1. Mechanisms
2. Gear and Gear Trains
3. Flywheels
4. Cam and followers
5. Gyroscope
6. Balancing of Masses
7. Governors
8. Vibrations

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9704968792

Expected Marks - (7 - 9) marks.

Reference Book

1. Ghosh and Malik (Theory of Mechanisms)
2. SS rattan
3. Sadhu Singh

Mechanism 1. Definition of link, Pair, Chain, Mechanism
2. Mobility Analysis (NAT)
3. Grashof law (NAT)
4. Performance Parameters
5. Inversions of Planer Mechanism
6. Analysis of Quick Return Mechanism
7. Kinematic Analysis of Planer Mechanism (velocity and acceleration analysis) (NAT)
8. Dynamic Analysis of Crank Slider Mechanism (NAT)

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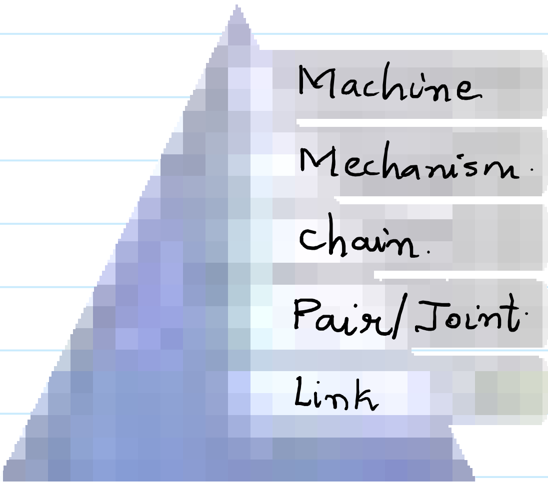


Link - It is the smallest element in the machine. Link need not be rigid but it is a resistant body which transfers relative motion.

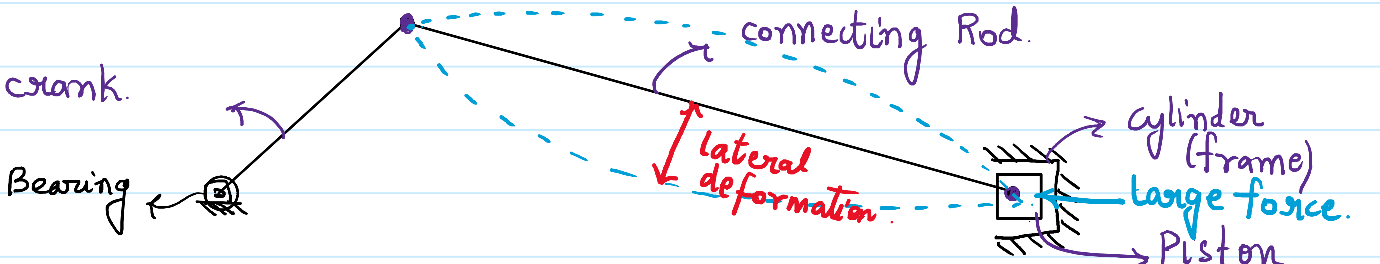
Types of links

↓

Solid

Rigid. | Deformable.

Rigid link doesnot deform while transmitting relative motion.



connecting rod can be assumed as a rigid link.

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Deformable link :- A body undergoes small amount of deformation while transmitting relative motion.

Ex:- Belt, chain.

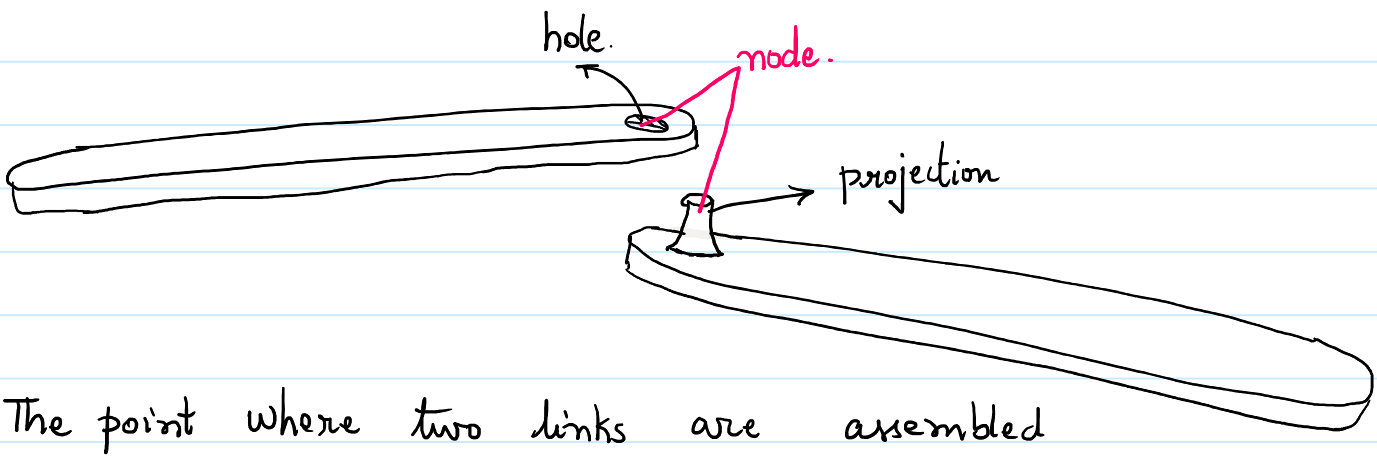
↓

Belt and Pulley arrangement.

Fluid link :- It will undergo very large deformation while transmitting relative motion.

Ex:- Hydraulic / Pneumatic Application.

- A link possess uniaxial rigidity it will be considered as link in a situation when it is able to transfer relative motion. In any other condition when it is not able to transfer relative motion it cannot be treated as link.
- Belt is able to transfer the relative motion under the action of Tensile force.
- Fluid link is able to transfer the relative motion under the action of compressive force.



The point where two links are assembled together is called as node.

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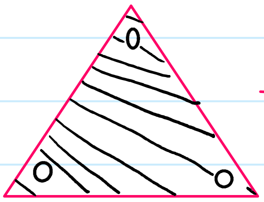
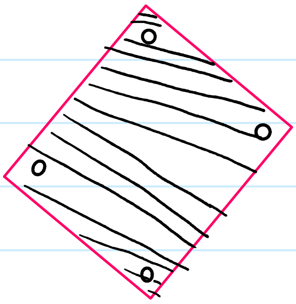
Classification of Links based on nodes

→ Binary link.

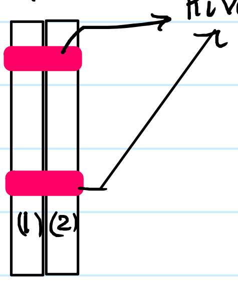


→ Ternary link.



→ Quaternary link.

Fig(1)

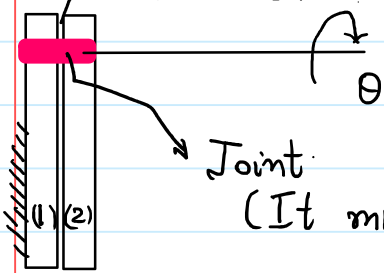


Riveted joint.

(1) (2)

If the two bodies are manufactured separately and joined/assembled In a way that there is no relative motion between them then they will Be treated as single link.

clearence



θ_x (Relative motion)

Joint.

(It must permit and transfer the relative motion)

(1) (2)

Degree of Freedom - Number of **independent co-ordinates** required to describe the motion of a body in space/plane.

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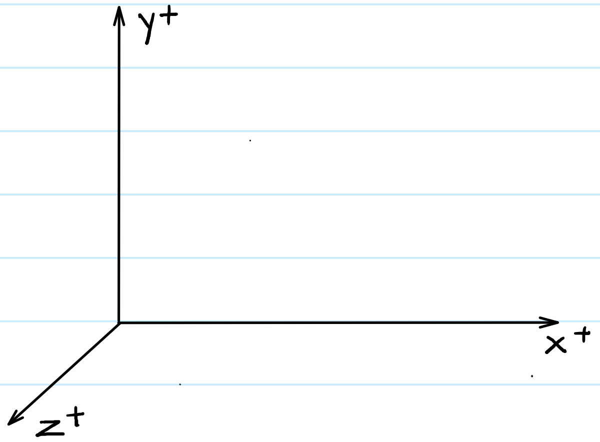
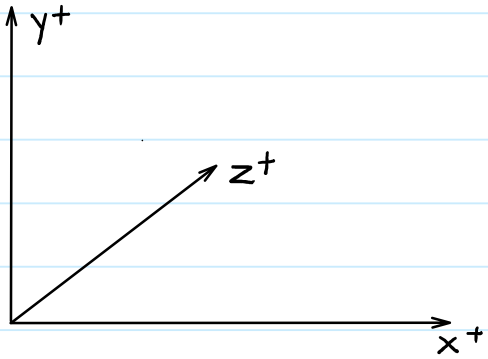
Right hand co-ordinate System.



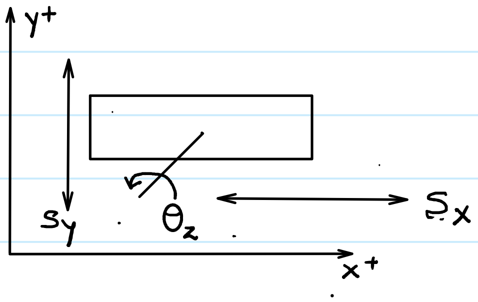
Left hand co-ordinate System.



DOF in space. 6 → 3 (Translation) + 3 (Rotation)

S_x	θ_x
S_y	θ_y
S_z	θ_z

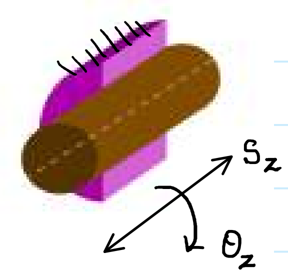
DOF in plane →



★ **Whenever the pair or contact is formed the degree of freedom are lost.**

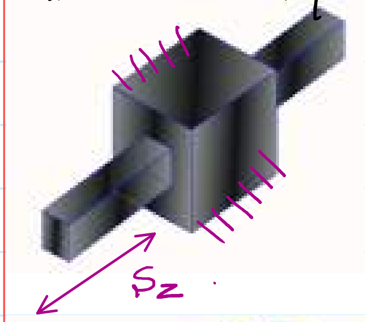
Lower Pairs.

Cylinderical Pair



Permitted DoF = 2
(S_z, θ_z)

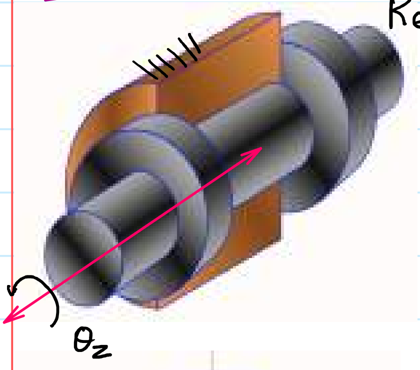
Prismatic Joint / Sliding pair.



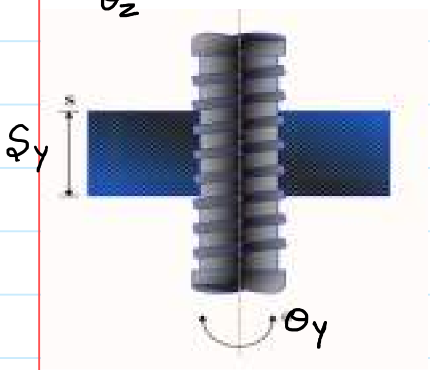
Permitted DoF = 1.
(S_z)

Revolute Joint / Pin joint.

Permitted DOF = 1. (θ_z)



Helical pair / Screw pair.



$S_y = f(\theta_y)$ → independent co-ordinate

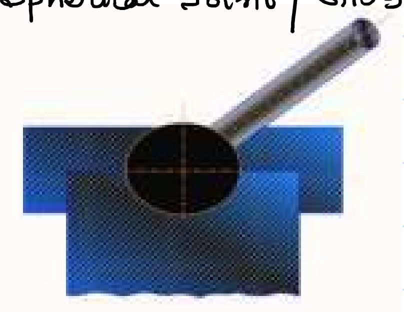
DoF = 1.

Lead = $n \times$ pitch

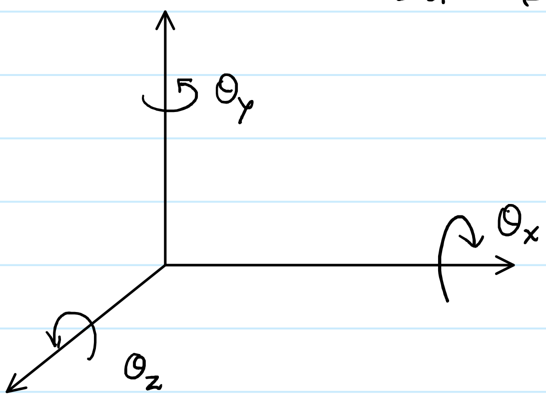
$n = 1$ single start
$n = 2$ double start
$n = 3$ triple start

$$\frac{\text{lead}}{\Delta S_y} = \frac{2\pi}{\Delta \theta_y}$$

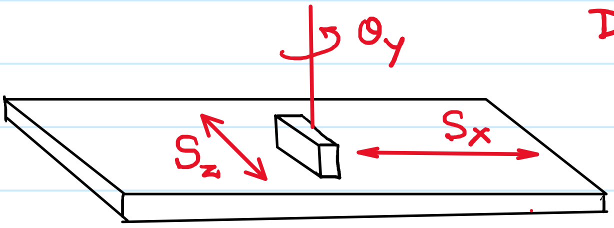
Spherical Joint / Globular pair.



DoF = 3



6. Evena pair / Planer Pair



DOF = 3

Lower pairs.

DOF = 1	linear motion pair.	Ex. Prismatic Pair, Revolute Pair, Helical Pair.
DOF > 1	Surface motion Pair (Robotics.)	Ex:- Cylinder Pair, Spherical Pair, Evena pair.

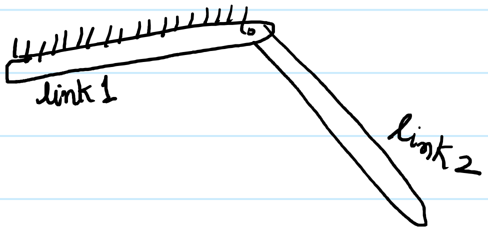
According to hartenberg and Reaulax if there is a surface contact b/w the mating elements then the pair is called lower pair.

If their is negligible surface contact b/w the mating elements the pair is called higher pair.

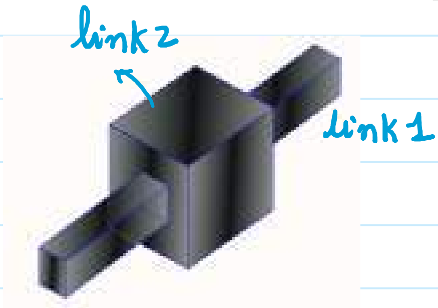
Ex:- 1. Cam and follower
2. Gear and Pinion
3. Wrapping Pair → Belt and Pulley, Chain and Sprocket.

According to hartenberg and Reaulax the inversion is possible only in case of lower pair. Inversion of higher pair is not possible.

Inversion is possible only when the locus of one moving link over another fixed link must be same or viceverse.



Locus of link.2 is circle.

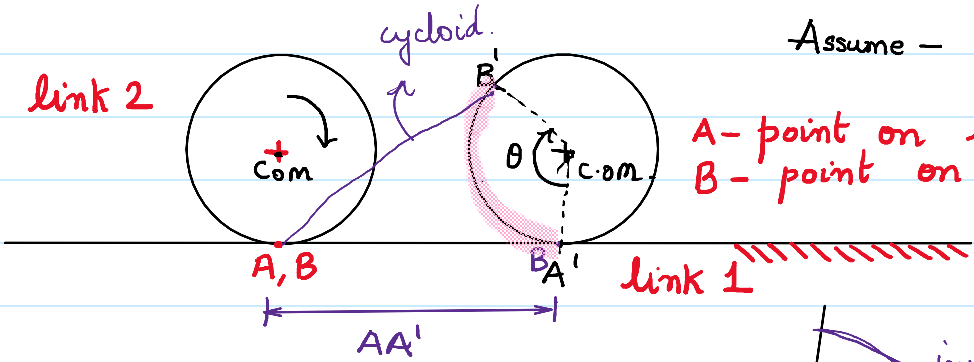


Fixed link — link 1/ link 2
locus of other moving link is straig-
-ht. line.

Higher Pair

★ **(In Higher pair Rolling or Rolling + Slipping will only occur)**

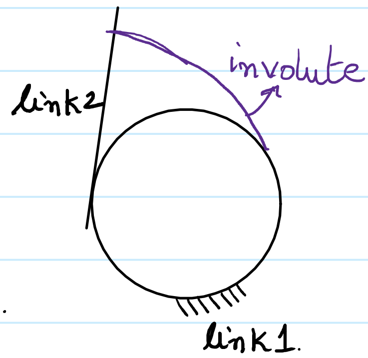
Assume — Pure Rolling.



A – point on link 1
B – point on link 2

For Pure Rolling $\bar{AA'} = \widehat{BB'}$

★ Distance covered by the points A and B must be same.



$$\bar{AA'} = S_x / S_{cm}.$$

$$\widehat{BB'} = r\theta_z$$

$$S_x = r\theta_z$$

$$S_{cm} = r.\theta_z \Rightarrow V_{cm} = r\omega.$$

If the circular disc moves on a fixed st. line (without slipping) the locus of any point on circumference of disc will be a cycloid.

If the st. line moves on a fixed circle without slipping the locus of any point on a st. line will be a involute.

$$\boxed{\text{Hence the inversion in the higher is not possible because the locus under different conditions are not same.}}$$

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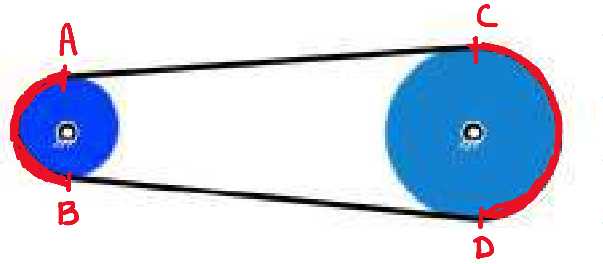
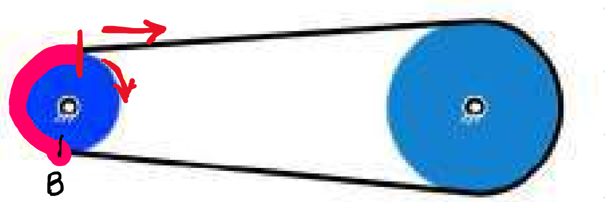


Case (i) If there is surface contact b/w Belt and pulley in segement $\widehat{AB}$ and $\widehat{CD}$ if the relative motion exists b/w the belt and pulley in segment $\widehat{AB}$ and $\widehat{CD}$ then the belt tends to slip over the pulley due to which required relat-ive motion is not transmitted.

Case (ii) In the segement $\widehat{AB}$ and $\widehat{CD}$ there is no relative motion b/w the belt and pulley they tend to behave as a single link. The relative exists at the points A, B, C & D.

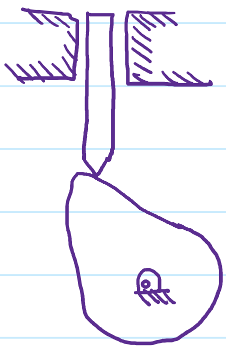
No. of higher pair = 4.



Classification of pair based Mechanical constraint.

1. Open Pair :- If one link is kept over another link and contact b/w them is due to gravity then it is called open pair.

Ex. Cam and follower.



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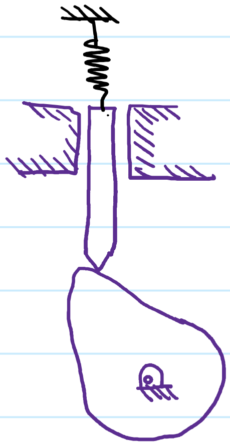
2. Closed pair :- If one link is kept inside another link and for dissassembly the failure of outer is required then it is called closed pair.

3. Form closed pair :- If the two link are connected by means of geometrical constraint then it is called form closed pair.
Ex :- Spherical joint.

4. Force closed Pair :- If the contact b/w the links is maintained by the application of force then it is called force closed pair



spring is used for exerting force to maintain contact.

Classification pair based on constraint

1. Incompletely constrained pair **(DOF > 1)**
2. Completely constrained pair **(DOF = 1)**
3. Successfully constrained pair **(DOF > 1)** $\xrightarrow{\text{link/force}}$ **(DOF = 1)**

Incompletely constrained pair - If the connected links have more than one relative motion between them then the pair is called as incompletely constrained pair.
Ex - (Cylinderical pair , Spherical pair and Evena pair)

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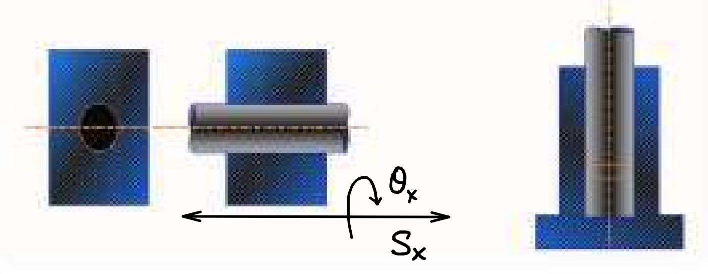
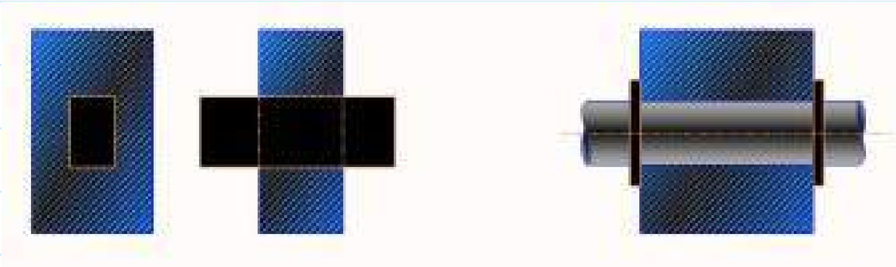
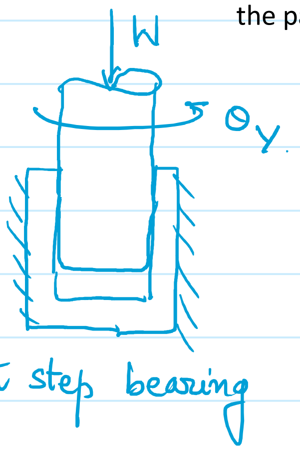


Completely Constrained Pair - If the connected links have a one definite motion between them then the pair is called as completely constrained pair.
Ex - Prismatic joint , Revolute joint and Helical pair

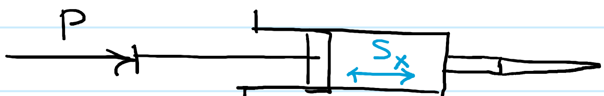


Successfully Constrained Pair - If the connected have more than one relative motion between them then by using some force/link it is converted to one definite motion then the pair is called successfully constrained pair.

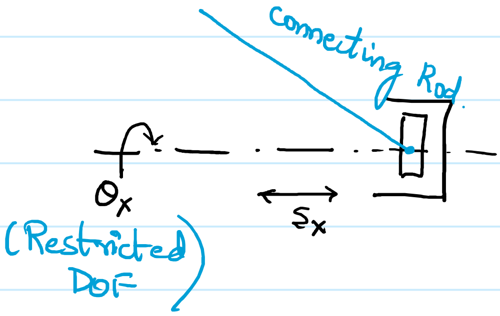


S_y – Restricted.

Syringe



Translation S_x – Permitted.
Rotation. θ_x – Restricted.



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Types of Pairs based on number of links connected

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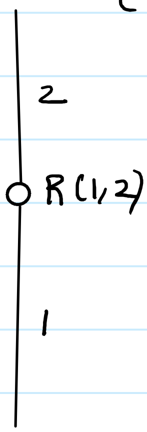
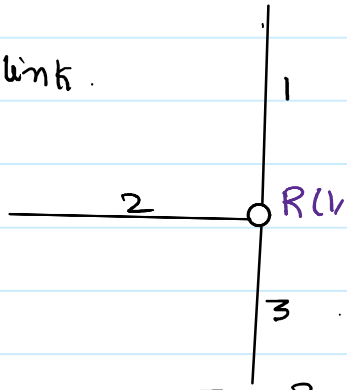
1. Binary pair. (Two link connected @ one node)



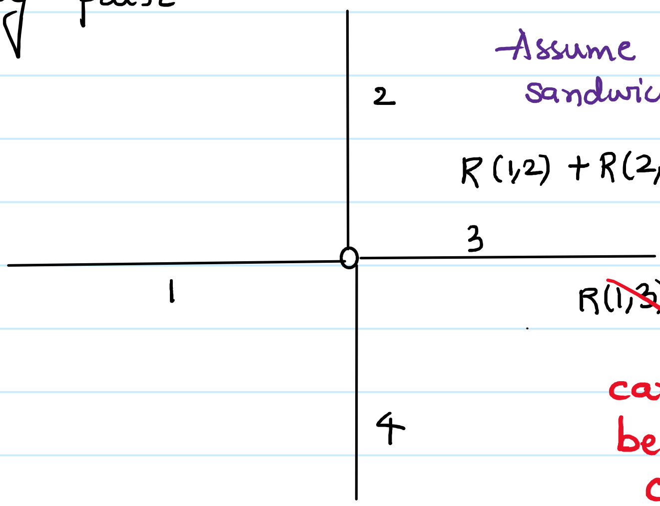
2. Ternary link.



R(1,2) + R(2,3) + ~~R(1,3)~~

1 Ternary pair ≡ 2 Binary pairs.

3. Quaternary pair



Assume link 2 & 3 are sandwiched b/w link 1 & 4.

R(1,2) + R(2,3) + R(3,4)

~~R(1,3)~~ + ~~R(1,4)~~ + ~~R(2,4)~~

cannot be counted because no physical contact.

★ **If there are n number of links connected at a node then the number of pairs formed is equal n-1**

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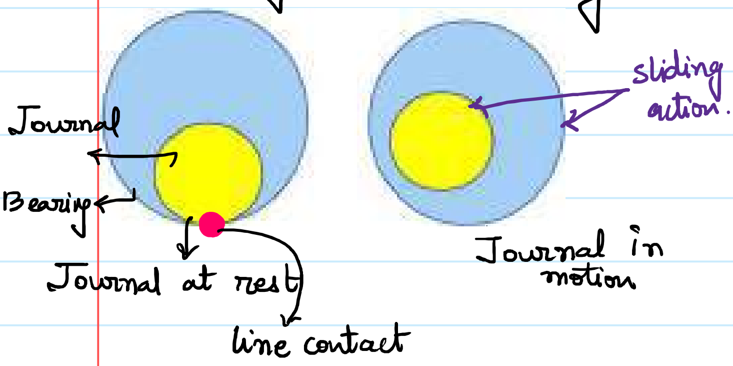
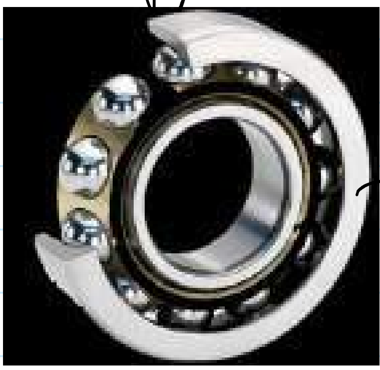
Bearings → sliding contact Bearings.
→ Rolling contact Bearing / Antifriction bearing.

Sliding contact bearing





→ Rolling contact Bearing

➢ Bearing is a mechanical element which is used to support the shaft and bear the various load acting on the shaft or rotor.

Bearing permit the relative motion of the shaft but bearing can't transfer the relative motion and hence bearing are not a kinematic pair.

Chain:

- Assembly of various links and pairs which permit the transfer of relative motion is known as chain.
- If the first link is connected directly or indirectly to the last link then it is knows as closed chain.
- If the first link and last link is not connected then it is known as open chain. It is mainly used in robotics. → Robotic Arm / Manipulator.
- **All the mechanisms are obtained from the closed chain**

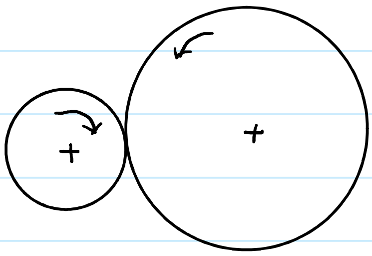
Mechanism/Linkage:

- If any one link of chain is fixed and it can transfer the relative motion with or without transformation it will be known as mechanism.

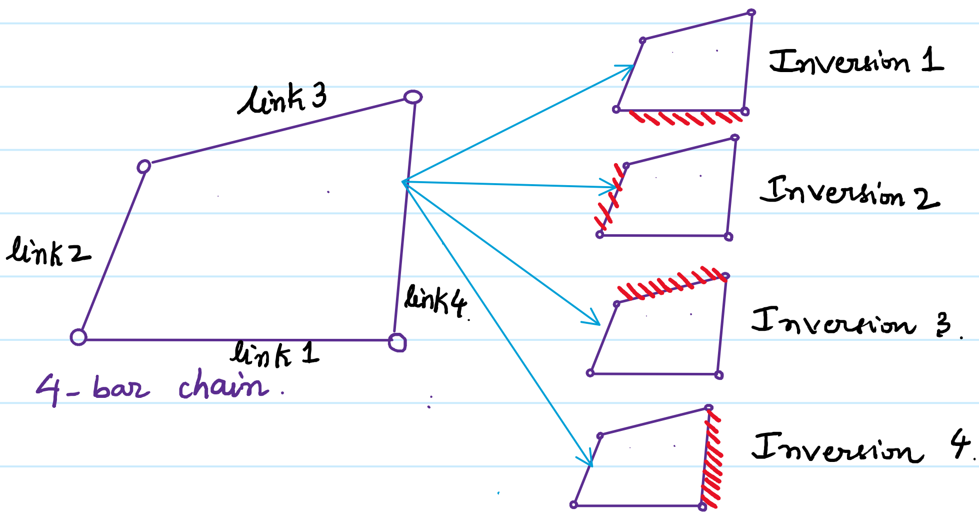
Crank slider Mechanism. → circular motion is transformed to Reciprocating motion.
(Transformation of motion)

Gear and Pinion. / Belt and Pulley. → Relative motion is transferred without transformation.

i/p motion – circular
o/p motion – circular.



Inversion of a Mechanism - Inversion of mechanism is obtained by fixing different links



Number of Inversions = Number of Different links

Inversion of Mechanism:

- The process of fixing different links of a mechanism or chain is known as inversion of mechanism.
- Every inversion results in a unique mechanism
- No of possible inversion is equal to the number of different link.
- Inversion can't effect the ability to transfer the relative motion as it is fundamental property of parent kinematic chain.
- Inversion of higher pair mechanism is not possible.
  → because the locus will be different.

Mechanism is the working model behind the machine.

Machine:

- Machine is a assembly of various link, pairs and mechanism such that it can transfer relative motion, force or power from source to the load with or without transformation in a **controlled manner**.
- Machine can transfer some form of available energy from source to some other form of available energy at the load.

Mechanism & Machine:

- Every machine is a mechanism in spirit, where as the reverse is not true.
- A machine may consist of one or several mechanism.
  - EX- clock, typewriter, keyboard, are mechanism only not machine.

$\boxed{\text{I.C. Engine.}}$

1. Crank slider mechanism.
2. Cam and follower
3. Governor.
4. Chain and Sprocket | Belt and Pulley | Gears.

Lathe Machine → Head stock (Gear Train)
→ Belt and Pulley.
→ Automatic feed Mechanism. (Lead screw)
→ Tumbler gear Mechanism.

Mobility analysis or degree of planar mechanism:

(c) Planar Mechanism: The mechanism in which locus of different points on different links lies on parallel plane is known as planar mechanism.

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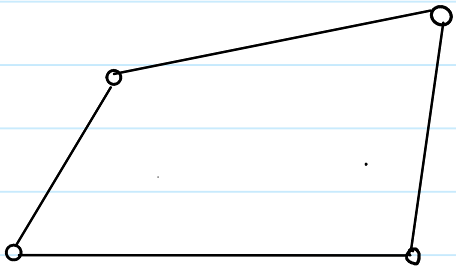
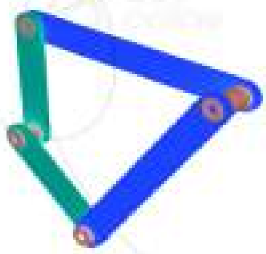
- The kinematic representation of any planar mechanism can be drawn a single plane.

Links are moving in the parallel planes.



(Representation)



Mobility Analysis of Chain:

Maximum DOF in a plane for a link = 3

Permitted DOF = Maximum DOF in a plane - Restricted DOF

If there are N number of links in a plane then maximum DOF is = 3N

Mechanism is formed by lower pair (Revolute/Prismatic) each of these pair restricts **2 DOF** and permits **1 DOF**

If there are j no. of such pair in the plane then restricted dof = 2j

$$(DOF)_{Permitted} = (DOF)_{Max} - (DOF)_{Restricted}$$

$$(DOF)_{chain} = 3N - 2j$$

In mechanism one link is fixed so DOF lost = 3.

$$(DOF)_{mechanism} = 3N - 2j - 3 = 3(N-1) - 2j$$

$$(DOF)_{chain} = (DOF)_{mechanism} + 3$$

$DOF = 3(N-1) - 2j \longrightarrow$ Grubler's criteria

Grubler's criteria for completely constrained Mechanism.

$$DOF = 1$$

$$3N - 3 - 2j = 1$$

$$3N - 2j - 4 = 0 \quad \text{or} \quad \boxed{3N - 2j - 4 \geq 0}$$

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$3N-2j-4=0$ Completely constrained Mechanism

$3N-2j-4>0$ Incompletely constrained Mechanism or Unconstrained Mechanism.

Higher Pair → Rolling (DOF = 1) $(S_{cm} = r\theta_z)$ → independent variable

Higher Pair → Rolling + Slipping (DOF = 2) $(S_{cm} \neq r\theta_z)$ → independent variable, independent variable.

In a plane if a higher pair permits pure rolling motion then it restricts 2 DOF and permits 1 DOF

If there are h no. of higher pairs in plane that permits pure rolling motion then restricted DOF = 2h.

$$\boxed{DOF = 3(N-1) - 2j - 2h} \rightsquigarrow \text{Pure Rolling motion.}$$

Restricted DOF

$= 3(N-1) - 2P_1$ → Permited DOF

In a plane if a higher pair permits Rolling + slipping motion then it restricts 1 DOF and permits 2 DOF

If there are h no of higher pair in plane that permit Rolling + slipping motion then restricted DOF = 1h

★★ $$\boxed{DOF = 3(N-1) - 2j - h} \rightsquigarrow \text{Rolling + slipping.}$$

→ Kutzbach's equation

$$DOF = 3(N-1) - 2P_1 - 1P_2$$

Restricted DOF: 2, 1. Permitted DOF: 1, 2.

N – Number of link.

P_1 – No. of pairs having 1 dof in plane.

P_2 – No. of pairs having 2 DOF in plane.

Effect of lower pair:
As there are assumptions:

- Linear motion pair (DOF = 1). (Prismatic / Revolute Joint)

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- Only binary joint.
- Each linear motion pair restrict 2 DOF in planar conditions if there are 'j' number of linear motion binary pair then restricted degree of freedom by them = 2j

Effect of higher pair:

- Since each higher pair in planar mechanism is equivalent to two linear motion pairs, therefore it will permit 2 DOF & restrict 1 DOF. → Rolling + Slipping
- If there are 'h' number of higher pair then restricted DOF by higher pair = h

Physical Significance of DOF

1. DOF = 0 No relative motion can be transmitted.
→ structure (Simply supported Beam, C.Beam, overhanging beam, Perfect)
statically determinate structures. → Force / Resisting Moment is transmitted in the structures
No. of equilibrium equations = No. of unknowns / Reactions.

2. DOF < 1 Super structure / Redundent structure / statically indeterminate structure
Ex. Redundent Truss, Fixed beam, Propped cantilever etc.
No. of unknown / Reactions > No. of Equilibrium equations.

3. DOF = 1 Completely Constrained Mechanism / Kinematic Mechanism.
4. DOF > 1 Incompletely constrained mechanism.

Significance:

- Structures are used to transfer the load where as mechanism are used to transfer the relative motion.
- Degree of freedom predicts about minimum number of output possible from a mechanism with respect to a given input.
- Degree of freedom also predict about number of equations required between input and output pair variables.
- Degree of freedom predicts about number of links or pair variable that should be control in order to obtained a constrained mechanism.

DOF = 1.

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DOF = 1 single i/p , single o/p.

DOF = 2 single i/p , two o/p's.
two i/p's , single o/p

★ **Degree of Freedom predicts the number of input variable/variables required to obtain a single/Multiple output motions.**

Synthesis of link.

For completely constrained Mechanism DOF = 1.
Lets assume only lower pair.

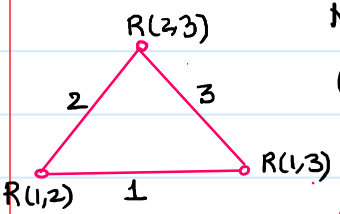
$$3N - 2j - 3 \geq 1.$$

$$N \geq \frac{4+2j}{3} \qquad 4+2j \rightarrow \text{even no.}$$

N → must be a +ve integer. (even no.)

N = 1 Single link.
N = 2 Pair/Joint.
N = 3 Not possible.
$\boxed{N = 4 \quad \text{Possible}}$ → Minimum no. of links required to obtain a Kinematic Mechanism.
N = 5 Not possible
N = 6 Possible.

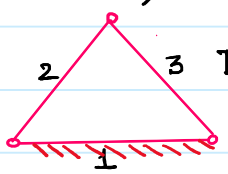
(1)



N = 3. j = 3.

$$(DOF) = 3N - 2j$$
$$= 3(3) - 2(3) = 3.$$

→ structure.



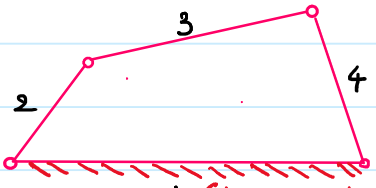
$$DoF = 3(N-1) - 2j$$
$$= 3(3-1) - 2(3)$$
$$= 0$$

(2)

N = 4 j = 4

$$DoF = 3N - 2j$$
$$= 3(4) - 2(4)$$
$$= 4$$

Kinematic chain.



1 (Kinematic Mechanism)

$$= 3(4-1) - 2(4) = 1.$$

★ The chain which can be converted into Kinematic Mechanism is called a Kinematic chain.

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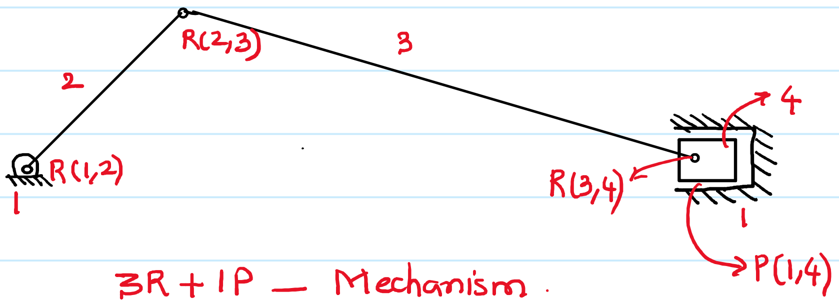
Crank slider Mechanism.

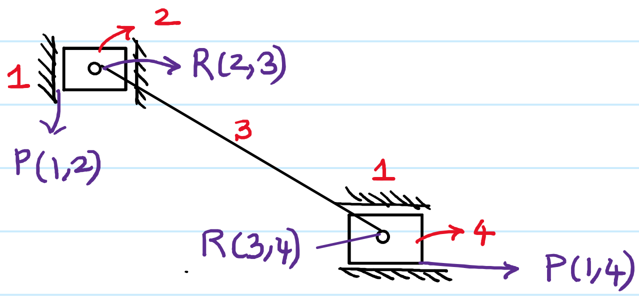


3R + 1P — Mechanism.

$N = 4 \quad j = 4 \qquad DOF = 3(N-1) - 2j - h.$

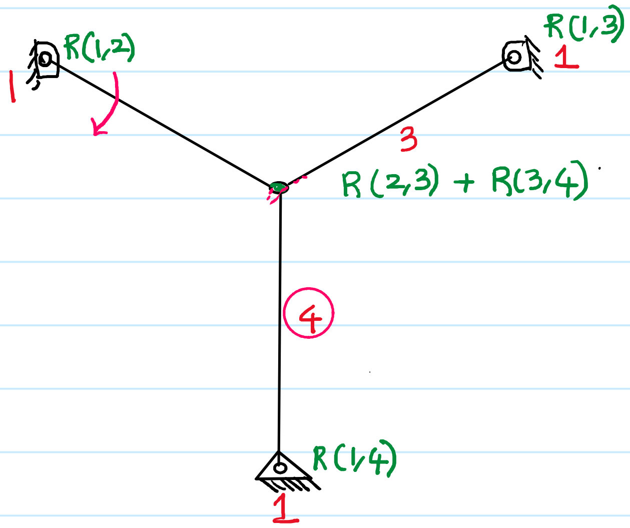
$h = 0 \qquad DOF = 3(4-1) - 2(4) - 0 = 1$

Double slider Mechanism. (2R + 2P) - Mechanism.



$N = 4 \quad j = 4 \quad h = 0$

$DoF = 3(4-1) - 2(4) - 0$
$= 1$

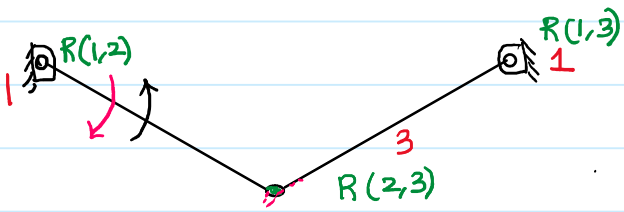


$N = 4$

$j = 5 \quad h = 0$

$DoF = 3(4-1) - 2(5)$
$= 9 - 10$
$= -1$

Super structure.

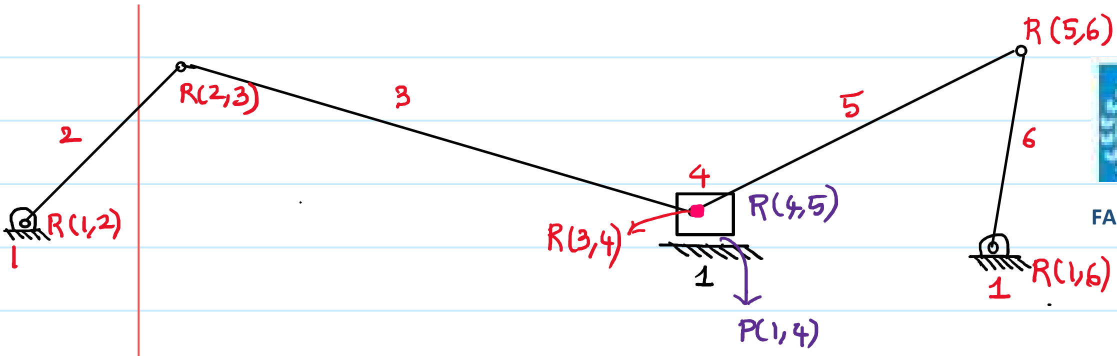


$N = 3$

$j = 3$

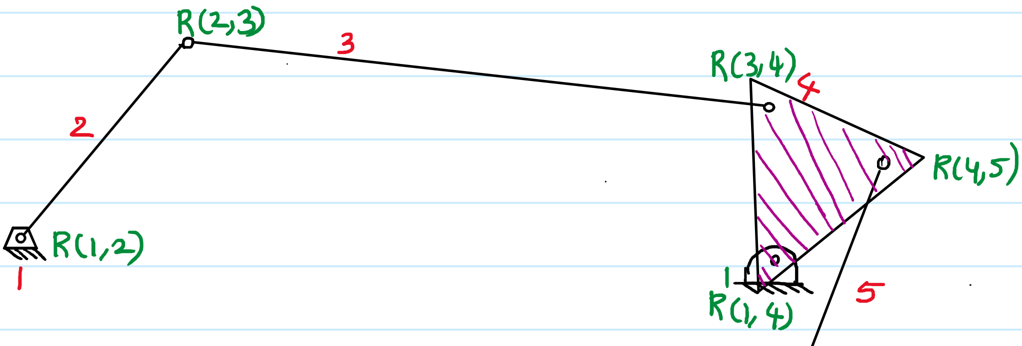
$DoF = 3(3-1) - 2(3)$
$= 0$

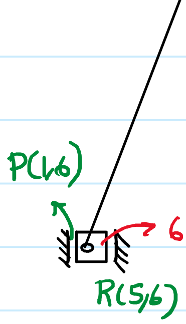
Structure.



$N = 6 \quad j = 7 \quad h = 0$

$DOF = 3(N-1) - 2j - h \Rightarrow 3(6-1) - 2(7) = 1$





$N = 6$

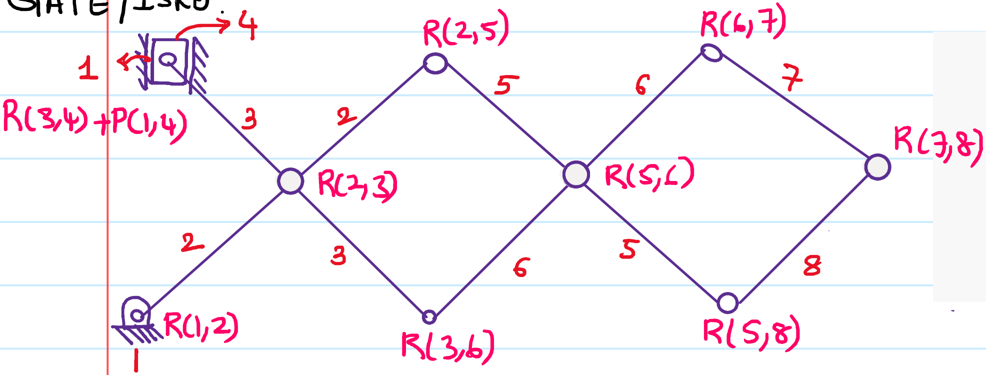
$j = 7$

$DOF = 3(N-1) - 2j - \cancel{h}$

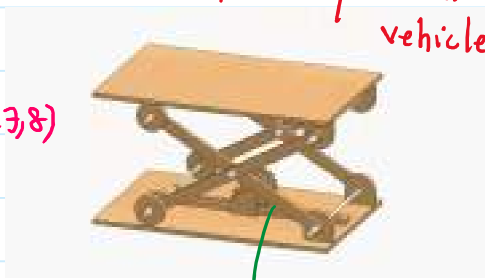
$= 3(6-1) - 2(7)$

$= 1.$

GATE/ISRO



lift door/ elevating vehicle



continous link.

link 2, 3, 5, 6 → continous link.

$N = 8 \quad j = 10 \quad h = 0$

$DOF = 3(8-1) - 2(10) - 0 = 1.$

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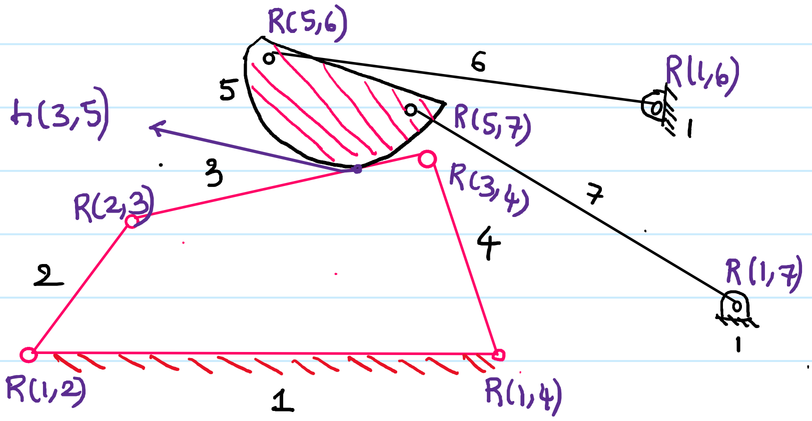
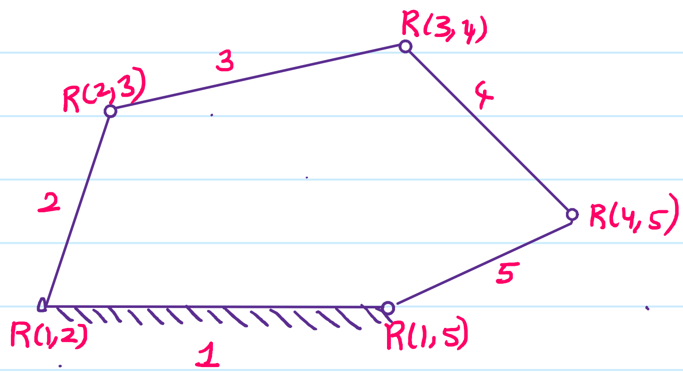


$N = 7 \quad j = 8 \quad h = 1$

Rolling + slipping

$$DoF = 3(7-1) - 2(8) - 1$$
$$= 18 - 16 - 1 = 1$$

case (i)



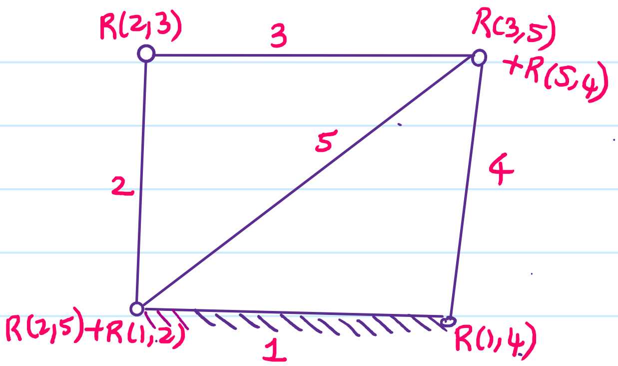
$N = 5 \quad h = 0$

$j = 5$

$$DoF = 3(5-1) - 2(5)$$
$$= 2$$

Unconstrained Mechanism.

case (ii)



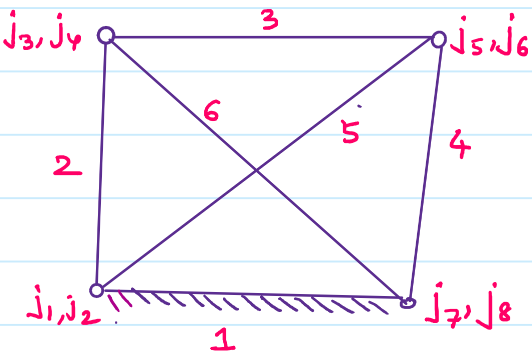
$N = 5$

$j = 6$

$$DoF = 3(5-1) - 2(6) = 0$$

Structure / Perfect truss.

case (iii)



$N = 6$

$j = 8$

$$DoF = 3(6-1) - 2(8) = -1$$

Superstructure.

Redundent truss.

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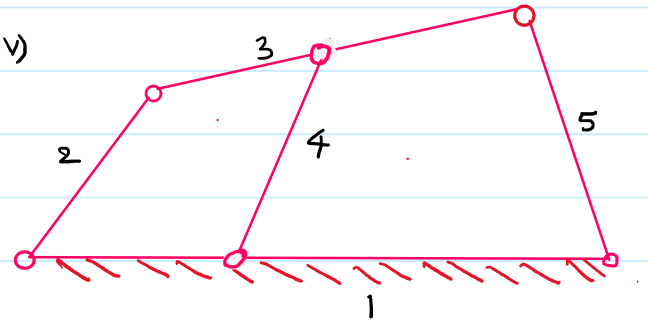
case(iV)

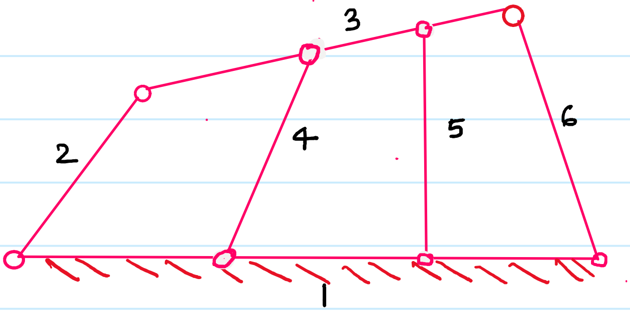


$N = 5$

$j = 6 \quad h = 0$

$DOF = 3(5-1) - 2(6) - 0 = 0$

case (V)



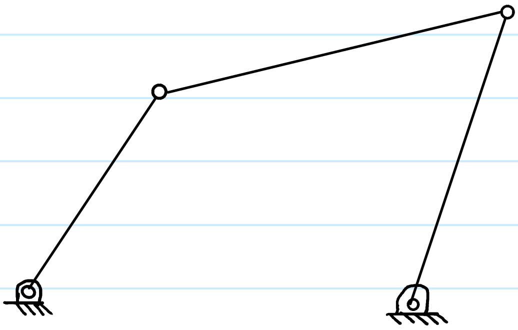
$N = 6 \quad j = 8 \quad h = 0$

$DOF = 3(6-1) - 2(8) - 0$

$= -1$

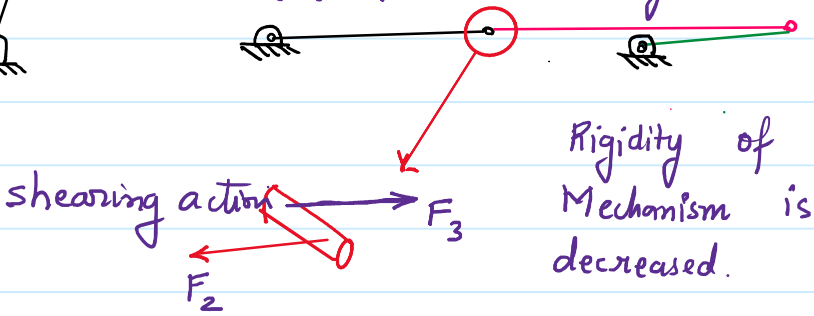
Observation -

Based on case 2,3,4,5 we can as the number of links increases the mobility(DOF) of mechanism decreases and stiffness of mechanism increases.



All the links are $\parallel^{le}$ at one instant.

Chances of failure of mechanism is increased for this configuration.



Rigidity of Mechanism is decreased.

We are required to increase the rigidity without effecting the mobility of mechanism.

In order to increase Rigidity without changing mobility we are required to make use of Redundent Parameters.

Redundent Parameters - The parameters whose presence or absence does not effect the mobility are called Redundant parameters

1. Redundant link
2. Redundant joint
3. Redundant DOF

For solving redundant parameters we are required to make use of modified Kutzbach's equation

$$DOF = 3(N-N_r-1)-2(j-j_r)-h-F_r$$

N— No. of links.

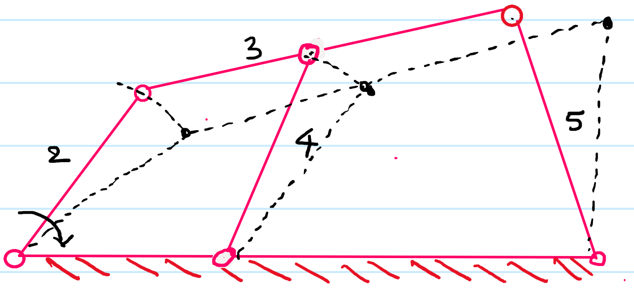
N_r – No. of Redundents.

j – No. of joints.

j_r – no. of redundent joints.

h— no. of higher pair.

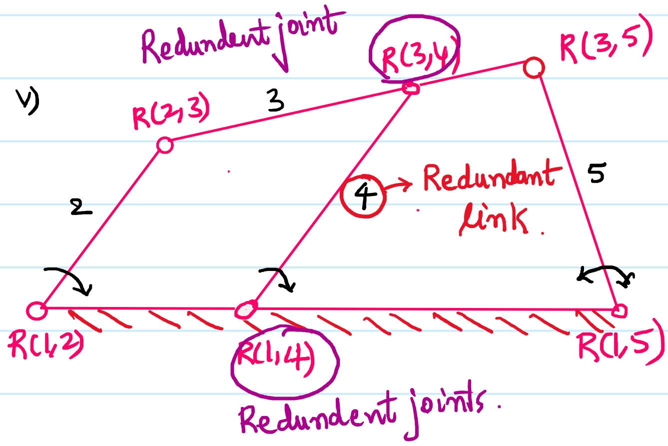
F_r – no. of redundent DOF.



link 4 has the tendency to lock the mechanism.

link 4 may experience tension or compression. & hence all the links are rigid.

v)

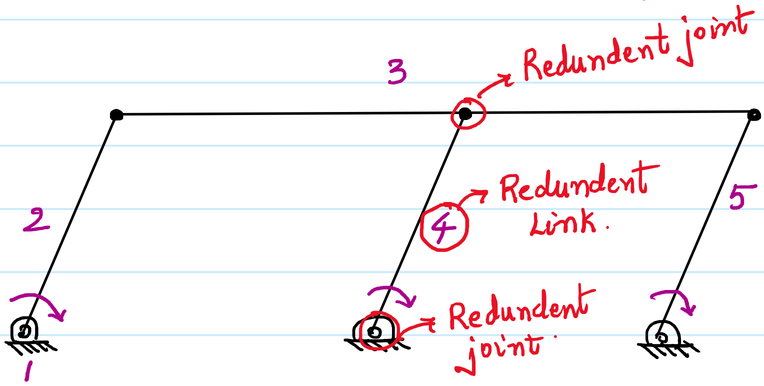


link 2 & 4 are $\parallel^{le}$ to each other.
link 2 & 4 will remain $\parallel^{le}$ to each other at all the instances of motion.
link 4 will neither experience tension or compression.

Link 4 can be called as Redundent link. & R(3,4) + R(1,4) are called as Redundent Joints.

$N=5$ $N_r=1$ $j=6$ $j_r=2$ $h=0$ $F_r=0$ $\Rightarrow$ DOF $=1$.

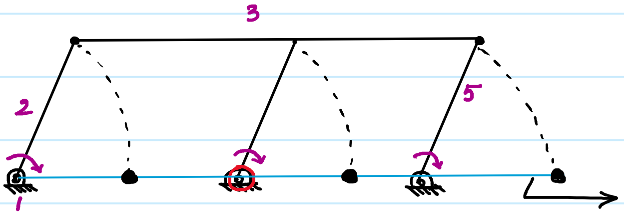
Gate.



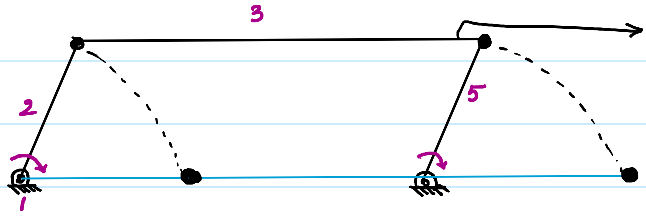
$N=5$ $j=6$ $h=0$ $N_r=1$ $j_r=2$ $h=0$ $F_r=0$

$$DOF = 3(N-N_r-1)-2(j-j_r)-h-F_r$$

$$= 3(5-1-1)-2(6-2)-0-0 = 9-8=1.$$



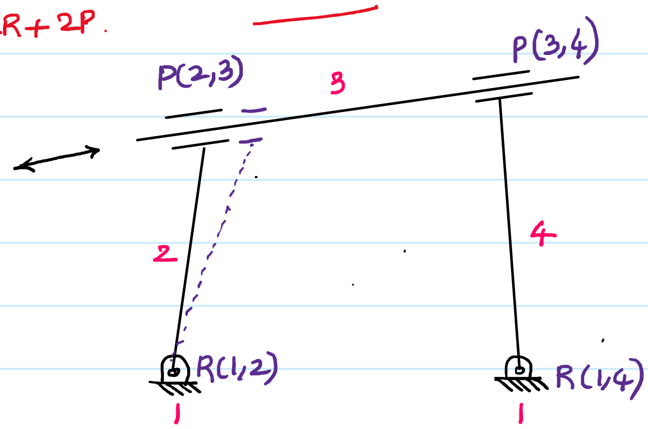
All the links are //le to each other.



This mechanism is more prone to failure.

Redundent link must be added to avoid failure.

2R+2P.



$N = 4 \quad j = 4 \quad h = 0$

$N_r = 0 \qquad F_r = 1.$

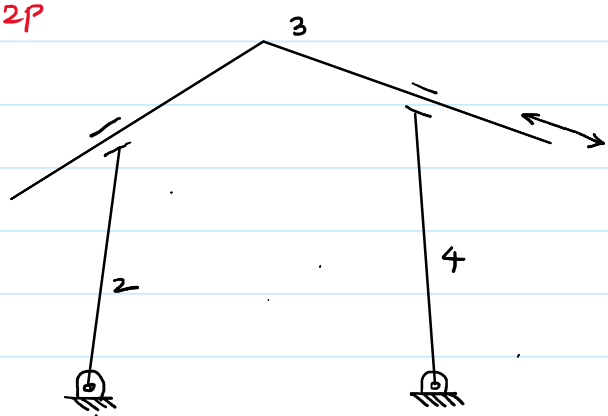
$j_r = 0$

$DOF = 3(N-N_r-1) - 2(j-j_r) - h - F_r$

$= 3(4-0-1) - 2(4-0) - 0 - 0$

$= 0$

2R+2P



No relative motion can be transmitted.

$N=4 \quad j=4 \quad N_r=0 \quad j_r=0 \quad F_r=0$

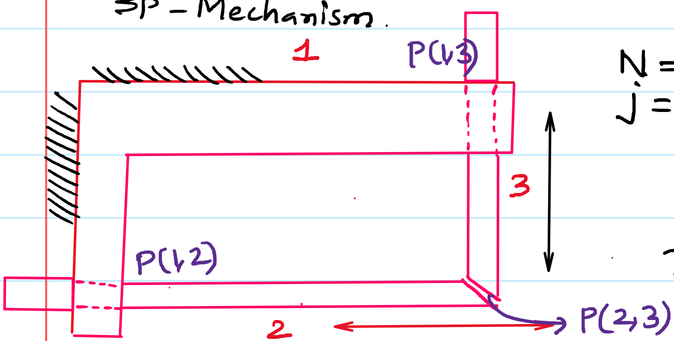
$h=0$

$DOF = 3(4-0-1) - 2(4-0) - 0 - 0$

$= 1$

Grubler's equation or Kutzbach's equation is only helpful for estimating mobility of linkage/Mechanism. They are unable to predict how the mechanism must be constructed.

3P – Mechanism.



$N = 3 \quad N_r = 0$

$j = 3 \quad j_r = 0$

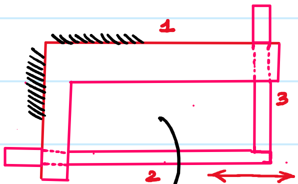
$h = 0$

$F_r = 0$

$DOF = 3(N-1) - 2j - h.$

$= 3(3-1) - 2(3) - 0$

$= 0$



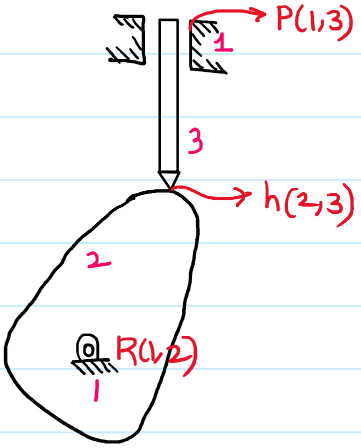
For this case link 3 doesnot move.

DOF = 1

$S_y = f(S_x)$ ⟶ independent variable

Kutzbach's eqn. is failing to predict whether the given arrangement is a mechanism.

Knife edge Cam and follower.



$N = 3$

$j = 2 \quad h = 1.$

$N_r = 0 \quad F_r = 0$

$j_r = 0 \quad DOF = 3(N-1) - 2j - h$

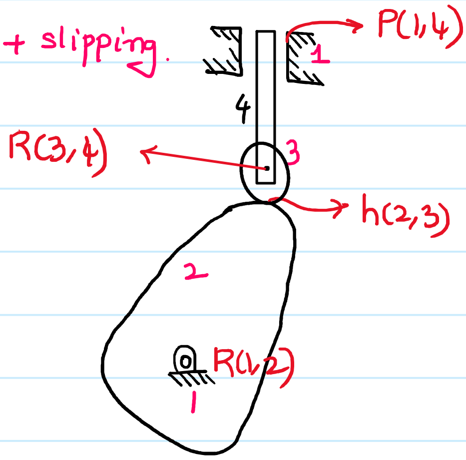
$DOF = 3(3-1) - 2(2) - 1$

$= 6 - 4 - 1 = 1.$

$\mu_{sliding} > \mu_{Rolling}$

Roller follower and cam.

Rolling + slipping.



$N = 4 \quad j = 3. \quad h = 1.$

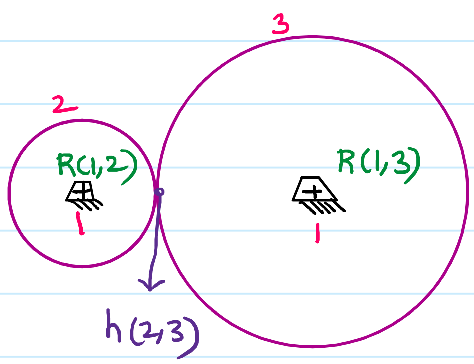
$N_r = 1 \quad j_r = 1. \quad F_r = 0$

$DOF = 3(N - N_r - 1) - 2(j - j_r) - h - F_r$

$= 3(4-1-1) - 2(3-1) - 1 - 0$

$= 6 - 4 - 1 = 1.$

Gear and Pinion.



$N = 3$

$j = 2$

$h = 1$

Pure Rolling $DOF = 3(N-1) - 2j - 2h$

$DOF = 3(3-1) - 2(2) - 2(1)$

$= 0$

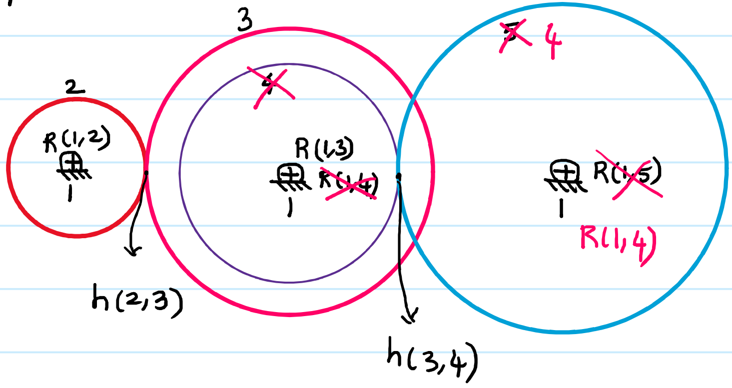
Gear 2 & Gear 3 rotate about the fixed axis.

Rolling + slipping

$DOF = 3(N-1) - 2j - h$

$= 3(3-1) - 2(2) - 1 = 1$ ✓

Compound Gear Train.



Gear 3 and 4 are compounded they will be treated as a single link.

$N = 5 \quad j = 4 \quad h = 2$

$DoF = 3(5-1) - 2(4) - 2$

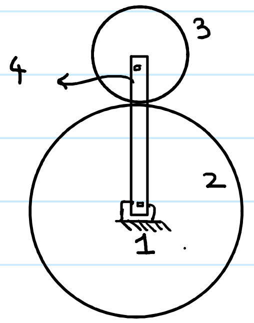
$= 12 - 8 - 2 = 2$

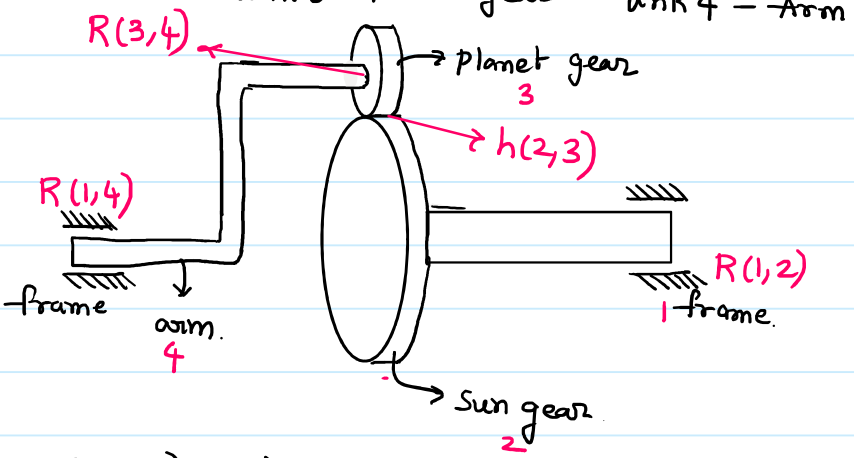
$DoF = 3(4-1) - 2(3) - 2$

$= 9 - 6 - 2 = 1$

Epicyclic Gear Train.

link 1 — frame link 2 — Sun gear
link 3 – Planet gear link 4 — Arm.





$N = 4$

$j = 3 \quad h = 1$

Rolling + slipping

$DoF = 3(N-1) - 2j - h.$

$= 3(4-1) - 2(3) - 1$

$= 2$

Gear 2 — fixed axis.

Gear 3 —

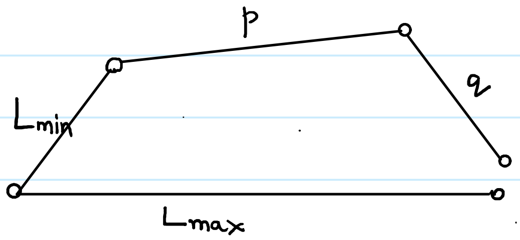
I/p	o/p
Sun.	Planet, Arm.
Sun, Arm.	Planet.

Observations

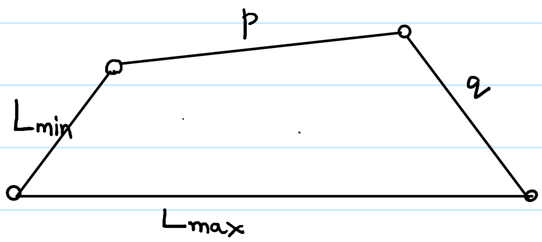
- If a mechanism consist of all revolute pairs then number of links required to form a kinematic/Completely constrained mechanism is 4.
- If a mechanism consist of all prismatic pairs then number of links required to form a kinematic/Completely constrained mechanism is 3.
- If a mechanism consist of lower and higher pair then number of required to form a kinematic /Completely constrained mechanism is 3.

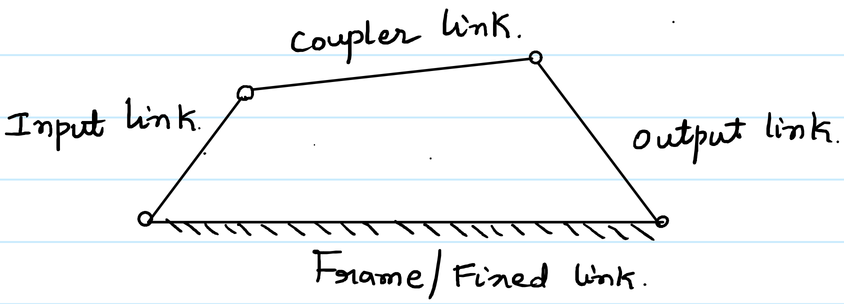
Types of chain.

$L_{max} > L_{min} + p + q \longrightarrow$ open chain. (Robotic Arm)



$L_{max} < L_{min} + p + q \longrightarrow$ Closed chain. $\xrightarrow{\text{by fixing any one link}}$ Mechanism.





I/p link & O/p link are connected to frame they move/ rotate about the fixed point.

Coupler link will float in the plane of rotation.

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Movability analysis of 4-bar mechanism:

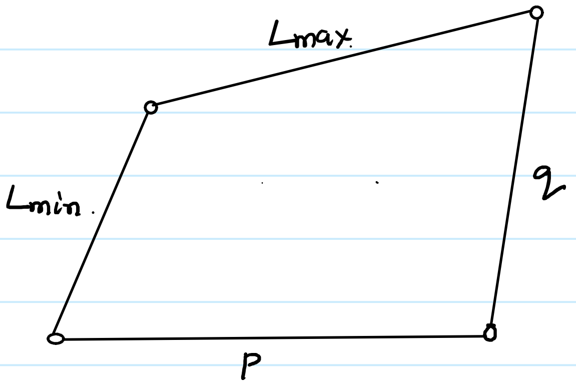
On the basis of range of movement the links of a 4-bar mechanism are classified as:

1. Frame or Fixed Link: The link which cannot move.
2. Crank: The link which can execute full circular motion.
3. Coupler: The link which is opposite to fixed link or the link which connects input and output.
4. Rocker: The link which oscillates.

- On the basis of input and output there are following four type of mechanism:

	Input		Output
1.	Crank	-	Crank or double crank mechanism
2.	Crank	-	Rocker mechanism
3.	Rocker	-	Rocker mechanism
4.	Rocker	-	Crank mechanism

Grasshof law:- It states that summation of shortest link and longest link lenghts must be less summation of lenghts of other links in order to transfer the relative motion continously.



$L_{max} + L_{min} < p + q$.

↳ class-I linkage.

In order to transfer relative motion continously there must be minimum one link undergoing circular motion.

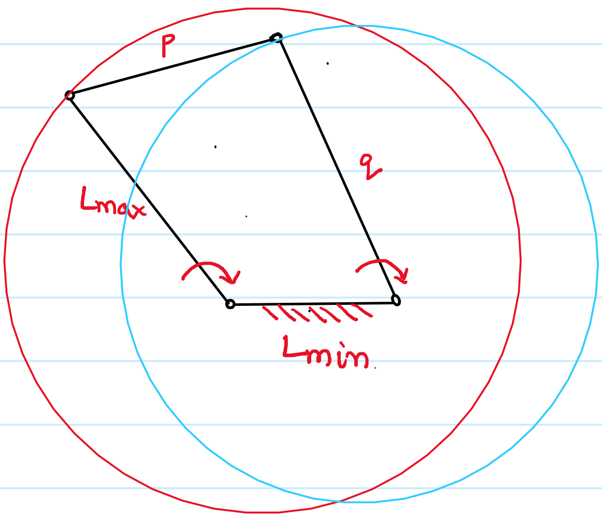
The type of inversion is decided on the basis of location of L_{min}.

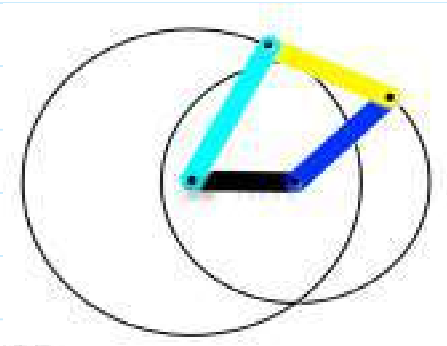
1. L_{min} can be fixed.
2. L_{min} can be adjacent to fixed.
3. L_{min} can be opposite to fixed.

Inversion 1. – L_{min} is fixed.

Crank – Crank Mechanism.

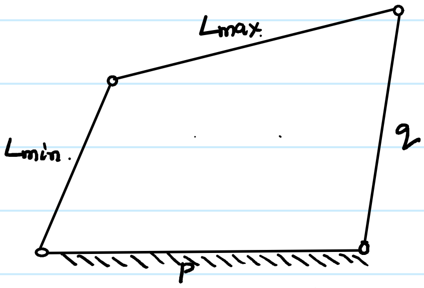
Locus of L_{max} and q links will be circle.

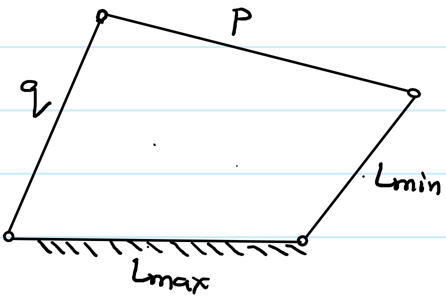


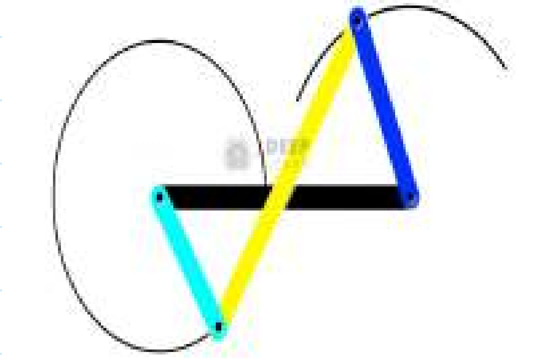


Inversion – II Link adjacent to L_{min} is fixed. fixed link (L_{max}/P)

Crank – Rocker / Rocker-crank Mechanism.





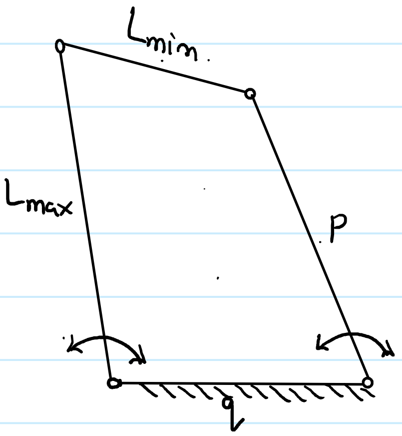


L_{min} completes circular motion / crank.

Link q oscillates / Rocker.

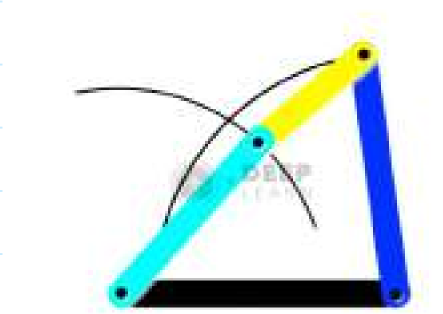
Inversion – III. Link opposite to L_{min} is fixed

Rocker – Rocker Mechanism.



links L_{max} & P will oscillate

L_{min} will complete circular motion.



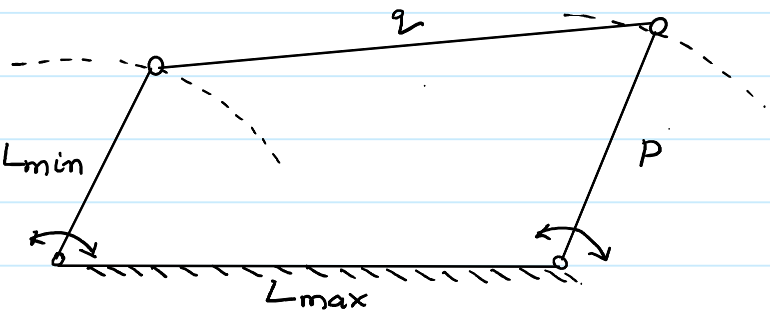
Class – II linkage / Non – Grashof linkage.

$$L_{min} + L_{max} > P + q$$

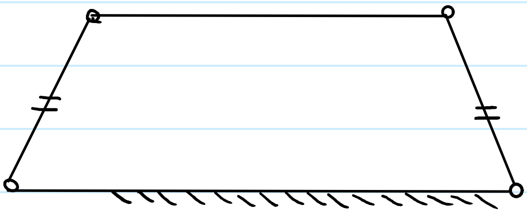
None of the links will complete circular motion. Continous Relative motion is not transmitted.

Fixed link – L_{min} / L_{max} / P / q ⟶ Rocker – Rocker Mechanism

Triple Rocker Mechanism.



Ackerman steering



Class – III linkage / Special Grashof linkage.

$$L_{min} + L_{max} = p + q.$$

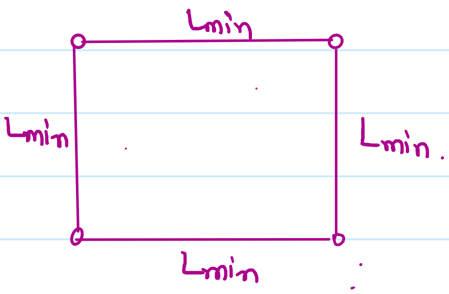
case (A) $L_{min} = L_{max} = p = q$ ⟶ Rhombus Linkage.

All the links are of same lenghts.

No. of different links = 1.

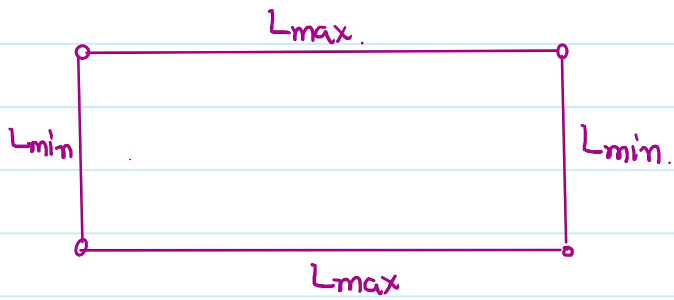
No. of inversions = 1

Fixed link :- $L_{min} / L_{max} / P / Q$. ⟶ Crank – Crank Mechanism.



case (B) $L_{min} = p$ $L_{max} = q$. ⟶ Parallelogram linkage.

case (i) If the links of equal lenghts are opposite to each other.



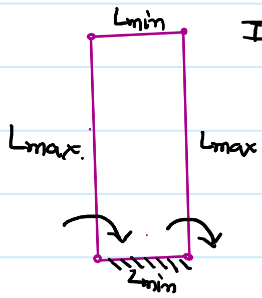
No. of inversions = No. of different links = 2

Inversion – I

Fixed link – L_{min}.

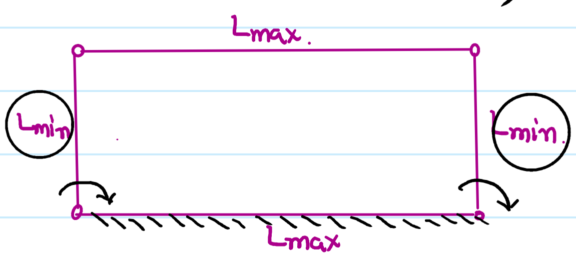
Crank – Crank Mechanism.

I/p & o/p → o/w motion.



Inversion – II.

Fixed link. – L_{max} (adjacent to L_{min})



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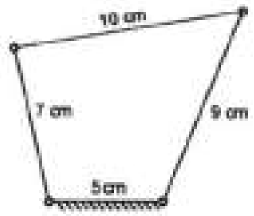
Identify the type of Mechanisms

(i)



(a)

$L_{min} = 5cm$

$L_{max} = 10cm.$

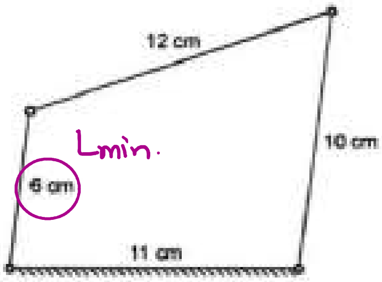
$P = 9\ cm. \quad Q = 7cm.$

$L_{max} + L_{min} < P + Q \longrightarrow$ class-I linkage.

$5 + 10 < 9 + 7$

L_{min} is fixed crank-crank.

(ii)



(c)

$L_{min} = 6cm \quad P = 10cm.$

$L_{max} = 12cm. \quad Q = 11\ cm.$

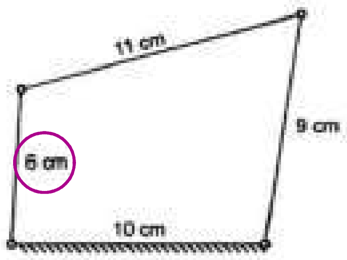
$L_{min} + L_{max} < P + Q$

$6 + 12 < 10 + 11 \longrightarrow$ class-I linkage.

Link adjacent to L_{min} is fixed.

Crank - Rocker Mechanism.

(iii)



(b)

$L_{min} = 6cm \quad L_{max} = 11cm.$

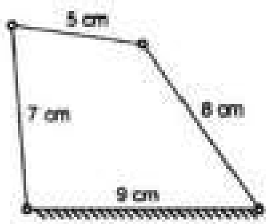
$P = 9cm \quad Q = 10cm.$

$L_{min} + L_{max} < P + Q.$

$6 + 11 < 9 + 10 \longrightarrow$ class-I. linkage

Crank - Rocker Mechanism

(iv)



(d)

$L_{min} = 5cm \quad L_{max} = 9cm.$

$P = 7cm \quad Q = 8cm.$

$L_{min} + L_{max} < P + Q.$

$5 + 9 < 7 + 8 \longrightarrow$ class I linkage.

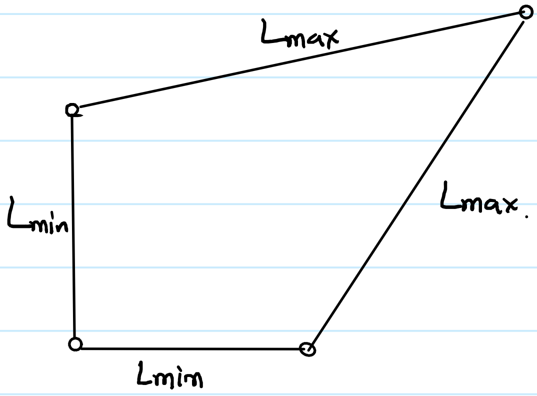
Rocker - Rocker Mechanism.

$L_{min} = 5cm$ will complete circular motion.

case (ii) $L_{min} = P$ $L_{max} = q$

Links of equal lenghts are adjacent to each other.

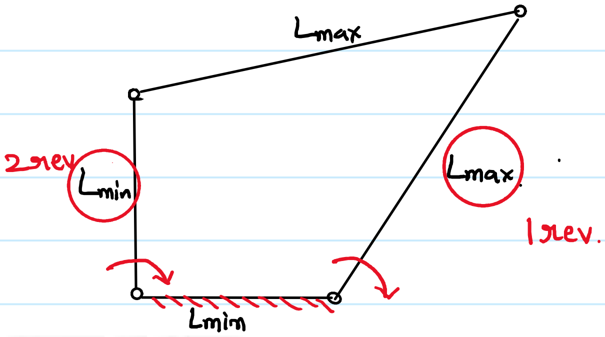
→ Deltoid / Kite linkage



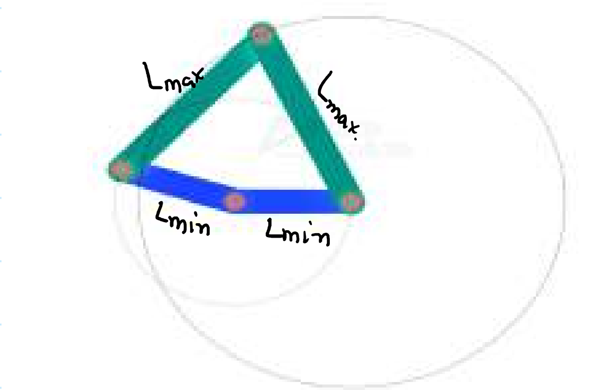
Inversion – I.

Fixed link – Lmin.

Crank-Crank Mechanism.



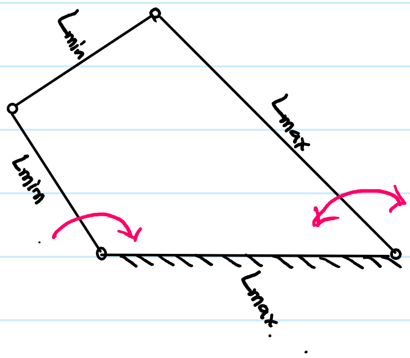
For 1 rev of Lmax the Lmin will complete 2 rev.



Inversion – II

Fixed link – Lmax. (Link adjacent to Lmin)

Crank – Rocker Mechanism.



From given arrangement of links identify which of the following is mechanism

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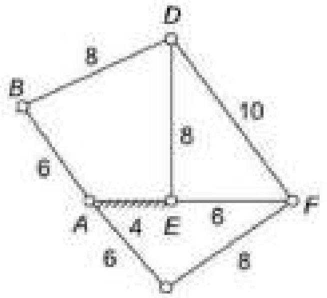
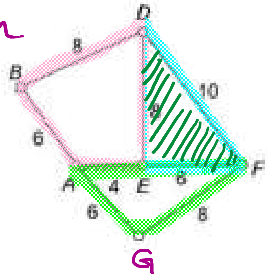


AEDB — 4 bar mechanism.

AEFG — 4 bar mechanism

EDF → Not a mechanism.

↳ It can be assumed as a single link.



Mechanism AEDB

$L_{min} = 4 \quad L_{max} = 8 \quad P = 8 \quad Q = 6$

$L_{min} + L_{max} < P + Q \rightarrow 4 + 8 < 8 + 6 \longrightarrow$ class – I. (Crank-crank)

Mechanism AEFG — $L_{min} = 4 \quad L_{max} = 8 \quad P = 6 \quad Q = 6$

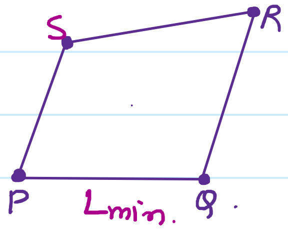
$L_{min} + L_{max} = P + Q \Rightarrow 4 + 8 = 6 + 6 \longrightarrow$ class – III (Crank-Crank)

02. A planar closed kinematic chain is formed with rigid links $\boxed{PQ = 2.0\text{ m}}$ (L_{min}), $\boxed{QR = 3.0\text{ m}}$ → L_{max}, RS = 2.5 m and SP = 2.7 m with all revolute joints. The link to be fixed to obtain a double rocker (rocker-rocker) mechanism is (GATE-13)

(a) $\boxed{PQ}$ → L_{min}.
(b) $\boxed{QR}$ → Crank-Rocker Mechanism.
(c) $\boxed{RS}$ → Rocker - Rocker (Link opposite to L_{min} is fixed)
(d) $\boxed{SP}$

Crank-crank. Mechanism. (L_{min} is fixed)

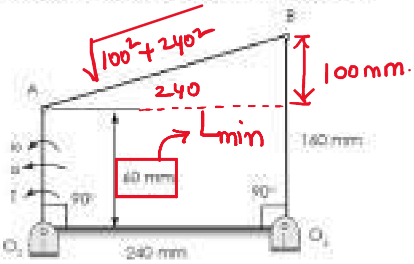
Link adjacent to L_{min} is fixed.



$L_{max} + L_{min} < P + Q$

$3 + 2 < 2.5 + 2.7 \longrightarrow$ class. I.

An instantaneous configuration of a four bar mechanism, whose plane is horizontal, is shown in the fig below.



$L_{min} = 60\text{ mm}$.

$L_{max} = 260\text{mm}$.

$P = 240\text{mm}. \quad Q = 160\text{mm}$.

At this instant, the angular velocity and angular acceleration of link O_1A are $\omega = 8$ rad/s and $\alpha = 0$, respectively, and the driving torque (τ) is zero. The link O_1A is balanced so that its center of mass falls at O_1. (GATE-05)

$L_{max} + L_{min} < P + Q$

$60 + 260 < 240 + 160 \longrightarrow$ class – I.

Link adjacent to L_{min} is fixed. so it is Crank – Rocker Mechanism.

42. Which kind of 4-bar mechanism is O_1ABO_2?
(a) Double-crank mechanism
(b) Crank-rocker mechanism
(c) Double-rocker mechanism
(d) Parallelogram mechanism

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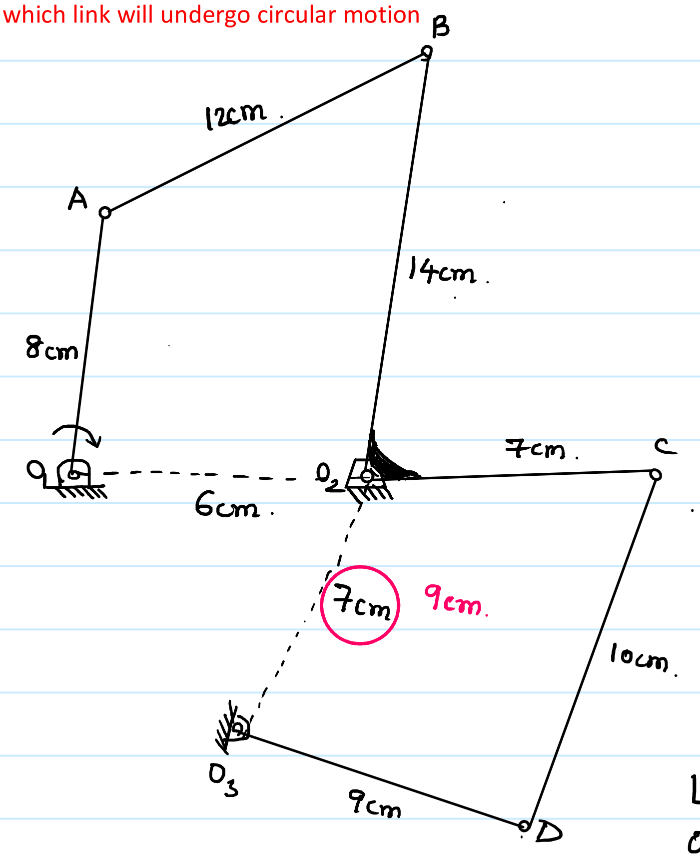
Identify which link will undergo circular motion



$BO_2C \longrightarrow$ compound link / L-shape link.

$O_1ABO_2 \rightarrow$ Mechanism.

$L_{min} = 6\,cm$, $L_{max} = 14\,cm$

$P = 8\,cm$ $\quad Q = 12\,cm$.

$L_{min} + L_{max} = P + Q$

$\underbrace{6 + 14 = 12 + 8}$

class – III linkage.

L_{min} is fixed so O_1A and O_2B will be cranks.

O_2CDO_3 – Mechanism.

$L_{min} = 7\,cm$ $\quad L_{max} = 10\,cm$. $\quad P = 7\,cm$ $\quad Q = 9\,cm$.

$L_{min} + L_{max} > P + Q$

$7 + 10 > 7 + 9 \longrightarrow$ class – II linkage.

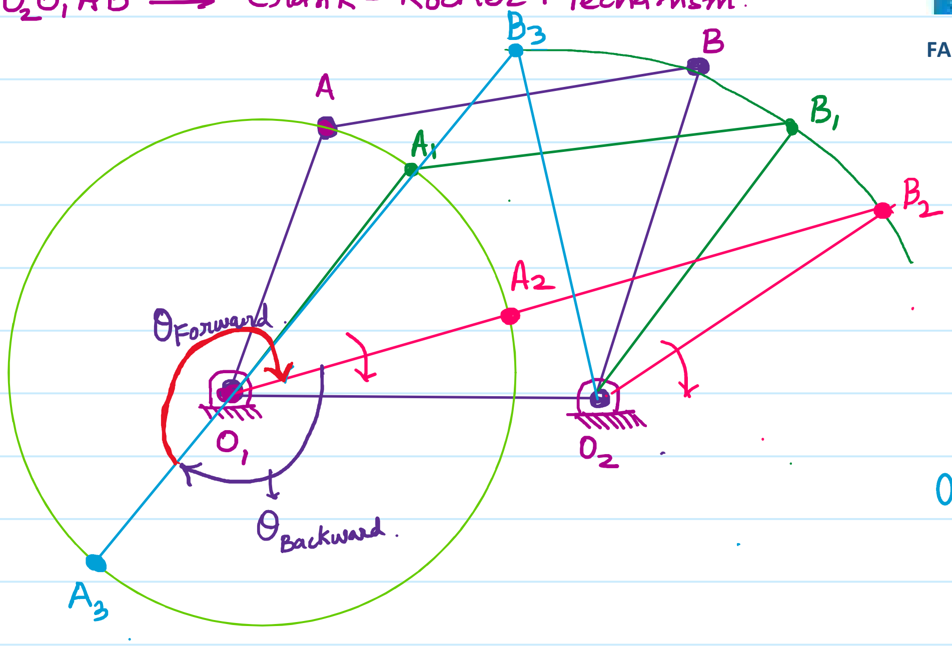
Link O_2C must be crank it cannot be rocker because BO_2C is a single link and Locus of B and C must be same.

$7 + 10 < 9 + 9 \longrightarrow$ class – I linkage.

Crank – Rocker mechanism.

link O_1A and BO_2C will complete circular motion

$O_2O_1AB \longrightarrow$ Crank-Rocker Mechanism.

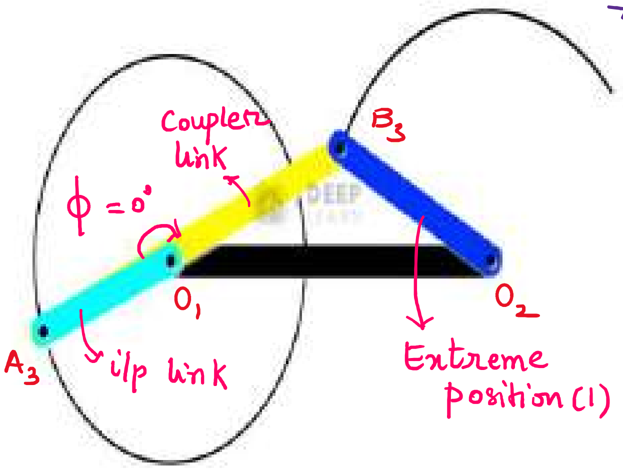


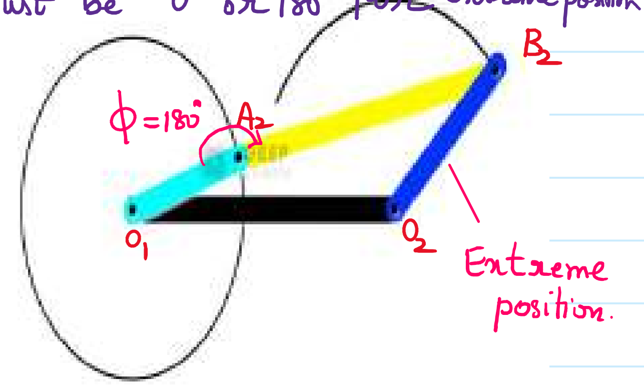
$O_1A_2B_2O_2$ – Extreme position.

$O_2A_3B_3O_2$ — Extreme position.

If crank O_1A moves from O_1A_3 to O_1A_2 the Rocker O_2B moves from point B_3 to B_2

Angle b/w i/p and coupler link. must be 0° or 180° for extreme position





$\theta_{Forward} > \theta_{Backward} \longrightarrow$ Quick Return. Mechanism.

$$\frac{\theta_{cutting}}{\theta_{Return}} > 1$$

Position Analysis of 4-bar mechanism:

1. Extreme positions of rocker in a Crank-Rocker mechanism of Class – I linkage

2. It is given by the angle between input and coupler of a 4-bar mechanism.

3. In a four bar mechanism of (C-R of class – I) whenever the angle between the input and coupler is 0 or 180 degree the rocker will be at its extreme position.

4. At the extreme position the angular velocity of rocker will be zero and angular retardation will be maximum.

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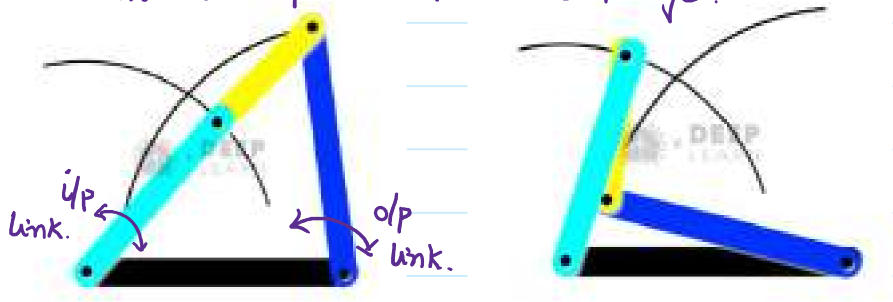
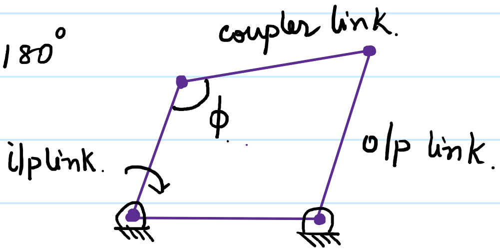
Extreme position of o/p link in Rocker-Rocker mechanism of class-I linkage.



Pressure Angle - It is angle subtended between the input link and coupler link

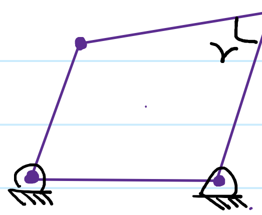
$\phi = 0^\circ$ or 180°



ϕ – Pressure Angle.

Transmission Angle - It is the angle subtended between the output link and the coupler link.

γ – Transmission Angle.

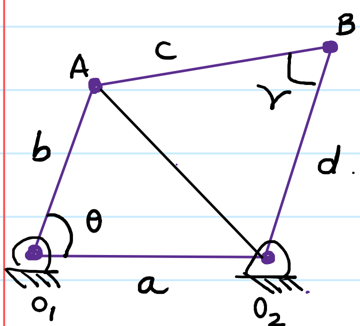


- It is a important performance parameter because it signifies about the Possibility of relative motion getting transmitted.

- Smaller values of transmission angle must be avoided because under The presence of friction forces the mechanism has the tendency to get Locked at the smaller values of transmission angle.

$$\boxed{35/40 \leq \gamma \leq 135/140}$$

θ - i/p Angle.



In Δ^{le} O_2O_1A

$$AO_2^2 = O_1A^2 + O_1O_2^2 - 2 \cdot O_1A \cdot O_1O_2 \cdot \cos\angle O_2O_1A$$

$$AO_2^2 = a^2 + b^2 - 2ab\cos\theta \longrightarrow (1)$$

In Δ^{le} ABO_2

$$AO_2^2 = AB^2 + BO_2^2 - 2 \cdot AB \cdot BO_2 \cos\angle ABO_2$$

$$AO_2^2 = c^2 + d^2 - 2dc\cos\gamma \longrightarrow (2)$$

(1) = (2)

$$a^2+b^2-2ab\cos\theta = c^2+d^2-2dc\cos r$$

$$\cos r = \frac{c^2+d^2-a^2-b^2+2ab\cos\theta}{2dc}$$

$$r = f(\underbrace{a, b, c, d}_{\text{constants}}, \theta) \longrightarrow \text{variable.}$$

$$r = f(\theta)$$

For r = Max or Min. $\dfrac{dr}{d\theta} = 0$

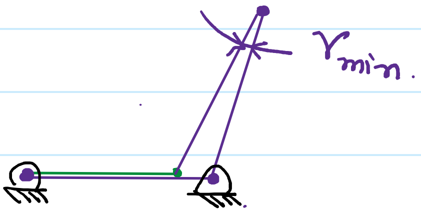
$$-\sin r \cdot \frac{dr}{d\theta} = \frac{0+0-0-0+2ab(-\sin\theta)}{2dc}$$

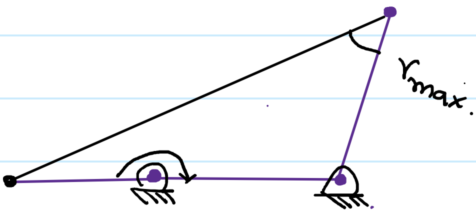
$$\frac{dr}{d\theta} = \frac{ab\sin\theta}{cd\sin r} \qquad \frac{dr}{d\theta} = 0 \qquad \begin{array}{l}\sin\theta = 0, 180 \\ \sin r = \infty \text{ (Not possible)}\end{array}$$

$$\sin\theta = 0 \Rightarrow \theta = 0°, 180°$$

θ – Angle b/w input and fixed link.

$\theta = 0°$





$\theta = 180$

- Maximum and Minimum values of transmission angle is dependent on the position of input link w.r.t fixed link.
- Extreme positions of output link (rocker) in crank rocker mechanism is dependent on the angle between input link and coupler link (pressure angle).

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Mechanical Advantage -

It is the performance parameter which measures the effectiveness of mechanism.

$$M.A. = \frac{\text{Load}}{\text{Effort}} > 1$$

$$M.A. = \frac{\text{Torque @ o/p}}{\text{Torque @ i/p}} > 1$$

$$M.A. = \frac{\sin\gamma}{\sin\phi}$$ [In terms of Transmission and Pressure Angle]

$M.A. \propto \sin\gamma$ — As $\gamma\uparrow$ $\sin\gamma\uparrow$ $M.A.\uparrow$

$M.A. \propto \frac{1}{\sin\phi}$ — As $\phi\downarrow$ $\sin\phi\downarrow$ $M.A.\uparrow$

Efficiency $\eta = \frac{\text{Power @ o/p}}{\text{Power @ i/p}} = \boxed{\frac{T_{o/p}}{T_{i/p}}}\boxed{\frac{\omega_{o/p}}{\omega_{i/p}}}$

M.A.

Velocity Ratio (V.R)

$$\eta = M.A \times (V.R)$$

Conservation of Energy $\eta = 100\%$

$$M.A. = \frac{1}{V.R.}$$

$$\boxed{V.R. = \frac{\omega_{o/p}}{\omega_{i/p}}}$$

$$\boxed{M.A. = \frac{\omega_{i/p}}{\omega_{o/p}}}$$

Also written as

$$\boxed{V.R. = \frac{D_{o/p}}{D_{i/p}} = \text{const.}}$$

- Velocity Ratio is a performance parameter for those mechanisms in which relative motion is transferred without transformation (ex - Gear and Pinion , Belt and Pulley & Chain and Sprocket).
- In the above mentioned mechanism relative motion is transferred at constant ratio.

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- Mechanical advantage is performance parameter for those mechanisms In which relative motion is transferred with transformation (ex- crank Slider mechanism).

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- The value of Mechanical advantage is dependent on pressure angle and Transmission angle , they vary based on the change in the configuration of Mechanism.

- Mechanical advantage is variable during the motion transmission.

→ At the extreme position of o/p link. (Rocker) the value of $\boxed{\phi = 0^\circ \text{ or } 180^\circ}$ → Toggle Position.

$M.A. = \infty$

$M.A. = \dfrac{T_{o/p}}{T_{i/p}}$ [Very small amount of Torque is required at input in order overcome very large value of Torque at the o/p].

As $M.A. \longrightarrow 0$ the mechanism is likely to get locked.

$\boxed{M.A. > 1.}$

$\eta = M.A \times \boxed{V.R}$ $(V.R. < 1)$

$\eta \leq 100\%$

$M.A. > 1$ $V.R. > 1.$

$\eta = \dfrac{M.A.}{V.R.}$ $M.A \propto \sin\gamma$

Observation:

- M.A = ∞ means a very small input torque is required to overcome a very large output torque.
- For a 4-bar mechanism M.A will be variable as pressure angle and transmission angle changes continuously.
- In a mechanism corresponding to large value of transmission angle mechanical advantage will be large.
- Velocity ratio is the performance parameter mainly for those mechanism which do not transform the type of relative motion.

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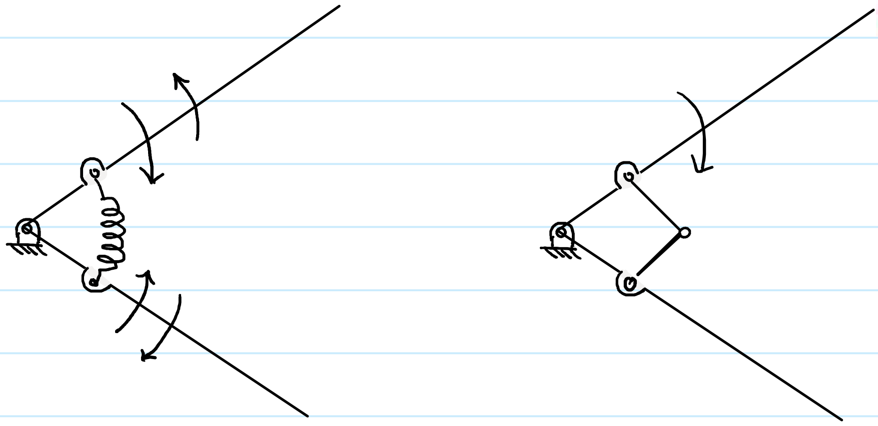
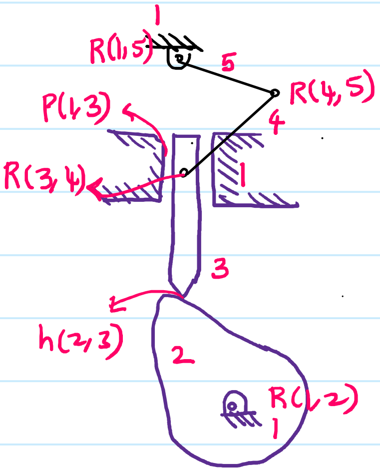
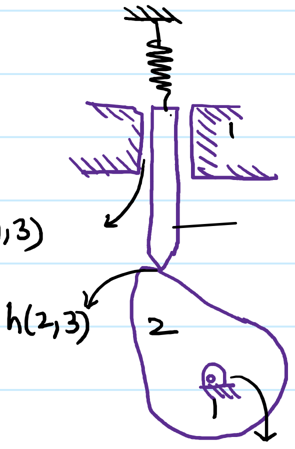
Equivalence of Links



Spring $\equiv$ 2 links. (Binary links)



$N = 5$
$j = 5$
$h = 1$



$N = 3$
$j = 2$
$h = 1$

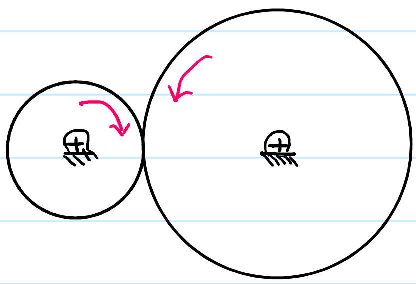
$$\begin{aligned} DOF &= 3(N-1) - 2j - h \\ &= 3(5-1) - 2(5) - 1 \\ &= 12 - 10 - 1 = 1 \end{aligned}$$

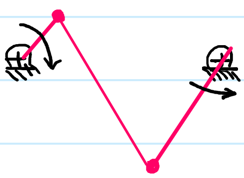
$$\begin{aligned} DOF &= 3(3-1) - 2(2) - 1 \\ &= 6 - 4 - 1 \\ &= 1 \end{aligned}$$

Spring is a mechanical element used for restoring force or energy.

Equivalence of Higher pair and Lower Pair

1 Higher pair $\equiv$ 2 lower pair
1 Higher Pair $\equiv$ 1 Binary link.





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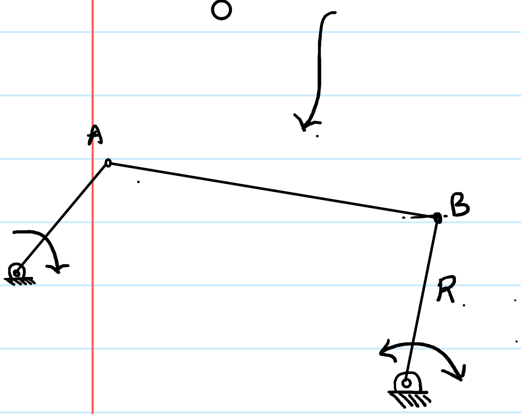
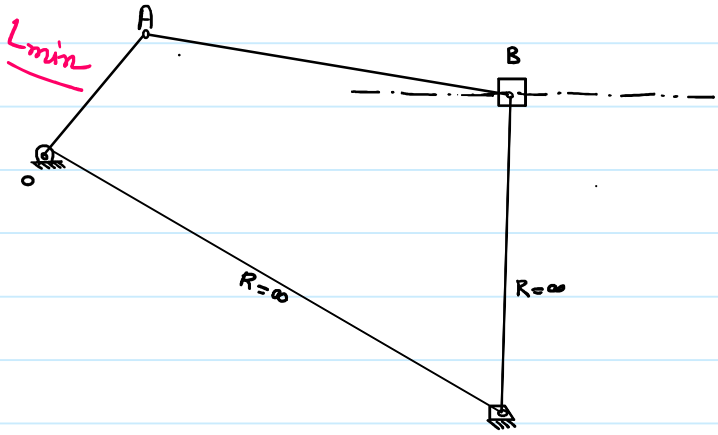
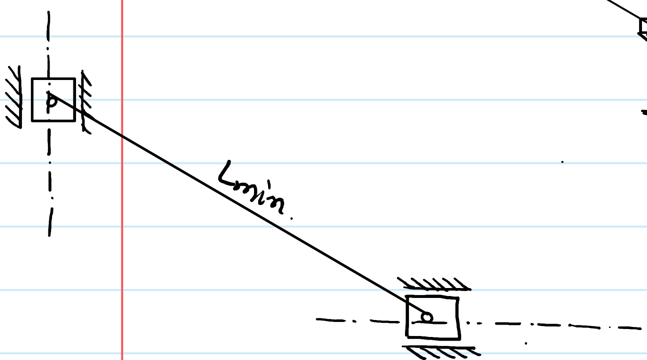
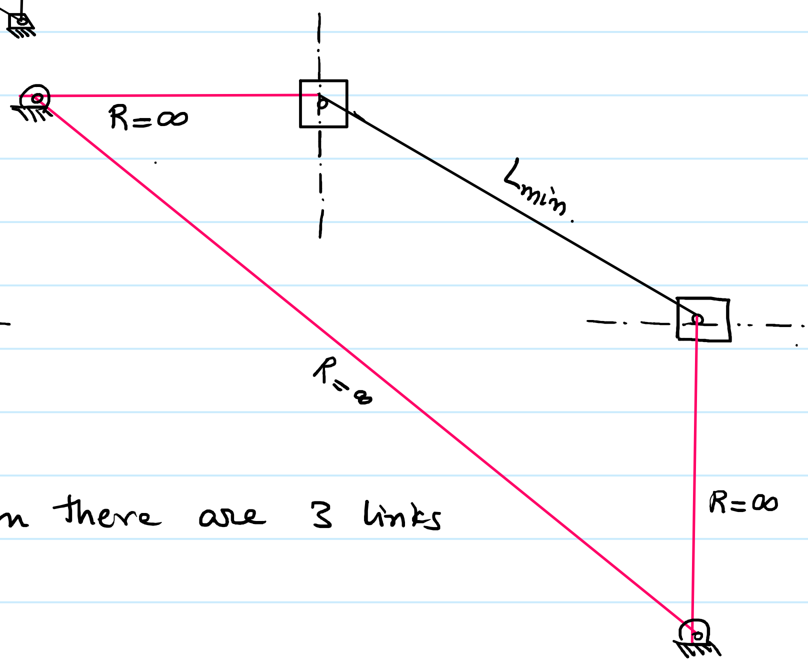


As $R \to \infty$ Locus of point B will become a st. line.

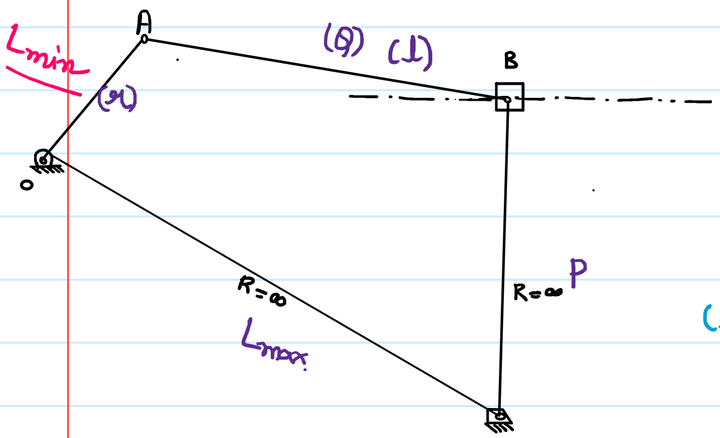


In crank slider there are 2 links of infinite lenght which it is converted to 4 bar mechanism.

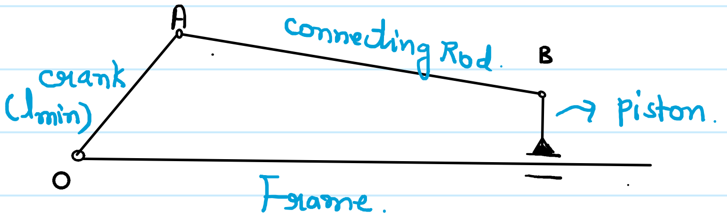




In double slider mechanism there are 3 links of infinite lenght.



Crank slider chain.



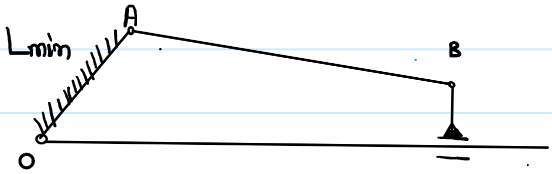
No. of inversions = No. of different links = 4.

According Grashof's law $L_{min} + L_{max} < p + q$

$\Rightarrow r + \infty \leq \infty + l.$

Inversion – I. Fixed link – Lmin.

→ Inversion – crank – crank mechanism.

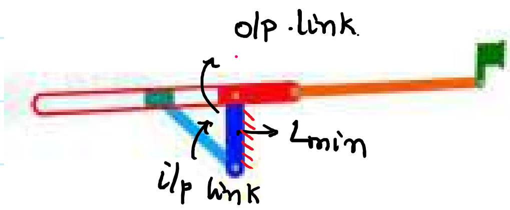


I/p & o/p link – o/lur motion.

Application
1. Radial Engine / Wrankel engine.
2. Whitworth Mechanism.

Radial Engine.

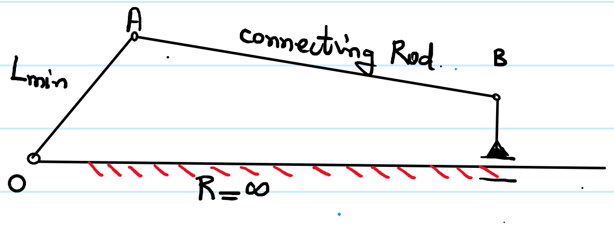
Whitworth Mechanism.



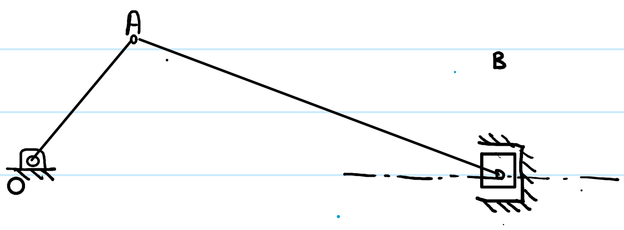
Inversion – II . case (A)

Fixed link – link adjacent to L_{min}.
(Link of infinite lenght)

Inversion – crank – Rocker Mechanism



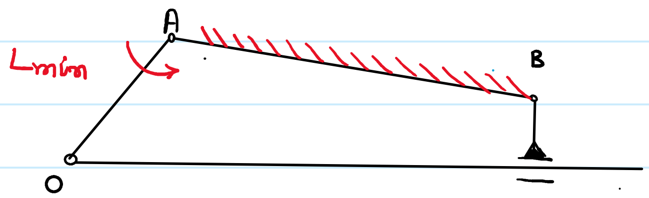
Application – I C Engine, Air-compressor, offset crank slider mechanism



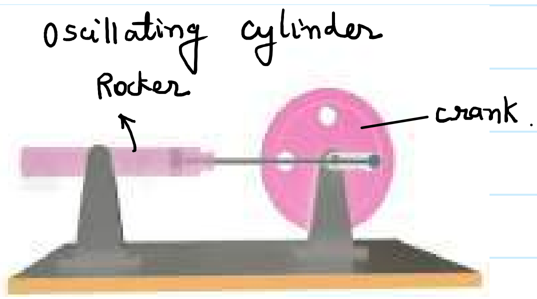
Case (B)

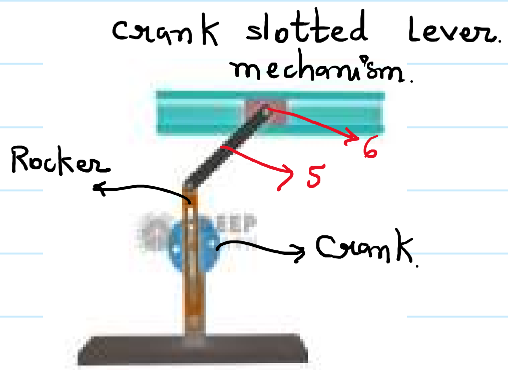
Fixed Link – Link adjacent to L_{min} / connecting Rod / Link 3 /
Link of finite lenght.

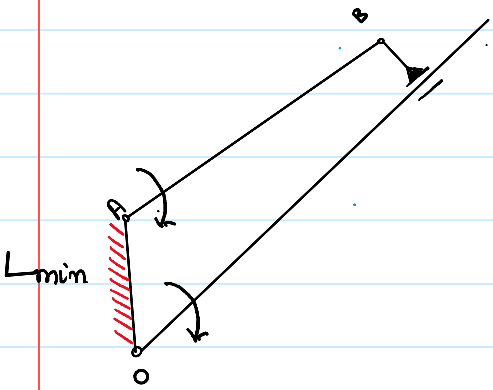
Inversion – Crank – Rocker mechanism.



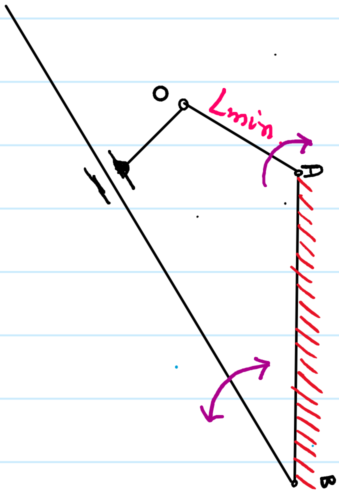
Application – 1. Crank Slotted lever mechanism.
2. Oscillating cylinder.







Whitworth Mechanism.



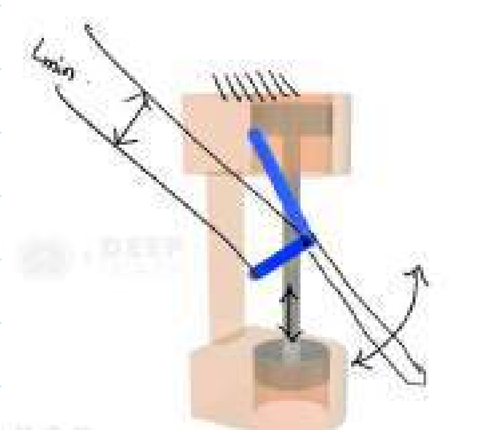
Crank slotted lever mechanism.

Inversion – III

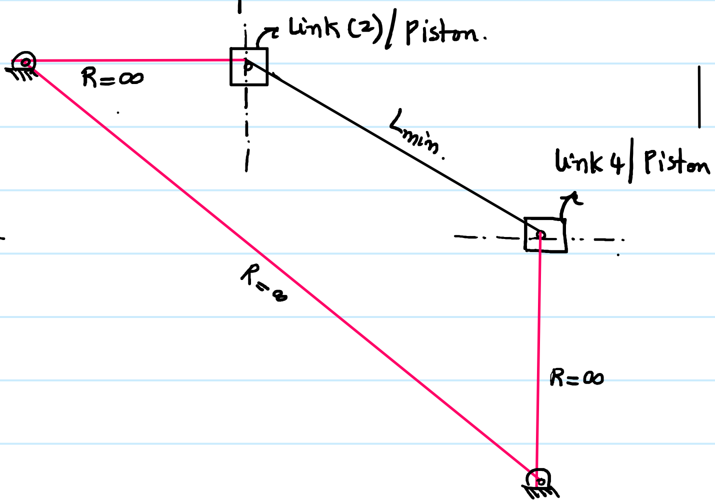
Fixed link – link opposite to Lmin / Link 4 / Link of infinite lenght / Piston Rod.

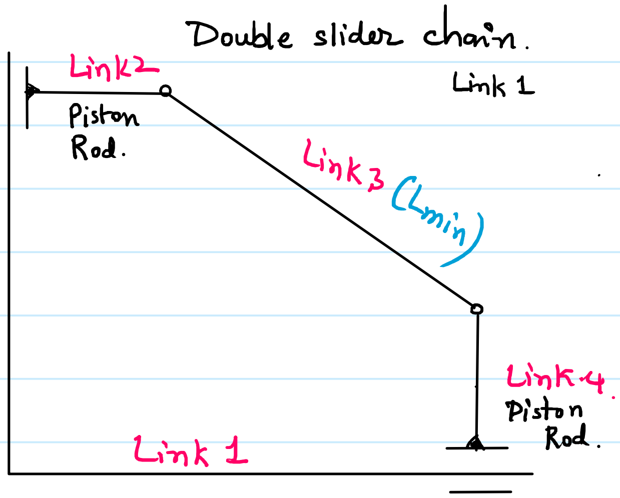
Inversion – Rocker – Rocker Mechanism

Application – Hand pump / Pendulum Pump.



Inversions of Double slider mechanism.





No. of different links = No. of inversions = 3.

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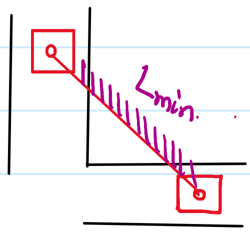
Inversion – I.

Fixed link. – L_{min}.

Inversion – Double crank Mechanism.



Application – Oldham Coupling

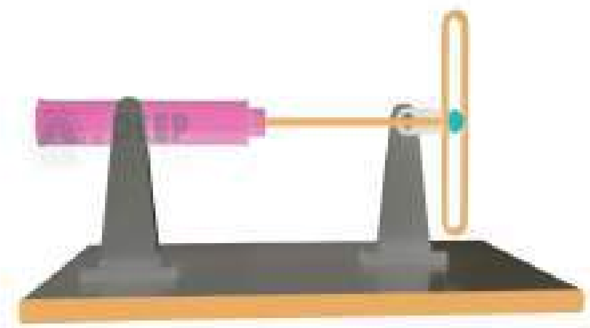
Inversion – II.

Fixed link – Link adjacent to L_{min} / Piston Rod / Piston.

Inversion – Crank – Rocker mechanism.

Application – Scotch Yoke Mechanism.

↳ This mechanism is used for generating sine and cosine functions.



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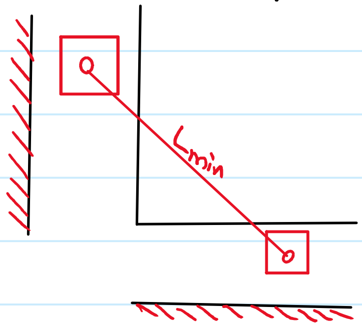
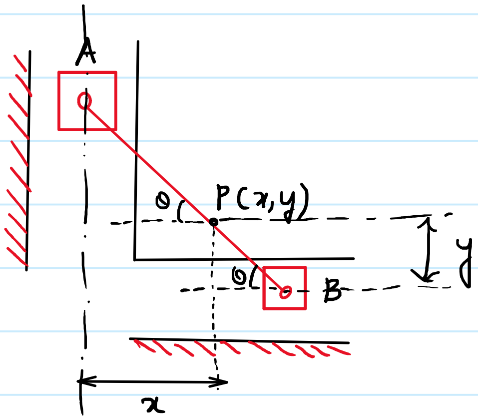
Inversion – III.

Fixed link — Link opposite to Lmin

Inversion – Rocker – Rocker mechanism.

Application – Elliptical trammel.





$$x = AP\cos\theta$$

$$y = BP\sin\theta.$$

$$\left(\frac{x}{AP}\right)^2 + \left(\frac{y}{BP}\right)^2 = 1$$

Locus of point P is ellipse.

AP– Lenght of semi major axis.

BP– Lenght of semi minor axis.

If $AP = BP$ [P is a mid point]

Locus of midpoint is circle.

$$x^2 + y^2 = AP^2$$

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Analysis of Quick Return Mechanism

The mechanism in which return stroke is completed in the shortest
Possible time is called Quick return Mechanism

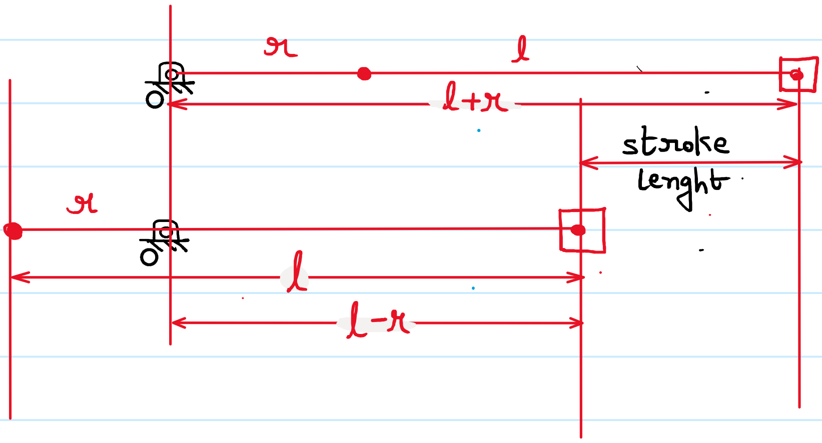
Types of Quick Return Mechanism

1. **Offset Crank slider Mechanism**
2. **Whitworth Mechanism**
3. **Crank Slotted lever Mechanism**

Parameters

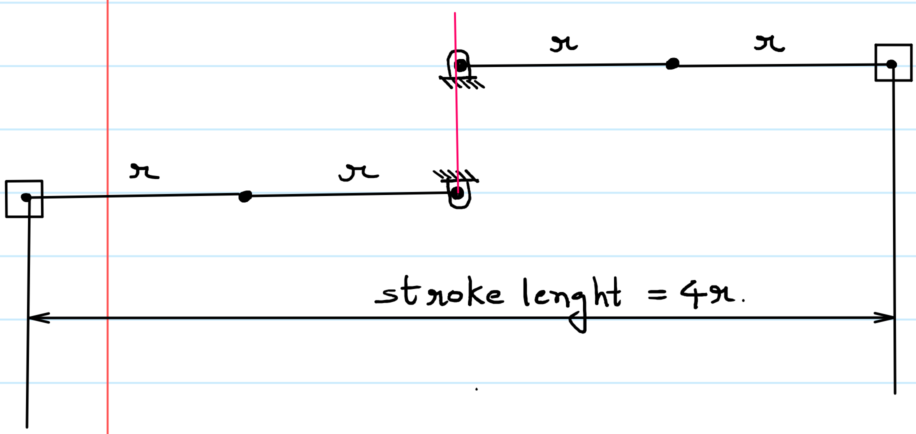
1. Stroke Length - It the distance between the extreme positions of cutting tool.
2. Quick Return Ratio - It is ratio of time required for cutting stroke to time required for return stroke.
It is ratio of angular displacement of input link responsible for cutting stroke to angular displacement of input link responsible for return stroke.

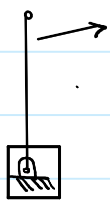
$$Q.R.R. = \frac{\theta_{cutting}}{\theta_{Return}} > 1 \qquad Q.R.R. = \frac{\theta_{Return}}{\theta_{cutting}} < 1$$



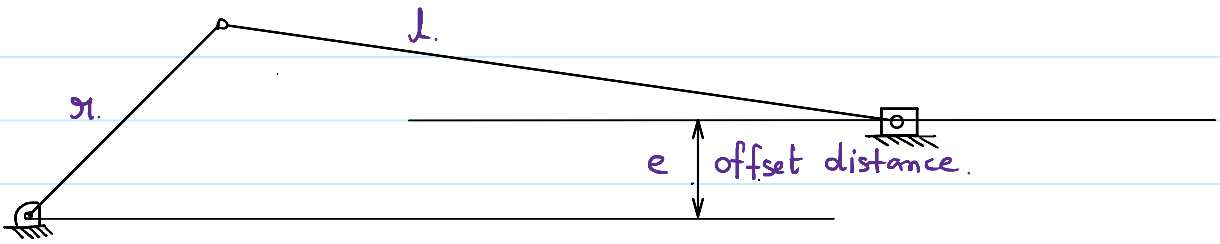
stroke lenght $= 2r$.

If $l = r$ then stroke lenght

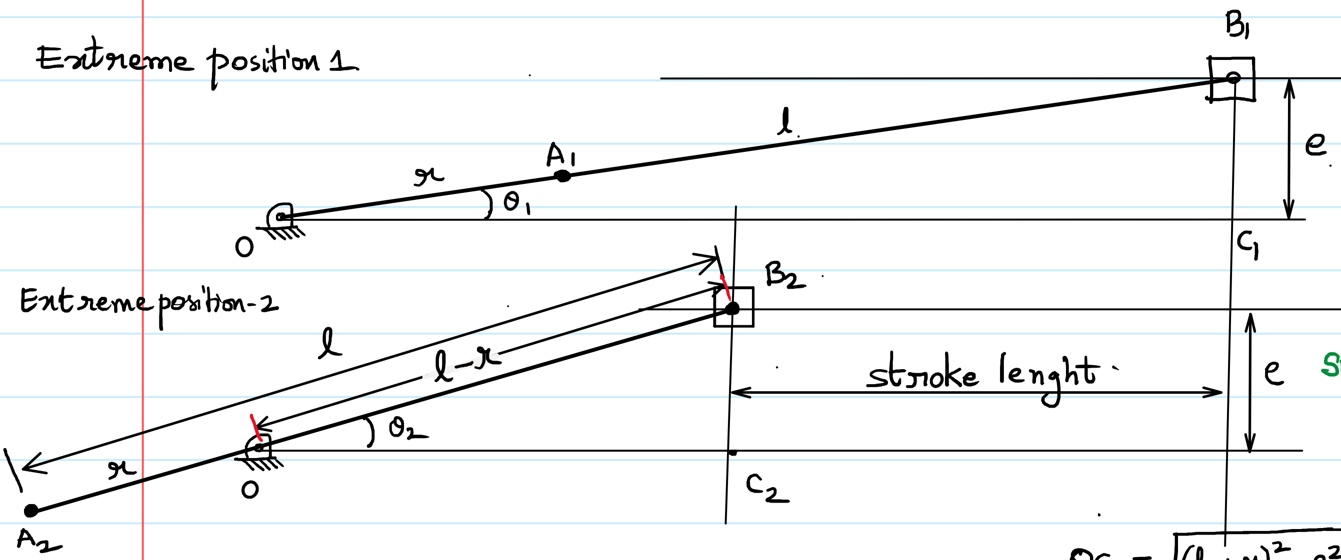




$\gamma = 0$
Mechanism is getting locked.
M.A. $= 0$



r – crank lenght l – lenght of connecting rod.
e – offset distance.



$\sin\theta_1 = \dfrac{e}{l+r}$

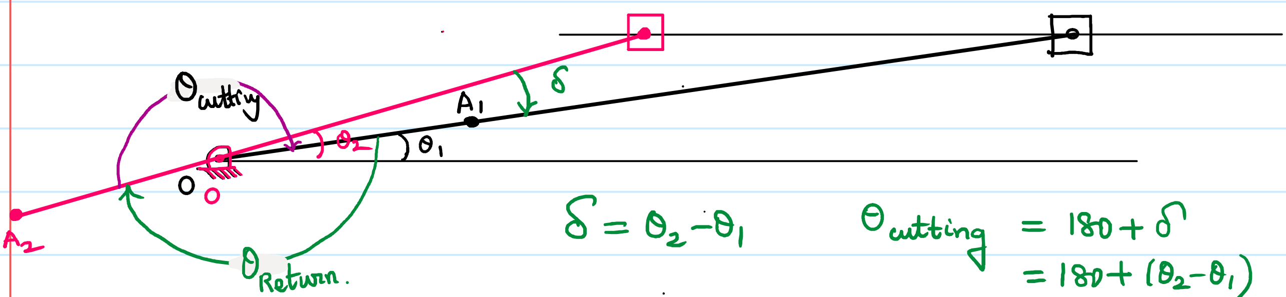
$\sin\theta_2 = \dfrac{e}{l-r}$

$OC_1 = \sqrt{(l+r)^2 - e^2}$

$OC_2 = \sqrt{(l-r)^2 - e^2}$

stroke lenght $= OC_1 - OC_2 = C_1C_2$

stroke lenght $= \left[\sqrt{(l+r)^2 - e^2}\right] - \left[\sqrt{(l-r)^2 - e^2}\right]$



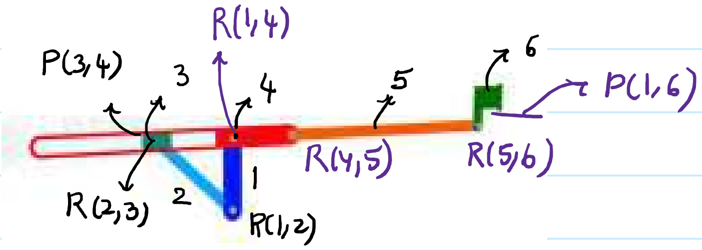
$\delta = \theta_2 - \theta_1$

$\theta_{cutting} = 180 + \delta$
$= 180 + (\theta_2 - \theta_1)$

$\theta_{Return} = 180 - \delta$
$= 180 - (\theta_2 - \theta_1)$

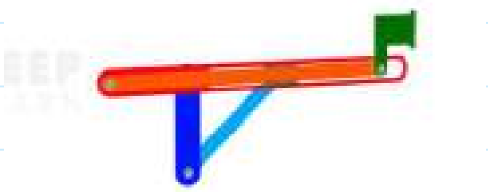
$$Q.R.R. = \frac{\theta_{cutting}}{\theta_{Return}} = \frac{180 + (\theta_2 - \theta_1)}{180 - (\theta_2 - \theta_1)}$$

6 link mechanism. 5 Revolute & 2 Prismatic Joints.

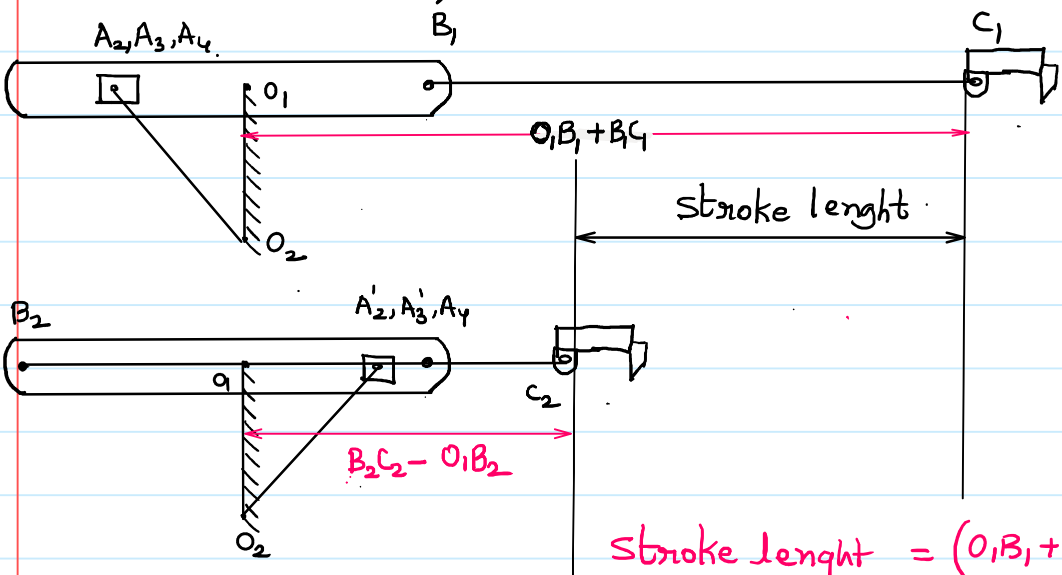


Extreme Position 1

Extreme position.

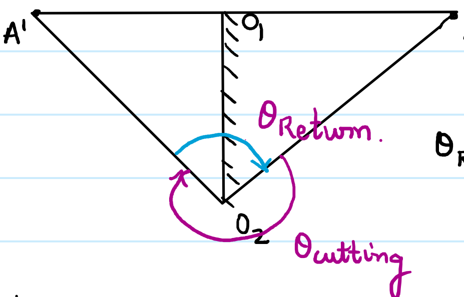


A_2, A_3, A_4 are points on link 2 (crank), link 3 (slider) link 4. (slotted bar)

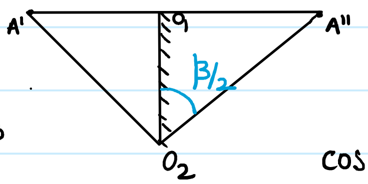


$B_1C_1 = B_2C_2$
$O_1B_1 = O_1B_2$

stroke lenght $= (O_1B_1 + B_1C_1) - (B_2C_2 - O_1B_2)$
$= 2 \times O_1B_1$



$\theta_{Return} = \beta$

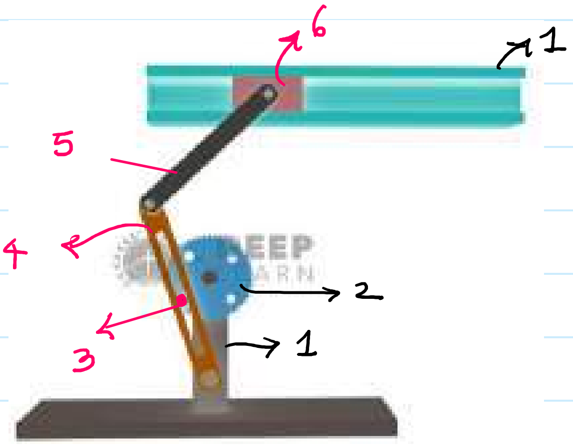


In $\Delta O_1O_2A''$

$$\cos\beta/2 = \frac{O_1O_2}{O_2A''}$$

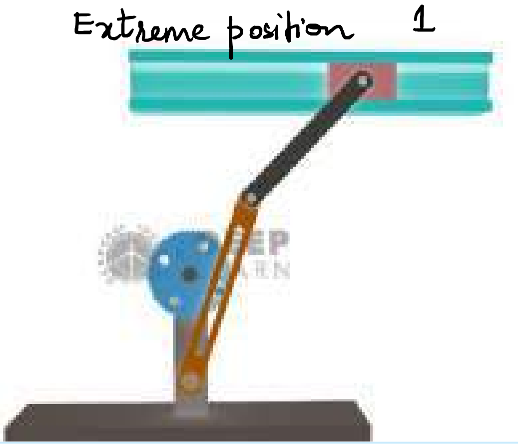
$$\text{Quick Return Ratio} = \frac{\theta_{cutting}}{\theta_{Return}} = \frac{360-\beta}{\beta}.$$

$$\cos\beta/2 = \frac{\text{lenght of fixed link}/(L_{min})}{\text{lenght of crank}}.$$

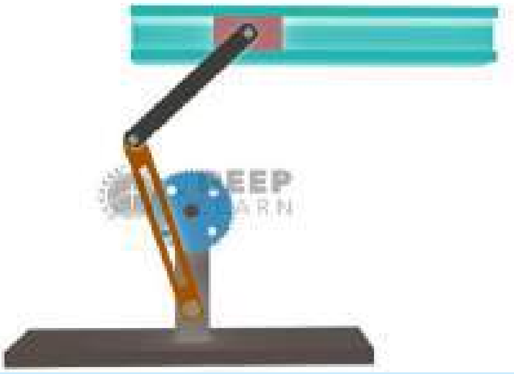


6 links mechanism.
5 revolute and 2 Prismatic joints.

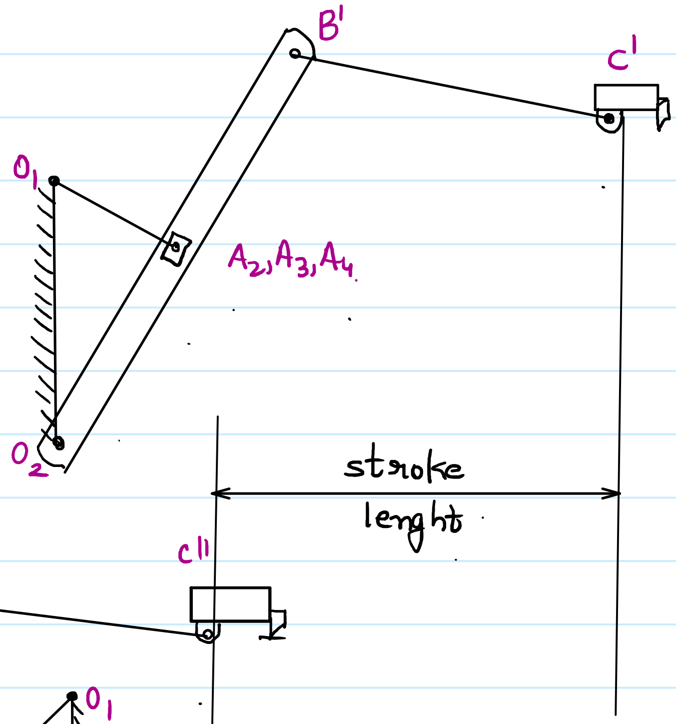
Extreme position

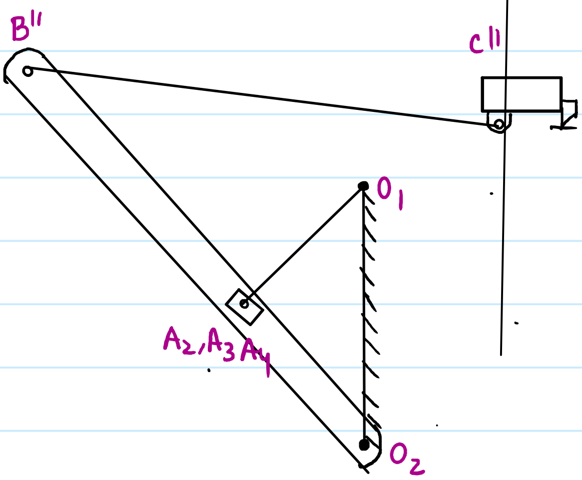


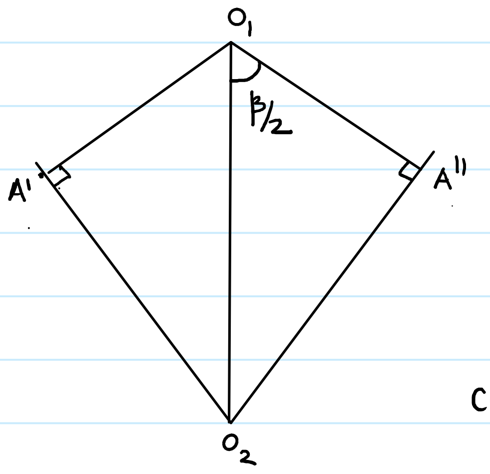
Extreme Position.



A₂, A₃, A₄ are points on link 2 (crank), link 3 (slider) link 4 (slotted bar)



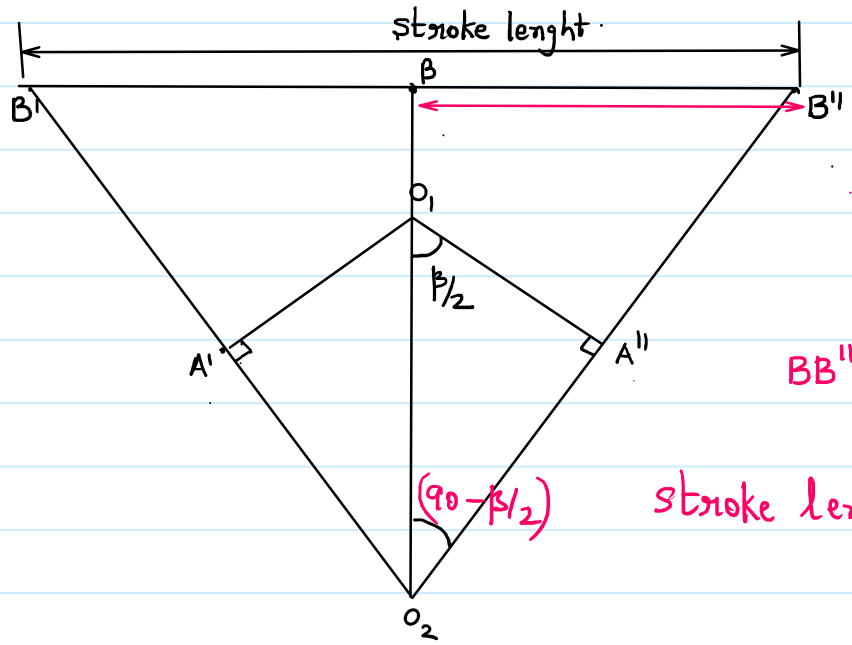




In $\triangle O_1O_2A''$

$$\cos\beta/2 = \frac{O_1A''}{O_1O_2}$$

$$\cos\beta/2 = \frac{\text{lenght of crank} / L_{min}}{\text{lenght of fixed link}}$$



In $\triangle^{le}\, O_2BB''$

$$\sin(90-\beta/2) = \frac{BB''}{O_2B''}$$

$$BB'' = O_2B'' \cdot \sin(90-\beta/2)$$

$$\text{stroke lenght} = 2\times BB''$$

$$\text{stroke lenght} = 2\times O_2B'' \times \cos(\beta/2)$$

$$= \frac{2\times \text{Lenght of slotted bar} \times \text{Lenght of crank}}{\text{lenght of fixed link}}$$

Observations

- The stroke length in Whitworth Quick Return Mechanism is dependent on **extension of slotted bar** diametrically opposite to the slider.

- In the Crank Slotted lever Mechanism the stroke length is dependent on **length of slotted bar , length of Crank and length of Fixed link**.

case. If Q.R.R. = 2 or Q.R.R. = 1/2

$$Q.R.R. = \frac{\theta_{cutting}}{\theta_{Return}} = 2 \checkmark$$

$$Q.R.R. = \frac{1}{2} = \frac{\theta_{Return}}{\theta_{cutting}}$$

$$Q.R.R. = \frac{360-\beta}{\beta} = 2 \Rightarrow \beta = 120^\circ$$

Whitworth mechanism.

$$\cos(\beta/2) = \frac{\text{lenght of fixed link}/L_{min}}{\text{lenght of crank}}$$

$$\cos\left(\frac{120}{2}\right) = \frac{1}{2} = \frac{\text{lenght of fixed link}/L_{min}}{\text{lenght of crank}} = \frac{1}{Q.R.R.}$$

Crank slotted lever mechanism.

$$\cos(\beta/2) = \frac{\text{lenght of crank}/L_{min}}{\text{lenght of fixed link}}$$

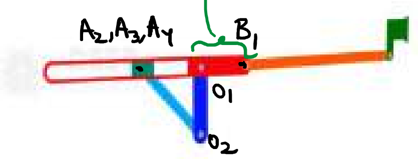
$$\cos\left(\frac{120}{2}\right) = \frac{1}{2} = \frac{\text{lenght of crank}/L_{min}}{\text{lenght of fixed link}} = \frac{1}{Q.R.R.}$$

stroke lenght

Whitworth Mechanism

stroke lenght = 2 x extension of slotted bar diametrically opposite to link3 / slider.

extension of slotted bar diametrically opposite to link3 / slider



Crank slotted Lever Mechanism

$$\text{stroke} = \frac{2 \times \text{lenght of slotted bar} \times \text{lenght of crank}}{\text{lenght of fixed link}}$$

If Q.R.R. = 2 $\frac{1}{2} = \frac{\text{lenght of crank}/L_{min}}{\text{lenght of fixed link}}$

stroke lenght = 2 x lenght of slotted bar x 1/2

stroke lenght = lenght of slotted bar.

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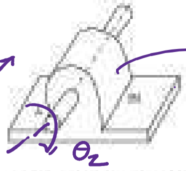
01. A round bar A passes through the cylindrical hole in B as shown in the given figure. Which one of the following statements is correct in this regard?



→ cylinderical pair

DOF = 2

(a) The two links shown form a kinematic pair
(b) The pair is completely constrained
(c) The pair has incomplete constraint — / Lower pair.
(d) The pair is successfully constrained

03. For a four – bar linkage in toggle position, the value of mechanical advantage is
(a) 0.0 (b) 0.5
(c) 1.0 (d) ∞ ✓

→ Extreme positions of Rocker

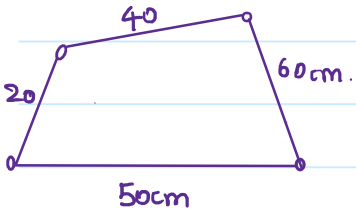
$\phi = 0°, 180°$

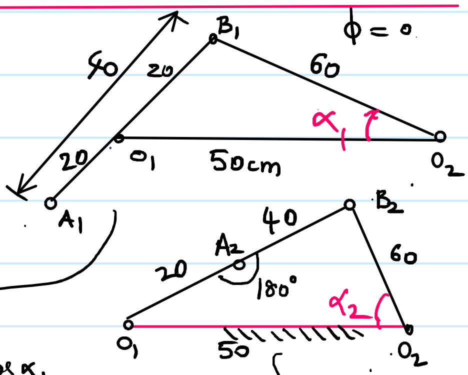
$M.A. = \frac{\sin\gamma}{\sin\phi} = \infty$

Statement for Common data Q 04 & 05

In a crank rocker mechanism the lengths of the fixed link, crank, coupler and rocker are respectively 50, 20, 40 and 60 cm.

$\phi = 0°, 180°$





04. Extreme positions of rocker measured with the fixed link are
(a) 60° & 134.43° (b) 60° & 13.77°
(c) 134.43° & 13.77° (d) 65.37° & 18.19°

In $\Delta^{le}\ O_1B_1O_2$

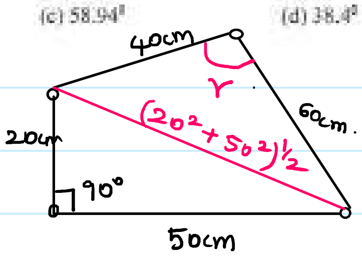
$20^2 = 50^2 + 60^2 - 2(50)(60)\cos\alpha_1$

$\alpha_1 = \underline{18.19°}$

$60^2 = 60^2 + 50^2 - 2(50)(60)\cos\alpha_2$

$\alpha_2 = \underline{65.37°}$

05. Transmission angle when the crank is perpendicular to the fixed link is
(a) 61.37° (b) 60°
(c) 58.94° (d) 38.4°



$20^2 + 50^2 = 40^2 + 60^2 - 2\cdot(40)(60)\cdot\cos\gamma$

$\gamma = 61.37°$

Statement for Common data Q. 06 & Q. 07

In an offset slider crank mechanism the length of crank is 20 cm and that of the connecting rod is 40 cm. The offset between the line of stroke and the centre of crank is 10 cm

offset $e = 10$ cm.

Crank lenght $r = 20$ cm.

lenght of connecting Rod $= 40$ cm.

06. The stroke of the slider is
(a) 40 cm (b) 20 cm
(c) 41.8 cm (d) 38.2 cm

$$\text{stroke lenght} = \sqrt{(l+r)^2 - e^2} - \sqrt{(l-r)^2 - e^2}$$

$$= \sqrt{(40+20)^2 - 10^2} - \sqrt{(40-20)^2 - 10^2}$$

$$= 41.8 \text{ cm.}$$

07. The quick return ratio is
(a) 1.0 (b) 1.25
(c) 1.12 (d) 1.414

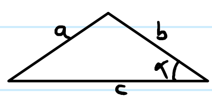
$\sin\theta_2 = \frac{e}{l-r} \Rightarrow \theta_1 = \sin^{-1}\left(\frac{10}{40-20}\right) = 30°$

$\sin\theta_1 = \frac{e}{l+r} \Rightarrow \theta_1 = \sin^{-1}\left(\frac{10}{40+20}\right) = 9.594$

$$Q.R.R. = \frac{180 + (\theta_2 - \theta_1)}{180 - (\theta_2 - \theta_1)}$$

$$Q.R.R. = \frac{180 + (30 - 9.594)}{180 - (30 - 9.594)}$$

$Q.R.R. = 1.25$



$a^2 = b^2 + c^2 - 2bc\cdot\cos\alpha$

08. In a quick return mechanism of the crank and slotted lever type the length of the crank is 20cm and that of the fixed link is also 40cm. The quick return ratio is

(a) 1.5 (b) 1.75
$\boxed{\text{(c) 2}}$ ✓ (d) None

lenght of crank = 20 cm.
lenght of fixed link = 40cm.

Ratio 1:2

Q.R.R. = 2

Kinematic Analysis of Planer Mechanisms

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It is the study of parameters that describe the relative motion between the connected bodies without considering the effect of force/Torque acting on the body/link.

Mass or inertia of the link is not considered.

1. Velocity Analysis
2. Acceleration Analysis. (Vector Alegbra)

Types of Tensors.

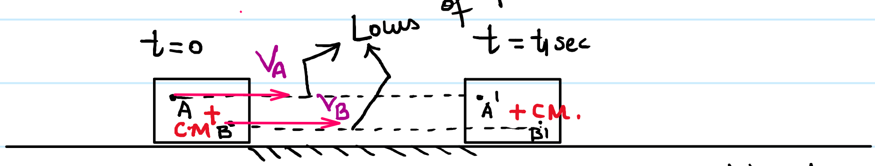
1. Zero Order Tensor :- Quantities which has only magnitude / Scalers.
   Ex:- time, lenght, Mass
2. First order Tensor :- Quantities which has magnitude & direction / Vectors.
   Ex:- Force, velocity.
3. Second Order Tensor :- Quantities which has magnitude, direction and plane of reference is called Tensor
   Ex:- stress, strain, Moment of Inertia.

Types of Motion

1. Rectilinear Motion
2. Curvilinear Motion
3. Angular Motion
4. General Plane Motion/Rolling Motion

Rectilinear Motion

Locus of points A & B (st. line)



$x_A = x_B = x_{cm}$.

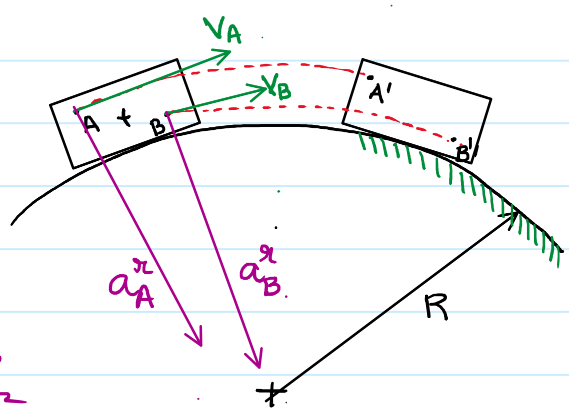
$V_A = V_B = V_{cm}$.

$a_A = a_B = a_{cm}$.

$a^{r}_{cm} = \frac{V_{cm}^2}{R}$.

As $R \to \infty$ $a^{r}_{cm} \to 0$

2. Curvilinear Motion



Locus of points A & B is arc of circle.

$$V_A = V_B = V_{cm}.$$

$$a_A = a_B = a_{cm}.$$

$$a^r_A = \frac{V_A^2}{R}. \qquad a^r_{cm} = \frac{V_{cm}^2}{R}.$$

$$a^r_B = \frac{V_B^2}{R}.$$

Radial acceleration

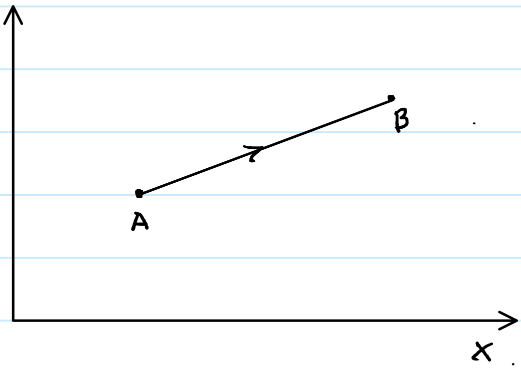
$$a^r_A = a^r_B = a^r_{cm}$$

Vectors

$$\vec{AB} = |\vec{AB}| \cdot \hat{AB}$$

$|\vec{AB}|$ → Magnitude.

$\hat{AB}$ → unit vector



$\vec{AB}$

B → point of interest

A → point of reference.

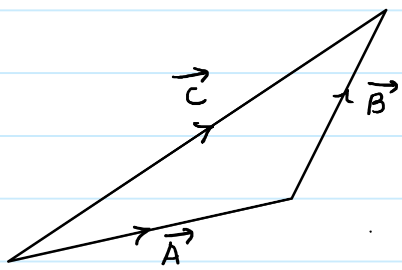
Addition of Vectors

$$\vec{A} + \vec{B} = \vec{C}$$

$\vec{C}$ → Resultant

Head

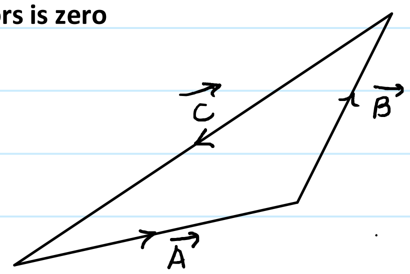
Tail



In a closed polygon the summation of all vectors is zero

$$\vec{A} + \vec{B} + (-\vec{C}) = 0$$

$(-\vec{C})$ → Equilibrant.

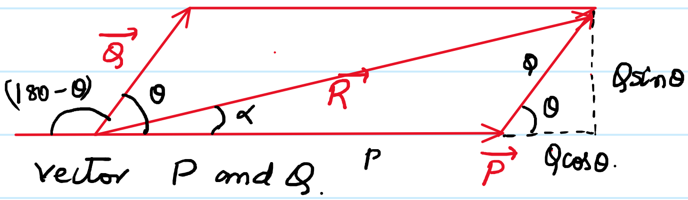


$$\vec{F} = \underbrace{\vec{A}}_{\text{Tail}} + \vec{B} + \vec{C} + \vec{D} + \vec{E}$$

Tail

Head.

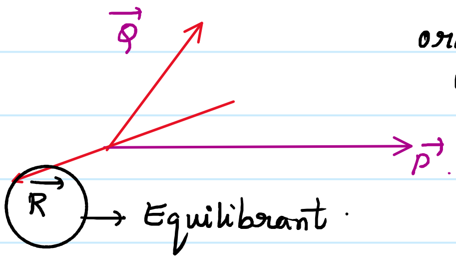
Parallelogram law of Vectors.



Resultant of vector P and Q.

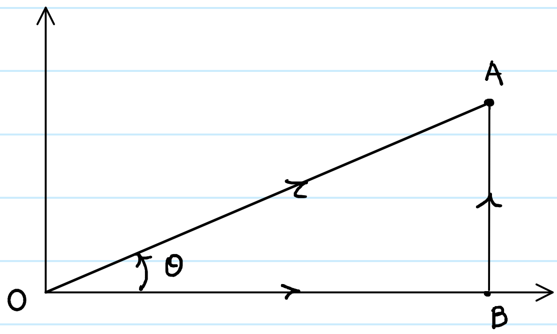
$$R^2 = P^2 + Q^2 + 2PQ.\cos\theta$$

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$



orientation of vectors when a body is in equilibrium.

Resolution of Vector

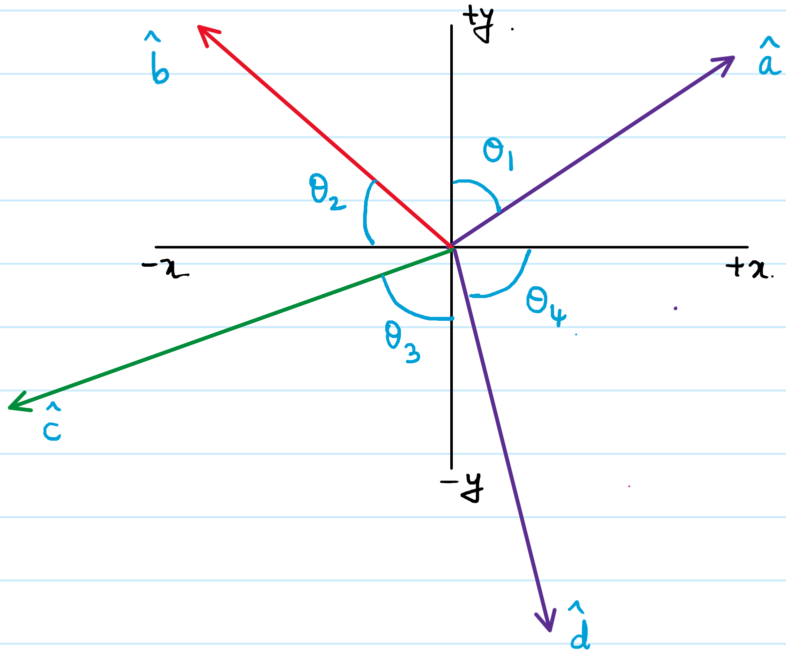


$$\vec{OA} = \vec{OB} + \vec{AB}$$

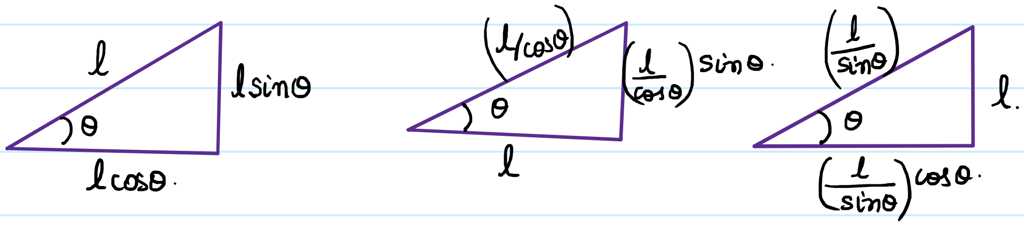
$$|\vec{OA}| \cdot \hat{OA} = |\vec{OB}| \cdot \hat{OB} + |\vec{AB}| \cdot \hat{AB}$$

$$\hat{OA} = \frac{|\vec{OB}|}{|\vec{OA}|} \cdot \hat{OB} + \frac{|\vec{AB}|}{|\vec{OA}|} \cdot \hat{AB}$$

$$\hat{OA} = \cos\theta\ \hat{i} + \sin\theta\ \hat{j}$$



$$\hat{a} = \sin\theta_1 \hat{i} + \cos\theta_1 \hat{j}$$
$$\hat{b} = -\cos\theta_2 \hat{i} + \sin\theta_2 \hat{j}$$
$$\hat{c} = -\sin\theta_3 \hat{i} - \cos\theta_3 \hat{j}$$
$$\hat{d} = \cos\theta_4 \hat{i} - \sin\theta_4 \hat{j}$$

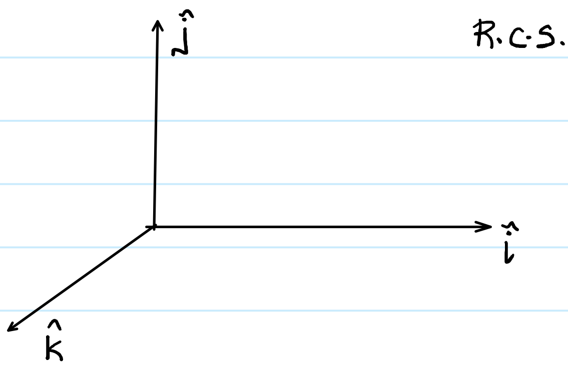


Dot product.

$$i \cdot i = j \cdot j = k \cdot k = 1$$
$$i \cdot j = j \cdot k = k \cdot i = 0$$

$$\cos\theta = \frac{a \cdot b}{|a||b|}$$

Cross. Product.

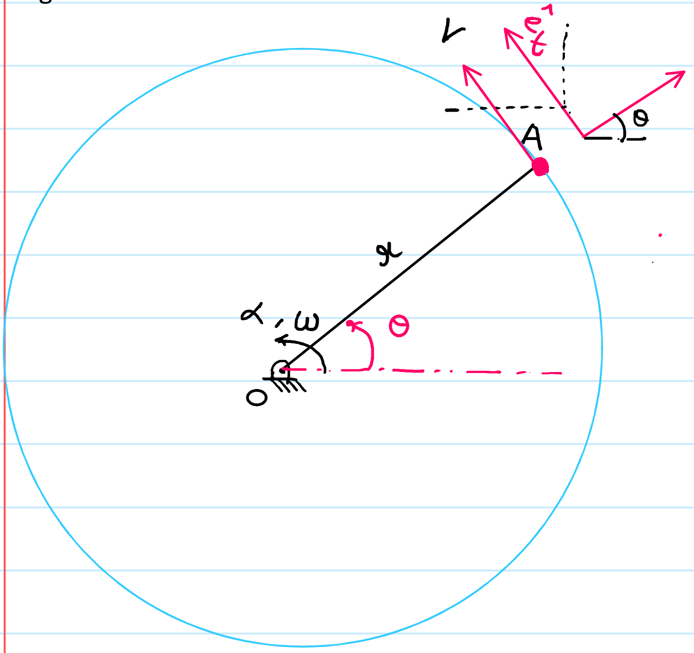


$$\hat{i} \times \hat{j} = \hat{k} \qquad \hat{j} \times \hat{k} = \hat{i}$$

$$-i \times j = -\hat{k}$$

$$-\hat{i} \times -\hat{k} = -\hat{j}$$

→ Angular Motion



ω – Angular Velocity

α – Angular Acceleration

A – point A is having relative motion along the rod.

OA = variable

r = variable

$\hat{e}_r, \hat{e}_t$ – unit vectors describing radial and tangential direction.

OA = r = variable.

$\hat{OA} = \hat{i}\cos\theta + \hat{j}\sin\theta$

$$\vec{OA} = |\vec{OA}| \cdot \hat{OA}$$

$$\vec{r} = r[i\cos\theta + j\sin\theta]$$

Velocity

$$\vec{V} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}[i\cos\theta + j\sin\theta] + r \cdot \frac{d}{dt}[i\cos\theta + j\sin\theta]$$

$$\vec{V} = \dot{r}\hat{e}_r + r \cdot \Big[\underbrace{-i\sin\theta + j\cos\theta}_{\hat{e}_t}\Big]\frac{d\theta}{dt} \rightarrow \dot{\theta}$$

$$\boxed{\vec{V} = \dot{r}\hat{e}_r + r\dot{\theta} \cdot \hat{e}_t}$$

$\dot{r}$ – Rate of change in position of point A along Rod.

$\dot{\theta}$ - Angular Velocity of Rod.

if r = constant $\quad \dot{r} = 0 \quad \boxed{\vec{V} = r\dot{\theta}\,\hat{e}_t = r\omega \cdot (\hat{e}_t)}$

$$\vec{V} = \dot{r}\hat{e}_r + r\dot{\theta}(\hat{e}_t)$$

$uv' + vu'$ uv

$uv \cdot w' + u'vw + uw \cdot v'$ uvw

Acceleration

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d}{dt}[\dot{r}\hat{e}_r] + \frac{d}{dt}[r\dot{\theta}\,\hat{e}_t]$$

$$\vec{a} = \ddot{r}\hat{e}_r + \dot{r}\frac{d}{dt}[\hat{e}_r] + \frac{dr}{dt} \cdot \dot{\theta} \cdot \hat{e}_t + r \cdot \frac{d\dot{\theta}}{dt} \cdot \hat{e}_t + r \cdot \dot{\theta} \cdot \frac{d}{dt}[\hat{e}_t]$$

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$$\vec{a} = \ddot{r}\hat{e}_r + \dot{r}\frac{d}{dt}[\hat{e}_r] + \frac{dr}{dt}\cdot\dot{\theta}\cdot\hat{e}_t + r\cdot\frac{d\dot{\theta}}{dt}\cdot\hat{e}_t + r\cdot\dot{\theta}\cdot\frac{d}{dt}[\hat{e}_t]$$

$$\vec{a} = \ddot{r}[\hat{i}\cos\theta+\hat{j}\sin\theta] + \dot{r}\frac{d}{dt}[\hat{i}\cos\theta+\hat{j}\sin\theta] + \dot{r}\dot{\theta}\cdot[-\hat{i}\sin\theta+\hat{j}\cos\theta] + r\cdot\ddot{\theta}[-\hat{i}\sin\theta+\hat{j}\cos\theta] + r\dot{\theta}\frac{d}{dt}[-\hat{i}\sin\theta+\hat{j}\cos\theta]$$

$$\vec{a} = \ddot{r}\underbrace{[\hat{i}\cos\theta+\hat{j}\sin\theta]}_{(\hat{e}_r)} + \dot{r}\underbrace{[-\hat{i}\sin\theta+\hat{j}\cos\theta]}_{\hat{e}_t}\dot{\theta} + \dot{r}\dot{\theta}\cdot\underbrace{[-\hat{i}\sin\theta+\hat{j}\cos\theta]}_{\hat{e}_t} + r\cdot\ddot{\theta}\underbrace{[-\hat{i}\sin\theta+\hat{j}\cos\theta]}_{\hat{e}_t} + r\dot{\theta}^2\underbrace{[-\hat{i}\cos\theta-\hat{j}\sin\theta]}_{-\hat{e}_r}$$

$$\vec{a} = \ddot{r}(\hat{e}_r) + \dot{r}\dot{\theta}(\hat{e}_t) + \dot{r}\dot{\theta}(\hat{e}_t) + r\cdot\ddot{\theta}(\hat{e}_t) + r\dot{\theta}^2(-\hat{e}_r)$$

$$\boxed{\vec{a} = \ddot{r}(\hat{e}_r) + r\dot{\theta}^2(-\hat{e}_r) + 2\dot{r}\dot{\theta}(\hat{e}_t) + r\ddot{\theta}(\hat{e}_t)}$$

Acceleration of point A along the Rod.

Radial acceleration of rod. OA.

Coriolis's acceleration.

Tangential Acceleration of Rod OA.

$r = \text{constant}$ $\quad \dot{r} = 0 \quad \ddot{r} = 0$

$$\boxed{\begin{aligned}\vec{a} &= r\dot{\theta}^2(-\hat{e}_r) + r\ddot{\theta}(\hat{e}_t)\\ \vec{a} &= r\omega^2(-\hat{e}_r) + r\alpha(\hat{e}_t)\end{aligned}}$$

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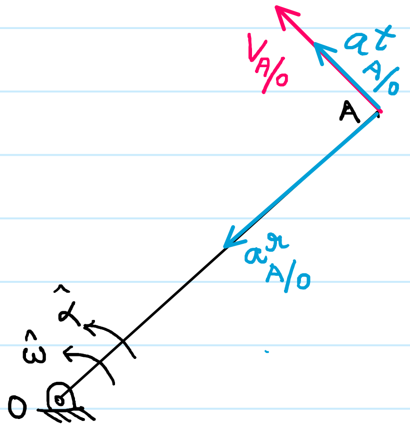
(i)



$\hat{\omega} = \hat{\alpha} = +\hat{k}$

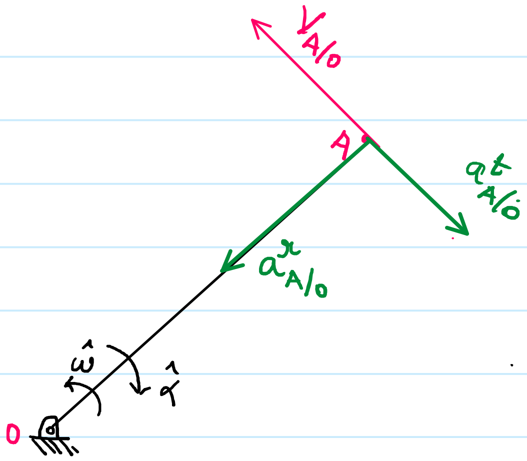
Link is accelerating

OA = constant.

$V_{A/O}$ → Reference point; → Point of interest.

$a^{\sigma}_{O/O}$ → Type of acceleration: radial / Tangent / Coriolis; → Reference point; → Point of interest.

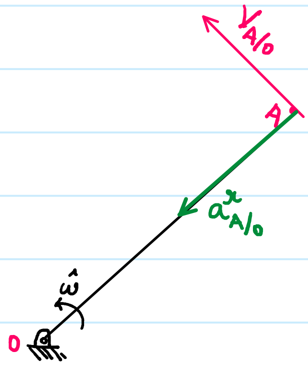
(ii)



$\hat{\omega} = +\hat{k}$, $\hat{\alpha} = -\hat{k}$

Link is retarding.

(iii)



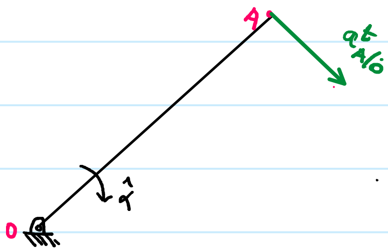
$\hat{\omega} = +\hat{k}$

ω = constant

$\alpha = 0$

$a^t_{A/O} = OA \cdot \alpha = 0$

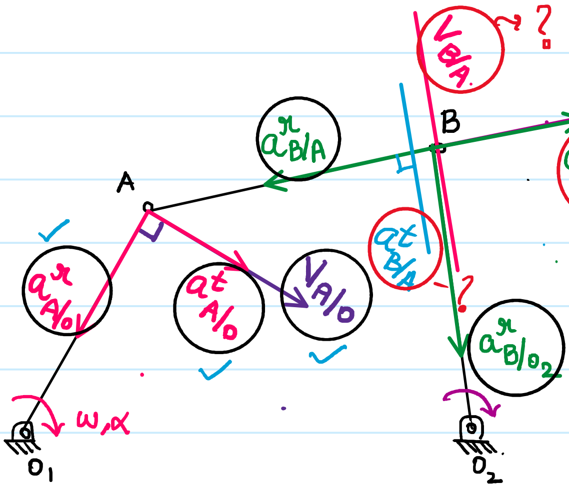
(iv)



ω = constant. $\alpha \neq 0$.

$\hat{\alpha} = -\hat{k}$

Maximum Retardation.



Magnitude and direction is unknown for $a^t_{B/A}$, a^t_{B/O_2}, $V_{B/A}$, V_{B/O_2}

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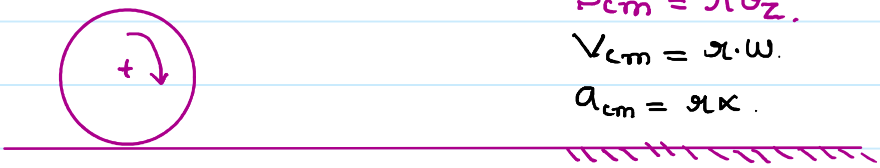
Rolling Motion

Velocity Analysis for Rolling motion

Rolling = Rotation + Translation



$S_{cm} = r\theta_z$

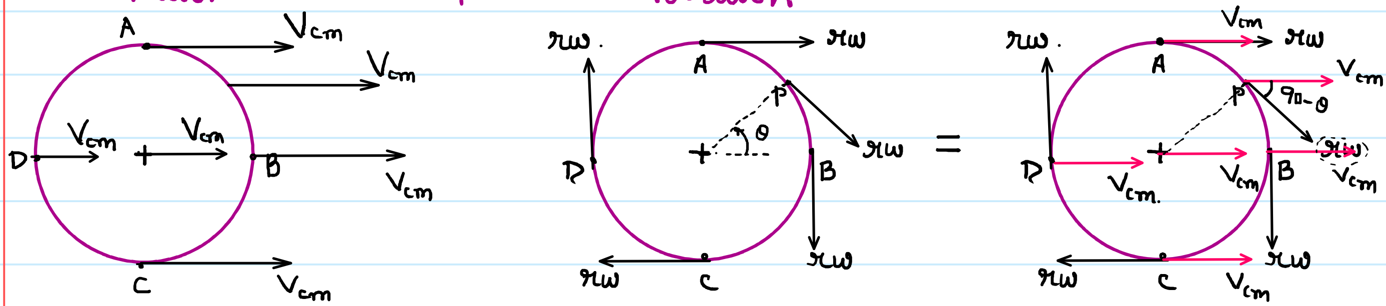
$V_{cm} = r \cdot \omega$

$a_{cm} = r\alpha$

$a^r = r\omega^2$

$a^t = r\alpha$

Translation + Rotation



Velocities at points A, B, C, D, P.

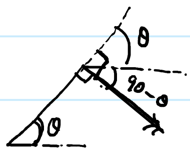
$$\vec{V_A} = V_{cm}(\hat{i}) + r\omega(\hat{i}) = 2V_{cm}.(\hat{i})$$

$$\vec{V_P} = V_{cm}.(\hat{i}) + r\omega.[\hat{i}\cos(90-\theta) - \hat{j}\sin(90-\theta)] = V_{cm}\hat{i} + V_{cm}[\hat{i}\sin\theta - \hat{j}\cos\theta]$$

$$\vec{V_B} = V_{cm}(\hat{i}) + r\omega(-\hat{j}) = V_{cm}[(\hat{i}) - (\hat{j})]$$

$$\vec{V_C} = V_{cm}(\hat{i}) + r\omega(-\hat{i}) = 0$$

$$\vec{V_D} = V_{cm}(\hat{i}) + r\omega.(\hat{j}) = V_{cm}.[\hat{i} + \hat{j}]$$



In pure rolling motion velocity at the point of contact is zero. About Point C the disc will undergo pure rotation.

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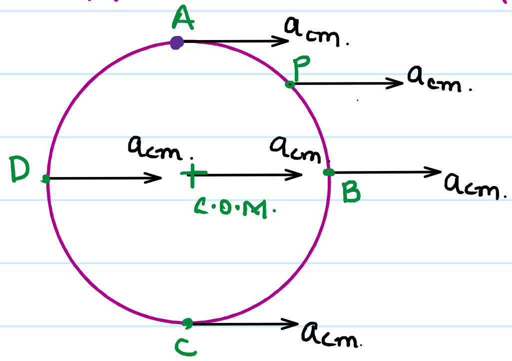
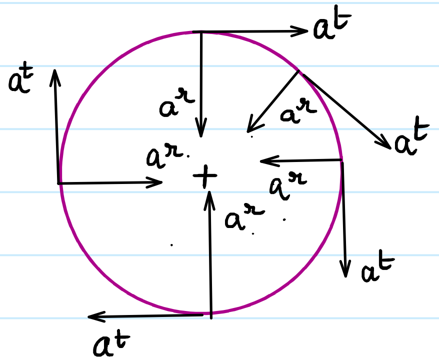
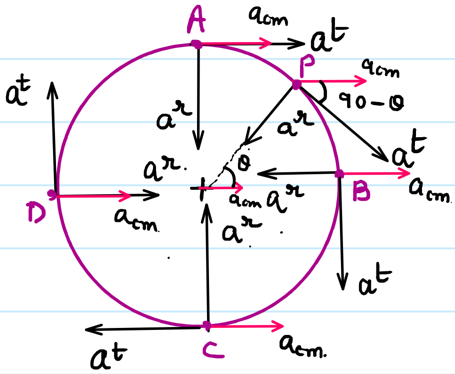
Translation + Rotation





Rotation + Translation.



Acceleration at points A, B, C, D & P.

$$\vec{a}_A = \vec{a}_{cm} + \vec{a}^t + \vec{a}^r = a_{cm}(\hat{i}) + r\alpha(\hat{i}) + r\omega^2(-\hat{j})$$
$$= r\alpha(\hat{i}) + r\alpha(\hat{i}) + r\omega^2(-\hat{j}) = 2r\alpha(\hat{i}) + r\omega^2(-\hat{j})$$

$$\vec{a}_B = \vec{a}_{cm} + \vec{a}^t + \vec{a}^r = a_{cm}(\hat{i}) + r\alpha(-\hat{j}) + r\omega^2(-\hat{i})$$
$$= r\alpha(\hat{i}) + r\omega^2(-\hat{i}) + r\alpha(-\hat{j})$$

$$\vec{a}_C = \vec{a}_{cm} + \vec{a}^t + \vec{a}^r = r\alpha(\hat{i}) + r\alpha(-\hat{i}) + r\omega^2(\hat{j}) = r\omega^2(\hat{j})$$

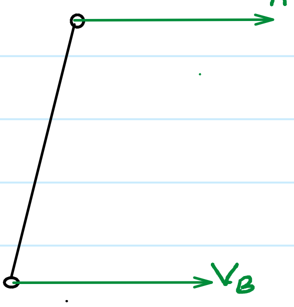
$$\vec{a}_D = \vec{a}_{cm} + \vec{a}^t + \vec{a}^r = r\alpha(\hat{i}) + r\alpha(\hat{j}) + r\omega^2(\hat{i})$$

Acceleration at point P.

$$\vec{a}_P = \vec{a}_{cm} + \vec{a}^r + \vec{a}^t = r\alpha(\hat{i}) + r\omega^2(-\hat{i}\cos\theta - \hat{j}\sin\theta) + r\alpha(\hat{i}\cos(90-\theta) - \hat{j}\sin(90-\theta))$$
$$= r\alpha(\hat{i}) + r\omega^2(-\hat{i}\cos\theta - \hat{j}\sin\theta) + r\alpha(\hat{i}\sin\theta - \hat{j}\cos\theta)$$

Recognition of Motion

1. Translation



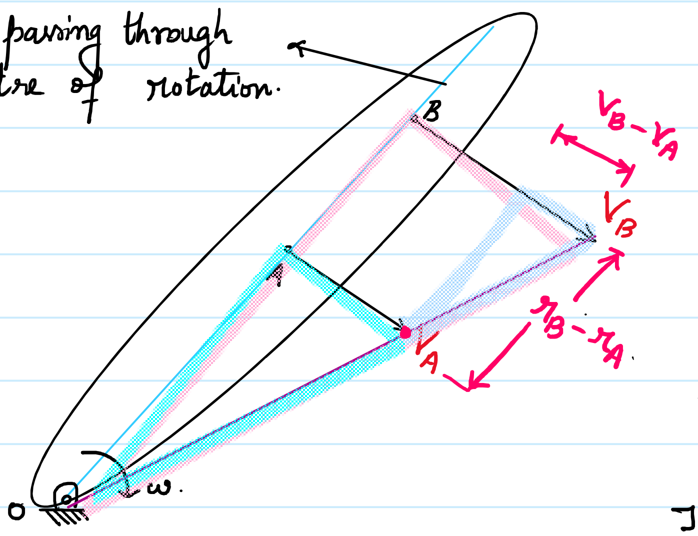
$$\left.\begin{array}{l} V_A = V_B \\ a_A = a_B \end{array}\right\} \text{Translation.}$$

$$V = u + at$$
$$S = ut + \tfrac{1}{2}at^2$$
$$V^2 - u^2 = 2as$$

2. Angular Motion

Angular Motion → oscillatory motion
→ circular motion

$V_{B/A}$ – Velocity of B wrt A

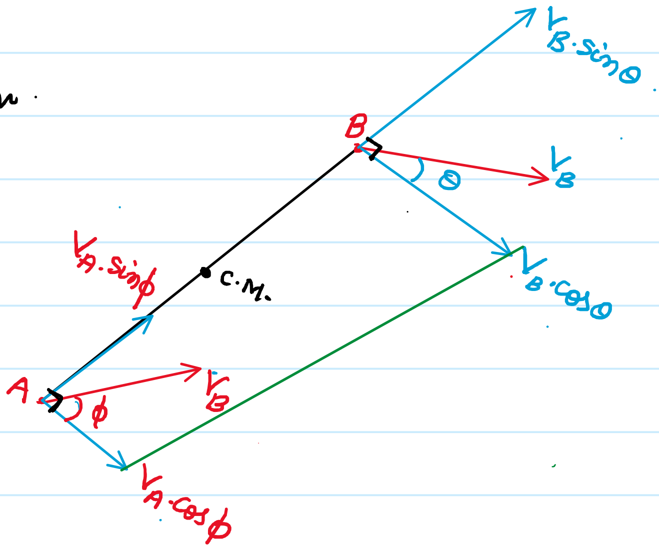


$$\boxed{\frac{V_B}{OB} = \frac{V_A}{OA} = \frac{V_B - V_A}{r_B - r_A} = \frac{V_{B/A}}{r_{B/A}} = \omega}$$

Point O is permanent center of rotation.
It is having zero velocity permanently!

- Line passing through the centre of rotation will be always perpendicular to the Velocity Vectors at any point.
- If there is velocity gradient across any dimension then the body will be subjected rotation

3. General Plane Motion

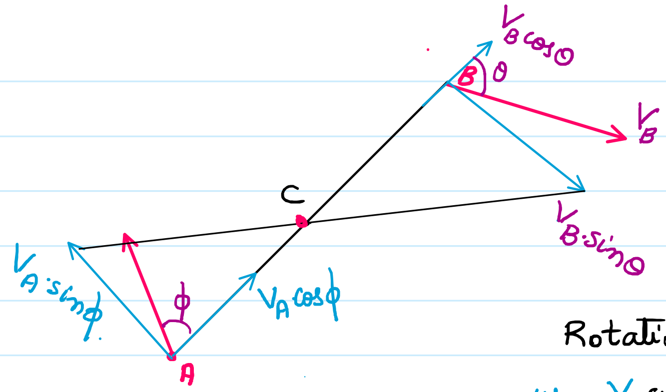


Rotation

$$\omega = \frac{V_B \cos\theta - V_A \cos\phi}{AB}$$

Translation.

$$V_A \sin\phi = V_B \sin\theta$$



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C – center of Rotation (having only Translation Velocity)

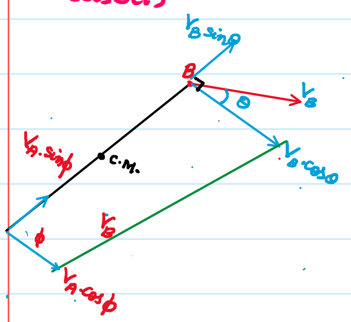
Rotation

$$\omega = \frac{V_B \sin\theta - (-V_A \sin\phi)}{AB}$$

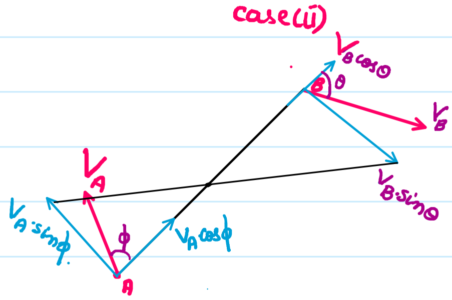
Translation

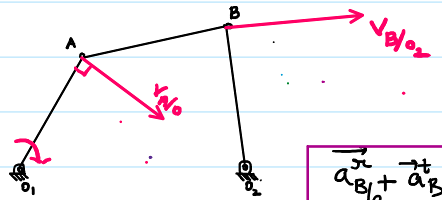
$$V_A \cos\phi = V_B \cdot \cos\theta$$

case(i)



case(ii)

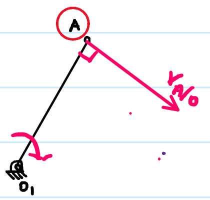




$$\vec{V_B} = \vec{V_A} + \vec{V_{B/A}}$$

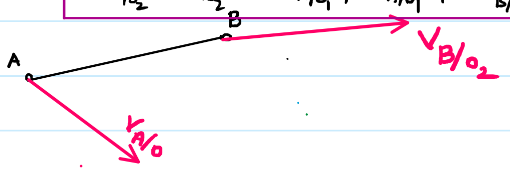
$$\vec{a_B} = \vec{a_A} + \vec{a}^r_{B/A} + \vec{a}^t_{B/A}$$

$$\boxed{\vec{a}^r_{B/O_2} + \vec{a}^t_{B/O_2} = a^r_{A/O_1} + a^t_{A/O_1} + \vec{a}^r_{B/A} + \vec{a}_{B/A}}$$



Angular motion

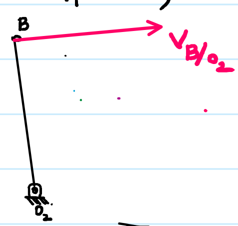
(i/p link)



General plane Motion

Coupler link

(o/p link)



Angular motion

Absolute velocity and acceleration of point A

$$V_{A/O_1} = O_1A \cdot \omega_{O_1A}$$

$$\vec{a}_{A/O_1} = \vec{a}^r_{A/O_1} + \vec{a}^t_{A/O_1}$$

$$\vec{a}_A = O_1A \cdot \omega^2_{O_1A} \cdot (-\hat{e}_r) + O_1A \cdot \alpha_{O_1A} \cdot (\hat{e}_t)$$

Absolute velocity and acceleration of point

$$V_{B/O_2} = O_2B \cdot \omega_{O_2B}$$

$$\vec{a}_{B/O_2} = \vec{a}^r_{B/O} + \vec{a}^t_{B/O_2}$$

$$\vec{a}_B = O_2B \cdot \omega^2_{O_2B} (-\hat{e}_r) + O_2B \cdot \alpha_{O_1} (\hat{e}_t)$$

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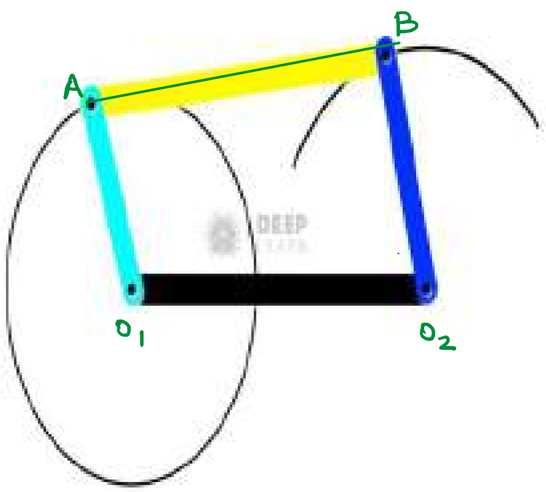
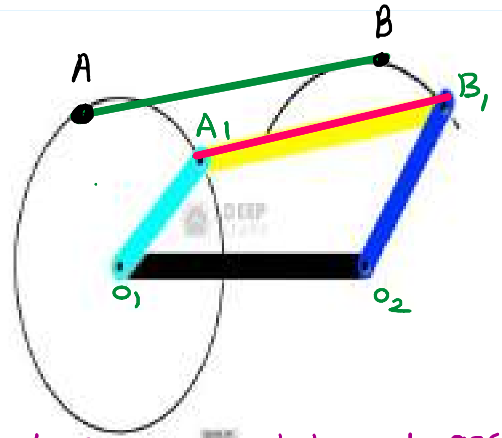
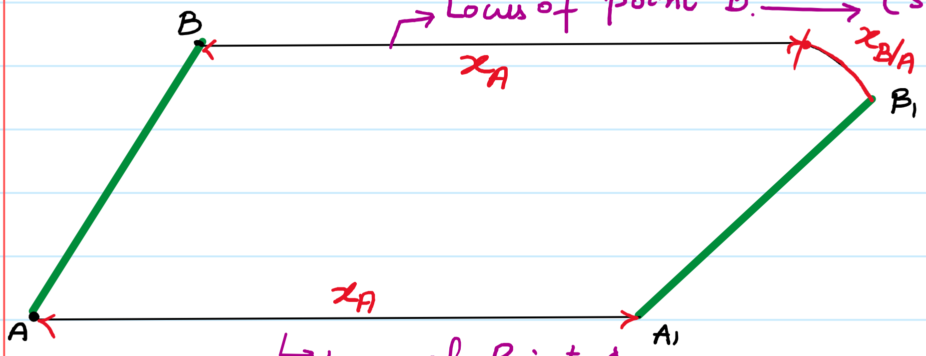




Locus of point B. → (st. line + arc of circle)



↳ Locus of Point A.
st line

Rotation/Angular motion

Displacement of point B $\Rightarrow$ $x_B = x_A + x_{B/A}$

Absolute velocity of A / Velocity of A wrt fixed point.

$$\vec{V_B} = \vec{V_A} + \vec{V_{B/A}}$$

Relative velocity of B wrt A.

Absolute velocity of point B / Velocity of B wrt fixed point

$$\vec{a_B} = \vec{a_A} + \vec{a_{B/A}}$$

$$\vec{a_B} = \vec{a_A} + \vec{a}^{r}_{B/A} + \vec{a}^{t}_{B/A}$$

Instanteous Centre - It is a imaginary point in plane about which the link will undergo pure rotation.

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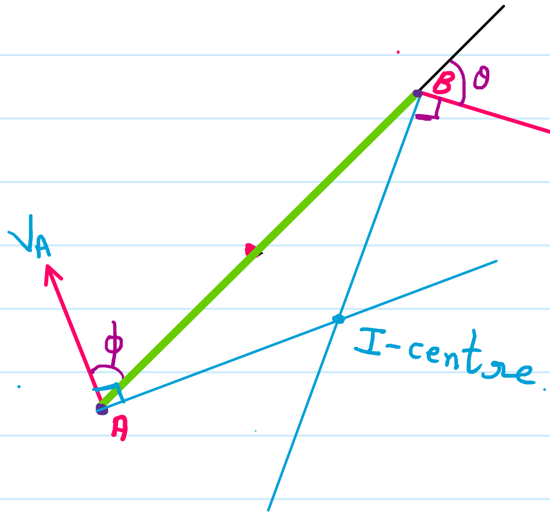
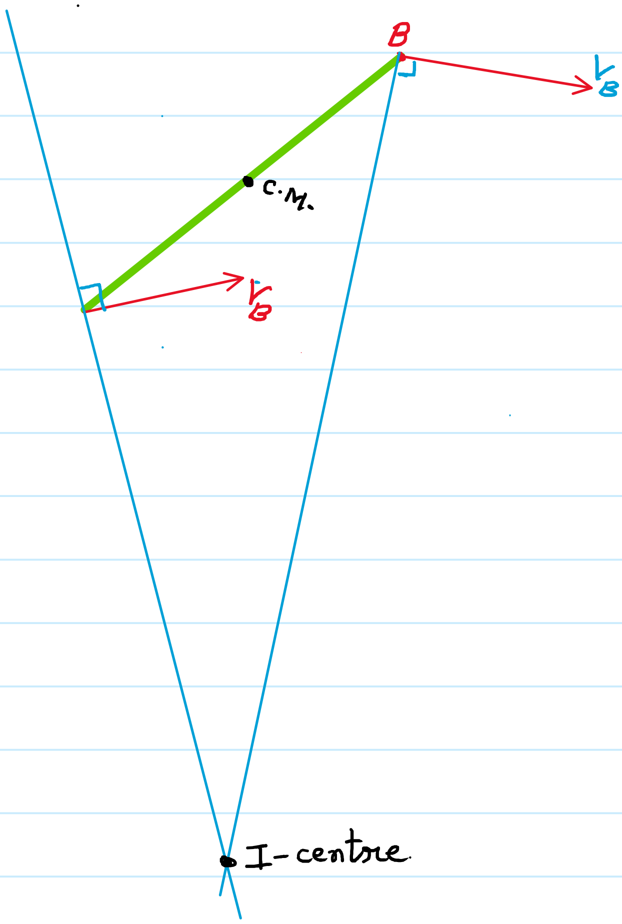
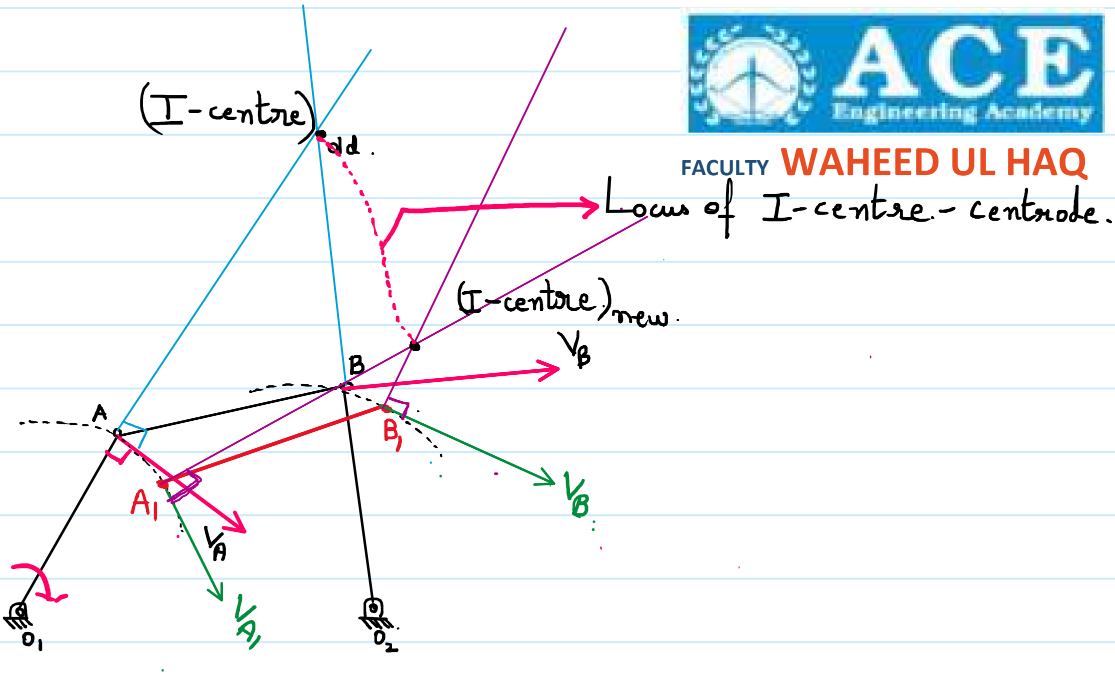


If a body is in general plane motion it will undergo pure Rotation about the I-center





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Line passing through the I-center is called I-axis of Rotation.
Locus of I-axis of Rotation is called as axode(plane).

No. of I-centres is given by $^{n}C_2 = \frac{n(n-1)}{2}$
n – no. of links.

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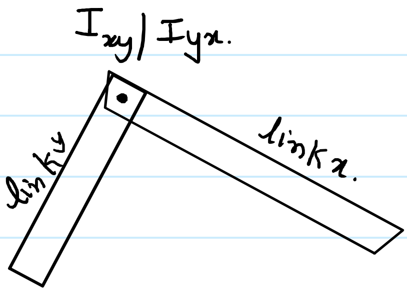
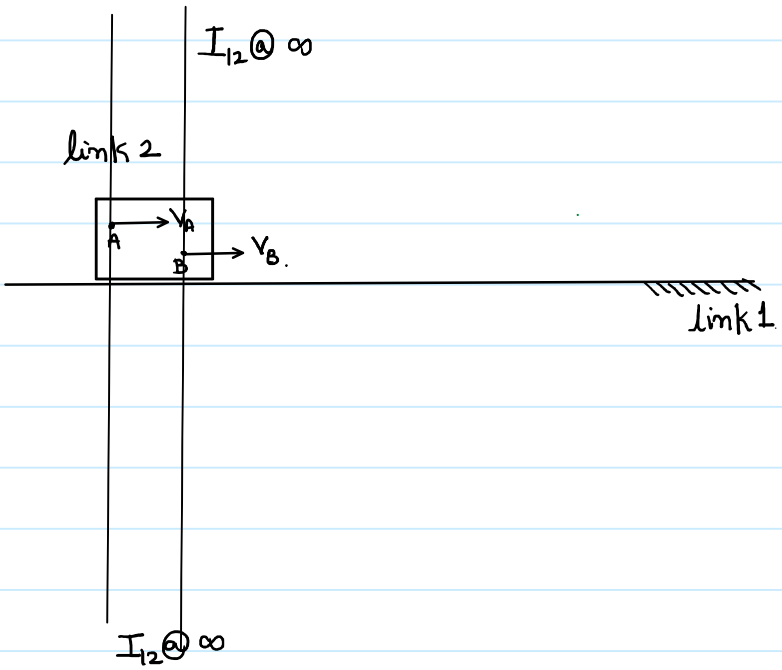
- If the body/link is in pure rotation the I -centre will lie on the physical boundary of the body/link.
- If the body/link is in translation the I -centre will lie at infinity in the plane of rotation.
- If the body/link is in General Plane Motion the I-centre will lie anywhere in the plane of rotation.

Some of the Common I-centres

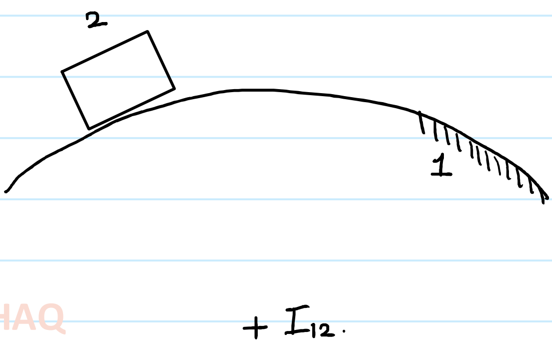
1. Revolute Joint.



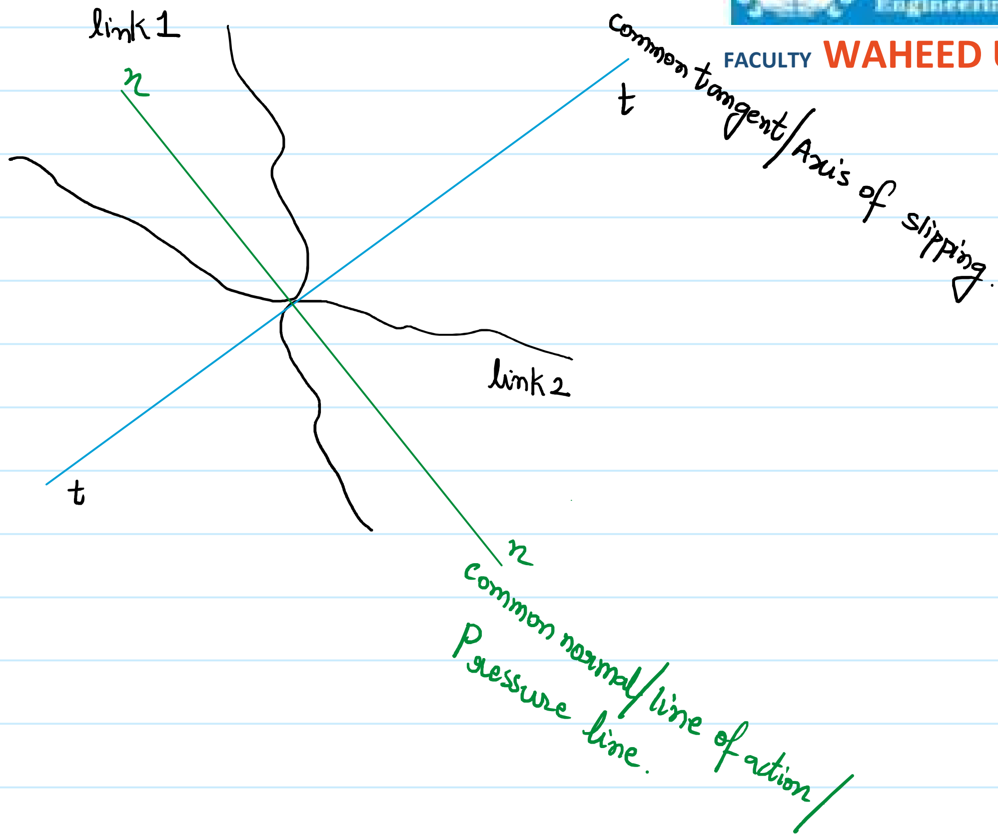
2. Prismatic Joint.



3. Curvilinear Motion – Centre of curvature becomes the I-centre.

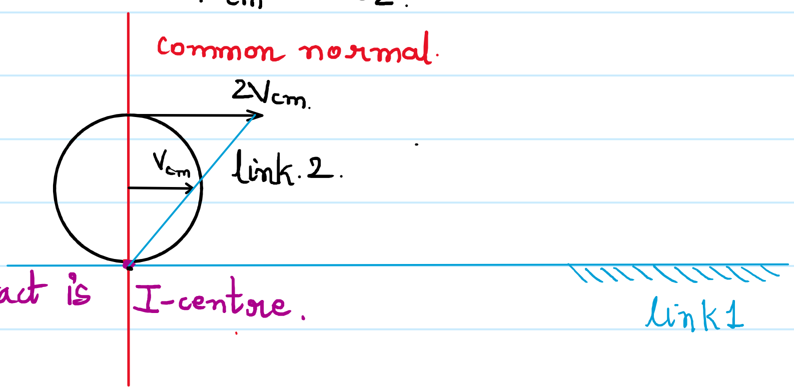


Higher Pair. – The I-centre lies somewhere on the common normal.



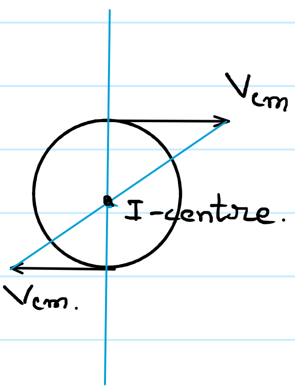
1. Pure Rolling Motion

$$S_{cm} = r \cdot \theta_z .$$



Point of contact is I-centre.

2. Pure Rotation



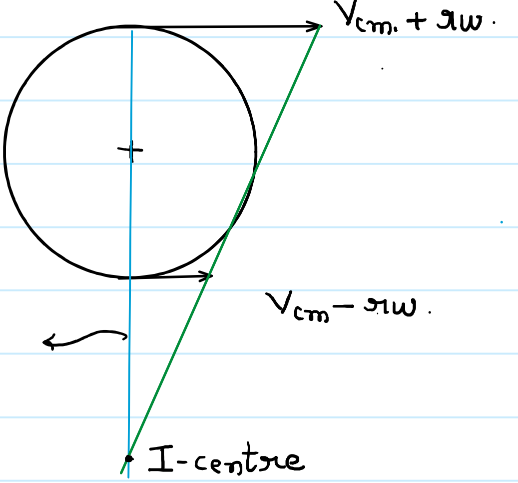
Rolling slipping

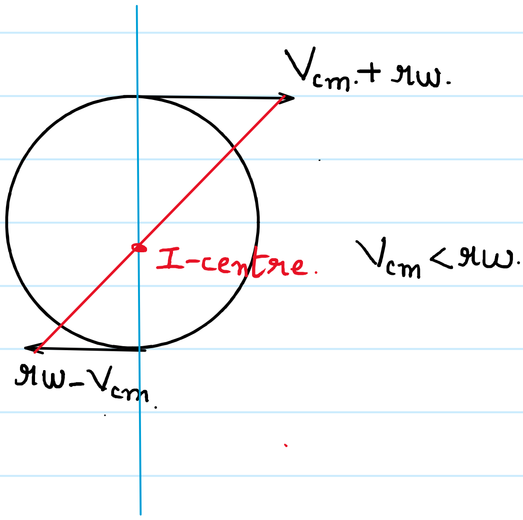
$S_{cm} \neq r\theta_z$

$S_{cm} > r\theta_z$ (Translation > Rotation) Forward slipping

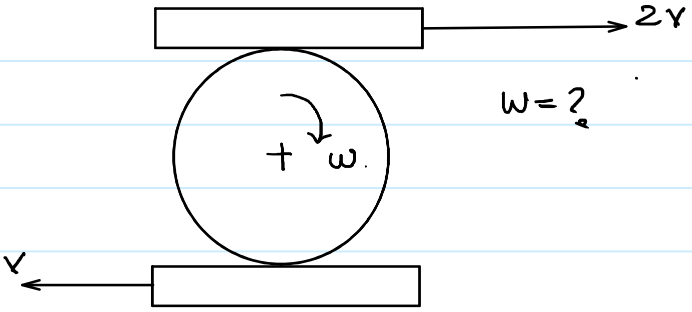
$S_{cm} < r\theta$ (Translation < Rotation) Backward slipping

Forward slipping

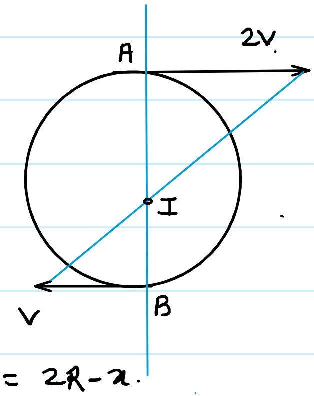




Gate - 2018/19.



$\omega = ?$



$IB = x$. $IA = 2R - x$.

Relative velocity

$$\omega = \frac{V_A - V_B}{AB} = \frac{2V - (-V)}{2R} = \frac{3V}{2R}.$$

$$\frac{2V}{IA} = \frac{V}{IB} = \omega. \Rightarrow \frac{2V}{2R - x} = \frac{V}{x} = \omega.$$

$$\omega = \frac{3V}{2R}.$$

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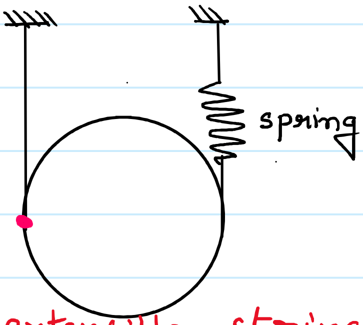
String and Pulley arrangement (Rigid)

(Pure Rolling) I-centre



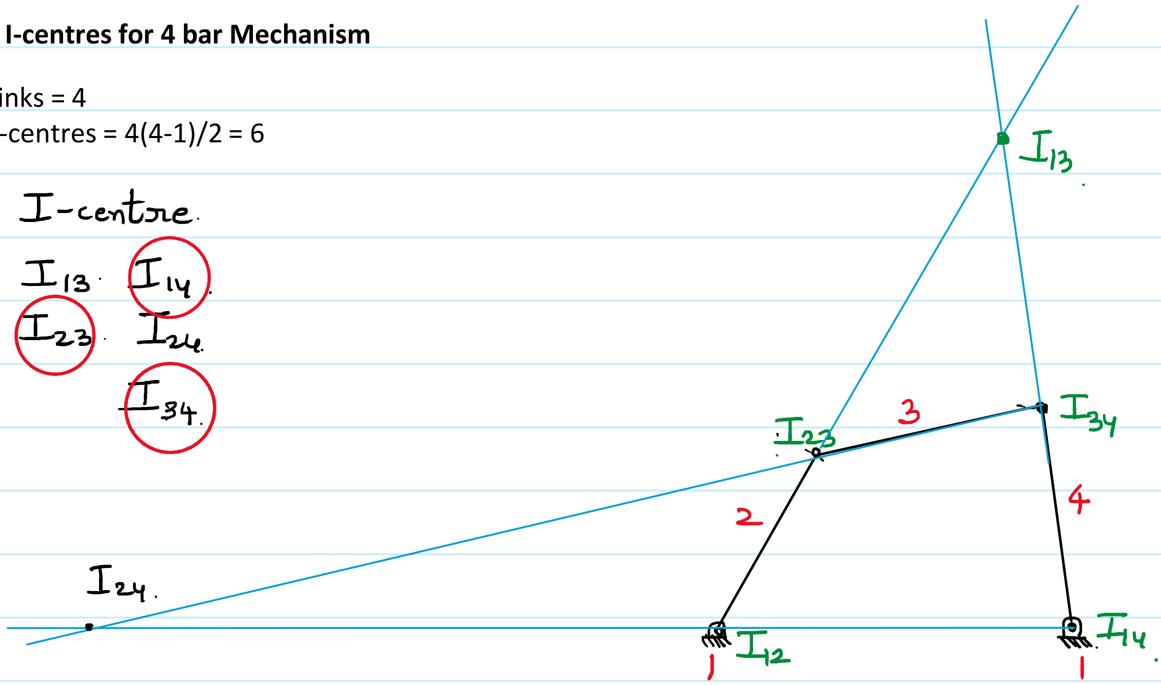
The point where inextensible string enters the pulley it can be assumed as a I-centre.

Locating the I-centres for 4 bar Mechanism

Number of Links = 4
Number of I -centres = 4(4-1)/2 = 6

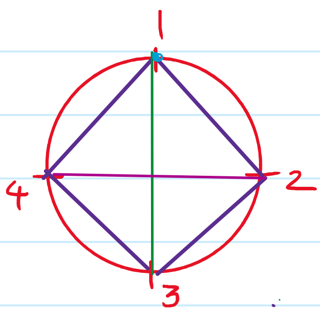
Possible I-centre.

I_{12} I_{13} I_{14}
I_{23} I_{24}
I_{34}



Arnhold Kennedy's Theorem - If the three bodies are connected directly or indirectly and there relative motion between them, then their I - centres will be colinear.

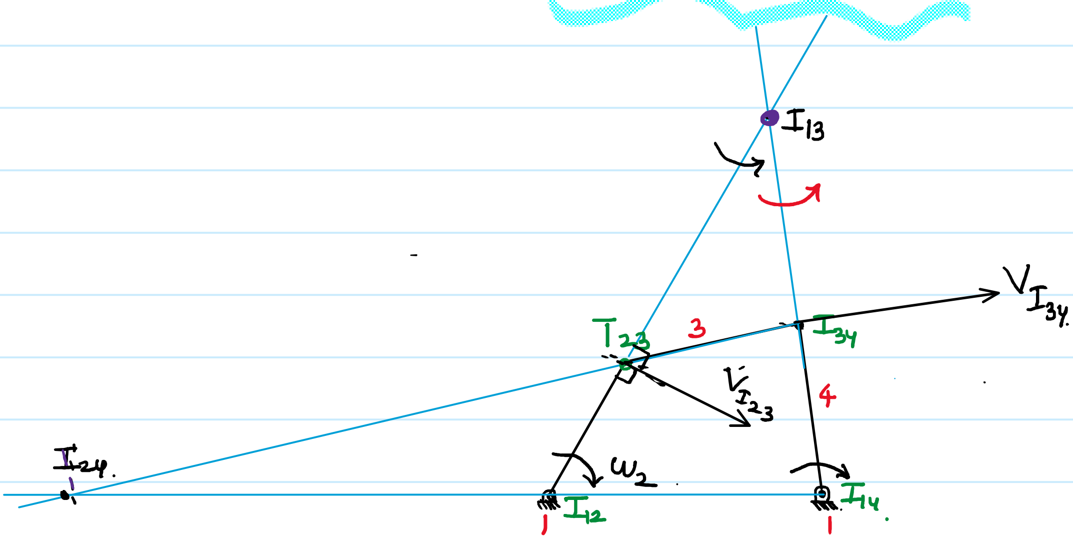
I_{24} must be colinear with I_{12} & I_{14}.
I_{24} must be colinear with I_{23} & I_{34}.



I_{13} must be colinear with I_{12} & I_{23}.
I_{13} must be colinear with I_{14} & I_{34}.

	Absolute I-centre	Relative I-centre
	Ex:- I_{12}, I_{13}, I_{14} (link 1) fixed link	Ex- I_{23}, I_{34}, I_{24}
1.	It is defined wrt fixed link.	It is defined on moving links.
2.	About this I-centre 1 link is assumed in rotation.	About this I-centre 2. links are assumed in rotation.
3.	It is used to predict the direction of angular velocity	It is used to calculate the magnitude of angular velocity.

Angular Velocity theorem - The velocity of an I-centre when it is assumed as a point on link 2 w.r.t link 1 **(fixed)** will be same when it is assumed as a point on link 3 w.r.t link 1 **(fixed).**



$$V_{I_{xy}} = I_{1x}\, I_{xy}\ \omega_x = I_{1y}\, I_{xy}\ \omega_y$$

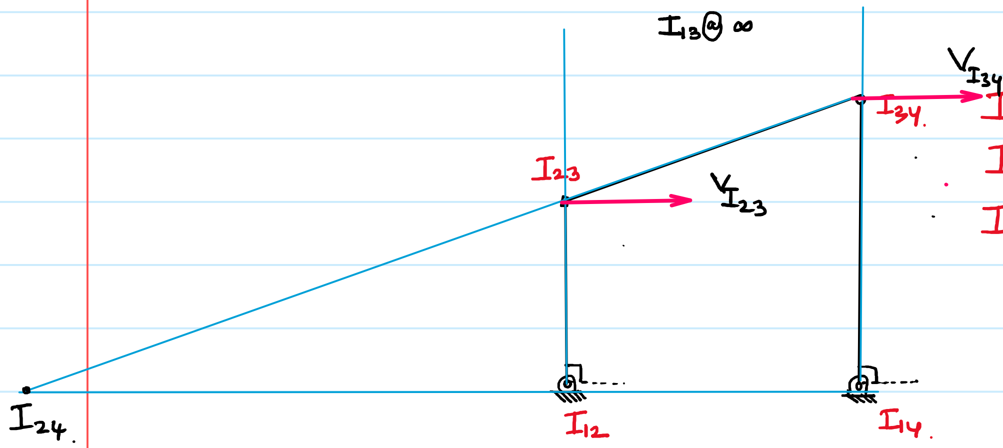
$$V_{I_{23}} = I_{12} \cdot I_{23}\ \omega_2 = I_{13} \cdot I_{23}\ \omega_3$$

$$V_{I_{34}} = I_{13}\, I_{34} \cdot \omega_3 = I_{14} \cdot I_{34} \cdot \omega_4$$

$$V_{I_{24}} = I_{12}\, I_{24} \cdot \omega_2 = I_{14}\, I_{24} \cdot \omega_4 .$$

- If the relative I - centre lies between two absolute I - centres then the sense of rotation of adjacent will be in opposite sense.
- If the relative I - centre lies on one side of two absolute I - centres then the sense of rotation of links will be same.

case(ii) I/p and o/p links are //le.



I_{13} is colinear with I_{12} & I_{23}.

I_{13} is colinear with I_{14} & I_{34}

link 3 is in Translation.

$$V_{I_{23}} = I_{12} \cdot I_{23} \cdot \omega_2 = I_{13} I_{23} \cdot \omega_3 \Rightarrow \boxed{\omega_3 = 0}$$

$\Delta^{le} I_{24} I_{14} I_{34}$ is similar to $\Delta^{le} I_{24} I_{12} I_{23}$.

$$\frac{I_{12} \cdot I_{24}}{I_{12} \cdot I_{23}} = \frac{I_{14} \cdot I_{24}}{I_{14} \cdot I_{34}}$$

$$\frac{I_{14} \cdot I_{24}}{I_{12} \cdot I_{24}} = \frac{I_{14} \cdot I_{34}}{I_{12} \cdot I_{23}}$$

$$V_{I_{24}} = I_{12} \cdot I_{24} \cdot \omega_2 = I_{14} I_{24} \cdot \omega_4$$

$$\frac{\omega_2}{\omega_4} = \frac{I_{14} \cdot I_{24}}{I_{12} \cdot I_{24}} = \frac{I_{14} \cdot I_{34}}{I_{12} \cdot I_{23}} \quad \begin{matrix} L_4 \\ L_2 \end{matrix}$$

$$\frac{\omega_2}{\omega_4} = \frac{L_4}{L_2} \Rightarrow \omega_2 \cdot L_2 = \omega_4 \cdot L_4$$

case(ii) I/p link and coupler link are colinear

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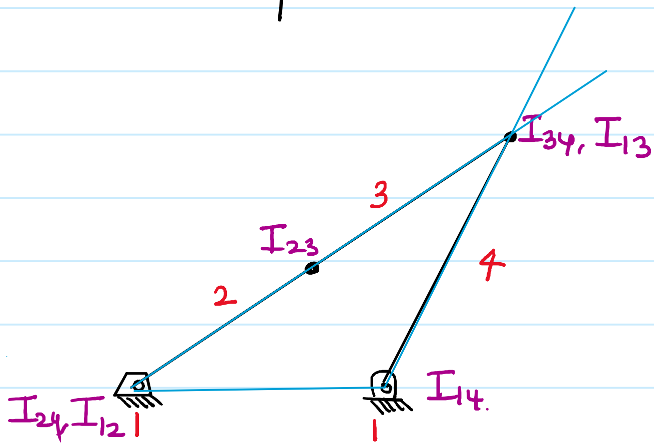
$\omega_4 = 0$



I_{13} / I_{12} & I_{23}

I_{13} / I_{14} & I_{34}

I_{24} / I_{12} & I_{14}

I_{24} / I_{23} & I_{34}

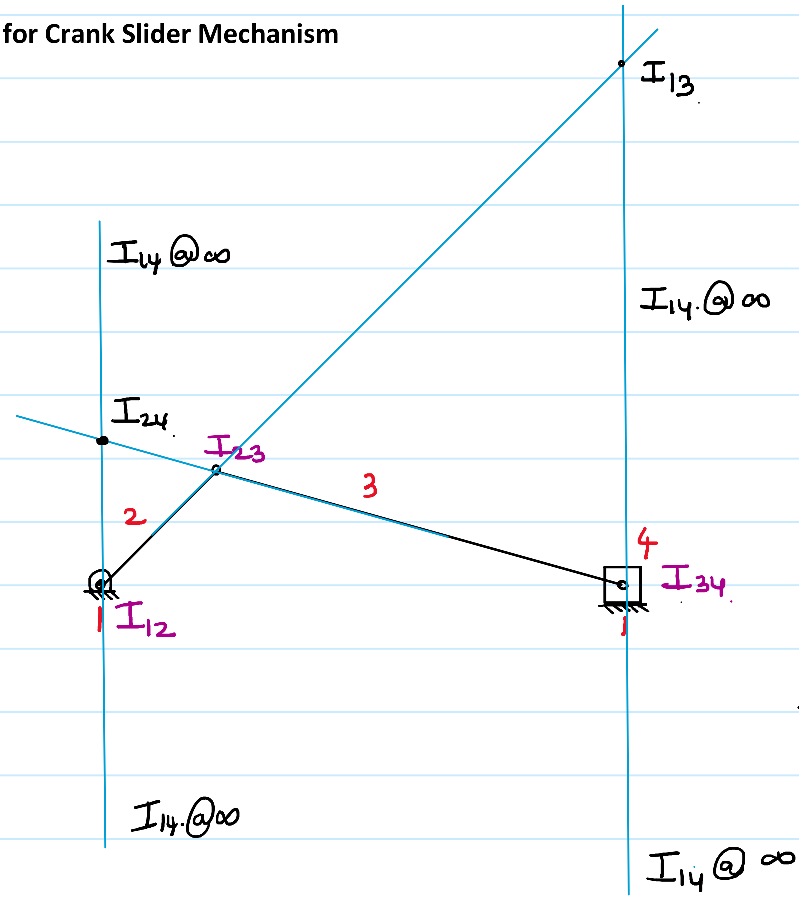
$$V_{I_{23}} = I_{12} \cdot I_{23} \cdot \omega_2 = I_{13} \cdot I_{23} \cdot \omega_3$$

$$= l_2 \cdot \omega_2 = l_3 \cdot \omega_3$$

$$V_{I_{34}} = \underbrace{\cancel{I_{13} \cdot I_{34}}}_{0} \cdot \omega_3 = I_{14} \cdot I_{34} \cdot \omega_4 \longrightarrow \omega_4 = 0$$

$$V_{I_{24}} = \underbrace{\cancel{I_{12} \cdot I_{24}}}_{0} \cdot \omega_2 = I_{14} \cdot I_{24} \cdot \omega_4 \longrightarrow \omega_4 = 0$$

Locating I - centres for Crank Slider Mechanism



I_{13} / I_{12} & I_{23}

I_{13} / I_{14} & I_{34}

I_{24} / I_{12} & I_{14}

I_{24} / I_{23} & I_{34}

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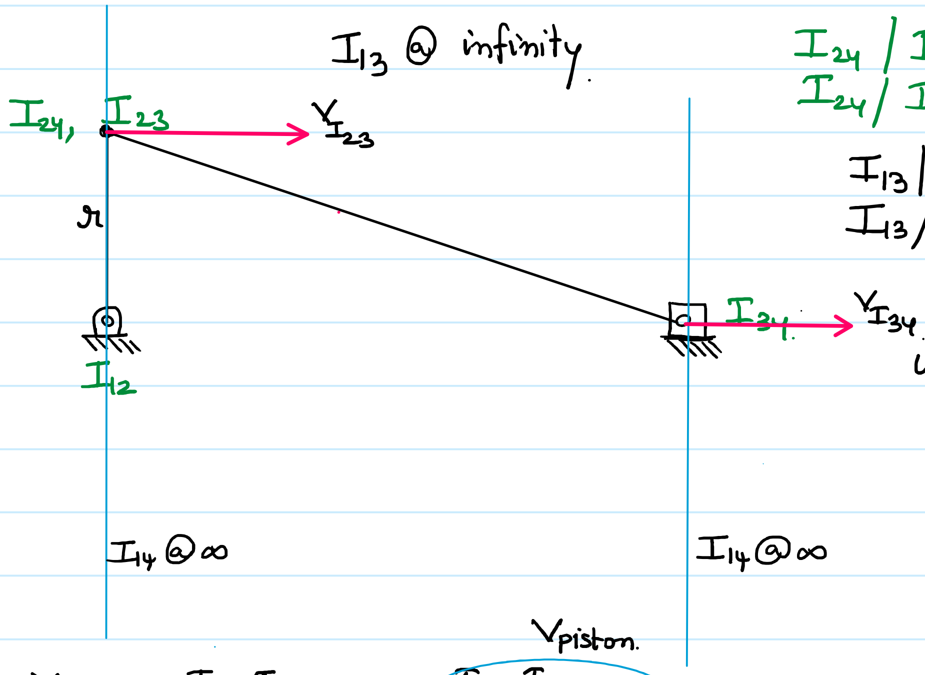
case(A) When the crank angle is 90°

I_{13} @ infinity



$I_{24} / I_{12} \& I_{14}$

$I_{24} / I_{23} \& I_{34}$

$I_{13} / I_{12} \& I_{23}$

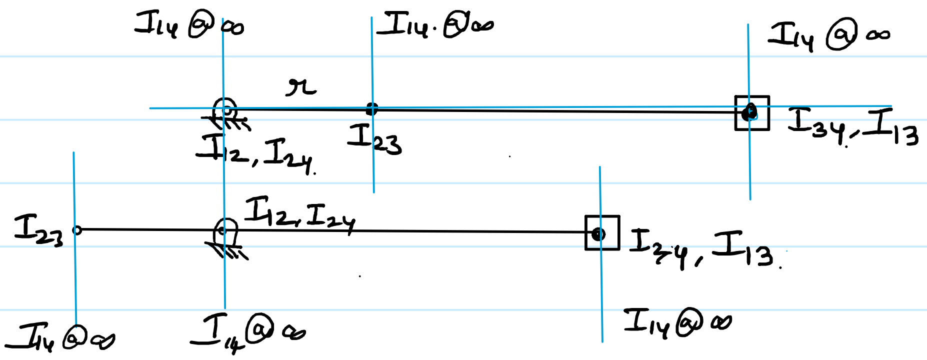
$I_{13} / I_{14} \& I_{34}$

$\omega_3 / \omega_{conn.Rod} = 0$

$$V_{I_{24}} = \underbrace{I_{12} \cdot I_{24}}_{r} \cdot \omega_2 = \overbrace{I_{14} \cdot I_{24} \cdot \omega_4}^{V_{piston}}$$

$$\boxed{V_{piston} = r\omega}$$

case(ii) When piston is at the extreme positions

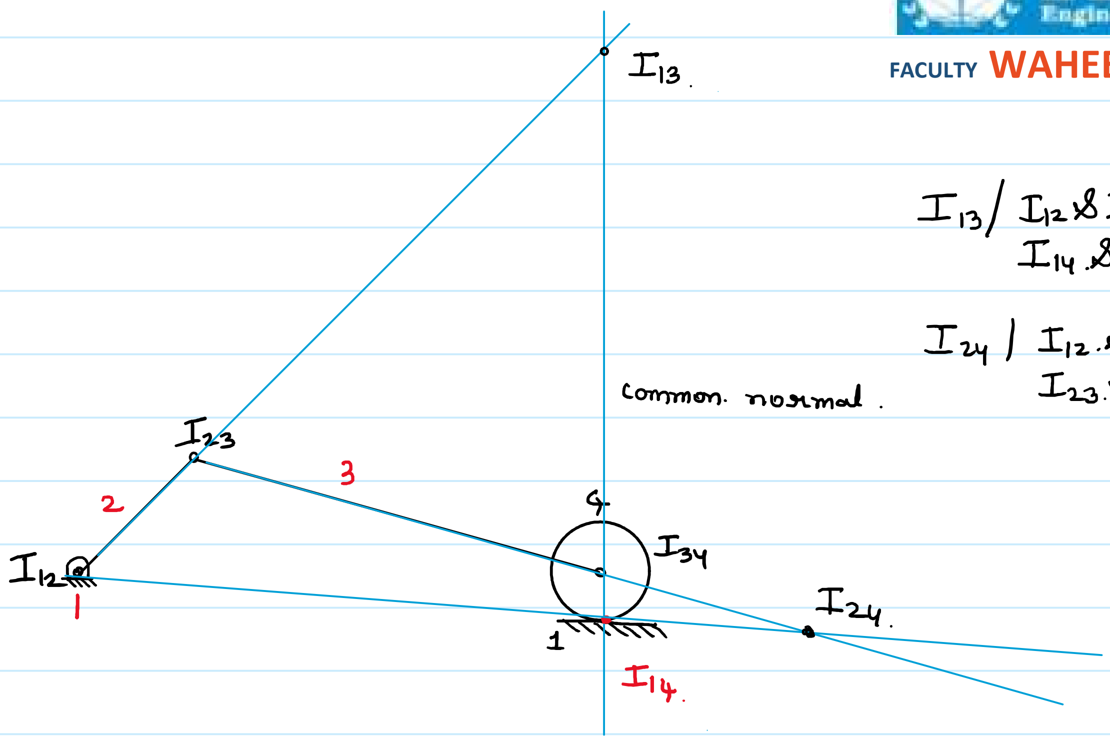


$$V_{I_{23}} = I_{12} \cdot I_{23} \cdot \omega_2 = I_{13} \cdot I_{23} \cdot \omega_3$$
$$= r \cdot \omega_2 = l \cdot \omega_3$$
$$l_2 \omega_2 = l_3 \cdot \omega_3$$

$$V_{I_{34}} = \cancel{I_{13} \cdot I_{34}} \cdot \omega_3 = V_{piston}$$

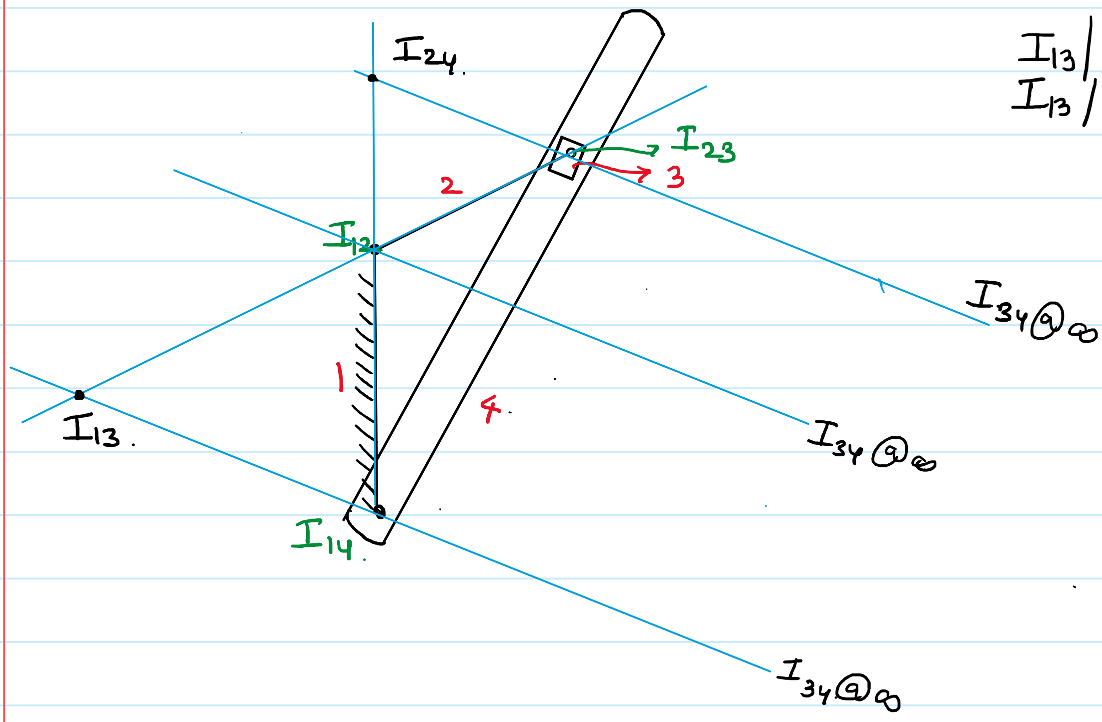
$\to 0$

$$V_{piston} = 0$$



I_{13} / I_{12} & I_{23}
I_{14} & I_{34}

I_{24} / I_{12} & I_{14}
I_{23} & I_{34}



I_{13} / I_{12} & I_{23}
I_{13} / I_{14} & I_{34}

I_{24} / I_{12} & I_{14}
I_{24} / I_{23} & I_{34}

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09. The input link O_2P of a four bar linkage is rotated at 2 rad/s counter clockwise direction as shown below. The angular velocity of the coupler PQ in rad/s, at an instant when $\boxed{\angle O_4O_2P = 180^\circ}$, is

(GATE-07)

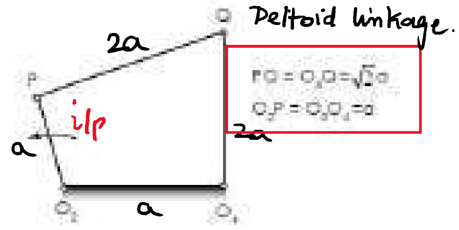
$I_{13} / I_{12}, I_{23}$

$I_{13} / I_{14}, I_{34}$

$I_{24} | I_{12} \,\&\, I_{14}$

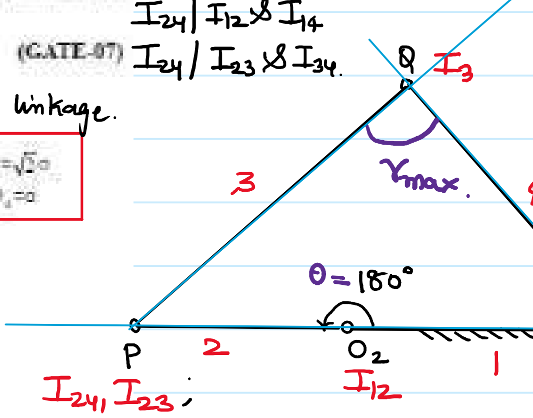
$I_{24} / I_{23} \,\&\, I_{34}$.

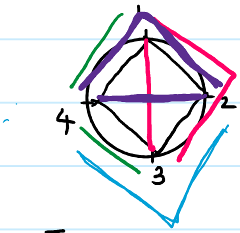
Deltoid linkage.



(a) 4 (b) $2\sqrt{2}$

(c) 1 (d) $\frac{1}{\sqrt{2}}$





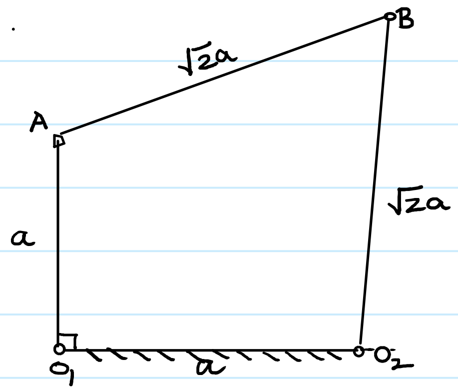
$$I_{12}.I_{23}.\omega_2 = I_{13}.I_{23}.\omega_3$$

$$a \times 2 = 2a \times \omega_3 \Rightarrow \omega_3 = 1\,rad/s.$$

$$I_{13}.I_{34}.\omega_3 = I_{14}.I_{34}.\omega_4$$

$$2a \times 1 = 2a \times \omega_4 \Rightarrow \omega_4 = 1\,rad/s.$$

Gate – 19/20.



$$\frac{\omega_4}{\omega_2} = ?$$

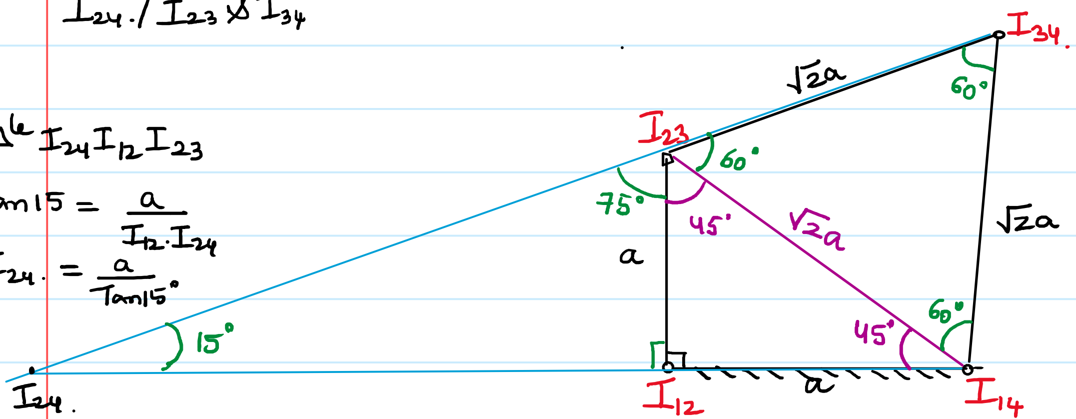
$I_{24} / I_{12} \,\&\, I_{14}$

$I_{24} / I_{23} \,\&\, I_{34}$

In Δ^{le} $I_{24} I_{12} I_{23}$

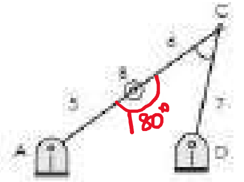
$$\tan 15 = \frac{a}{I_{12}.I_{24}}$$

$$I_{12}.I_{24}. = \frac{a}{\tan 15^\circ}$$



$$\frac{\omega_4}{\omega_2} = \frac{I_{12}.I_{24}}{I_{14}.I_{24}} \Rightarrow \frac{\omega_4}{\omega_2} = \frac{a/\tan 15^\circ}{a + a/\tan 15^\circ} = \frac{1}{\tan 15^\circ + 1} = 0.788$$

11. In a four bar mechanism for the position shown the velocity of the crank is 1 rad/sec (cc). The angular velocity of the link DC is



(a) 0
(b) 6/7 CCW
(c) 5/7 CW
(d) 7/5 CW

$\phi = 180^\circ \longrightarrow$ Toggle position.

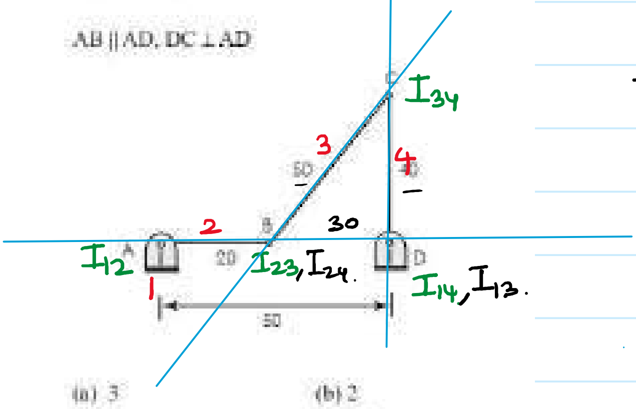
$\omega_{o/p.} = 0$

$l_2 . \omega_2 = l_3 . \omega_3 \Rightarrow \quad 5 \times 1 = 6 \times \omega_3$

$\omega_3 = 5/6 \text{ rad/s.}$

24. Consider the four bar mechanism shown in figure. At the given position the output rocker is to be driven in clockwise direction at 2 rad/sec. Determine the required velocity of the input crank AB in rad/sec. The length of the links AB, BC, CD and DC are respectively 20 cm, 50 cm, 40 cm and 50 cm.

AB || AD, DC ⊥ AD



(a) 3
(b) 2
(c) 5.5
(d) 12

$\omega_4 = 2 \text{ rad/s.} \quad (C.W)$

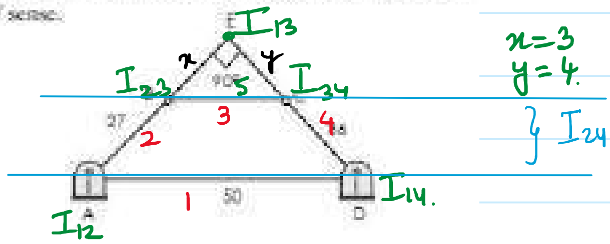
$\omega_2 = ?$

$$\frac{\omega_4}{\omega_2} = \frac{I_{12}.I_{24}}{I_{14}.I_{24}} \qquad \frac{\omega_4}{\omega_3} = \frac{I_{13}.I_{34}}{I_{14}.I_{34}}$$

$$\frac{2}{\omega_2} = \frac{20}{30} \qquad \frac{3}{\omega_3} = \frac{40}{40}$$

$\omega_2 = 3 \text{ rad/s.} \ (C.C.W) \qquad \omega_3 = 1 \text{ rad/s.} \ (C.W)$

The four bar mechanism is shown in figure at the configuration. If AB and DC are extended they meet at right angles and the lengths of the sides of the formed triangle AED are integers . The length of the links AB = 27cm, $\boxed{BC = 5 \text{ cm}}$ CD = 36 cm and AD = 50 cm. The link AB has an instantaneous angular velocity of 1 rad/sec in CW sense.



$x=3$
$y=4.$
$\} I_{24} @ \infty$

$\Delta^{le} I_{12}.I_{13}.I_{14}$ & $\Delta^{le} I_{23} I_{13}.I_{34}$ are similar

$$\frac{I_{23}.I_{34}}{I_{12}.I_{14}} = \frac{I_{23}.I_{13}}{I_{12}.I_{13}} = \frac{I_{13}.I_{34}}{I_{14}.I_{13}.}$$

$$\frac{5}{50} = \frac{x}{27+x} = \frac{y}{36+y}$$

$x = 3$

$y = 4.$

25. Angular velocity of two link BC is
(a) 3 rad/sec
(b) 6 rad/sec
$\boxed{\text{(c) 9 rad/sec}}$
(d) zero

$V_{I_{23}} = I_{12}.I_{23}.\omega_2 = I_{13}.I_{23}.\omega_3$

$(27 \times 1) = 3 \times \omega_3$

$\omega_3 = 9 \text{ rad/s.}$

26. Angular velocity of link DC is
(a) 1 rad/sec
(b) 1.6 rad/sec
(c) 2.4 rad/sec
(d) zero

$V_{I_{34}} = I_{14}.I_{34}.\omega_4 = I_{13}.I_{34}.\omega_3$

$36 \times \omega_4 = 4 \times 9 \Rightarrow \omega_4 = 1 \text{ rad/s.}$

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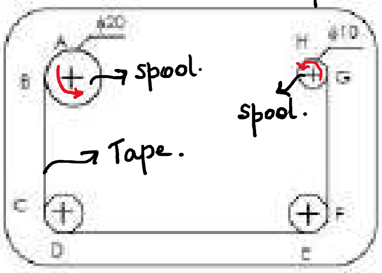
28. For the audio cassette mechanism shown in figure, where is the instantaneous center of rotation (point P) of the two spools? → $I_{24} = ?$

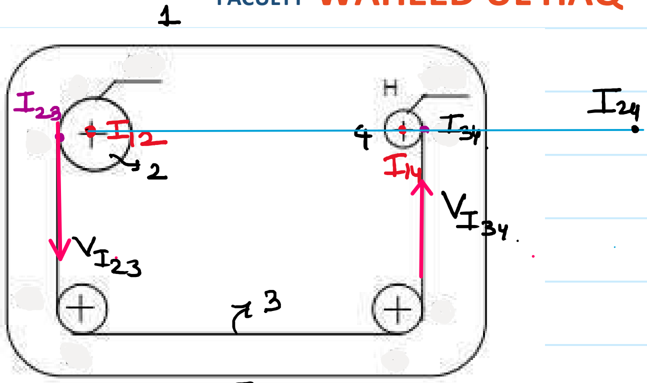


(a) Point 'P' lies to the left of both the spools but at infinity along the line joining 'A' and 'H'

(b) ~~Point 'P' lies in between the two spools on the line joining 'A' and 'H' such that PH = 2AP~~

(c) Point 'P' lines to the right of both the spools on the line joining 'A' and 'H' such that AH = HP

(d) ~~Point 'P' lies at the intersection of the line joining 'B' and 'C' and the line joining 'G' and 'F'~~



Pure Rolling

$V_{I_{23}} = I_{12} I_{23} \cdot \omega_2$

$V_{I_{23}} = 10 \times \omega_2$

$V_{I_{34}} = I_{14} I_{34} \cdot \omega_4 = 5 \times \omega_4$

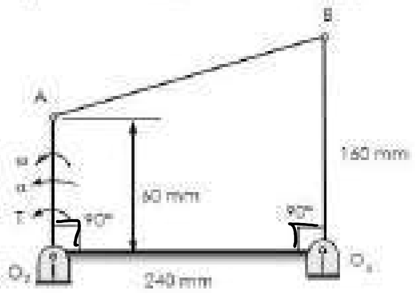
$V_{I_{23}} = V_{I_{34}} \Rightarrow 10 \times \omega_2 = 5 \times \omega_4 \Rightarrow \dfrac{\omega_2}{\omega_4} = \dfrac{1}{2}$

$$\frac{\omega_2}{\omega_4} = \frac{I_{14} \cdot I_{24}}{I_{12} \cdot I_{24}} = \frac{1}{2}$$

$$\frac{\omega_2}{\omega_4} = \frac{I_{14} \cdot I_{24}}{I_{12} I_{14} + I_{14} \cdot I_{24}} = \frac{1}{2}$$

$I_{12} \cdot I_{14} = I_{14} \cdot I_{24}$

An instantaneous configuration of a four bar mechanism, whose plane is horizontal, is shown in the fig below.



At this instant, the angular velocity and angular acceleration of link O_2A are $\omega = 8$ rad/s and $\boxed{\alpha = 0}$ → ω = constant. respectively, and the $\boxed{\text{driving torque } (\tau) \text{ is zero}}$. The link O_2A is balanced so that its center of mass falls at O_2.

(GATE-05)

$$\boxed{l_2 \cdot \omega_2 = l_4 \cdot \omega_4}$$

$60 \times 8 = 160 \times \omega_4$

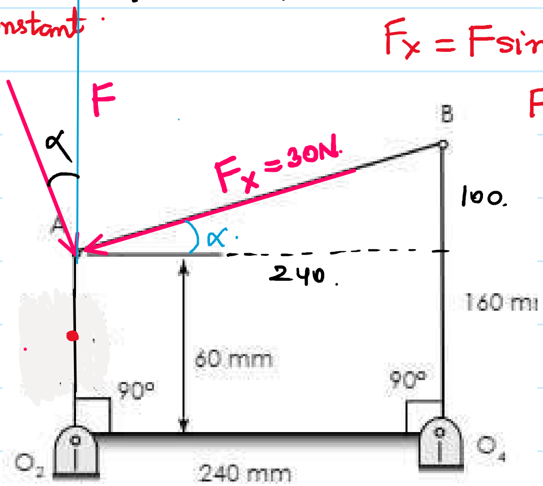
$\omega_4 = 3$ rad/s.

43. At the instant considered, what is the magnitude of the angular velocity of O_4B? $\omega_{O_4B} = ?$

(a) 1 rad/s (b) 3 rad/s
(c) 8 rad/s (d) 64/3 rad/s

44. At the same instant, if the component of the force at joint A along AB is 30N, then the magnitude of the joint reaction at O_2

(a) is zero (b) is 30 N
(c) is 78 N (d) cannot be determined



$F_x = F \sin\alpha = 30$

$F = \dfrac{30}{\sin 22.61}$

$F = 78$ N.

$\tan\alpha = \dfrac{100}{240} \Rightarrow \alpha = 22.61°$

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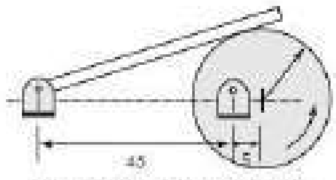
29. In the mechanism given below, if the angular velocity of the eccentric circular disc is 1 rad/s, the angular velocity (rad/s) of the follower link for the instant shown in the figure is (GATE-12)



(a) 0.05 (b) 0.1
(c) 5.0 (d) 10.0

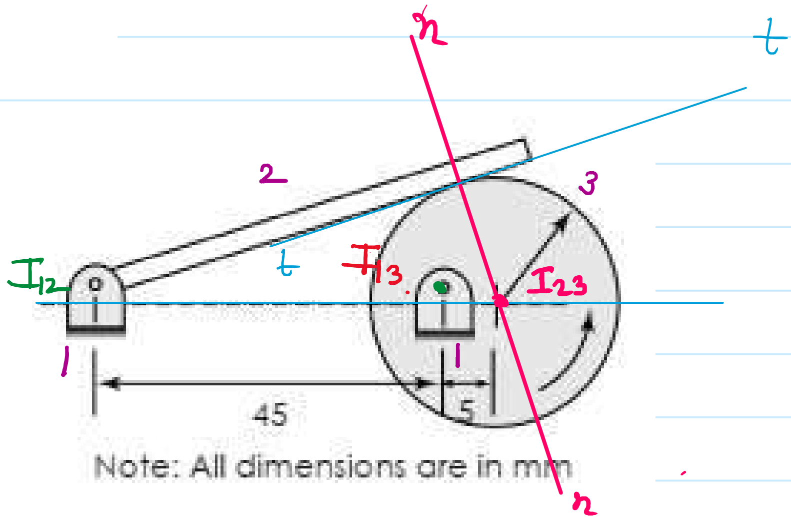
$\omega_3 / \omega_{Disc} = 1 rad/s.$

$\omega_2 / \omega_{Rod} = ?$

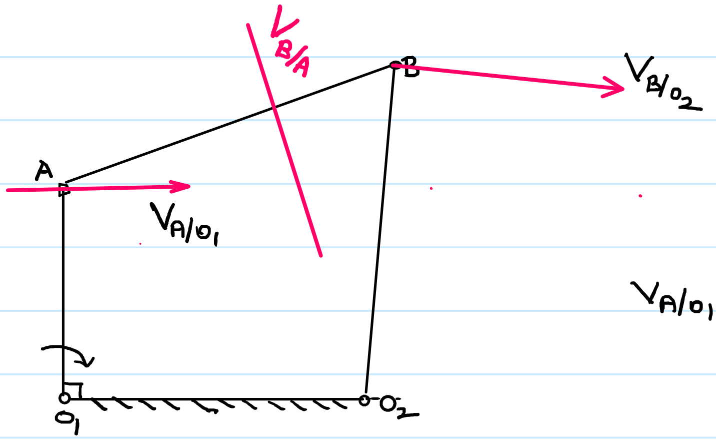
$$\frac{\omega_2}{\omega_3} = \frac{I_{13} \cdot I_{23}}{I_{12} \cdot I_{23}}$$

$$\frac{\omega_2}{1} = \frac{5}{50}$$

$\omega_2 = 0.1 rad/s.$

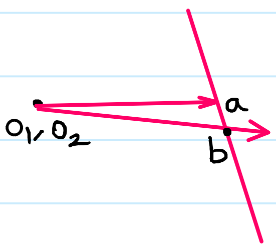


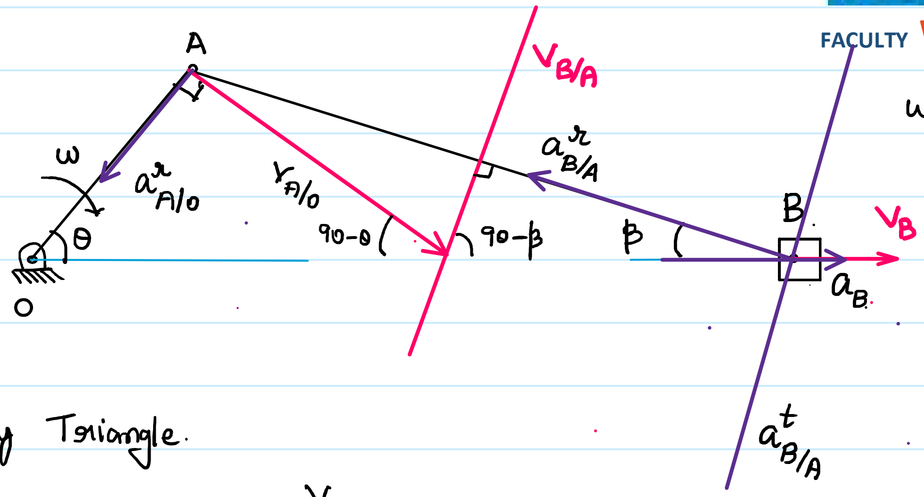
Velocity Analysis Using Relative Velocity Method (Graphical Method)



$V_{A/O_1} = O_1A \cdot \times \omega.$

Velocity Triangle.





$\omega_{crank} = \text{constant}$.

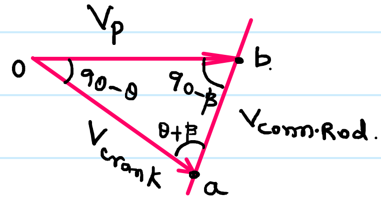
$\alpha_{crank} = 0$

$OA = r = \text{crank lenght}$

$AB = l = \text{connecting Rod lenght}$

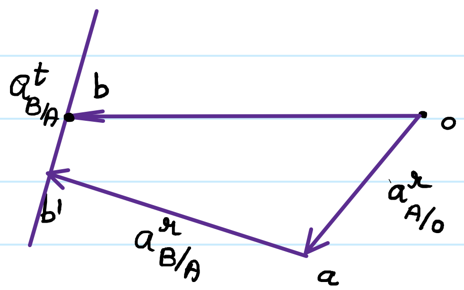
$OA \sin\theta = AB \sin\beta$

Velocity Triangle.



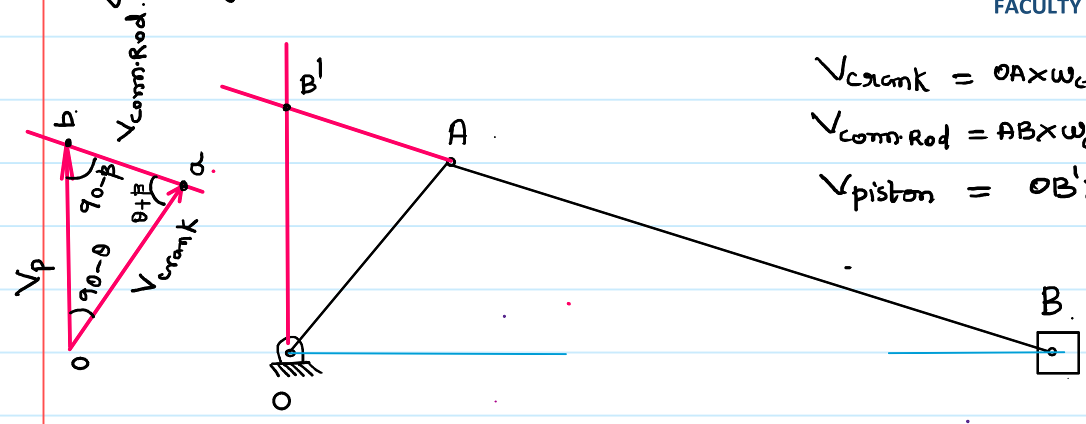
$$\frac{V_{piston}}{\sin(\theta+\beta)} = \frac{V_{crank}}{\sin(90-\beta)} = \frac{V_{conn.Rod}}{\sin(90-\theta)}$$

Acceleration Diagram.



Klein's Construction is the graphical method for performing velocity and acceleration analysis on the configuration diagram.

Velocity triangle.



$$V_{crank} = OA \times \omega_{crank}$$

$$V_{conn.Rod} = AB \times \omega_{conn.Rod} = AB' \times \omega_{crank}$$

$$V_{piston} = OB' \times \omega_{crank}$$

1. Draw a line from the point O perpendicular to the line of stroke.
2. Extend the line of connecting rod on the other side such that it intersects at a point B'.

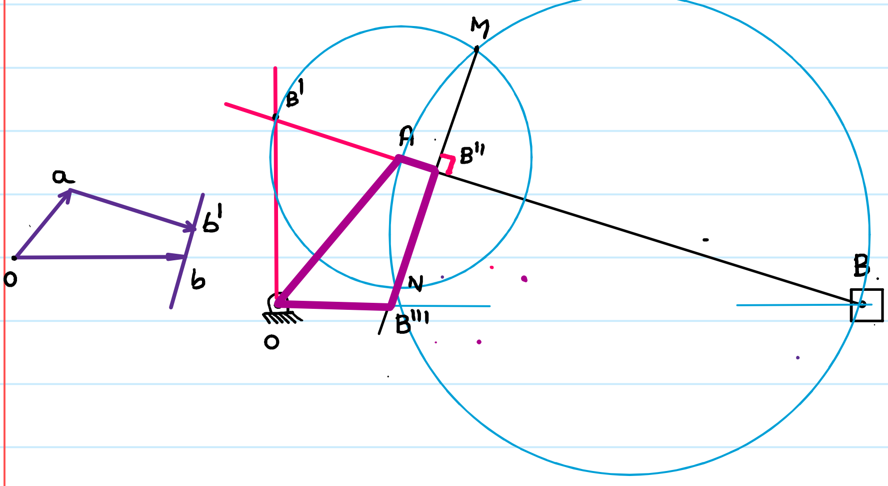
$$\frac{\text{lenght of Klein Diag.}}{\text{lenght of Vel. Diag.}} = \frac{OA}{oa} = \frac{OB'}{ob} = \frac{AB'}{ab} = \frac{1}{\omega_{crank}}$$

$oa - V_{crank} = OA \times \omega_{OA}$

$ab - V_{conn.Rod} = AB \times \omega_{AB}$

$ob - V_{piston}$

Velocity Triangle is scaled by a factor ω_{crank} when it is compared with the Klein diagram for Velocity.



$$\frac{OA}{oa} = \frac{AB''}{ab'} = \frac{B''B'''}{b'b} = \frac{OB'''}{ob} = \frac{1}{\omega_c^2}$$

$oa = a^r_{A/O} = OA \cdot \omega^2_{crank}$

$ab' = a^r_{B/A} = AB \cdot \omega^2_{conn.Rod}$

$b'b = a^t_{B/A} = AB \cdot \alpha_{conn.Rod}$

$ob = a_{piston}$

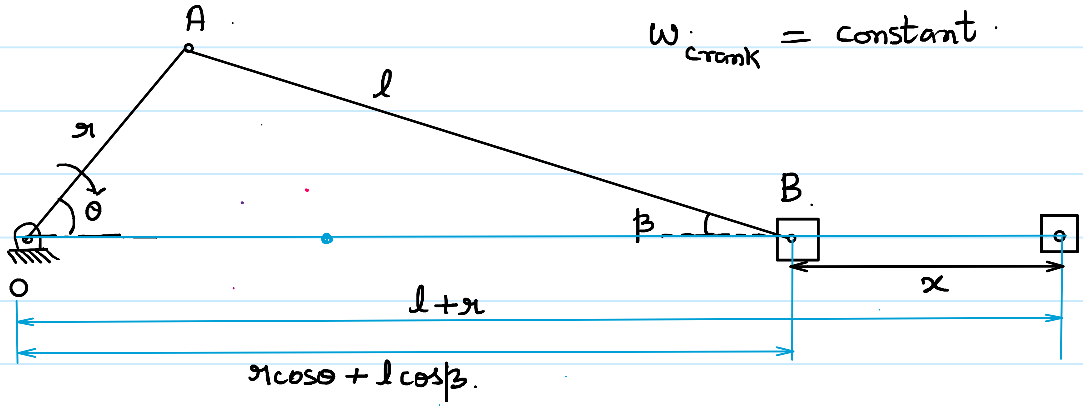
1. With A as centre and AB' as radius draw a circle.
2. With midpoint of AB as centre and AB as diameter draw another circle.
3. The two circles intersects at points M and N.
4. Join M , N and extend the line up till line of stroke of piston.

$$a^r_{crank} = OA \times \omega^2_{crank}$$

$$a^r_{conn.Rod} = AB \times \omega^2_{conn.Rod} = AB'' \times \omega^2_{crank}$$

$$a^t_{conn.Rod} = AB\,\alpha_{conn.Rod} = B''B''' \times \omega^2_{crank}$$

$$a_{piston} = OB''' \times \omega^2_{crank}$$



ω_{crank} = constant.

$OA \sin\theta = AB \sin\beta$

$$\boxed{r \sin\theta = l \sin\beta}$$

$$\boxed{\sin\beta = \frac{\sin\theta}{(l/r)} = \frac{\sin\theta}{n}} \Rightarrow \beta = f(\theta)$$

Analysis of Connecting Rod.

$$\frac{d\beta}{dt} \cos\beta = \cos\theta \cdot \frac{d\theta}{dt} \times \frac{1}{n}.$$

$$\omega_{conn.Rod} = \frac{\omega_{crank} \cdot \cos\theta}{n \cdot \cos\beta}.$$

$$\cos\beta = \sqrt{1-\sin^2\beta}$$

$$= \sqrt{1-\left(\frac{\sin\theta}{n}\right)^2} = \frac{1}{n}\sqrt{n^2-\sin^2\theta}$$

$$\omega_{conn.Rod} = \frac{\omega_{crank} \cdot \cos\theta}{n \cdot \frac{1}{n} \cdot \sqrt{n^2-\sin^2\theta}}$$

Angular Velocity of connecting Rod → $\boxed{\omega_{conn.Rod} = \frac{\omega_{crank} \cdot \cos\theta}{\sqrt{n^2-\sin^2\theta}}}$

Diff. wrt. t.

$$\frac{d\omega_{conn.Rod}}{dt} = \alpha_{conn.Rod} = \frac{d}{dt}\left[\frac{\overset{\text{constant}}{\omega_{crank}} \cdot \cos\theta}{\sqrt{n^2-\sin^2\theta}}\right]$$

$$\alpha_{conn.Rod} = \frac{\omega_{crank}}{\sqrt{n^2-\sin^2\theta}} \frac{d}{dt}(\cos\theta) + \omega_{crank} \cdot \cos\theta \frac{d}{dt}\left(\frac{1}{\sqrt{n^2-\sin^2\theta}}\right)$$

$n^2 - \sin^2\theta \approx n^2$

$$\boxed{\alpha_{conn.Rod.} = \frac{\omega^2_{crank} \cdot (-\sin\theta)}{n}}$$

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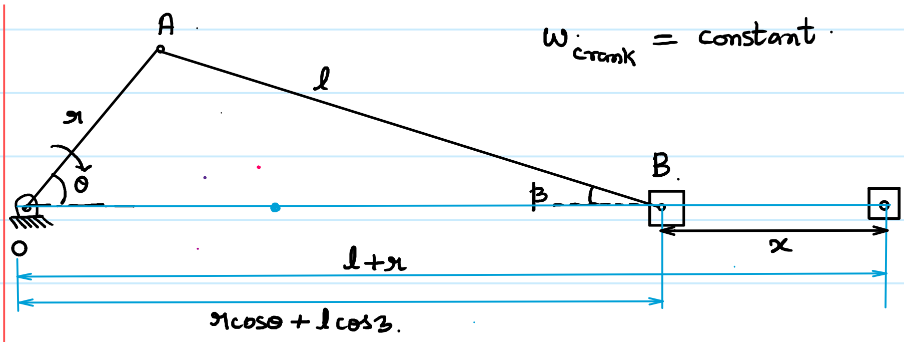
$\omega_{crank} = \text{constant}$



Piston displacement $\quad x = (l+r) - (l\cos\beta + r\cos\theta)$

$$x = r\left[\frac{l}{r}+1\right] - \left(\frac{l}{r}\cdot\cos\beta + \cos\theta\right)$$

$$x = r\cdot\left[(n+1) - (n\cos\beta + \cos\theta)\right]$$

$$\boxed{x = r\cdot\left[(n+1) - \left(n\cdot\frac{1}{n}\sqrt{n^2-\sin^2\theta} + \cos\theta\right)\right] = r\left[(n+1) - \left(\sqrt{n^2-\sin^2\theta} + \cos\theta\right)\right]}$$

$$x = f(\theta)$$

Velocity of Piston $\quad \frac{dx}{dt} = V_{piston} = r\left[0 - \left(\frac{1}{2\sqrt{n^2-\sin^2\theta}}(0 - 2\sin\theta\cos\theta) - \sin\theta\right)\right]\frac{d\theta}{dt}$

$$V_{piston} = r\cdot\omega\cdot\left[\sin\theta + \frac{\sin 2\theta}{2\cdot\sqrt{n^2-\sin^2\theta}}\right]$$

$n^2 - \sin^2\theta \simeq n^2$

$$\boxed{V_{piston} = r\omega\cdot\left[\sin\theta + \frac{\sin 2\theta}{2n}\right]}$$

Acceleration of Piston

$$a_{piston} = \frac{dV_p}{dt} = \frac{d}{dt}\left[r\omega\cdot\left[\sin\theta + \frac{\sin 2\theta}{2n}\right]\right]$$

$$a_{piston} = r\omega\cdot\left[\cos\theta + \frac{2\cos 2\theta}{2n}\right]\cdot\frac{d\theta}{dt}$$

$$a_{piston} = r\omega^2\cdot\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

Velocity of piston is maximum when $\left[\sin\theta + \frac{\sin 2\theta}{2n}\right] = 1$.

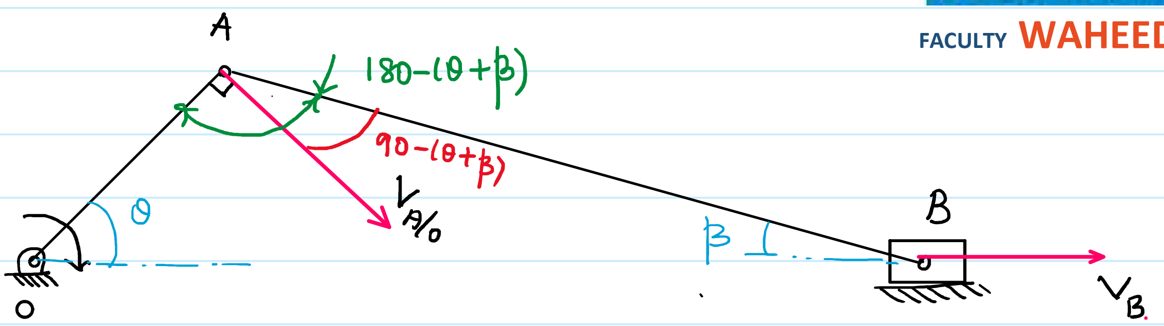
$(V_p)_{max} = r\omega. \quad \boxed{\theta \le 90^\circ}$

Max.

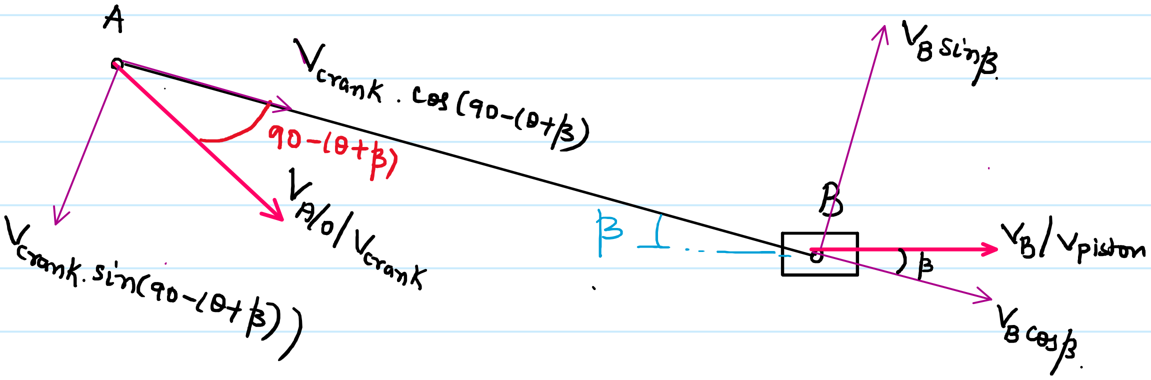
Acceleration of piston @ $\theta = 0^\circ \quad a_p = r\omega^2\cdot\left[1 + \frac{1}{n}\right]$

Acceleration of piston @ $\theta = 90^\circ \quad a_p = r\omega^2\cdot\left[0 - \frac{1}{n}\right]$

Acceleration of piston @ $\theta = 180^\circ \quad a_p = r\omega^2\cdot\left[-1 + 1/n\right]$



109 CRPQ.



Translation $\quad V_{crank} \cdot \cos(90-(\theta+\beta)) = V_B \cos\beta$

Velocity of Piston / X-head. $V_B = \dfrac{V_{crank} \cdot \sin(\theta+\beta)}{\cos\beta}$

Rotation $\quad \omega_{conn \cdot Rod} = \dfrac{V_{crank} \cdot \sin(90-(\theta+\beta)) - (-V_B \cdot \sin\beta)}{AB}$

$$\omega_{conn \cdot Rod} = \frac{V_{crank} \cdot \cos(\theta+\beta) + V_B \cdot \sin\beta}{AB}$$

I_{13} / I_{12} & I_{23}, I_{13} / I_{34} & I_{14}

I_{24} / I_{12} & I_{14} , I_{24} / I_{23} & I_{34}

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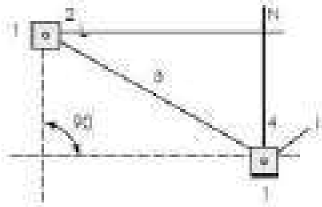
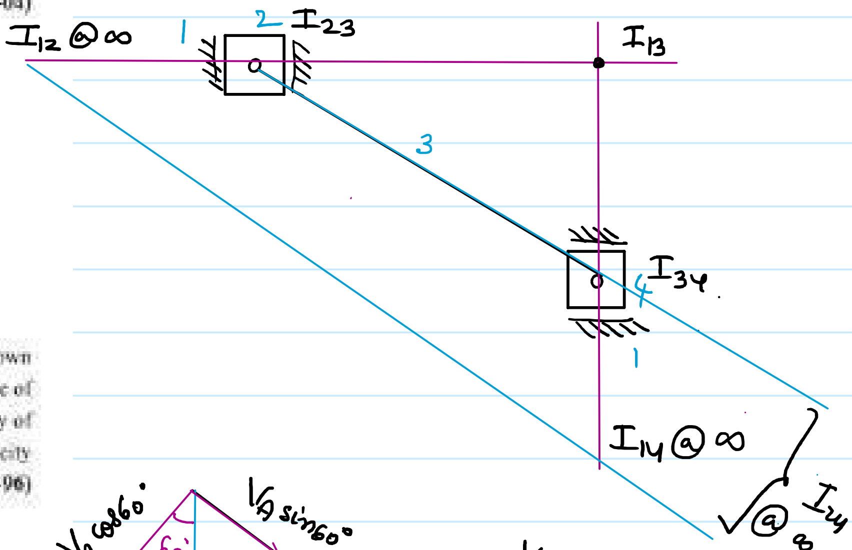
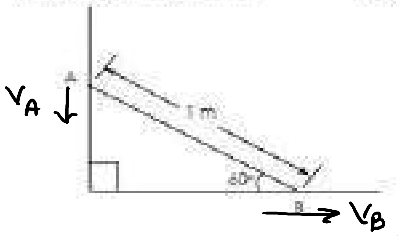
22. The figure below shows a planar mechanism with single degree of freedom. The instant center 24 for the given configuration is located at a position (GATE-04)

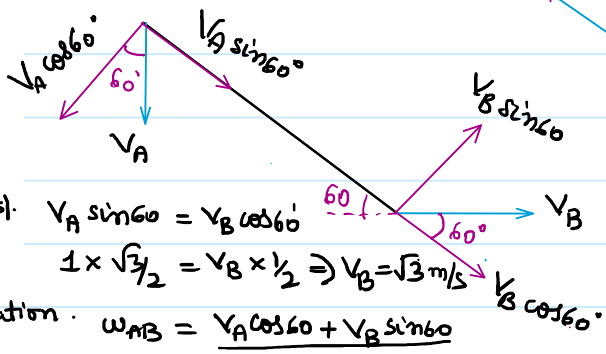


(a) L (b) M
(c) N (d) ∞



23. A rod of length 1 m is sliding in a corner as shown in fig. At an instant when the rod makes an angle of 60 degree with the horizontal plane, the velocity of point 'A' on the rod is 1 m/s. The angular velocity of the rod at this instant is (GATE-96)



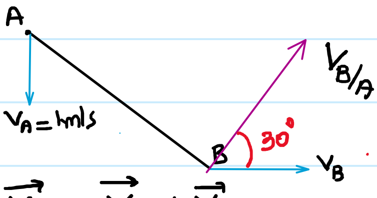


Transl. $V_A \sin 60 = V_B \cos 60$

$1 \times \sqrt{3}/2 = V_B \times 1/2 \Rightarrow V_B = \sqrt{3}$ m/s

Rotation. $\omega_{AB} = \dfrac{V_A \cos 60 + V_B \sin 60}{AB}$

$\omega_{AB} = \dfrac{(1 \times 1/2) + (\sqrt{3} \times \sqrt{3}/2)}{1} = 2$ rad/s



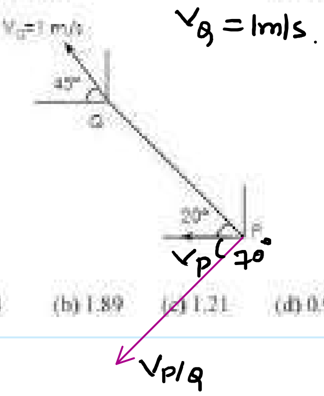
$\vec{V_B} = \vec{V_A} + \vec{V_{B/A}}$

$V_B(\hat{i}) = 1(-\hat{j}) + AB \cdot \omega_{AB} \cdot (i\cos 30 + j \sin 30)$

i-coff. $V_B = AB \cdot \omega_{AB} \cdot \cos 30° = 1 \times 2 \times \sqrt{3}/2 = \sqrt{3}$ m/s

j-coff $0 = -1 + 1 \cdot \omega_{AB} \cdot \sin 30 \Rightarrow \omega_{AB} = 1/\sin 30 = +2$ rad/s.

27. A rigid link PQ is 2 m long and oriented at 20° to the horizontal as shown in the figure. The magnitude and direction of velocity V_Q, and the direction of velocity V_P are given. The magnitude of V_P (in m/s) at this instant is (GATE-14)



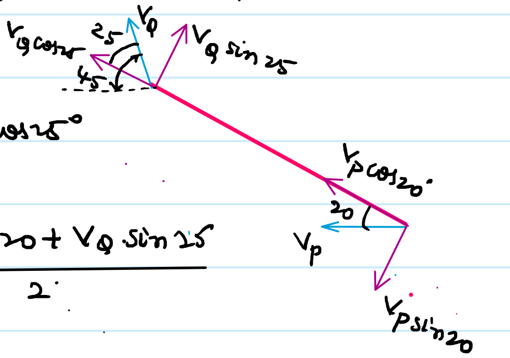
(a) 2.14 (b) 1.89 (c) 1.21 (d) 0.96

- Method 1

$V_P \cos 20° = V_Q \cos 25°$

Rotation

$\omega_{PQ} = \dfrac{V_P \cdot \sin 20 + V_Q \sin 25}{2}$



Method 2

$\vec{V_P} = \vec{V_Q} + \vec{V_{P/Q}}$

$V_P \cdot (-\hat{i}) = 1(-i\cos 45° + j \sin 45) + PQ \cdot \omega_{PQ} (-i\cos 70° - j \sin 70)$

$V_P(-\hat{i}) = 1(-i\cos 45° + j \sin 45) + 2 \cdot \omega_{PQ} \cdot (-i\cos 70 - j \sin 70)$

$V_P = 0.96$ m/s.

$\omega_{PQ} = 0.37$ rad/s.

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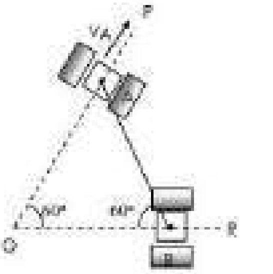
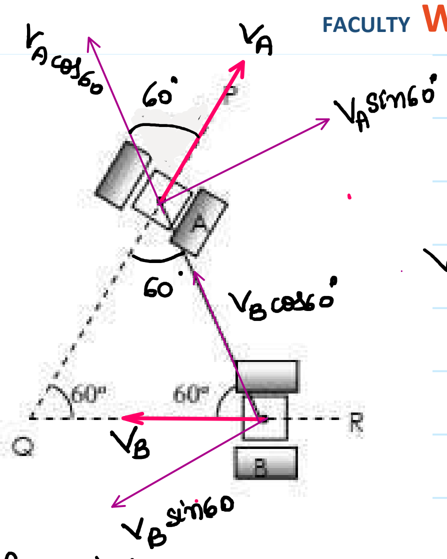
30. The rod AB, of length 1 m, shown in the figure is connected to two sliders at each end through pins. The sliders can slide along QP and QR. If the velocity V_A of the slider at A is 2 m/s, the velocity of the midpoint of the rod at this instant is ____ m/s.



(GATE – 16)



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$V_{midpoint} = V^{Trans.} =$

$V_A \cdot \cos 60° = 1\,m/s.$

Translation

$V_A \cos 60 = V_B \cos 60°$

$V_A = V_B = 2\,m/s.$

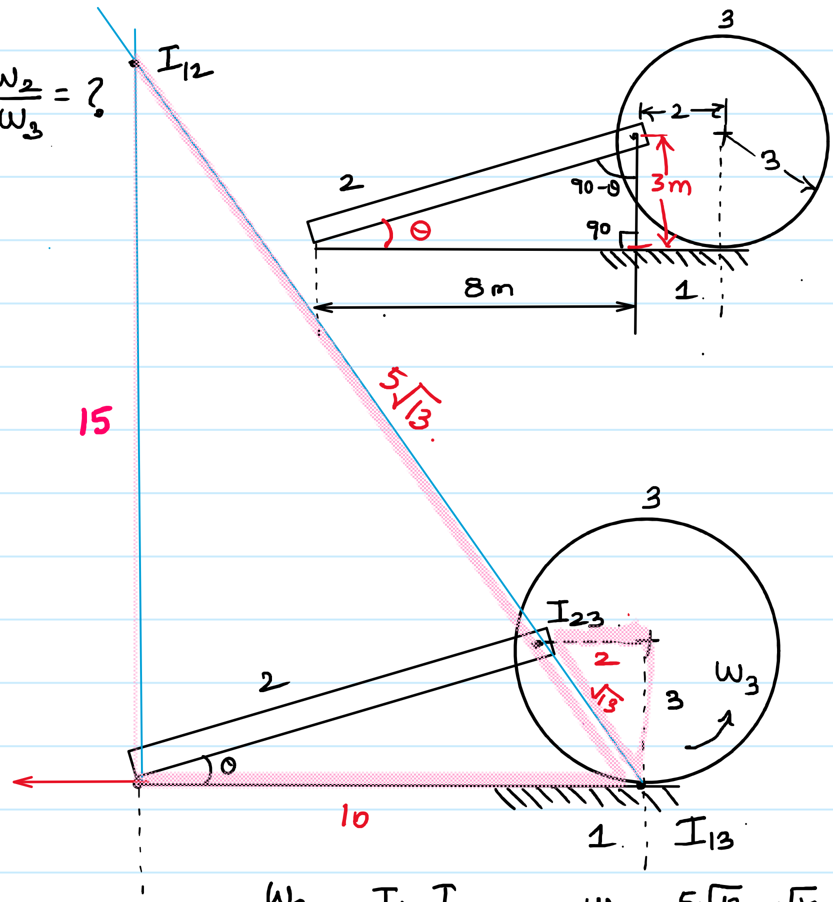
$V_A \sin 60° = V_B \cdot \sin 60°$

Mid point of AB is also centre of rotation of Rod AB

@ centre of rotation only Translational velocity is present.

@ this point velocity is minimum.

G-23.



$\frac{\omega_2}{\omega_3} = ?$

$\tan\theta = 3/8$

$\theta = 20.55°$

$V_{cm} = r\omega_3$

$V_{cm} = 3 \cdot \omega_3.$

Resultant Velocity

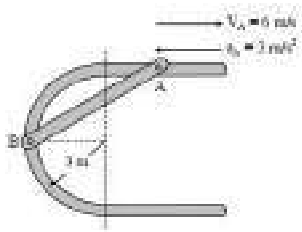
$= \left(V_{cm}^2 + r\omega)^{1/2}\right.$

$= \sqrt{(3\omega_3)^2 + (2\omega_3)^2} = \sqrt{13}\,\omega_3.$

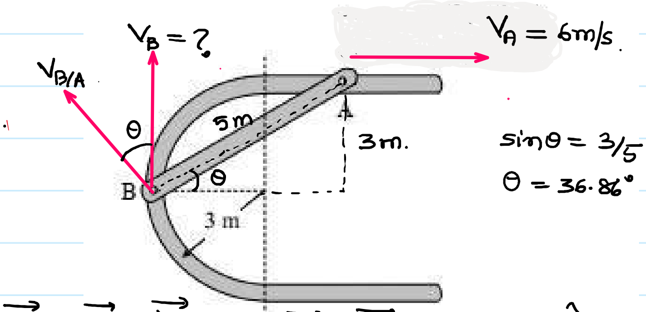
$\frac{\omega_2}{\omega_3} = \frac{I_{13} \cdot I_{23}}{I_{12} \cdot I_{23}} \Rightarrow \frac{\omega_2}{\omega_3} = \frac{5\sqrt{13} - \sqrt{13}}{\sqrt{13}} = 4.$

47. The rigid rod (AB) of length 5 m is confined to move along the path due to the pins at its ends. At the instant shown, point A has the motion shown below. Which of the following calculated results related to the above problem statement is/are correct ?

$V_B = ?$
$a_B = ?$



(a) The magnitude of velocity of point B at this instant is 8 m/s
(b) The magnitude of velocity of point B at this instant is 4 m/s
(c) The magnitude of acceleration of point B at this instant is 24.8 m/s²
(d) The magnitude of acceleration of point B at this instant is 21.4 m/s²



$\sin\theta = 3/5$
$\theta = 36.86°$

$\vec{V_B} = \vec{V_A} + \vec{V_{B/A}} \Rightarrow \vec{V_B} = \vec{V_A} + AB\cdot\omega_{AB}\,(\hat{e_t})$

$V_B(\hat{j}) = 6(\hat{i}) + 5\,\omega_{AB}(-\hat{i}\sin 36.86 + \hat{j}\cos 36.86)$

i-coff $\quad 0 = 6 - 5\,\omega_{AB}\cdot\sin 36.86$

$\omega_{AB} = 2\,rad/s.$

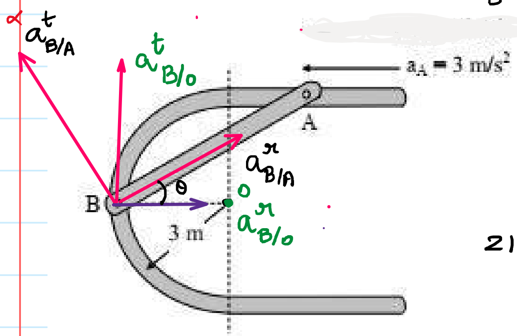
j-coff. $\quad V_B = 5\,\omega_{AB}\cos 36.86 = 5\times 2\times 0.8 = 8\,m/s.$

$V_B = OB\cdot\omega_B \Rightarrow \omega_B = 8/3 = 2.667\,rad/s.$

Acceleration Analysis

$\vec{a_B} = \vec{a_A} + \vec{a_{B/A}}$

$a_B^r + a_B^t = 3(-\hat{i}) + \vec{a}^r_{B/A} + \vec{a}^t_{B/A}$



$a_B^t = OB\cdot\alpha_{OB} \qquad a^t_{B/A} = AB\cdot\alpha_{AB} = 5\alpha_{AB}$

$a_B^r = \frac{V_B^2}{OB} = \frac{8^2}{3} = \frac{64}{3} = 21.33$

$a^r_{B/A} = AB\cdot\omega_{AB}^2 = 5\times 2^2 = 20\,m/s^2$

$21.33(\hat{i}) + 3\alpha_{OB}(\hat{j}) = 3(-\hat{i}) + 20(\hat{i}\cos\theta + \hat{j}\sin\theta)$
$+ 5\alpha_{AB}\cdot(-\hat{i}\sin\theta + \hat{j}\cos\theta)$

$21.33(\hat{i}) + 3\alpha_{OB}(\hat{j}) = -3\hat{i} + 20\cos 36.86\cdot\hat{i} + 20\sin 36.86\,\hat{j} - 5\alpha_{AB}\sin 36.86 + 5\alpha_{AB}\cos 36.86$

i-coff. $\quad 21.33 = -3 + 20\cos 36.86 - 5\,\alpha_{AB}\sin 36.86 \qquad \alpha_{AB} = -2.77\,rad/s^2$

$a^t_{B/A}$ must be in 4^{th} quadrant.

j-coff. $\quad 3\alpha_{OB} = 20\sin 36.86 + 5\alpha_{AB}\cos 36.86 \Rightarrow 3\alpha_{OB} = 20\sin 36.86 + 5(-2.77)\cos 36.86$

$\alpha_{OB} = 0.305\,rad/s^2$

$a_B = \sqrt{(a_B^r)^2 + (a_B^t)^2} \Rightarrow a_B = \sqrt{(21.33)^2 + (3\times 0.305)^2} = 21.34\,m/s^2$

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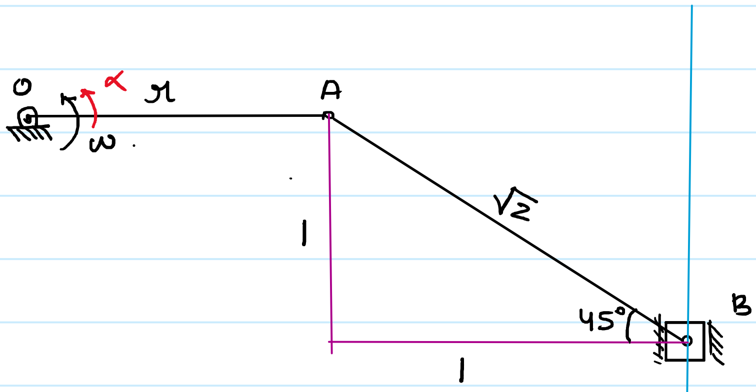
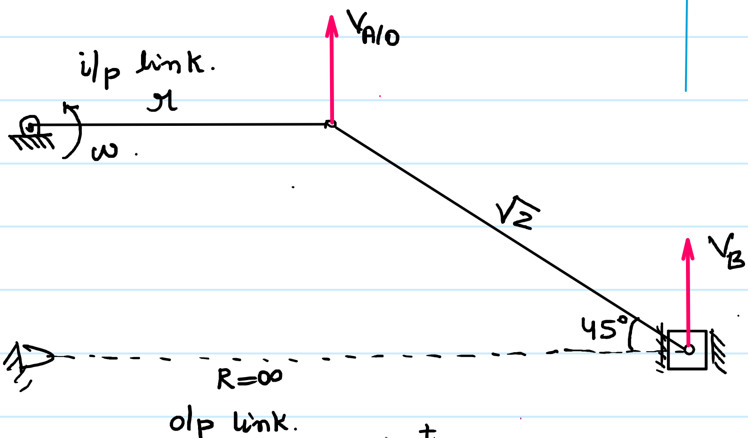
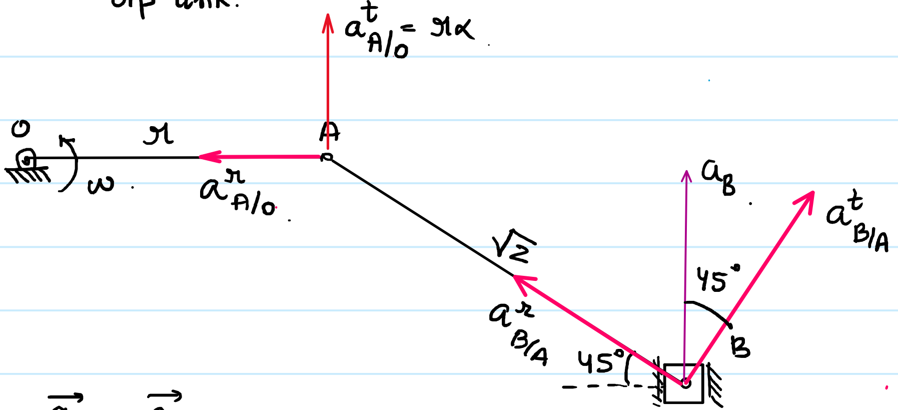
G-19

$a_{piston} = ?$ $\omega = \text{constant}$





$$V_{A/O} = V_B = r\omega$$
$$V_{AB} = 0$$



$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$$a_B(\hat{j}) = r\cdot\omega^2(-\hat{i}) + r\alpha(\hat{j}) + \vec{a}^{\,r}_{B/A} + \vec{a}^{\,t}_{B/A}$$

$$a_B(\hat{j}) = r\omega^2(-\hat{i}) + r\alpha(\hat{j}) \quad \sqrt{2}\cdot\underbrace{\cancel{\omega^2_{AB}}}_{0}(-\hat{i}\cos 45^\circ + \hat{j}\sin 45) + \sqrt{2}\cdot\alpha_{AB}(\hat{i}\sin 45^\circ + \hat{j}\cos 45^\circ)$$

$\hat{i}$-coff $0 = -r\omega^2 + \sqrt{2}\cdot\alpha_{AB}\sin 45$ $\alpha_{AB} = r\omega^2$

$\hat{j}$-coff $a_B = \sqrt{2}\cdot\alpha_{AB}\cdot\cos 45^\circ + r\alpha = \sqrt{2}\times r\omega^2\times\cos 45^\circ = r\omega^2 + r\alpha$

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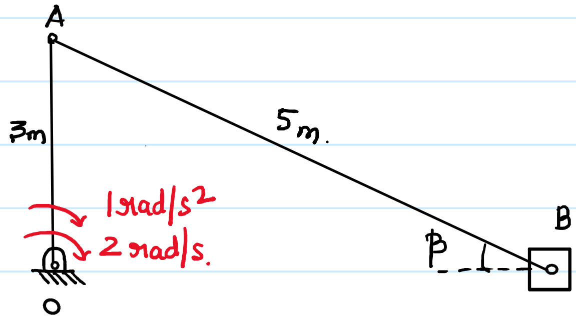


$\theta = 90^\circ$

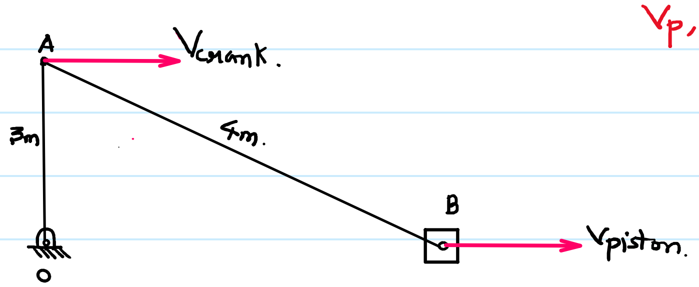
$\beta = \sin^{-1}(3/5)$

$\beta = 36.86^\circ$

$\omega_{conn.Rod}\,,\ \alpha_{Conn.Rod} = ?$

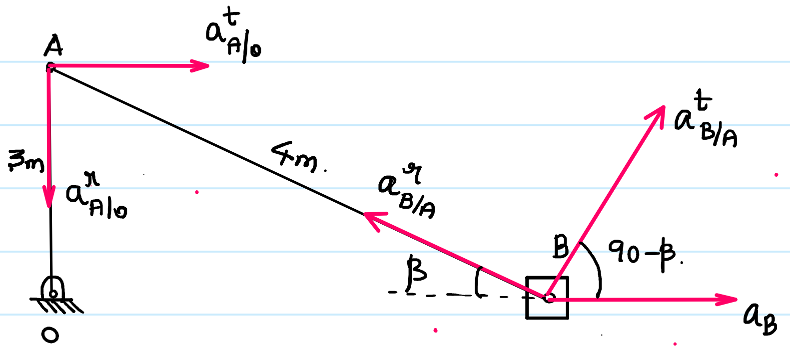
$V_p,\ a_p = ?$

Velocity Analysis.



$V_{piston} = r \cdot \omega = 3 \times 2 = 6\ m/s.$

$V_{conn.Rod} = 0 \Rightarrow \omega_{conn.Rod} = 0$



$a^r_{A/o} = OA \cdot \omega^2_{OA}$

$= 3 \times 2^2 = 12\ m/s^2$

$a^t_{A/o} = OA \cdot \alpha.$

$= 3 \times 1 = 3\ m/s^2$

$a^r_{B/A} = AB \cdot \cancel{\omega^2_{AB}} = 0$

$a^t_{B/A} = AB \cdot \alpha_{AB} = 5\alpha_{AB}$

$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

$a_B(\hat{i}) = \vec{a}^r_{A/o} + \vec{a}^t_{A/o} + \cancel{\vec{a}^r_{B/A}} + a^t_{B/A}$

$a_B(\hat{i}) = 12(-\hat{j}) + 3(\hat{i}) + 5\alpha_{AB}(\hat{i}\cos(90-\beta) + \hat{j}\sin(90-\beta))$

$a_B(\hat{i}) = -12\hat{j} + 3\hat{i} + 5\alpha_{AB}.(\hat{i}\sin 36.86 + \hat{j}\cos 36.86)$

j-coff. $\quad 0 = -12 + 5\alpha_{AB}.\cos 36.86$

$\alpha_{AB} = 3\ rad/s^2$

i-coff. $\quad a_B = 3 + 5\alpha_{AB}.\sin 36.86 \Rightarrow a_B = 3 + 5(3) \times 0.6$

$a_B = 12\ m/s^2$

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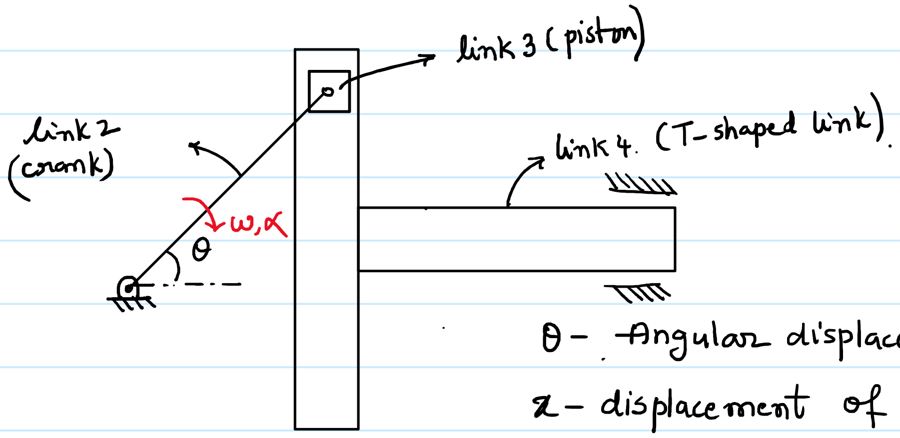
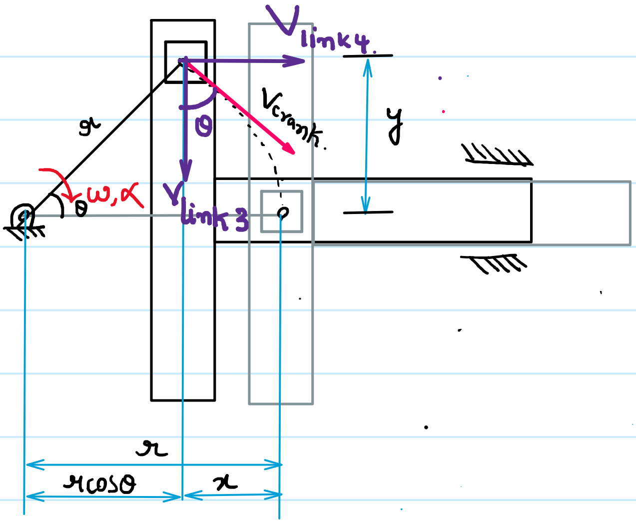
Scotch Yoke Mechanism.



θ – Angular displacement of link 2.

x – displacement of link 4.

y – displacement of link.



$$x = r - r\cos\theta$$

$$x = r[1-\cos\theta]$$

$$y = r\sin\theta$$

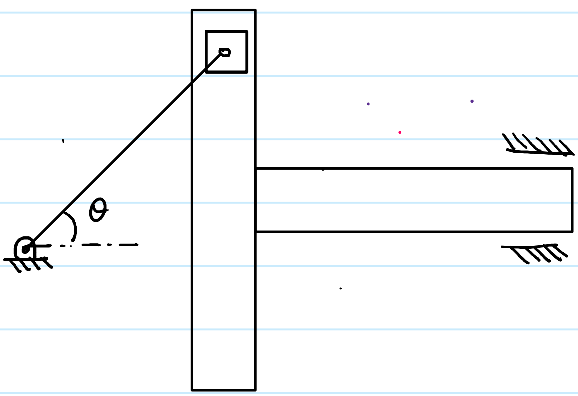
Velocity of link 3

$$V_{link\,3} = \frac{dy}{dt} = r\cos\theta \cdot \frac{d\theta}{dt}$$

$$V_{link\,3} = r \cdot \omega \cdot \cos\theta$$

$$V_{link\,4} = r \cdot \left[0 - (-\sin\theta)\right] \frac{d\theta}{dt}$$

$$= r\omega \sin\theta$$



Acceleration of link 3/slider

$$a_{link3} = \frac{d}{dt}\left(r\omega \cdot \cos\theta\right)$$

$$a_{link\,3} = r \cdot \omega \cdot (-\sin\theta)\frac{d\theta}{dt} + r \cdot \cos\theta \cdot \frac{d\omega}{dt}$$

$$a_{link\,3} = r\alpha\cos\theta - r\omega^2\sin\theta$$

Acceleration of link 4.

$$a_{link\,4} = \frac{d}{dt}(r\omega\sin\theta) = r\omega\cos\theta \cdot \frac{d\theta}{dt} + r\sin\theta \cdot \frac{d\omega}{dt} = r\omega^2\cos\theta + r\alpha\sin\theta$$

12. In a four bar mechanism at a particular configuration the angular velocity of the coupler is zero. The input link is half the length of the output link. The input link rotates at an angular velocity of 2 rad/sec. The pins at all the joints have a radius of 10cm. The rubbing velocity at the pins starting from the joint 1 (connecting fixed link to input link) to joint 4 (connecting output link to fixed link) are respectively in cm/sec.

(a) 20, 0, 0, 40 (b) 20, 20, 10, 0
(c) 20, 20, 10, 10 (d) 20, 0, 0, 10

i/p and o/p link are ‖le.

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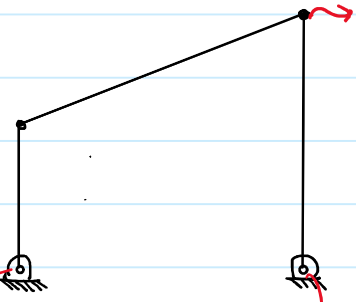
$l_{i/p} = \frac{1}{2} \cdot l_{o/p}$

$\omega_{i/p} = 2\,rad/s$

$r_{pin} = 10\,cm$

rubbing velocity @ all joints.



$V_R = r_{pin} \cdot (\cancel{\omega_3}^{0} \pm \omega_4)$

$= 10 \times 1 = 10\,cm/s$

$l_2 \cdot \omega_2 = l_4 \cdot \omega_4$

$\frac{1}{2} \cdot l_{o/p} \times 2 = l_{o/p} \times \omega_4$

$\omega_4 = 1\,rad/s$

$V_R = r_{pin} \cdot (\omega_2 \pm \cancel{\omega_3}^{0})$

$= 10 \times 2$

$= 20\,cm/s$

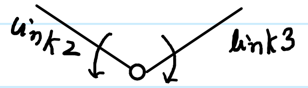
$V_R = r_{pin} (\cancel{\omega_1}^{0} \pm \omega_2)$

$= 10 \times 2 = 20\,cm/s$

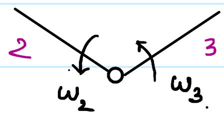
$V_R = r_{pin} \cdot (\omega_4 \pm \cancel{\omega_1}^{0})$

$= 10 \times 1 = 10\,cm/s$

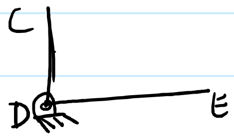
Rubbing velocity



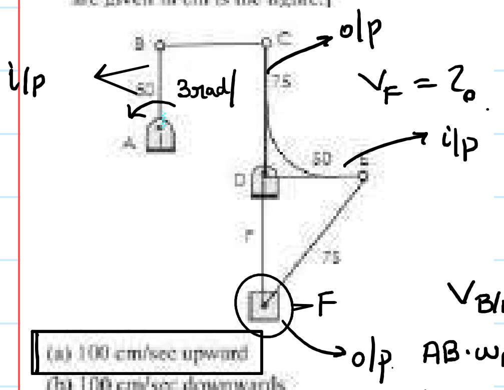
$V_{Rubbing} = r_{pin} \times (\omega_2 - (-\omega_3))$



$V_{Rubbing} = r_{pin} \cdot (\omega_2 - \omega_3)$

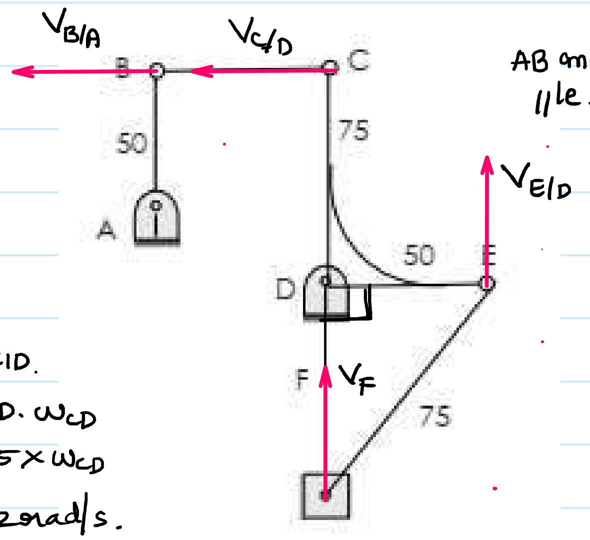


13. In a four bar mechanism shown in figure if the link AB has an instantaneous velocity of 3 rad/sec CCW. The velocity of the slider F is. [All the dimensions are given in cm is the figure.]



$V_F = ?$

(a) 100 cm/sec upward
(b) 100 cm/sec downwards
(c) 150 cm/sec upward
(d) 125 cm/sec upwards



AB and CD are ‖le. ⇒ $\boxed{\omega_{BC} = 0}$

$\boxed{\omega_{EF} = 0}$

$V_{B/A} = V_{C/D}$

$AB \cdot \omega_{AB} = CD \cdot \omega_{CD}$

$50 \times 3 = 75 \times \omega_{CD}$

$\omega_{CD} = 2\,rad/s$

$\omega_{DE} = \omega_{CD} = 2\,rad/s$

$V_{E/D} = V_F$

$ED \cdot \omega_{CE} = V_F$

$50 \times 2 = V_F$

$V_F = 100\,cm/s$

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14. In a slider crank mechanism the length of the crank is 1m and that of the connecting rod is 2 m. When the velocity of the slider is 2 m/sec. The angular velocity of the connecting rod is zero. The angular velocity of the crank, assuming to be uniform is

(a) 2 rad/sec (b) 1 rad/sec
(c) 0.5 rad/sec (d) 4 rad/sec

$r = 1m.$ $V_{piston} = 2 m/s.$

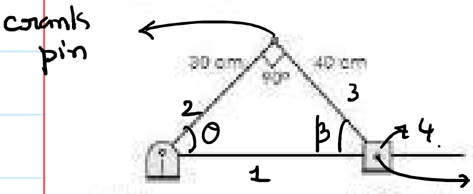
$\theta = 90^\circ$

$V_{crank} = V_{piston}.$

$r\omega = 2$

$\omega = 2/1 = 2 rad/s.$

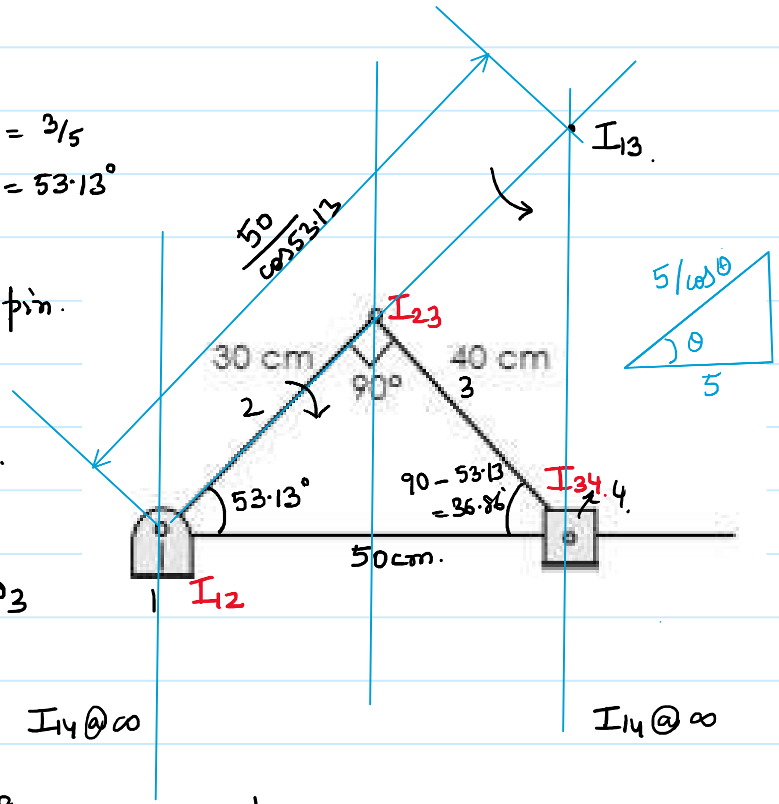
15. In a slider crank mechanism shown in figure at the instant the crank is perpendicular to the connecting rod. If the angular velocity of the crank is 10 rad/sec CCW. The rubbing velocity at the crank pin if its radius is 2.5 cm.



(a) 0 (b) 25 cm/sec
(c) 50 cm/sec (d) 39 cm/sec

$\cos\theta = 3/5$

$\theta = 53.13^\circ$



$r = 30cm$ $\omega_2 = 10 rad/s.$ c.c.w.

$V_{I_{23}} = I_{12}I_{23} \cdot \omega_2 = I_{13} \cdot I_{23} \cdot \omega_3$

$\Rightarrow 30 \times 10 = \left(\frac{50}{\cos 53.13} - 30\right) \times \omega_3$

$\omega_3 = \frac{300 \times 0.8 + 30 \times 0.8}{50} = 5.625 rad/s.$

@ Crank Pin

$V_{Rubbing} = r_{pin} \cdot (\omega_2 - (-\omega_3))$

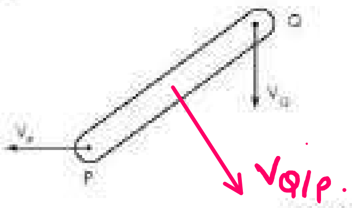
$= 2.5(10 + 5.625) = 39 cm/s.$

@ piston pin.

$V_{Rubbing} = r_{pin}(\omega_3 \pm \cancel{\omega_4}^{0})$

$= 2.5 \times 5.625 = 14.06 cm/s.$

19. A rigid link PQ is undergoing plane motion as shown in the figure (V_P and V_Q are non-zero). V_{QP} is the relative velocity of points Q with respect to point P.



(GATE – 16)

Which one of the following is TRUE?

(a) V_{QP} has components along and perpendicular PQ

(b) V_{QP} has only one component directed from P to Q

(c) V_{QP} has only one component directed from Q to P

$\boxed{\text{(d) } V_{QP} \text{ has only one component perpendicular to PQ}}$

$$\vec{V_Q} = \vec{V_P} + \vec{V_{Q/P}}$$

$$\vec{V_{Q/P}} = \vec{V_Q} - \vec{V_P}$$

$$\hat{V}_{Q/P} = \hat{V}_Q - \hat{V}_P$$

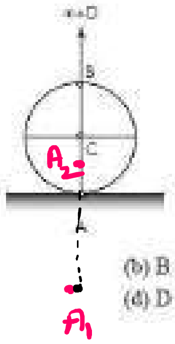
$$= -\hat{j} - (-\hat{i})$$

$$= \hat{i} - \hat{j}$$

Velocity analysis:

Instantaneous center method

20. The instantaneous centre of motion of a rigid-thin-disc-wheel rolling on plane rigid surface shown in the figure is located at the point.



(a) A (b) B

(c) C (d) D

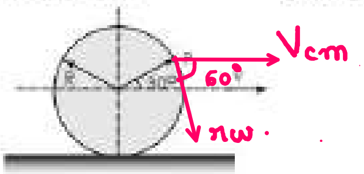
Higher Pair.

Pure Rolling — (A) — I-centre.

Rolling + slipping / Foward slipping. (A_1) — I-centre. (Translation > Rotation)

A_2. — Rolling + Slipping | Backward slipping (Rotation > Translation)

21. A circular disk of radius 'R' rolls without slipping at a velocity v. The magnitude of the velocity at point 'P' (see figure) is (GATE-08)



(a) $\sqrt{3}\,v$ (b) $\frac{\sqrt{3}\,v}{2}$

(c) $\frac{v}{2}$ (d) $\frac{2v}{\sqrt{3}}$

$$\vec{V_p} = V_{cm}(\hat{i}) + r\omega.\left[\hat{i}\cos 60 - \hat{j}\sin 60\right]$$

$$\vec{V_p} = V_{cm}(\hat{i}) + V_{cm}\times \tfrac{1}{2}.\hat{i} - V_{cm}.\frac{\sqrt{3}}{2}\hat{j}$$

$$\vec{V_p} = V_{cm}.\left[\frac{3}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right]$$

$$V_p = \sqrt{3}\,V_{cm}.$$

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