Thermodynamic and dynamic analysis of an internal combustion engine with a noncircular-gear based modified crank-slider mechanism

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Abstract—*This paper presents a model for the calculation of in-cylinder parameters in an internal combustion engine with a noncircular gear based modified crank-slider mechanism. With the introduction of noncircular gears, the instantaneous velocity of the piston can be accommodated to improve combustion performance. The displacement law of the noncircular gears is obtained using a B-spline curve, so that the appropriate instantaneous velocity of the piston is obtained. The gas pressure and temperature required for the determination of mechanical and thermal loads on engine components are found. The influence of the noncircular gears on the loads that act on all the components of the crank-slider mechanism, as well as the theoretical output torque for a given geometrical structure and inertial properties, are presented. To obtain the pressure and temperature inside the cylinder, under different operating parameters, such as air fuel ratio and spark angle advance, a Zero dimensional model is applied. The proposed mechanism enables the optimisation of the combustion cycle; therefore, greater power may be achieved.*

Keywords: noncircular gears, internal combustion engine

I. Introduction

Non-uniform rotation mechanisms are required in many applications. Noncircular gear wheels can be used to produce rotary motion with variable transmission ratio and, compared to linkages, provide a number of design advantages such as accurate transmission, ease of balancing, and compact size. Furthermore, they are very versatile because of the great flexibility to obtain a desired transmission function.

Research on noncircular gears has been very limited. Most of the research on these tooth bodies have concentrated on (i) the development of pitch curves for different applications or to satisfy specific requirements [1-9], as reviewed in the next paragraph; (ii) the development of new tooth profiles; and (iii) the derivation of mathematical models to describe and manufacture teeth of noncircular gears and their cutters. Reviews on noncircular gears have been presented in previous works [9,10].

Classical applications of noncircular gears are found in automatic machinery, packaging machines, quick return mechanisms, pumps, flow meters, and instruments. New applications have also been reported. Doege *et. al.* [1] present a new press concept using noncircular gears in the driving mechanism. Dooner [2] and Yao and Yan [3] propose using noncircular gears to reduce any undesired torque and speed fluctuations in rotating shafts. Fam *et. al.* [4] design a mechanical device consisting of a noncircular gear pair that acts as a variable-ratio transmission between an electro-mechanical actuator and a flexible structure. Han *et. al.* [5] design a noncircular front gear to maximize the mechanical power output of a driving system for a conventional bicycle. Dooner [6] uses a noncircular gear pair to achieve a two degree of freedom function generator. Librovich [7] uses noncircular gears in a torque transmission mechanism of a rotary vane engine. Voelkner [8] explains the advantages of using these tooth bodies in press-driving mechanisms in the metal-forming field. Vanegas Useche *et. al.* [9] develop a noncircular gear pair for minimising shaft accelerations of the driven gear.

This paper proposes a novel modified crank-slider mechanism of an internal combustion engine, by introducing a noncircular gear pair. The noncircular tooth bodies enable to adjust the piston speed throughout the entire cycle, so that the performance of the engine can be improved.

In spark ignition engines, the improvement of performance is constrained by the non-variability of the piston velocity law in accordance with the needs of the combustion process. With the introduction of a noncircular gear pair in the engine mechanism, the duration of the portion during which the non burned charge is subjected to high pressures and temperatures can be diminished. Thus, the knock tendency of the engine would be reduced. This modification also reduces the rejected heat.

Since, to the knowledge of the authors, a mathematical model for piston velocity that optimizes the combustion process has not been developed, this work proposes a design for the displacement law of the noncircular gear set based on B-spline curves. These curves provide a powerful tool for designing displacement laws, because they give the designer a higher-level interface and the curve design is thus more intuitive.

The primary input in mechanical design analyses is the data of the dynamic pressure of the cylinder. In the engine

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design process, a predictive model for the combustion process has to be selected. For simplicity, a Zero-dimensional or single zone model has been chosen in this work, in accordance with the approach found in Zhelezko [11]. With a Zero-dimensional model, the cylinder charge is assumed to be homogeneous in both temperature and composition.

Models for in-cylinder thermodynamics and dynamics of the crank-slider mechanism are integrated in this work to configure a concise methodology for an easy simulation of an internal combustion engine. Based on this methodology, a computer program to analyze pressure, temperature, heat release, forces, and torques is developed. The program is written in the Mathematica^{TM} software language. Results for an example case are presented, with an angular resolution of 0,25 degree of crank angle (2880 data points per engine cycle) and under steady operation conditions. Finally, a noncircular gear pair is designed in order to optimise the operation of the engine.

II. Thermodynamic modelling

The first law of thermodynamics for engine cylinder systems states that

$$\mathrm{d}U_s = \mathrm{d}Q + \mathrm{d}W \tag{1}$$

where

$$\begin{gathered} \mathrm{d}W = p\,\mathrm{d}V \\ \mathrm{d}U_s = mc_v\,\mathrm{d}T \\ \mathrm{d}T = \mathrm{d}(pV)/mR \\ R/c_v = k - 1 \end{gathered} \tag{2}$$

where $\mathrm{d}U_s$ is the change in internal energy, $\mathrm{d}Q$ is the heat added to the system, $\mathrm{d}W$ is the mechanical work done by the system, m is the working charge mass, c_v is the constant volume specific heat, p is the pressure, V is volume, T is temperature, k is adiabatic constant, and R is the gas constant.

Using the ideal gas law (neglecting the change in gas constant R and gas leakages), and after some transformations, the following expression for the heat release is obtained:

$$\mathrm{d}Q_{hr} = \frac{k}{k-1}p\,\mathrm{d}V + \frac{1}{k-1}V\,\mathrm{d}p + \mathrm{d}Q_{rech} \tag{3}$$

This is the traditional equation for the evaluation of the heat release, which can be inferred from Gatowski *et al.* [12] and Brunt and Platts [13].

For the average overall heat transfer from the gas to the cylinder coolant, convection type heat transfer equations are used:

$$\mathrm{d}Q_{rech} = Ah_g\left(T_g - T_{cool}\right) \tag{4}$$

where:
$\mathrm{d}Q_{rech}$ is the overall rejected heat transfer (W/m^2)
A is the cylinder area (m^2)
T_g is the effective gas temperature, typically 800 °C
T_{cool} is the coolant temperature, typically 80 °C
h_g is the film coefficient or heat transfer coefficient (W/ m^2 ºC).

The heat transfer coefficient depends on the engine geometric parameters, such as the exposed cylinder area and bore, as well as the piston speed. The coefficient varies with location and piston position. In this research, to model the heat exchange between gas and cylinder wall, the Woschni equation has been used [14]. In this model, applied to the internal combustion engine, the equation has the form:

$$h_g = 1,2\cdot 10^{-2}\cdot D^{-0.2}\cdot p^{0.8}\cdot T_g^{-0.53}\cdot w^{0.8} \tag{5}$$

where

$$w = (C_{w1}\cdot c_m + C_{w2}\cdot c_u) + C_2\left(\frac{V_T\cdot T_{CA}}{p_{CA}\cdot V_{CA}}\right)\cdot\left(p - p_0\right) \tag{6}$$

D is the cylinder diameter in m
p is the instantaneous pressure in N/m^2
T_g is the instantaneous temperature of the gas in K
c_m is the mean velocity of the piston in m/s
c_u is specific heat of the gas in J/ kg K
V_T is the displaced volume in m^3
T_{CA} is the charge temperature at intake valve closing in K
p_{CA} is the charge pressure at intake valve closing in N/m^2
V_{CA} is the charge volume at intake valve closing in m^3
p_0 is the instantaneous pressure for motored engine in N/m^2.

The constants C_{w1}, C_{w2}, and C_2 take the values given in Tables 1 and 2.

Table 1. Coefficients C_{w1} and C_{w2} (Source: [15])

	C_{w1}	C_{w2}
Gas exchange process	6,18	0,417
Compression-expansion process	2,28	0,308

Table 2. Coefficient C_2 (Source: [15])

	C_2
Open chamber	$3,24 \times 10^{-3}$
Divided chamber	$6,22 \times 10^{-3}$

The combustion process is dealt with in accordance with the approach in Zhelezko [11].

The combustion process starts with the spark ignition (neglecting the retarding period of the combustion process), point y in Fig. 1. During this phase, the pressure increases as a result of two factors: the geometrical compression and the heat release corresponding to the fraction of the mass burned [15].

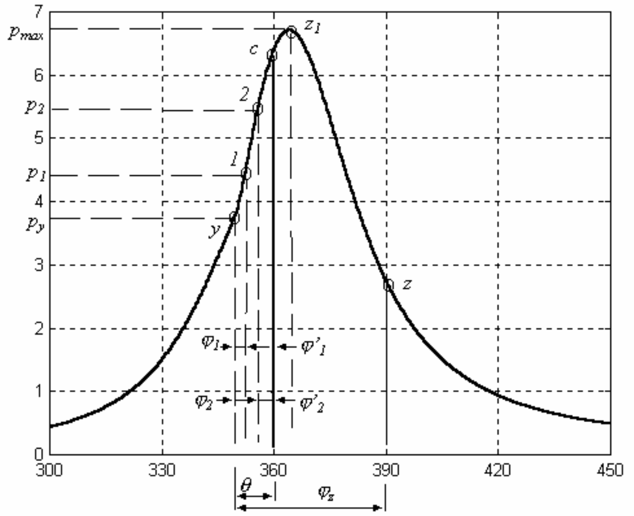

Fig. 1. Indicator diagram, p-φ.

The combustion heat release can be expressed in terms of the lower heating value of the fuel, H_i, and the fuel burning rate; the lower heating value can be found in fuel tables. The burning fuel rate is calculated as the product of induced fuel mass, m_f, and mass fraction burned. The induced fuel mass can be calculated from the specific fuel consumption and maximum power at a given speed, while the mass fraction burned is estimated by a Wiebe function [11]:

$$x = 1 - \exp\left[-6,908\left(\frac{\varphi - \varphi_0}{\varphi_z}\right)\right] \tag{7}$$

In expressions (1) and (6), it is important to note that since the gas pressure in the cylinder is dependent on the piston displacement law, the heat release, and the heat losses, any variation in the piston displacement law affects the in-cylinder pressure and heat losses, which in turn affect the output performance of the engine. Therefore, a manner in which the piston displacement law can be modified is needed.

III. Noncircular gear based modified crank slider mechanism

Nowadays big efforts are devoted to the improvement of the combustion process of internal combustion engines. Although combustion models have been refined, few movements have been made towards changing the piston kinematics. In order to improve the performance of the internal combustion engine, a novel concept is explored in this work: the introduction of noncircular gears in the transmission of the engine. Figure 2 presents the schematic representation of the proposed modified crank-slider mechanism that includes a noncircular gear pair. The driven gear rotates with the crankshaft, and the driving gear rotates with the power shaft. With the proposed modification, the curve of the piston speed can be defined as a function of the angle of rotation of the crankshaft, and it is not limited to the modification of the dimensions of the crank-slider mechanism.

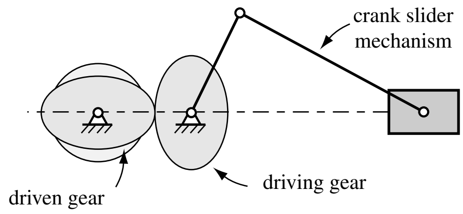

Fig. 2. Modified crank slider mechanism

Following Lagrangian analysis, as in [16], the vector of generalized coordinates of the mechanism is $\boldsymbol{q} = \{\varphi, \beta, s_p\}^T$, where φ, β and s_p are the angular position of the crank, the angular position of the connecting rod, and the position of the piston respectively. These and other variables are shown in Figure 3. Vector $\boldsymbol{q}$ gives the configurations of the mechanism. The constraint equation vector, $\boldsymbol{f}(\boldsymbol{q}) = 0$, is the set of equations that impose the geometrical constraints of the linkage mechanism:

$$\begin{aligned} f_1 &: r\cos\varphi + L\cos\beta - s_p = 0 \\ f_2 &: r\sin\varphi - L\sin\beta = 0 \end{aligned} \tag{8}$$

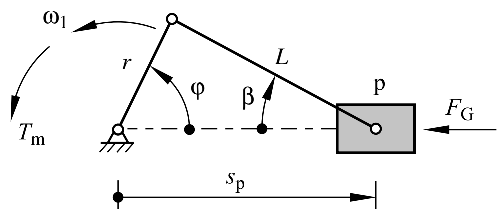

Fig. 3. Kinematics of a crank-slider mechanism

The vector $\boldsymbol{q}$ is usually subdivided into an independent coordinates vector $\{q_i\} = \{\varphi\}$ and a dependent coordinates vector $\{q_d\} = \{\beta, s_p\}^T$.

The velocity analysis can be carried out after differentiating the system of constraint equations with respect to time:

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{f}(\boldsymbol{q},t) = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{q}} \cdot \dot{q} = 0 \tag{9}$$

The Jacobian matrix is the partial derivation of the constraint equation with respect to the generalized coordinates vector, $J_q = \partial f_i / \partial q_j$:

$$J_q = \begin{bmatrix} -r\sin\varphi & -L\sin\beta & -1 \\ r\cos\varphi & -L\cos\beta & 0 \end{bmatrix} \tag{10}$$

The angular velocities of crank link and connecting rod (Fig. 2) are obtained by expressing the generalized velocity vector in terms of two components: a generalized dependent vector, $\{\dot{q}_{\mathrm{d}}\}$, and a generalized independent vector, $\{\dot{q}_{\mathrm{i}}\}$:

$$\left[J_{\mathrm{q,d}} \mid J_{\mathrm{q,i}} \mid\right]\begin{Bmatrix}\dot{q}_{\mathrm{d}} \\ \dot{q}_{\mathrm{i}}\end{Bmatrix}=\left[J_{\mathrm{q,d}}\right]\{\dot{q}_{\mathrm{d}}\}+\left[J_{\mathrm{q,i}}\right]\{\dot{q}_{\mathrm{i}}\}=0$$

$$\{\dot{q}_d\}=-\left[J_{q,\mathrm{d}}\right]^{-1}\cdot\left[J_{q,\mathrm{i}}\right]\cdot\omega_1 \tag{11}$$

$$\begin{Bmatrix}\omega_2 \\ v_{\mathrm{p}}\end{Bmatrix}=-\begin{bmatrix}-L\sin\beta & -1 \\ -L\cos\beta & 0\end{bmatrix}^{-1}\cdot\begin{bmatrix}-r\sin\varphi \\ r\cos\varphi\end{bmatrix}\cdot\omega_1$$

Differentiation of Equation (11) with respect to time allows finding the angular accelerations of both links:

$$\left[J_{\mathrm{q,d}}\right]\{\ddot{q}_{\mathrm{d}}\}+\{\dot{q}_{\mathrm{d}}\}^T\cdot\left[\dot{J}_{q,\mathrm{d}}\right]\cdot\{\dot{q}_{\mathrm{d}}\}$$

$$+\{\dot{q}_{\mathrm{i}}\}^T\left[\dot{J}_{q,\mathrm{i}}\right]\cdot\{\dot{q}_{\mathrm{i}}\}=0$$

$$\begin{Bmatrix}\alpha_2 \\ a_{\mathrm{p}}\end{Bmatrix}=-\begin{bmatrix}-L\sin\beta & -1 \\ -L\cos\beta & 0\end{bmatrix}^{-1}\cdot \tag{12}$$

$$\left(\begin{bmatrix}-\omega_2 L\cos\beta & 0 \\ \omega_2 L\sin\beta & 0\end{bmatrix}\begin{Bmatrix}\omega_2 \\ \omega_3\end{Bmatrix}+\begin{bmatrix}-r\cos\varphi \\ -r\sin\varphi\end{bmatrix}\cdot\omega_1^2\right)$$

IV. Displacement law design

In this section, a noncircular gear set is designed. The gear pair is positioned between the crankshaft and the new output shaft with the aim of increasing the piston velocity before and after the top dead centre. This has the twofold objective of reducing the area of convective heat transfer during the main combustion period and enabling the compression ratio to increase beyond the limits imposed by the knocking phenomena to conventional engines.

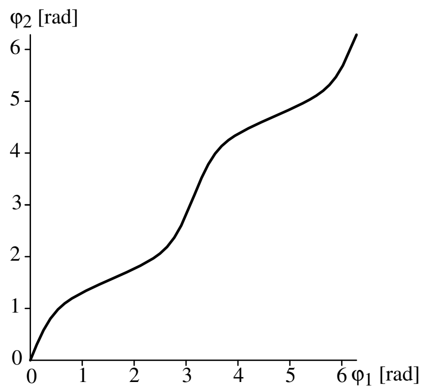

Fig. 4. Displacement law of the noncircular gear pair

Figure 4 shows a displacement law of noncircular gears, where φ_1 and φ_2 are the angles of rotation of the driving and driven gear wheels, respectively. This work proposes the design for the displacement law of the noncircular gear set based on B-spline curves. The objective of the curve designed is to obtain a higher piston velocity around the top dead centre.

Figure 5 presents a comparison of the piston displacement curves of the conventional crank-slider mechanism and the mechanism proposed in this paper. In the traditional mechanism (dashed line), the angular speed of the crank is considered constant and equal to that of the crankshaft. This produces an S_{p}-φ_1 curve of sinusoidal form. The coordinate φ_1 represents the angular position of the crankshaft.

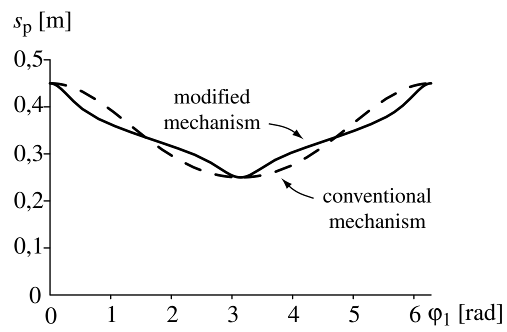

Fig. 5. Piston displacement

In the proposed mechanism with noncircular gears, it is considered that the angular speed of the crankshaft is constant. Based on this, the angular velocity of the crank would be given by the product of the angular velocity of the crankshaft and the gear ratio. In this case, the coordinate φ_1 represents the angular position of the driving gear.

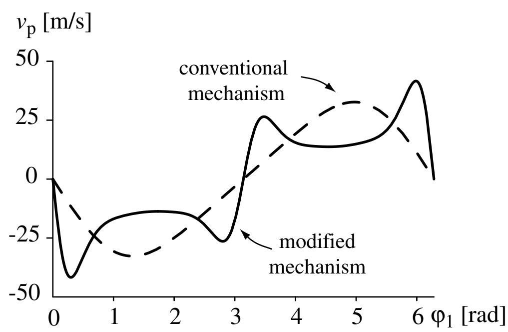

Fig. 6. Piston speed

V. Dynamic loads

The primary input needed for mechanical design analyses is the dynamic cylinder pressure data. In the thermodynamic model, the combustion process is considered to occur in the same displacement interval of the piston for both the conventional and modified mechanisms.

The torque of the engine is obtained from the study of the power in the system. Neglecting the friction forces, the forces that act in the mechanism are the inertial force

$$\boldsymbol{F}_0 = -\sum m_i \boldsymbol{a}_{Gi} \tag{13}$$

and the force due to the pressure of the gas, F_g; the torque that acts in the mechanism is the torque in the crankshaft, T_m. Therefore:

$$\frac{\mathrm{d}E_c}{\mathrm{d}t} = -\boldsymbol{F}_g \cdot \boldsymbol{v}_p + T_m \omega_1 \tag{14}$$

On the one hand, the total kinetic energy of the conventional mechanism is the sum of the kinetic energy of the crank, the connecting rod, and the piston:

$$E_c = \frac{1}{2} J_{01}\omega_1^2 + \left(\frac{1}{2} J_2 \omega_2^2 + \frac{1}{2} m_2 v_2^2\right) + \frac{1}{2} m_p v_p^2 \tag{15}$$

The derivative of the kinetic energy is:

$$\frac{\mathrm{d}E_c}{\mathrm{d}t} = \left(J_2 \omega_2 \alpha_2 + m_2 \boldsymbol{v}_{G2} \cdot \boldsymbol{a}_{G2}\right) + m_p \boldsymbol{v}_p \cdot \boldsymbol{a}_p \tag{16}$$

On the other hand, assuming that the crankshaft and, consequently, the driving gear rotate at constant speed, the total kinetic energy of the proposed mechanism is:

$$\begin{aligned} E_c &= \frac{1}{2} J_{dr}\omega_{dr}^2 + \frac{1}{2} J_{driven}\omega_{driven}^2 \\ &\frac{1}{2} J_{01}\omega_1^2 + \left(\frac{1}{2} J_2 \omega_2^2 + \frac{1}{2} m_2 v_2^2\right) + \frac{1}{2} m_p v_p^2 \end{aligned} \tag{17}$$

The derivative of the kinetic energy is:

$$\begin{aligned} \frac{\mathrm{d}E_c}{\mathrm{d}t} &= \left(J_{driven} + J_{01}\right)\omega_1 \alpha_1 + \\ &\left(J_2 \omega_2 \alpha_2 + m_2 \boldsymbol{v}_{G2} \cdot \boldsymbol{a}_{G2}\right) + m_p \boldsymbol{v}_p \cdot \boldsymbol{a}_p \end{aligned} \tag{18}$$

The external force that acts in the mechanism is the force produced by the pressure of the gas, F_g; the torque that acts in the mechanism is the torque in the crankshaft, T_m. Hence:

$$\frac{\mathrm{d}E_c}{\mathrm{d}t} = -\boldsymbol{F}_g \cdot \boldsymbol{v}_p + T_m \omega_{dr} \tag{19}$$

VI. Results

Figure 7 shows the curves of the torque in the crankshaft against φ_1 for both the conventional and the modified mechanism.

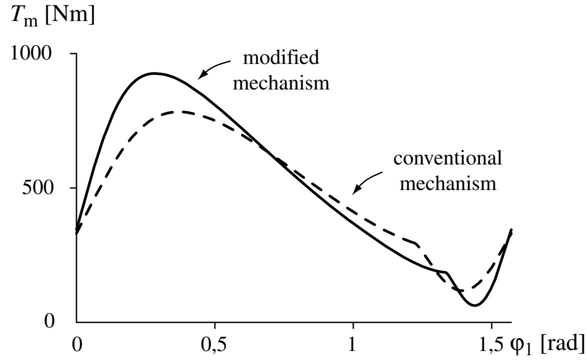

Fig. 7. Torque in the engine for both conventional and modified mechanisms

Considering an engine of 8 cylinders, the energy available in a thermodynamic cycle is 482 N m, for the conventional engine, and 498 N m, for the modified engine. Therefore, there is an increase of the energy available in a cycle.

The curves for the pressure against φ_1 for both configurations are shown in Figure 8. The pressure in the modified mechanism configuration is slightly higher than that in the conventional configuration.

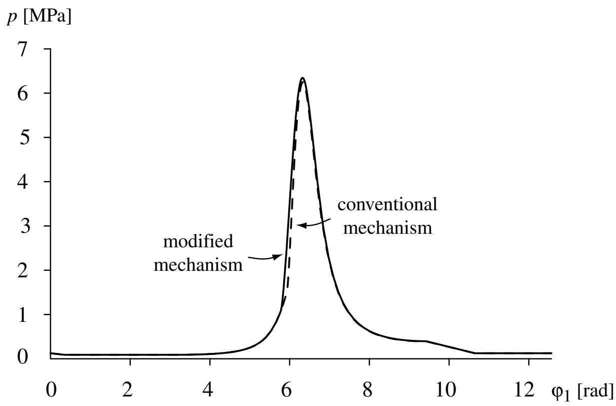

Fig. 8. Pressure

Figure 9 presents the heat flux due to losses by convection at the engine. As may be inferred from Figure 9, the magnitude of heat transferred to the combustion chamber walls has the biggest changes during combustion and expansion. With the modified mechanism, the behaviour of the heat flux has the same sharp rise as that of the conventional mechanism, but the maximum is followed by a more rapid decrease, resulting in a lower amount of heat loss.

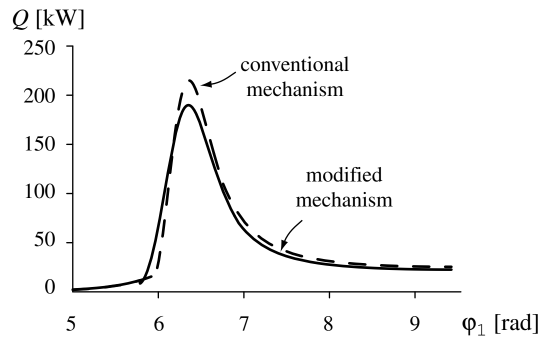

Fig 9 Heat lost by convection

Even though the amount of energy saved in this case is not impressive, an optimized design can be attempted to reduce further heat losses.

In Figure 10, the noncircular gears designed are illustrated. The number of teeth of each gear is 40, and the pressure angle of the rack cutter is 30º.

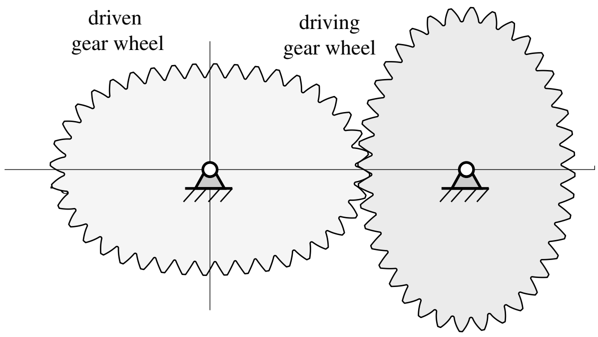

Fig. 10. Gear wheels

VII. Conclusion

This paper proposed a modified crank-slider mechanism of an internal combustion engine, through the introduction of a noncircular gear pair; the driven gear rotates with the crankshaft and the driving gear is coupled to the output power shaft. With these gears, the piston speed can be adjusted to obtain the desired performance of the engine. The thermodynamic and kinematic analyses of the proposed mechanism were presented. A noncircular gear pair was designed using B-spline curves, based on the optimisation of engine performance. The results of the example presented indicate that the performance of the engine can be improved with the proposed mechanism.

VIII. Acknowledgements

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