Design of Crankshaft Bearing :-

The bearing pressure[2] is given by :-

$$p_b = \frac{\sqrt{\left[(R_2)_v + \left(R_2'\right)_v\right]^2 + \left[(R_2)_h + \left(R_2'\right)_h\right]^2}}{d_{s1} l_2} \tag{1.29}$$

So, we get the bearing pressure as, $p_b = 5.02$ MPa which is less than the allowed bearing pressure of 10 MPa whcih means that the design is safe.

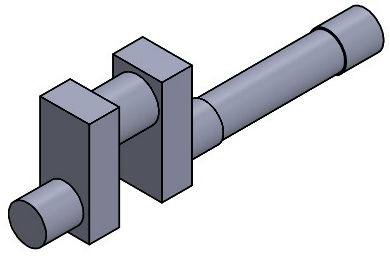
★Result and Discussion :-

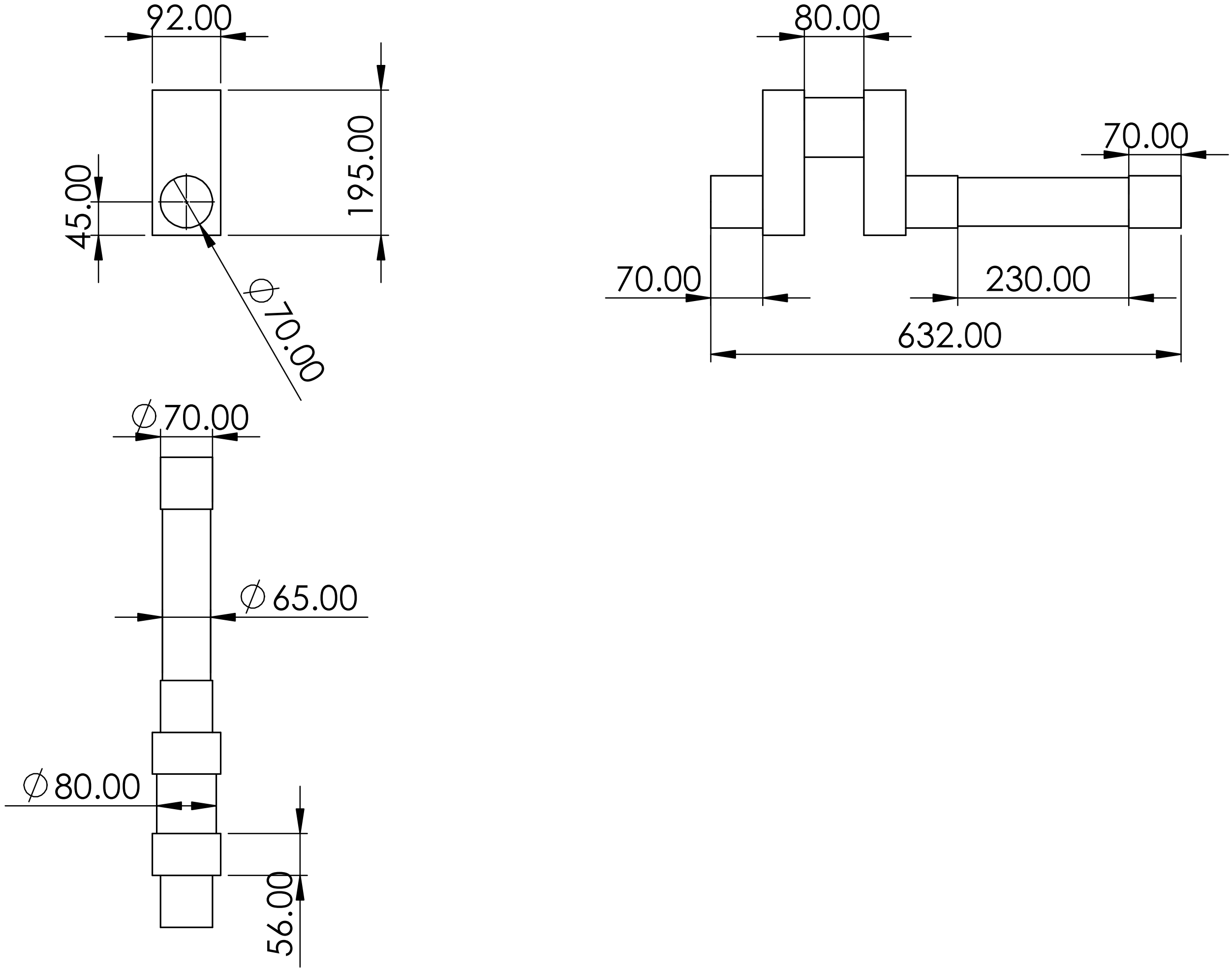
A crankshaft for an internal combustion 4 stroke petrol engine was designed having a shaft under flywheel diameter of 65 mm, shaft at the juncture of the right hand crank web diameter as 70 mm. The crank web dimensions were obtained as 56 mm thickness and 92 mm width. The crank pin diameter and length were obtained as 80 mm.

★References :-

[1] K. Lingaiah, *Machine Design Data Book*, 2nd Edition, New Delhi: Tata McGraw-Hill Education, 1994

[2] V.B Bhandari, *Design of Machine Elements*, 3rd Edition, New Delhi: Tata McGraw-Hill Education, 2010





NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA

TITLE : Crankshaft

MATERIAL : Plain Carbon Steel

THIRD ANGLE PROJECTION
(All Dimensions are in mm)

SCALE : 1:10

MACHINE ELEMENT DESIGN PRACTICE - II
ME-381

ME-I(GROUP-6)