# Chapter No:06, Crank Shaft

**Objective:**

The student will learn that crankshaft design is the estimation of the shaft diameter, crankpin dimensions to ensure satisfactory strength and rigidity when the crankshaft is transmitting power under various operating conditions.

**Outcomes:**

The student should be able to design crankshafts for various operating and loading conditions.

**Prerequisites:**

This topic requires the student to know about, the fundamentals of Engineering Mathematics, Engg physics, Strength of Materials, Engineering Drawing, Workshop Processes, Theory of Machines, Material Science and fundamentals of Machine Design.

**Number of Question/s expected in examination: 01 [20Marks]**

**INTRODUCTION:**

Before studying the actual crankshaft and design details, we shall study briefly the basics of Power Transmission of shafts.

## 6.1 Power Transmitting Shaft:

Shaft Design consists primarily of the determination of the correct shaft diameter to ensure satisfactory **strength** and **rigidity** when the shaft is transmitting power under various operating and loading conditions. Shafts are usually circular in cross section, and may be either hollow or solid.

Design of shafts of **ductile materials**, based on **strength**, is controlled by the **maximum shear theory**. And the shafts of **brittle material** would be designed on the basis of the **maximum normal stress theory**.

Various loads subjected on Shafting are torsion, bending and axial loads.

### 6.1.1 Basics of Design for solving Shaft problems:

6.1.1a. Maximum Principal Stress:($\sigma_1$)

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left[\frac{\sigma_x - \sigma_y}{2}\right]^2 + \tau_{xy}^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(1.11a/2)$$

Here 1.11a and 2 refers to the formula number and page number from ***Design data handbook by K Mahadevan and K Balaveera Reddy***, CBS Publications, INDIA, 1989.
Same Data handbook and similar procedure is adopted in further discussion.

Where,

$\sigma_x$ --- Stress in x direction, in MPa or N/mm$^2$

$\sigma_y$ --- Stress in y direction, in MPa

$\tau_{xy}$ ---Shear stress, in MPa.

6.1.1b.Minimum Principal Stress: ($\sigma_2$)

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left[\frac{\sigma_x - \sigma_y}{2}\right]^2 + \tau_{xy}^2} \quad \text{.................................(1.11b/2)}$$

6.1.1c. Maximum Shearing Stress:($\tau_{max}$)

$$\tau_{\max} = \pm\sqrt{\left[\frac{\sigma_x - \sigma_y}{2}\right]^2 + \tau_{xy}^2} \quad \text{.........................................(1.12/2)}$$

6.1.1d.Torsional stresses: ($\tau$)

The Torsional formula is given by,

$$\frac{T}{J} = \frac{G\theta}{l} = \frac{\tau}{r} \quad \text{........................................................(1.15/3)}$$

Here T=torque or Torsional moment, N-mm

J=polar moment of inertia, $mm^4$

$= \frac{\pi}{32} d^4$, Where d is the solid shaft diameter.

$= \frac{\pi}{32}\left(d_o^{\,4} - d_i^{\,4}\right)$, Where $d_o$ and $d_i$ are outer and inner diameter of the hollow shaft respectively.

G=Modulus of elasticity in shear or modulus of rigidity, MPa

$\theta$=Angle of twist, radians

l= Length of shaft , mm

r= Distance from the Neutral axis to the top most fibre, mm

$= \frac{d}{2}$ (For solid shaft)

$= \frac{d_o}{2}$ (For hollow shaft)

6.1.1d.Bending Stresses:($\sigma_b$)

The bending equation is given by

$$\frac{M}{I} = \frac{E}{R} = \frac{\sigma_b}{c} \quad \text{......................................................(1.16/3)}$$

Here M=bending moment, N-mm

I= Second moment of area, $mm^4$

$$= \frac{\pi}{64} d^4 \text{ (For solid shaft)}$$

$$= \frac{\pi}{64}\left(d_o^4 - d_i^4\right), \qquad \text{(For hollow shaft)}$$

E=modulus of elasticity or Young' modulus for the material, MPa
θ=Angle of twist, radians
R= radius of curvature, mm
c= Distance from the Neutral axis to the extreme fibre, mm

$$= \frac{d}{2} \text{ (For solid shaft)}$$

$$= \frac{d_o}{2} \text{ (For hollow shaft)}$$

## <u>6.1.2 Methods of obtaining the Twisting moment and Bending Moment.</u>

### 6.1.2a Twisting Moment:

i) <u>Power transmitted :</u>

$$P = \frac{2\pi nT}{60000} \text{ kW}$$

Where T - twisting moment in N-m= ($10^3$) N-mm
n – speed of the shaft, rpm

Hence $T = \dfrac{60000(P)(10^3)}{2\pi n} = \dfrac{9.55(10^6)(P)}{n}$ ……………………………..(3.3a/42)

ii) <u>In case of belt drives</u>

Power transmitted

$$P = \frac{(T_1 - T_2)v}{1000} \text{ kW} \quad \text{…………………………………………….(14.9a/239)}$$

Where $T_1$- tension of belt on tight side, N
$T_2$- tension of belt on slack side, N
v- velocity of belt, m/s [Student should take care of units here, it is in m/sec not in mm/sec]

$$\frac{T_1}{T_2} = e^{\mu\theta} \quad \text{………………………………………………………….(14.6a/238)}$$

θ ---arc of contact, rad
μ---coefficient of friction between belt and pulley.
From equation (14.9a/239) and (14.6a/238) get $T_1$ and $T_2$.

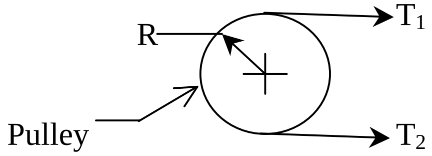


Figure 1

Knowing the value of radius of the pulley (R) twisting moment can be found by using the following equation:[Refer Figure 1]

$T = (T_1 - T_2)\, R$, N-mm.

iii) In case of Gear drives.

Power transmitting capacity of gears is given by

$P = \dfrac{F_t v}{1000}$ kW……………………………..………… …...…(12.14a/163)

$F_t$ = driving force or tangential load at pitch line, N

The torque is given by, $T = F_t(\dfrac{d}{2})$, N-mm……………………….(12.22/165)

Where d is the pitch circle diameter of Gear.

**6.1.2b Bending Moment.**

i) Cantilever, end load [Figure 2]

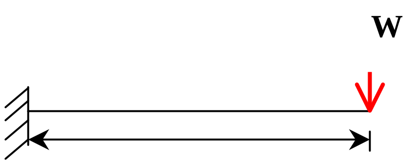


Figure 2

Maximum Bending Moment, M=W(l), N-mm …………………………..[Table 1.4/1/10]

Where W is the concentrated load, N

l is the length of the beam, mm

ii) Simply supported beam [End support, center load] [Figure 3]

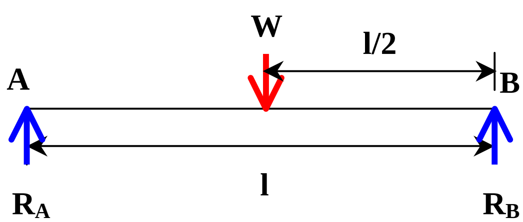


Figure 3

To find the reactions $R_A$ and $R_B$

$\sum M_A = 0$,

For the convenient of calculations, Clock Wise direction is taken as positive bending moment and Counter Clockwise as negative bending moment.

$$W(\frac{l}{2}) - R_B(l) = 0$$

Hence $R_B = \frac{W}{2}, N$

$\sum F = 0$, $\downarrow -$ $\uparrow +$ , Upward force is taken as positive and downward is taken as negative.

→ $R_A + R_B - W = 0$

Hence $R_A = W - R_B$

Maximum bending Moment, $M = R_A(\frac{l}{2}) = \frac{Wl}{4}$ ……………………….[Table 1.4/4/10].

<u>iii) Simply supported beam [End support, Intermediate][Figure 4]</u>

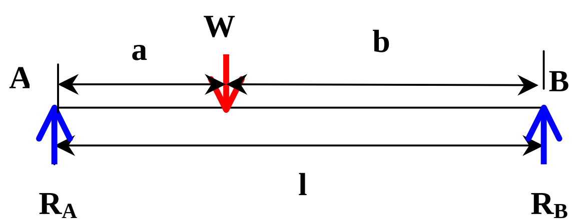


Figure 4

To find the reactions $R_A$ and $R_B$

$\sum M_A = 0$, $\circlearrowleft +$ $- \circlearrowright$

→ $W(a) - R_B(l) = 0$

Hence $R_B = \frac{W(a)}{l}, N$

$\sum F = 0$, $\downarrow -$ $\uparrow +$

→ $R_A + R_B - W = 0$

Hence $R_A = W - R_B, N$

$$R_A = W - \frac{W(a)}{l} = W(1 - \frac{a}{l}) = W(\frac{l-a}{l}) = \frac{W(b)}{l}, N$$

Maximum bending Moment, $M = R_A(a) = \frac{W(a)(b)}{l}$ …………………[Table 1.4/5/10].

For different kinds of loading and support students are advised to refer Table 1.4 from page no 10-12 of the Design Data Book.

## 6.2. Crank Shaft:

A crankshaft is used to convert reciprocating motion of the piston into rotary motion or vice versa. The crankshaft consists of the shaft parts, which revolve in the main bearings, the crank pins to which the big ends of the connecting rod are connected, the crank arms or webs, which connect the crankpins, and the shaft parts. The crankshaft, depending upon the position of crank, may be divided into the following two types.

The crankshaft is the principal member of the crank train or crank assembly, which latter converts the reciprocating motion of the pistons into rotary motion. It is subjected to both torsional and bending stresses, and in modern high-speed, multi-cylinder engines these stresses may be greatly increased by resonance, which not only renders the engine noisy, but also may fracture the shaft. In addition, the crankshaft has both supporting bearings (or main bearings) and crankpin bearings, and all of its bearing surfaces must be sufficiently large so that the unit bearing load cannot become excessive even under the most unfavorable conditions. At high speeds the bearing loads are due in large part to dynamic forces-inertia and centrifugal. Fortunately, loads on main bearings due to centrifugal force can be reduced, and even completely eliminated, by the provision of suitable counterweights. All dynamic forces increase as the square of the speed of rotation. (i.e. $F_{Dynamic}\uparrow \Rightarrow Speed^2\uparrow$)

### 6.2.1 TYPES OF CRANKSHAFT

A crankshaft is composed of the crankpins, crank arms, crank journals, and driving ends. As a rule, crankshafts are forged in a single piece, but occasionally they are built up. Built-up crankshafts are used in small single- and double-cylinder motorcycle engines. The enclosed flywheels of these engines take the place of the crank arms, the crankpin and crank journals being bolted to the flywheels, which latter are cast with solid webs. The built-up construction also has advantages when it is desired to support the crankshaft in three or more ball bearings, as with a one-piece shaft all intermediate bearings would have to be stripped over the crank arms, and therefore would have to be made extraor-dinarily large.

A crankpin together with the two crank arms on opposite sides of it is frequently referred to as a "throw." In some crankshafts there is only a single throw between a pair of main journals or supporting bearings, while in others there are two and even three or four throws between main bearings.

6.2.1a Based on the position of the crank pin

i) Side crankshaft or overhung crankshaft. (Figure. 5)

ii) Centre crankshaft (Figure. 6)

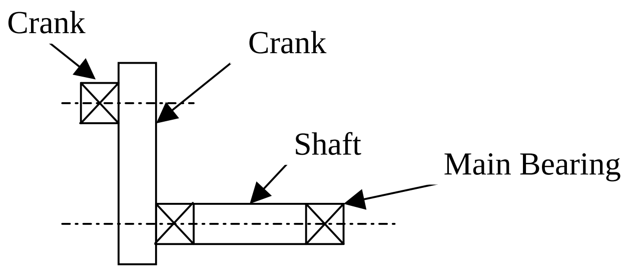


Figure.5 Side Crank

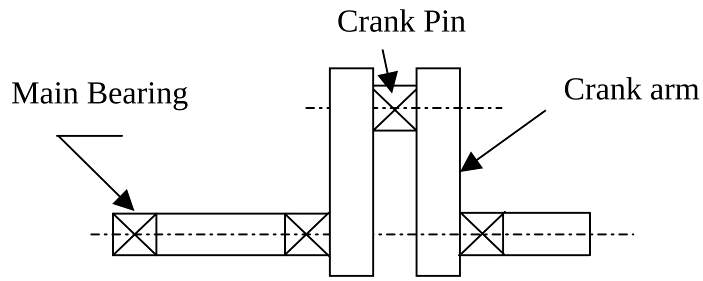


Figure.6 Centre Crank Shaft

6.2.1b Based on the number of throw

The other classification is based on the number of cranks in the shaft are:

I. Single throw crankshafts
II. Multi throw crankshafts

A crankshaft with only one side crank or centre crank is called a single throw crankshaft. A crankshaft with two-side cranks, one on each end or more centre cranks is called as multi throw crankshaft.

## 6.2.2. FEW TYPICAL CRANKSHAFTS

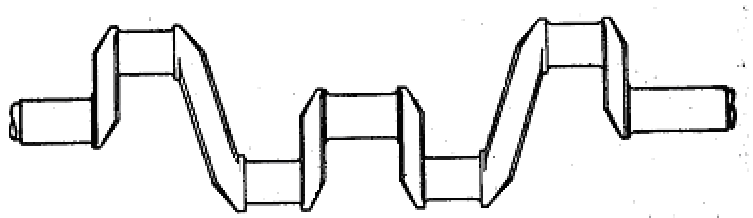

Figure 7. Proportions of four cylinder Crank Shaft in 1911

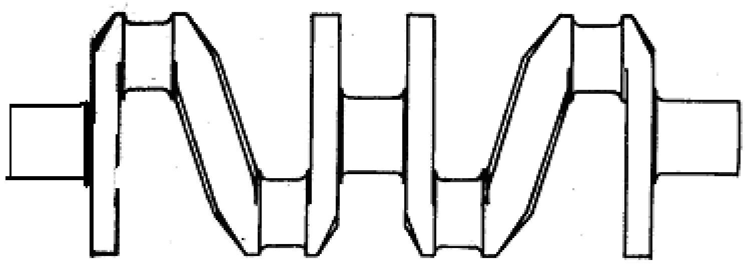

Figure 8. Proportions of four cylinder Crank Shaft in 1948

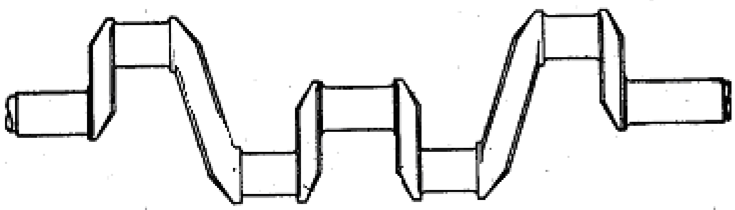

Figure 9.Four cylinder, three bearing Crank Shaft

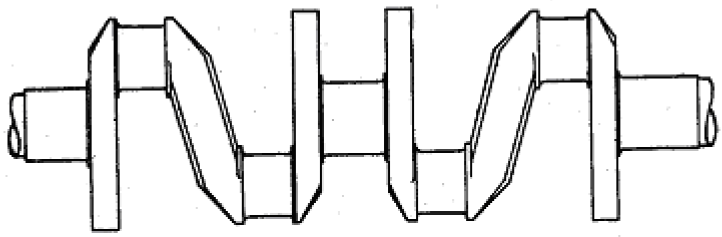

Figure 10.Crank shaft for Four cylinder opposed engine

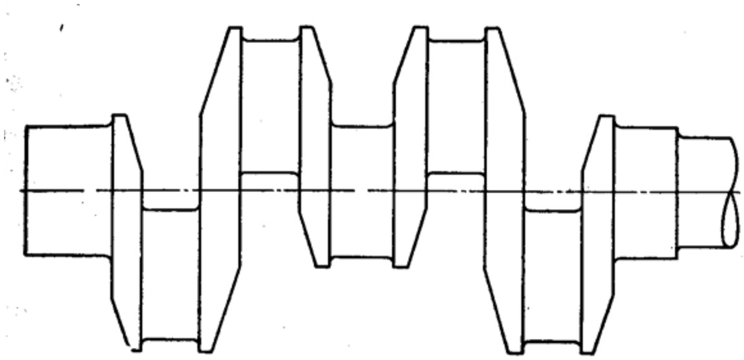

Figure 11.Crank shaft of Volkswagan Four cylinder-opposed engine

**6.2.3 Materials**

For the proper functioning, the crankshaft should full fill the following conditions:

1. Enough strength to withstand the forces to which it is subjected i.e. the bending and twisting moments.
2. Enough rigidity to keep the distortion a minimum.
3. Stiffness to minimize, and strength to resist, the stresses due to torsional vibrations of the shaft.
4. Sufficient mass properly distributed to see that it does not vibrate critically at the speeds at which it is operated.
5. Sufficient projected areas of crankpins and journals to keep down the bearing pressure to a value dependent on the lubrication available.
6. Minimum weight, especially in aero engines.

The crankshafts are made much heavier and stronger than necessary from the strength point of view so as to meet the requirements of rigidity and vibrations. Therefore the weight cannot be reduced appreciably by using a material with a very high strength. The material to be selected will also depend upon the method of manufacture i.e. cast, forged, or built up. Built up crank shafts are sometimes used in aero engines where light weight is very important.

In industrial engines, 0.35 Carbon steel of ultimate tensile strength 500MPa to 525 MPa and 0.45 Carbon steel of ultimate tensile strength of about 627 to 780 MPa are commonly used.

In transport engines, alloy steel e.g. manganese steel having ultimate tensile strength of about 784 to 940 MPa is generally used.

In aero engines, nickel chromium steel having ultimate tensile of about 940 to 1100 MPa is generally used. Heavy duty Cast iron is being successfully used for crankshafts, especially for industrial engines of comparatively low speed and it can replace easily the plain carbon steels. Cast iron usually used for crankshafts is nickel cast iron with ultimate tensile strength ranging from 350 to 525 MPa. Cast steel is also used as material for crankshafts, its ultimate tensile strength ranging from 560 to 600 MPa.

*Students are requested to go through the Table3.2/47, Table3.3/48, Table 3.5b/48, and page no 412 to 430 of the data handbook for different material properties.*

Medium-carbon steel is the material most extensively used. The blanks from which the crankshafts are machined are produced by the drop forging process. This process requires several heatings to a good red heat, which impairs the physical qualities of the steel; and the latter have to be restored and the latter have to be restored by suitable heat treatment, consisting of normalizing, reheating and quenching. The carbon steel generally used for crankshafts is **S.A.E. steel No. 1045** [Refer T 1.18/428] which has the following composition and physical properties:

Chemical Composition

| | PerCent |
|---|---|
| Carbon | 0.43-0.50 |
| Manganese | 0.60-0.90 |
| Sulphur | Not over 0.050 |
| Phosphorus | Not over 0.040 |

A suitable heat treatment for drop forgings of this grade of steel is as follows: Normalize at $571^0$C to $927^0$C, reheat to $789^0$C to $843^0$C, quench in oil and draw at $842^0$C. When thus heat-treated the steel has approximately the following physical properties:

| | |
|---|---|
| Tensile strength | 759MPa |
| Elastic limit | 517MPa |
| Elongation in 50mm | 18 per cent |
| Reduction of area | 45 per cent |
| Brinell hardness | 225-235 |

In some of the higher-grade automotive engines, chrome-nickel steel (S.A.E. 3140) is used for the crankshaft. The composition and the approximate mechanical properties of this steel (after heat treatment) are as follows:

Chemical Composition

| | Percent |
|---|---|
| Carbon | 0.38-0.43 |
| Manganese | 0.70-0.90 |
| Phosphorus | Not over 0.040 |
| Sulphur | Not over 0.040 |
| Nickel | 1.1 0-1.40 |
| Chromium | 0.55-0.75 |

Mechanical Properties (After Heat Treatment)

| | |
|---|---|
| Tensile strength | 1069MPa |
| Elastic limit | 910MPa |
| Elongation in 50mm | 16 per cent |
| Reduction of area | 50 per cent |
| Brinell hardness | 295-305 |

The heat treatment for this steel consists in normalizing at $871^0$C -$927^0$C, annealing to the desired structure or machinability; heating to $788^0$C - $816^0$C, quenching in oil, and tempering at $483^0$C

Other materials used for crankshafts include chrome-vanadium and chrome-molybdenum steels. All of these binary alloy steels have excellent mechanical properties, the tensile strength in the heat-treated condition usually running above 1034MPa

## 6.2.4 Manufacturing:

Great care must be observed in the manufacture of crankshafts since it is the most important part of the engine. Small crankshafts are drop forged. Larger shafts are forged and machined to shape. Casting of the crankshafts allows a theoretically desirable but complicated shape with a minimum amount of machining and at the smallest cost. These are cast in permanent moulds for maximum accuracy and a minimum of machining. While machining, the shaft must be properly supported between centers and special precautions should be taken to avoid springing. The journals and crankpins are ground to exact size after turning. After this, the crankshaft is balanced. Large shafts of low speed engines are balanced statically; Crankshafts of high-speed engines are balanced dynamically on special balancing machines. Most crankshafts are ground at the journals and crankpins. In some cases grounding is followed by hand lapping with emery cloth.

## 6.2.6. Bearing pressures:

The bearing pressures are very important in the design of crankshafts. The allowable bearing pressure depends upon the journal velocity, change of direction of bearing pressures, amount and method of lubrication and the maximum gas pressures and space limitations. Maximum allowable bearing pressures are given below Table 1.

**6.2.6.Stresses:**

The stresses induced in the crankshafts are bending and also shear stresses due to torsional moment of the shaft. Most crankshafts fail due to progressive fracture due to repeated bending or reversed torsional stresses. Thus the type of loading on the crankshafts is fatigue loading therefore, the design should be based on endurance limit.

To avoid stress concentration and fatigue failure, abrupt changes in the section of shaft connection should be avoided. Two different cross sections must be blended with a large fillet 'r', if possible, **r should not be less than 0.2d.** [d is the diameter of the shaft]. If there is no space for fillet, the crank web should be under cut to obtain the fillet. This will make the web weak and to compensate for it, the width is increased. (Figure12)

Since the failure of the crankshaft is serious for the engine, and also because of the inaccuracy in determining all the forces and stresses, a high factor of safety based on endurance limit from 3 to 4 should be used. To be on the safe side, the endurance limits for complete reversal of bending and torsional stresses are taken.

For chrome nickel and other alloy steels, the endurance limit is about 525 N/mm$^2$ in bending and about 290 N/mm$^2$ in shear.

Table 1. DESIGN DATA FOR BEARING *

| Machinery (1) | Bearing (2) | Maximum $p^{**}$ MN/m$^2$ (3) | Suitable Z** Ns/m$^2$ (4) | Minimum $(ZN/p)^*$ (5) | $\frac{c}{r}$ (6) | $\frac{l}{d}$ (7) |
|---|---|---|---|---|---|---|
| 1. Automobile and aircraft engines | Main | 5.5-12.0 | 0.007-0.008 | 2.18 | — | 0.8-1.8 |
| | Crank pin | 10.0-24.0 | | 1.45 | — | 0.7-1.4 |
| | Wrist pin | 16.0-34.5 | | 1.16 | — | 1.5-2.2 |
| 2. Gas and oil engines, four stroke | Main | 4.8- 8.2 | 0.020-0.065 | 2.90 | 0.001 | 0.6-2.0 |
| | Crankpin | 9.6-12.4 | | 1.45 | <0.001 | 0.6-1.5 |
| | wrist pin | 12.4-15.2 | | 0.73 | <0.001 | 1.5-2.0 |
| 3. Gas and oil engines, two stroke | Main | 3.4- 5.5 | 0.020-0.065 | 3.63 | 0.001 | 0.6-2.0 |
| | Crank pin | 6.9-10.3 | | 1.74 | <0.001 | 0.6-1.5 |
| | Wrist pin | 8.2-12.4 | | 1.45 | <0.001 | 1.5-2.0 |

** The above table is extracted from "Design Data Book ", K Mahadevan and K Balaveera Reddy, CBS Publications, INDIA, 1989, TABLE 15.11, Page No 314*
** Refer Table 3.6/49 for Allowable Bearing pressures*

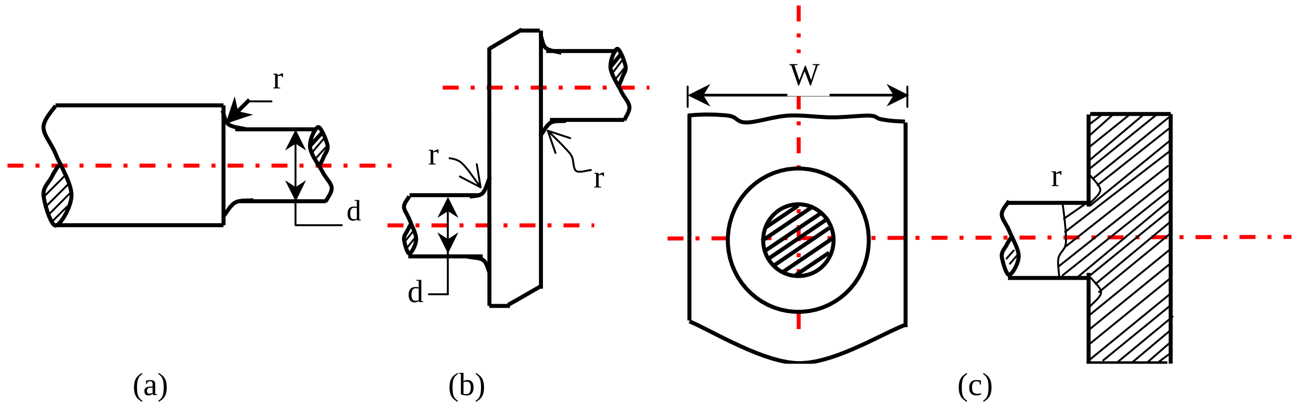


**Figure 12, Use of Fillets**

For carbon steel and cast steel, the endurance limit is about 220 N/mm$^2$ in bending and about 120 N/mm$^2$ in shear. For alloy cast iron, the endurance limit is about 140 N/mm$^2$ in bending and in shear.

Thus the allowable stress is:

For Carbon steel: bending = 55 to 75 N/mm$^2$

Shear = 30 to 40 N/mm$^2$

Combined stress = $\frac{1}{2}$ (elastic limit in tension)

For alloy Cast Iron:

Bending= shear = 34 to 45 N/mm$^2$

Combined stress = elastic limit in pure tension.

For Chrome nickel and other alloy steel:

Bending = 130 to 175 N/mm$^2$

Shear = 70 to 97 N/mm$^2$

Combined stress = $\frac{1}{2}$ (elastic limit in pure tension)

### 6.2.7 Balance Weights:

In a single-cylinder crankshaft the centrifugal force on the crank arms, crankpin, and part of the connecting rod forms an unbalanced rotating force, which would cause the engine to vibrate if no means were provided to balance it. Therefore, balance weights are applied to the crank arms.

In a high-speed engine the balance weights are preferably forged integral with the crank arms. If made separate, they must be very securely applied, since the stresses on the fastenings due to the centrifugal force at "racing" speeds are very considerable, and if one of the weights should come loose, it would be sure to do serious damage. Alloy steel bolts or studs should be used, or the counterweights should be fitted to the crank arms in such a way that the centrifugal force produces shearing stresses in the parts, instead of tensile stresses in the bolts.

In a double-cylinder opposed engine the crankshaft is always made with two throws set at $180^0$ relative to each other. One set of reciprocating parts then always moves in opposition to the other set, and at exactly the same speed, so that the reciprocating parts are perfectly balanced, except for the fact that the two sets are not quite in line with each other. The rotating parts also are very nearly balanced, since the centrifugal force acting on one throw is equal and opposite in direction to the centrifugal force acting on the other throw, and there is only a small rotating couple due to the centrifugal forces acting at the ends of an arm equal in length to the distance between the centers of the two. This rotating couple can be balanced by applying balance weights to the two short crank arms.

In a four-cylinder vertical engine the four throws are always in the same plane, the two outer throws being on the same side of the crankshaft axis, and the two inner throws on the opposite side. The centrifugal force acting on each throw is a radial rotating force, which may be considered to act at the center of that throw. The centrifugal forces F on the two outer throws **(Figure 13)** naturally are always in the same plane, and. as they are equal, their resultant *R* is a force equal to twice that acting on the individual throw, acting at a point midway between the two throws. The resultant $R_l$ of the centrifugal forces $F_1$ acting on the two inner throws is exactly equal to the resultant of the centrifugal forces acting on the two outer throws, and acts at the same point but in the opposite direction to the latter; consequently it neutralizes or balances it.

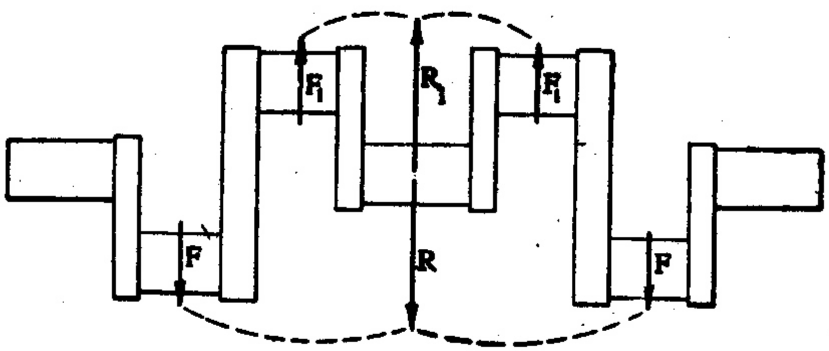


Figure 13 Centrifugal forces on four throw crankshaft

**6.2.8 Local Balance:**

But while a crankshaft of this type is perfectly balanced as a whole, its individual throws are unbalanced, and since the crankshaft is more or less flexible, the centrifugal force acting on the individual throw presses the crank journals adjacent to it against their bearings, adding to the loads on these bearings.

To reduce this bearing load it is now customary to provide crank arms of high-speed engines with balance weights. A rather serious degree of local unbalance occurs in a four-cylinder, two-bearing crankshaft, because in the conventional design there is nothing to balance the two inner crank pins and the intervening portion of the crankshaft. Sometimes balance weights are welded to crankshafts of this type, as well as to others, but the majority of all crankshafts for high-speed engines have balance weights forged on all crank arms that are not symmetrical with relation to the crankshaft axis, which balance weights either completely or partly balance the rotating parts of the crank throw. A crankshaft of such design that the centrifugal forces acting on it would vanish if all acted in the same plane perpendicular to the crankshaft axis is said to be in static balance.

In a six-throw crankshaft the throws are arranged in pairs, the two inner ones being in line with each other, as are also the two outer ones and the two intermediate ones, respectively. Each pair of throws is located at an angular distance of $120^0$ from the other two pairs. By reference to Figure 14, which is an end view of a six throw, seven-bearing crankshaft, it can be seen that if all of the throws are identical, such a crankshaft is in static balance. The resultant F of the centrifugal forces on the two throws of each pair acts at the middle of the length of the crankshaft. The three resultants therefore all act in the same plane radially outward from the center of the crankshaft, at angles of $120^0$, and they exactly balance each other. The highest degree of balance, of course, is obtained if each throw is balanced separately, which involves the use of a balance weight on each arm.

Six-cylinder crankshafts with either three or four main bearings are not inherently balanced. In fact, as sometimes made, they are not even in static balance. In the most primitive form, the long crank arms extend straight across from one crankpin to another at angular distance of $120^0$ therefrom. This crank arm then lies wholly to one side of the axis of rotation, and in operation produces an unbalanced rotating force whose direction is at all times along a line through the axis of rotation and the center of gravity of the crank arm. The magnitude of this unbalanced force is proportional to the distance of the center of gravity of the crank arm from the axis of rotation, and therefore can be reduced by reducing this distance, by curving the crank arm inward, as shown in Figure 15. The rotating force, of course, can be entirely eliminated by bolting a balance weight against the side of the long crank arm.

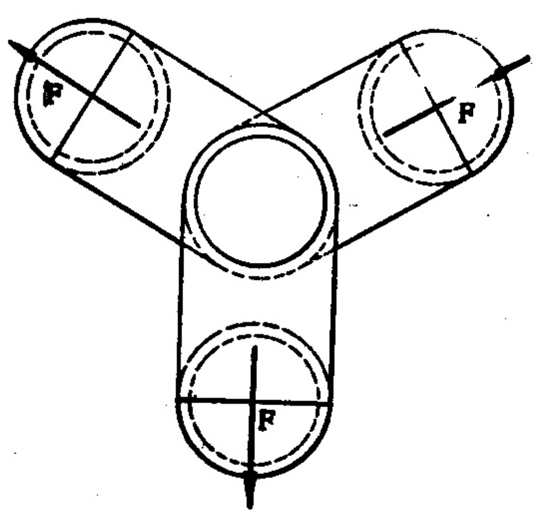


Figure 14. End view of Six-throw, seven-bearing crankshaft.

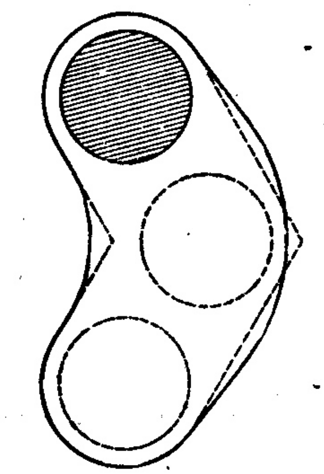

Figure 15. Curved long arm of six-throw, three or four-bearing crankshaft.

### 6.2.9 Empirical Rules for Crankshaft Dimensions

In making a preliminary lay-out, all dimensions of the crankshaft can be made proportional to the cylinder bore. Of course, the proportion between any given dimension of the crankshaft and the cylinder bore varies with the number and arrangement of cylinders and with the type of crankshaft as defined by the number of its main bearings.

In all drop forged crankshafts in which the arms are left unfinished, the arm section, of course, is not a rectangle, as draft has to be allowed on the two long sides, usually about $7^0$. In applying the rules for arm thickness the calculated thickness can be taken as the mean between the minimum and maximum actual thicknesses.

### 6.2.10 Six Cylinder Crankshafts

Crankshafts for six-cylinder in line engines are made with three, four, or seven main journals. The greater the number of main journals the better the support for the crankshaft and the smoother the operation of the engine at high speeds. On the other hand, manufacturing costs increase somewhat with the number of main bearings. Whatever the number of main journals, the angular spacing of the throws is the same; that is, throws Nos. 1 and 6 are in line, as are throws Nos. 2 and 5, and throws Nos. 3 and 4. [Firing order 1-6-2-5-3-4] Both the three-bearing and the four-bearing crankshafts have short arms connecting a crankpin with the adjacent main journal, and long arms connecting two crankpins spaced *120°* apart. In Figure 16 is shown sides view of the rear half of a four-bearing, six-cylinder crankshaft (the front half being omitted because it is an inverse duplicate of the rear). At the center between crankpins Nos. 3 and 4 there is a balance weight, of which a transverse section is shown. The short crank arms have the usual integral counterweights, while the long crank arm is in the form of a disc, of which a side view is shown at the right in the illustration. Back of the junction with the crankpins the disc is rounded off so as to eliminate unnecessary weight. Long crank arms of the same general form are used in three-bearing crankshafts for six-cylinder engines, which latter have four of them. Seven bearing, six-cylinder crankshafts usually are finished all over; at least the crank arms, which are all similar in shape, are finished on both sides, so as to make the engine as compact as possible length wise.

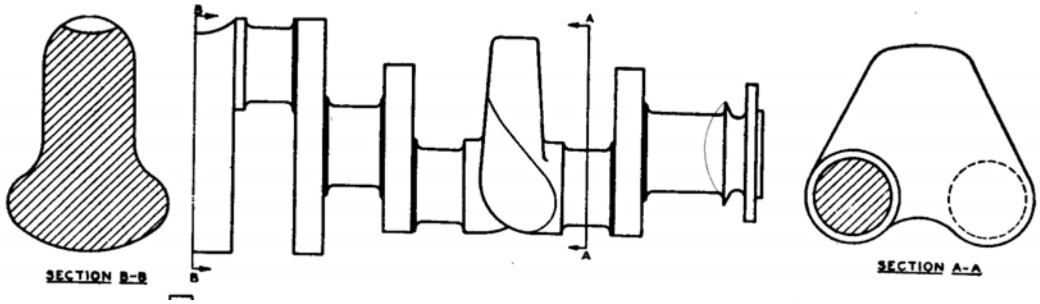


Figure 16. Rear half of four bearing, six-cylinder crankshaft

## 6.2.11 Eight Cylinder Crankshafts

Crankshafts for eight-cylinder in line engines are designed with either five or nine main journals. In the first case there are two crank throws between supports, while in the last there is only one. Here, too, the angular spacing of the various throws is the same, regardless of the number of main journals, cranks Nos. 1 and 8 being in line, also Nos. 2 and 7, 3 and 6, and 4 and 5, and the angular spacing between one pair of cranks and the next is always 90°. The crankshaft of an eight-cylinder in-line engine consists essentially of two conventional four-cylinder crank shafts, one of these being cut in halves, and each half joined to one end of the other crankshaft, in a plane at right angles to it. Figure is a side view of such a crankshaft. The center bearing is made considerably longer than the two intermediate bearings, be cause it carries the inertia loads from two sets of reciprocating parts that are in phase, while in the case of the intermediate bearings the two *sets* of reciprocating parts on opposite sides of them are 90° out of phase.

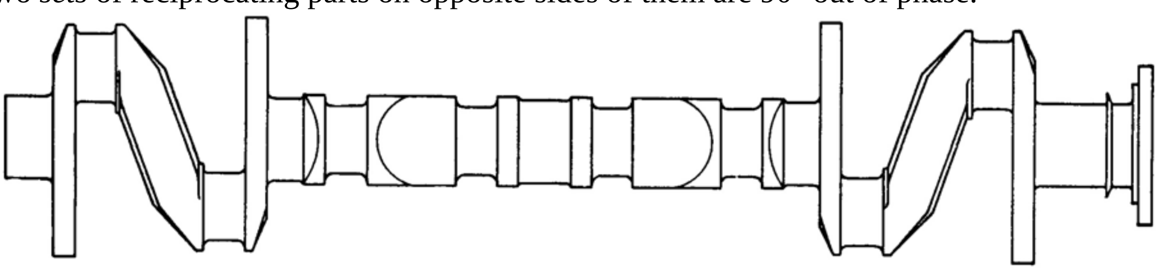

Figure 17 Five bearing crankshaft for eight cylinder crankshaft in line engine

## 6.2.12 Oil Holes Drilled in Crankshafts

In engines provided with pressure lubrication, oil holes are drilled through the crankshaft journals and arms to permit oil to flow from the main to the crankpin bearings. Sometimes radial holes are drilled through the crank arms, and communicating axial and radial holes through the journals, the holes through the crank arms and the axial holes through the journals later having their ends plugged. However, the preferred practice is to drill single inclined holes through the main journals, crank arms and crankpins, as illustrated in Figure 18. In the illustration, a second hole is shown drilled halfway through main journal, so that there are two oilcrankshaft inlets, but this practice is not common.

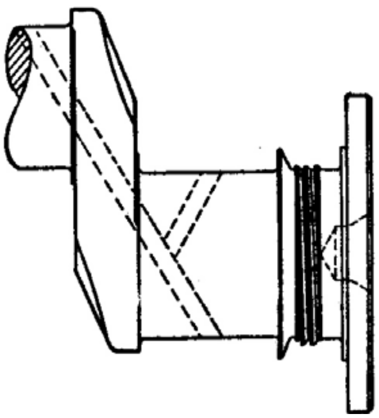

Figure 18 Inclined Oil holes in crankshaft

**6.2.13 Balancing Crankshaft**

Crankshafts, after they are completely machined, must be balanced both statically and dynamically. A crankshaft is in static balance if, when placed on horizontal steel balance ways or their equivalent, it will remain in any position to which it is turned. A simple static balancing machine consists of two pairs of discs freely supported either on hardened centers or on ball bearings (Figure 19), the two pairs being sufficiently far apart so the crankshaft can be placed upon them with its end main bearings. If the center of gravity of the crankshaft does not lie in the mechanical axis, then the crankshaft will turn until the center of gravity is directly underneath the mechanical axis. By removing material from the heavy side, with a drill or emery wheel, until the crankshaft will remain in any angular position in which it is placed on the discs, static balance may be attained.

Dynamically the shaft may still be unbalanced. For instance, there may be excess weight on one side of the shaft at one end, which is balanced statically by an equivalent weight on the other side at the opposite end. In that case, when the engine is running, there is what is known as a centrifugal couple, and this must be eliminated before the crank can be expected to run without vibration at high speeds. Unbalance of this kind can be determined only in a dynamic balancing machine.

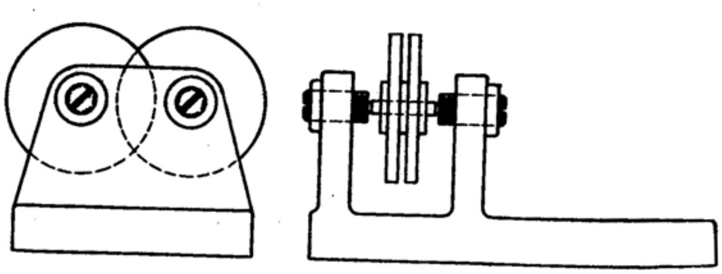

Figure 19 One end of static balancing machine

## 6.3 Design Procedure:

1. Determine the magnitudes of the various loads acting on the crankshaft.
2. Determine the distance between supports. The distances will depend upon the lengths of the bearing. The lengths & diameters of the bearings are determined on the basis of maximum permissible bearing pressures, l/d ratios and the acting loads. (TABLE 1 and Table 3.6/49)
3. For the sake of simplicity and safety, the shaft is considered to be supported at the centers of the bearings.
4. The thickness of the crank webs is assumed, about 0.5d to 0.6d, where d is the shaft diameter, or from 0.22D to 0.32D, where D is the cylinder bore.
5. Now calculate the distance between supports.
6. Assume allowable bending and shearing stresses.
7. Compute the necessary dimensions of the crankshaft.

The above procedure is general design procedure. It may change as per the requirements and definition of the given problem.
Note: All the forces and reactions are assumed to be acting at the centers of the bearings.

### 6.3.1 DESIGN CALCULATIONS:

In the design of the crankshafts, it is assumed that the crankshaft is a **beam** with two or more supports. Every crankshaft must be designed or checked at least for two crank positions, one when the bending moment is maximum, and the other when the twisting moment is a maximum. In addition, the additional moments due to the flywheel weight, belt tension and other forces must be considered.

To make the calculations simpler, without loosing accuracy, it is assumed that the effect of the bending forces does not extend two bearings between which a force is applied.

There are two considerations, which determine the necessary dimensions of the crankpin. One is that its projected bearing area (diameter times length) must be large enough so it will safely sustain the bearing loads imposed upon it by gas pressure, inertia and centrifugal force; the second, that the crankshaft as a whole must be sufficiently rigid so that it will not vibrate perceptibly under the periodic forces to which it is subjected in service. When the crankshaft of a given engine is made more rigid, the so-called critical speeds-that is, speeds at which there is synchronous vibration-are raised, and in this way at least the most important critical speeds can be moved outside the normal operating range.

## 6.4 Analysis of Center Crank Shaft:

The crank shaft is to analysed for two positions:

i) Crank on Dead Center: and ii) Crank at angle of maximum Twisting Moment

### 6.4a Crank at Dead Center

When the crank is on dead center, maximum bending moment will act in the crankshaft. The thrust in the connecting rod will be equal to the piston gas load (F), W is the weight of the flywheel acting downward and $T_1$ and $T_2$ is the belt pull acting horizontally.

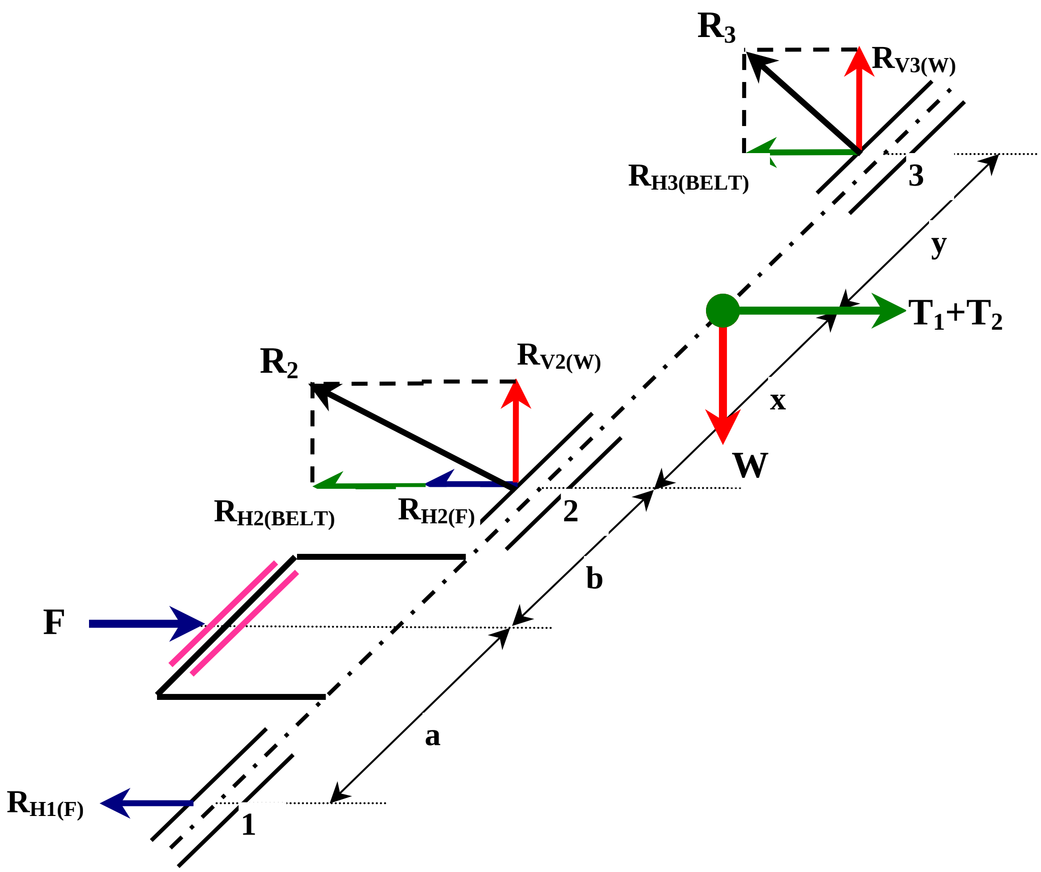


Figure 20. Force Analysis of Crank at Dead Center

In between bearings 1 and 2, Gas Load F, acts as shown in Figure 7

Now Gas Load,

$F = \frac{\pi}{4} D^2 * p_{max}$ , Where D is the diameter of the piston in mm and $p_{max}$ is the maximum gas pressure

Due to this there will be two horizontal reactions, $R_{H1(F)}$ at bearing 1, and $R_{H2(F)}$ , at bearing 2, so that,

To find the reactions $R_{H1(F)}$ and $R_{H2(F)}$

$\sum M_1 = 0$, + −

Clock Wise direction is taken as positive bending moment and Counter Clockwise as negative bending moment.

$$F(a) - R_{H2(F)}(a+b) = 0 \quad \text{.......................................................}(1)$$

Hence $R_{H2(F)} = \dfrac{F(a)}{(a+b)}, N$ ..........................................................(2)

$\sum F_y = 0$, − + , Upward force is taken as positive and downward is taken as negative.

$$R_{H1(F)} + R_{H2(F)} - F = 0 \quad \text{..........................................................}(3)$$

By substituting equation 2 in equation 3 we get,

$$R_{H1(F)} = F - \frac{F(a)}{(a+b)} = \frac{F(b)}{(a+b)}, N$$

If a=b, then, $R_{H1(F)} = R_{H2(F)} = \dfrac{F}{(2)}, N$

In between bearings 2 and 3, we have two loads

i) Belt pull ($T_1 + T_2$ ), acting horizontally as shown in Figure 8

ii) Weight of the Flywheel (W), acting vertically as shown in Figure 8

<u>Reactions at bearing 2 and 3 due to Belt Pull,</u>

Due to this there will be two horizontal reactions, $R_{H2(belt)}$ at bearing 2, and $R_{H3(belt)}$ at bearing 3, so that ,

To find the reactions $R_{H2(belt)}$ and $R_{H3(belt)}$

$\sum M_2 = 0$, + −

Clock Wise direction is taken as positive bending moment and Counter Clockwise as negative bending moment.

$$(T_1 + T_2)(x) - R_{H3(belt)}(x+y) = 0$$

Hence $R_{H3(belt)} = \dfrac{(T_1 + T_2)(x)}{(x+y)}, N$ .......................................................(4)

$\sum F_y = 0$, $\downarrow -$ $\uparrow +$ , Upward force is taken as positive and downward is taken as negative.

$\rightarrow$ $R_{H2(belt)} + R_{H3(belt)} - (T_1+T_2) = 0$……………………………………………(5)

By substituting equation (4) in equation(5) we get,

$$R_{H2b(elt)} = (T_1 + T_2) - \frac{(T_1 + T_2)(x)}{(x+y)} = \frac{(T_1 + T_2)(y)}{(x+y)}$$

If x=y, then, $R_{H2(belt)} = R_{H3(bel)t} = \dfrac{(T_1 + T_2)}{(2)}$

<u>Reactions at bearing 2 and 3 due to Weight of the Flywheel,</u>

Due to this there will be two Vertical reactions, $R_{v2(W)}$ at bearing 2, and $R_{v3(W),}$ at bearing 3, so that,
To find the reactions $R_{v2(W0}$ and $R_{v3(W)}$

$\sum M_2 = 0$, $\curvearrowright +$ $\curvearrowleft -$

Clock Wise direction is taken as positive bending moment and Counter Clockwise as negative bending moment.

$\rightarrow$ $(W)(x) - R_{v3(w)}(x+y) = 0$

Hence $R_{v3(W)} = \dfrac{(W)(x)}{(x+y)}, N$ ……………………………………………………(6)

$\sum F_y = 0$, $\downarrow -$ $\uparrow +$ , Upward force is taken as positive and downward is taken as negative.

$\rightarrow$ $R_{v3(W)} + R_{V2(W)} - (W) = 0$……………………………………………(7)
By substituting equation (6) in equation (7) we get,

$$R_{v2(W)} = (W_2) - \frac{(W)(x)}{(x+y)} = \frac{(W)(y)}{(x+y)}$$

If x=y, then $R_{V2(W)} = R_{V3(W)} = \dfrac{(W)}{(2)}$

In this position of the crank, there will be no twisting moment, and the various parts will be designed for bending only.

6.4a.1 CRANKPIN:

The bending moment at the centre of the crankpin is,
$M = R_{H1(F)}(a)$, N-mm
We know that,
$$\frac{M}{I} = \frac{\sigma}{c} \text{ ........................................................(1.16/3)}$$

Substituting the values of $C = \frac{d_p}{2}$ and $I = \frac{\pi d_p^{\ 4}}{64}$ in Equation 1.16 and solving for M we get,
$$M = \frac{\sigma_b}{c}(I) = \frac{\sigma_b}{\left(\frac{d_p}{2}\right)}\left(\frac{\pi}{64}\right)(d_p^{\ 4})$$
$$M = \frac{\pi}{32} d_p^{\ 3}(\sigma_b) \text{, N-mm}$$
Knowing the value of Bending moment M and allowable stress in bending, $\sigma_b$, diameter of the Crankpin $d_p$ can be obtained.

Length of the crankpin ($l_p$) can be obtained by suitably choosing/assuming the value of allowable bearing pressure and using the following formula;
Bearing pressure, $p_b = \frac{F}{(l_p)(d_p)}$, MPa
Length of the crankpin, $l_p = \frac{F}{(d_p)(p_b)}$, mm
Or we can use empirical relation as $l_p$=(0.8 to 1.3) $d_p$ ……………………….(Page No 50)

6.4a.2 Left Hand Crank Web:
The crank web is designed for eccentric loading. There will be two stresses on it, one *direct compressive stress* and the other *bending stress* due to the gas load F.

The thickness h=0.22D to 0.32 D or
=0.5 $d_p$ to 0.9 $d_p$ ……………………………….………(Page No 50)
=0.65 $d_p$ + 6.35mm………………………………………(Page No 50)

The width 'w'may be assumed to be as follows:
$$w = \frac{9}{8} d_p + 12.7, mm$$
= (1.1 to 1.2) $d_p$, mm…………………………………………………(Page No 50)

Since the empirical relations are used it is advised to check the developed stresses against the given values.

Direct stresses($\sigma_d$)

$$\sigma_d = \frac{R_{1H(F)}}{(w)(h)}, MPa$$

Bending stresses: ($\sigma_b$)

$$\frac{M}{I} = \frac{\sigma_b}{c}; \quad \text{………………………………………………………..(1.16/3)}$$

$$M = R_{1H(F)}\left(a - \frac{l_p}{2} - \frac{h}{2}\right)$$

$$I = \frac{wh^3}{12} \text{ And } c = \frac{h}{2}$$

Substituting the values of M, c and I in bending equation (1.16/3) we get

$$\sigma_b = R_{1H(F)}\left(a - \frac{l_p}{2} - \frac{h}{2}\right)\left(\frac{6}{wh^2}\right), MPa$$

Superimposing the direct and bending stresses we get total stress on the web, which must not exceed the allowable stress in bending. Otherwise increase the value of thickness and width and recheck the design.

6.4a.3Right Hand Crank Web:

Since the bearing 1 and 2 are usually of the same length and symmetrical to the cylinder centerline, therefore $R_{H1(F)}$ and $R_{H2F()}$ are equal. Hence normally same dimensions are adopted for both crank webs.

Otherwise, providing the dimensions empirically as used in Left hand crank web and check is made as follows:

$$M = R_{1H1(F)}\left(a + \frac{l_p}{2} - \frac{h}{2}\right)$$

$$\sigma_b = R_{H1(F)}\left(a + \frac{l_p}{2} - \frac{h}{2}\right)\left(\frac{6}{wh^2}\right), MPa$$

Superimposing the direct and bending stresses we get total stress on the web, and check against the allowable stresses. To avoid the manufacturing difficulties keep the same dimensions for both Webs by taking higher h and w values.

6.4a.4 Shaft Under the Flywheel: [Diameter of the shaft between bearing 2 and 3]

Bending moment due to flywheel weight is $M_{FLY} = (R_{V3(W)})(y)$

Bending moment due to the belt pull is $M_{belt} = (R_{H3(BELT)})(y)$

Since these bending moments act at right angles to each other, the combined bending moment is given by;

$$M_{Total} = \sqrt{M_{FLY}^2 + M_{belt}^2}$$

We know that $M_{Total} = \frac{\sigma_b}{c}(I) = \frac{\sigma_b}{\left(\frac{d_W}{2}\right)}(\frac{\pi}{64})(d_W{}^4)$

$M_{total} = \frac{\pi}{32} d_W{}^3 (\sigma_b)$, N-mm,

Where $d_W$ is diameter of the shaft under flywheel and

$\sigma_b$ is allowable stress in bending. Its value should be taken low to take care of reversal of stresses in each revolution and to ensure necessary rigidity.

**6.4b Crank at an angle of maximum twisting moment**

The twisting moment on the crankshaft will be maximum, when the tangential force $F_t$ is maximum and this can be calculated graphically by taking pressures from the net effort diagram for different crank angles. The angle usually lies between $25^0$ to $35^0$from the dead center for a constant volume combustion engines and between $30^0$ to $40^0$ for constant pressure combustion engines. At this angle, the gas pressure will not be maximum. If $F_p$ is the gas load along the cylinder centerline, then the thrust $F_C$ along the connecting rod is given by (Ref Figure 21 or same as FIG 3.1/50 in design data book)

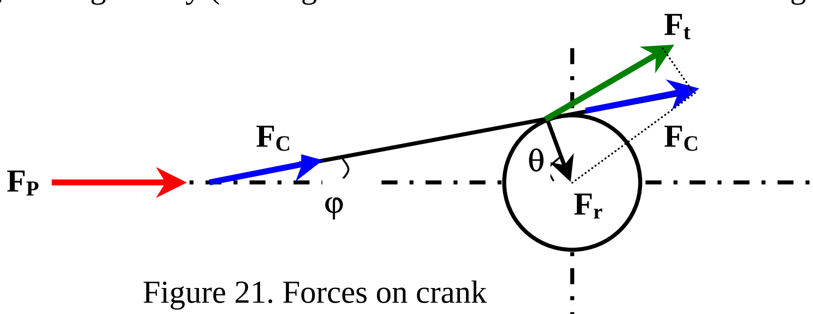


Figure 21. Forces on crank Arm

The force on the connecting Rod or thrust force

$$F_C = \frac{F_P}{\cos(\phi)} \quad \ldots\ldots (3.12/45)$$

The tangential force or the rotative effort on the crank

$$Ft = F_C \sin(\phi + \theta) = \frac{F_P \sin(\phi + \theta)}{\cos(\phi)} \quad \ldots\ldots (3.13/45)$$

The radial force along the crank

$$Fr = F_C \cos(\phi + \theta) = \frac{F_P \cos(\phi + \theta)}{\cos(\phi)} \quad \ldots\ldots (3.14/45)$$

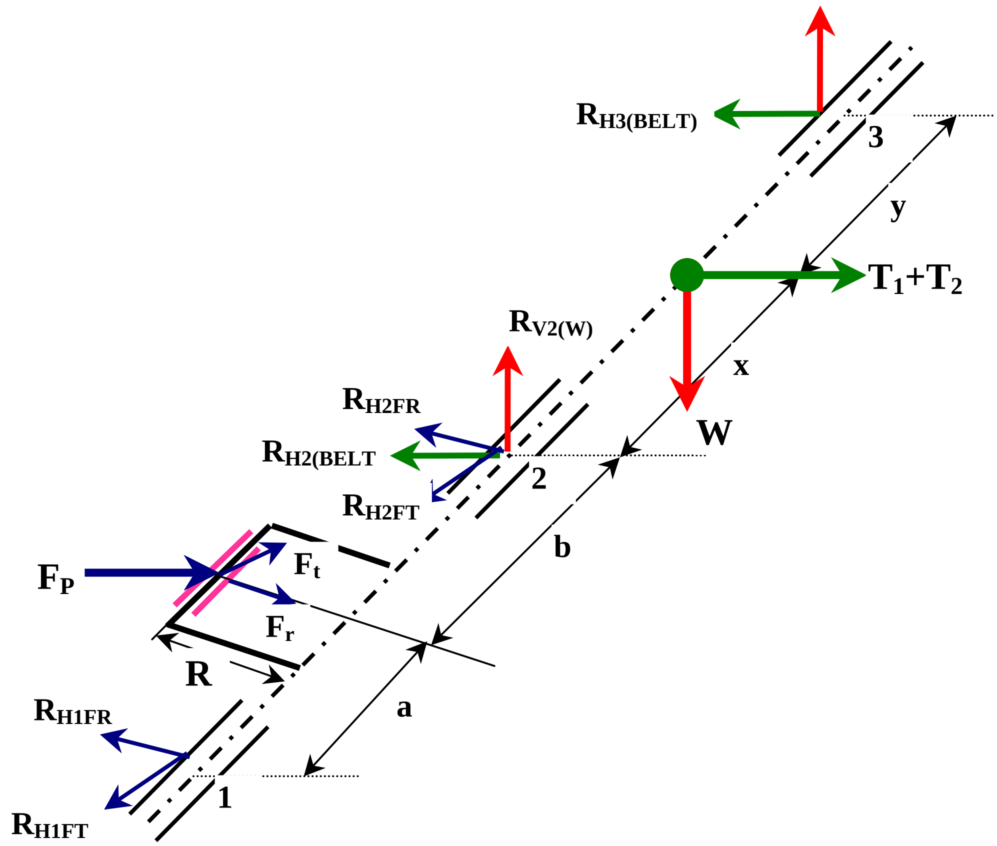


Figure 22. Force Analysis of Crank at angle of maximum twisting Moment

Tangential force $F_t$ will have two reactions $R_{H1FT}$ and $R_{H2FT}$ at bearing 1 and 2 respectively.

Radial force $F_r$ will have two reactions $R_{H1FR}$ and $R_{H2FR}$ at bearing 1 and 2 respectively.

The reactions at the bearings 2 and 3 due to belt pull ($T_1$+$T_2$) and Flywheel W will be same as before.

In this position of the crankshaft, the different sections will be subjected to both bending and torsional moments and these must be checked for combined stress. At this point, Shear stress is taken as failure criteria for crankshaft.

<u>The reactions due Radial force ($F_r$):</u>
To find the reactions $R_{H1FR}$ and $R_{H2FR}$

$$\sum M_2 = 0, \qquad + \qquad -$$

Clock Wise direction is taken as positive bending moment and Counter Clockwise as negative bending moment.

$$-F_r(b) + R_{H1FR}(a+b) = 0$$

$$R_{H1FR} = \frac{F_r(b)}{(a+b)}$$

$\sum F_y = 0$ , $-$ $\uparrow$ , Upward force is taken as positive and downward is taken as negative.

$$-F_r + R_{H1FR} + R_{H2FR} = 0$$

$$R_{H2FR} = F_r - R_{H1FR} = F_r - \frac{F_r(b)}{(a+b)}$$

$$R_{H2FR} = \frac{F_r(a)}{(a+b)}$$

<u>The reactions due tangential force ($F_t$):</u>

To find the reactions $R_{H1FT}$ and $R_{H2FT}$

$\sum M_2 = 0$ , $+$ $-$

Clock Wise direction is taken as positive bending moment and Counter Clockwise as negative bending moment.

$$\rightarrow \quad -F_T(b) + R_{H1FT}(a+b) = 0$$

$$R_{H1FT} = \frac{F_T(b)}{(a+b)}$$

$\sum F_y = 0$ , $-$ $\uparrow$ , Upward force is taken as positive and downward is taken as negative.

$$-F_T + R_{H1FT} + R_{H2FT} = 0$$

$$R_{H2FT} = F_T - R_{H1FT} = F_T - \frac{F_T(b)}{(a+b)}$$

$$R_{H2FT} = \frac{F_T(a)}{(a+b)}$$

The reactions at the bearings 2 and 3due to Flywheel weight (W) and resultant belt pull ($T_1$+$T_2$) will be as discussed earlier.

<u>6.4b.1 Crank pin:</u>

The bending moment at the centre of the crankpin is, $M_b = R_{H1FR}(a)$, N-mm

The Twisting moment is, $T = R_{H1FT}(R)$, N-mm

Equivalent twisting moment, $T_e = \sqrt{T^2 + M_b^2}, N-mm$

We know that

$$\frac{T_e}{J}=\frac{\tau}{r} \quad \text{……………………………………………………………..(1.15/3)}$$

Here $T_e$=torque or Torsional moment, N-mm

J=polar moment of inertia, $mm^4$

$= \frac{\pi}{32} d_p^{\ 4}$, Where $d_p$ is the solid shaft diameter.

τ= allowable shear stress, MPa

r= Distance from the Neutral axis to the top most fibre, mm $= \frac{d_P}{2}$

Substituting the values of J and r in equation 1.15 and simplifying we get,

$$T_e = \frac{\pi}{16} d_P^{\ 3}(\tau), \text{ N-mm}$$

From this equation the diameter of the crank pin can be obtained.
Length of the crankpin ($l_p$) can be obtained by suitably choosing/assuming the value of allowable bearing pressure and using the following formula;

Bearing pressure, $p_b = \frac{F}{(l_p)(d_p)}$, MPa

Length of the crankpin, $l_p = \frac{F}{(d_p)(p_b)}$, mm

Or we can use empirical relation as $l_p$=(0.8 to 1.3) $d_p$ ………………………(Page No 50)

6.4b.2 Shaft under the Flywheel: [Diameter of the shaft between bearing 2 and 3]

The collective bending moment due to flywheel and the belt pull will be the same as earlier.

Bending moment due to flywheel weight is $M_{FLY} = (R_{V3(W)})(y)$

Bending moment due to the belt pull is $M_{belt} = (R_{H3(BELT)})(y)$

Since these bending moments act at right angles to each other, the combined bending moment is given by;

$$M_{Total} = \sqrt{M_{FLY}^2 + M_{belt}^2}, \text{ N-mm}$$

In addition to this moment there will be a twisting moment because of tangential force $F_t$.

The twisting moment, $T = F_t(R)$, N-mm

Therefore Equivalent twisting moment,

$$T_e = \sqrt{T^2 + M_{Total}^2}, N - mm$$

We have, $T_e = \frac{\pi}{16} d_W^{\ 3}(\tau)$, N-mm, Diameter of the shaft under flywheel $d_W$ can be obtained.

6.4b.3 Right hand Crank Web:

We have used empirical formulae to obtain the values of crank web dimensions. And also we know that the Right hand Crank Web is severely stressed. In order to find the correctness of the dimensions of the web it is necessary to check the developed stresses against the allowable stresses. This web is subjected to bending stresses in two planes normal to each other, due to radial and tangential components of $F_P$; to direct compression; and to torsion.

The bending moment due to radial component is

$M_{rad} = R_{H2FR}(b - \frac{l_p}{2} - \frac{h}{2})$ , N-mm

$$\sigma_{rad} = M_{rad}(\frac{6}{bh^2}), MPa \quad \ldots\ldots(8)$$

The bending moment due to tangential component is maximum at the juncture of the crank and shaft.

$M_{Tang} = F_t(R)$, N-mm (Since here shaft diameter at junction is not considered for calculation. By doing so the bending moment increases and hence the stresses, which leads to safer side.)

$$\sigma_{Tang} = M_{Tang}(\frac{6}{w^2 h}), MPa \quad \ldots\ldots(9)$$

The stress due to direct compression, $\sigma_d = \frac{F_r}{2bh}, MPa$ ……(10)

Superimposing the stresses (At the upper left corner to the cross section of the crank) will be equal to (Addition of equation 8,8 and 10)

$$\sigma_{total} = \sigma_{rad} + \sigma_{Tang} + \sigma_d, MPa \quad \ldots\ldots(11)$$

Now the twisting moment, on the arm is

$$T = R_{HIFT}(a + \frac{l_p}{2}) - F_T(\frac{l_p}{2}) = R_{H2FT}(b - \frac{l_p}{2}), N - mm$$

We know that,
Shear stress,

$$\tau = \frac{T}{J}(c) = \frac{T}{Z} \quad \ldots\ldots(12)$$

Where Z – polar section modulus, = $\frac{bh^2}{4.5}, mm^3$

Therefore maximum combined stress is given by,
Where, $Z_p$=polar section modulus
Total combined stress,

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left[\frac{\sigma_x - \sigma_y}{2}\right]^2 + \tau_{xy}^2} \quad \ldots\ldots(1.11a/2)$$

Here $\sigma_1 = \sigma_{max}$; $\tau_{xy} = \tau$ ; $\sigma_x = \sigma_{total}$; $\sigma_y = 0$;

Calculated $\sigma_{max}$ must be within limits. If it exceeds the safe limit, b can be increased since it does not affect any other dimension.

6.4b.4 Left hand Crank Web:

This crank web is less severely stressed than the right hand crank since it is not to transmit any power while the right hand crank transmits the power to the flywheel and to the power take off. Hence there is no need to check the left hand crank and its dimensions may be taken as that of the right hand crank.

6.4b.4 Crankshaft bearings**:**

The distance between bearing 1 and bearing 2 may be assumed to be equal to twice the cylinder diameter. From the length of the crankpin and the thickness of the arm, the lengths of the bearings can be found out. Bearing 2 is the most heavily loaded, therefore, only this bearing may be checked for the safe bearing pressure.

We know that the total reaction at the bearing 2,

$$R_2 = \frac{F_p}{2} + \frac{W}{2} + \frac{T_1 + T_2}{2}, \text{ N}$$

Therefore bearing pressure $p_b = \dfrac{R_2}{(L)(d)}, MPa$

Where L and d are the length and diameter of the bearing. The bearing design details are not discussed here, as it is beyond the scope of this subject.

**6.5 Analysis of side Crank Shaft:**

The analysis of the side crankshaft is on the same lines as for centre crankshaft. Before the crankshaft is checked for the positions of maximum bending moment and that of maximum twisting moment, the approximate dimensions for the crank pin and the journals may be found.

The side or overhung crankshafts are used for medium size and large horizontal engines. Their main advantage is that it requires only two bearings in either the single or two crank constructions.

The analysis of the shaft is done at two positions:-

1. When the crank is at dead centre (maximum bending moment)
2. When the crank is at angle of maximum twisting moment.

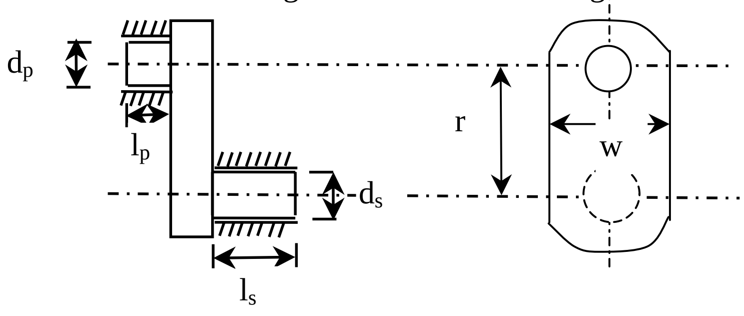


Figure.23 Simple Side Crank Shaft

### 6.5.1 When the crank is at dead centre

Consider a side crankshaft at dead centre with its loads and distances of their application.(Ref Figure 12). The notations used are same as that of center crank shaft.

We know that

Gas Load, $F = \frac{\pi}{4} D^2 * p_{max}$ , Where D is the diameter of the piston in mm and $p_{max}$ is the maximum gas pressure

Due to this piston gas load there will be two horizontal reactions, $R_{H1(F)}$ at bearing 1, and $R_{H2(F)}$ , at bearing 2.

To find the reactions $R_{H1(F)}$ and $R_{H2(F)}$

$\sum M_1 = 0$, $\circlearrowright +$ $\circlearrowleft -$

Clock Wise direction is taken as positive bending moment and Counter Clockwise as negative bending moment.

$\longrightarrow$ $-F(b) + R_{H2(F)}(x + y) = 0$ ……………………………………..………..(13)

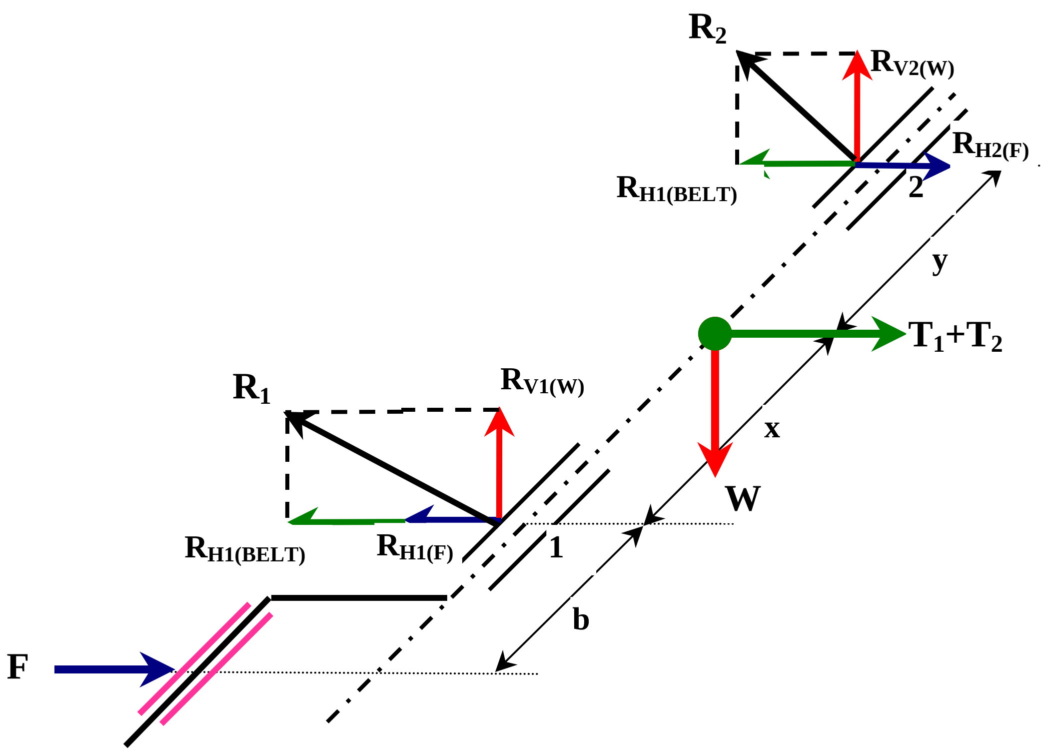


Figure 24. Force Analysis of side Crank at Dead Center

Hence $R_{H2(F)} = \dfrac{F(b)}{(x+y)}, N$ ………...……………………………………………….(14)

$\sum F_y = 0$, $\downarrow -$ $\uparrow +$ , Upward force is taken as positive and downward is taken as negative.

$\rightarrow$ $R_{H1(F)} - R_{H2(F)} - F = 0$………………………………………………………….(15)

By substituting equation 14 in equation 15 we get,

$$R_{H1(F)} = F + \frac{F(b)}{(x+y)} = \frac{F(b+x+y)}{(x+y)}, N$$

Besides gas load, in between bearings 1 and 2, we have two loads

iii) Belt pull ($T_1 + T_2$), acting horizontally as shown in Figure 12

iv) Weight of the Flywheel (W), acting vertically as shown in Figure 12

<u>Reactions at bearing 1 and 2 due to Belt Pull,</u>

Due to this there will be two horizontal reactions, $R_{H1(belt)}$ at bearing 1, and $R_{H2(belt)}$ at bearing 2.

To find the reactions $R_{H2(belt)}$ and $R_{H3(belt)}$

$\sum M_2 = 0$, $\circlearrowright +$ $\circlearrowleft -$

Clock Wise direction is taken as positive bending moment and Counter Clockwise as negative bending moment.

$\rightarrow$ $-(T_1 + T_2)(y) + R_{H1(belt)}(x+y) = 0$

Hence $R_{H1(belt)} = \dfrac{(T_1+T_2)(y)}{(x+y)}, N$ …………………………………………..……...(16)

$\sum F_y = 0$, $\downarrow -$ $\uparrow +$ , Upward force is taken as positive and downward is taken as negative.

$\rightarrow$ $R_{H1(belt)} + R_{H2(belt)} - (T_1+T_2) = 0$……………………………………….(17)

By substituting equation (16) in equation(17) we get,

$$R_{H2b(elt)} = (T_1+T_2) - \frac{(T_1+T_2)(y)}{(x+y)} = \frac{(T_1+T_2)(x)}{(x+y)}$$

If x=y, then, $R_{H2(belt)} = R_{H3(bel)t} = \dfrac{(T_1+T_2)}{(2)}$

Reactions at bearing 1 and 2 due to Weight of the Flywheel (W),

Due to this there will be two Vertical reactions, $R_{v1(W)}$ at bearing1, and $R_{v2(W)}$, at bearing2. To find the reactions $R_{v1(W)}$ and $R_{v2(W)}$

$$\sum M_2 = 0,$$

Clock Wise direction is taken as positive bending moment and Counter Clockwise as negative bending moment.

$$\rightarrow \quad -(W)(y) + R_{v1(W)}(x+y) = 0$$

Hence $R_{v1(W)} = \dfrac{(W)(y)}{(x+y)}, N$ ……………………………………………………..…….(18)

$\sum F_y = 0$, , Upward force is taken as positive and downward is taken as negative.

$$\rightarrow \quad R_{v2(W)} + R_{V1(W)} - (W) = 0 \text{……………………………………………...….(19)}$$

By substituting equation (18) in equation (19) we get,

$$R_{v2(W)} = (W) - \frac{(W)(y)}{(x+y)} = \frac{(W)(x)}{(x+y)}$$

If x=y, then $R_{V1(W)} = R_{V2(W)} = \dfrac{(W)}{(2)}$

In this position of the crank, there will be no twisting moment, and the various parts will be designed for bending only.

If the student choose a wrong direction for $R_{H2(F)}$ due to gas force F, then after solving we get negative value of $R_{H2(F)}$. It clearly indicates that the chosen direction is wrong and further calculations are carried out by changing the direction of the reaction.

6.5.1a CRANKPIN:

The dimensions of the crankpin are obtained by considering the crankpin in bearing and then checked for bending stress.

We know that bearing pressure $p_b = \dfrac{F}{(l_p)(d_p)}$ …………………………………..(20)

Where $l_p$ and $d_p$ are length and diameter of the crankpin respectively.

$p_b$ is the allowable bearing pressure on the pin, MPa

The value of Allowable bearing pressure is available in Table 3.6/49 for various class of work. For example $p_b$ = 2.5 to 2.75 MPa for Automobile Engines.

The length of the crankpin is approximately taken as (0.8 to 1.1) diameter of the crankpin.[Refer page no 50 of the data hand book]

By taking suitable ratio of length to crank pin, and using equation (20), length and diameter of the pin can be obtained.

The crankpin is now checked for bending stress. If it is assumed that the crankpin acts as a cantilever and the load on the crankpin is uniformly distributed, then maximum bending moment will be

$M = \frac{(F)(l_p)}{2}$. But in actual practice, the bearing pressure on the crankpin is not uniformly distributed and may, therefore, give a greater value of bending moment ranging between $M = \frac{(F)(l_p)}{2}$ and $M = (F)(l_p)$.

So a mean value of bending moment i.e. $M = \left(\frac{3}{4}\right)(F)(l_p)$ may be used.

We know that,

$$\frac{M}{I} = \frac{\sigma_b}{c} \quad \text{.........................................................(1.16/3)}$$

Substituting the values of $c = \frac{d_p}{2}$ and $I = \frac{\pi d_p^{\,4}}{64}$ in Equation 1.16 and solving for $\sigma_b$ we get,

$$M = \frac{\sigma_b}{c}(I) = \frac{\sigma_b}{\left(\frac{d_p}{2}\right)}\left(\frac{\pi}{64}\right)(d_p^{\,4})$$

$$(\sigma_b) = \frac{32M}{\pi d^3_p}, \text{ MPa.}$$

This induced bending stress should be within the permissible limits.

## 6.5.1b Design of Bearing:

The bending moment at the center of the bearing 1 is given by

$M = F(b)$, N-mm, (Assuming that the gas force is acting at 0.75 times of the crankpin length from the crank web.)

We know that,

$$\frac{M}{I} = \frac{\sigma_b}{c} \quad \text{.........................................................(1.16/3)}$$

Substituting the values of $c = \frac{d_1}{2}$ and $I = \frac{\pi d_1^{\,4}}{64}$ in Equation 1.16 and solving for $d_1$ we get,

$$M = \frac{\sigma_b}{c}(I) = \frac{\sigma_b}{\left(\frac{d_1}{2}\right)}\left(\frac{\pi}{64}\right)(d_1^{\ 4})$$

$$M = \left(\frac{\pi}{32}\right)\sigma_b(d_1^{\ 3}) \quad \ldots\ldots(21)$$

From equation (21) the diameter of the bearing can be obtained. Length of the bearing can be found by taking, $l_1=1.5d_1$ to $2d_1$.

The bearing 2 is also made of the same diameter. The length of the bearing is found on the basis of allowable bearing pressure and maximum reactions at the bearings.

6.5.1c Design of Shaft under the flywheel

The total bending moment at the flywheel location will be the resultant of horizontal bending moment due to gas load and belt pull and the vertical bending moment due to the flywheel weight.

Horizontal Bending Moment due to ;

i) Piston Gas load

$$M_{Gas} = F(b+x) - R_{H1(F)}(x)$$

ii) Belt Pull

$$M_{Belt} = R_{H2(BELT)}(y)$$

Therefore total horizontal bending moment is

$M_{HOR}=M_{Gas}+M_{Belt}$……(22)

Vertical Bending Moment due to ;

i) Flywheel

$$M_{Vert} = R_{v2(W)}(y)$$

Resultant Bending Moment

$$M_R = \sqrt{(M_{HOR}^2 + M^2_{Vert}}$$

We know that,

$$M_R = \left(\frac{\pi}{32}\right)\sigma_b(d_S^{\ 3}) \quad \ldots\ldots(23)$$

From equation diameter of the shaft $d_S$ can be obtained.

6.5.1d Design of Crank Web:

When the crank is dead centre, the crank web is subjected to a bending moment and a direct compressive stress.

The thickness and width of the crank web is fixed by empirical relations and checked for induced stresses.

Thickness of crank web t=(0.5 to 0.9) $d_p$

Width of crank web b=(1.1 to 1.2 )$d_s$
Where $d_p$ and $d_s$ are diameter of crankpin and crank shaft respectively.
Check:
Maximum bending moment on the crank web

$$M = F(0.5t + 0.75 * l_p)$$

$$I = \frac{bt^3}{12} \text{ And } c = \frac{t}{2}$$

We know that

$$(\sigma_b) = \frac{M}{I}(c) \quad \text{..........(22)}$$

Substituting the values of M,c and I in the equation (22), we get

$$(\sigma_b) = \frac{6M}{bt^2} \quad \text{..........(23)}$$

Direct stress $(\sigma_d) = \dfrac{F}{bt}$ ..........(24)

Superimposing the stresses we get,

$$(\sigma_{Total}) = (\sigma_b) + (\sigma_d) \le (\sigma_{allowable})$$

**6.5.2 When the crank is at an angle of maximum twisting moment:**

Consider a position of the crank at angle of maximum twisting moment as shown in Figure 25.

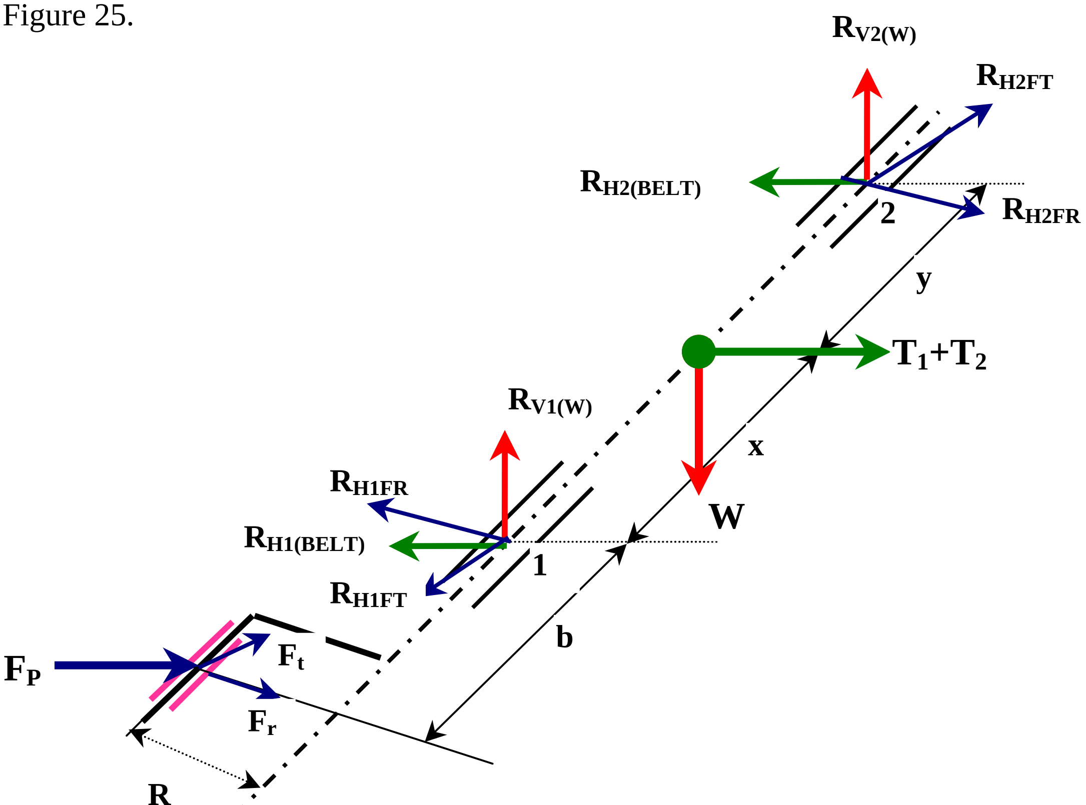


Figure 25. Force Analysis of Side Crank at angle of maximum twisting Moment

The twisting moment on the crankshaft will be maximum, when the tangential force $F_t$ is maximum and this can be calculated graphically by taking pressures from the net effort diagram for different crank angles. The angle usually lies between $25^0$ to $35^0$from the dead center for a constant volume combustion engines and between $30^0$ to $40^0$ for constant pressure combustion engines. At this angle, the gas pressure will not be maximum. If $F_p$ is the gas load along the cylinder centerline, then the thrust $F_C$ along the connecting rod is given by (Ref Figure 8 or same as FIG 3.1/50 in design data book)

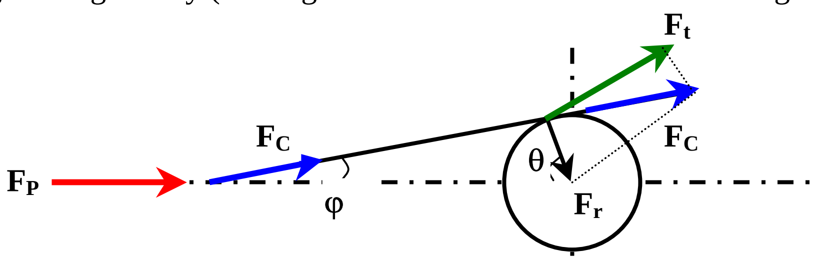


Figure 26. Forces on crank Arm

The force on the connecting Rod or thrust force

$$F_C = \frac{F_P}{\cos(\phi)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.12/45)$$

The tangential force or the rotative effort on the crank

$$Ft = F_C \sin(\phi + \theta) = \frac{F_P \sin(\phi + \theta)}{\cos(\phi)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.13/45)$$

The radial force along the crank

$$Fr = F_C \cos(\phi + \theta) = \frac{F_P \cos(\phi + \theta)}{\cos(\phi)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.14/45)$$

Tangential force $F_t$ will have two reactions $R_{H1FT}$ and $R_{H2FT}$ at bearing 1 and 2 respectively.

Radial force $F_r$ will have two reactions $R_{H1FR}$ and $R_{H2FR}$ at bearing 1 and 2 respectively.

The reactions at the bearings 1 and 2 due to belt pull ($T_1$+$T_2$) and Flywheel W will be same as before.

In this position of the crankshaft, the different sections will be subjected to both bending and torsional moments and these must be checked for combined stress. At this point, Shear stress is taken as failure criteria for crankshaft.

<u>The reactions due Radial force ($F_r$):</u>

To find the reactions $R_{H1FR}$ and $R_{H2FR}$

$$\sum M_2 = 0, \qquad + \qquad -$$

Clock Wise direction is taken as positive bending moment and Counter Clockwise as negative bending moment.

$$-F_r(b+x+y)+R_{H1FR}(x+y)=0$$
$$R_{H1FR}=\frac{F_r(b+x+y)}{(x+y)}$$

$\sum F_y=0$, , Upward force is taken as positive and downward is taken as negative.

$$-F_r+R_{H1FR}-R_{H2FR}=0$$
$$R_{H2FR}=F_r-R_{H1FR}=F_r-\frac{F_r(b+x+y)}{(x+y)}$$
$$R_{H2FR}=\frac{F_r(b)}{(x+y)}$$

The reactions due tangential force ($F_t$):

To find the reactions $R_{H1FT}$ and $R_{H2FT}$

$\sum M_2=0$, + −

Clock Wise direction is taken as positive bending moment and Counter Clockwise as negative bending moment.

$$-F_T(b+x+y)+R_{H1FT}(x+y)=0$$
$$R_{H1FT}=\frac{F_T(b+x+y)}{(x+y)}$$

$\sum F_y=0$, , Upward force is taken as positive and downward is taken as negative.

$$-F_T+R_{H1FT}-R_{H2FT}=0$$
$$R_{H2FT}=F_T-R_{H1FT}=F_T-\frac{F_T(b+x+y)}{(x+y)}$$
$$R_{H2FT}=\frac{F_T(b)}{(x+y)}$$

The reactions at the bearings 1 and 2 due to Flywheel weight (W) and resultant belt pull ($T_1$+$T_2$) will be same as discussed earlier.

6.5.2a Design of Crank Web:

The dimensions of the crank pin and Crank web are taken same as obtained in crank at dead centre.

The most critical section is where the web joins the shaft. This section is subjected to the following stresses:

i) Bending stress due to the tangential force $F_T$
ii) Bending stress due to the radial force $F_r$
iii) Direct compressive stress due to radial force $F_r$ and
iv) Shear stress due to the twisting moment of $F_T$.

Bending stress due to the tangential force $F_T$

Bending moment due to tangential force, $M_{bT} = F_T(R - \frac{d_p}{2}), N-mm$

Therefore bending stress due to tangential force $\sigma_{bT} = \frac{6M_{bT}}{hw^2}$

Bending stress due to the radial force $F_r$

Bending moment due to the radial force, $M_{bR} = F_R(0.75l_p + 0.5h)$

Therefore bending stress due to radial force $\sigma_{bT} = \frac{6M_{bR}}{hw^2}$

Direct compressive stress due to radial force $F_r$

We know that, direct compressive stress, $\sigma_d = \frac{F_R}{wh}$

Shear stress due to the twisting moment of $F_T$.

Twisting moment due to the Tangential force, $T = F_T(0.75l_p + 0.5h)$

Therefore shearing stress due to Tangential l force $\tau = \frac{T}{I}r = \frac{T}{Z} = \frac{4.5T}{wh^2} = \tau_{xy}$

Where Z-Polar section modulus, $= \frac{wh^2}{4.5}$

Superimposing the stresses we get,

Total compressive stress, $\sigma_C = \sigma_{bT} + \sigma_{bR} + \sigma_d = \sigma_x$

Now the total or maximum normal and maximum shear stresses are given by,

$$\sigma_{max} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left[\frac{\sigma_x - \sigma_y}{2}\right]^2 + \tau_{xy}^2} \quad \text{.....................................(1.11b/2)}$$

$$\tau_{\max} = \pm\sqrt{\left[\frac{\sigma_x - \sigma_y}{2}\right]^2 + \tau_{xy}^2} \quad \text{.........................................(1.12/2)}$$

Here $\sigma_y$=0;

This total maximum stress should be less than the maximum allowable stress.

6.5.2b Design of Shaft under the flywheel:

Horizontal bending moment acting on the shaft due to piston gas load,

$$M_{H1} = F_P(b+x) - \left[\sqrt{(R_{H1FR})^2 + (R_{H1FT})^2}\right]x$$

Horizontal bending moment acting on the shaft due to belt pull,

$$M_{Hbelt} = (R_{H2(BELT)})(y)$$

Therefore total horizontal bending moment, $M_H = M_{H1} + M_{Hbelt}$

Vertical bending moment due to flywheel,

$$M_{VFLY} = (R_{V2(W)})(y)$$

Since these bending moments act at right angles to each other, the combined bending moment is given by;

$$M_{Total} = \sqrt{M_{VFLY}^2 + M_{Ht}^2}, \text{ N-mm}$$

In addition to this moment there will be a twisting moment because of tangential force $F_t$.
The twisting moment, $T = F_t(R)$, N-mm

Therefore Equivalent twisting moment,

$$T_e = \sqrt{T^2 + M_{Total}^2}, N - mm$$

We have, $T_e = \frac{\pi}{16} d_W{}^3(\tau)$, N-mm, Diameter of the shaft under flywheel $d_W$ can be obtained.

**Problems:**

**Problem No 1**

Design an overhung crank pin for an engine having the following particulars:

| | |
|---|---|
| Cylinder diameter | =300mm |
| Stroke | =500mm |
| Maximum explosion pressure in the cylinder | =1.8MPa |
| Engine Speed | =200rpm |
| Permissible bending stress for pin | =1000MPa |
| Permissible Bending stress | =85MPa |

Given data:

| | |
|---|---|
| Cylinder diameter | D=300mm |
| Stroke | L=500mm |
| Maximum explosion pressure in the cylinder | $P_{max}$=1.8MPa |
| Engine Speed | N=200rpm |
| Permissible bending stress for pin | $\sigma_b$=800MPa |
| Permissible Bearing stress | $p_b$=85MPa |

Solution:

We know that bearing pressure $p_b = \dfrac{F}{(l_p)(d_p)}$ -----------------------------(P1.1)

Where $l_p$ and $d_p$ are length and diameter of the crankpin respectively.
$p_b$ is the allowable bearing pressure on the pin, MPa

The length of the crankpin is approximately taken as (0.8 to 1.1) diameter of the crankpin.[Refer page no 50 of the data hand book]

Let us take $l_p$=1.1$d_p$

We know that gas load $F = \dfrac{\pi}{4} D^2 * p_{max}$

$$F = \frac{\pi}{4} 300^2 * 1.8 \quad = 127234.50 N$$

Substituting the values in equation (P1.1) we get

$$85 = \frac{127234.50}{(1.1 d_p)(d_p)}$$

Diameter of the crank pin = $d_p$=36.88mm

Referring the table 3.5a/48, standard diameter of $\mathbf{d_p=40mm}$ is taken.

Length of the crankpin $\mathbf{l_p}$=(1.1)(40)=**44mm**

<u>Check:</u>

$$M=\left(\frac{3}{4}\right)(F)(l_p)$$

We know that,

$$\frac{M}{I}=\frac{\sigma_b}{c} \quad \text{.........................................................(1.16/3)}$$

Substituting the values of $c=\frac{d_p}{2}$ and $I=\frac{\pi d_p^{\ 4}}{64}$ in Equation 1.16 and solving for $\sigma_b$ we get,

$$(\sigma_b)=\frac{32M}{\pi d^3{}_p}\text{, MPa.}$$

$$(\sigma_b)=\frac{32(127234.50)(44)(0.75)}{\pi(40)^3}=668.25MPa<800\text{MPa, hence safe.}$$

**<u>Problem No 2</u>**

A force of 120kN acts tangentially on the crank pin of an overhang crank. The axial distance between the centre of the crankshaft journal and the crank pin is 400mm and the crank is 500mm long. Determine

a) Diameter and length of the crankpin journal.

b) Diameter of the shaft journal

Given that:

| | |
|---|---|
| Safe bearing pressure | : 5MPa |
| Bending stress | : 65MPa |
| Principal stress in the shaft journal | : 65 MPa FEB 2005, [12M] VTU |

Given Data:

Referring to Figure 24,

b=400mm and R=500mm

$p_b$=5MPa, $\sigma_b$=65MPa, $\sigma_{max}$ =65MPa, F=120(10)$^3$N

Solution:

a) We know that, Bearing pressure $p_b = \frac{F}{l_p * d_p}$

And assuming ratio of length to diameter of the crank pin as 1.3,

$$5 = \frac{120(10)^3}{1.3(d_p)*d_p}$$

Solving we get, diameter of the crank pin $d_p$= 135.87mm

Adopting the standard diameter **$d_p$= 140mm** [ T3.4/48]

Minimum length of the crankpin,

$$l_p = \frac{F}{p_b * d_p} = \frac{120(10)^3}{5*140} = \mathbf{171.4mm}$$

Check:

$M = \left(\frac{3}{4}\right)(F)(l_p)$ ; We know that, $\frac{M}{I} = \frac{\sigma_b}{c}$ ............................................(1.16/3)

Substituting the values of $c = \frac{d_p}{2}$ and $I = \frac{\pi d_p^4}{64}$ in Equation 1.16 and solving for $\sigma_b$ we get,

$(\sigma_b) = \frac{32M}{\pi d^3_p}$, MPa; $(\sigma_b) = \frac{32(120)(10)^3(171.4)(0.75)}{\pi(140)^3} = 57.26MPa < 65MPa$ ,hence safe.

b) Bending moment at the shaft journal

$M=F(b)=120(10)^3(400)=48(10)^6$, N-mm

Twisting moment at the shaft journal,

$T=F(R)=120(10)^3(500)=60(10)^6$, N-mm

According to maximum normal stress theory,

$$d_S = \left[\frac{16}{\pi\sigma_{\max}}(M + \sqrt{M^2 + T^2})X\frac{1}{1-K^4}\right]^{\frac{1}{3}}$$ ----------------------------(3.5a/42)

Here, because of solid shaft, K=0,

Substituting the values of M, T and $\sigma_{max}$ in equation 3.5a we get

$$d_S = \left[\frac{16}{\pi(65)}(48(10)^6 + \sqrt{(48(10)^6)^2 + \left(60(10)^6\right)^2})\right]^{\frac{1}{3}}$$

= 213.85mm

Taking **$d_s$=220mm** as standard diameter (T3.4/48)

**Problem No 3**

Determine the maximum normal stress and the maximum shear stress at section A-A for the crank shown in Figure 15 when a load of 10kN is assumed to be concentrated at the center of the crank pin.

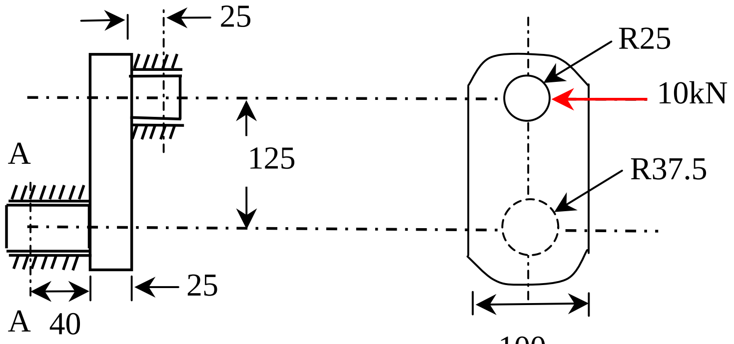


Figure.27

Bending moment M=$10(10)^3(40+25+25)=9(10)^5$, N-mm
Twisting moment T=$10(10)^3(125)=12.5(10)^5$, N-mm

$$\sigma_x = \frac{M(y)}{I} = \frac{9(10)^5(37.5)(64)}{\pi(75)^4} = 21.73MPa$$

$$\tau_{xy} = \frac{T(c)}{J} = \frac{12.5(10)^5(37.5)(32)}{\pi(75)^4} = 15.10MPa$$

Maximum Principal Stress:($\sigma_1$)

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left[\frac{\sigma_x - \sigma_y}{2}\right]^2 + \tau_{xy}^2} \quad \text{………………………………..(1.11a/2)}$$

$$\sigma_1 = \frac{21.73+0}{2} + \sqrt{\left[\frac{21.72 \quad -0}{2}\right]^2 + 15.10^2} = \underline{\mathbf{29.46MPa}}$$

Maximum Shearing Stress:($\tau_{max}$)

$$\tau_{\max} = \pm\sqrt{\left[\frac{\sigma_x - \sigma_y}{2}\right]^2 + \tau_{xy}^2} \quad \text{………………………………………..(1.12/2)}$$

$$\tau_{\max} = \pm\sqrt{\left[\frac{21.72-0}{2}\right]^2 + 15.10^2} = \underline{\mathbf{18.60MPa}}$$

**Problem No 4**

Design a plain carbon steel centre crankshaft for a single acting four stroke single cylinder engine for the following data:

Bore = 400 mm; Stroke = 600 mm; Engine speed = 200 rpm.; Mean effective pressure = 0.5 N/mm$^2$; Maximum combustion pressure = 2.5 M/mm$^2$; Weight of flywheel used a pulley = 50 kN; Total belt pull = 6.5 kN.

When the crack has turned through 35$^0$ from the top dead centre, the pressure on the piston is 1N/mm$^2$ and the torque on the crank is maximum. The ratio of the connecting rod length to the crank radius is 5. Assume any other date required for the design.

Given Data:

D=400mm, L=600mm or R=300mm, $p_{mean}$=0.5MPa, $p_{max}$=2.5MPa, W=50(10)$^3$ N, $T_1+T_2$=6.5(10)$^3$ N, θ=35$^0$, $p_{35}$=1MPa, (l/R)=5

Crankshaft is designed for the two positions:

a) Crank is at dead center; b) Angle of maximum twist;

**a) Design of the crankshaft when the crank is at the dead center**

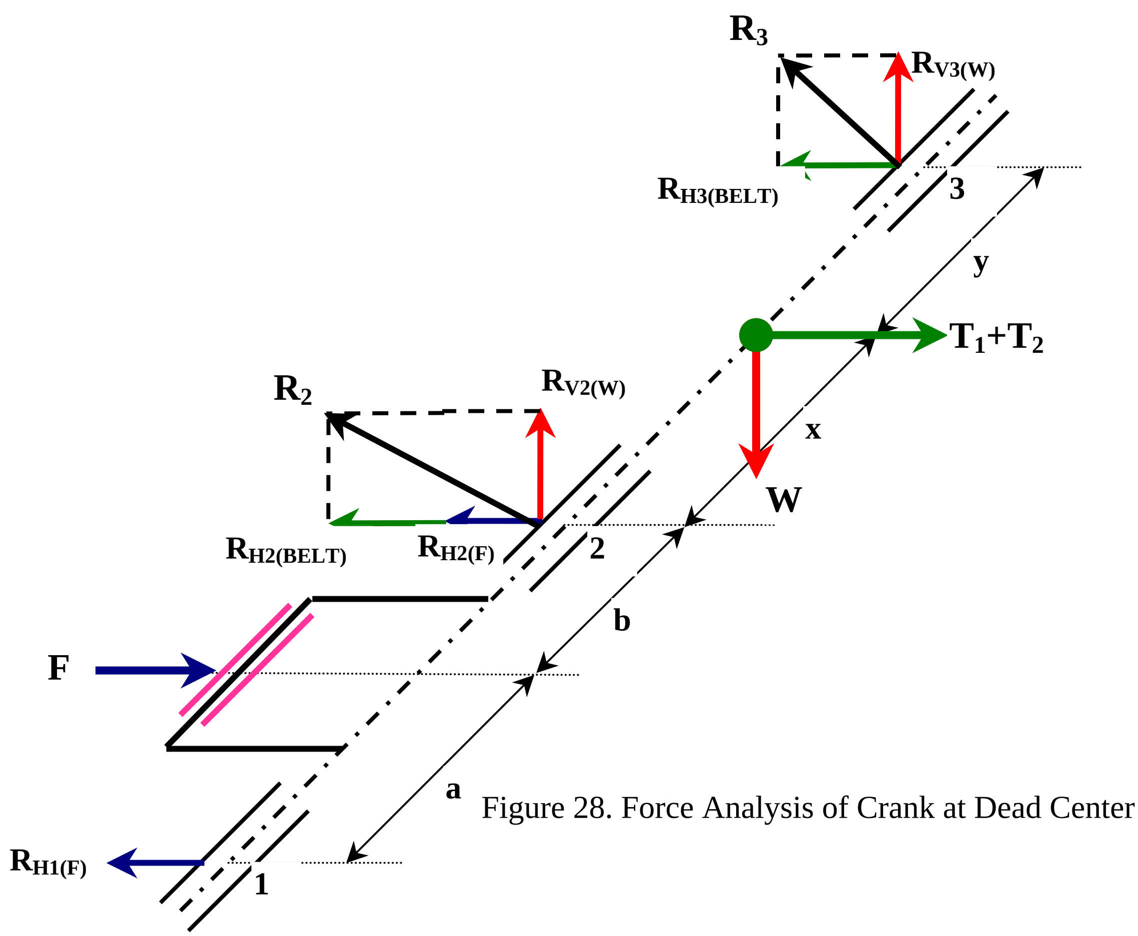


Figure 28. Force Analysis of Crank at Dead Center

Piston Gas load

$$F = \frac{\pi}{4} D^2 * p_{\max} \; = \frac{\pi}{4} 400^2 * 2.5 = \underline{\mathbf{314.16(10)^3\ N}}$$

*Assume that the distance between bearing 1 and 2 is equal to twice the piston diameter (D) and distance a=b.*

Therefore $a = b = \frac{2 * D}{2} = \frac{2 * 400}{2} = 400mm$

Due to gas load, there will be two horizontal reactions, $R_{H1(F)}$ at bearing 1, and $R_{H2(F)}$ , at bearing 2, so that,

To find the reactions $R_{H1(F)}$ and $R_{H2(F)}$

Since a=b, then, $R_{H1(F)} = R_{H2(F)} = \frac{F}{(2)}, N$

$$R_{H1(F)} = R_{H2(F)} = \frac{314.16(10)^3}{(2)}, N = 157.08(10)^3, N$$

In between bearings 2 and 3, we have two loads

i) Belt pull ($T_1 + T_2$ ), acting horizontally as shown in Figure 28

ii) Weight of the Flywheel (W), acting vertically as shown in Figure 28

<u>Reactions at bearing 2 and 3 due to Belt Pull,</u>

Due to this there will be two horizontal reactions, $R_{H2(belt)}$ at bearing 2, and $R_{H3(belt)}$ at bearing 3, so that ,

*Taking x=y; Its value is computed after calculating the crankpin length.*

Since, x=y, then, $R_{H2(belt)} = R_{H3(bel)t} = \frac{(T_1 + T_2)}{(2)}$

$$R_{H2(belt)} = R_{H3(bel)t} = \frac{(6.5(10)^3)}{(2)} = 3.25(10)^3, N$$

<u>Reactions at bearing 2 and 3 due to Weight of the Flywheel,</u>

Since, x=y, therefore $R_{V2(W)} = R_{V3(W)} = \frac{(W)}{(2)}$

then $R_{V2(W)} = R_{V3(W)} = \frac{(50(10)^3)}{(2)} = 25(10)^3, N$

In this position of the crank, there will be no twisting moment, and the various parts will be designed for bending only.

CRANKPIN:

The bending moment at the centre of the crankpin is,

$M = R_{H1(F)}(a)$ , N-mm

$=157.08(10)^3(400)$

$= 62832(10)^3$, N-mm

We know that,

$$\frac{M}{I} = \frac{\sigma_b}{c} \text{ ..........................................................(1.16/3)}$$

$\sigma_b$=allowable bending stress for the crankpin. It may be assumed as 83MPa. (Refer T3.5b/48)

Substituting the values of $c = \frac{d_p}{2}$ and $I = \frac{\pi d_p^{\ 4}}{64}$ in Equation 1.16 and solving for M we get,

$$M = \frac{\sigma_b}{c}(I) = \frac{\sigma_b}{\left(\frac{d_p}{2}\right)}\left(\frac{\pi}{64}\right)(d_p^{\ 4})$$

$$M = \frac{\pi}{32} d_p^{\ 3}(\sigma_b) \text{, N-mm}$$

$$62832(10)^3 = \frac{\pi}{32} d_p^{\ 3}(83)$$

We get $d_p$=197.56mm.

Standard value of diameter **$d_p$=200mm** is adopted. (Refer T3.5a/48)

Length of the crankpin ($l_p$) can be obtained by suitably choosing the value of allowable bearing pressure.

We know that bearing pressure for the given type of engine is between 9.6 MPa to 12.4 MPa. Let us take $p_b$=10MPa. (Refer T15.11/314)

Bearing pressure, $p_b = \frac{F}{(l_p)(d_p)}$ , MPa

Length of the crankpin, $l_p = \frac{F}{(d_p)(p_b)}$ , mm $= \frac{314.16(10)^3}{(200)(10)}$ =**157mm**

6.4a.2 Left Hand Crank Web:

The crank web is designed for eccentric loading. There will be two stresses on it, one *direct compressive stress* and the other *bending stress* due to the gas load F.

The thickness h=0.65 $d_p$ + 6.35mm……………………………………(Page No 50)

=0.65(200)+6.35

=136.35mm

Let us take **h=137mm**

The width 'w'may be assumed to be as follows:

$$w = \frac{9}{8} d_p + 12.7, mm \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(Page No 50)}$$

$$= \frac{9}{8} 200 + 12.7, mm$$

w=237.7mm

Let us take**w=238mm**

Since the empirical relations are used it is advised to check the developed stresses against the given values.

Direct stresses ($\sigma_d$)

$$\sigma_d = \frac{R_{1H(F)}}{(w)(h)}, MPa$$

$$\sigma_d = \frac{157.08(10)^3}{(238)(137)}, MPa = \mathbf{\underline{4.82MPa}}$$

Bending stresses: ($\sigma_b$)

$$\frac{M}{I} = \frac{\sigma_b}{c}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.16/3)$$

$$M = R_{1H(F)}(a - \frac{l_p}{2} - \frac{h}{2})$$

$$I = \frac{bh^3}{12} \text{ And } c = \frac{h}{2}$$

Substituting the values of M, c and I in bending equation (1.16/3) we get

$$\sigma_b = R_{1H(F)}(a - \frac{l_p}{2} - \frac{h}{2})(\frac{6}{bh^2}), MPa$$

$$\sigma_b = 157.08(10)^3(400 - \frac{157}{2} - \frac{137}{2})(\frac{6}{238(137)^2}), MPa$$

=**53.38MPa**

Superimposing the direct and bending stresses, we get

Total stress on the crank web=$\sigma_d$+$\sigma_b$=4.85+53.38=**58.23MPa<83MPa,**

Hence Design is safe.

Right Hand Crank Web:

From the balancing point of view, the dimensions of the right hand crank web **h=137mm** and **w=238mm** are taken equal to the dimensions of the left hand crank web.

Shaft Under the Flywheel: [Diameter of the shaft between bearing 2 and 3]

Length of the bearing, $l_1 = l_2 = l_3 = 2\left(a - \frac{l_p}{2} - h\right)$

$$l_1 = l_2 = l_3 = 2\left(400 - \frac{157}{2} - 137\right) = \mathbf{369mm}$$

Assuming width of the flywheel as 300mm, we have

x+y=369+300+clearance=369+300+131(to make it round off)

=800mm.

Taking x=y, we have x=y=400mm

Bending moment due to flywheel weight is $M_{FLY} = (R_{V3(W)})(y)$

$=25(10)^3(400)$

$=\mathbf{10(10)^6}$**, N-mm**

Bending moment due to the belt pull is $M_{belt} = (R_{H3(BELT)})(y)$

$=3.25\ (10)^3(400)$

$=\mathbf{1.3(10)^6}$**, N-mm**

Since these bending moments act at right angles to each other, the combined bending moment is given by;

$$M_{Total} = \sqrt{M_{FLY}^2 + M_{belt}^2}$$

$$= \sqrt{(10*10^6)^2 + (1.3*10^6)^2}$$

$=\mathbf{10.08(10)^6}$**, N-mm**

We know that $M_{Total} = \frac{\sigma_b}{c}(I) = \frac{\sigma_b}{\left(\frac{d_W}{2}\right)}\left(\frac{\pi}{64}\right)(d_W{}^4)$

$$M_{total} = \frac{\pi}{32} d_W{}^3(\sigma_b), \text{ N-mm,}$$

For plain carbon steel taking $\sigma_b$=65MPa

[Ref T1.8/418, taking FOS n=4, Yield stress=196MPa]

$$10.08(10)^6 = \frac{\pi}{32} d_W{}^3(65)$$

Diameter $d_W$=116.46mm, Use standard diameter as **$d_W$=125mm** [Ref. T3/48]

**b. Crank at an angle of maximum twisting moment**

We know that piston gas load $F_p = \frac{\pi}{4} D^2 * p_{35} = \frac{\pi}{4} 400^2 * 1 =$ **125.66(10)$^3$ N**

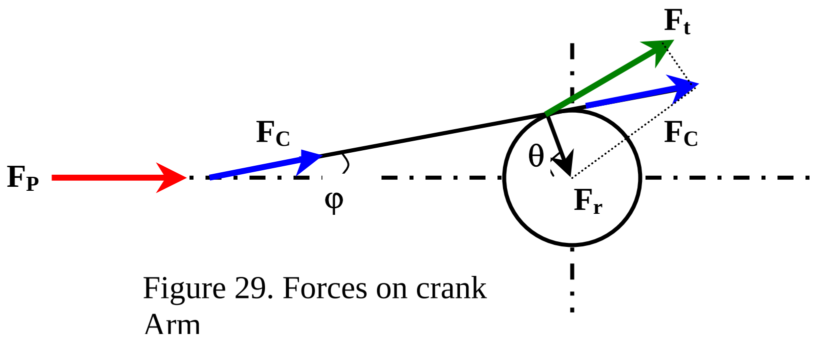


Figure 29. Forces on crank Arm

We know that, $\sin(\phi) = \frac{\sin(\theta)}{L/R}$ --------------------------<Remember>

Where ϕ is the angle of inclination of the connecting rod with the line of stroke.

$\sin(\phi) = \frac{\sin(35)}{5}$ =0.1147

Therefore **ϕ=6.58$^0$**

The force on the connecting Rod or thrust force

$$F_C = \frac{F_P}{\cos(\phi)} \quad \text{....................................................................(3.12/45)}$$

$F_C = \frac{125.66(10)^3}{\cos(6.58)}$ =**126.50(10)$^3$ N**

The tangential force or the rotative effort on the crank

$$Ft = F_C \sin(\phi + \theta) \quad \text{.......................................................(3.13/45)}$$

$Ft = 126.50(10)^3 \sin(6.58 + 35)$ =**83.95(10)$^3$, N**

The radial force along the crank

$$Fr = F_C \cos(\phi + \theta) \quad \text{.......................................................(3.14/45)}$$

$Fr = 126.50(10)^3 \cos(6.58 + 35)$ =**94.63(10)$^3$, N**

Tangential force $F_t$ will have two reactions $R_{H1FT}$ and $R_{H2FT}$ at bearing 1 and 2 respectively.

Radial force $F_r$ will have two reactions $R_{H1FR}$ and $R_{H2FR}$ at bearing 1 and 2 respectively. The reactions at the bearings 2 and 3 due to belt pull ($T_1$+$T_2$) and Flywheel W will be same as before.

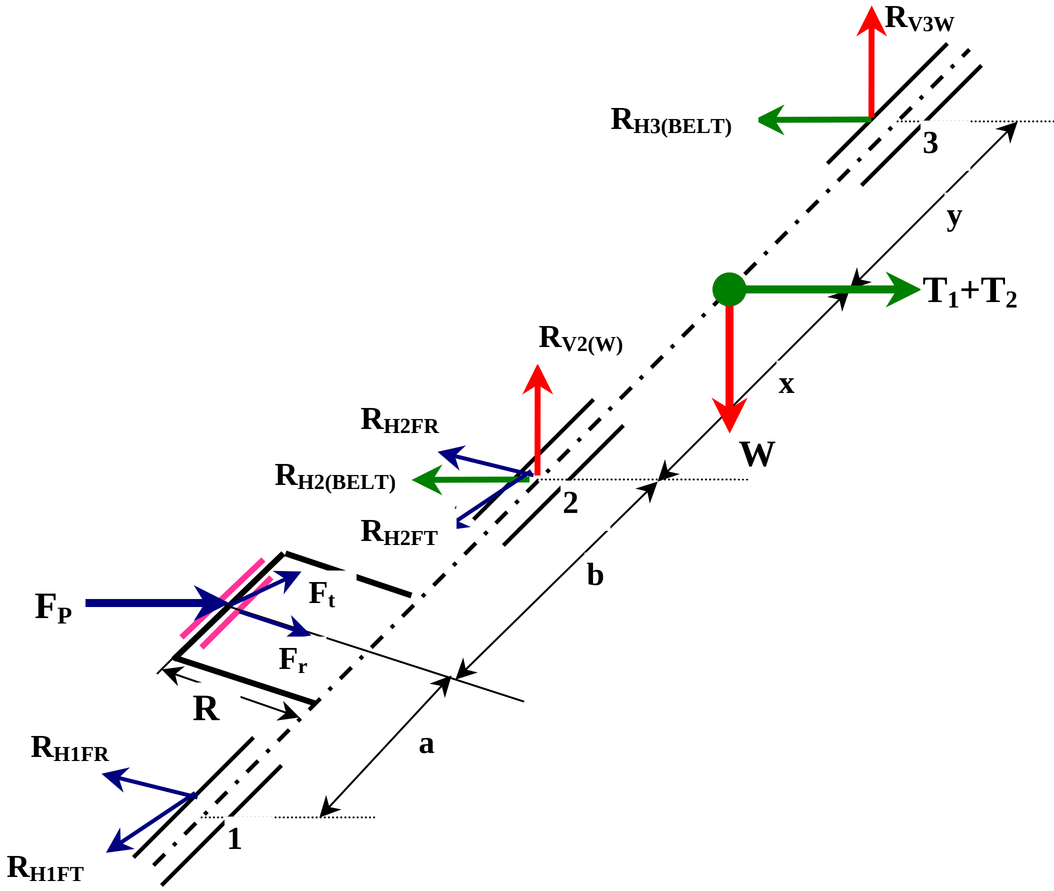


Figure 30. Force Analysis of Crank at angle of maximum twisting Moment

In this position of the crankshaft, the different sections will be subjected to both bending and torsional moments and these must be checked for combined stress. At this point, Shear stress is taken as failure criteria for crankshaft.

The reactions due Radial force ($F_r$):

$$R_{H1FR} = R_{H2FR} = \frac{F_r}{(2)} = \frac{94.63(10)^3}{2} = \underline{\mathbf{47.315(10)^3,\ N}}$$

The reactions due tangential force ($F_t$):

$$R_{H1FT} = R_{H2FT} = \frac{F_T}{(2)} = \frac{83.95(10)^3}{(2)} = \underline{\mathbf{41.975(10)^3,\ N}}$$

The reactions at the bearings 1 and 2 due to Flywheel weight (W) and resultant belt pull ($T_1$+$T_2$) will be as discussed earlier.

Crank pin:

The bending moment at the centre of the crankpin is, $M_b = R_{H1FR}(a)$, N-mm

$M_b = 47.315(10)^3(400)$ =**$18.926(10)^6$, N-mm**

The Twisting moment is, $T = R_{H1FT}(R)$, N-mm

$T = 41.975(10)^3(300)$ = **$12.60(10)^6$, N-mm**

Equivalent twisting moment, $T_e = \sqrt{T^2 + M_b^2}, N-mm$

$T_e = \sqrt{(12.60*10^6)^2 + \left(18.926*10^6\right)^2}, N-mm$ =**$22.737(10)^6$, N-mm**

We know that $T_e = \frac{\pi}{16} d_P{}^3 (\tau)$, N-mm

$22.737(10)^6 = \frac{\pi}{16} d_P{}^3 (42)$ *(The value of $\tau$=0.4 to 0.6$\sigma$)*

Solving we get, $d_p$=139.1mm.

Since this value of crankpin is less than the already calculated value of $d_p$=200mm,(i.e higher among the two).

We shall take **$d_p$=200mm and $l_p$=157mm**

Shaft under the Flywheel: [Diameter of the shaft between bearing 2 and 3]

The collective bending moment due to flywheel and the belt pull will be the same as earlier.

Bending moment due to flywheel weight is $M_{FLY} = (R_{V3(W)})(y)$

Bending moment due to the belt pull is $M_{belt} = (R_{H3(BELT)})(y)$

Since these bending moments act at right angles to each other, the combined bending moment is given by;

$$M_{Total} = \sqrt{M_{FLY}^2 + M_{belt}^2}, \text{ N-mm}$$

$$= \sqrt{(10*10^6)^2 + (1.3*10^6)^2}$$

=**$10.08(10)^6$, N-mm**

In addition to this moment there will be a twisting moment because of tangential force $F_t$.

The twisting moment, $T = F_t(R)$, N-mm

$T = 83.95(10)^3(300)$ =**$25.185(10)^6$, N-mm**

Therefore Equivalent twisting moment,

$$T_e = \sqrt{T^2 + M_{Total}^2}, N-mm$$

$T_e = \sqrt{(25.185(10)^6)^2 + (10.08(10)^6)^2}, N-mm$ =**$27.13(10)^6$, N-mm**

We have, $T_e = \frac{\pi}{16} d_W{}^3 (\tau)$, N-mm,

τ=(0.5 to 0.6)*σ=(0.5 to 0.6)*65=32.5MPa to 39MPa.
Let us take τ=35MPa

$$27.13(10)^6 = \frac{\pi}{16} d_W{}^3 (35)$$

$d_W$=157.25mm
Standard value of $d_W$=160mm is adopted.
Earlier value of $d_W$ is 125mm is less than $d_{W=}$160mm.
Hence **$d_W$=160mm**

6.4b.3 Right hand Crank Web:

We have used empirical formulae to obtain the values of crank web dimensions. And also we know that the Right hand Crank Web is severely stressed. In order to find the correctness of the dimensions of the web it is necessary to check the developed stresses against the allowable stresses. This web is subjected to bending stresses in two planes normal to each other, due to radial and tangential components of $F_P$; to direct compression; and to torsion.

The various dimensions obtained are
w= 238mm; h=137mm; $l_p$=157mm; $d_p$=200mm;

The bending moment due to radial component is

$$M_{rad} = R_{H2FR}(b - \frac{l_p}{2} - \frac{h}{2}) \text{, N-mm}$$

$$M_{rad} = 47.315(10)^3 (400 - \frac{157}{2} - \frac{137}{2}) = \mathbf{11.97(10)^6, N\text{-}mm}$$

Bending stress in radial direction

$$\sigma_{rad} = M_{rad} (\frac{6}{wh^2}), MPa$$

$$\sigma_{rad} = 11.97(10)^6 (\frac{6}{238(137)^2}), MPa\ \mathbf{=16.08MPa}$$

The bending moment due to tangential component is maximum at the juncture of the crank and shaft.

$M_{Tang} = F_t(R)$, N-mm (Since here shaft diameter at junction is not considered for calculation. By doing so the bending moment increases and hence the stresses, which leads to safer side.)

$$M_{Tang} = 83.95(10)^3 (300) = \mathbf{25.185(10)^6, N\text{-}mm}$$

$$\sigma_{Tang} = M_{Tang} \left( \frac{6}{w^2 h} \right) \text{, MPa}$$

$$\sigma_{Tang} = 25.185(10)^6 \left(\frac{6}{238^2(137)}\right), MPa = \mathbf{\underline{19.47 MPa}}$$

The stress due to direct compression, $\sigma_d = \frac{F_r}{2wh}, MPa$

$$\sigma_d = \frac{94.63(10)^3}{2(238)(137)}, MPa = \mathbf{\underline{1.45 MPa}}$$

Superimposing the stresses (At the upper left corner to the cross section of the crank) will be equal to

$$\sigma_{total} = \sigma_{rad} + \sigma_{Tang} + \sigma_d, MPa$$

$$\sigma_{total} = 16.08 + 19.47 + 1.45, MPa = \mathbf{\underline{37 MPa}}$$

Now the twisting moment, on the arm is

$$T = R_{HIFT}\left(a + \frac{l_p}{2}\right) - F_T\left(\frac{l_p}{2}\right) = R_{H2FT}\left(b - \frac{l_p}{2}\right), N-mm$$

$$T = 41.975(10)^3\left(400 - \frac{157}{2}\right), N-mm = \mathbf{\underline{13.49(10)^6}}$$

We know that,
Shear stress,

$$\tau_{xy} = \frac{T}{J}(c) = \frac{T}{Z}$$

Where Z – polar section modulus, = $\frac{wh^2}{4.5}, mm^3$

$$\tau_{xy} = \frac{T}{J}(c) = \frac{T}{Z} = \frac{13.49(10)^6(4.5)}{238(137)^2} = \mathbf{\underline{13.60 MPa}}$$

Therefore maximum combined stress is given by,

Total combined stress,

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left[\frac{\sigma_x - \sigma_y}{2}\right]^2 + \tau_{xy}^2} \quad \text{……………………………….(1.11a/2)}$$

Here $\sigma_1 = \sigma_{max}$; $\tau_{xy} = \tau$ ; $\sigma_x = \sigma_{total}$; $\sigma_y = 0$;

$$\sigma_1 = \frac{37+0}{2} + \sqrt{\left[\frac{37-0}{2}\right]^2 + 13.60^2}$$ =41.46MPa<83MPa, Design is safe.

<u>Left hand Crank Web:</u>

This crank web is less severely stressed than the right hand crank since it is not to transmit any power while the right hand crank transmits the power to the flywheel and to the power take off. Hence there is no need to check the left hand crank and its dimensions may be taken as that of the right hand crank.

Crankshaft bearings**:**

The distance between bearing 1 and bearing 2 may be assumed to be equal to twice the cylinder diameter. From the length of the crankpin and the thickness of the arm, the lengths of the bearings can be found out. Bearing 2 is the most heavily loaded, therefore, only this bearing may be checked for the safe bearing pressure.

We know that the total reaction at the bearing 2,

$R_2 = \frac{F_p}{2} + \frac{W}{2} + \frac{T_1 + T_2}{2}$, N here $F_P$ to taken as maximum, i.e.

**$314.16(10)^3$ N** instead of **$125.66(10)^3$ N**

$R_2 = \frac{314.16(10)^3}{2} + \frac{50(10)^3}{2} + \frac{6.5(10)^2}{2}$ = **$185.33(10)^3$, N**

Therefore bearing pressure $p_b = \frac{R_2}{(L)(d)}, MPa$ here $d=d_w$=160mm, L=369mm

$p_b = \frac{185.33(10)^3}{(369)(160)}, MPa$ =3.14MPa<10MPa, hence the design of bearing is safe.

**Problem No 5**

Design a side or overhung crankshaft for a 250mm X 300 mm gas engine. The weight of the flywheel is 30kN and the explosion pressure is 2.1 MPa. The gas pressure at the maximum torque is 0.9 MPa, when the crank angle is $35^0$ from I.D.C. The connecting rod is 4.5 times the crank radius.

Given Data:

D=250mm, L=300mm, or R=150mm, $W=30(10)^3$N, $p_{max}$=2.1MPa and $p_{35}$=0.9MPa, l/r=4.5

Material taken: $\sigma_b$=allowable bending stress for the crankpin= 83MPa. (Refer T3.5b/48)

Solution:

Crankshaft is designed for the two positions:

a) Crank is at dead center;
b) Angle of maximum twist;

a) Design of the crankshaft when the crank is at the dead center

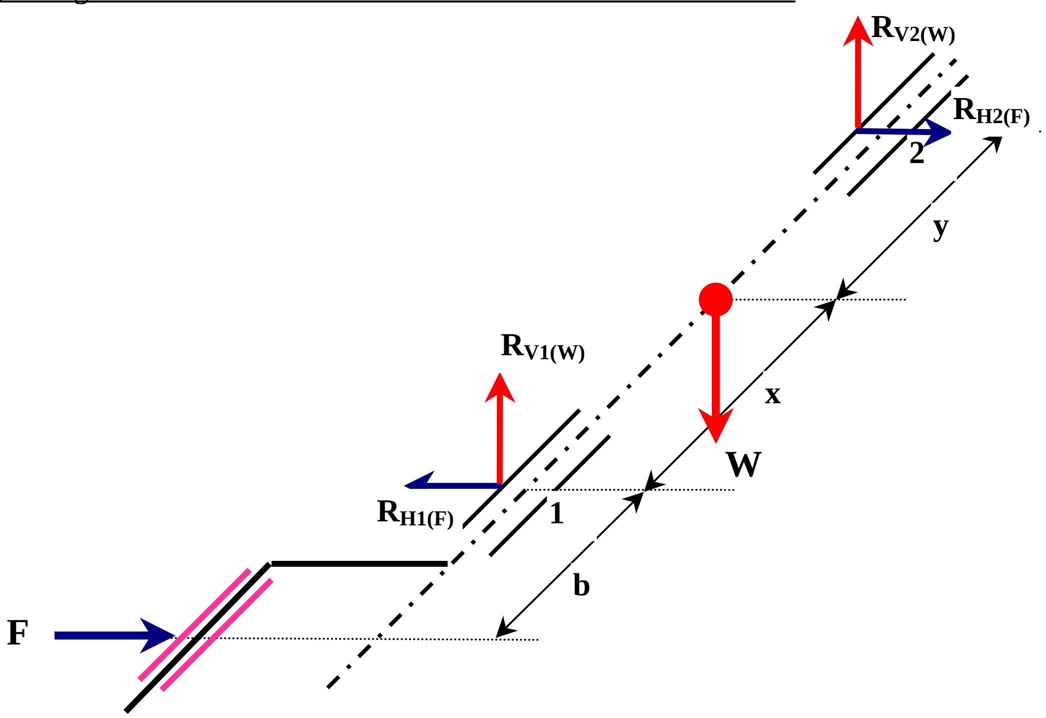


Figure 31. Force Analysis of side Crank at Dead Center

Gas Load, $F = \frac{\pi}{4} D^2 * p_{\max}$

$F = \frac{\pi}{4} 250^2 * 2.1 =$ **$103.1(10)^3$, N**

**Crankpin:**

The dimensions of the crankpin are obtained by considering the crankpin in bearing and then checked for bending stress.

We know that bearing pressure $p_b = \frac{F}{(l_p)(d_p)}$

Where $l_p$ and $d_p$ are length and diameter of the crankpin respectively.
$p_b$ is the allowable bearing pressure on the pin, MPa

The length of the crankpin is approximately taken as (0.8 to 1.1) diameter of the crankpin.[Refer page no 50 of the data hand book]

Let us take, $l_p=d_p$ And solving for the dimensions of crankpin, we get

$$10 = \frac{103.1(10)^3}{(d_p)(d_p)}$$

Diameter of the crankpin $d_p$=101.54mm

Standard diameter **$d_p$= 110mm** is adopted (Refer T3.5a/48)

Length of the crankpin **$l_p$=110mm**

Check:

Bending moment $M = \left(\frac{3}{4}\right)(F)(l_p)$ may be used.

$M = \left(\frac{3}{4}\right)(103.1(10)^3)(110)$ =**$8.51(10)^6$, N-mm**

We know that,

$$\frac{M}{I} = \frac{\sigma_b}{c} \qquad \text{..........................................................(1.16/3)}$$

Substituting the values of $c = \frac{d_p}{2}$ and $I = \frac{\pi d_p^{\ 4}}{64}$ in Equation 1.16 and solving for $\sigma_b$ we get,

$$(\sigma_b) = \frac{32M}{\pi d^3_p}, \text{ MPa.}$$

$(\sigma_b) = \frac{32(8.51)(10)^6}{\pi(110)^3}$ =**65.13MPa.<83MPa.**

This induced bending stress should be within the permissible limits, Hence design is safe.

**Design of bearings:**

Let $d_1$ be the diameter of the bearing 1.
Thickness of web t=h=(0.5 to 0.9) $d_p$ ………………………………………..(Page No 50)
Let us take h=0.6$d_p$=0.6(110)=**66mm**
Length of the bearing $l_1$=1.7$d_p$=1.7(110)=**187mm**

We know that bending moment,

$$M = F\ (0.75l_p + h + 0.5(l_1), N-mm$$

$$M = 103(10)^3\left[0.75(110) + 66 + 0.5(187)\right], N-mm$$

M=**$25(10)^6$, N-mm**

We know that bending stress, $(\sigma_b) = \dfrac{32M}{\pi d^3{}_1}$

Assuming bearing material as Phosphor bronze, $\sigma_b$=68.65MPa [Refer T15.2/309]
Solving for $d_1$, we get

$$(68.65) = \frac{32(25(10)^6)}{\pi d^3{}_1}$$

The diameter of the bearing $d_1$=154.72mm
Let us take $d_1$=**155mm**
The bearing dimensions are taken same for bearing 2. i.e **$l_1$=$l_2$=187mm**

**Design of crank web**

w=Width of the crank web, mm
We know that bending moment,

$$M = F\ (0.75l_p + 0.5(h), N-mm$$

$$M = 103(10)^3\left[0.75(110) + (0.5)66\right], N-mm$$

M=**$11.9(10)^6$, N-mm**

Bending stress $\sigma_b = M\left(\dfrac{6}{w\ h^2}\right)$, MPa

$$\sigma_b = 11.9(10)^6\left(\frac{6}{w(66)^2}\right), MPa = \frac{16.39(10)^3}{w}, MPa$$

The direct Stress, $\sigma_d = \dfrac{F_p}{wh}, MPa$

$$\sigma_d = \frac{103(10)^3}{(w)(66)}, MPa = \frac{1.56(10)^3}{w}, MPa$$

Superimposing the stresses and equating to allowable stress we get

$$\sigma_{all} = \sigma_b + \sigma_d, MPa$$

$$83 = \frac{16.39(10)^3}{w} + \frac{1.56(10)^3}{w}$$

The width of crank web **w=216.3mm**

**Design of shaft under the flywheel**

Let $d_S$ be the Diameter of shaft under the flywheel.
Assuming the width of the flywheel as 250mm

Length $(x+y) = 250 + \frac{l_1}{2} + \frac{l_2}{2} + Clearance$

$$(x+y) = 250 + \frac{187}{2} + \frac{187}{2} + 23 = 460mm$$

$$b = 0.75 l_p + h + \frac{l_1}{2}$$

$$b = 0.75(110) + 66 + \frac{187}{2} = 242\text{mm}$$

Taking x=y=230mm

**Reactions:**

Reactions at bearing 1 and 2 due to Weight of the Flywheel (W),

Due to this there will be two Vertical reactions, $R_{v1(W)}$ at bearing1, and $R_{v2(W)}$, at bearing2.

Here x=y, then $R_{V1(W)} = R_{V2(W)} = \frac{(W)}{(2)}$

$$R_{V1(W)} = R_{V2(W)} = \frac{(30(10)^3)}{(2)} = 15(10)^3, N$$

Reactions at bearing 1 and 2 due to Piston Gas Load(F),

Due to this piston gas load there will be two horizontal reactions, $R_{H1(F)}$ at bearing 1, and $R_{H2(F)}$, at bearing 2.

To find the reactions $R_{H1(F)}$ and $R_{H2(F)}$

$\sum M_1 = 0$, $\quad +$ $\quad -$

Clock Wise direction is taken as positive bending moment and Counter Clockwise as negative bending moment.

$$\rightarrow \quad -F(b) + R_{H2(F)}(x+y) = 0$$

$$R_{H2(F)} = \frac{F(b)}{(x+y)} = \frac{103(10)^3(242)}{(230+230)} = 54.2(10)^3, N$$

$\sum F_y = 0$, $\uparrow$ , Upward force is taken as positive and downward is taken as negative.

$$\rightarrow \quad R_{H1(F)} - R_{H2(F)} - F = 0$$

$$R_{H1(F)} = F + \frac{F(b)}{(x+y)} = \frac{F(b+x+y)}{(x+y)}, N$$

$$R_{H1(F)} = \frac{103(10)^3(242+230+230)}{(230+230)}, N = 157.2(10)^3, N$$

Since there is no belt tension, therefore the horizontal reactions due to the belt tension are not taken.

In this position of the crank, there will be no twisting moment, and the various parts will be designed for bending only.

Horizontal Bending Moment due to Piston Gas load

$$M_{Gas} = F(b+x) - R_{H1(F)}(x)$$

$$M_{Gas} = 103(10)^3(242+230) - 157.2(10)^3(230) = 12.46(10)^6, N-mm$$

$M_{Belt}=0$;

Therefore total horizontal bending moment is
$M_{HOR}=M_{Gas}+M_{Belt}=12.46(10)^6+0=$**$12.46(10)^6$, N-mm**

Vertical Bending Moment due to ;
Flywheel

$$M_{Vert} = R_{v2(W)}(y)$$

$M_{Vert} = 15(10)^3(230) =$ **$3.45(10)^6$, N-mm**

Resultant Bending Moment

$$M_R = \sqrt{(M^2_{HOR} + M^2_{Vert}}$$

$M_R = \sqrt{(12.46*10^6)^2 + (3.45*10^6)^2}$ =**$12.93(10)^6$, N-mm**

We know that,

$$M_R = (\frac{\pi}{32})\sigma_b(d_S{}^3)$$

$$12.93(10)^6 = (\frac{\pi}{32})83(d_S{}^3)$$

The diameter of the shaft under flywheel $d_S$=116.64mm
Since the diameter of the bearing is 155mm> 116.64mm.
Hence **$d_S$=155mm** is adopted

**b. Crank at an angle of maximum twisting moment**

We know that piston gas load $F_p = \frac{\pi}{4} D^2 * p_{35} = \frac{\pi}{4} 250^2 * 0.9 =$ **$44.18(10)^3$ N**

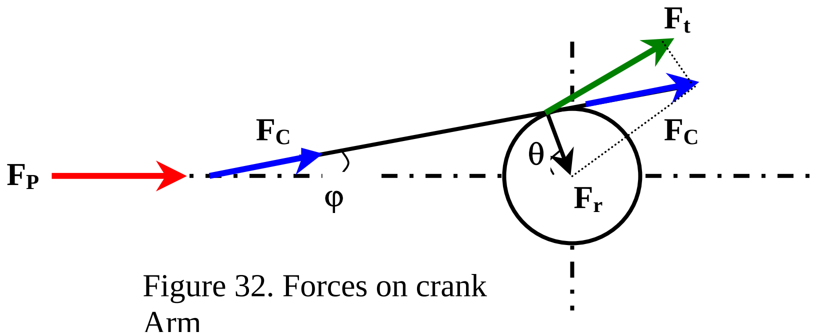


Figure 32. Forces on crank Arm

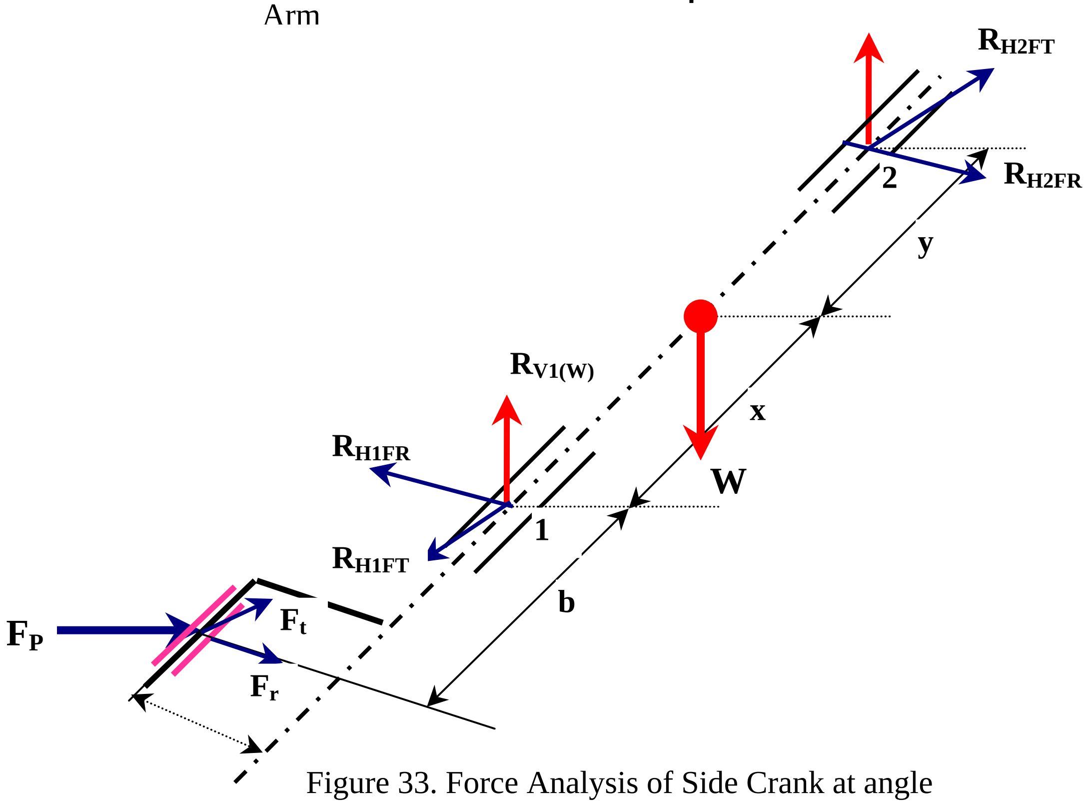


Figure 33. Force Analysis of Side Crank at angle of maximum twisting Moment

We know that, $\sin(\phi) = \frac{\sin(\theta)}{L/R}$ --------------------------<Remember>

Where ϕ is the angle of inclination of the connecting rod with the line of stroke.

$$\sin(\phi) = \frac{\sin(35)}{4.5} = 0.1275$$

Therefore **$\phi = 7.32^0$**

The force on the connecting Rod or thrust force

$$F_C = \frac{F_P}{\cos(\phi)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.12/45)$$

$$F_C = \frac{44.18(10)^3}{\cos(7.32)} = \mathbf{\underline{44543N}}$$

The tangential force or the rotative effort on the crank

$$Ft = F_C \sin(\phi+\theta) = \frac{F_P \sin(\phi+\theta)}{\cos(\phi)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.13/45)$$

$$Ft = 44543 \sin(7.32+35) = \mathbf{\underline{29989.50N}}$$

The radial force along the crank

$$Fr = F_C \cos(\phi+\theta) = \frac{F_P \cos(\phi+\theta)}{\cos(\phi)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.14/45)$$

$$Fr = 44543 \cos(7.32+35) = \mathbf{\underline{32935N}}$$

Tangential force $F_t$ will have two reactions $R_{H1FT}$ and $R_{H2FT}$ at bearing 1 and 2 respectively.

Radial force $F_r$ will have two reactions $R_{H1FR}$ and $R_{H2FR}$ at bearing 1 and 2 respectively.

The reactions at the bearings 1 and 2 due to Flywheel Weight W will be same as before.

In this position of the crankshaft, the different sections will be subjected to both bending and torsional moments and these must be checked for combined stress. At this point, Shear stress is taken as failure criteria for crankshaft.

<u>The reactions due Radial force ($F_r$):</u>

To find the reactions $R_{H1FR}$ and $R_{H2FR}$

$$\sum M_2 = 0, \quad \circlearrowright + \qquad - \circlearrowleft$$

Clock Wise direction is taken as positive bending moment and Counter Clockwise as negative bending moment.

$$\rightarrow \quad -F_r(b+x+y) + R_{H1FR}(x+y) = 0$$

$$R_{H1FR} = \frac{F_r(b+x+y)}{(x+y)}$$

$$R_{H1FR} = \frac{32935\ (242+230+230)}{(230+230)} = \mathbf{\underline{50261.67N}}$$

$\sum F_y = 0$, , Upward force is taken as positive and downward is taken as negative.

$$-F_r + R_{H1FR} - R_{H2FR} = 0$$

$$R_{H2FR} = F_r - R_{H1FR} = F_r - \frac{F_r(b+x+y)}{(x+y)}$$

$$R_{H2FR} = \frac{F_r(b)}{(x+y)}$$

$$R_{H2FR} = \frac{32935(242)}{(230+230)}$$ =**17326.67N**

The reactions due tangential force ($F_t$):

To find the reactions $R_{H1FT}$ and $R_{H2FT}$

$\sum M_2 = 0$, + —

Clock Wise direction is taken as positive bending moment and Counter Clockwise as negative bending moment.

$$-F_T(b+x+y) + R_{H1FT}(x+y) = 0$$

$$R_{H1FT} = \frac{F_T(b+x+y)}{(x+y)}$$

$$R_{H1FT} = \frac{29989.50(242+230+230)}{(230+230)}$$ =**45766.58N**

$\sum F_y = 0$, , Upward force is taken as positive and downward is taken as negative.

$$-F_T + R_{H1FT} - R_{H2FT} = 0$$

$$R_{H2FT} = F_T - R_{H1FT} = F_T - \frac{F_T(b+x+y)}{(x+y)}$$

$$R_{H2FT} = \frac{F_T(b)}{(x+y)}$$

$$R_{H2FT} = \frac{29989.50(242)}{(230+230)}$$ =**15777.08N**

The reactions at the bearings 1 and 2 due to Flywheel weight (W) will be same as discussed earlier.

$$R_{V1(W)} = R_{V2(W)} = \frac{(30(10)^3)}{(2)} = 15(10)^3, N$$

**Design of crank web**

The dimensions are taken same as calculated in crank at dead center.
The same dimensions are checked here for combined stress.
Width of crank web w=216.3mm
Thickness of crank web h=66mm

The most critical section is where the web joins the shaft. This section is subjected to the following stresses:

i) Bending stress due to the tangential force $F_T$
ii) Bending stress due to the radial force $F_r$
iii) Direct compressive stress due to radial force $F_r$ and
iv) Shear stress due to the twisting moment of $F_T$.

Bending stress due to the tangential force $F_T$

Bending moment due to tangential force, $M_{bT} = F_T(R - \frac{d_p}{2}), N-mm$

$M_{bT} = 29989.50(150 - \frac{110}{2}), N-mm$ =**$2.85(10)^6$, N-mm**

Therefore bending stress due to tangential force $\sigma_{bT} = \frac{6M_{bT}}{hw^2}$

$\sigma_{bT} = \frac{6*2.85(10)^6}{66(216.3)^2}$ =**5.54MPa**

Bending stress due to the radial force $F_r$

Bending moment due to the radial force, $M_{bR} = F_R(0.75l_p + 0.5h)$

$M_{bR} = 32953(0.75*110 + 0.5*66)$ =**$3.81(10)^6$ N-mm**

Therefore bending stress due to radial force $\sigma_{bR} = \frac{6M_{bR}}{wh^2}$

$\sigma_{bR} = \frac{6(3.81*10^6)}{(216.3)(66)^2}$ =**24.26MPa**

Direct compressive stress due to radial force $F_r$

We know that, direct compressive stress, $\sigma_d = \frac{F_R}{wh}$

$\sigma_d = \frac{32953}{(66)(216.3)}$ =**2.31MPa**

Shear stress due to the twisting moment of $F_T$.

Twisting moment due to the Tangential force, $T = F_T(0.75l_p + 0.5h)$

$T = 29989.50(0.75(110) + 0.5(66))$ =**3.46(10)⁶, N-mm**

Therefore shearing stress due to Tangential l force $\tau = \frac{T}{I} r = \frac{T}{Z} = \frac{4.5T}{wh^2} = \tau_{xy}$

Where Z-Polar section modulus, $= \frac{wh^2}{4.5}$

$\tau = \frac{4.5(3.46 * 10^6)}{(216..3)(66)^2} = \tau_{xy}$ =**16.53MPa**

Superimposing the stresses we get,

Total compressive stress, $\sigma_C = \sigma_{bT} + \sigma_{bR} + \sigma_d = \sigma_x$

$\sigma_x = 5.54 + 24.26 + 2.31$=**32.11MPa**

Now the total or maximum normal and maximum shear stresses are given by,

$$\sigma_{max} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left[\frac{\sigma_x - \sigma_y}{2}\right]^2 + \tau_{xy}^2} \quad \text{.....................................(1.11b/2)}$$

Here $\sigma_y$=0;

$\sigma_{max} = \frac{32.11 + 0}{2} + \sqrt{\left[\frac{32.11 - 0}{2}\right]^2 + 16.53^2}$ =**39.10MPa<83MPa**

Hence the calculated values of dimensions of crank web are safe

**Design of Shaft under the flywheel:**

Horizontal bending moment acting on the shaft due to piston gas load,

$M_{H1} = F_P(b + x) - \left[\sqrt{(R_{H1FR})^2 + (R_{H1FT})^2}\right] x$

$M_{H1} = 44.18(10)^3(242 + 230) - \left[\sqrt{(50261.67)^2 + (45766.58)^2}\right] * 230$

= **5.22(10)⁶, N-mm**

Therefore total horizontal bending moment, $M_H = M_{H1} + M_{Hbelt}$

$M_H = 5.22(10)^6 + 0 =$ **5.22(10)⁶, N-mm**

Vertical bending moment due to flywheel,

$M_{VFLY} = (R_{V2(W)})(y)$

$M_{VFLY} = (15 * 10^3)(230)$ =**3.45(10)⁶, N-mm**

Since these bending moments act at right angles to each other, the combined bending moment is given by;

$M_{Total} = \sqrt{M_{VFLY}^2 + M_{Ht}^2}$ , N-mm

$M_{Total} = \sqrt{(3.45*10^6)^2 + (5.22*10^6)^2}$ =**<u>$6.26(10)^6$, N-mm</u>**

In addition to this moment there will be a twisting moment because of tangential force $F_t$.
The twisting moment, $T = F_t(R)$ , N-mm

$T = 29989.50(150)$ =**<u>$4.5(10)^6$, N-mm</u>**

Therefore Equivalent twisting moment,

$T_e = \sqrt{T^2 + M_{Total}^2}, N - mm$

$T_e = \sqrt{(4.5(10)^6)^2 + (6.26(10)^6)^2}, N - mm$ =**<u>$7.71(10)^6$, N-mm</u>**

We have, $T_e = \frac{\pi}{16} d_S{}^3(\tau)$ , N-mm,

$7.71(10)^6 = \frac{\pi}{16} d_S{}^3(42)$ Here $\tau_{max}=0.5\sigma_b$

Diameter of the shaft under flywheel **$d_S$=97.78mm** can be obtained.

Since the diameter of the bearing is 155mm> 97.78mm.
Hence **<u>$d_S$=155mm</u>** is adopted

References:

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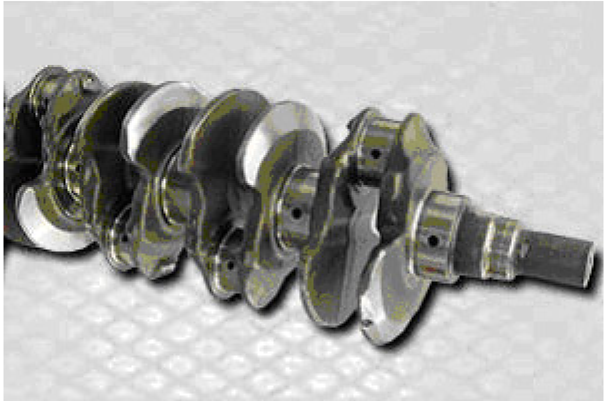

**Questions from Previous University Question Papers.**

1. Explain the methods of manufacturing crank shaft? (05M) July 2006. VTU
2. Design a overhung crankshaft for the steam engine to the following specifications:
   Diameter of piston = 400mm
   Stroke of piston = 600mm
   Maximum steam pressure =1.0 N/mm$^2$
   Speed of the engine= 100rpm
   Design shear stress for the crank shaft and crank pin = 3.5 N/mm$^2$
   Design tensile stress for the crank shaft and key = 6.0 N/mm$^2$
   The horizontal distance between crank shaft and crank pin=350 mm
   (15M) July 2006.VTU
3. Write a note on balancing of crankshafts. (04M) FEB 2006 VTU
4. Sketch a typical crankshaft used for a four cylinder engine. Indicate clearly the positions of pins & journals and the provision for fabrication. What are the materials used for the crankshaft. (8M) FEB 2006 VTU
5. Design & draw the sketch of an overhung crankpin for an engine having the following particulars.
   Cylinder diameter = 300 mm; Stroke = 500 mm;
   Maximum explosion pressure in the cylinder = 1.8 N/mm$^2$
   Engine speed = 200 rpm. Permissible bending stress for pin = 9.81 N / mm$^2$ &
   Bearing stress = 83.4 N / mm$^2$ (8M) FEB 2006 VTU
6. Distinguish between i. Center Crankshaft and Overhung Crankshaft.
   ii Built-up Crankshaft and Integral Crankshaft.
   (6M) Model QP VTU

7. Design a plain carbon steel crankshaft for a 0.40 m by 0.60 m single acting four stroke single Cylinder engine to operate at 200 rev/min. The mean effective pressure is 0.49 MPa and the maximum combustion pressure is 2.625 MPa. At maximum torsional moment, when the crank angle is 36$^0$, the gas pressure is 0.975 MPa. The ratio of the connecting rod length to the crank radius is 4.8. The flywheel is used as a pulley. The weight of the flywheel is 54.50 KN. And the total belt pull is 6.75 KN. Assume suitable values for the missing data.
   (14M) Model QP VTU

## Model Questions

1. Design a plain carbon steel centre crankshaft for a single acting four stroke, single cylinder engine for the following data.

| | |
|---|---|
| Piston Diameter | 250mm |
| Stroke | 400mm |
| Maximum Combustion Pressure | 2.5MPa |
| Weight of the flywheel | 16kN |
| Total Belt Pull | 3kN |
| Length of the connecting rod | 950mm |

   When the crank has turned through $30^0$ from the top dead center, the pressure on the piston is 1 MPa and the torque on the crank is maximum.
   Any other data required for the design may be assumed.

2. Design aside crank shaft for a 500mmX600mm gas engine. The weight of the flywheel is 80kN and the explosion pressure is 2.5MPa. The gas pressure at maximum torque is 0.9MPa, when the crank angle is $30^0$. The connecting rod is 4.5 times the crank radius.
   Any other data required for the design may be assumed.

3. Explain the various types of crank shafts
4. What are the methods and materials used in the manufacture of crankshafts.
5. How a crankshaft is balanced?