Effect of lower pair:
As there are assumptions:

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- Linear motion pair (DOF = 1). (Prismatic / Revolute Joint)
- Only binary joint.
- Each linear motion pair restrict 2 DOF in planar conditions if there are 'j' number of linear motion binary pair then restricted degree of freedom by them = 2j

Effect of higher pair:

- Since each higher pair in planar mechanism is equivalent to two linear motion pairs, therefore it will permit 2 DOF & restrict 1 DOF. → Rolling + Slipping
- If there are 'h' number of higher pair then restricted DOF by higher pair = h

Physical Significance of DOF

1. DOF = 0 No relative motion can be transmitted.
→ structure (Simply supported Beam, C.Beam, overhanging beam, Perfect)
→ Force / Resisting Moment is transmitted in the structures
→ statically determinate structures.
→ No. of equilibrium equations = No. of unknowns / Reactions.

2. DOF < 1 Super structure / Redundent structure / statically indeterminate structure.
Ex. Redundent Truss, Fixed beam, Propped cantilever etc.
No. of unknown / Reactions > No. of Equilibrium equations.

3. DOF = 1 Completely Constrained Mechanism / Kinematic Mechanism.
4. DOF > 1 Incompletely constrained mechanism.

Significance:

- Structures are used to transfer the load where as mechanism are used to transfer the relative motion.
- Degree of freedom predicts about minimum number of output possible from a mechanism with respect to a given input.
- Degree of freedom also predict about number of equations required between input and output pair variables.
- Degree of freedom predicts about number of links or pair variable that should be control in order to obtained a constrained mechanism. → DOF = 1.

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DOF = 1   single i/p , single o/p.

DOF = 2   single i/p , two o/p's.
two i/p's , single o/p

★ **Degree of Freedom predicts the number of input variable/variables required to obtain a single/Multiple output motions.**

Synthesis of link.

For completely constrained Mechanism DOF = 1.
Lets assume only lower pair.

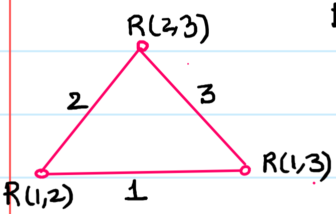
$$3N - 2j - 3 \geq 1.$$

$$N \geq \frac{4+2j}{3} \qquad 4+2j \rightarrow \text{even no.}$$

N → must be a +ve integer. (even no.)

N = 1   single link.
N = 2   Pair/Joint.
N = 3   Not possible.
$\boxed{N = 4 \quad \text{Possible}}$ → Minimum no. of links required to obtain a Kinematic Mechanism.
N = 5   Not possible
N = 6   Possible.

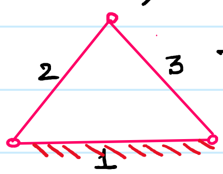
(1)



N = 3. j = 3.

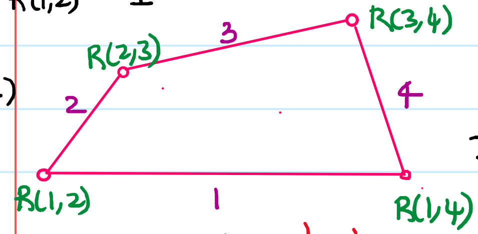
$(DOF) = 3N - 2j$
$= 3(3) - 2(3) = 3$

→ structure.



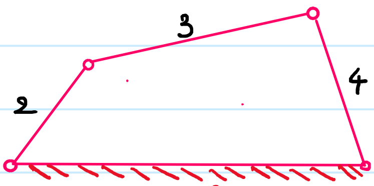
$DOF = 3(N-1) - 2j$
$= 3(3-1) - 2(3)$
$= 0$

(2)



Kinematic chain.

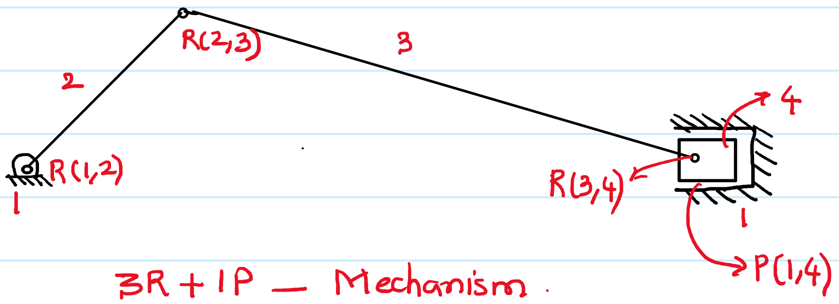
N = 4   j = 4
$DOF = 3N - 2j$
$= 3(4) - 2(4)$
$= 4$



1 (Kinematic Mechanism)
$= 3(4-1) - 2(4) = 1.$

★ The chain which can be converted into Kinematic Mechanism is called a Kinematic chain.

Crank slider Mechanism.



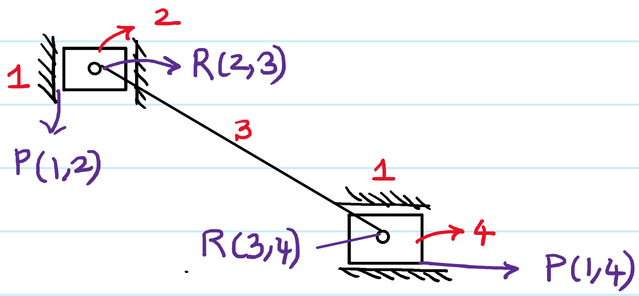
3R + 1P — Mechanism.

$N = 4 \quad j = 4 \quad h = 0$

$DOF = 3(N-1) - 2j - h$

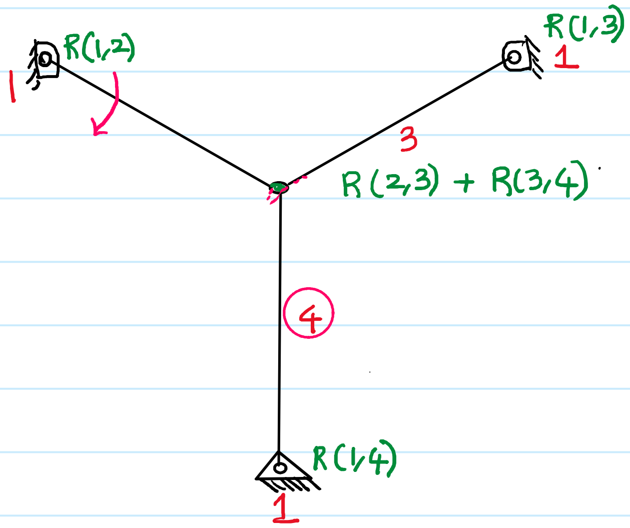
$DOF = 3(4-1) - 2(4) - 0 = 1$

Double slider Mechanism (2R + 2P) - Mechanism.



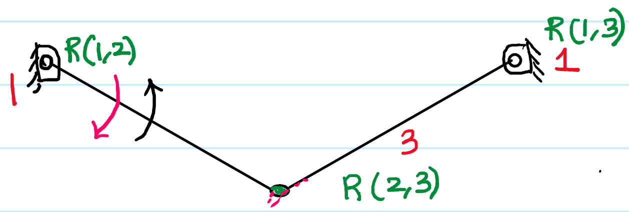
$N = 4 \quad j = 4 \quad h = 0$

$DoF = 3(4-1) - 2(4) - 0$
$= 1$



$N = 4$
$j = 5 \quad h = 0$
$DoF = 3(4-1) - 2(5)$
$= 9 - 10$
$= -1$

Super structure.



$N = 3$
$j = 3$
$DoF = 3(3-1) - 2(3)$
$= 0$

Structure.

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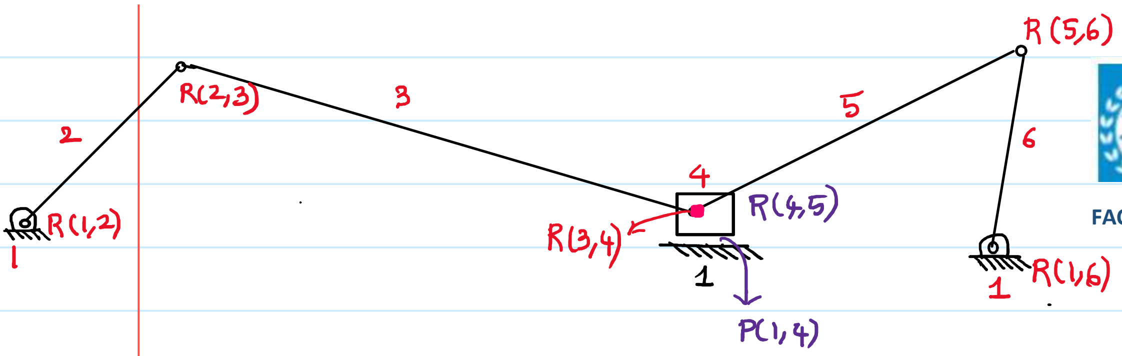
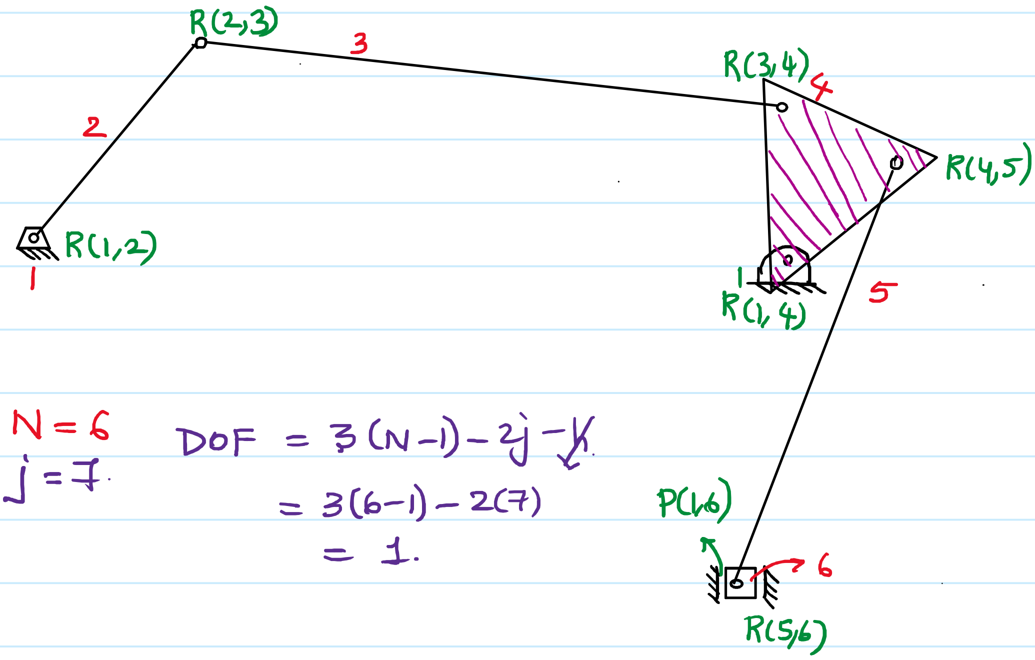


$N = 6 \quad j = 7 \quad h = 0$

$DOF = 3(N-1) - 2j - h \Rightarrow 3(6-1) - 2(7) = 1$

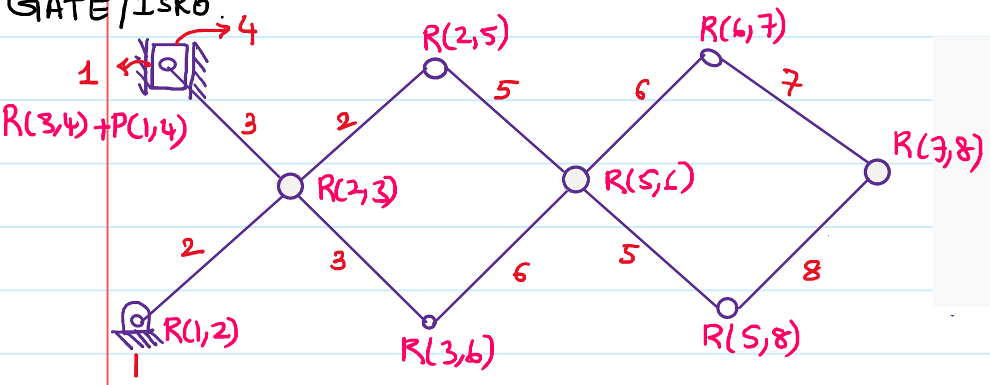


$N = 6$ $\quad DOF = 3(N-1) - 2j - \cancel{h}$

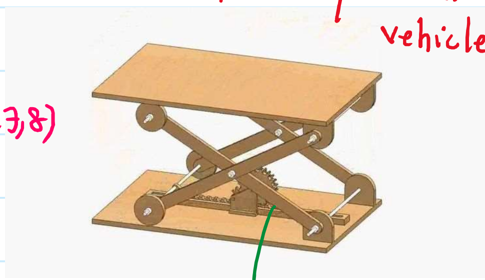
$j = 7$ $\quad = 3(6-1) - 2(7)$

$= 1.$

GATE/ISRO



lift door/ elevating vehicle.

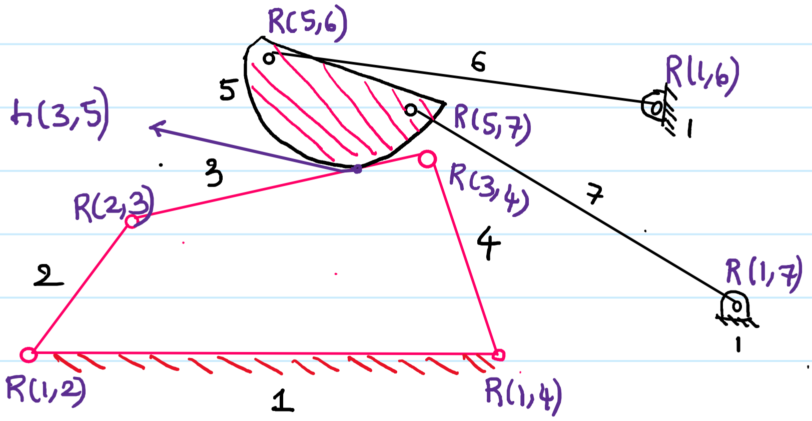


continous link.

$N = 8 \quad j = 10 \quad h = 0$

$DOF = 3(8-1) - 2(10) - 0 = 1.$

link 2, 3, 5, 6 → continous link.

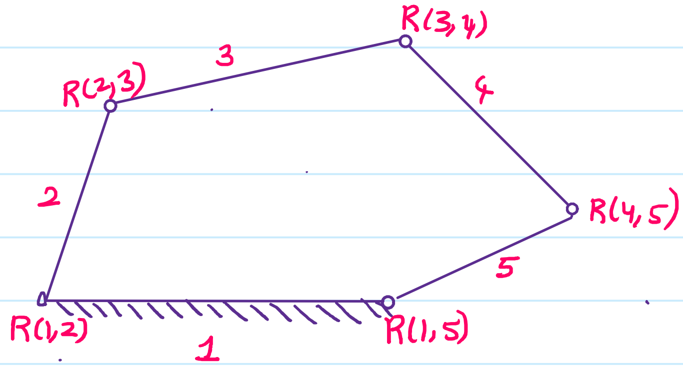


$N = 7 \quad j = 8 \quad h = 1$

Rolling + slipping

$$DoF = 3(7-1) - 2(8) - 1$$
$$= 18 - 16 - 1 = 1$$

case (i)



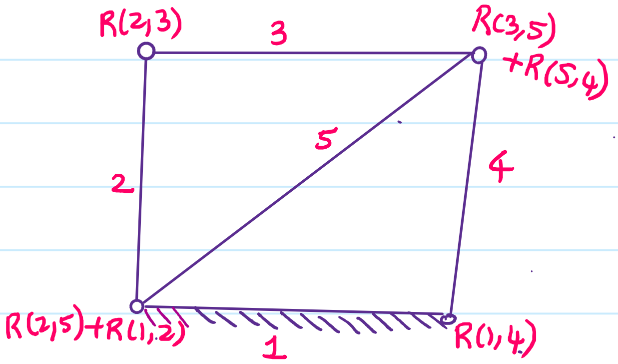
$N = 5 \quad h = 0$

$j = 5$

$$DoF = 3(5-1) - 2(5)$$
$$= 2$$

Unconstrained Mechanism.

case (ii)



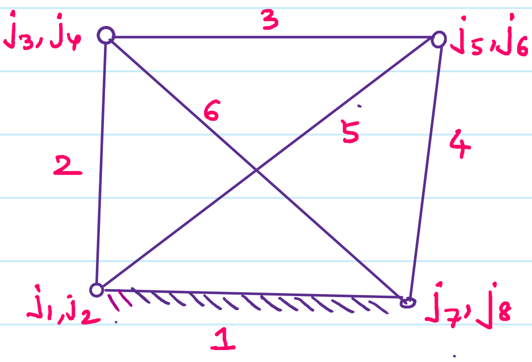
$N = 5$

$j = 6$

$$DoF = 3(5-1) - 2(6) = 0$$

Structure / Perfect truss.

case (iii)



$N = 6$

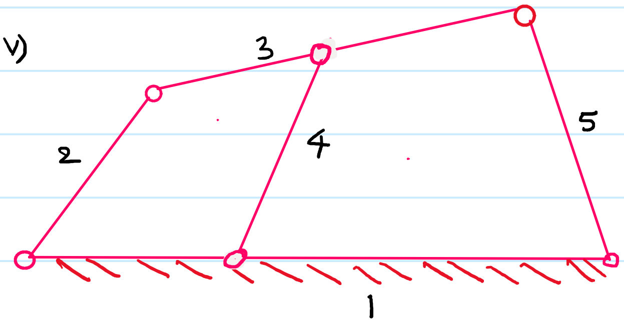
$j = 8$

$$DoF = 3(6-1) - 2(8) = -1$$

Superstructure.

Redundent truss.

case (iV)

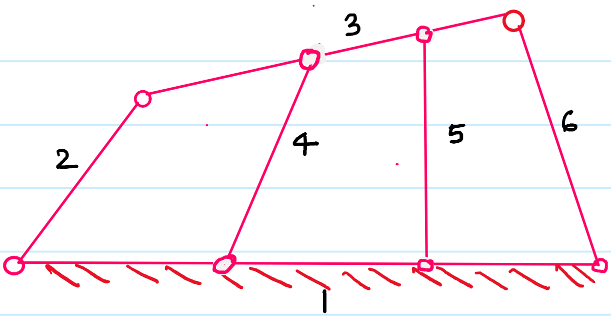


$N = 5$

$j = 6 \quad h = 0$

$DOF = 3(5-1) - 2(6) - 0 = 0$

case (V)



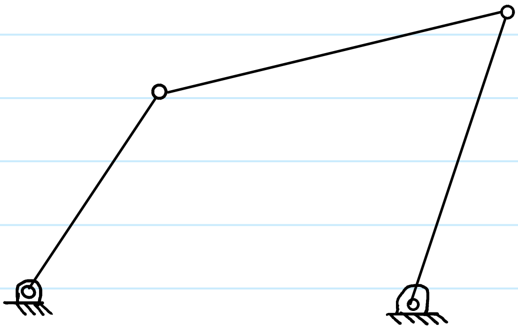
$N = 6 \quad j = 8 \quad h = 0$

$DOF = 3(6-1) - 2(8) - 0$

$= -1$

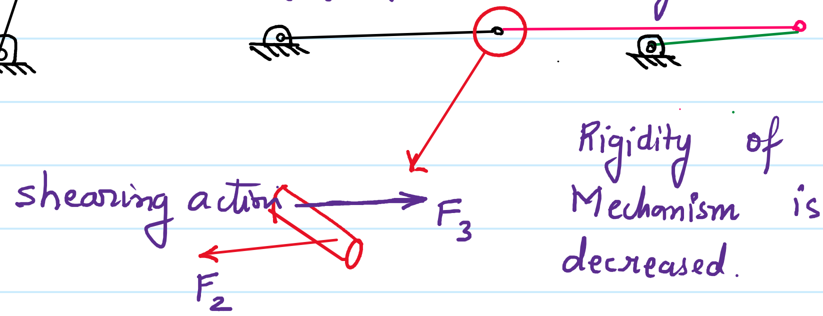
Observation -

Based on case 2,3,4,5 we can as the number of links increases the mobility(DOF) of mechanism decreases and stiffness of mechanism increases.



All the links are ||le at one instant.

Chances of failure of mechanism is increased for this configuration.



Rigidity of Mechanism is decreased.

We are required to increase the rigidity without effecting the mobility of mechanism.

In order to increase Rigidity without changing mobility we are required to make use of Redundent Parameters.

Redundent Parameters - The parameters whose presence or absence does not effect the mobility are called Redundant parameters

1. Redundant link
2. Redundant joint
3. Redundant DOF

For solving redundant parameters we are required to make use of modified Kutzbach's equation

$$DOF = 3(N - N_r - 1) - 2(j - j_r) - h - F_r$$

N — No. of links.

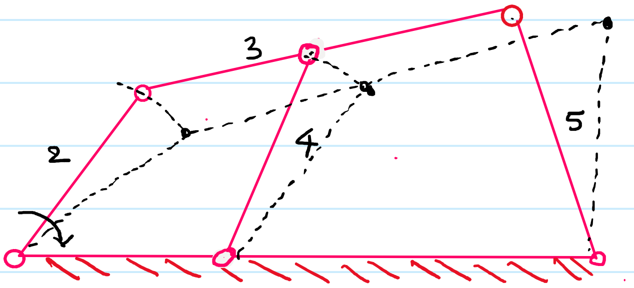
N_r — No. of Redundents.

j — No. of joints.

j_r — no. of redundent joints.

h — no. of higher pair.

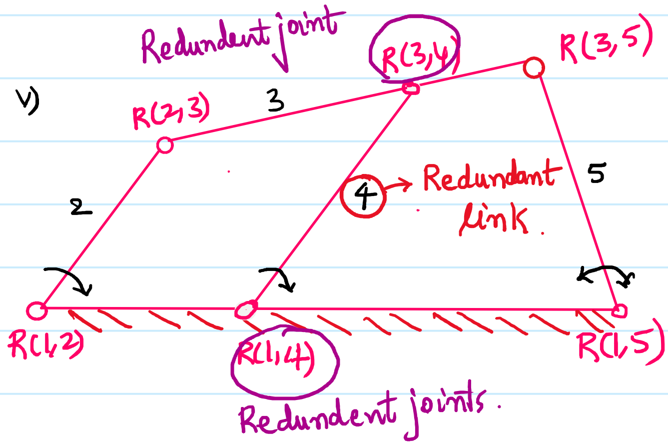
F_r — no. of redundent DOF.



Link 4 has the tendency to lock the mechanism.

Link 4 may experience tension or compression & hence all the links are rigid.

v)

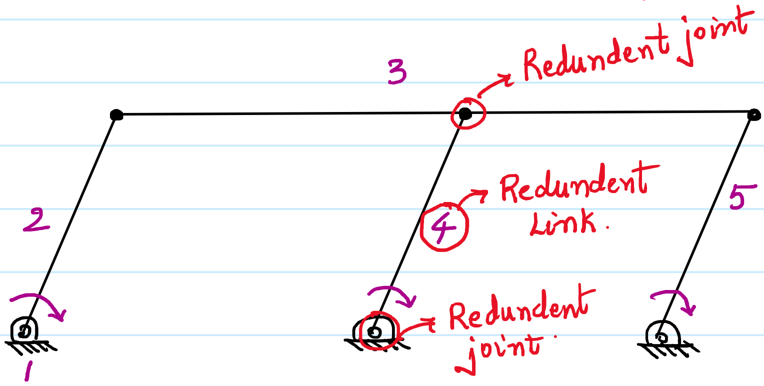


Link 2 & 4 are ||le to each other.
Link 2 & 4 will remain ||le to each other at all the instances of motion.
Link 4 will neither experience tension or compression.

Link 4 can be called as Redundent link. & R(3,4) + R(1,4) are called as Redundent Joints.

$N=5 \quad N_r=1 \quad j=6 \quad j_r=2 \quad h=0 \quad F_r=0 \Rightarrow DOF = 1.$

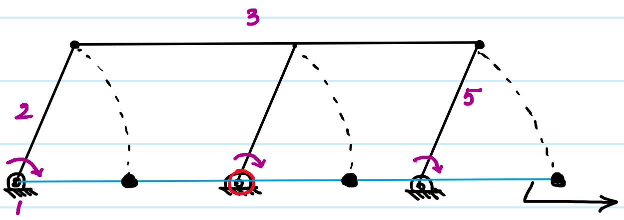
Gate.



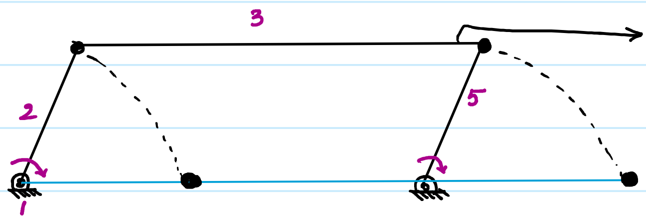
$N=5 \quad j=6 \quad h=0 \quad N_r=1 \quad j_r=2 \quad h=0 \quad F_r=0$

$$DOF = 3(N-N_r-1) - 2(j-j_r) - h - F_r$$

$$= 3(5-1-1) - 2(6-2) - 0 - 0 = 9-8 = 1.$$



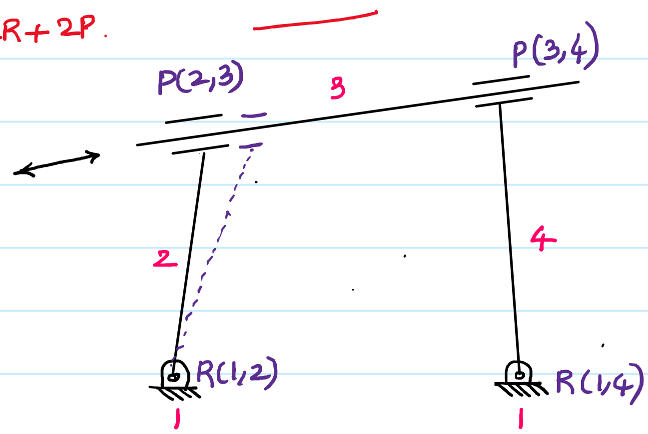
All the links are $//^{le}$ to each other.



This mechanism is more prone to failure.

Redundent link must be added to avoid failure.

2R+2P.



$N = 4 \quad j = 4 \quad h = 0$

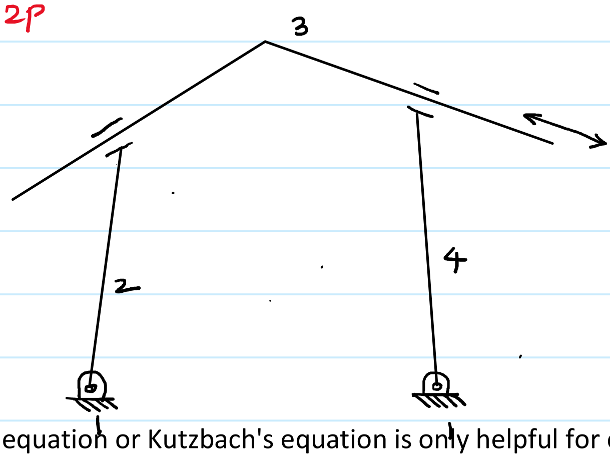
$N_r = 0 \qquad F_r = 1.$

$j_r = 0$

$$DOF = 3(N-N_r-1) - 2(j-j_r) - h - F_r$$
$$= 3(4-0-1) - 2(4-0) - 0 - 0$$
$$= 0$$

No relative motion can be transmitted.

2R+2P



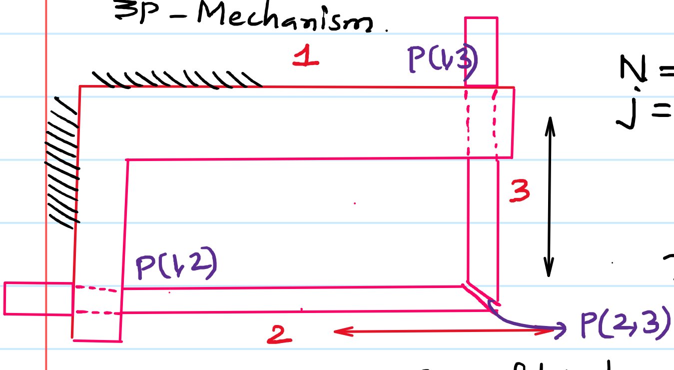
$N=4 \quad j=4 \quad N_r=0 \quad j_r=0 \quad F_r=0$

$h=0$

$$DOF = 3(4-0-1) - 2(4-0) - 0 - 0$$
$$= 1.$$

Grubler's equation or Kutzbach's equation is only helpful for estimating mobility of linkage/Mechanism. They are unable to predict how the mechanism must be constructed.

3P – Mechanism.



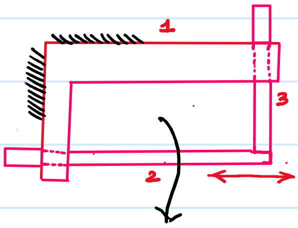
$N = 3 \quad N_r = 0$

$j = 3 \quad j_r = 0$

$h = 0$

$F_r = 0$

$$DOF = 3(N-1) - 2j - h.$$
$$= 3(3-1) - 2(3) - 0$$
$$= 0$$



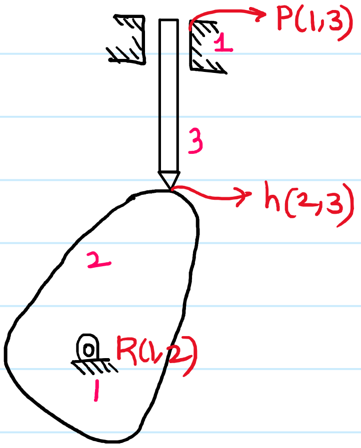
For this case link 3 doesnot move.

DOF = 1

$S_y = f(S_x)$ → independent variable

Kutzbach's eqn. is failing to predict whether the given arrangement is a mechanism.

Knife edge Cam and follower.



$N = 3$

$j = 2 \quad h = 1.$

$N_r = 0 \qquad F_r = 0$

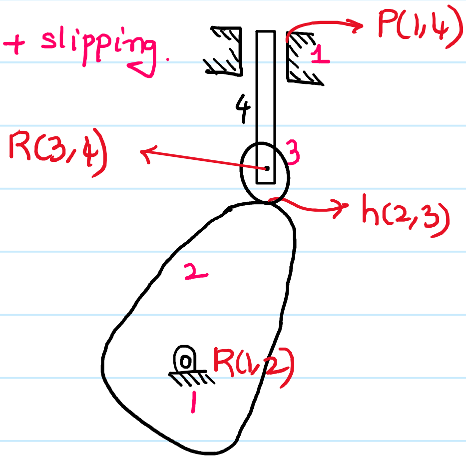
$j_r = 0 \qquad DOF = 3(N-1) - 2j - h$

$$DOF = 3(3-1) - 2(2) - 1$$
$$= 6 - 4 - 1 = 1.$$

$\mu_{sliding} > \mu_{Rolling}$

Roller follower and cam.

Rolling + slipping.

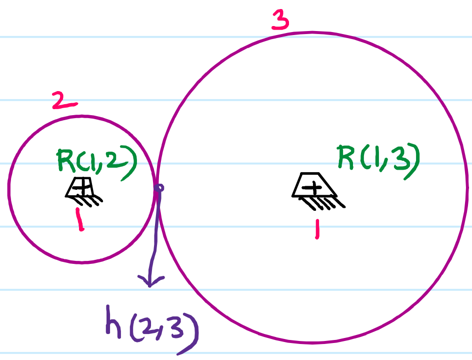


$N = 4 \quad j = 3. \quad h = 1.$

$N_r = 1 \quad j_r = 1. \quad F_r = 0$

$$DOF = 3(N - N_r - 1) - 2(j - j_r) - h - F_r$$
$$= 3(4-1-1) - 2(3-1) - 1 - 0$$
$$= 6 - 4 - 1 = 1.$$

Gear and Pinion.



$N = 3$

$j = 2$

$h = 1$

Pure Rolling $DOF = 3(N-1) - 2j - 2h$

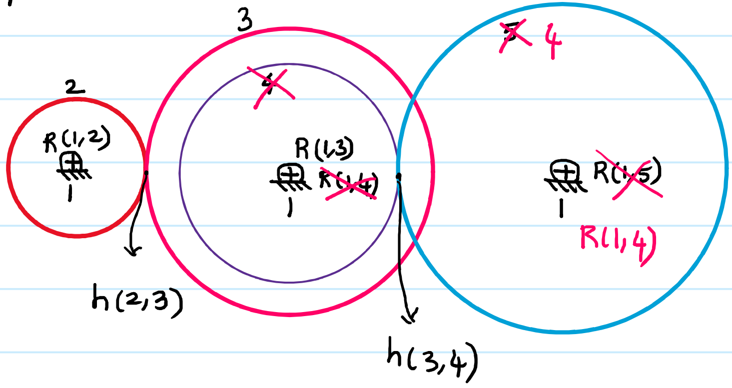
$$DOF = 3(3-1) - 2(2) - 2(1)$$
$$= 0$$

Gear 2 & Gear 3 rotate about the fixed axis.

Rolling + slipping

$$DOF = 3(N-1) - 2j - h$$
$$= 3(3-1) - 2(2) - 1 = 1$$

Compound Gear Train.



Gear 3 and 4 are compounded they will be treated as a single link.

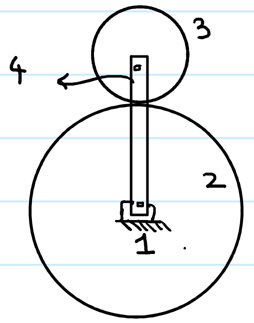
$N = 5 \quad j = 4 \quad h = 2$

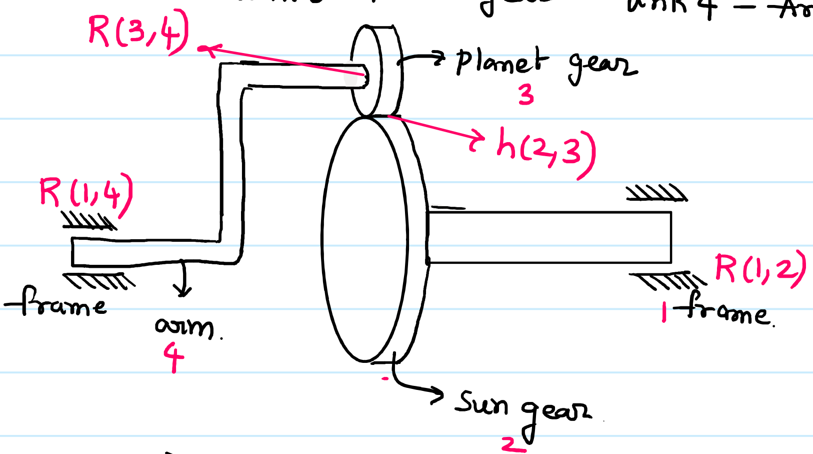
$$DoF = 3(5-1) - 2(4) - 2$$
$$= 12 - 8 - 2 = 2$$

$$DoF = 3(4-1) - 2(3) - 2$$
$$= 9 - 6 - 2 = 1$$

Epicyclic Gear Train.

link 1 – frame link 2 – Sun gear
link 3 – Planet gear link 4 – Arm.





$N = 4$

$j = 3 \quad h = 1$

Rolling + slipping

$$DoF = 3(N-1) - 2j - h.$$
$$= 3(4-1) - 2(3) - 1$$
$$= 2$$

Gear 2 – fixed axis.

Gear 3 –

I/p	o/p
Sun.	Planet, Arm.
Sun, Arm.	Planet.

Observations

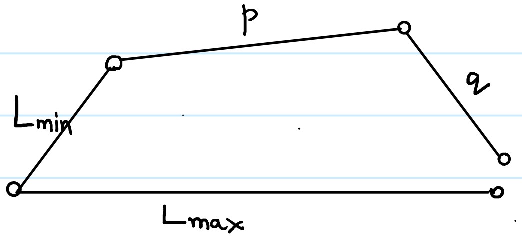
- If a mechanism consist of all revolute pairs then number of links required to form a kinematic/Completely constrained mechanism is 4.

- If a mechanism consist of all prismatic pairs then number of links required to form a kinematic/Completely constrained mechanism is 3.

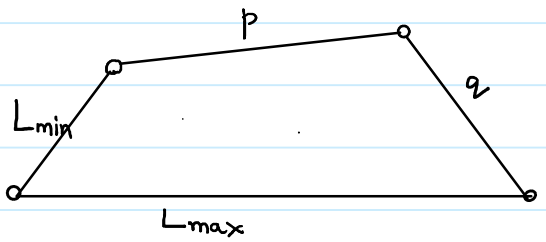
- If a mechanism consist of lower and higher pair then number of required to form a kinematic /Completely constrained mechanism is 3.

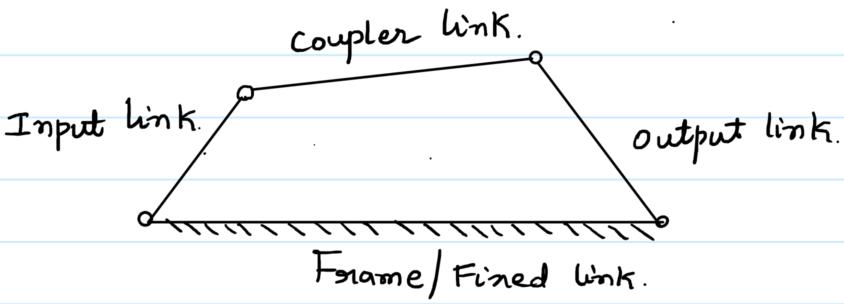
Types of chain.

$L_{max} > L_{min} + p + q \longrightarrow$ open chain. (Robotic Arm)



$L_{max} < L_{min} + p + q \longrightarrow$ Closed chain $\xrightarrow{\text{by fixing any one link}}$ Mechanism.





I/p link & O/p link are connected to frame they move/rotate about the fixed point.

Coupler link will float in the plane of rotation.

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Movability analysis of 4-bar mechanism:

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On the basis of range of movement the links of a 4-bar mechanism are classified as:

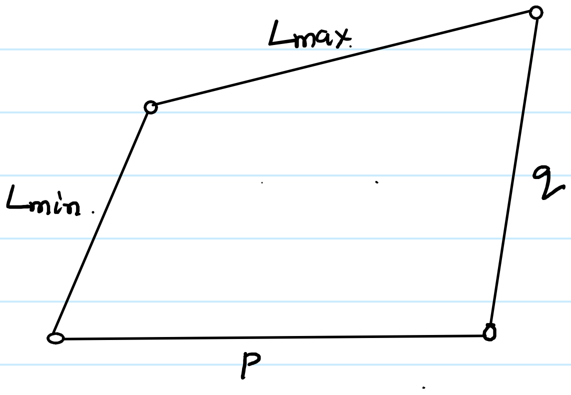
1. Frame or Fixed Link: The link which cannot move.
2. Crank: The link which can execute full circular motion.
3. Coupler: The link which is opposite to fixed link or the link which connects input and output.
4. Rocker: The link which oscillates.

- On the basis of input and output there are following four type of mechanism:

Input Output

1. Crank - Crank or double crank mechanism
2. Crank - Rocker mechanism
3. Rocker - Rocker mechanism
4. Rocker - Crank mechanism

Grasshof law:- It states that summation of shortest link and longest link lenghts must be less summation of lenghts of other links in order to transfer the relative motion continously.



$L_{max} + L_{min} < p + q$.

→ class-I linkage.

In order to transfer relative motion continously there must be minimum one link undergoing circular motion.

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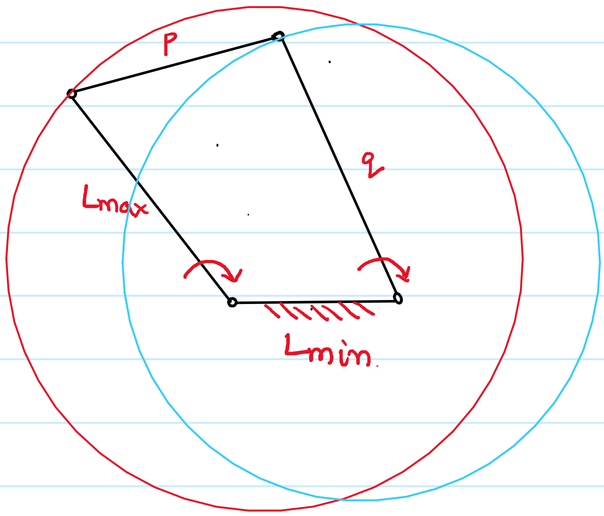
The type of inversion is decided on the basis of location of L_{min}.

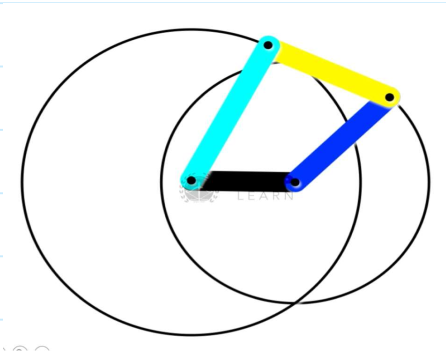
1. L_{min} can be fixed.
2. L_{min} can be adjacent to fixed.
3. L_{min} can be opposite to fixed.

Inversion 1. — L_{min} is fixed.

Crank – Crank Mechanism.

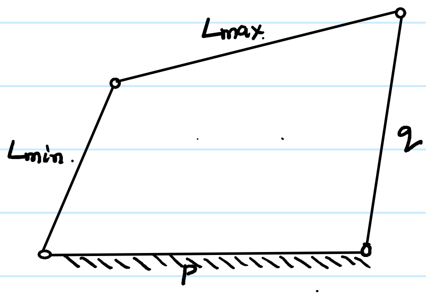
Locus of L_{max} and q links will be circle.

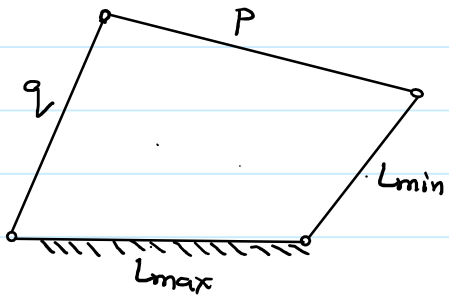


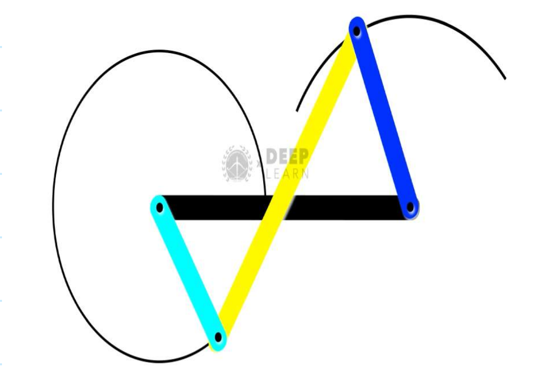


Inversion – II Link adjacent to L_{min} is fixed. fixed link (L_{max} / P)

Crank – Rocker / Rocker-crank Mechanism.



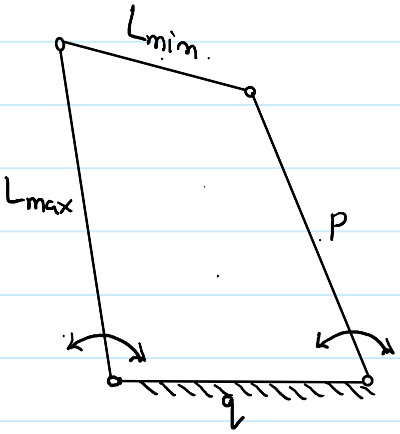




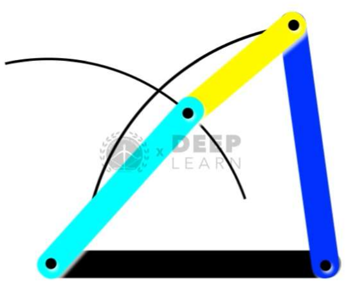
L_{min} completes circular motion / crank.

Link q oscillates / Rocker.

Inversion -III. Link opposite to L_{min} is fixed
Rocker - Rocker Mechanism.



links L_{max} & P will oscillate
L_{min} will complete circular motion.

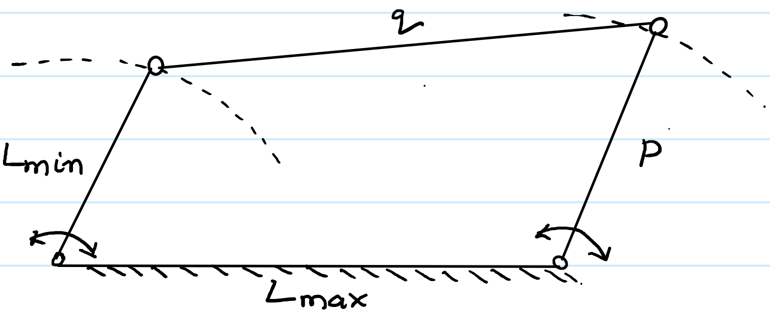


Class-II linkage/Non-Grashof linkage.

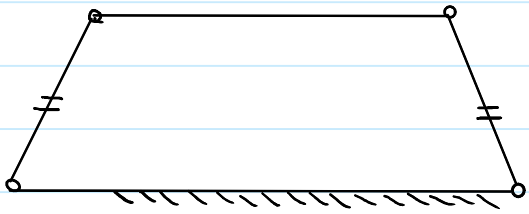
$$L_{min} + L_{max} > P + q.$$

None of the links will complete circular motion. Continous Relative motion is not transmitted.

Fixed link - L_{min} / L_{max} / P / q $\longrightarrow$ Rocker-Rocker Mechanism
Triple Rocker Mechanism.



Ackerman steering



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Class – III linkage / Special Grashof linkage.

$$L_{min} + L_{max} = p + q.$$

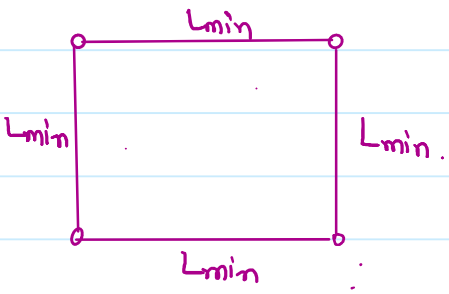
case (A) $L_{min} = L_{max} = p = q$ ⟶ Rhombus Linkage.

All the links are of same lenghts.

No. of different links = 1.

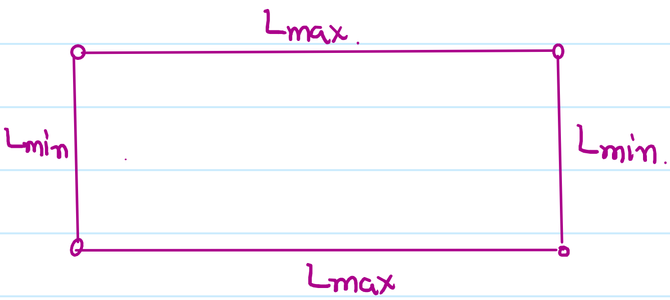
No. of inversions = 1

Fixed link :- $L_{min} / L_{max} / P / Q$ ⟶ Crank – Crank Mechanism.



case (B) $L_{min} = p$ $L_{max} = q$. ⟶ Parallelogram linkage.

case (i) If the links of equal lenghts are opposite to each other.

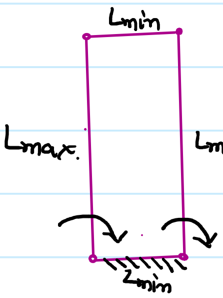


No. of inversions = No. of different links = 2

Inversion – I

Fixed link – L_{min}.

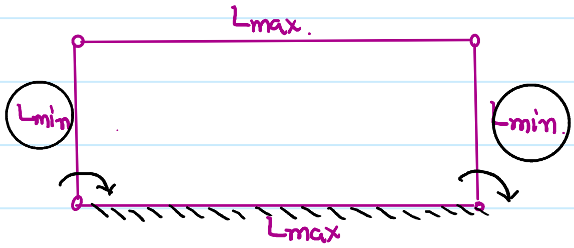
Crank – Crank Mechanism.



I/p & o/p → o'lur motion.

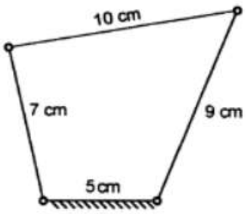
Inversion – II.

Fixed link. – L_{max} (adjacent to L_{min})



Identify the type of Mechanisms

(i)



(a)

$L_{min} = 5cm$

$L_{max} = 10cm.$

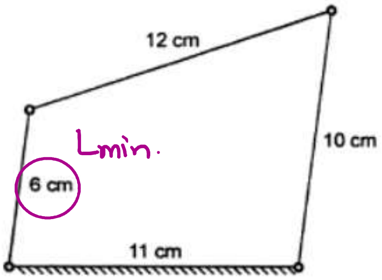
$P = 9cm. \quad Q = 7cm.$

$L_{max} + L_{min} < P+Q \longrightarrow$ class-I linkage.

$5+10 < 9+7$

L_{min} is fixed crank-crank.

(ii)



(c)

$L_{min} = 6cm \qquad P = 10cm.$

$L_{max} = 12cm. \qquad Q = 11cm.$

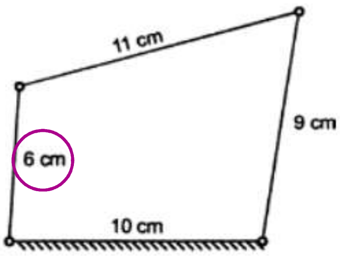
$L_{min} + L_{max} < P+Q$

$6+12 < 10+11 \longrightarrow$ class-I linkage.

Link adjacent to L_{min} is fixed.

Crank - Rocker Mechanism.

(iii)



(b)

$L_{min} = 6cm \qquad L_{max} = 11cm.$

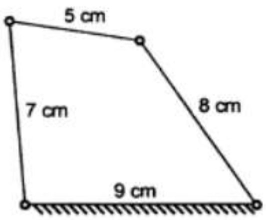
$P = 9cm \qquad Q = 10cm.$

$L_{min} + L_{max} < P + Q.$

$6+11 < 9+10 \longrightarrow$ class-I. linkage

Crank - Rocker Mechanism

(iv)



(d)

$L_{min} = 5cm \quad L_{max} = 9cm.$

$P = 7cm \qquad Q = 8cm.$

$L_{min} + L_{max} < P+Q.$

$5+9 < 7+8 \longrightarrow$ Class I linkage.

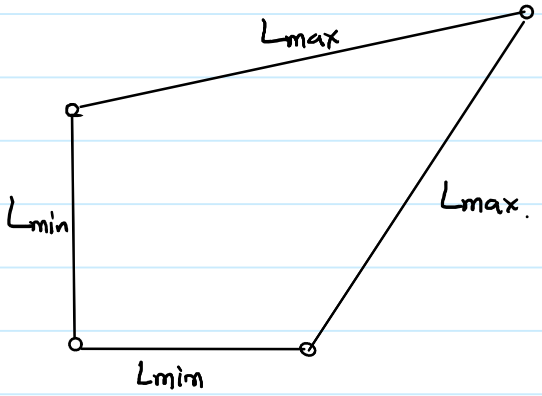
Rocker-Rocker Mechanism.

$L_{min} = 5cm$ will complete circular motion.

case (ii) $L_{min} = P$ $L_{max} = q$

Links of equal lenghts are adjacent to each other.

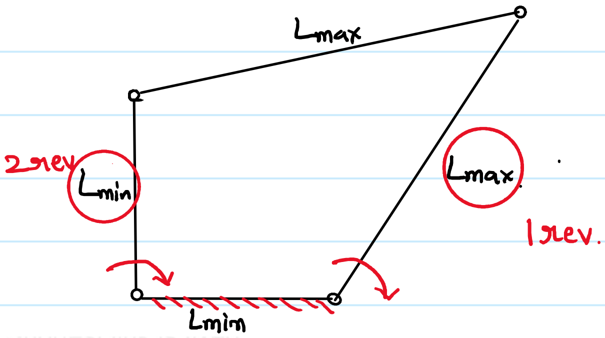
⟶ Deltoid / Kite linkage



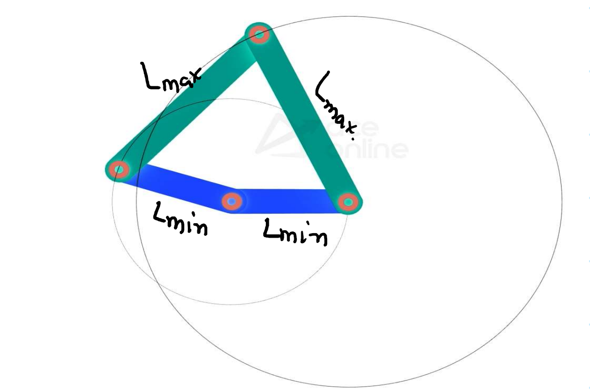
Inversion – I.

Fixed link – Lmin.

Crank-Crank Mechanism.



For 1 rev of Lmax the Lmin will complete 2 rev.



Inversion – II

Fixed link – Lmax. (Link adjacent to Lmin)

Crank – Rocker Mechanism.