Vibration - A motion which repeats after a fixed interval of time period is called as **vibratory motion.**

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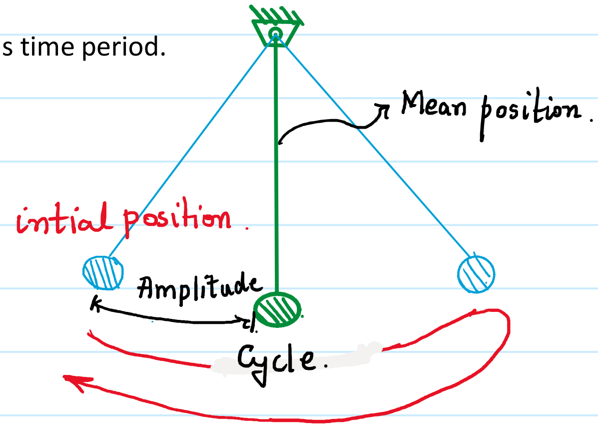
Cycle - Motion of body to reach the same starting point when is displaced is called as cycle.

Time Period - The time required to complete one cycle is called as time period.



Frequency/Speed - Number of Cycles completed per unit time.

circular frequency

ω — rad/sec.

linear frequency f - cycle/sec./Hz.

$$\omega = 2\pi f$$

$$f = \frac{1}{T} \qquad T - \text{Time period}$$

$$\boxed{\omega = \frac{2\pi}{T}}$$

Amplitude - Maximum Deviation from the mean position is called Amplitude.

Mean Position - About this position the body oscillates and at mean position the system will be in equilibrium.

Vibratory motion will never start on its own, for the vibratory to be started some disturbance/external source is required.

For the vibratory motion to occur the system must have

1. Elasticity/Stiffness
2. Inertia.

Vibrations will continue until the imparted energy is fully lost.

If the system continues to oscillate without loss in energy, then system is executing undamped vibrations.**(Conservation of Energy is valid)**.

If there is loss in energy while executing oscillations then the damping is present.

Damping represent the loss in energy.

Classification of Vibration on the basis of excitation.

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1. Free Vibrations - System continues to oscillate under the influence of initial condition (disturbance/input/Displacement/Velocity).
2. Forced Vibration - System continues to oscillate under the influence of some external source.

Free Vibration →
- Free undamped Vibration – under the influence of intial condition only. No loss in energy after each cycle. (conservation of Energy is valid)
- Free damped Vibrations – under the influence of intial condition only. Loss in energy occurs after each cycle.

Free undamped Vibrations – No. loss in energy / System will never return to rest; Hypothetical situation.

Free damped Vibration – Loss in energy after each cycle. System will come to rest finally.

Types of External Excitation/Forced Vibration

1.Excitation due to Unbalance masses
   a. Unbalance Rotating Mass
   b. Unbalance Reciprocating Mass

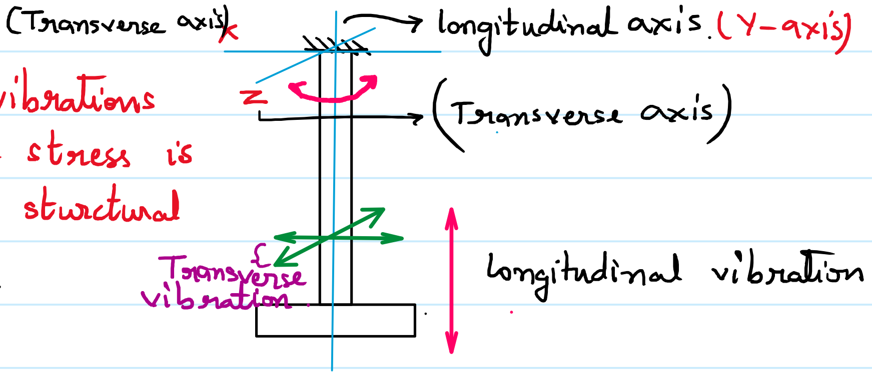
2. Base Excitation.

Classification of Vibrations on the basis of direction.

1.Longitudinal Vibrations
2.Transverse Vibrations
3.Torsional Vibrations

Longitudinal Vibrations - If the particles tend to move along the longitudinal axis thereby causing the elongation and shortening of the structural member , these oscillations are called Longitudinal Vibrations

Due to these vibrations variable axial stress is setup in the structural member (shaft).



Transverse Vibrations - If the particles tend to move laterally along the Transverse axis thereby causing the bending of the structural member , these oscillations are called Transverse Vibrations.

Due to these the structural member will be subjected to variable bending stress.

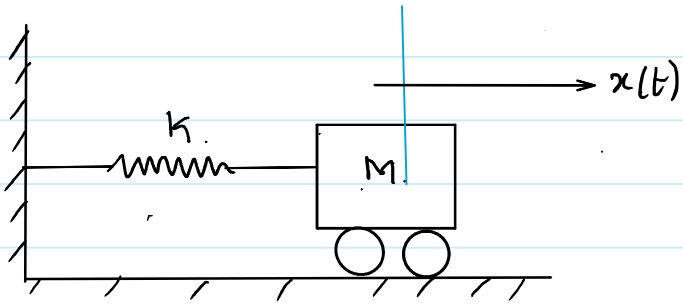
Torsional Vibrations - If the particles tend to move about the longitudinal axis thereby causing the twisting and untwisting of the structural member , these oscillations are called Torsional Vibrations.
The structural member is subjected to variable torsional shear stress.

Classification of Vibrations on the basis of DOF.

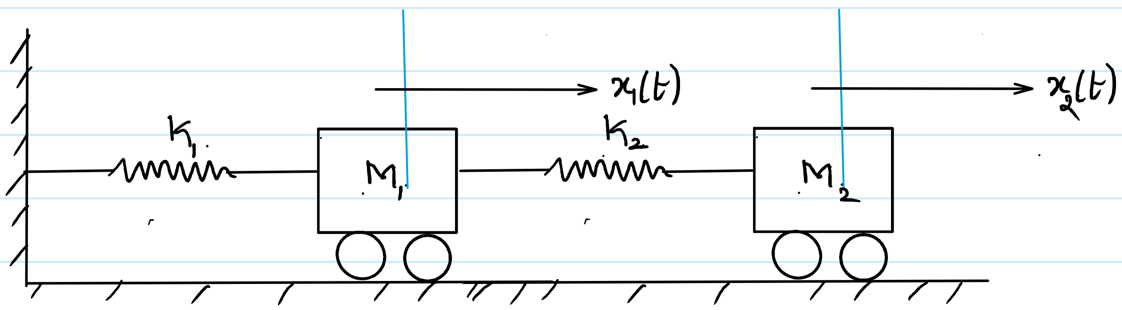
1. Single DOF system (SDOF) — U.G. level.
2. Multiple DOF system (MDOF) — ✓
3. Infinite DOF system.— [elementary level].

No. of DOF = No. of equation of motion.

DOF = 1.

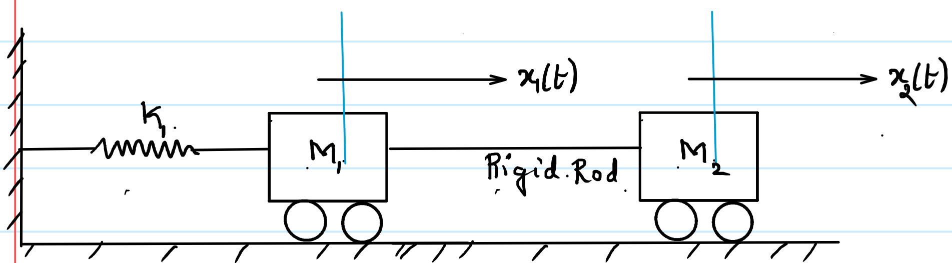


x_1, x_2 – independent variable.



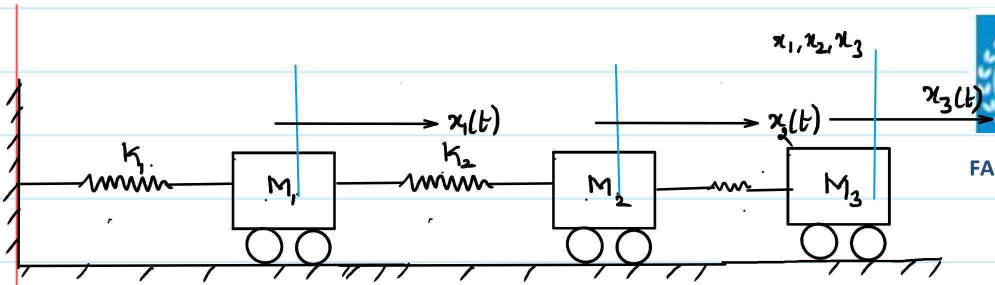
DOF = 2.

x_1, x_2



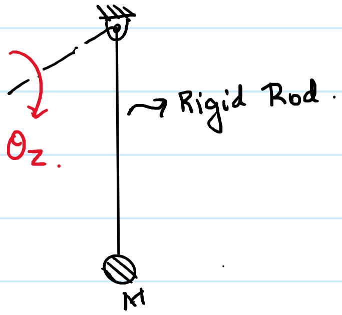
$x_1 = x_2$

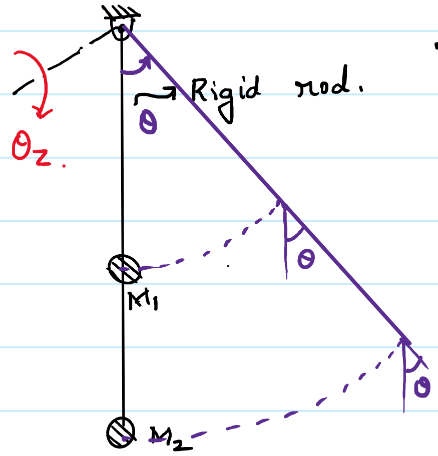
DOF = 1.



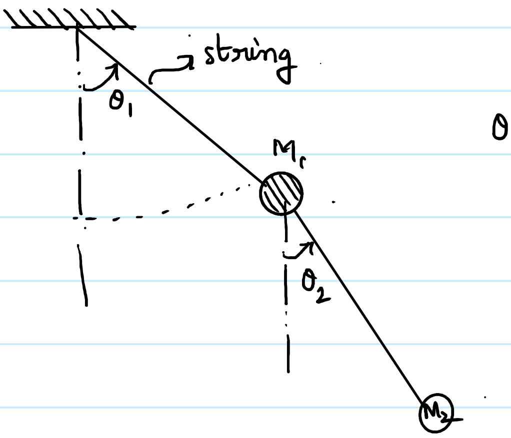
x_1, x_2, x_3 — independent variables

DoF = 3



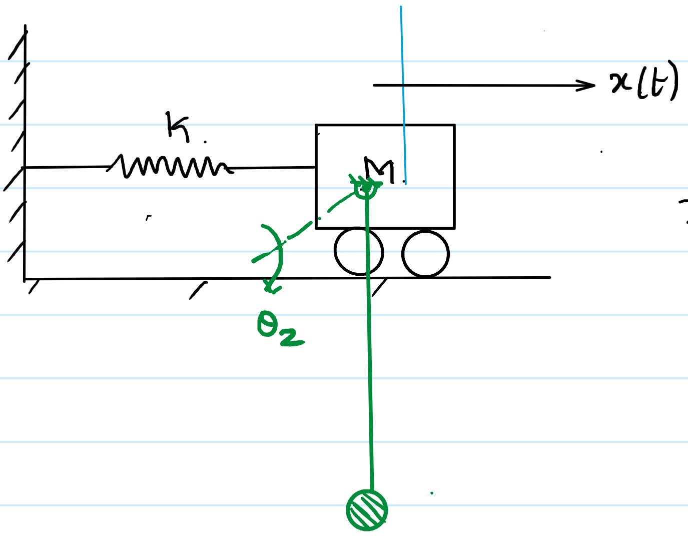


DoF = 1



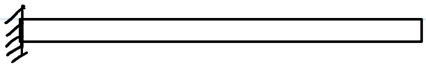
θ_1, θ_2 are independent variable

DoF = 2



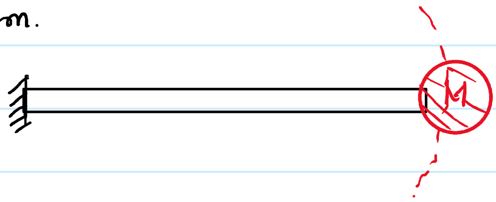
DoF = 2

Rigid Beam.



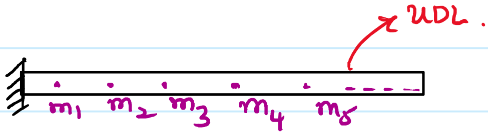
$\boxed{DoF = 0}$

Flexible Massless beam.



DoF = 1.

Flexible beam of mass "m"



DoF $= \infty$

System Parameters

1. Inertia
2. Restoration
3. Damping

Inertia – It is the tendency of a object to be rest. } Newton 1st law.
Resistance against the change in state.

Motion → Translation – Mass of object is the measure of inertia
Motion → Rotation/Angular motion – Second moment of mass is the measure of inertia.

Mass Moment of Inertia $= \int r^2 \cdot dm.$

r → ⊥lar distance from axis of rotation.

$I = \bar{I} + md^2$

$I = mk^2$

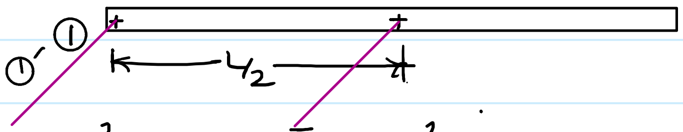
K – Radius of gyration.

$\bar{I}$ – Moment of Inertia about the centriodal axis.

m – mass.

d – ⊥lar distance b/w. (C.A) and other axis.

Slender Rod.

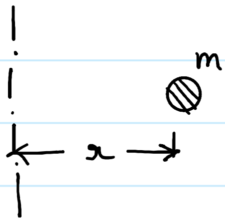


$$I = \frac{ML^2}{3} \qquad \bar{I} = \frac{ML^2}{12}$$

$$I = \bar{I} + Md^2$$

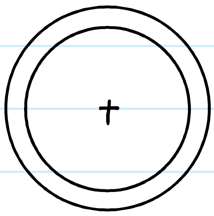
$$I_{1-1} = \frac{ML^2}{12} + M \cdot \left(\frac{L}{2}\right)^2 = \frac{ML^2}{3}.$$

Point Mass.



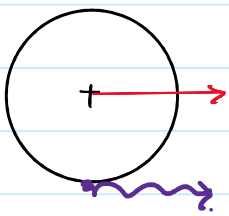
$$I = mr^2$$

Rim.



$$I = mR^2$$

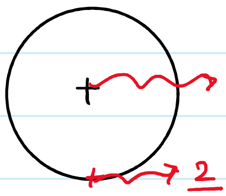
Cylinderical Disc.



$$\frac{MR^2}{2}$$

$$\frac{MR^2}{2} + MR^2 = \frac{3MR^2}{2}.$$

Spherical Roller

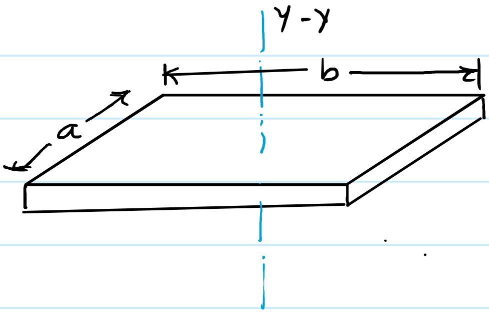


$$\frac{2}{5} mR^2$$

$$\frac{2}{5} mR^2 + mR^2$$

$$= \frac{7}{5} mR^2.$$

Rectangular plate.



$$\bar{I}_{y-y} = \frac{1}{12} \cdot (a^2 + b^2)$$

2. Restoration – It is the ability of system to return back to the initial position.

1. Springs.
2. Gravity
3. Fluid.

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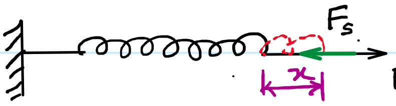
Hooke's law of spring.



F – Applied force / External force

F_s – Spring force / Internal force.

$F \propto x$.

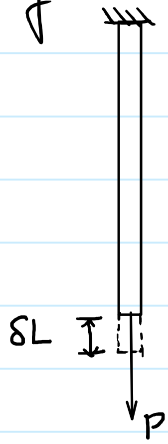
$\boxed{F_s \propto -x.}$

$F_s = -K \cdot x$

↳ spring constant.

F_s will be resisting the displacement or elongation.

Axial loading



elongation $\delta L = \dfrac{PL}{AE}$

stiffness – $\dfrac{\text{load}}{\text{deflection}} = \dfrac{P}{\delta L} = \dfrac{AE}{L}$

axial stiffness.

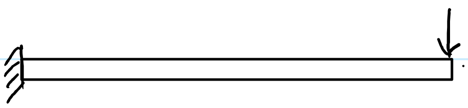
stiffness – Rigidity per unit lenght.

AE – Axial Rigidity. A – c/s area. E – Young Modulus.

Rigidity = f (c/s area, Material constant)

Rigidity = f (c/s property, Material property)

Bending



$\text{stiffness} = \dfrac{\text{load}}{\text{Deflection}} = \dfrac{W}{\dfrac{WL^3}{EI \times \text{const}}} = \dfrac{EI}{L^3} \times \text{constant}$.

EI – flexural Rigidity. Rigidity = f (c/s. property, Material property)

I – Area moment of inertia.

Twisting / Torsion.

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$$

Torsional stiffness

$$\Rightarrow \frac{T}{\theta} = \frac{GJ}{L}$$

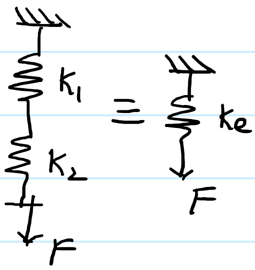
GJ – Torsional Rigidity.

J – Area Polar moment of inertia.

Spring in series and Parallel combinations.

$F_{s_1} = F_{s_2} = F.$

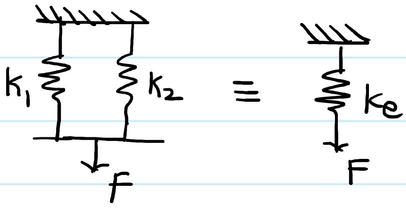
$F = k_1\delta_1 = k_2\delta_2 = k_e\delta_e$



$$\delta_e = \delta_1 + \delta_2$$

$$\frac{F}{k_e} = \frac{F}{k_1} + \frac{F}{k_2}$$

$$k_e = \frac{k_1 k_2}{k_1 + k_2}$$

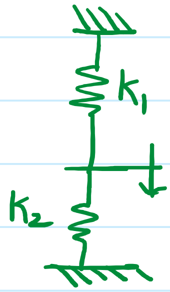


$$\delta_1 = \delta_2 = \delta_e$$

$$F_{s_1} + F_{s_2} = F$$

$$k_1\delta_1 + k_2\delta_2 = k_e \cdot \delta_e$$

$$k_e = k_1 + k_2$$

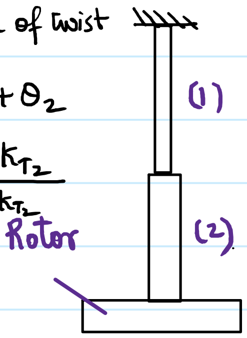


Shafts in series.

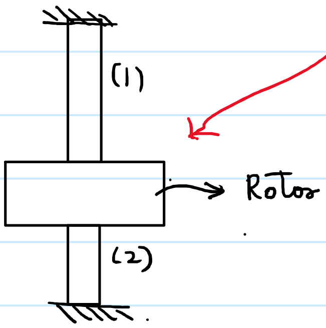
θ – Angle of twist

$$\theta_e = \theta_1 + \theta_2$$

$$k_e = \frac{k_{T_1} \cdot k_{T_2}}{k_{T_1} + k_{T_2}}$$



Shafts in parallel.



$$\theta_e = \theta_1 = \theta_2$$

$$k_{T_e} = k_{T_1} + k_{T_2}$$

k_T – Torsional stiffness.

$$k_T = \frac{T}{\theta} = \frac{GJ}{L}$$

$$K_{axial} = \frac{AE}{L}$$

$$K_{bending} = \frac{EI}{L^3}$$

$$K_{Torsional} = \frac{GJ}{L}$$

$$\boxed{\text{spring stiffness} \quad K = \frac{Gd^4}{8D^3 n}}$$

$$K \propto \frac{1}{\text{Lenght of member}}$$

$$K \propto \frac{1}{\text{no. of turns}}$$

If the Spring is cut into 'n' no. of pieces. then the stiffness of each Spring will be increased by n times.

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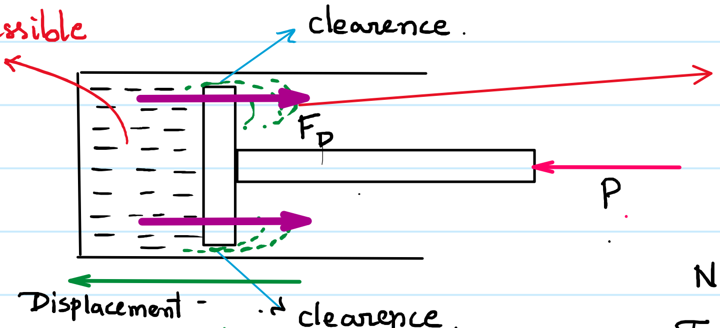
Damping – Ability of system to resistance the relative motion.

1. Coloumb's Damping / Frictional Damping → frictional force ∝ –velocity.
2. Hysteresis Damping.
3. Viscous Damping ✓

Cylinder – Piston



F_D – Damping force.

Newton's law of Viscosity

$$\tau \propto \frac{du}{dy}$$

$$\tau = \frac{\mu \cdot V}{h}$$

$$P = \tau \times A = \frac{\mu \cdot V}{h} \times A$$

μ – constant

h – constant / clearence

A – Area – constant

$$F_D = \frac{\mu \cdot V}{h} \times A$$

$$F_D \propto -\text{velocity}$$

$$F_D = \underbrace{\left(\frac{\mu \cdot A}{h}\right)}_{\text{Damping coefficient}} \cdot V = CV = C\dot{x}$$

$$\boxed{F_D \propto -\dot{x}}$$

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The process of deriving the mathematical equation for a system which is having motion of masses is **mathematical modelling.**

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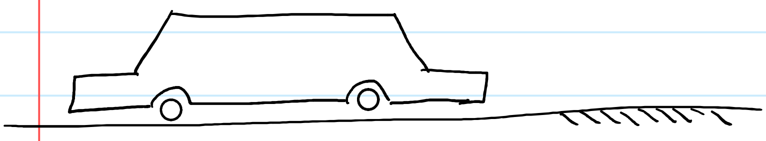
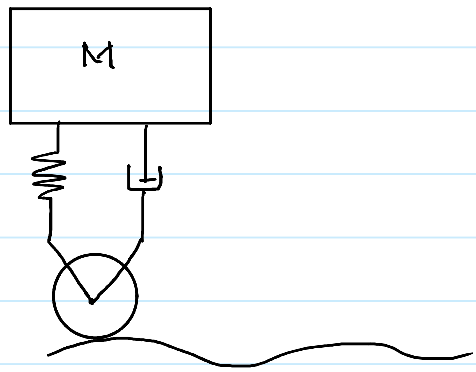


Lumped model of car.



Methods for deriving equation of motion

1. D'Alembert's Principle (SDOF)
2. Conservation of Energy (SDOF | MDOF)
3. Rayleigh Method
4. Deflection Method
5. Galerkin Method
6. Lagrangian Method.

D'Alembert Principle - method of converting a **kinematic behaviour to static** by applying a **virtual force in direction opposite to the direction** of motion.

Inertia force is in equilibrium with the net forces acting on the system.

Newton's 2^{nd} law. $\Sigma F = ma$.

Newton's 1^{st} law. Inertia $\propto$ – change in state.

Translation $F_I = -ma$.

Rotation $T_I = -I\alpha$.

D'Alembert Principle $\Sigma F - ma = 0$

$\Sigma F - (-ma) = 0$

$\Sigma F + ma = 0$

By the reversing the direction of force causing motion and adding with net forces acting on system the dynamic problem is converted into a equivalent static equilibrium problem.

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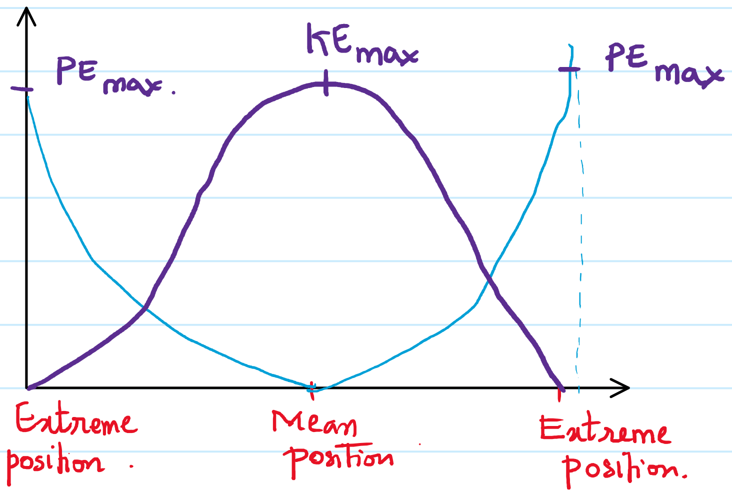
(2) Conservation of Energy.

$$E = P.E + KE = \text{constant}$$

(3) Rayliegh Method.

$$(KE)_{max} = (PE)_{max}$$

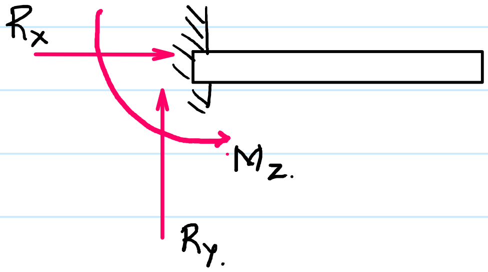


Rules for Drawing FBD

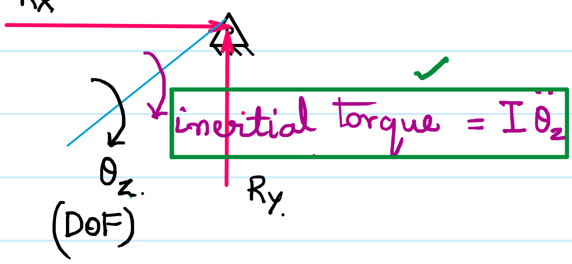
1. Identify the type of system (**SDOF**/MDOF).
2. Identify the displacement variable (x - translation / θ - Rotation(or) Angular Motion)
3. Identify the mean position or equilibrium position.
4. Provide the initial displacement to the body.
5. Isolate the body from the system.
6. Replace the connections with equivalent Reaction forces.
7. Write the equation of motion.

F.B.D.

Fixed end.

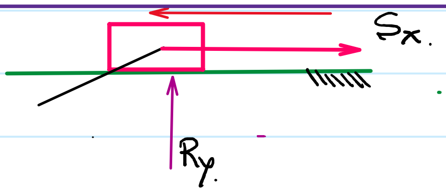


Hinge support.



Block sliding on horizontal surface.

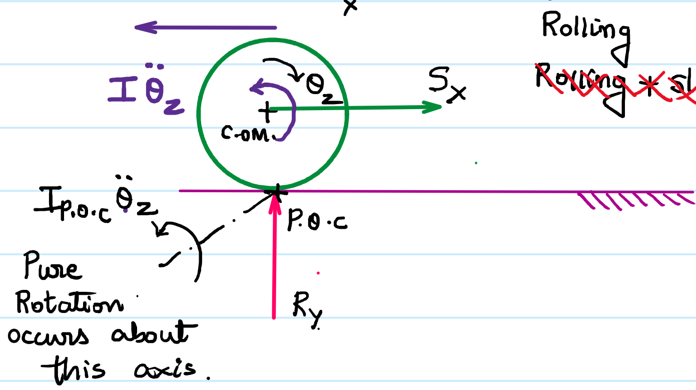
Inertia force. $F = ma_x$.



Higher Pair.

Rolling

~~Rolling~~ ~~+~~ ~~slipping~~



P.O.C – point of contact.

Rolling = Translation + Rotation @ C.O.M.

Rolling = Rotation about P.O.C.

Spring force / Restoring force.

$F_s \propto -x$.

F_s will resist the displacement x

Restoring Torque.

$\tau_s \propto -\theta$

τ_s will resist the displacement θ.

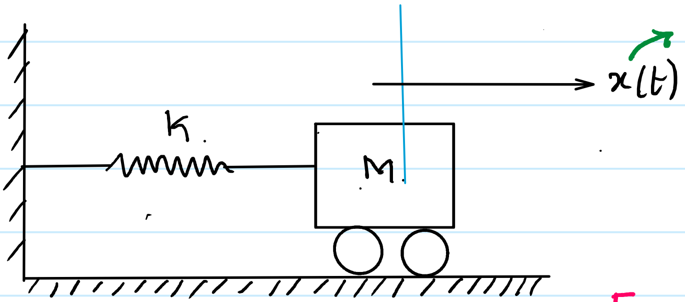
Damping force. – $F_D \propto -\dot{x}$ F_D will resist the relative motion.

Damping Torque. – $\tau_D \propto -\dot{\theta}$ τ_D will resist the relative motion

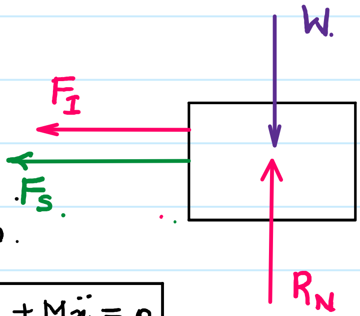
Inertia force $F_I \propto -\ddot{x}$ F_I will resist the acceleration.

Inertial torque $\tau_I \propto -\ddot{\theta}$ τ_I will resist the acceleration.

1.



intial displacement



$\ddot{x} = \frac{d^2x}{dt}$

$\underbrace{x, \dot{x}, \ddot{x}}$ variables.

$\Sigma F_y = 0 \qquad R_N - W = 0$

$R_N = W \rightsquigarrow$ constant

In Y-direction it is static equilibrium.

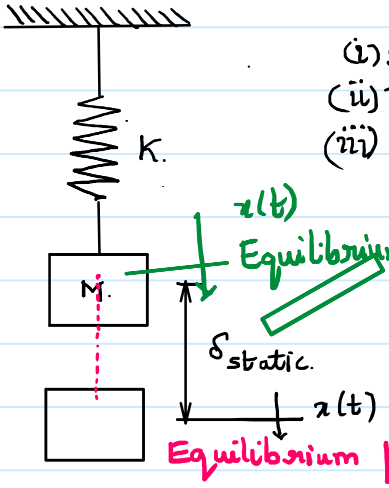
$\Sigma F_x = 0 \qquad F_S + F_I = 0 \rightsquigarrow \boxed{kx + M\ddot{x} = 0}$

$-Kx + (-M\ddot{x}) = 0$

$\boxed{\underbrace{M\ddot{x}}_{\text{Variable}} + \underbrace{kx}_{\text{Variable}} = 0} \rightsquigarrow$ Equation of motion.

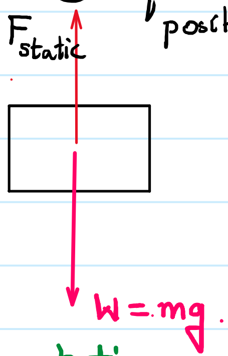
$M\ddot{x}(t) + K.x(t) = 0$

2.



(i) SDoF
(ii) Displacement variable $- x$.
(iii) Equilibrium posn - identified.

F.B.D. @ Equilibrium position.



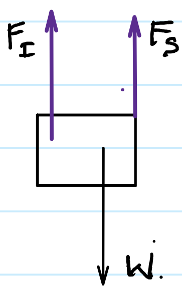
$\Sigma F_y = 0$

$F_{static} = W$

$\boxed{K.\delta_{static} = W.}$

$\hookrightarrow$ System is in static equilibrium.

F.B.D. after giving intial displacement.



$\Sigma F_y = 0$

$F_I + F_S - W = 0$

$M\ddot{x} + K.(\delta_{static} + x) - W = 0$

$M\ddot{x} + Kx. + \boxed{K\delta_{static} - W} = 0 \rightarrow$ static terms

$M\ddot{x} + kx = 0$

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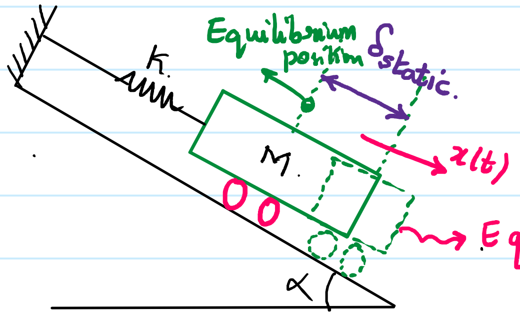

DOF = 1

Disp. variable – x

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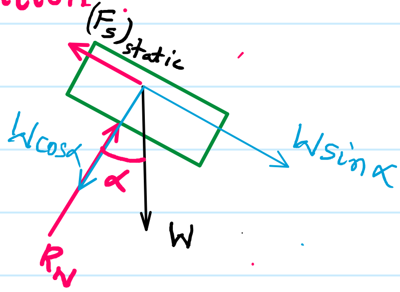
W – constant

α – constant

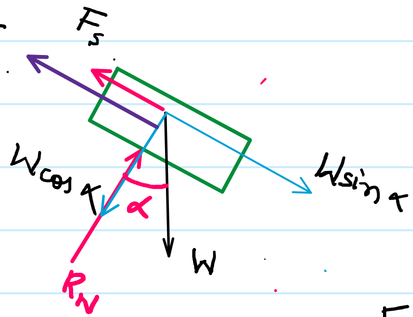
@ Equilibrium position



→ both in x and y - direction it is in static equilibrium.

$$\Sigma F_x = 0$$
$$(F_s)_{static} = W\sin\alpha$$
$$K\cdot\delta_{static} = W\sin\alpha$$
$$\Sigma F_y = 0$$
$$R_N - W\cos\alpha = 0$$
$$R_N = W\cos\alpha$$

F.B.D. after giving intial displacement.



$$\Sigma F_y = 0 \Rightarrow R_N = W\cos\alpha$$

$$\Sigma F_x = 0$$
$$F_I + F_s - W\sin\alpha = 0$$
$$M\ddot{x} + K(\delta_{static} + x) - W\sin\alpha = 0$$
$$M\ddot{x} + Kx = 0$$

★ If Mg is responsible for δ_{static} then the given position itself can be assumed as equilibrium position, by giving intial displacement we can write the equation of motion.

★ If Mg is responsible for δ_{static} then in the final equation of motion Mg will not appear.

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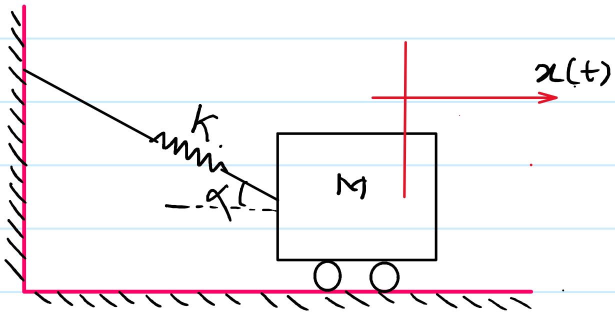
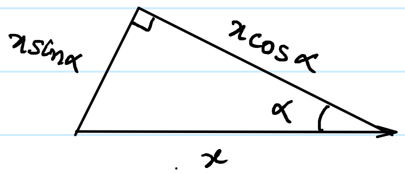
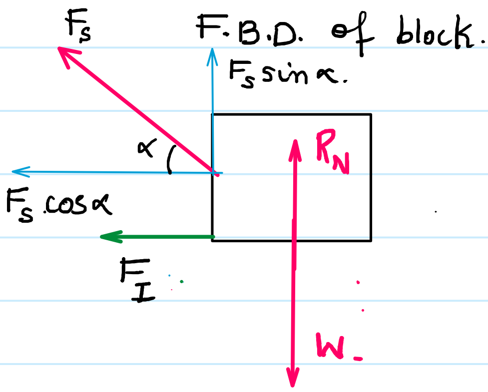


Resolving displacement along spring.



F.B.D. of block.



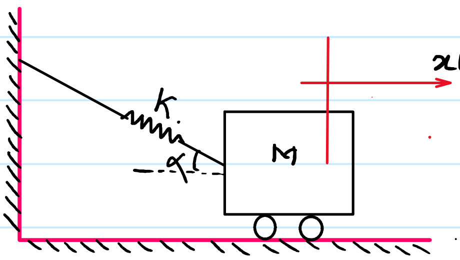
$$F_s = k \cdot x_s = k \cdot x \cdot \cos\alpha$$

$$\Sigma F_y = 0 \qquad R_N + F_s \sin\alpha = W$$

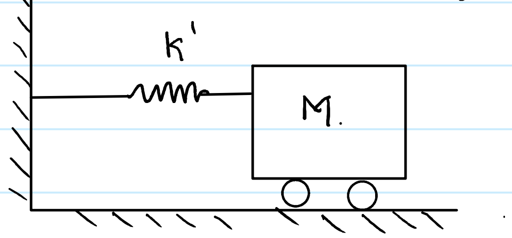
$$-F_s \cdot \cos\alpha - F_I = 0$$

$$F_s \cos\alpha + F_I = 0 \Rightarrow k \cdot x \cdot \cos^2\alpha + M\ddot{x} = 0$$

$$\boxed{M\ddot{x} + k x \cdot \cos^2\alpha = 0}$$



$$K' = K \cdot \cos^2\alpha$$



Equation of motion

$$M\ddot{x} + Kx = 0$$

$$M \cdot \frac{d^2x}{dt^2} + Kx = 0$$

$$\frac{d}{dt} = D$$

$$\frac{d^2x}{dt^2} + \frac{K}{M}x = 0$$

$$\left(D^2 + \frac{K}{M}\right)x = 0$$

$$D = \pm i\sqrt{\frac{K}{M}}$$

$$\sqrt{\frac{K}{M}} = \omega_n \longrightarrow \text{natural frequency.}$$

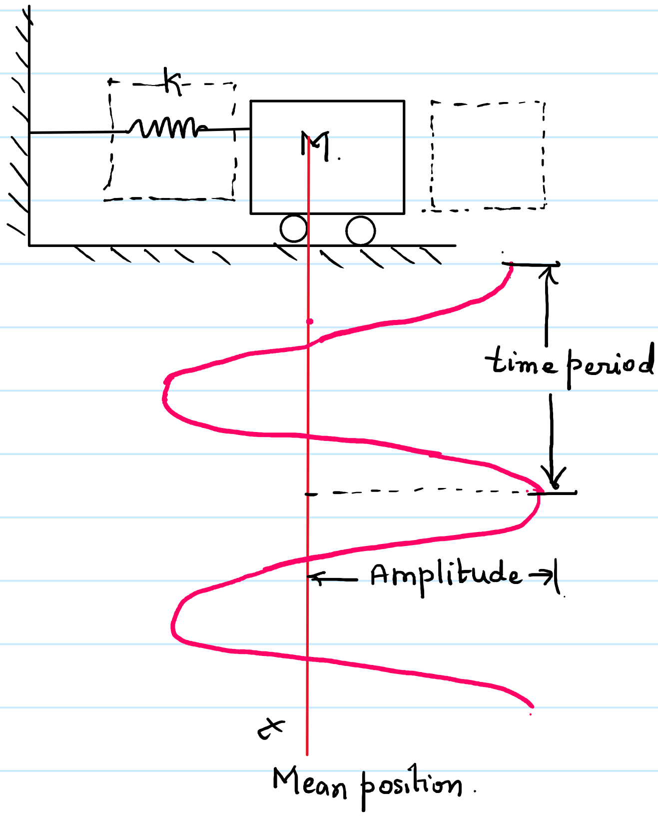
Complimentary function $x(t) = A \cdot \sin\left(\sqrt{\frac{K}{M}} \cdot t\right) + B\cos\left(\sqrt{\frac{K}{M}} \cdot t\right)$

$$A = X_0 \cdot \cos\phi \qquad B = X_0 \cdot \sin\phi$$

$$x(t) = X_0 \cdot \cos\phi \cdot \sin\left(\sqrt{\frac{K}{M}}\,t\right) + X_0 \sin\phi \cdot \cos\left(\sqrt{\frac{K}{M}}\,t\right)$$

$$x(t) = X_0 \cdot \sin(\omega_n t + \phi)$$

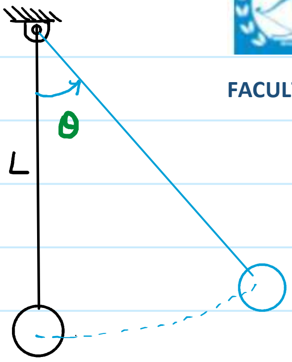
Displacement, Velocity, Acceleration are harmonic in nature.



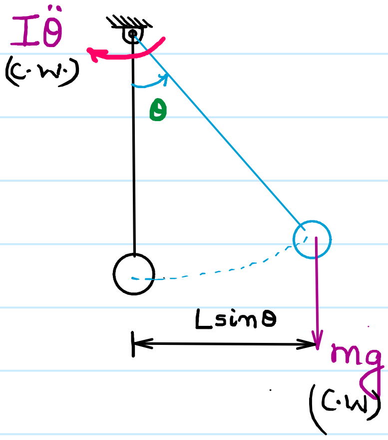
Simple Pendulum.

DoF = 1.

Displacement Variable = θ_z.



$\theta \rightarrow \sin\theta \approx \theta, \ \cos\theta = 1.$
$\tan\theta \approx \theta.$



$$I_{Hinge} = ML^2$$

$$\Sigma M_{Hinge} = 0$$

$$I_{Hinge}\ddot{\theta} + MgL\sin\theta = 0$$

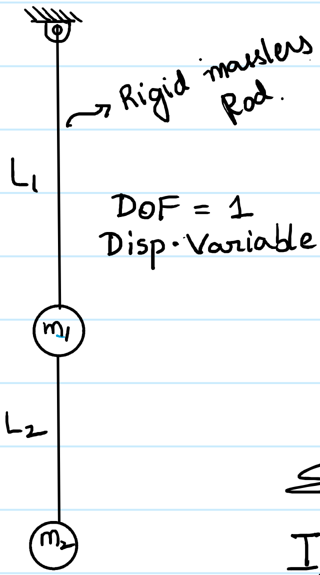
$$I_{Hinge}\ddot{\theta} + MgL\theta = 0$$

$$ML^2\ddot{\theta} + MgL\theta = 0$$

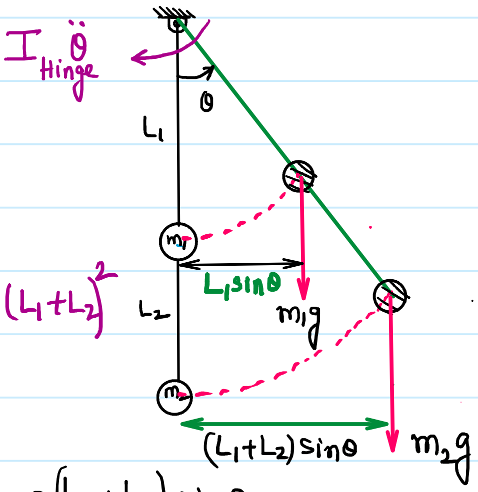
$$\ddot{\theta} + \frac{g}{L}\theta = 0$$

$$\omega_n = \sqrt{\frac{g}{L}}$$

time period. $\quad t_n = \dfrac{2\pi}{\omega_n} = 2\pi\sqrt{L/g}$



DoF = 1
Disp. Variable = θ.



$$I_{Hinge} = m_1L_1^2 + m_2.(L_1+L_2)^2$$

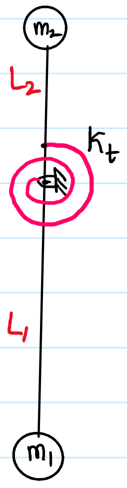
$$\Sigma M_{Hinge} = 0$$

$$I_{Hinge}\ddot{\theta} + m_1gL_1\sin\theta + m_2g(L_1+L_2)\sin\theta = 0$$

$\sin\theta \approx \theta.$

$$\left[m_1L_1^2 + m_2.(L_1+L_2)^2\right]\ddot{\theta} + \left[m_1L_1 + m_2.(L_1+L_2)\right]g.\theta = 0$$

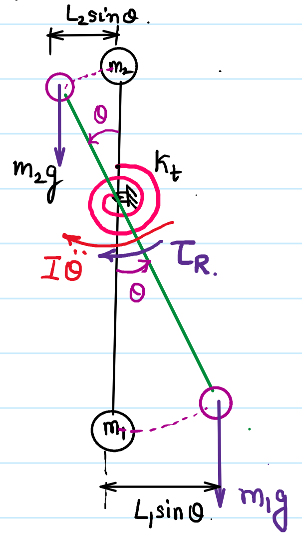
$$\omega_n = \sqrt{\frac{\left[m_1L_1 + m_2.(L_1+L_2)\right]g}{m_1L_1^2 + m_2.(L_1+L_2)^2}}$$



k_t - Torsional stiffness.

DoF = 1.

Disp. variable = θ.



$$\boxed{\tau_R = k_T \theta}$$

$\sin\theta = \theta$

$$I_{Hinge} = m_1 L_1^2 + m_2 L_2^2$$

$$\Sigma M_{Hinge} = 0$$

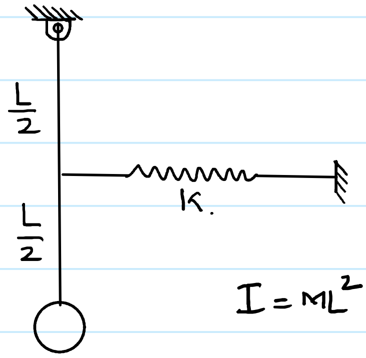
$$I_{Hinge}\ddot{\theta} + \tau_R + m_1 g L_1 \sin\theta - m_2 g L_2 \sin\theta = 0$$

$$(m_1 L_1^2 + m_2 L_2^2)\ddot{\theta} + k_T\theta + m_1 g L_1 \theta - m_2 g L_2 \theta = 0$$

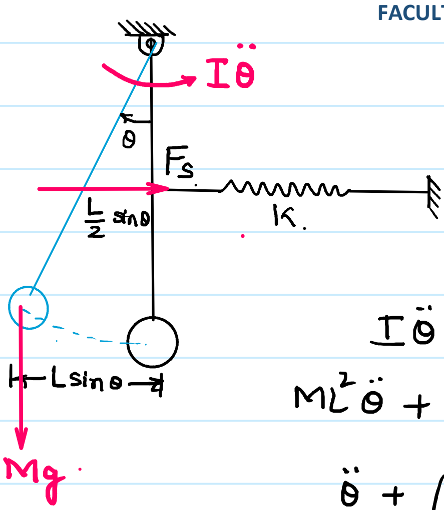
$$(m_1 L_1^2 + m_2 L_2^2)\ddot{\theta} + (k_T + m_1 g L_1 - m_2 g L_2)\theta = 0$$

$$\ddot{\theta} + \frac{(k_T + m_1 g L_1 - m_2 g L_2)\theta}{m_1 L_1^2 + m_2 L_2^2} = 0$$

$$\omega_n = \sqrt{\frac{(k_T + m_1 g L_1 - m_2 g L_2)}{m_1 L_1^2 + m_2 L_2^2}}$$



$I = ML^2$



$F_s = k \cdot L/2 \sin\theta$

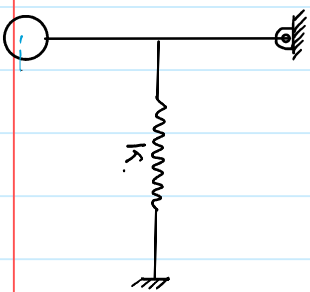
$= \frac{k.L}{2}.\theta.$

$\Sigma M_{Hinge} = 0$

$I\ddot{\theta} + F_s \times L/2 + Mg.L\sin\theta = 0$

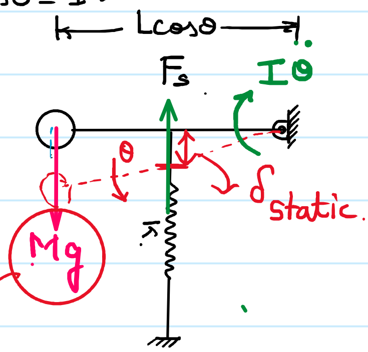
$ML^2\ddot{\theta} + \frac{k.L^2}{4}\theta + Mg L\theta = 0$

$\ddot{\theta} + \left(\frac{k}{4M} + \frac{g}{L}\right)\theta = 0.$



$\sin\theta = \theta$

$\cos\theta = 1$ ✓



$\Sigma M_{Hinge} = 0$

$I\ddot{\theta} + F_s \times L/2 - MgL\cos\theta = 0$

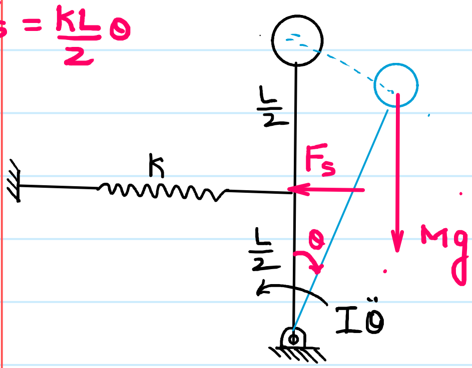
$ML^2\ddot{\theta} + \frac{kL^2}{4}\theta - \cancel{MgL} + \cancel{k\delta_s \cdot L/2} = 0$

$$\boxed{\ddot{\theta} + \frac{k.}{4M}\theta = 0}$$

It will cause δ_{static}.

Mg is not responsible for restoring.

$F_s = \frac{KL}{2}\theta$



$\Sigma M_{Hinge} = 0$

$I\ddot{\theta} - Mg.L\sin\theta + F_s \times L/2 = 0$

$ML^2\ddot{\theta} + \frac{KL^2}{4}\theta - Mg.L\theta = 0$

$\ddot{\theta} + \left(\frac{K}{4M} - g/L\right)\theta = 0$

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01. In a cuckoo clock the pendulum has a mass of 50gms and has a time period of 0.5 sec. Its length is approximately

→ simple pendulum.

(a) 52 mm (b) 62 mm
(c) 48 mm (d) 24 mm

$$\omega_n = \sqrt{g/l}$$

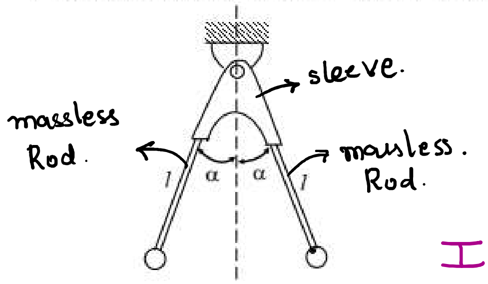
$$t_n = 2\pi\sqrt{L/g} \Rightarrow 0.5 = 2\pi\sqrt{\frac{L}{9.81}}$$

$$L = \frac{0.5^2}{4\pi^2} \times 9.81 \text{ m}.$$

$$L = 62\text{mm}.$$

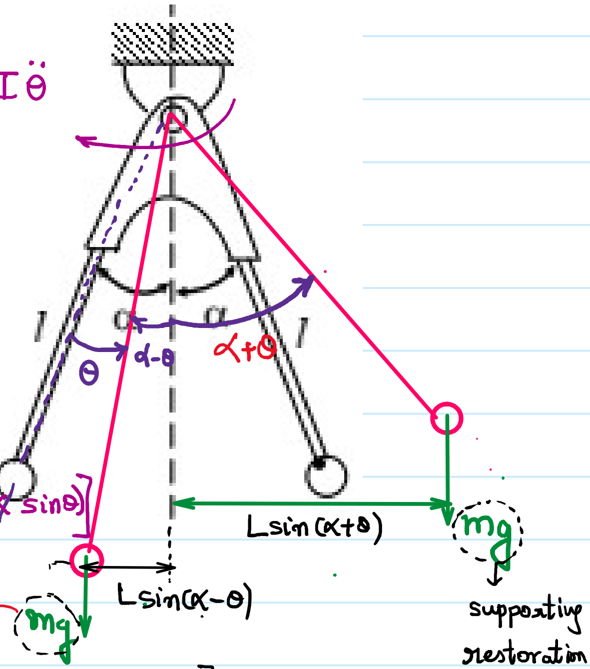
02. The assembly shown in the figure is composed of two mass less rods of length l with two particles, each of mass m. The natural frequency of this assembly for small oscillations is (GATE-2001)

DoF = 1   Disp. variable = θ.



(a) $\sqrt{g/\ell}$ (b) $\sqrt{2g/(\ell\cos\alpha)}$
(c) $\sqrt{g/(\ell\cos\alpha)}$ (d) $\sqrt{(g\cos\alpha)/\ell}$

$$I = mL^2 + mL^2 = 2mL^2$$



$$\Sigma M_{Hinge} = 0$$

$$I\ddot{\theta} + mgL\sin(\alpha+\theta) - mgL\sin(\alpha-\theta) = 0$$

$$2mL^2\ddot{\theta} + mgL[(\sin\alpha\cos\theta + \cos\alpha\sin\theta) - (\sin\alpha\cos\theta - \cos\alpha\sin\theta)]$$

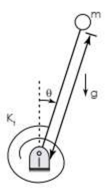
$\sin\theta \approx \theta$

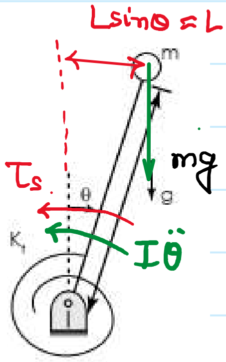
$$2mL^2\ddot{\theta} + mgL[\cancel{\sin\alpha\cos\theta} + \cos\alpha\sin\theta - \cancel{\sin\alpha\cos\theta} + \cos\alpha\sin\theta]$$

$$2mL^2\ddot{\theta} + 2mgL\cos\alpha\cdot\theta = 0 \Rightarrow \boxed{\ddot{\theta} + \frac{g\cos\alpha}{L}\theta = 0}$$

Common data for Q. 11 & 12

For the system shown in figure $\theta = 0$ is the static equilibrium position and the torsional spring of stiffness K_t is uncompressed at this position. Assume small oscillations about this position and consider θ as the co ordinate.





$$T_s = K_T\theta$$

11. The Equivalent Inertia is
(a) m kg (b) m / l kgm
(c) ml^2 kgm^2 (d) m / l kgm

12. The Equivalent torsional stiffness is
(a) K_t (b) $K_t + gl$
(c) $K_t - mgl$ (d) $K_t + mgl$

ΣM_{Hinge}.   against Restoration.

$$I\ddot{\theta} + T_s \boxed{- mgL\theta} = 0 \Rightarrow mL^2\ddot{\theta} + K_T\theta - mgL\theta = 0$$

$$\underbrace{mL^2}_{I_{eq}}\ddot{\theta} + \underbrace{(K_T - mgL)}_{K_{eq}}\theta = 0$$

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