MEDJEE

DIGITEXT | DEXTERITY | DEVELOPMENT

XII- STD | WT -1 | JEE - KEYS & HINTS

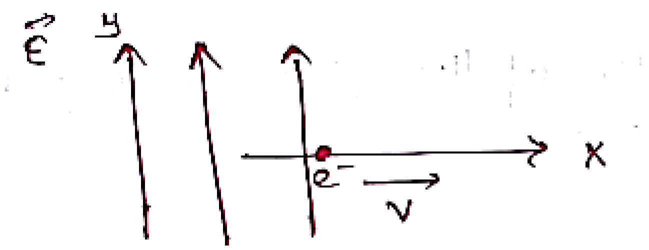
PHYSICS				CHEMISTRY				MATHS			
1	c	**14**	b	**26**	d	**39**	c	**51**	c	**64**	b
2	b	**15**	c	**27**	b	**40**	d	**52**	b	**65**	c
3	a	**16**	b	**28**	c	**41**	c	**53**	b	**66**	b
4	b	**17**	d	**29**	d	**42**	d	**54**	d	**67**	b
5	a	**18**	a	**30**	a	**43**	d	**55**	a	**68**	d
6	b	**19**	d	**31**	d	**44**	a	**56**	a	**69**	d
7	c	**20**	a	**32**	a	**45**	d	**57**	d	**70**	c
8	c	**21**	2	**33**	a	**46**	20	**58**	a	**71**	99
9	c	**22**	5	**34**	b	**47**	6	**59**	c	**72**	5
10	c	**23**	2	**35**	b	**48**	1	**60**	c	**73**	2
11	a	**24**	17	**36**	c	**49**	3	**61**	b	**74**	7
12	d	**25**	3	**37**	b	**50**	50	**62**	b	**75**	-3
13	b			**38**	a			**63**	b		

HINTS

1. The charge on disc A is 10^{-6} C. The charge on disc B is 10×10^{-6} C. The total charge on both $= 11 \times 10^{-6}$ C. When touched, this charge will be distributed equally, i.e., 5.5×10^{-6} C or 5.5 ∝C on each disc.

2. The force on the electron in a electric field is always in the direction opposite to the direction of E.F.

$\therefore$ It will experience force in negative Y –axis.



3.

The force on the charge Q is $\vec{F} = Q\vec{E}$

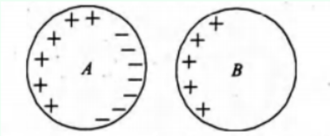
The work done ,

$$W = \vec{F}.\vec{r} = Q\vec{E}.\vec{r} = Q(E_1\hat{i} + E_2\hat{j}).(a\hat{i} + b\hat{j}) = Q(E_1a + E_2b)$$

4.

1. In the given figure, we see when the first charge is very large as compared to the second, we find the positive charges are displaced at the opposite end and negative charges are concentrated on the end nearer to the first charge.
2. As negative charges come nearer due to electrostatic induction there occurs attraction between the charges.

Statement -1 is false, Statement -2 is true.



Although both A and B are positively charged, it is possible that due to neighbouring induced charges of opposite nature, attractions dominates repulsions.

5.

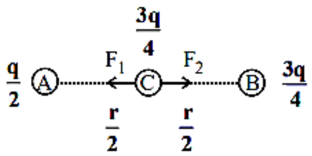
- **Statement 1:** Electric charge is additive in nature.
  True - The net charge of a system is the sum of all charges present, taking into account their signs.
- **Statement 2:** The total charge of a system is the algebraic sum of individual charges.
  True - This explains that charges add algebraically (considering signs) to give the total charge.
- **Relationship:** Statement 2 correctly explains Statement 1 because it describes the additivity of charges, which is the property mentioned in Statement 1.

Correct Option: (a) Both Statement 1 and Statement 2 are true, and Statement 2 is the correct explanation of Statement 1.

6.

C will be $= \dfrac{q + \frac{q}{2}}{2} = \dfrac{3q}{4}$

Now,



$$F' = F_2 - F_1 = \frac{\left(K\frac{3q}{4} - K\frac{q}{2}\right)}{\frac{r^2}{4}} . \frac{3q}{4}$$

$$= \frac{3Kq^2}{4r^2} = \frac{3F}{4}$$

7.

Solve for n

(c) q = ne

$80 \times 10^{-6} = n \times 1.6 \times 10^{-19}$

$$n = \frac{80 \times 10^{-6}}{1.6 \times 10^{-19}}$$

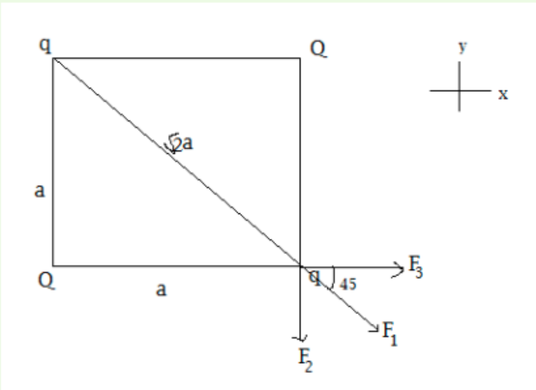
$$n = \frac{80}{1.6} \times 10^{13}$$

$$n = 50 \times 10^{13}$$

$$n = 5 \times 10^{14}$$

8.

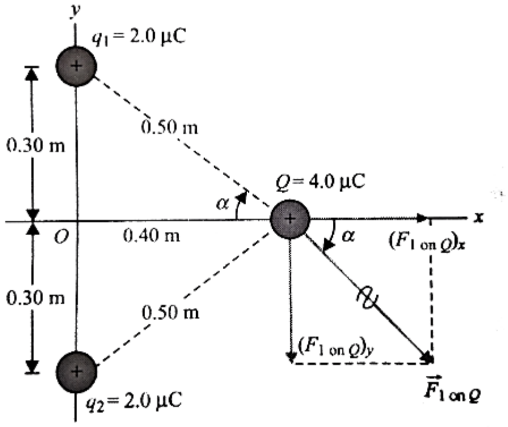
c)

	$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$ This shows that the electric field **varies inversely with** r: $$E \propto \frac{1}{r}$$
9.	The force depends on both the magnitude of the negative charge and its position relative to the positive charges. If the negative charge is closer to one of the positive charges, it will experience a stronger force from that charge, affecting the net force direction. Therefore, both the **magnitude and position** of the negative charge influence the force experienced. **Correct answer:** **(c) Both the magnitude and position of the negative charge.**
10.	The correct option is **C** $\frac{1}{4\pi\epsilon}\frac{\sqrt{3}}{4}\left(\frac{q}{R}\right)^2$ $F_{net} = \frac{Kq^2}{(2R)^2}.2\cos\theta \frac{Kq^2}{4R^2}.2\frac{R\sqrt{3}}{2R}$ $\frac{\sqrt{3}Kq^2}{4R^2}$ $\frac{1}{4\pi\varepsilon_0}\frac{\sqrt{3}}{4}\left(\frac{q}{R}\right)^2$
11.	Each charge experiences two forces inclined at $60°$. Therefore, $$R = \sqrt{(2F)^2 + (2F)^2 + 2\times 2F\times 2F\cos 60^\circ}$$ $$= 2\sqrt{3F^2}$$ $$= 2\sqrt{3}F$$
12.	The net force on q at one corner is zero if $\vec{F_1} + \vec{F_2} + \vec{F_3} = 0$ or $F_1\cos 45^\circ\hat{i} - F_1\sin 45^\circ\hat{j} - F_2\hat{j} + F_3\hat{i} = 0$ so, $F_1\cos 45^\circ = -F_3 \ldots (1)$ and $F_1 \sin 45^\circ = -F_2 \ldots (2)$ using (1), $\frac{kq^2}{(\sqrt{2}a)^2}\times\frac{1}{\sqrt{2}} = -\frac{kQ}{a^2}$ or $q = -2\sqrt{2}Q$ 

13

$F_{net} = 2|F_{31}|\cos\alpha$

$= 2 \times \frac{1}{4\pi\epsilon_0} \times \frac{2 \times 4 \times 10^{-12}}{(0.5)^2} \times \frac{4}{5} = 0.46N$



14.

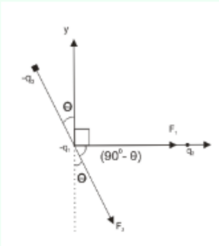
Force on ($-q_1$) due to $q_2 = \frac{q_1 q_2}{4\pi\epsilon_0 b^2}$

$\therefore F_1 = \frac{q_1 q_2}{4\pi\epsilon_0 b^2}$ along $(q_1 q_2)$

Force on $(-q_1)$ due to $(-q_3) = \frac{(q_1)(q_3)}{4\pi\varepsilon_0 a^2}$

$F_2 = \frac{q_1 q_3}{4\pi\epsilon_0 a^2}$ as shown

F_2 > makes an angle of $F_2 >$ makes an angle of $(90^\circ - \theta)$ with $(q_1 q_2)$



Resolved part of F_2 along $q_1 q_2$

$= F_2 cos\left((90)^0 - \theta\right)$

$= \frac{q_1 q_3 \sin\theta}{4\pi\epsilon_0 a^2}$ along $(q_1 q_2)$

$\therefore$ Total force on $(-q_1)$

$= [\ \frac{q_1 q_2}{4\pi\epsilon_0 b^2} + \frac{q_1 q_3 \sin\theta}{4\pi\epsilon_0 a^2}\ ]$ along x-axis

$\therefore$ x-component of force $\propto [\ \frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta\ ]$.

15.

As electric field due to charge q at distance r from r_o

$$\vec{E} = \frac{1}{4\pi\varepsilon_o|\vec{r} - \vec{r}_o|^3}\frac{q}{}(\vec{r} - \vec{r}_o)$$

$\vec{MN} = \vec{r} - \vec{r}_o = (8\hat{i} - 5\hat{j}) - (2\hat{i} + 3\hat{j}) = (6\hat{i} - 8\hat{j})$

$|\vec{r} - \vec{r}_o| = \sqrt{6^2 + 8^2} = 10m$

$$\vec{E} = 9 \times 10^9 \times \frac{50 \times 10^{-6}}{(10)^3}(6\hat{i} - 8\hat{j})$$

$= (2.7\hat{i} - 3.6\hat{j})kNC^{-1}$

16.	The electric field vector at due to (2) $\overrightarrow{\mathrm{E}}_2 = \frac{\mathrm{k}\lambda}{\mathrm{a}}(\hat{\mathrm{i}} + \hat{\mathrm{k}})$ due to (3) $\vec{E}_3 = \frac{k\lambda}{a}(\hat{i} + \hat{j})$ (1) $\vec{E}_1 = \frac{k\lambda}{a}(\hat{j} + \hat{k})$ So $\vec{E}_P = \frac{2k\lambda}{a}(\hat{i} + \hat{j} + \hat{k})$
17.	Correct Answer - d Electric field at a point on z-axis distant r from origin is $E = \frac{1}{4\pi\varepsilon_0}\left(\frac{Qr}{(r^2+R^2)^{3/2}} - \frac{\sqrt{8}Qr}{(r^2+4R^2)^{3/2}}\right) = 0$ Solving we get $r = \sqrt{2}R$
18.	The cavity in the sphere can be replaced by superposition of a sphere with density $+\rho$ and a sphere of density $-\rho$. Thus, the electric field at B due to the large sphere of density $+\rho$ is $E_1 = \frac{1}{4\pi\epsilon_o}\frac{Q_1}{r_o^2}$ towards left and $Q_1 = \frac{4}{3}\pi r_o^3\rho$ Thus, $E_1 = \frac{1}{4\pi\epsilon_o r_o^2}\frac{4\pi r_o^3\rho}{3} = \frac{\rho r_o}{3\epsilon_o}$ towards left. Similarly, the field at B due to negatively charged sphere is $E_2 = \frac{1}{4\pi\epsilon_o}\frac{Q_2}{(3r_o/2)^2}$ towards right and $Q_2 = \frac{4}{3}\pi\left(\frac{r_o}{2}\right)^3\rho$ Thus, $E_2 = \frac{1}{9\pi\epsilon_o r_o^2}\frac{\pi r_o^3\rho}{6} = \frac{\rho r_o}{54\epsilon_o}$ towards right. Thus, the net electric field at B is $\lvert E_1\rvert - \lvert E_2\rvert = \frac{\rho r_o}{3\epsilon_o} - \frac{\rho r_o}{54\epsilon_o} = \frac{17\rho r_o}{54\epsilon_o}$ leftwards.
19.	1. **Inside the sphere** ($r < R$): • The electric field increases linearly with r, following $E \propto r$. • This is because the field inside a uniformly charged sphere is given by $$E = \frac{Q}{4\pi\epsilon_0 R^3}r$$ which is directly proportional to r. 2. **Outside the sphere** ($r > R$): • The sphere behaves like a point charge with field decreasing as $E \propto \frac{1}{r^2}$. • This follows Coulomb's law: $$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$ From the given options, **option (d) is correct** because:

20.	**Explanation:** • (A) Charges at the corners of a square create forces that are balanced along the diagonals due to symmetry. • (B) Different magnitudes and signs determine the strength and direction of force between charges on a line. • (C) The net force is calculated as the vector sum of all forces acting on a particular charge. • (D) Symmetrical placement of charges ensures that the forces cancel each other, resulting in zero net force.

21.

$$dV = 4\pi r^2 dr$$

$$Q = \int_0^R kr^a(4\pi r^2)dr$$

$$Q = 4\pi k \int_0^R r^{a+2} dr$$

$$Q = 4\pi k \left[\frac{r^{a+3}}{a+3}\right]_0^R$$

$$Q = \frac{4\pi k R^{a+3}}{a+3}$$

ıe charge enclosed within a sphere of radius r is:

$$Q_r = 4\pi k \int_0^r r'^{a+2} dr'$$

$$Q_r = 4\pi k \left[\frac{r'^{a+3}}{a+3}\right]_0^r$$

$$Q_r = \frac{4\pi k r^{a+3}}{a+3}$$

At $r = R$:

$$E_R = \frac{kR^{a+1}}{\varepsilon_0(a+3)}$$

At $r = \frac{R}{2}$:

$$E_{R/2} = \frac{k(R/2)^{a+1}}{\varepsilon_0(a+3)}$$

$$E_{R/2} = \frac{kR^{a+1}}{\varepsilon_0(a+3)} \cdot \frac{1}{2^{a+1}}$$

Given that $E_{R/2} = \frac{1}{8}E_R$:

$$\frac{kR^{a+1}}{\varepsilon_0(a+3)} \cdot \frac{1}{2^{a+1}} = \frac{1}{8} \times \frac{kR^{a+1}}{\varepsilon_0(a+3)}$$

$$\frac{1}{2^{a+1}} = \frac{1}{8}$$

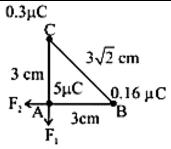
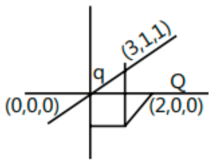
$$2^{a+1} = 8$$

$$2^{a+1} = 2^3$$

$$a + 1 = 3$$

$$a = 2$$

22.	Here, $q = \pm 10\mu C = \pm 10^{-5} C$ $2a = 5 \cdot 0mm = 5 \times 10^{3} m$ $r = OP = 15cm = 15 \times 10^{-2} m$ $\lvert p \rvert = q \times 2a = 10^{-5} \times 5 \times 10^{-3}$ $5 \times 10^{-8} C - m$ (a) As P lies on axial line of diople. $\therefore = E_1 = \frac{2\lvert\vec{p}\rvert r}{4\pi \in_0 (r^2 - a^2)^2}$, along BP $Asa < < r, \therefore$ $\frac{2\lvert\vec{p}\rvert r}{4\pi \in_0 r^3} = \frac{2 \times 5 \times 10^{-8} \times 9 \times 10^{9}}{(15 \times 10^{-2})^3}$ $2 \cdot 67 \times 10^{5} N/C$, along BP (b) As Q lies on equatorail line of dipole, $\therefore E_2 = \frac{\lvert\vec{p}\rvert}{4\pi \in_0 (r^2 + a^2)^{3/2}} = \frac{\lvert\vec{p}\rvert}{4\pi \in_0 r^3}$ $(\because a < < r)$ $\frac{1}{2} E_1 = \frac{1}{2} \times 2 \cdot 67 \times 10^{5} = 1 \cdot 33 \times 10^{5} N/C$
23.	**Charge Redistribution:** • After touching A, sphere C gets charge $Q_C = \frac{Q}{2}$. • Charge on A reduces to $Q_A = \frac{Q}{2}$. **Forces on C:** 1. **Due to A** (distance $d/2$): $$F_{CA} = \frac{kQ_C Q_A}{(d/2)^2} = 2 \times 10^{-5}\ \text{N (away from A)}$$ 2. **Due to B** (distance $d/2$): $$F_{CB} = \frac{kQ_C Q_B}{(d/2)^2} = 4 \times 10^{-5}\ \text{N (away from B)}$$ **Net Force on C:** $$F_{\text{net}} = F_{CB} - F_{CA} = 4 \times 10^{-5} - 2 \times 10^{-5} = 2 \times 10^{-5}\ \text{N (away from B)}$$ **Final Answer:** $$\boxed{2}$$ (Net force on C is 2×10^{-5} N, away from sphere B.)

24	 $F_1 = \frac{k \times 5 \times 0.3 \times 10^{-12}}{9 \times 10^{-4}}$ $= \frac{9 \times 10^{9} \times 5 \times 0.3 \times 10^{-12}}{9 \times 10^{-4}}$ $= 1.5 \times 10 = 15$ N $F_2 = \frac{9 \times 10^{9} \times 5 \times 0.16 \times 10^{-12}}{9 \times 10^{-4}} = 8N$ force experienced by charge at $A = \sqrt{F_1^2 + F_2^2}$ $= \sqrt{15^2 + 8^2}$ $= \sqrt{289} = 17$ N
25	**3**  $\vec{E}_q = \frac{kQ}{\sqrt{(3^2+1^2+1^2)}}\left(\frac{3\hat{i}+\hat{j}+\hat{k}}{\sqrt{11}}\right)$ $\vec{E}_Q = \frac{kQ}{\left(\sqrt{1^2+1^2+1^2}\right)}\left(\frac{\hat{i}-\hat{j}-\hat{k}}{\sqrt{3}}\right)$ At P x-Component of field is zero $\Rightarrow (\vec{E}_P + \vec{E}_Q)_x = 0 \Rightarrow \frac{3kq}{\left(\sqrt{11}\right)^3} = \frac{-QK}{\left(\sqrt{3}\right)^3}$ $\Rightarrow Q = -3\left(\sqrt{\frac{3}{11}}\right)^3 \times 10^{-9}$ C y-component has zero field.
26	$p = p^0 x_A$ $32 = 40 \times x_A$ or $x_A = 0.8]$
27	$x_A = \frac{2}{5}, x_B = \frac{3}{5}$ $p = p_A^0 x_A + p_B^0 x_B$ $= 100 \times \frac{2}{5} + 150 \times \frac{3}{5} = 40 + 90 = 130]$
28	$m = \frac{M \times 1000}{1000d - Mm_B} = \frac{2.05 \times 1000}{1000 \times 1.02 - 2.05 \times 60}$ $= 2.28$ mol kg^{-1}

29	$N_{\text{HCl}} = \dfrac{w_B \times 1000}{E_B \times V} = \dfrac{0.04 \times 1000}{36.5 \times 1} = 1.095$ $N_{\text{NaOH}} \equiv N_{\text{HCl}}$ $1.095 = \dfrac{w_B \times 1000}{40 \times 1}$ $w_B = 0.0438\ \text{g / mL}$
30	Strength of the solution $= \text{Molarity} \times \text{mol. mass}$ $= 2.03 \times 60 = 121.8\ \text{g / L}$ Density of solution $= 1.017\ \text{g / mL}$ Mass of 1 litre of solution $= 1000\ \text{mL} \times 1.017\ \text{g / mL}$ $= 1017\ \text{g}$ Mass of water $= 1017 - 121.8 = 895.2\ \text{g} = \dfrac{895.2}{1000}\ \text{kg}$ Molality $= \dfrac{2.03}{895.2} \times 1000 = 2.267\ m$
31	$M = \dfrac{x \times d \times 10}{m_B}$ $d = \dfrac{M \times m_B}{x \times 10} = \dfrac{3.6 \times 98}{29 \times 10} \approx 1.22\ \text{g mL}^{-1}$]
32	$n_A = \dfrac{w_A}{m_A} = \dfrac{25}{100} = 0.25; \quad n_B = \dfrac{35}{114} = 0.3$ $x_A = \dfrac{0.25}{0.25 + 0.30}; \quad x_B = \dfrac{0.3}{0.25 + 0.30}$ $= 0.45 \quad\quad = 0.55$ $p = p_A^0 x_A + p_B^0 x_B$ $= 105.2 \times 0.45 + 46.8 \times 0.55$ $= 47.34 + 25.74 = 73.08\ \text{kPa}$
33	$\Delta V_{mix} = 0$, hence the solution is ideal.
34	P_{total} = Mole fraction of $A \times p_A^0$ + Mole fraction of $B \times p_B^0$ No. of moles of $A = \dfrac{28}{140} = 0.2$ Liquid B is water. Its mass is $(100 - 28)$, *i.e.*, 72. No. of moles of $B = \dfrac{72}{18} = 4.0$ Total no. of moles $= 0.2 + 4.0 = 4.2$ Given, $P_{\text{total}} = 160\ \text{mm}$ $p_B^0 = 150\ \text{mm}$ So, $160 = \dfrac{0.2}{4.2} \times p_A^0 + \dfrac{4.0}{4.2} \times 150$ $p_A^0 = \dfrac{17.15 \times 4.2}{0.2} = 360.15\ \text{mm}$

35	[Hint: 10^6 g water contains (0.002×1000) mol $MgSO_4$ 1 mol $MgSO_4 \cong$ 1 mol $CaCO_3$ $\therefore$ 2 mol $MgSO_4 \cong$ 2 mol $CaCO_3$, *i.e.*, 2×100 g $CaCO_3$ $\therefore$ Hardness of water = 200 ppm]
36	$P_{Total} = 120 - 75X_B = P_A^oX_A + P_B^oX_B$ But $X_A = 1 - X_B$ Hence, $P_{Total} = 120 - 75X_B = P_A^o(1 - X_B) + P_B^oX_B = P_A^o - (P_A^o - P_B^o)X_B$ Hence, $P_A^o = 120$ and $P_B^o = 120 - 75 = 45$
37	[Hint: $(\Delta p)_{glucose} = (\Delta p)_{urea}$ $(x_B)_{glucose} = (x_B)_{urea}$ *i.e.*, $\left(\frac{n_B}{n_A}\right)_{glucose} = \left(\frac{n_B}{n_A}\right)_{urea}$ $\frac{w_B}{50} \times \frac{18}{180} = \frac{1 \times 18}{50 \times 60}$ $w_B = 3$ g]
38	$\text{Volume of solution} = \frac{\text{Total mass}}{\text{density}} = \frac{32x + 18y}{0.994}$ mL $= \frac{32x + 18y}{0.994 \times 1000}$ litre $= \frac{32x + 18y}{994}$ litre $\text{Molarity} = \frac{x}{32x + 18y} \times 994$ $= \frac{994}{32 + 18 \times y/x} = \frac{994}{32 + 18 \times 49} = 1.0875\,M$
39	We are to prepare 0 · 4M NaCl solution and we are starting with 100 ml, 0 · 3M NaCl solution. So, we only need the moles corresponding to the difference in molarity of the two solutions. Required moles = $(0.4 - 0.3) \times 0.1 = 0.01$
40	$N_a = M_a \times$ basicity = 1x2=2 $M_b = N_b = 1M$ $V_aN_a = V_bN_b$ $V_b = 10mL$ $V_a = \frac{10}{2} = 5mL$

<table>
<tr><td>41</td><td>
$$\text{Mol. mass of ethyl alcohol} = C_2H_5OH = 46$$
$$\text{No. of moles of ethyl alcohol} = \frac{60}{46} = 1.304$$
$$\text{Mol. mass of methyl alcohol} = CH_3OH = 32$$
$$\text{No. of moles of methyl alcohol} = \frac{40}{32} = 1.25$$
$$\text{'}X_A\text{', mole fraction of ethyl alcohol} = \frac{1.304}{1.304 + 1.25} = 0.5107$$
$$\text{'}X_B\text{', mole fraction of methyl alcohol} = \frac{1.25}{1.304 + 1.25} = 0.4893$$
$$\text{Partial pressure of ethyl alcohol} = X_A \cdot p_A^0 = 0.5107 \times 44.5 = 22.73 \text{ mm Hg}$$
$$\text{Partial pressure of methyl alcohol} = X_B \cdot p_B^0 = 0.4893 \times 88.7 = 43.40 \text{ mm Hg}$$
$$\text{Total vapour pressure of solution} = 22.73 + 43.40 = 66.13 \text{ mm Hg}$$
</td></tr>
<tr><td>42</td><td>
$$\text{Normality} = \frac{\text{number of equivalent of solute}}{\text{1 liter of solution}}$$
If we solution is diluted, means volume is changed.

Hence normality of solute will also change.
</td></tr>
<tr><td>43</td><td>
The density of NH_4OH solution is 0.6 g/mL.
Thus, 1 L of solution contains 600 g of NH_4OH.
It contains 34% by weight of NH_4OH.
Hence, the mass of NH_4OH present in 1 L is 600 × 0.34 = 204g
The molar mass of NH_4OH is 35 g/mol.
Hence, 204 g of NH_4OH coresponds to $\frac{204}{35} = 5.8$mol
Thus, the normality of the solution is 5.8 N
</td></tr>
</table>

44	Molar mass of $CH_2Cl_2 = 12\times1 + 1\times2 + 35.5\times2$ $= 85\ g\ mol^{-1}$ Molar mass of $CHCl_3 = 12\times1 + 1\times1 + 35.5\times3$ $= 119.5\ g\ mol^{-1}$ Moles of $CH_2Cl_2 = \frac{40\ g}{85\ g\ mol^{-1}} = 0.47\ mol$ Moles of $CHCl_3 = \frac{25.5\ g}{119.5\ g\ mol^{-1}} = 0.213\ mol$ Total number of moles $= 0.47 + 0.213 = 0.683\ mol$ $\chi_{CH_2Cl_2} = \frac{0.47\ mol}{0.683\ mol} = 0.688$ $\chi_{CHCl_3} = 1.00 - 0.688 = 0.312$ $p_{total} = p^{\circ}_{CHCl_3} + (p^{\circ}_{CH_2Cl_2} - p^{\circ}_{CHCl_3})\ \chi_{CH_2CH_2}$ $= 200 + (415 - 200) \times 0.688$ $= 347.9\ mmHg$ To calculate the mole fraction of component in vapour phase, $\chi_i^V = p_i / p_{total}$ $\therefore$ $p_{CH_2Cl_2} = 0.688 \times 415\ mmHg = 285.5\ mmHg$ $p_{CHCl_3} = 0.312 \times 200\ mmHg = 62.4\ mmHg$ $\chi^V_{CH_2Cl_2} = \frac{285.5}{347.9} = 0.82$ and $\chi^V_{CHCl_3} = \frac{62.4}{347.9} = 0.18$
45	$M = \frac{1000 \times d}{(1000/m) + M'}$ M → molarity. m → molality d → density of solution. M' → Molecular weight of solute. $M = \frac{1000 \times 1.02}{(1000/1) + 40}$ $= \frac{1020}{1040}$ $= 0.98$

46	$N_1V_1 = N_2V_2$ Normality = Molarity × Basicity For sulphuric acid, the basicity is 2. $\therefore N_1 = 0.1 \times 2 = 0.2N$ $V_1 = ?$ $N_2 = 0.1 \times 1 = 0.1N$ $V_2 = 40ml$ $0.2 \times V_1 = 0.1 \times 40$ $V_1 = 20ml$
47	$P_{Total} = \frac{2\times500\times750}{500+750} = \frac{7.5\times10^5}{1.25\times10^3} = 6\times10^2$ mm
48	Equivalent weight of dibasic acid $= \frac{\text{mol, wt.}}{2} = \frac{200}{2} = 100$ Strength = 0.1 N, m = ?, V= 100 ml. From, $m = \frac{E\ V\ N}{1000} = \frac{100 \times 100 \times 0.1}{1000} = 1\,g$
49	(m_1) initial molality $= \frac{\frac{1}{2}}{1-\frac{1}{2}} \times \frac{1000}{M_{solvent}}$ (m_2) final molality $= \frac{\frac{1}{4}}{1-\frac{1}{4}} \times \frac{1000}{M_{solvent}}$ $\frac{m_1}{m_2} = \frac{1}{\frac{1}{3}} = 3$ $m_1 = 3m_2$
50	Oxalic acid, $H_2C_2O_4$, has 2 acidic hydrogen present. Normality =Molarity × acidic hydrogen No of equivalent = N × V $\Rightarrow (M \times 2) \times V$ $\Rightarrow 0.1 \times 2 \times \frac{250}{1000}$ $= \frac{50}{1000}$ No of milli equivalent = 50

51	$\because \quad A^2 = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix}$ and $\quad 8A + kI_2 = 8\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} + k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $= \begin{bmatrix} 8+k & 0 \\ -8 & 56+k \end{bmatrix}$ $\therefore \quad A^2 = 8A + kI_2$ $\therefore \quad \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} = \begin{bmatrix} 8+k & 0 \\ -8 & 56+k \end{bmatrix}$ On comparing, we get $8 + k = 1 \Rightarrow k = -7.$
52	Let $C = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}, D = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$ $DC = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$ $B = CAD$ $B^n = \underbrace{(CAD)(CAD)(CAD)....(CAD)}_{n-\text{times}}$ $\Rightarrow B^n = CA^nD \qquad ....(1)$ $A^2 = \begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{2}{51} \\ 0 & 1 \end{bmatrix}$ $A^3 = \begin{bmatrix} 1 & \frac{3}{51} \\ 0 & 1 \end{bmatrix}$ similarly $A^n = \begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix}$

	$B^n = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$ $= \begin{bmatrix} 1 & \frac{n}{51}+2 \\ -1 & -\frac{n}{51}-1 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$ $= \begin{bmatrix} \frac{n}{51}+1 & \frac{n}{51} \\ -\frac{n}{51} & 1-\frac{n}{51} \end{bmatrix}$ $\sum_{n=1}^{50} B^n = \begin{bmatrix} 25+50 & 25 \\ -25 & -25+50 \end{bmatrix}$ $= \begin{bmatrix} 75 & 25 \\ -25 & 25 \end{bmatrix}$ Sum of the elements = 100
53	$A` = A \Rightarrow \begin{bmatrix} 4 & 2x-3 \\ x+2 & x+1 \end{bmatrix} = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ On comparing, we get $2x - 3 = x + 2 \Rightarrow x = 5$.
54	a unit matrix $A^2 = AA$ $\Rightarrow A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$ $\Rightarrow A^2 = \begin{bmatrix} 1+0+0 & 0+0+0 & 0+0-0 \\ 0+0+0 & 0+1+0 & 0+0-0 \\ a+0-a & 0+b-b & 0+0+1 \end{bmatrix}$ $\Rightarrow A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
55	$\therefore$ A2 = 2A – I $A^3 = A^2 . A = 2A^2 - IA = 2A^2 - A = 2(2A - I) - A$

	$= 3A - 2I = 3A - (3-1)I$ $A^n = nA - (n-1)I$.
56	If A and B are square matrices of equal degree, then A + B = B + A
57	*(d)* We have, $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix}$ Then, $A^2 = A \cdot A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix}$ $= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} = A$ $\therefore \quad A^3 = A^2 \cdot A = A \cdot A = A^2 = A$ Similarly, $A^4 = A^5 = ... = A^{11} = A$. Now, $(A+I)^{11} = {}^{11}C_0 A^{11} + {}^{11}C_1 A^{10} I + {}^{11}C_2 A^9 I^2 + ... + {}^{11}C_{11} A^0 I^{11}$ $= {}^{11}C_0 A + {}^{11}C_1 A + {}^{11}C_2 A + ... + {}^{11}C_{10} A + {}^{11}C_{11} I$ $= A({}^{11}C_0 + {}^{11}C_1 + {}^{11}C_2 + ... + {}^{11}C_{10}) + I$ $= A(2^{11} - 1) + I$ $= (2^{11} - 1)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $= \begin{bmatrix} 2^{11} - 1 + 1 & 0 & 0 \\ 0 & 4(2^{11}-1)+1 & -(2^{11}-1) \\ 0 & 12(2^{11}-1) & -3(2^{11}-1)+1 \end{bmatrix}$ $\therefore$ Sum of all diagonal elements $= 2^{11} + 4(2^{11}-1) + 1 - 3(2^{11}-1) + 1$ $= 2 \times 2^{11} + 1 = 2^{12} + 1 = 4097$

58	*(a)* Now, $P^TP=\begin{bmatrix}\sqrt{3}/2 & -1/2\\ 1/2 & \sqrt{3}/2\end{bmatrix}\begin{bmatrix}\sqrt{3}/2 & 1/2\\ -1/2 & \sqrt{3}/2\end{bmatrix}$ $\Rightarrow \quad P^TP=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ $\Rightarrow \quad P^TP=I \quad \Rightarrow \quad P^T=P^{-1}$ Since, $Q=PAP^T$ $\therefore \; P^TQ^{2005}P=P^T[(PAP^T)(PAP^T)\ldots 2005 \text{ times}]P$ $=\underbrace{(P^TP)A(P^TP)A(P^TP)\ldots(P^TP)A(P^TP)}_{2005\text{ times}}$ $=IA^{2005}=A^{2005}$ $\therefore \quad A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ $A^2=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$ $A^3=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}$ ... ... $A^{2005}=\begin{bmatrix}1 & 2005\\ 0 & 1\end{bmatrix}$ $\therefore \; P^TQ^{2005}P=\begin{bmatrix}1 & 2005\\ 0 & 1\end{bmatrix}$
59	· *(c)* Given, A and B are any two 3×3 symmetric and skew-symmetric matrix, respectively. $\Rightarrow \quad A^T=A, B^T=-B$ Let $C=A^4-B^4$ $\Rightarrow C^T=(A^4-B^4)^T=(A^4)^T-(B^4)^T$ $=(A^T)^4-(B^T)^4=A^4-(-B)^4$ $[\because A^T=A \text{ and } B^T=-A]$ $=A^4-B^4 \qquad [\because (-B)^4=B^4]$ $=C$ $\therefore$ **C is a symmetric matrix.**

	Let $D = AB - BA$ $\Rightarrow D^T = (AB - BA)^T = (AB)^T - (BA)^T$ $= B^T A^T - A^T B^T$ $[\because (AB)^T = B^T A^T]$ $= -BA + AB = D$ $[\because A^T = A \text{ and } B^T = -B]$ $\therefore D$ is a symmetric matrix. Let $E = B^5 - A^5$ $\Rightarrow E^T = (B^5 - A^5)^T = (B^5)^T - (A^5)^T$ $= (B^T)^5 - (A^T)^5 = (-B)^5 - A^5$ $[\because A^T = A, B^T = -B]$ $= -B^5 - A^5 = -[B^5 + A^5] \neq -E$ $\therefore E$ is neither symmetric nor skew-symmetric. Let $F = AB + BA$ $\Rightarrow F^T = (AB + BA)^T = (AB)^T + (BA)^T$ $\Rightarrow \quad = B^T A^T + A^T B^T = -BA - AB$ $\Rightarrow \quad = -(AB + BA) = -F$ $\therefore F$ is a skew-symmetric.
60	(c) We have, $P^2 = I - P$ ...(i) $\Rightarrow \quad P^4 = (I - P)^2 = I + P^2 - 2P$ $= 2I - 3P$ [using Eq. (i)] $\Rightarrow \quad P^8 = (2I - 3P)^2 = 4I + 9P^2 - 12P$ $= 13I - 21P$ [using Eq. (i)] and $P^6 = (I - P)(2I - 3P)$ $= 2I - 3P - 2P + 3P^2$ $= 5I - 8P$ [using Eq. (i)] Now, $P^8 + P^6 = 18I - 29P$ and $P^8 - P^6 = 8I - 13P$ $\therefore \alpha = 8, \beta = 6, \gamma = 18, \delta = 8$ $\therefore \alpha + \beta + \gamma - \delta = 8 + 6 + 18 - 8 = 24$

61	$\begin{bmatrix} 2/3 & 1 \\ 1 & 1 \end{bmatrix}$
62	$B = \begin{pmatrix} -2 & 3 \\ 1 & -2 \end{pmatrix} - \begin{pmatrix} 2 & -4 \\ 6 & 0 \end{pmatrix}$ $B = \begin{pmatrix} -2-2 & 3+4 \\ 1-6 & -2-0 \end{pmatrix}$ $B = \begin{pmatrix} -4 & 7 \\ -5 & -2 \end{pmatrix}$ $\boxed{\begin{pmatrix} -4 & 7 \\ -5 & -2 \end{pmatrix}}$
63	2. **Subtract the third matrix:** $\begin{bmatrix} 4 & 4 & 5 \\ 5 & 5 & 13 \\ \sqrt{2}-2 & 4 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 2 & 3 \\ 2 & 0 & 4 \\ \sqrt{2}-2 & 4 & 4 \end{bmatrix} = \begin{bmatrix} 4-3 & 4-2 & 5-3 \\ 5-2 & 5-0 & 13-4 \\ (\sqrt{2}-2)-(\sqrt{2}-2) & 4-4 & 5-4 \end{bmatrix}$ $= \begin{bmatrix} 1 & 2 & 2 \\ 3 & 5 & 9 \\ 0 & 0 & 1 \end{bmatrix}$
64	). *(204)* Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, such that $a_{ij} \in \{0, 1, 2, 3, 4\}$ Now, $a_{11} + a_{12} + a_{21} + a_{22} = p$, where p is prime number $p \in (2, 13)$. $\therefore a_{11} + a_{12} + a_{21} + a_{22} = 3$ or 5 or 7 or 11 Now, $(x^0 + x^1 + x^2 + x^3 + x^4)^4$ $= \left(\frac{1-x^5}{1-x}\right)^4 = (1-x^5)^4 (1-x)^{-4}$ $= {}^4C_{r_1} (-x^5)^{r_1} \times {}^{4+r_2-1}C_{r_2} x^{r_2}$ $= {}^4C_{r_1} \times {}^{4+r_2-1}C_{r_2} (-1)^{r_1} x^{5r_1+r_2}$

$\therefore\ 5r_1 + r_2 = 3$ or 5 or 7 or 11

Now, when, $5r_1 + r_2 = 3$

$\Rightarrow \quad r_1 = 0, r_2 = 3$

When, $5r_1 + r_2 = 5$

$\Rightarrow \quad r_1 = 0, r_2 = 5$

or $\quad r_1 = 1, r_2 = 0$

When, $5r_1 + r_2 = 7$

$\Rightarrow \quad r_1 = 0, r_2 = 7, r_1 = 1, r_2 = 2,$

When, $5r_1 + r_2 = 11 \Rightarrow r_1 = 0, r_2 = 11,$

$r_1 = 1, r_2 = 6, r_1 = 2, r_2 = 1$

Sum of all coefficients

$$= {}^4C_0 \times {}^6C_3 + {}^4C_0 \times {}^8C_5 - {}^4C_1 \times {}^3C_0$$
$$+ {}^4C_0 \times {}^{10}C_7 - {}^4C_1 \times {}^5C_2 + {}^4C_0 \times {}^{14}C_{11}$$
$$- {}^4C_1 \times {}^9C_6 + {}^4C_2 \times {}^4C_1 = 204$$

65

Let A, B and C be 3×3 matrices, where A is symmetric, B and C are skew-symmetric matrices.

S1 $A^{13} B^{26} - B^{26} A^{13}$ is symmetric

Let $\quad P = A^{13} B^{26} - B^{26} A^{13}$

Now, taking transpose both the sides, we get

$$P^T = (A^{13} B^{26} - B^{26} A^{13})^T$$

$$\left[\begin{array}{c} \because (M+N)^T = M^T + N^T \\ \text{and } (AB)^T = B^T A^T \\ \text{and } (M^n)^T = (M^T)^n \end{array}\right]$$

So, $P^T = (A^{13} B^{26})^T - (B^{26} A^{13})^T$

$$\boldsymbol{P^T} = (B^{26})^T (A^{13})^T - (A^{13})^T (B^{26})^T$$

$$\boldsymbol{P^T} = (B^T)^{26} (A^T)^{13} - (A^T)^{13} (B^T)^{26}$$

	[$\because$ if A is a symmetric matrix, so $A^T = A$ but B is a skew-symmetric matrix, so $B^T = -B$] Now, $P^T = (-B)^{26}(A)^{13} - (A^{13})(-B)^{26}$ $\Rightarrow P^T = B^{26}A^{13} - A^{13}B^{26}$ $P^T = -(A^{13}B^{26} - A^{13}B^{26})$ $\Rightarrow P^T = -P$ skew-symmetric matrix, So, S1 is a wrong statement. Now, S2 is symmetric. Let $Q = A^{26}C^{13} - C^{13}A^{26}$ Now, taking transpose both the sides, we get $Q^T = (A^{26}C^{13} - C^{13}A^{26})^T$ $\Rightarrow Q^T = (A^{26}C^{13})^T - (C^{13}A^{26})^T$ $\Rightarrow Q^T = (C^{13})^T(A^{26})^T - (A^{26})^T(C^{13})^T$ $\Rightarrow Q^T = (C^T)^{13}(A^T)^{26} - (A^T)^{26}(C^T)^{13}$ $\Rightarrow Q^T = (-C)^{13}(A)^{26} - (A)^{26}(-C)^{13}$ $\Rightarrow Q^T = -C^{13}A^{26} + A^{26}C^{13}$ $Q^T = A^{26}C^{13} - C^{13}A^{26}$ $Q^T = Q$ symmetric matrix So, $S2$ is a correct statement. So, only $S2$ is true. V
66	given, $A = \begin{bmatrix} \sqrt{2} & 1 \\ -1 & \sqrt{2} \end{bmatrix} \Rightarrow \det(A) = 3 \Rightarrow \|A\| = 3$ and $B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \Rightarrow \det(B) = 1 \Rightarrow \|B\| = 1$ $\because$ $C = ABA^T \Rightarrow \det(C) = (\det(A))^2 . \det(B)$ i.e. $\|C\| = \|A\|^2 . \|B\| = 9$ Now $\|X\| = \|A^T C^2 A\|$ $= \|A^T\|\|C\|^2\|A\| = \|A\|^2\|C\|^2$ $(\because \|A^T\| = \|A\|)$ $= (9)(81) = 729$

67	(b) $\begin{vmatrix} 1 & \frac{3}{2} & \alpha+\frac{3}{2} \\ 1 & \frac{1}{3} & \alpha+\frac{1}{3} \\ 2\alpha+3 & 3\alpha+1 & 0 \end{vmatrix}=0$ $(2\alpha+3)\left(\frac{3}{2}\left(\alpha+\frac{1}{3}\right)-\frac{1}{3}\left(\alpha+\frac{3}{2}\right)\right)$ $-(3\alpha+1)\left[\left(\alpha+\frac{1}{3}\right)-\left(\alpha+\frac{3}{2}\right)\right]=0$ $\Rightarrow \quad (2\alpha+3)\left\{\frac{7\alpha}{6}\right\}-(3\alpha+1)\left\{\frac{-7}{6}\right\}=0$ $\Rightarrow \quad (2\alpha+3).\frac{7\alpha}{6}+(3\alpha+1).\frac{7}{6}=0$ $\Rightarrow \quad 2\alpha^2+6\alpha+1=0 \Rightarrow \alpha=\frac{-3+\sqrt{7}}{2}, \frac{-3-\sqrt{7}}{2}$
68	The minor of the element multiplied by (-1)i+j where i and j are row and column indices of the element
69	The **determinant** of a matrix is calculated as the sum of the **products of the elements** of any row or column of the matrix with their corresponding **cofactors**.
70	a=4,b=2,c=2
71	$\because \quad A^2=\begin{bmatrix}1&1\\1&1\end{bmatrix}\begin{bmatrix}1&1\\1&1\end{bmatrix}=\begin{bmatrix}2&2\\2&2\end{bmatrix}=2\begin{bmatrix}1&1\\1&1\end{bmatrix}$ $A^3=A^2\cdot A=2\begin{bmatrix}1&1\\1&1\end{bmatrix}^2=2^2\begin{bmatrix}1&1\\1&1\end{bmatrix}$ .... $A^n=2^{n-1}\begin{bmatrix}1&1\\1&1\end{bmatrix}$ $\therefore \quad A^{100}=2^{99}\begin{bmatrix}1&1\\1&1\end{bmatrix}=2^{99}A.$
72	*(b)* We have, $\|2A\|^3=2^{21}$ $\Rightarrow \quad \|2A\|=2^7 \Rightarrow 8\|A\|=2^7$ $\Rightarrow \quad \|A\|=2^4$ Also, $A=\begin{bmatrix}1&0&0\\0&\alpha&\beta\\0&\beta&\alpha\end{bmatrix}$ $\Rightarrow \quad \|A\|=\alpha^2-\beta^2=2^4$ $\Rightarrow (\alpha-\beta)(\alpha+\beta)=16$ $\Rightarrow \quad \alpha-\beta=2 \text{ and } \alpha+\beta=8$ $\Rightarrow \quad \alpha=5 \text{ and } \beta=3$

73	The order of a matrix is determined by its dimensions, specifically the number of rows and columns. If a matrix has 19 elements, then the product of the number of rows (m) and columns (n) must equal 19: $$m \times n = 19$$ Since 19 is a prime number, the possible pairs (m, n) that satisfy this equation are: 1. $(1, 19)$ 2. $(19, 1)$ These are the only possible orders for a matrix with 19 elements, meaning there are 2 different orders. Therefore, the answer is: $$\boxed{2}$$
74	**(7)** Given $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$ $$A^2 = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ 3 & 0 \end{bmatrix}$$ $$A^3 = \begin{bmatrix} 3 & -3 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 6 & -3 \end{bmatrix}$$ $$A^4 = \begin{bmatrix} 3 & -6 \\ 6 & -3 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -9 \\ 9 & -9 \end{bmatrix}$$ $$A^5 = \begin{bmatrix} 0 & -9 \\ 9 & -9 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -9 & -9 \\ 9 & -18 \end{bmatrix}$$ $$A^6 = \begin{bmatrix} -9 & -9 \\ 9 & -18 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -27 & 0 \\ 0 & -27 \end{bmatrix}$$ $$A^7 = \begin{bmatrix} -54 & 27 \\ -27 & -27 \end{bmatrix}$$ $A^6 . A^7 = A^{13}$ $$A = \begin{bmatrix} -27 & 0 \\ -0 & -27 \end{bmatrix}\begin{bmatrix} -54 & 27 \\ -27 & -27 \end{bmatrix} = \begin{bmatrix} 3^6 \times 2 & -27^2 \\ 27^2 & 3^6 \end{bmatrix}$$ $3^7 = 3^n \Rightarrow n = 7$

75	Given, $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix}$ $AA^T = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix}$ $= \begin{bmatrix} 9 & 0 & a+4+2b \\ 0 & 9 & 2a+2-2b \\ a+4+2b & 2a+2-2b & a^2+4+b^2 \end{bmatrix}$ It is given that $AA^T = 9I$ $\Rightarrow \begin{bmatrix} 9 & 0 & a+4+2b \\ 0 & 9 & 2a+2-2b \\ a+4+2b & 2a+2-2b & a^2+4+b^2 \end{bmatrix}$ $= 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 9 & 0 & a+4+2b \\ 0 & 9 & 2a+2-2b \\ a+4+2b & 2a+2-2b & a^2+4+b^2 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

On comparing, we get

$$a+4+2b=0 \Rightarrow a+2b=-4 \quad \ldots\text{(i)}$$

$$2a+2-2b=0 \Rightarrow a-b=-1 \quad \ldots\text{(ii)}$$

and $$a^2+4+b^2=9 \quad \ldots\text{(iii)}$$

On solving Eqs. (i) and (ii), we get

$$a=-2,\ b=-1$$

This satisfies Eq. (iii)

Hence, $(a, b) \equiv (-2, -1)$