02. Ans: (c)

Sol: $\dfrac{\omega}{\omega_n} = 0.5$

For undamped isolation system the transmissibility ratio

$$TR = \frac{1}{1-\left(\frac{\omega}{\omega_n}\right)^2} = \frac{4}{3}$$

03. Ans: (c)

Sol: The sample for which $\dfrac{\omega}{\omega_n}$ is closest to 1 will have more response,

Given, $f = 144$ Hz execution frequency.

f_{R_n} (Natural frequency) is 128.

$$\frac{\omega}{\omega_{R_n}} = \frac{f}{f_{R_n}} = \frac{144}{128} = 1.125$$

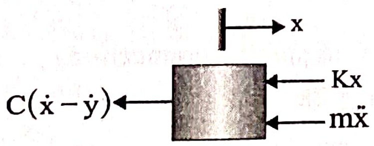
It is close to 1, so sample R will show most perceptible to vibration.

04. Ans: (c)

Sol: The differential equation governing the vibrating system

$$m\ddot{x} + c(\dot{x} - \dot{y}) + kx = 0$$

Free body diagram:



05. Ans: (c)

Sol: Resonance occurs when forcing frequency is equal to natural frequency,

i.e., $\omega = \omega_n$

06. Ans: (b)

Sol: Critical or whirling speed

$$\omega_c = \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{\delta}}\ \text{rad/sec}$$

If N_C is the critical or whirling speed in rpm then

$$\frac{2\pi N_C}{60} = \sqrt{\frac{g}{\delta}}$$

$$\Rightarrow \frac{2\pi N_C}{60} = \sqrt{\frac{9.81\ \text{m/s}^2}{1.8\times10^{-3}\text{m}}}$$

$$\Rightarrow N_C = 705.32\ \text{rpm} \approx 705\ \text{rpm}$$

07. Ans: (a)

Sol: **Note:** ω_n depends on mass of the system not on gravity

$$\therefore\ \omega_n \propto \frac{1}{\sqrt{m}}$$

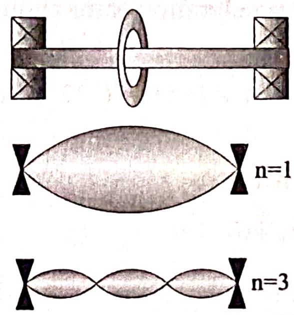
If $\omega_n = \sqrt{\dfrac{g}{\delta}}$ $\qquad \delta = \dfrac{mg}{K}$

$$\therefore\ \omega_n = \sqrt{\frac{g}{\left(\frac{mg}{k}\right)}} = \sqrt{\frac{k}{m}}$$

$\therefore\ \omega_n$ is constant every where.

08. Ans: (a)

Sol: Number of nodes observed at a frequency of 1800 rpm is 2



n-mode number

The whirling frequency of shaft, $f = \dfrac{\pi}{2} \times n^2 \sqrt{\dfrac{gEI}{WL^4}}$

For 1st mode frequency, $f_1 = \dfrac{\pi}{2} \times \sqrt{\dfrac{gEI}{WL^4}}$

$$f_n = n^2 f_1$$

As there are two nodes present in 3rd mode,

$$f_3 = 3^2 f_1 = 1800\ \text{rpm}$$

$$\therefore\ f_1 = \frac{1800}{9} = 200\ \text{rpm}$$

$\therefore$ The first critical speed of the shaft = 200 rpm

09. Ans: (b)

Sol: In damped free vibrations the oscillatory motion becomes non-oscillatory at critical damping.

Hence critical damping is the smallest damping at which no oscillation occurs in free vibration

10. Ans: (b)

Sol: Transmissibility (T) reduces with increase in damping up to the frequency ratio of $\sqrt{2}$. Beyond $\sqrt{2}$, T increases with increase in damping

11. Ans: (c)

Sol: At resonance for all the damping values the phase is 90°.

12. Ans: 6.32 m/sec²

Sol: In simple harmonic motion, $x = X \sin \omega t$

$$\ddot{x} = -X\omega^2 \sin \omega t$$

Given $X = 10$ mm $= 0.01$ m; $f = 4$ Hz

$\therefore \quad \omega = 2\pi f = 8\pi$ rad/sec

Magnitude of acceleration or maximum acceleration

$= X\omega^2$

$= 0.01 \times (8\pi)^2 = 6.32$ m/sec²

13. Ans: 0.38 to 0.42

Sol: Given $C_c = 2\sqrt{km} = 0.1$ kg/sec

$m^1 = 2m \; k^1 = 8k$

$C_c^1 = 2\sqrt{k^1 m^1}$

$= 2 \times \sqrt{8k \times 2m}$

$= 4 \times 2\sqrt{km} = 4.C_c = 0.4$ kg/sec

14. Ans: (c)

Sol: In case of viscous damping drag force is

$F_D = c\dot{x}$

where, c = damping coefficient and

$\dot{x}$ = relative velocity.

Thus drag force is directly proportional to relative velocity.

15. Ans: 20 (range 19.9 to 20.1)

Sol: $k = 10$ kN/m; $F_0 = 100$ N; $\xi = 0.25$

$$X = \frac{(F_0/k)}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$\frac{\omega}{\omega_n} = 1$ at resonance

$$X = \frac{F_0}{2k\xi} = \frac{100}{2 \times 10 \times 0.25 \times 10^3} = 20 \text{ mm}$$

16. Ans: 100 (range 99 to 101)

Sol: $\omega_n = \sqrt{\frac{g}{\delta}} = \sqrt{\frac{10}{1 \times 10^{-3}}} = 100$ rad/sec

17. Ans: (b)

Sol: Critical Damping Constant,

$C_c = 2\sqrt{km}$

$q = 2\sqrt{km}$

18. Ans: (b)

Sol: $m\ddot{x} + c\dot{x} + kx = F(t)$

We have $\frac{c}{m} = \frac{2c}{2\sqrt{km}}\omega_n = 2\frac{c}{C_c}\omega_n$

Damping ratio $= \frac{c}{C_c} = \frac{c}{2\sqrt{km}}$

19. Ans: (a)

Sol: Springs are in parallel connections

$k_{eq} = k + k = 2k$

$\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{2k}{m}}$ rad/sec

20. Ans: (d)

Sol: Given equation of motion is $M\ddot{x} + Kx = F\cos(\omega t)$

ω = excitation frequency

Its natural frequency is $\omega_n = \sqrt{\frac{K}{M}}$

Resonance occurs when excitation frequency coincides/matches with system's natural frequency (and amplitude of forced excitation linearly increases with time)

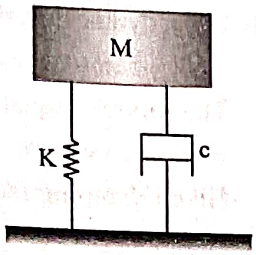
21. Ans: (c)

Sol: If additional damper is added parallel to existing damper.

$\therefore$ Equation damping coefficient increases.

$\therefore \xi = \frac{c}{c_c} = \frac{c}{2\sqrt{km}} \uparrow$

$\omega_d = \omega_n \sqrt{1-\xi^2}$



As $\xi \uparrow \omega_d \downarrow$

$T_d = \frac{2\pi}{\omega_d}(\uparrow)$

As ξ increases, damped natural frequency decreases and time period increases.

22. Ans: (a)

Sol: System – I :

$k_{eq} = \frac{k.k}{k+k} = \frac{k}{2}$

$\omega_{n_1} = \sqrt{\frac{k}{2m}}$

System – II :

$k_{eq} = 2k\ ;\ \ \omega_{n_2} = \sqrt{\frac{2k}{m}}$

$$\frac{\omega_{n_1}}{\omega_{n_2}} = \frac{\sqrt{\frac{k}{2m}}}{\sqrt{\frac{2k}{m}}} = \sqrt{\frac{k}{2m} \times \frac{m}{2k}} = \frac{1}{2}$$

23. Ans: (c)

Sol: $\left.\begin{matrix}\epsilon = 1 \\ F_T = F_0\end{matrix}\right\}$ either @ $r = 0$ or $r = \sqrt{2}$

so at $r = \sqrt{2}$

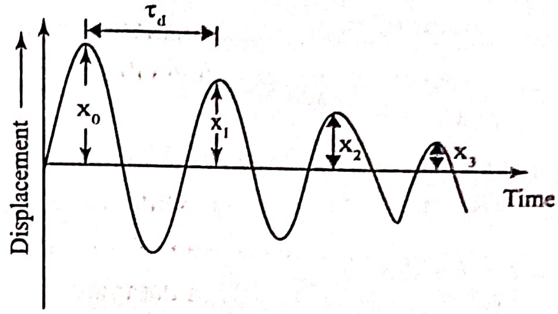
$\frac{\omega}{\omega_n} = \sqrt{2}$

$\omega = \sqrt{2}\,\omega_n$

$\omega = \sqrt{\frac{2k}{m}}$

24. Ans: (0.088)

Sol:



$x_o = 8.0$ mm

$x_3 = 1.5$ mm

$\frac{x_o}{x_3} = \frac{x_o}{x_1} \times \frac{x_1}{x_2} \times \frac{x_2}{x_3}$

$= e^{\delta} . e^{\delta} . e^{\delta}$

$\Rightarrow \frac{x_o}{x_3} = e^{3\delta}$

$\Rightarrow \ln\left(\frac{x_o}{x_3}\right) = 3\delta$

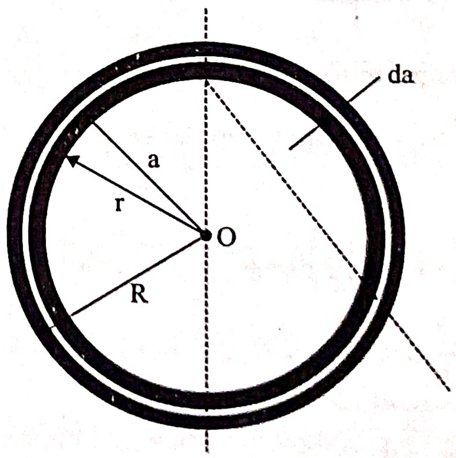
$\delta = \frac{1}{3}\ln\left(\frac{8}{1.5}\right)$

$\frac{2\pi\xi}{\sqrt{1-\xi^2}} = \frac{1}{3}\ln\left(\frac{8}{1.5}\right)$

$\Rightarrow \xi = 0.0884$

25. Ans: 2.65

Sol:



Considering the differential ring mass of differential ring

$$dm = \frac{\text{mass of ring}}{\text{volume of ring}}(dV)$$

$$dm = \frac{M}{\pi(R^2 - r^2)t} 2\pi a da \times t$$

$$dm = \frac{2Mada}{(R^2 - r^2)}$$

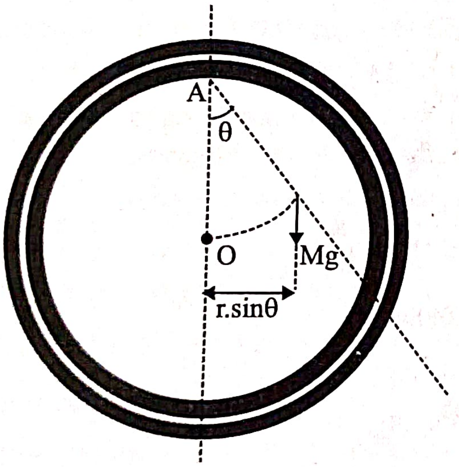
$$I_0 = \bar{I} = \int a^2 dm$$

$$= \int_r^R \frac{2Ma^3 da}{(R^2 - r^2)}$$

$$\bar{I} = \frac{2M}{(R^2 - r^2)} \times \left[\frac{a^4}{4}\right]_r^R = \frac{M}{2}\frac{(R^4 - r^4)}{(R^2 - r^2)} = \frac{M}{2}(R^2 + r^2)$$

$$I = \bar{I} + md^2 = \frac{M}{2}(R^2 + r^2) + Mr^2 = 3$$

$$= \frac{3M}{4}(R^2) + Mr^2 = \frac{5MR^2}{4}$$



Taking moments about A

$\Sigma M_A = 0$

$I\ddot{\theta} + Mg(r\sin\theta) = 0$

$\sin\theta \cong \theta$

$$\frac{5MR^2}{4}\ddot{\theta} + Mg\left(\frac{R}{\sqrt{2}}\right)\theta = 0$$

$$\ddot{\theta} + \frac{4g}{5\sqrt{2}R}\theta = 0$$

$$\omega_n = \sqrt{\frac{4g}{5\sqrt{2}R}}$$

$$T = \frac{2\pi}{\omega_n} = \pi\left(2 \times \sqrt{\frac{5\sqrt{2}}{5}} \times \sqrt{\frac{R}{g}}\right)$$

$\beta = 2.65$

26. Ans: 2

Sol: For a dynamic system,

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$$

The above equation represents a damped free vibration system.

Given damping factor, $\zeta = \frac{1}{2\pi}\ln(2)$

Given a peak $x_{n-1} = 4$ mm, $x_n = ?$

Logarithmic decrement is given by

$$\ln\left(\frac{x_{n-1}}{x_n}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

As question is asking to neglect the higher power of ζ, so neglecting ζ^2, as $\zeta < 1$.

$$\ln\left(\frac{x_{n-1}}{x_n}\right) = 2\pi\zeta$$

$$2\pi\zeta = \ln(2)$$

$$\Rightarrow \ln\left(\frac{x_{n-1}}{x_n}\right) = \ln(2)$$

$$\Rightarrow \frac{4}{x_n} = 2$$

$$\Rightarrow x_n = 2 \text{ mm}$$

27. Ans: 6.28

Sol: Given data, $m = 20$ kg, $k = 1000$ N/m

Equivalent stiffness, $k_e = k + k = 2k$

Natural frequency,

$$\omega_n = \sqrt{\frac{k_e}{m}} = \sqrt{\frac{2 \times 1000}{20}} = 10 \text{ rad/sec}$$

Time period, $T = \frac{2\pi}{\omega_n} = \frac{2\pi}{10}$ sec

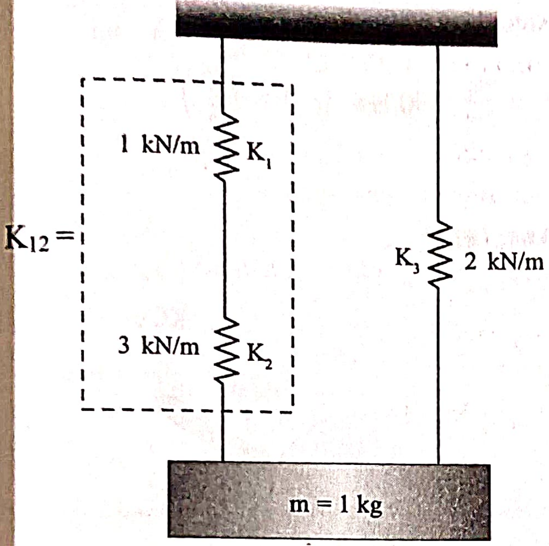
Time taken for 10 oscillations

$$= 10 \times \frac{2\pi}{10} = 2\pi \text{ sec} = 6.28 \text{ sec}$$

Two Marks Solutions

01. Ans: (b)

Sol:



K_1 & K_2 springs are in series

Resultant spring stiffness (K_{12})

$$\frac{1}{K_{12}} = \frac{1}{K_1} + \frac{1}{K_2} = \frac{1}{1} + \frac{1}{3} = 1.33$$

$K_{12} = 0.75$ kN/m

K_{12} & K_3 are in parallel

$\therefore$ Resultant $K = K_{12} + K_3$

$= 0.75 + 2$

$= 2.75$ kN/m

$\therefore$ Natural frequency $= \sqrt{\frac{K}{M}} = \sqrt{\frac{2.75 \times 1000}{1}}$

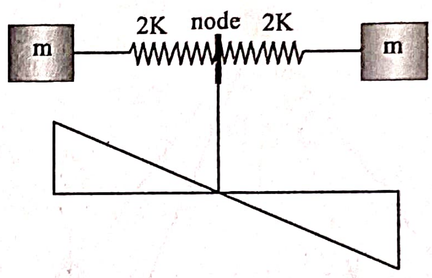
$= 52.44$ rad/sec

$= 8.35$ Hz

02. Ans: (a)

Sol: The given system is a 2 D.O.F one without constraints and exhibits a rigid body motion for which the frequency is zero. The node shape corresponding to the non zero frequency is as shown in figure. As the masses are equal in both sides, the node will be at the middle. By fixing the spring at the node we can separate into two single D.O.F systems and both will have same natural frequency. As the node falls in the middle of the spring, the spring is divided into two equal halves and each will have stiffness of 2K. So the frequency for each system is equal to $\sqrt{\frac{2K}{m}}$.

Hence the frequencies for the given system are 0 and $\sqrt{\frac{2K}{m}}$



Alternative Method:

For the above diagram the equation can be written as

$$m_1\ddot{x} + K(x_1 - x_2) = 0$$

$$m_2\ddot{x} + K(x_2 - x_1) = 0$$

Assuming the solution of the form.

$x_1 = A_1 \sin\omega t$

$x_2 = A_2 \sin\omega t$

$$\Rightarrow -m_1\omega^2 A_1 + K(A_1 - A_2) = 0$$

$$\Rightarrow -m_2\omega^2 A_2 + K(A_2 - A_1) = 0$$

Amplitude ratio

$$\frac{A_1}{A_2} = \frac{K}{K - m_1\omega^2} = \frac{K - m_2\omega^2}{K}$$

$$\Rightarrow \frac{K}{K - m_1\omega^2} = \frac{K - m_2\omega^2}{K}$$

$$\Rightarrow -K(m_2\omega^2 + m_1\omega^2) + m_1 m_2 \omega^4 = 0$$

$$\Rightarrow m_1 m_2\left\{\omega^4 - K\omega^2\left(\frac{1}{m_1} + \frac{1}{m_2}\right)\right\} = 0$$

......($m_1 = m_2 = m$)

Solving this equation we get $\omega_1 = 0$ and

$\omega_2 = \sqrt{\frac{2K}{m}}$

03. Ans: (c)

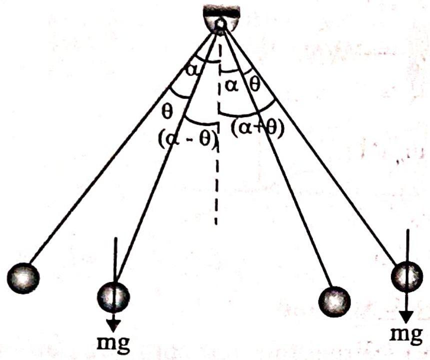
Sol: Springs are in parallel, since deflection is same

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K_1+K_2}{m}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{40\times1000}{100}} = \frac{20}{2\pi} = \frac{10}{\pi}\text{ Hz}$$

04. Ans: (d)

Sol:



Let the system be displaced by θ from the equilibrium position. It's position will be as shown in figure.

By considering moment equilibrium about the axis of rotation (Hinge)

$$I\ddot{\theta} + mg\ell\sin(\alpha+\theta) - mg\ell\sin(\alpha-\theta) = 0$$

$$I = m\ell^2 + m\ell^2 = 2m\ell^2$$

After simplification

$$2m\ell^2\ddot{\theta} + 2mg\ell\cos\alpha\sin\theta = 0$$

For small oscillations (θ is small) sin θ = θ

$$\therefore\quad 2m\ell^2\ddot{\theta} + 2mg\ell\cos\alpha.\theta = 0$$

$$\omega_n = \sqrt{\frac{2mg\ell\cos\alpha}{2m\ell^2}} = \sqrt{\frac{g\cos\alpha}{\ell}}$$

05. Ans: (b)

Sol: Here, m = 10 kg = Mass of Rotor

d = Diameter of shaft

= 0.03 m

l = Length of shaft

$= 500 \times 10^{-3}$ m = 0.5 m

E for steel $= 2.1 \times 10^{11}$ N/m²

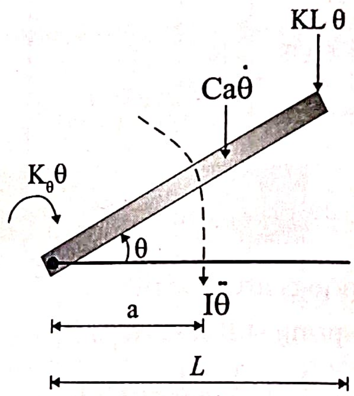
The Given shaft can be considered as a simply supported beam for which the stiffness $K = \frac{48EI}{\ell^3}$

Natural frequency $\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{48EI}{m\ell^3}}$

$$f_n = \frac{\omega_n}{2\pi} = 90\text{ Hz}\quad\left(\because I = \frac{\pi d^4}{64}\right)$$

06. Ans: (a)

Sol:



By moment equilibrium about the hinge

$$I\ddot{\theta} + Ca^2\dot{\theta} + KL^2\theta + K_\theta\theta = 0$$

$$\frac{mL^2}{3}\ddot{\theta} + Ca^2\dot{\theta} + (KL^2 + K_\theta)\theta = 0$$

$$\omega_n = \sqrt{\frac{K_{eq}}{m_{eq}}} = \sqrt{\frac{KL^2+K_\theta}{mL^2/3}}$$

$$\omega_n = \sqrt{\frac{1500}{0.833}}$$

= 42.26 rad/sec

07. Ans: (c)

Sol: Refer to the above equilibrium equation

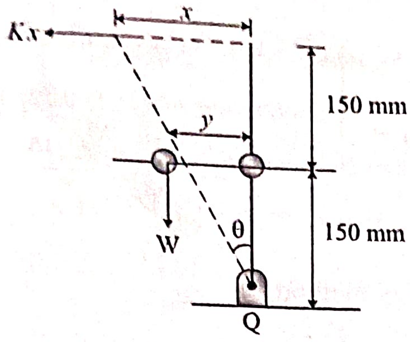
$C_{eq} = Ca^2$

$= 500 \times 0.4^2 = 80$ N m.s/rad

$\Rightarrow$ C = 80 N m.s/rad

08. Ans: (c)

Sol:



Let the weight be displaced by x from equilibrium position. Then

$$\tan\theta = \frac{x}{300} = \frac{y}{150}$$

$$\therefore\ y = \frac{x}{2}$$

Taking moment about hinged point

$$Kx.(0.3) = W.y = W.\frac{x}{2}$$

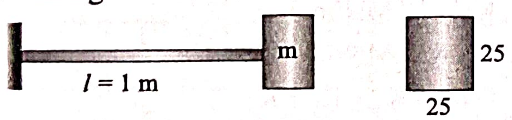
$$\therefore K = \frac{300}{2\times 0.3} = 500 \text{ N/m}$$

09. Ans: (a)

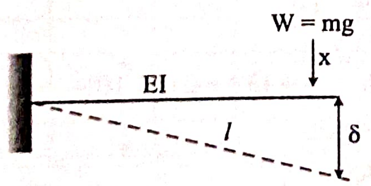
Sol: Given length of cantilever beam,

$l = 1000$ mm = 1 m,

m = 20 kg



Cross section of beam is square,



Moment of inertia of the shaft,

$$I = \frac{1}{12}bd^3 = \frac{25\times(25)^3}{12} = 3.25\times 10^{-8}\ \text{m}^4$$

$E_{steel} = 200 \times 10^9$ Pa

Mass, M = 20kg

Stiffness $K = \dfrac{3EI}{\ell^3}$

Critical damping coefficient, $C_C = 2\sqrt{Km}$

≈ 1250 N.sec/m

10. Ans: (c)

Sol: Given, m = 0.1 kg,

$f_d = 0.9 f_n$

$\Rightarrow\ \omega_d = 0.9\ \omega_n$

K = 1 kN/m

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\Rightarrow 0.9\omega_n = \omega_n\sqrt{1-\xi^2}$$

$$\Rightarrow \xi = 0.435$$

$$\xi = \frac{C}{2\sqrt{Km}}$$

$$\Rightarrow C = \xi\times 2\sqrt{km} = 8.7 \text{ N.sec/m}$$

11. Ans: (c)

Sol: Given, m = 250 kg, k = 100,000 N/m

N = 3600 rpm, $\xi = 0.15$

$$\omega_n = \sqrt{\frac{k}{m}} = 20 \text{ rad/sec}$$

$$\omega = \frac{2\pi\times N}{60} = 377 \text{ rad/sec}$$

$$TR = \frac{\sqrt{1+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}} = 0.0162$$

12. Ans: (d)

Sol: Given, m = 12.5 kg,

k = 1000 N/m,

C = 15 Ns/m

$\therefore$ C_c = critical damping coefficient

We know that

$$C_c = 2\sqrt{k\,m} = 2\times\sqrt{12500} = 223.6 \text{ Ns/m}$$

13. Ans: (b)

Sol: $\xi = \frac{C}{C_C} = \frac{15\text{ Ns/m}}{223.6\text{ Ns/m}} = 0.0670$

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} = 0.4222$$

Near value is selected

14. Ans: (a)

Sol: $X_1 = Xe^{-\xi\omega_n T_d}$

$$X_n = Xe^{-\xi\omega_n n T_d}$$

$$T_d = \frac{2\pi}{\omega_d} \Rightarrow \frac{2\pi}{[\sqrt{1-\xi^2}]\omega_n}$$

$$\therefore X_n = Xe^{-\xi\omega_n \times n \times \frac{2\pi}{\sqrt{(1-\xi^2)}\omega_n}}$$

$$= Xe^{-2\pi n \frac{\xi}{(\sqrt{1-\xi^2})}} = Xe^{-\xi\omega_n \times n \times \frac{2\pi}{(\sqrt{1-\xi^2})\omega_n}}$$

15. Ans: (a)

Sol:

(1) when springs are in parallel

$$\therefore K = K_1 + K_2 \Rightarrow \frac{K}{2} + \frac{K}{2} = K$$

(2) Springs are in series then

$$\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2}$$

$$\frac{1}{K_e} = \frac{1}{K} + \frac{1}{K} \Rightarrow \frac{1}{K_e} = \frac{2}{K}$$

$$\Rightarrow K_e = \frac{K}{2}$$

$$\Rightarrow \omega_n = \sqrt{\frac{K_e}{m}} \Rightarrow \sqrt{\frac{K}{2m}}$$

16. Ans: (b)

Sol: Springs are parallel

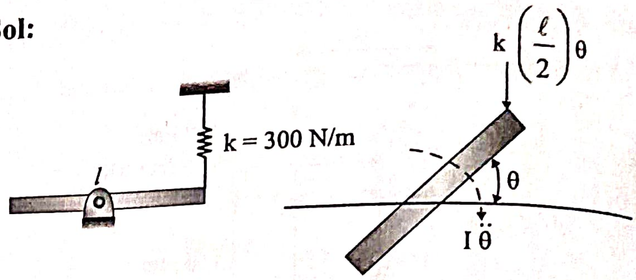
$$f_n = \frac{1}{2\pi}\sqrt{\frac{K_e}{m}}$$

$$K_e = K_1 + K_2 = 4000 + 1600 = 5600\text{ N/m}$$

$$f_n = \frac{1}{2\pi} = \sqrt{\frac{5600}{1.4}} = 10\text{ Hz}$$

17. Ans: (c)

Sol:



By energy method

$$E = \frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}kx^2 = \text{constant}$$

$$E = \frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}k \times \left(\frac{\ell}{2}\theta\right)^2 = \text{constant}$$

Differentiating w.r.t 't'

$$\frac{dE}{dt} = I\ddot{\theta} + \frac{k}{2} \times \frac{\ell^2}{4} \times 2\dot{\theta} = 0$$

$$I = \frac{m\ell^2}{12}$$

$$\frac{m\ell^2}{12}\ddot{\theta} + \frac{K\ell^2}{4}\dot{\theta} = 0$$

$$\Rightarrow \ddot{\theta} + \frac{3k}{m}\dot{\theta} = 0$$

$$\Rightarrow \omega_n = \sqrt{\frac{3k}{m}} = 30\text{ rad/sec}$$

18. Ans: (a)

Sol: $K_1, K_2 = 16$ MN/m

$K_3, K_4 = 32$ MN/m

$K_{eq} = K_1 + K_2 + K_3 + K_4$

$m = 240$ kg

$$\omega_n = \sqrt{\frac{K_e}{m}}$$

$$K_{eq} = [(16 \times 2) + (32 \times 2)] \times 10^6$$
$$= 96 \times 10^6$$

$m = 240$ kg

$$\omega_n = \sqrt{\frac{96 \times 10^6}{240}} = 632.455\text{ rad/sec}$$

$$N = \frac{\omega_n \times 60}{2\pi} = 6040\text{ rpm}$$

19. Ans: (a)

Sol: Dampling factor$(\xi) = \dfrac{C}{C_c}$

$$= \frac{\text{Actual damping coefficient}}{\text{critical damping coeffieient}}$$

$$C_c = 2\sqrt{Km} = 2\times\sqrt{3.6\times10^3\times50}$$
$$= 848.528 \text{ N-s/m}$$

$$\text{Damping factor } \xi = \frac{C}{C_c} = \frac{400}{848.52} = 0.47$$

$$\omega_n = \sqrt{\frac{K}{m}} = 8.48 \text{ rad/ sec}$$

$$\omega_d = (\sqrt{1-\xi^2})\omega_n = 7.4833 \text{ rad/sec}$$

$$f_n = \frac{\omega_d}{2\pi} = 1.19100 \text{ Hz}$$

$$\therefore f_n = 1.191 \text{ Hz}$$

20. Ans: (a & d)

Sol: $m\ddot{x} + Kx = F\cos\omega t$

$m = ?$ $\qquad K = 3000$ N/m,

$X = 50$ mm $= 0.05$ m

$F = 100$ N, $\quad \omega = 100$ rad/sec

$$X = \frac{F}{K - m\omega^2}$$

$$\Rightarrow m = \frac{K}{\omega^2} - \frac{F}{X\omega^2} = 0.1 \text{ kg}$$

$$\text{or } X = \frac{F}{m\omega^2 - K} \quad \Rightarrow m = 0.5 \text{ kg}$$

21. Ans: (c)

Sol: Applying Energy method

$$\frac{d}{dt}(KE + PE) = 0$$

$$\Rightarrow \frac{d}{dt}\left(\frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}Kx^2\right) = 0 \quad (\because x = r\theta)$$

$$\Rightarrow \frac{d}{dt}\left(\frac{1}{2}\left(\frac{3}{2}mr^2\right)\dot{\theta}^2 + \frac{1}{2}K(r\theta)^2\right) = 0$$

$$\therefore \left(I_0 = I_G + mr^2 = \frac{1}{2}mr^2 + mr^2 = \frac{3}{2}mr^2\right)$$

$$\Rightarrow \frac{d}{dt}\left[\frac{3}{4}mr^2\dot{\theta}^2 + \frac{1}{2}Kr^2\theta^2\right] = 0$$

$$\Rightarrow \ddot{\theta} + \frac{2K}{3m}\theta = 0$$

$$\Rightarrow \omega_n^2 = \frac{2K}{3m}$$

$$\Rightarrow f_n = \frac{1}{2\pi}\sqrt{\frac{2K}{3m}}$$

22. Ans: (a)

Sol: $\omega_n = \sqrt{\dfrac{K_{eq}}{m}}$

$K_{eq} = K_1 + K_2 \quad \rightarrow$ for parallel

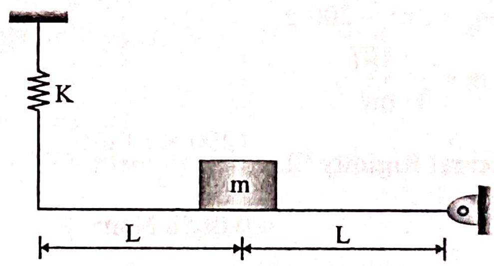
$\Rightarrow K_{eq} = 20 + 20 = 40$ kN/m

$$\omega_n = \sqrt{\frac{40\times1000}{1}} = 200 \text{ rad/sec}$$

$$\Rightarrow f_n = \frac{1}{2\pi}\times 200 = \frac{100}{\pi} = 32 \text{ Hz}$$

23. Ans: (d)

Sol:



$$\text{Equation of motion} = mL^2\ddot{\theta} + K(2L)^2\theta = 0$$

$$\ddot{\theta} + \frac{K4L^2}{mL^2}\theta = 0$$

$$\ddot{\theta} + \left(\frac{4K}{m}\right)\theta = 0$$

$$\therefore \omega_n = \sqrt{\frac{4K}{m}} \text{ rad/sec}$$

24. Ans: (c)

Sol: Let,

V_o is the initial velocity, 'm' is the mass

Equating Impulse = momentum

$$mV_0 = 5\ kN \times 10^{-4}\ sec$$
$$= 5 \times 10^3 \times 10^{-4} = 0.5\ sec$$

$$\therefore\ V_0 = \frac{0.5}{m} = 0.5\ m/sec$$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{10000}{1}} = 100\ rad/sec$$

When the free vibrations are initiated with initial velocity,

The amplitude

$$X = \frac{V_0}{\omega_n}\ \text{(Initial displacement is 0)}$$

$$\therefore\ X = \frac{V_0}{\omega_n} = \frac{0.5 \times 10^3}{100} = 5\ mm$$

25. Ans: 0.0658 N.m²

Sol: For a cantilever beam stiffness, $K = \frac{3EI}{\ell^3}$

Natural frequency $\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{3EI}{m\ell^3}}$

Given $f_n = 100$ Hz

$\Rightarrow \omega_n = 2\pi f_n = 200\,\pi$

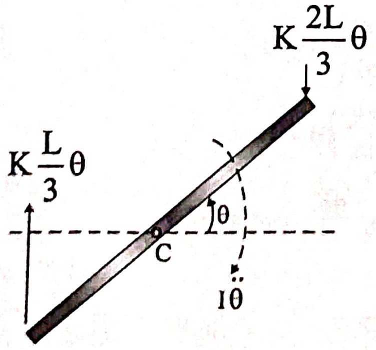
$$200\pi = \sqrt{\frac{3EI}{m\ell^3}}$$

Flexural Rigidity $EI = \frac{(200 \times \pi)^2 m\ell^3}{3}$

$= 0.0658\ N.m^2$

26. Ans: (d)

Sol:



Moment equilibrium about the pivot gives

$$I\ddot{\theta} + K \times \frac{2L}{3} \times \frac{2L}{3}\theta + K \times \frac{L}{3} \times \frac{L}{3} \times \theta = 0$$

$$I\ddot{\theta} + \left(\frac{4KL^2}{9} + \frac{KL^2}{9}\right)\theta = 0$$

Mass moment of Inertia $I = \frac{mL^2}{9}$

$$\therefore\ \frac{mL^2}{9}\ddot{\theta} + \frac{5KL^2}{9}\theta = 0$$

$$\omega_n = \sqrt{\frac{K_{eq}}{m_{eq}}}$$

$$\omega_n = \sqrt{\frac{\frac{5KL^2}{9}}{\frac{mL^2}{9}}} = \sqrt{\frac{5K}{m}}$$

27. Ans: (d)

Sol: The springs are in parallel

$$K_{eq} = K_1 + K_2$$

$$\omega_n = \sqrt{\frac{K_{eq}}{m}} = \sqrt{\frac{K_1 + K_2}{m}}$$

28. Ans: (1.25)

Sol: Given, $m = 1$ kg, $K = 100$ N/m
$C = 25$ N-sec/m

Critical damping $C_C = 2\sqrt{Km} = 20$ N-sec/m

Damping Ratio $= \frac{C}{C_C} = \frac{25}{20} = 1.25$

29. Ans: 10 N.sec/m

Sol: Given systems represented by

$$m\ddot{x} + c\dot{x} + kx = F\cos\omega t$$

For which $X = \frac{F}{\sqrt{(K - m\omega^2)^2 + (C\omega)^2}}$

Given $K = 6250$ N/m, $m = 10$ kg,
$F = 10$ N, $\omega = 25$ rad/sec,
$X = 40 \times 10^{-3}$

$$\omega_n = \sqrt{\frac{K}{m}} = 25\ rad/sec$$

$\omega t = 25t \Rightarrow \omega = 25$ rad/sec

$\omega = \omega_n$ or $K = m\omega_n^2$

$\therefore\ X = \dfrac{F}{C\omega} \Rightarrow C = \dfrac{F}{X\omega}$

$$= \frac{10}{40\times10^{-3}\times25}$$

$= 10$ N-sec/m

30. Ans: (2.28)

Sol: Given, $m = 2$ kg, $K = 8$ N/m,

$\xi = 0.02$, $\omega = 1.5$ rad/sec

$$\omega_n = \sqrt{\frac{K}{m}} = 2$$

$$\frac{\omega}{\omega_n} = \frac{1.5}{2} = \frac{3}{4}$$

Magnification factor (M.F)

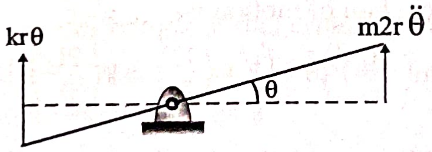
$$= \sqrt{\frac{1}{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$$= \sqrt{\frac{1}{\left\{1-\left(\frac{3}{4}\right)^2\right\}^2 + \left(2\times0.02\times\frac{3}{4}\right)^2}}$$

$= 2.28$

31. Ans: (d)

Sol: Free body diagram



Moment equilibrium about hinge

$m2r\ddot{\theta}.2r + kr\theta.r = 0$

$4mr^2\ddot{\theta} + kr^2\theta = 0$

$$\omega_n = \sqrt{\frac{kr^2}{4mr^2}} = \sqrt{\frac{k}{4m}} = \sqrt{\frac{400}{4}}$$

32. Ans: (b)

Sol: When lifted from ground at Q, reaction = 0.

Taking the moments about 'P' and equating to zero.

$90\times10^{-3}\times9.81\times6\times10^{-2} = me\omega^2\times9\times10^{-2}$

$90\times10^{-3}\times9.81\times6\times10^{-2} = 2\times10^{-3}\times2.19\times 10^{-3}\times\omega^2\times9\times10^{-2}$

$\Rightarrow \omega = 366.58$ rad/sec

$$\omega = \frac{2\pi N}{60}$$

$$\Rightarrow N = \frac{60\times366.58}{2\pi} = 3500 \text{ rpm}$$

33. Ans: 6750 to 7150

Sol: Given, $f = 60$ Hz,

$m = 1$ kg

$\omega = 2\pi f = 120\ \pi$rad/sec

Transmissibility ratio, $TR = 0.05$

Damping is negligible, $C = 0$, $K = ?$

We know $TR = \dfrac{K}{K - m\omega^2}$ when $C = 0$

As TR is less than 1 $\Rightarrow \omega/\omega_n >> \sqrt{2}$

TR is negative $\Rightarrow -0.05 = \dfrac{K}{K - m\omega^2}$

Solving we get, $K = 6767.7$ N/m

34. Ans: (b)

Sol: Given Problem of the type

$$m\ddot{x} + c\dot{x} + kx = F\cos\omega t$$

For which, $X = \dfrac{F}{(k - m\omega^2)^2 + (c\omega)^2}$

$$\text{or}\quad X = \frac{F/K}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right) + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

Given, $F = 10$, $\omega_n = 10\omega$

$k = 150$ N/m or $\dfrac{\omega}{\omega_n} = \dfrac{1}{10} = 0.1$

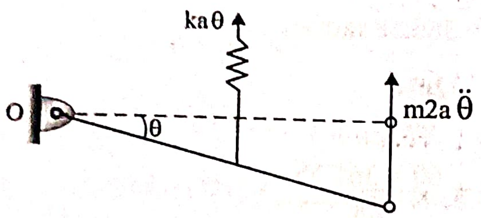
$\xi = 0.2$

$$X = \frac{10/150}{\sqrt{(1-0.1)^2 + (2\times0.2\times0.1)^2}}$$

$= 0.0669 \cong 0.07$

35. Ans: (a)

Sol:



By taking the moment about 'O',

$\Sigma M_o = 0$

$(m2a\ddot{\theta} \times 2a) + (ka\theta \times a) = 0$

$\Rightarrow 4a^2 m\ddot{\theta} + ka^2\theta = 0$

Where, $m_{eq} = 4a^2 m$, $k_{eq} = ka^2$

Natural frequency, $\omega_n = \sqrt{\dfrac{k_{eq}}{m_{eq}}}$

$$= \sqrt{\frac{ka^2}{4a^2 m}} = \sqrt{\frac{k}{4m}}\ \frac{rad}{sec}$$

$\because\ \omega_n = 2\pi f$

$$\Rightarrow f = \frac{\omega_n}{2\pi} = \frac{1}{2\pi} \times \sqrt{\frac{k}{4m}}\ Hz$$

36. Ans: 10 (range 9.9 to 10.1)

Sol: $KE = \dfrac{1}{2}m\dot{x}^2 + \dfrac{1}{2}I\dot{\theta}^2$

$m = 5\ kg,\ \theta = \dfrac{x}{r}$

$I = \dfrac{20 \times r^2}{2} = 10r^2$

$KE = \dfrac{1}{2}5\dot{x}^2 + \dfrac{1}{2}10r^2.\dfrac{\dot{x}^2}{r^2} = \dfrac{1}{2}(15)\dot{x}^2$

$\therefore m_{eq} = 15$

$PE = \dfrac{1}{2}kx^2$

$\therefore k_{eq} = k = 1500\ N/m$

Natural frequency $\omega_n = \sqrt{\dfrac{k_{eq}}{m_{eq}}} = \sqrt{\dfrac{1500}{15}}$

$= 10\ rad/sec$

37. Ans: 0.5 (range 0.49 to 0.51)

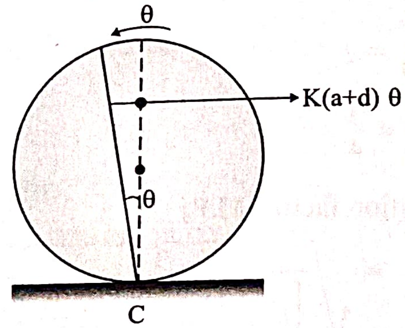
Sol: $\dfrac{\omega}{\omega_n} = \dfrac{\sqrt{3k/m}}{\sqrt{\dfrac{k}{m}}} = \sqrt{3}$

$$M.F = \sqrt{\frac{1}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2}} = \sqrt{\frac{1}{(1-3)^2}}$$

$$= \frac{1}{2} = 0.5$$

38. Ans: (a)

Sol: Moment equilibrium above instantaneous centre (contact point) $-k(a+d)\theta.(a+d) = I_c\ddot{\theta}$



$I_c = \dfrac{3}{2}Ma^2$, $\omega_a = \sqrt{\dfrac{k(a+d)^2}{\frac{3}{2}Ma^2}}$

$\omega_n = \sqrt{\dfrac{2k(a+d)^2}{3Ma^2}}$

39 Ans: (b)

Sol: Take the moment about the hinge 'O'

The equation of motion is

$$\left(I_0 + m\left(\frac{2L}{3}\right)^2\right)\ddot{\theta} + \left(k \times \left(\frac{L}{3}\right)^2 + k\left(\frac{2L}{3}\right)^2\right)\theta = 0$$

$$I_0 = \frac{ML^2}{12} + M\left(\frac{L}{6}\right)^2 = \frac{ML^2}{9},\quad m = \frac{M}{4}$$

$$\left(\frac{ML^2}{9} + \frac{4ML^2}{9 \times 4}\right)\ddot{\theta} + \frac{5kL^2}{9}\theta = 0$$

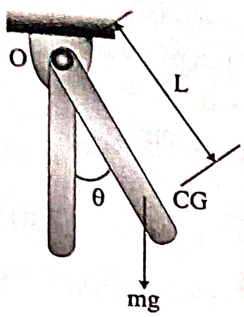
$$(2ML^2)\ddot{\theta} + \frac{5kL^2}{9}\theta = 0$$

Natural frequency,

$$\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{\left(\frac{5KL^2}{9}\right)}{\frac{2ML^2}{9}}} = \sqrt{\frac{5k}{2M}}$$

40. Ans: 15.66

Sol:



Radius of gyration, $r = 100$ mm $= 0.1$ m,

$L = 250$ mm $= 0.25$ m,

$g = 9.81$ m/sec^2

$\omega_n = ?$

The equation of motion is

$$I_o\ddot{\theta} + mgL\sin\theta = 0$$

For small value of θ, $\sin\theta \simeq \theta$

$$\therefore\ \omega_n = \sqrt{\frac{mgL}{I_o}}$$

$$\therefore\ I_o = mr^2$$

$$\therefore\ \omega_n = \sqrt{\frac{mgL}{mr^2}} = \sqrt{\frac{gL}{r^2}} = \sqrt{\frac{9.81\times0.25}{0.1^2}}\ \text{rad/s}$$

$$= 15.66\ \text{rad/s}$$

41. Ans: 33.33

Sol: Given, $m = 200$ kg

Since springs are in parallel arrangement

$K = k_1 + k_2 = k + k = 2k = 20$ kN/m

Harmonic excitation force

$F = 50\cos(5t) = F_0\cos(\omega t)$

$F_0 = 50$ N,

$\omega = 5$ rad/s

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{20000}{200}} = 10\ \text{rad/s}$$

Frequency ratio $r = \dfrac{\omega}{\omega_n} = 0.5$

Damping ratio $\xi = 0$

$$\frac{F_t}{F_0} = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}} = \frac{1}{1-r^2} = 1.333$$

$F_t = 1.333\,F_0 = 66.66$ N

Force transmitted from each mount to the ground

$$\frac{F_t}{2} = 33.33\ \text{N}$$

42. Ans: 23.09

Sol: $I_P\ddot{\theta} + F_{s_1}.r\cos\theta + F_{s_2}(2r\cos\theta) = 0$

$$I_P\ddot{\theta} + k_1.x_{s_1}.r + k_2x_{s_2}.2r = 0$$

$$I_P\ddot{\theta} + k_1r^2\theta + k_2 4r^2\theta = 0$$

$$I_P\ddot{\theta} + (k_1r^2 + 4k_2r^2)\theta = 0$$

$$I_{eq} = I_P = I_{cm} + m(r)^2$$

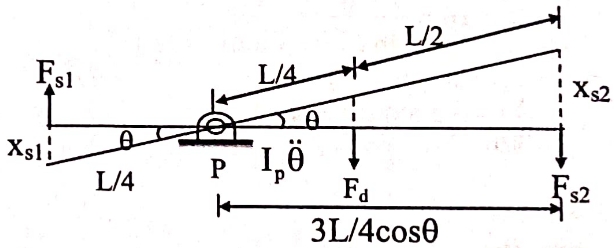
$$= \frac{mr^2}{2} + mr^2 = \frac{3}{2}mr^2$$

$$\omega_n = \sqrt{\frac{k_1r^2+4k_2r^2}{\frac{3}{2}mr^2}} = \sqrt{\frac{400+(4\times100)}{\frac{3}{2}\times1}}$$

$$= 23.09\ \text{rad/s}$$

43. Ans: (b)

Sol:



$$x_{s_1} = \frac{L}{4}\sin\theta \approx \frac{L}{4}\theta$$

$$x_{s_2} = \frac{3L}{4}\sin\theta \approx \frac{3L}{4}\theta$$

$$x_d = \frac{L}{4}\sin\theta \approx \frac{L}{4}\theta$$

$$\dot{x}_d = \frac{L}{4}\dot{\theta}$$

Taking moment about 'P'

$$\Rightarrow I_P\ddot{\theta} + c\dot{x}_d\frac{L}{4} + kx_{s_2}.\frac{3L}{4} + 3kx_{s_1}\frac{L}{4} = 0$$

$$\Rightarrow I_P\ddot{\theta} + c\left(\frac{L}{4}\right)^2\dot{\theta} + k\left(\frac{3L}{4}\right)^2\theta + 3k\left(\frac{L}{4}\right)^2\theta = 0$$

$$\Rightarrow I_P\ddot{\theta}+\frac{CL^2}{16}\dot{\theta}+\left(\frac{9L^2k}{16}+\frac{3kL^2}{16}\right)\theta=0$$

$$\Rightarrow I_P\ddot{\theta}+\frac{CL^2}{16}\dot{\theta}+\frac{3}{4}kL^2\theta=0$$

$$I_{eq}=I_p=\frac{m\ell^2}{12}+\frac{m\ell^2}{16}$$

$$=I_p=\frac{7}{48}m\ell^2,$$

$$C_{eq}=\frac{C\ell^2}{16}$$

$$k_{eq}=\frac{3k\ell^2}{4}$$

$$\omega_n=\sqrt{\frac{k_{eq}}{I_p}}=\sqrt{\frac{3k\ell^2}{4\times\frac{7m\ell^2}{48}}}=\sqrt{\frac{36k}{7m}}$$

$$\omega_n=6\sqrt{\frac{k}{7m}}$$

$$\zeta=\frac{C_{eq}}{2I_{eq}\omega_n}=\frac{C\ell^2}{2\times16\times\frac{7\ell^2}{48}\times6\sqrt{\frac{k}{7m}}}$$

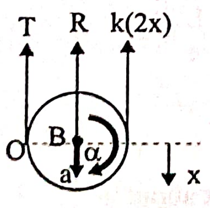
$$=2\times16\times\frac{7\times6^2}{48\sqrt{7}}=\frac{C}{\sqrt{k.m}}$$

$$\frac{C}{\sqrt{km}}=2\times2\sqrt{7}=4\sqrt{7}$$

44. Ans: (b)

Sol: Let the small displacement of the disk centre be x (downwards) as shown below then the extension in spring will be '2x'.

F.B.D. of disc



Writing equation of motion for disk in displaced direction,

$-T-R-2kx=ma$ .... (i)

[a is acceleration of centre of disk]

For no slip between disk and rope,

$a=r\alpha$ .... (ii)

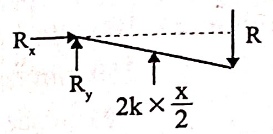
Writing equation of rotational motion for disk,

$$\Sigma M_B=I_B\times\alpha$$

$$T\times r-2kx\times r=\frac{mr^2}{2}\times\alpha$$

$$T-2kx=\frac{ma}{2}\ \text{.................. (iii) [using (ii)]}$$

F.B.D. of rod



[∵ If centre of disk is displaced by x then spring of stiffness 2k will deflect by x/2]

Writing equation of rotational motion for rod,

$\Sigma M_A=I_A\times\alpha=0$ [∵ Rod is massless]

$$\therefore\quad 2k\times\frac{x}{2}\times\frac{L}{2}=R\times L$$

$$\Rightarrow\quad R=\frac{kx}{2}\ \text{.................. (iv)}$$

Putting the values in (i) we get,

$$-\frac{ma}{2}-2kx-\frac{kx}{2}-2kx=ma$$

$$\Rightarrow\quad \frac{3ma}{2}=-\frac{9kx}{2}$$

$$a=-\frac{3kx}{m}$$

$$\therefore\quad \omega=\sqrt{\frac{3k}{m}}$$

Method-II [Energy method]

When the centre of the disk is displaced by x, then the energy of the system is written as

E = Energy of both springs + Translational kinetic energy of disk + Rotational kinetic energy of disk

$$E=\frac{1}{2}\times2k\times\left(\frac{x}{2}\right)^2+\frac{1}{2}\times k\times(2x)^2+\frac{1}{2}mv^2+\frac{1}{2}I\omega^2$$

[∵ If centre of disk is displaced by x then spring of stiffness 2k will deflect by x/2]

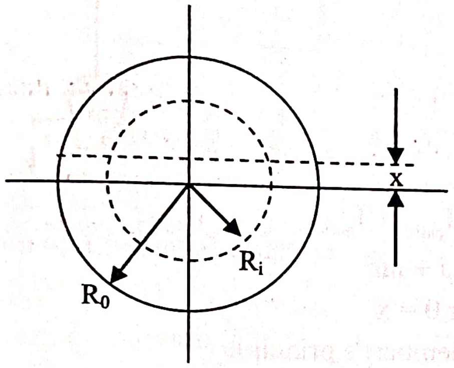
$$E = \frac{9kx^2}{4} + \frac{3}{4}mv^2 \quad \left[I = \frac{mr^2}{2} \ \& \ \omega = \frac{v}{r}\right]$$

$$\frac{dE}{dt} = \frac{9}{4} \times k \times 2x\frac{dx}{dt} + \frac{3}{4}m \times 2v\frac{dv}{dt} = 0$$

$$\Rightarrow a = -\frac{3kx}{m} \quad \Rightarrow \omega = \sqrt{\frac{3k}{m}}$$

45. Ans: 8.66

Sol: When the ball is displaced by small distance 'x' in vertical direction then the displaced volume is changed by $\pi R_0^2 x$ as shown in figure.



This leads to unbalanced buoyancy force of $\rho\pi R_0^2 xg$. The unbalanced buoyancy force tries to restore the ball in equilibrium position.

$\therefore$ Restoring force per unit displacement

$$= \frac{\rho\pi R_0^2 xg}{x}$$

i.e., $K = \rho\pi R_0^2 g$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{\rho\pi R_0^2 g}{\rho_s \times \frac{4}{3}\pi(R_0^3 - R_i^3)}} \quad \text{........(1)}$$

In equilibrium position weight = buoyancy force

$$\rho_s \times \frac{4}{3}\pi(R_0^3 - R_i^3) \times g = \rho \times \frac{4}{3}\pi R_0^3 \times \frac{1}{2} \times g$$

$$\therefore \rho_s \times \frac{4}{3}\pi(R_0^3 - R_i^3) = \frac{2}{3}\rho\pi R_0^3 \quad \text{......... (2)}$$

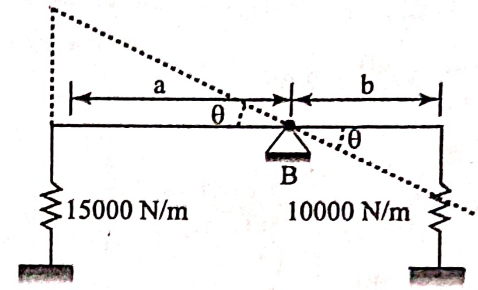
Substitute in equation (1)

$$\therefore \omega_n = \sqrt{\frac{\rho\pi R_0^2 \times g}{\frac{2}{3} \times \pi R_0^3}} = \sqrt{\frac{3}{2} \times \frac{g}{R_0}}$$

$$\sqrt{\frac{3}{2} \times \frac{10}{0.2}} = 8.66 \text{ rad/s}$$

46. Ans: (b)

Sol:



Given,

$k_1 = 15$ N/mm $= 15 \times 10^3$ N/m

$k_2 = 10$ N/mm $= 10 \times 10^3$ N/m

Mass moment of inertia of rocker, $I_B = 10^{-4}$ kg-m²

$a = 3.5$ cm $= 0.035$ m ,

$b = 2.5$ cm $= 0.025$ m

Applying moment about point B

$$I_B\ddot{\theta} + k_1 a^2\theta + k_2 b^2\theta = 0$$

The resonance occurs at natural frequency,

$$\omega_n = \sqrt{\frac{k_1 a^2 + k_2 b^2}{I_B}}$$

$$\omega_n = \sqrt{\frac{15 \times 10^3 \times 0.035^2 + 10 \times 10^3 \times 0.025^2}{10^{-4}}}$$

$$= \sqrt{246250} = 496.235 \text{ rad/s}$$

$$N = \frac{60 \times \omega_n}{2\pi} = 4738.7 \text{ rpm}$$

47. Ans: (c)

Sol: Given data, $m = 100$ kg, $\omega = 40$ rad/s

(i) $\xi_1 = 0.7$

$$\omega_{n1} = \sqrt{\frac{k_1}{m}} = \sqrt{\frac{640 \times 10^3}{100}} = 80 \text{ rad/s}$$

$$r_1 = \frac{\omega}{\omega_{n1}} = \frac{40}{80} = 0.5$$

(ii) $\xi_2 = 0.07$

$$\omega_{n2} = \sqrt{\frac{k_2}{m}} = \sqrt{\frac{640 \times 10^3}{100}} = 80 \text{ rad/s}$$

$$r_2 = \frac{\omega}{\omega_{n2}} = \frac{40}{80} = 0.5$$

(iii) $\xi_2 = 0.7$

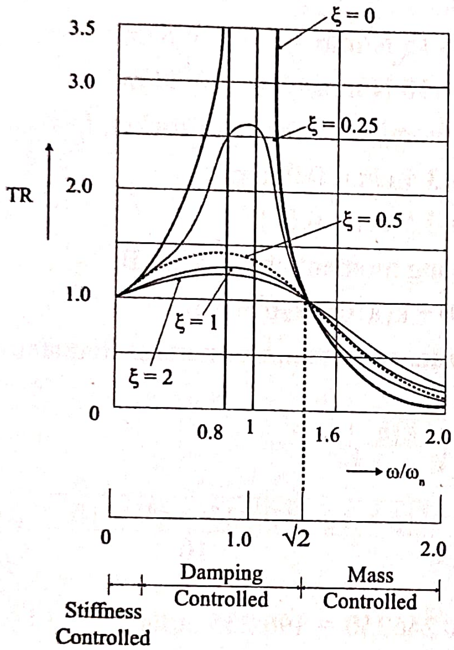
$$\omega_{n3} = \sqrt{\frac{k_3}{m}} = \sqrt{\frac{22.5 \times 10^3}{100}} = 15 \text{ rad/s}$$

$$r_3 = \frac{\omega}{\omega_{n3}} = \frac{40}{15} = 2.66$$

(iv) $\xi_2 = 0.07$

$$\omega_{n4} = \sqrt{\frac{k_4}{m}} = \sqrt{\frac{22.5 \times 10^3}{100}} = 15 \text{ rad/s}$$

$$r_4 = \frac{\omega}{\omega_{n4}} = \frac{40}{15} = 2.66$$



From this above graph,

(1) $0 < \frac{\omega}{\omega_n} < 1$; TR (Symbol) > 1

- TR increases when $\frac{\omega}{\omega_n}$ increases.
- TR decreases when ξ increases.

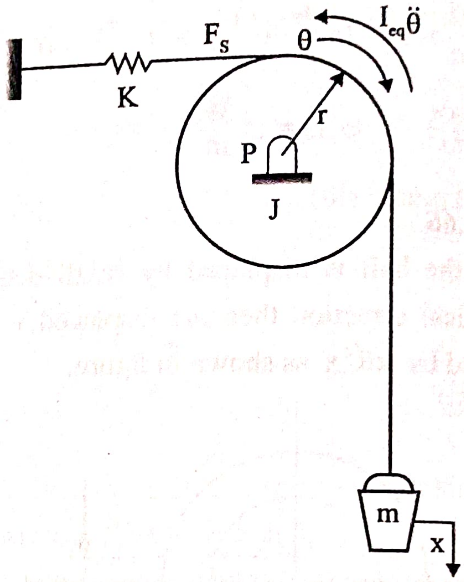
(2) when $\frac{\omega}{\omega_n} > \sqrt{2}$, TR $(\in) < 1$, i.e., the transmitted is always less than the exciting force.

- TR decreases when $\frac{\omega}{\omega_n}$ increases
- TR increases when ξ increases

$\Rightarrow \in_4 < \in_3 < \in_1 < \in_2$

48. Ans: (c)

Sol:



$I_{eq} = I_{pulley} + I_{mass}$

$= J + mr^2$

$x = r\theta = x_s$

D-Alembert's principle

Taking moment about pivoted point 'P' (let CCW couples are +ve)

$\because\ I_{eq}\ddot{\theta} + F_s.r = 0$

$\Rightarrow I_{eq}\ddot{\theta} + k\, x_s\, r = 0$

$\Rightarrow I_{eq}\ddot{\theta} + k\,(r\theta)\, r = 0$

$\Rightarrow I_{eq}\ddot{\theta} + k\,(r^2)\,\theta = 0$ equation of motion

$I_{eq}\ddot{\theta} + k_{eq}\theta = 0 \Rightarrow$ governing equation

$I_{eq} = J + mr^2$

$k_{eq} = kr^2$

$$\omega_n = \sqrt{\frac{k_{eq}}{I_{eq}}} = \sqrt{\frac{kr^2}{J + mr^2}}$$

49. Ans: (a)

Sol: The relation between the frequency at peak and natural frequency is given

$\omega_p = \omega_n.\sqrt{1 - 2\zeta^2}$ ....(1)

$\omega_p > \omega_n$

The relation between the damped frequency and natural frequency

$\omega_d = \omega_n\sqrt{1 - \zeta^2}$ ....(2)