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Department of Automobile Engineering

Subject: Theory and Design of Automotive Engines
[Sub Code - AU51]
V – Semester, Automobile Engineering

Syllabus Covered:

1 Connecting rod – design, effects of whipping, bearing materials, lubrication

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CONNECTING RODS

Definition: A Connecting rod is the link between the reciprocating piston and rotating crank shaft. Small end of the connecting rod is connected to the piston by means of gudgeon pin. The big end of the connecting rod is connected to the crankshaft.

Function: The function of the connecting rod is to convert the reciprocating motion of the piston into the rotary motion of the crankshaft.

Materials: The connecting rods are usually forged out of the open hearth steel or sometimes even nickel steel or vanadium steel. For low to medium capacity high speed engines, these are often made of duraluminium or other alluminium alloys. However, with the progress of technology, the connecting rods these days are also cast from malleable or spheroidal graphite cast iron. The different connecting rod steels are (4OC8, 37Mn6, 35Mn6 MO3, 35Mn6 Mo4, 40Cr4, 40Cr4 Mo3, 40NiCr4MO2) etc.

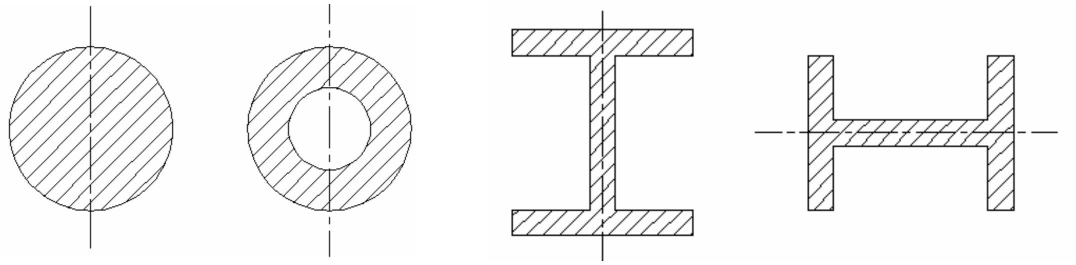
In general, forged connecting rods are compact and light weight which is an advantage from inertia view point, whereas cast connecting rods are comparatively cheaper, but on account of lesser strength their use limited to small and medium size petrol engines.

Construction: A typical connecting rod is shown in fig1. A combination of axial and bending stresses act on the rod in operation. The axial stresses are product due to cylinder gas pressure and the inertia force arising on account of reciprocating motion. Where as bending stresses are caused due to the centrifugal effects. To provide the maximum rigidity with minimum weight, the cross section of the connecting rod is made as and I – section end of the rod is a solid eye or a split eye this end holding the piston pin. The big end works on the crank pin and is always split. In some connecting rods, a hole is drilled between two ends for carrying lubricating oil from the big end to the small end for lubrication of piston and the piston pin.

Classification: The classification of connecting rod is made by the cross sectional point of view i.e. I – section, H – section, Tabular section, Circular section.

In low speed engines, the section of the rod is circular, with flattened sides.

In high speed engines either an H – section or Tabular section is used because of their lightness. The rod usually tapers slightly from the big end to the small end.



Forces acting on the Connecting Rod:

1. The combined effect (or joint effect) of,
   a) The pressure on the piston, combined with the inertia of the reciprocating parts.
   b) The friction of the piston rings, piston, piston rod and the cross head.
2. The longitudinal component of the inertia of the rod.
3. The transverse component of the inertia of the rod.
4. The friction of the two end bearings.

Design of Connecting Rod:

In designing a connecting rod the following dimensions are required to be determined.

1. Dimension of cross section of connecting rod
2. Dimension of the crank pin at the big end and the piston pin at the small end.
3. Size of the bolts for securing the big end cap and
4. Thickness of the big end cap.

According to **Rankine's – Gordon** formula,

$$F \text{ about x-axis } = \frac{f_c A}{1 + a\left(\frac{l}{K_{xx}}\right)}$$

Let,

A = C/s area of connecting rod, L = Length of connecting rod

f_c = Compressive yield stress, F = Buckling load

I_{xx} and I_{yy} = Radius of gyration of the section about x – x and y – y axis respectively

and K_{xx} and K_{yy} = Radius of gyration of the section about x – x and y – y axis respectively.

for both ends hinged or free, *l* = 1*l* data from Pg. 5, Eq. 1.29

$$\text{F about y-axis} = \frac{f_c A}{1 + a\left(\frac{l}{K_{yy}}\right)}$$

for both ends fixed, $l = \frac{l}{2}$ data from Pg. 5, Eq. 1.29

In order to have a connecting rod equally strong in buckling about both the axes, the buckling loads must be equal,

ie. $$\frac{f_c A}{1 + a\left(\frac{l}{K_{xx}}\right)^2} = \frac{f_c A}{1 + a\left(\frac{l}{2K_{yy}}\right)^2}$$

or $$\left(\frac{l}{K_{xx}}\right)^2 = \left(\frac{l}{2K_{yy}}\right)^2$$

$\therefore K_{xx}{}^2 = 4K_{yy}{}^2$

Or $I_{xx} = 4I_{yy}$

Design a connecting rod for a semi diesel engine with the following data.

Diameter of the piston = 88 mm

Weight of the reciprocating parts = 1.6 Kg

Length of the connecting rod = 30 cm = 300 mm (center to center)

Stroke = 125 mm

RPM = 2200 when developing 70 HP i.e. 52.2 KW

= 3000 is possible over speed

Compression ratio = 6.8:1

Probable maximum explosion pressure = 35 Kgf/cm^2 = 3.44 N/mm^2

1. Cross section of the Connecting Rod:

Since in all high speed engines connected rods,

i. Lightness is essential in order to keep the inertia forces as small as possible and
ii. Ample strength is required to withstand the momentary high gas pressure in the cylinder.

Therefore, the I – section is generally found most suitable for this type of connecting rod.

The connecting rod is under alternating tension and compression and since compression corresponds to the power and compression strokes, the compressive stress is much greater numerically than the tensile stress. The connecting rod is therefore, designed mainly as a strut. The inertia force due to change of motion of the reciprocating parts will be considered and checked later.

In the plane of motion of the connecting rod, the ends are direction free at the crank and the gudgeon pins, and the strut is therefore, Hinged for buckling about "neutral axis" (x-x Axis)

In the plane perpendicular to the motion plane (NA), (i.e. y-y axis) when buckling tends to occur about y – y axis, the strut has almost fixed ends due to the constraining effect of the bearing at crank and gudgeon pins.

For buckling about y – y axis,

The connecting is therefore 4 times as strong about y – y for buckling as for, the buckling about x – x due to constraining effect of the fixed ends.

i.e. 4 $I_{yy} = I_{xx}$

The result is a convincing evidence of the suitability of I – section.

It can be noticed that, a circular section connecting rod, is un-necessarily strong for buckling about the y – y axis.

The proportions given in the figure are assumed for the section as representing a typical connecting rod. It is needed to check the relationship of the equation ------ 1.

Area A = $(4t^2+4t^2)+3t^2 = 11t^2$

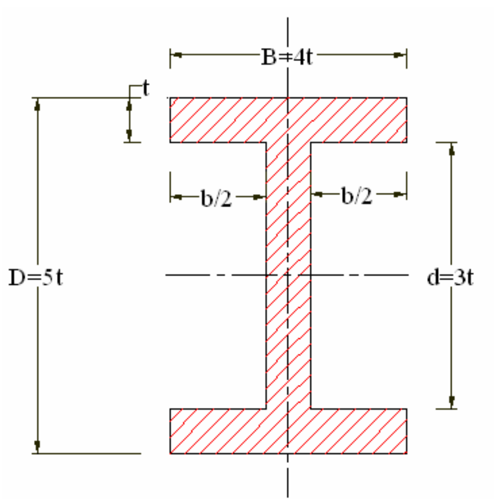
$$I_{xx} = \frac{1}{12}\left(BD^3 - bd^3\right)$$

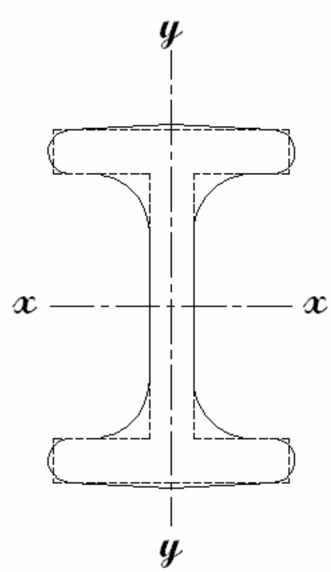
$$= \frac{1}{12}\left(4t(5t)^3 - 3t(3t)^3\right)$$

$$= 10.91\, t^4$$

$$\therefore \frac{I_{xx}}{I_{yy}} = 3.2 \text{ approx.}$$

So, in the case of this section (assumed section) proportions shown above will be satisfactory.





(Problem No.1)

Design a connecting rod for a petrol engine for the following data,

Diameter of the piston (d)= 110 mm, length of the connecting rod(2L) = 325 mm

Stroke length(L) = 150 mm, Speed (n) = 1500 rpm, Over speed = 2500 rpm

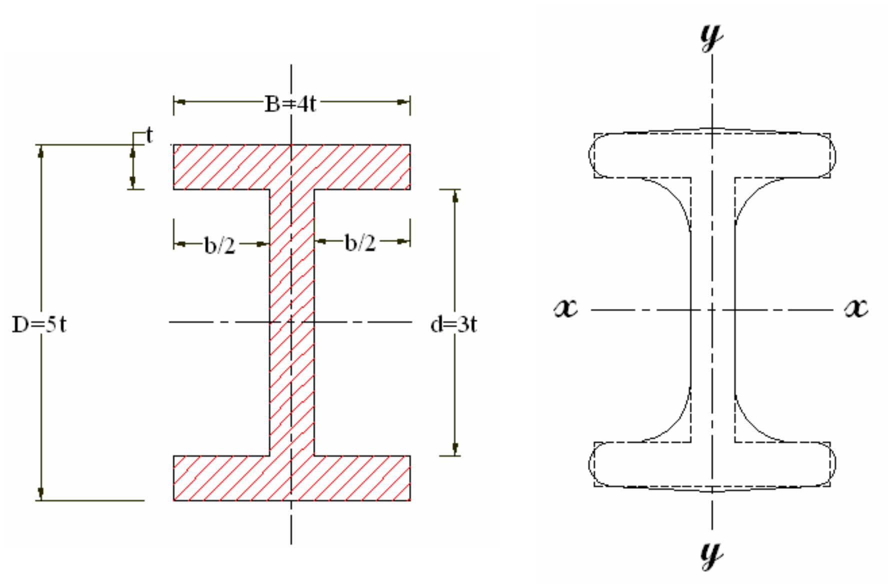
compression ratio = 4 : 1, Maximum explosion pressure = 2.5 MPa.

Solution:

Step 1. Dimensions of cross section of the connecting rod:

Let us consider an I – section of the connecting rod as shown in figure, with the following proportions, so that the connecting rod to be equally resistant to buckling in either plane, the relation between moment of inertia must be,

$I_{xx} = 4I_{yy}$.



From pg. 431,

Moment of inertia of the I – cross section abut x-x is given by,

$$I_{xx} = \frac{1}{12}\left(BD^3 - bd^3\right) = \frac{1}{12}\left(4t(5t)^3 - 3t(3t)^3\right) = 34.91t^4$$

Moment of inertia of the I – cross section about yy is given by,

$$I_{yy} = \frac{1}{12}\left(bD^3 - Bd^3\right) = \frac{1}{12}\left(2t(4t)^3 + 3t(t)^3\right) = 10.91t^4$$

$\therefore$ Ratio of I_{xx} to I_{yy} i.e. $\frac{I_{xx}}{I_{yy}} = \frac{34.91t^4}{10.91t^4} = 3.2$

$\therefore$ The section chosen is quite satisfactory

Area of cross section (A)

A = (5t x 4t) – (3t x 3t) = 11 t^2

Radius of gyration K_{xx} (K) is given by,

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{34.19t^4}{11t^2}} = 1.78\ \text{t}$$

w.k.t.

Stroke length = L = 150 mm

$\therefore$ crank radius $r = \frac{storkeofpiston}{2} = \frac{L}{2} = \frac{150}{2} = 75mm$

$$n^1 = \frac{l}{r} = \frac{lengthofconnectingrod}{crankradius} = \frac{325}{75} = 4.33$$

w = angular speed $= \frac{2\pi N}{60} = \frac{2\pi 1500}{60} = 157.1$ rad / sec.

Step 2. Inertia force of Reciprocating Parts (F) :

$$F = \frac{1000WrV^2}{gr} Cos\theta \pm \frac{Cos2\theta}{n^1}$$ …………………… 19.8 (a), 370

Wr = mg = Weight of reciprocating parts……….. N

= 2 x 9.81 = 19.62 N

r = Crank radius = 75 mm

θ = Crank angle from the dead center

= 0 considering that connecting rod is at the TDC position

n^1 = 4.33

g = Acceleration due to gravity = 9.81 m/s^2

V = Crank velocity m/s

= rw = 75 x 10^{-3} x 157.1

= 11.78 m/s

Substituting,

$$F = \frac{1000 \times 19.62 \times (11.78)^2}{9.81 \times 75}\left(Cos0 + \frac{Cos20}{4.33}\right)$$

= 4555 N

Step 3: Total force on the connecting rod :

$$Fc = F_p - F_j = F_p - F$$

= 23.76 x 10^3 – 4555

= 19205 N

Step 4: To find the thickness of the connecting rod flange and web:

By using **Rankine's – Gordan** formula, The stress due to axial load,

$$fcr = \frac{Fc}{A} = \frac{fc}{1 + K\left(\frac{l}{k}\right)^2}$$ ……………………Eq. 19.5 Pg. 369

$$\therefore Fc = \frac{fcA}{1 + K\left(\frac{l}{k}\right)^2}$$

Fc = Total force on the connecting rod i.e. axial load on the rod

= 19205 N

K = Constant

$= \frac{4}{25000}$ for steel rod, pin connected at both ends, so that the rod is free to bend in any plane.

A = Area of cross section

= 11 t^2

l = Length of connecting rod

= 325 mm

k = Radius of gyration about x – x axis

= 1.78 t

fc = Allowable unit stress for designing n/mm^2

$= \frac{Yieldpoint\,stress}{FOS}$ Assume FOS = 4

= 378/4 Yield point stress, from T – 19.1 Pg. 371

= 94.5 MPa   378 MPa

Substituting,

$$Fc = \frac{94.5 \times 11t^2}{1 + \left(\frac{4}{25000}\right)\left(\frac{325}{1.78t}\right)^2}$$

$$19205 = \frac{1.39.5t^4}{t^2 + 5.34}$$

$$\therefore 1040t^4 = 19205t^2 + 102554.7$$

==> $1040t^4 - 19205t^2 - 102554.7 = 0$

$$t^2 = \frac{19205 + \sqrt{(19205)^2 + (4 \times 1040 \times 102554.7)}}{2 \times 1040}$$

=> $t^2 = 22.8$

$\therefore t = 4.775$ Say 5 mm

Take *t* = 10 mm

Note the dimensions, width = 4t = 40 mm

Depth = 5t = 5o mm

Flange and web thickness = t = 10 mm

Step 5: Design of small end:

We know that,

Load on the piston pin or small end bearing (Fp) = Projected area x Bearing pressure

$\therefore Fp = dplp \times P_{bp}$

Fp = 23760 N force or load on the piston pin, d_p = Diameter of piston pin

P_{bp} = Bearing pressure ……………. From Pg. 362

= 12.4 for gas engines.

= 15.0 for oil engines.

= 15.7 for automotive engines.

We assume

P_{bp} = 10 MPa

l_p = length of piston pin

= 1.5 d_p ……… from Pg. 362

Substituting,

$23760 = 1.5\ d_p\ .\ d_p \times 10$

$\therefore d_p = 39.79 \cong 40mm$

$\therefore l_p = 1.5 d_p = 60mm$

Step6: Design of Big end:

w.k.t

load on the crankpin or big end bearing (Fp)

= Projected Area x Bearing pressure

$\therefore F_p = d_c l_c P_{bc}$

Fp = 23760 N forces or load on the piston pin

d_c = diameter of crankpin

l_c = length of crankpin

= $1.25\ d_c$

P_{bc} = 7.5 MPa Assume

Substituting,

$23760 = 1.25\ d_c\ d_c\ 7.5$

$\therefore d_c = 50mm$

$l_c = 62.5mm$

Step 7: Design of Big end Bolts:

w.k.t.,

Force on the bolts = $\frac{\pi}{4}(d_{cb})^2 \times \sigma_t \times n_b$

d_{cb} = Core diameter of the bolts

σ_t = Allowable tensile stress for the material of bolts

= 12 MPa assume

n_b = Number of bolts usually 2 bolts are used

$= \frac{\pi}{4} \times 12 \times 2 \times (d_{cb})^2$

$= 18.85\ d_{cb}^2$

Also,

The bolts and the big end cap are subjected to a tensile force which corresponds to the inertia force of the reciprocating parts at the TDC on the exhaust stroke.

We Know that inertia force on the reciprocating parts

$$F = \frac{1000WrV^2}{gr}\left(Cos\theta \pm \frac{Cos2\theta}{n^1}\right)$$

As calculated earlier

F = 4555 N

Equating the Inertia force, to the force on the bolts,

$4555 = 18.85\, d_{cb}^2$

$\therefore d_{cb} = 15.55mm$

$\therefore$ Normal diameter of the bolts (d_{cb})

$$d_{cb} = \frac{d_{cb}}{0.84} = 18.50mm$$

$\cong say 20mm$

$\therefore$ use M20 sized bolts

Step 8: Design of Big end cap:

The big end cap is designed as a beam freely supported at the cap bolt centers and loaded by the inertia force at the TDC on the exhaust stroke (Fj at θ=0)

Since load is assumed to act in between the UDL (Uniformly distributed load) and the centrally concentrated load,

$\therefore$ Maximum Bending moment is taken as,

$$M\max = \frac{Fi \times l_o}{6}$$

Fi = Magnitude of Inertia force

= 4555 N

l_o = Distance between bolt centers.

= Dia of crank pin or Big end bearing + Nominal dia of bolt

+ (2 x thickness of bearing liner) + Clearance

= $d_c + d_b + (2 \times (0.05\, d_c + 1)) + 3$

= 80 mm

Substituting,

$$M_{max} = \frac{4555 \times 80}{6}$$

= 60734 N-mm

Section modulus for the cap,

$$Z = \frac{bh^2}{6}$$

Z = Section modulus

b = width of the big end cap

it is taken equal to the length of the crankpin or Big end bearing (l_c)

l_c = b = 62.5 mm

Substituting, h = thickness of big end cap

$$= \frac{62.5 \times h^2}{6}$$

= 10.42 h^2

We know that bearing stress

$$\sigma_b = \frac{M_{\max}}{Z}$$

σ_b = Allowable bending stress for the material of the cap

= 120 MPa Assume

Substituting,

$$120 = \frac{60734}{10.42h^2}$$

$\therefore h = 6.97$ say 7 mm

Step 9: Check for stresses:

The magnitude of Inertia force (F_i)

$$F_i = \frac{W \times A \times w^2 \times r \times l \times 10^{-2}}{2g}$$

W = Weight density per unit volume of the rod N/m^3

= 7800 x 9.81 N/m^3 assume

r = Crank radius = 75 mm

l = length of connecting rod = 325 mm

A = Area of cross section (I – section)

w = Angular speed = 157.1 rad / sec

g = Acceleration due to gravity = 9.81 m/sec^2

Substituting,

$$F_i = \frac{7800 \times 9.81 \times 1100 \times (157.1)^2 \times 75 \times 325 \times 10^{-12}}{2 \times 9.81}$$

F_i = 2580.8 N

The max. bending moment (M_{max})

$$M_{max} = \frac{ZF_i l}{9\sqrt{3}}$$

$$= \frac{2 \times 2580.8 \times 325}{9\sqrt{3}}$$

= 107612.95 N-mm

The maximum Inertia bending stress or whipping stress (σ_b)

$$\sigma_b = \frac{M_{max}}{Z}$$

From Eq. 19.3 Pg. 369

$$\sigma_b = \frac{0.2854 \times 10^{-12} \times n^2 \times r \times A \times w \times l^2}{Z}$$

Z = I/y

I = 34.91 t^4 = 34.91 x 10^4 mm^4

y = D/2 = 50/2 = 25 mm

n = rev/sec = Speed of crank = 2200/60 = 36.67 r/sec

Substitute,

$$\sigma_b = \frac{0.2854 \times 10^{-12} \times (36.67)^2 \times 75 \times 1100 \times 7800 \times 9.81 \times 325^2}{34.91 \times 10^4 / 25}$$

= 18.32 MPa Which is Safe

Maximum compressive stress in the connecting rod,

i.e. = Stress due to axial load + The max. Inertia bending stress or whipping stress

$= f_{cr} + \sigma_b$

$= \frac{Fc}{A} + \sigma_b$

$= \frac{19205}{1100} + 18.32$

= 35.78 MPa Which is safe

(Problem 2)

Design the connecting rod of a steam engine to the following data

Length of the connecting rod = 825 mm, Dia of the crankpin = 155 mm

Dia of the cross head pin = 95 mm, Maximum load on the pin = 15160 Kg = 148720 N, The rod is to be made of circular cross section and made hallow by boring a central hole of 28 mm dia, throughout the length.

Calculations should be made for,

1. External dia at the centre
2. Length of the cross head pin
3. Diameter of the big end bolts
4. Length of the crankpin
5. Width and thickness of the cap

1. Calculation of External dia at the center.

Let us assume that, at the middle of the connecting rod,

D = Outside dia = ?

d = Inside dia = 28 mm

$\therefore$ cross section area $A = \frac{\pi}{4}\left[D^2 - d^2\right]$

MOI $I_{xx} = \frac{\pi}{4}\left[D^4 - d^4\right]$

K_{xx} = Radius of Gyration = $\sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{\pi \times 4\left(D^2 - d^2\right)\left(D^2 + d^2\right)}{64 \times \pi\left(D^2 - d^2\right)}}$

$= \frac{1}{4}\sqrt{\left(D^2 + d^2\right)}$

By using **Rankine – Gordon** formula,

Eq. 19.5, Pg. 369

Crippling load i.e. Axial load on the rod due to steam or gas pressure

$$F_c = \frac{f_c A}{1 + K\left(\frac{l}{k}\right)^2}$$

f_c = Yield point stress / FOS

= 324 / 7

= 46.3 MPa

Yield point stress = 324 MPa

for forged M.S rod connecting rod material

Assume FOS as 7

K = $\frac{1}{7500}$ for M.S when both ends are rounded

F_c = 148720 N

k = $\frac{\sqrt{D^2 + d^2}}{4}$

l = length of the connecting rod = 825 mm

A = $\frac{\pi\left(D^2 - d^2\right)}{4}$

Substituting in **Rankine – Gordon** equation

$$148720 = \frac{46.3 \times 0.7854\left(D^2 - d^2\right)}{1 + \frac{1}{7500}\left(\frac{(825)^2 \times 16}{\left(D^2 + d^2\right)}\right)} = \frac{36.4\left(D^2 - d^2\right)}{1 + \frac{1452}{\left(D^2 + d^2\right)}}$$

$$148720 = \frac{36.4\left(D^2 - d^2\right)}{\frac{\left(D^2 + d^2\right) + 1452}{\left(D^2 + d^2\right)}} = \frac{36.4\left(D^4 - d^4\right)}{\left(D^2 + d^2\right) + 1452}$$

148720 D^2 + 148720 d^2 + 215.94x10^6 = 36.4 D^4 – 36.4 d^4

Substitute, D^2 = C,

148720 C + 148720 x $(28)^2$ + 215.94x10^6 = 36.4 C^2 – 36.4 x $(28)^4$

148720 C + 116.6 x 10^6 + 215.94x10^6 = 36.4 C^2 – 223.74 x 10^6

36.4C^2 – 148720 C – 223.74 x10^6 – 116.6 x 10^6 – 215.94 x 10^6 = 0

36.4 C^2 – 148720 C – 556.3 x 10^6 = 0

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2.a} = \frac{148720 + \sqrt{(148720)^2 + 4(36.4)(556.3 \times 10^6)}}{2 \times 36.4}$$

$$= \frac{148720 + 321.115 \times 10^3}{72.8} = \frac{469.84 \times 10^3}{72.8}$$

C = 6454

$\therefore$ D = $\sqrt{6454}$

$= 80.33 mm$ s

2. Calculation for length of the cross head pin (Gudgeon pin)

W.K.T, force on the piston (F_p) = l_p d_p P_{bpin}

F_p = 148720 N

d_p = 95 mm

P_b = 8.25 MPa Assume

$$\therefore l_p = \frac{148720}{95 \times 8.25} = 190 mm$$

3. Calculation for length of the crank pin

W.K.T., Force on the piston (F_p) = l_c d_c $P_{b\ crank}$

F_p = 148720 N

d_c = 155 mm

P_b = 6.2 MPa Assume

$$\therefore l_c = \frac{148720}{155 \times 6.2} = 155 mm$$

4. Diameter of Big end Bolts :

As the bearing length of the big end is 155 mm,

Assuming 4 nos. of bolts, these 4 bolts have to take this load i.e.

F_p

$\therefore F_b$ = load on each bolt = $\frac{148720}{4} = 37180 N$

$\therefore$ Magnitude of load $F_b = \frac{\pi}{4} d_b^2 \times \sigma_t$

37180 = 0.7854 x 69 x d_b^2

$\therefore d_b$ = 26.2 mm

Full dia = $\frac{27}{0.84} = 32 mm$

The nearest standard size is 33 mm and may be adopted.

5. Calculations for width and thickness of cap

The effective width of the cap will be equal to,

The length of the big end } – {2 x thickness of the flange of the brass brasses

length of the big end brasses = 155 mm

Thickness of the flange of the brass = 6 mm

$b = 155 - (2 \times 6) = 143$ mm

$l_o = d_c + d_b +$ (2 x thickness of the liner)

= 155 + 33 + (2 x 6) = 200 mm

M_{max} = Moment of Resistance = Z x σ_b

w.k.t.,

$M_{max} = \frac{Fil_o}{6}$ (Check)

$Z = \frac{l_c h^2}{6} = \frac{bh^2}{6}$

$\sigma_b = 68.67 \cong 69$ MPa

$\therefore$ Substituting,

$$\frac{148720 \times 200}{6} = \frac{143 \times h^2}{6} \times 69$$

$\therefore h = 55$ mm

(Problem No.3) Determine the maximum stress in the connecting rod of I – section, as shown in fig., due to inertia. The length of the connecting rod is 360 mm and the piston stroke is 180 mm. The speed is 200 rpm. Density of the material of the connecting rod may be taken as 7800 Kg/m^3

Solution:

Ans. Cross section Area of I – section,

A = 2 x 6 x 30 + (45 – 12) x 6

= 558 mm^2

The maximum inertia bending stress or whipping stress (σ_b)

$$\sigma_b = \frac{0.2854\times10^{-12}\times n^2\times r\times A\times w\times l^2}{Z}$$

n = Speed of crank in rev/sec

= 200/60 = 3.34 rev/s

r = Crank radius = $\frac{Storkelength}{2} = \frac{180}{2} = 90$ mm

w = 7800 x 9.81 76518 N/m^3

= weight density of rod material

l = length of connecting rod

= 360 mm

Z = section modulus of mean section in mm^3

$$= \frac{I}{y} = \frac{MoIaboutxx}{y} = \frac{{}^1\!/\!{}_2\left(BD^3 - bd^3\right)}{{}^{45}\!/\!{}_2}$$

$$= \frac{\frac{1}{2}\left[30\times45^3 - 24\times33^3\right]}{22.5}$$

= 6930.6 mm^3

Substituting,

$$\sigma_b = \frac{0.2854\times10^{-12}\times3.34^2\times90\times558\times76518\times360^2}{6930.6}$$

= 0.228 MPa

(Problem No.4) Find the diameter of a connecting rod 250 mm long for a stroke speed diesel engine. Cylinder diameter is 100 cm = 1000 mm and stroke is 125 cm = 1250 mm. Maximum combustion pressure is 4.905 N/mm^2, FOS = 20, E = 2.06 x 10^5 N / mm^2

Solution:

Max. load on the piston

$$F_p = \frac{\pi}{4} D^2 \times P_{max} = \frac{\pi}{4} \times 1000^2 \times 4.905 = 3.85 \times 10^6 \text{ N}$$

We neglect the Inertia effect of the reciprocating mass as for the slow speed engine.

Let 'd' be the diameter of the connecting rod. Then by EULER'S FORMULA Eq. 1.29 Pg. 5,

$$F_{cr} = \frac{n\pi^2 EI}{l^2}$$

n = Constant = 1 for both ends hinged

E = 2.06 x 10^5 N/mm^2

I = MOI = $\frac{\pi d^4}{64}$ for circular section (Solid)

$= \frac{\pi d^4}{64}$

l = length of connecting rod = 250 mm

$F_{cr} = F_p \times FOS = 3.85 \times 10^6 \times 20$ N

Substituting,

$$3.65 \times 10^6 \times 20 = \frac{1 \times \pi^2 \times 2.06 \times 10^5 \times \pi \times d^4}{(250)^2 \times 64}$$

$\therefore d$ = 83.33 mm

(Problem No.5.) A reciprocating pump is used to raise the water against a heap of 165 Kg. Pump diameter is 450 mm and piston rod is 1400 mm long. Calculate the diameter of the piston rod. Use Rankine constant a=1/7500, FOS = 10, pressure on the piston = 1.61 N/mm^2

Solution:

Load on the piston = $\frac{\pi}{4}(450)^2 \times 1.61 = 256.06$ KN

Design load = load on the piston x FOS

= 256.06 x 10^3 x 10

= 2560.6 x 10^3 N

From equation,

$$F = \frac{f_c \times A}{1 + K\left(\frac{l}{k}\right)^2}$$

$$A = \frac{\pi d^2}{4}$$

$2560.6 \times 10^3 = \dfrac{323.73 \times 0.7854 \times d^2}{1 + \frac{1}{7500}\left(\frac{1400^2}{d^2}\right)}$ d = dia of connecting rod

K = d Assume

$1 + \dfrac{261.4}{d^2}$ K = 1/7500

$2560.6 \times 10^3 = \dfrac{254.25 \times d^2}{\frac{d^2 + 261.4}{d^2}}$ l = 1400 mm

f_c = 323.73 N/mm^2

$2560.6 \times 10^3 d^2 + 669.34 \times 10^6 = 254.25 d^4$

$254.25 d^4 - 2560.6 \times 10^3 d^2 - 669.34 \times 10^6 = 0$

$$d^2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{2560.6 \times 10^6 + \sqrt{\left(2560.6 \times 10^3\right)^2 + \left(4 \times 254.25 \times 669.34 \times 10^6\right)}}{2 \times 254.25}$$

$= 10325$

$\therefore d = 201.61 mm$

(ProblemNo.6) Design a connecting rod for a petrol engine for the following data Diameter (d) = 110 mm, Mass of reciprocating die of piston (M) = 2 Kg Length of connecting rod = 325 mm, Stroke length L = 150 mm, Speed *n* = 1500 rpm, Over speed = 2500 rpm, Connecting rod = 4 : 1, Max. Exp. Pressure = 2.5 MPa

Solution:

1. Stroke length = L = 150 mm
2. Crank radius = L/2 = 150/2 = 75 mm
3. *n* = length of connecting rod / crank radius = 325/75 = 4.33 = l/r
4. Angular speed = $\dfrac{2\pi N}{60} = \dfrac{2\pi \times 1500}{60} = 157.07$

Inertia force of reciprocating parts

$$F\frac{100W_r v^2}{g\times r}\left(Cos\theta \pm \frac{Cos2\theta}{n^1}\right)$$ 19.8 (a) 370

W = Mxg = Weight of reciprocating parts

v = Crank velocity m/sec

r = crank radius mm

θ = Crank angle from the dead center

g = 9.81 m/s^2

$n^1 = l/r$

$v = rw$

= 75 x 10^{-3} x 157.1 = 11.78 m/s

take, θ = 0, Considering that connecting rod is at the TDC.

$\therefore$ Inertia force

$$F = \frac{1000\times(2\times 9.81)\times(11.78)^2}{9.81\times 75}\left(Cos0 \pm \frac{Cos20}{4.33}\right)$$
$$= 3700.49\left(1\pm\frac{1}{4.33}\right)$$
$$= 3700.49(1.230)$$
$$= 4555N$$

Force on the piston (F_p)

$F_p = P \times A$

= 2.5 x $\left(\frac{\pi}{4}\times(110)^2\right)$

= 23758.3 N

Total force on the connecting rod :

$F_T = F_P - F_i = F_P - F$

= 23758.3 – 4555

= 19203.3 N

Cross section of the connecting rod :

In order that connecting rod to be equally resistant to buckling in either plane, the relation between the moment of inertias must be,

$I_{xx} = 4I_{yy}$ ……………19.6

Now the cross section satisfying the condition is the I section as shown in fig.

b = 4t – t = 3t

B = 4t

d = 5t – 2t = 3t

D = 5t

About x – x axis.

From Pg.-431,

Moment of inertia 'I_{xx}' for the above I – section about x x is given by,

$$I_{xx} = \frac{1}{12}\left(BD^3 - bd^3\right)$$

Moment of Inertia 'I_{yy}' for the above I – section about yy is given by,

$$I_{yy} = \frac{1}{12}\left(bD^3 + Bd^3\right)$$

Substituting the values in I_{xx},

$$I_{xx} = \frac{1}{12}\left(4t \times (5t)^3 - 3t(3t)^3\right)$$

$$= \frac{1}{12}\left(\left(4t \times 125t^3\right) - \left(3t \times 27t^3\right)\right)$$

$$= \frac{1}{12}\left((500 - 81)t^4\right) = 34.91t^4$$

$$I_{yy} = \frac{1}{12}\left(2t(4t)^3 + 3t\left(t^3\right)\right)$$

$$= \frac{1}{12}\left(128t^4 + 3t^4\right) = \frac{131}{12}t^4 = 10.91t^4$$

$\therefore$ Ratio of I_{xx} to I_{yy}

=> $\frac{I_{xx}}{I_{yy}} = \frac{34.19t^4}{10.91t^4} = 3.197 \cong 4$

$$\therefore I_{xx} \cong 4I_{yy}$$

<u>Area of cross section</u>

A = (5t x 4t) – (3t x 3t)

= $20t^2 - 9t^2$

= $11\ t^2$

To find 't'

By using **Rankine – Gordon** formula,

The stress due to axial load (Crippling load or buckling load)

$$f_{cr} = \frac{F_c}{A} = \frac{f_c}{1 + K\left(\frac{l}{k}\right)^2}$$ Eq. 19.5 Pg. 369

K = Constant = 4/25000 for steel rod pin connected at both ends rod is free to bend in any plane

l = length of connecting rod = 325 mm

K_{xx} = radius of gyration

$$= \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{34.91t^4}{11t^2}} = \sqrt{3.17t^2}$$

K_{xx} = 1.78t

$$f_{cr} = \frac{F_c}{A}$$

$$also, \frac{F_c}{A} = \frac{f_c}{1 + K\left(l/k\right)^2}$$

$$\therefore F_c = \frac{f_c}{\left(1 + K\left(l/2\right)^2\right)}$$

Also f_c = Allowable unit stress for designing MN/m^2

= Yield point stress / FOS (assume) = 378/4

= 94.5 MPa

Yield point stress from table

T 19.1 pg. 371

$$19203.3 = \frac{94.5 \times 11t^2}{1 + \frac{4}{25000}\left(\frac{325}{1.78t}\right)^2}$$

$$19203.3 = \frac{1039.5t^2}{1 + \frac{5.33}{t^2}} = \frac{1039.5t^2}{\left(\frac{t^2 + 5.33}{t^2}\right)}$$

$$19203.3 = \frac{1039.5t^4}{\left(t^2 + 5.33\right)}$$

$$19203.3t^2 + 1022535 = 1039.5t^4$$

$$1039.5t^4 - 19203.3t^2 - 102253.5 = 0$$

$$t^4 - 18.47t^2 - 98.37 = 0\left(ax^2 + bx + c = 0\right)$$

w.k.t.

roots, i.e. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{18.47 \pm \sqrt{18.47 + \left(4 \times 1 \times 98.37\right)}}{2 \times 1}$

x => $t^2 = \frac{18.47 \pm \sqrt{18.47 \pm \left(4 \times 1 \times 98.37\right)}}{2} = 19.38$

$$\therefore t = \sqrt{19.38}$$

$$= 4.78 \cong 5mm$$

Take t = 10 mm

<u>Note the dimensions</u>

$\therefore$ width = 4t = 40 mm

Depth = 5t = 50 mm

Flange and web thickness = 10 mm

Area = 11 t^2 = 11 x 10^2 = 1100 mm^2

Check for stresses.

From equation 19.1   Pg. 369

The magnitude of inertia force (F_i)   (Max. force in the crank pin) x L/2

i.e. Resultant normal force on the CP

$$= \left(\frac{1}{2} \times F_{\max} \times R\right)$$

$$F_i = \frac{W \times A}{g \times 2} w^2 \times r \times l \times 10^{-12}$$

$$= \frac{1}{2} \times \frac{W}{g} \times A \times w^2 \times r \times l \times 10^{-12}$$

=> W = weight per unit volume of rod in N/m^3

= density in Kg/m^3

= 7800 Kg/m^3 Assume

r = Crank radius mm = 75 mm

l = length of connecting rod = 325 mm

A = Area of cross section = 1100 mm^2

w = Angular speed = 157.1 rad/sec

g = Acceleration due to gravity = 9.81 m/s

Substituting the values,

$$F_i = \frac{7800 \times 9.81 \times 1100 \times (157.1)^2 \times 75 \times 325 \times 10^{-12}}{9.81 \times 2}$$

$$F_i = 2580.8N$$

From equation 19.2 Pg. 369

The max. bending moment (M_{max})

$$M_{\max} = \frac{2F_i \times l}{9\sqrt{3}}$$

$$= \frac{2 \times 2580.8 \times 325}{9\sqrt{3}}$$

$$= 107612.95N - mm$$

$\therefore$ The maximum inertia bending stress or whipping stress, (MPa) N/mm^2

$$\sigma_b = \frac{M_{\max}}{Z}$$

From equation 19.3, Pg. 369

$$\sigma_b = \frac{0.2854 \times 10^{-12} \times n^2 \times r \times A \times w \times l^2}{Z} \qquad \text{Equation}$$

W.K.T. $Z = \dfrac{I}{y}$ $\quad y = \dfrac{depth}{2} = \dfrac{5t}{2} = 25mm$

$$= \frac{349.1 \times 10^3}{25} \qquad I = I_{xx} = 34.91t^4 = 349.1 \times 10^3 \text{ mm}^4$$

= 13964 mm^3

n = Speed of crank = $\dfrac{2200}{60} = 36.67$ rev/sec

r = Crank radius = 75 mm

A = Area in mm^2 = 1100 mm^2

w = density of rod material = 7800 x 9.81 N/m^3

l = length of the rod = 325 mm

$$\sigma_b = \frac{0.2854 \times 10^{-12} \times 36.67^2 \times 75 \times 1100 \times 7800 \times 9.81 \times 325^2}{13964}$$

$$= \frac{255894}{13964} = 18.32MPa$$

Which is Safe

Maximum compressive stress in the connecting rod

i.e. = stress due to axial load + The max. Inertia bending stress or whipping stress

$$= f_{cr} + \sigma_b$$

$$= \frac{F_c}{A} + 18.32MPa$$

$$= \frac{19203.3}{1100} + 18.32$$

$$= 35.78MPa$$

Which is safe.

Design of small end

Force on piston $F_p = l_p \, d_p \, P_b$

$F_p = 23758.3$ N

l_p = length of piston pin

d_p = dia of piston pin

P_b = bearing pressure

$23758.3 = 1.5 \, d_p \times d_p \times 10$

$\therefore d_p^2 = 1583.9$

$d_p = 39.79 \cong 40$ mm

$\therefore l_p = 1.5 \, d_p = 60$ mm

From Pg. 362

$l_1 = K_1 d$

d = dia of piston pin

$K_1 = 1.5$ for petrol and gas ends

$\therefore l_p = 1.5 \, d_p$ from Pg. 362

$P_b = 12.4$ for gas engine

15.0 for oil engine

15.7 for automotive engine

Here we take,

$P_b = 10$ MPa from Pg. 362

Design of Big end

$F_p = l_c \, d_c \times P_b$

From Pg. 45

Equation 3.17,

$P_b = 7.5$ MPa assume

Also assume, $l_c = 1.25 \, d_c$

Where,

d_c = diameter of crank pin

l_c = length of crank pin

P_b = bearing pressure

F_p = force on the piston

= 23758.3 N

$23758.3 = 1.25 \, d_c \times d_c \; 7.5$

$\therefore d_c = 50$ mm

$l_c = 62.5$ mm

Design of Big end Bolts

Magnitude of Inertia force F_i = $2\left(\frac{\pi}{4}d_b^{\,2}\right)\sigma_t$

$F_i = 4555$ N

d_b = dia of the bolts

σ_t = tension stress assume as 12 MPa

As this inertia force is supported by 2 bolts which hold the big end side,

$\therefore 4555 = 2\times\frac{\pi}{4}\times d_b^{\,2}\times 12$

$\therefore d_b = 15.50mm$

$\therefore$ Use M20 sized bolts.

Design of big end cap

$$M_{\max} = \frac{F_i l_o}{6}$$

F_i = Magnitude inertia force = 4555 N

$l_o = d_c$ + (2 x thickness of liner) + d_b + Clearance (say 1.5 mm)

= 50+2x(0.05x50+1) + 21.5

= 50 + 7 + 21.5

= 78.5 mm

M_{max} = 78.5 mm

$$M_{\max} = \frac{4555\times 78.5}{6} = 59595 \text{ N-mm}$$

To find Cap thickness

w.k.t., $\sigma_b = \frac{M_{\max}}{Z}$

$$Z = \frac{l_c\times h^2}{6}$$

Z = Section modulus

l_c = length of crankpin = 62.5 mm

Assume σ_b = 120 MPa

Substituting,

$$120 = \frac{59595}{\frac{62.5 \times h^2}{6}} = \frac{5721}{h^2}$$

$$\therefore h = \sqrt{\frac{5721}{120}} = 6.9 \cong 7 \text{ mm}$$

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