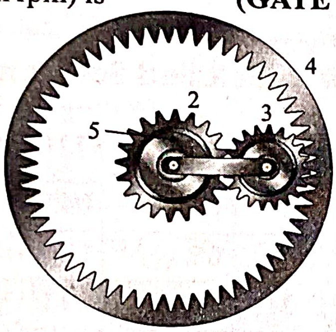
(c) $F_1 = F_2;\ \tau_1 = I_1\ddot{\theta}_1;\ F_2 = I_2\frac{1}{r_2}\ddot{\theta}_2$

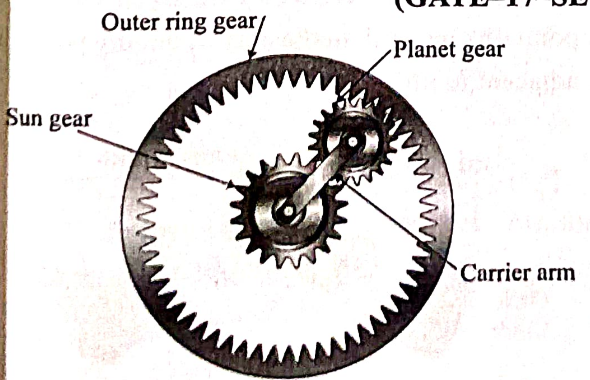
(d) $F_1 \neq F_2;\ \tau_1 = \left[I_1 + I_2\left(\frac{r_1}{r_2}\right)^2\right]\ddot{\theta}_1;\ F_2 = I_2\frac{1}{r_2}\ddot{\theta}_2$

15. In the gear train shown, gear 3 is carried on arm 5. Gear 3 meshes with gear 2 and gear 4. The number of teeth on gear 2, 3, and 4 are 60, 20, and 100, respectively. If gear 2 is fixed and gear 4 rotates with an angular velocity of 100 rpm in the counterclockwise direction, the angular speed of arm 5 (in rpm) is **(GATE – 16 – SET – 1)**

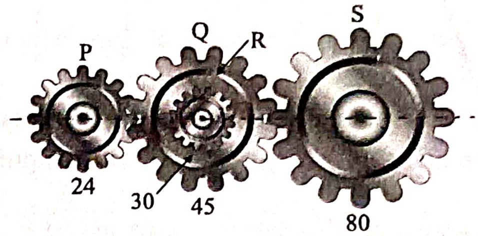


(a) 166.7 counterclockwise
(b) 166.7 clockwise
(c) 62.5 counterclockwise
(d) 62.5 clockwise

16. In an epicyclic gear train, shown in the figure, the outer ring gear is fixed, while the sun gear rotates counterclockwise at 100 rpm. Let the number of teeth on the sun, planet and outer gears to be 50, 25 and 100 respectively. The ratio of magnitudes of angular velocity of the planet gear to the angular velocity of the carrier arm is _____. **(GATE–17–SET–1)**

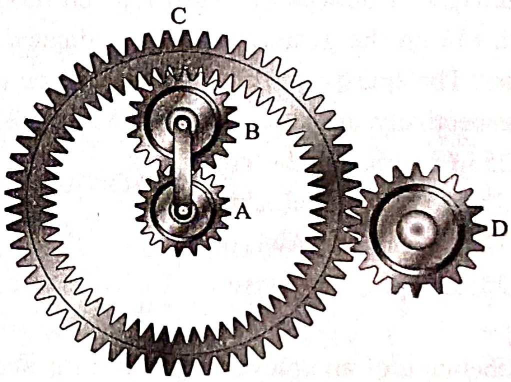


17. A gear train shown in the figure consists of gears P, Q, R and S. Gear Q and gear R are mounted on the same shaft. All the gears are mounted on parallel shafts and the number of teeth of P, Q, R and S are 24, 45, 30 and 80, respectively. Gear P is rotating at 400 rpm. The speed (in rpm) of the gear S is _____



(GATE – 17 – SET – 2)

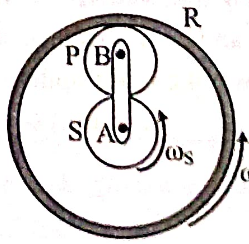
18. An eipcyclic gear train is shown in the figure below. The number of teeth on the gears A, B and D are 20, 30 and 20, respectively. Gear C has 80 teeth on the inner surface and 100 teeth on the outer surface. If the carrier arm AB is fixed and the sun gear A rotates at 300 rpm in the clockwise direction, then the rpm of D in the clockwise direction is



(a) 240 (b) –240 (c) 375 (d) –375

(GATE – 18 – SET – 1)

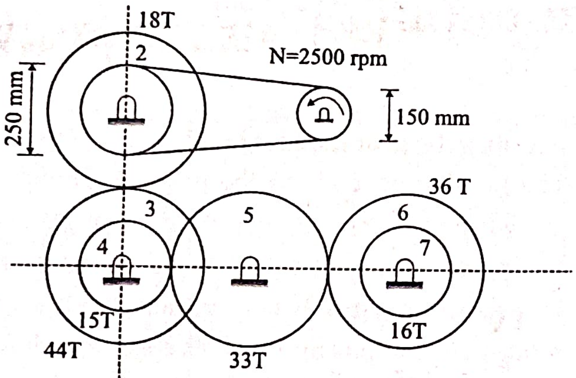
*19. The sun (S) and the planet (P) of an epicyclic gear train shown in the figure have identical number of teeth.



If the sun (S) and the outer ring (R) gears are rotated in the same direction with angular speed ω_S and ω_R, respectively, then the angular speed of the arm AB is (GATE–20–SET–2)

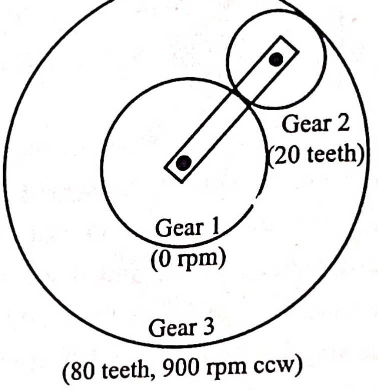
(a) $\frac{3}{4}\omega_R+\frac{1}{4}\omega_S$
(b) $\frac{1}{2}\omega_R-\frac{1}{2}\omega_S$
(c) $\frac{3}{4}\omega_R-\frac{1}{4}\omega_S$
(d) $\frac{1}{4}\omega_R+\frac{3}{4}\omega_S$

20. A power transmission mechanism consists of a belt drive and a gear train as shown in the figure.



Diameters of pulleys of belt drive and number of teeth (T) on the gears 2 to 7 are indicated in the figure. The speed and direction of rotation of gear 7, respectively, are (GATE–21_SET–2)
(a) 255.68 rpm, anticlockwise
(b) 575.28 rpm, anticlockwise
(c) 575.28 rpm, clockwise
(d) 255.68 rpm, clockwise

21. A schematic of an epicyclic gear train is shown in the figure. The sun (gear 1) and planet (gear 2) are external, and the ring gear (gear 3) is internal. Gear 1, gear 3 and arm OP are pivoted to the ground at O. Gear 2 is carried on the arm OP via the pivot joint at P, and is in mesh with the other two gears. Gear 2 has 20 teeth and gear 3 has 80 teeth. If gear 1 is kept fixed at 0 rpm and gear 3 rotates at 900 rpm counter clockwise (ccw), the magnitude of angular velocity of arm OP is ________ rpm (in integer).



(GATE-22_SET-1)

KEY & Detailed Solutions

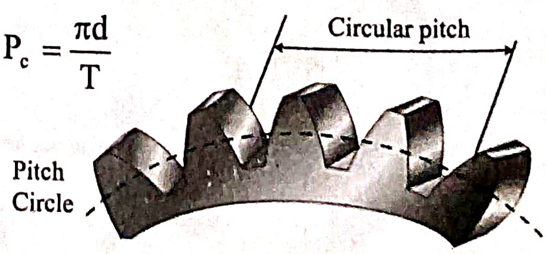
ONE MARK QUESTIONS				
01. (*)	02. (d)	03. (d)	04. (b)	05. (d)
06. (c)	07. (d)	08. (c)	09. (d)	10. 96
TWO MARKS QUESTIONS				
01. (*)	02. (b)	03. (b)	04. (b)	05. (a)
06. (b)	07. (a)	08. (d)	09. (a)	10. (c)
11. (b)	12. (a)	13. (a)	14. (b)	15. (c)
16. 3	17. 120	18. (c)	19. (a)	20. (d)
21. 600				

One Mark Solutions

01.

Sol: Circular pitch (P_c): It is the distance measured along the circumference of the pitch circle from a point on one tooth to the corresponding point on the adjacent tooth

$$P_c=\frac{\pi d}{T}$$



Diametral Pitch (P_d): It is the number of teeth per unit length of the pitch circle diameter in inches.

$$P_d = \frac{T}{d}$$

$\therefore$ Product of P_c (circular pitch) $\times$ P_d (diametral pitch)

$$\therefore\ P_c.P_d = \frac{\pi d}{T} \times \frac{T}{d} = \pi$$

02. Ans: (d)

Sol: Velocity ratio: $\frac{1440}{36} = 40$

For 35 to 45 velocity ratios, suitable gear train is "worm gear".

03. Ans: (d)

Sol: Tooth interference in an external involute spur gear pair can be reduced by increasing number of gear teeth. The minimum number of teeth required to avoid interference is

$$T_{min} = \frac{2}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1}$$

04. Ans: (b)

Sol: Gear 1: ϕ 80, $\alpha_1 = 30$

Gear 2: ϕ120, $\alpha_2 = 22.5$

$d'_1 = d_1\cos\alpha_1 = 80 \times \cos 30° = 40\sqrt{3}$ mm

$d'_2 = d_2 \cos\alpha_2 = d_2\cos\alpha_2$

$= 120 \times \cos 22.5° = 110.87$ mm

$$\frac{N_1}{N_2} = \frac{d'_2}{d'_1}$$

$$\frac{1440}{N_2} = \frac{110.87}{40\sqrt{3}}$$

$\therefore N_2 = 900$ rpm

05. Ans: (d)

Sol: Rack and pinion converts Rotary Motion to translating motion.

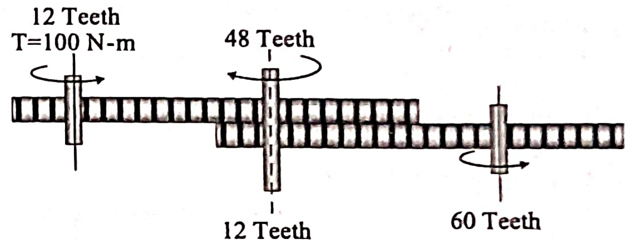
06. Ans: (c)

Sol: $\frac{N_2}{N_6} = \frac{N_3N_5N_6}{N_2N_4N_5} = \frac{N_3N_6}{N_2N_4}$

Wheel 5 is the only Idler gear as the number of teeth on wheel '5' does not appear in the velocity ratio.

07. Ans: (d)

Sol: Gear is compound gear train



$$\frac{N_A}{N_D} = \frac{N_B \times N_D}{Z_A \times Z_A} = \frac{T_D}{T_A}$$

$$\frac{T_D}{100} = \frac{48\times 60}{12\times 12}$$

$T_D = 2000$ N-m

08. Ans: (c)

Sol: $\phi = 20°$, $P = 20$ kW,

$\omega = 200$ rad/s, $d = 100$ mm $= 0.1$ m

Torque = Power / ω

$$T = \frac{20000}{200} = 100 \text{ N-m}$$

Now, $T = F_T \times \frac{d}{2}$

$\Rightarrow 100 = F_T \times \frac{0.1}{2}$

$\Rightarrow F_T = 2000$ N

$\frac{F_R}{F_T} = \tan\phi$

$\Rightarrow F_R = 2000 \times \tan 20°$

$\Rightarrow F_R = 727.94$ N $= 0.73$ kN

09. Ans: (d)

Sol: **Circular pitch :** It is the distance between two similar points on adjacent teeth measured along pitch circle circumference, Circular pitch (P_c) $= \dfrac{\text{Pitch circle circumference}}{\text{Number of teeth}}$

$= \dfrac{\pi D}{T}$

10. Ans: 96

Sol: Diametral pitch $= \dfrac{T}{d} = 8\,\dfrac{\text{teeth}}{\text{mm}}$

Module, $m = \dfrac{1}{8}$ mm/teeth

Let T_1 and T_2 = No. of teeth in driver and driven gears respectively.

$$\frac{\omega_2}{\omega_1} = \frac{T_1}{T_2} = \frac{1}{4}$$

Centre distance, $C = r_1 + r_2 = 30$ mm

$$\frac{m}{2}(T_1 + T_2) = 30$$

$$\frac{1}{8}(T_1 + 4T_1) = 60$$

$$\Rightarrow T_1 = \frac{60 \times 8}{5} = 96 \text{ teeth}$$

Two Marks Solutions

01. Ans: a– 4, b–3, c–2, d–1

Sol:

- Worm gears → Large speed ratios
- Cross helical gears → Non-parallel, non intersecting shafts
- Bevel gears → Non-parallel, intersecting shafts
- Spur gears → Parallel shafts

02. Ans: (b)

Sol: $\dfrac{\omega_C - \omega_a}{\omega_B - \omega_a} = -\dfrac{T_B}{T_C}$

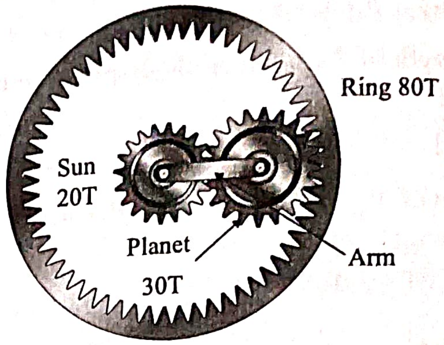
Given $T_B = T_C$, $\omega_B = 0$,

$\omega_a = 4$ rad/sec

$\therefore \dfrac{\omega_C - 4}{0 - 4} = -1 \Rightarrow \omega_C = 8$ rad/sec

03. Ans: (b)

Sol:



$T_S = 20$, $T_P = 30$, $T_R = 80$,

$N_S = 100$ rpm CW, $N_R = 0$, $N_a = ?$

$$\frac{N_S - N_a}{N_R - N_a} = \frac{-T_R}{T_S}$$

$$\frac{100 - N_a}{0 - N_a} = \frac{-80}{20}$$

$\Rightarrow N_a = 20$ rpm CW

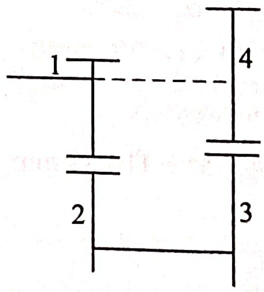
04. Ans: (b)

Sol:

- Addendum – Gear
- Instantaneous centre of velocity – Linkage
- Section modulus – Beam
- Prime circle – Cam

05. Ans: (a)

Sol:



$Z_1 = 16$, $Z_3 = 15$,

$Z_2 = ?$, $Z_4 = ?$

First stage gear ratio, $G_1 = 4$,

Second stage gear ratio, $G_2 = 3$,

$m_{12} = 3$, $m_{34} = 4$

$Z_2 = 16 \times 4 = 64$

$Z_4 = 15 \times 3 = 45$

06. Ans: (b)

Sol: $\text{Centredistance} = \frac{m_{12}}{2}\times(Z_1+Z_2) = \frac{m_{34}}{2}\times(Z_3+Z_4)$

$$= \frac{4}{2}\times(15+45) = 120\text{ mm}$$

07. Ans: (a)

Sol: By Analytical Approach

$$\frac{\omega_1-\omega_5}{\omega_4-\omega_5} = \frac{-T_2}{T_1}\times\frac{-T_4}{T_3} = \frac{45}{15}\times\frac{40}{20}$$

$$\frac{\omega_1-\omega_5}{\omega_4-\omega_5} = 6$$

08. Ans: (d)

Sol: Data given :

$\omega_1 = 60$ rpm (CW)

$\omega_4 = -120$ rpm [2 times speed of gear - 1]

We have

$$\therefore \quad \frac{\omega_1-\omega_5}{\omega_4-\omega_5} = 6$$

$$\Rightarrow \frac{60-\omega_5}{-120-\omega_5} = 6\text{, simplifying}$$

$$60-\omega_5 = -720-6\omega_5$$

$$\omega_5 = -156\text{ rpm}$$

$$\Rightarrow \omega_5 = 156\text{ rpm. CCW}$$

09. Ans: (a)

Sol: An epicyclic gear train is shown schematically the gear 5 is fixed and gear 2 is rotating 60 rpm (CCW) Then the arm 4 attached to the output shaft will rotate at

$$\frac{N_2-N_4}{N_5-N_4} = \frac{T_5}{T_3}\times\frac{T_3}{T_2}$$

$$\frac{N_2-N_4}{N_5-N_4} = \frac{T_5}{T_2}$$

$T_2 = 20$, $T_3 = 40$, $T_5 = 100$

$N_2 = 60$ CCW (+ve), $N_5 = 0$, $N_4 = ?$

$$\frac{N_2-N_4}{N_5-N_4} = \frac{-T_5}{T_2}$$

$$\frac{60-N_4}{0-N_4} = -\frac{100}{20} \Rightarrow N_4 = 10\text{ CCW}$$

10. Ans: (c)

Sol: $\omega_2 = 100$ rad/sec,

$\omega_{arm} = 80$ rad/s (CCW) [Assume as CW rotation is positive, CCW rotation is negative].

$$\frac{\omega_5-\omega_a}{\omega_2-\omega_a} = \frac{-T_2}{T_3}\times\frac{T_4}{T_5}$$

$$\frac{\omega_5-(-80)}{100-(-80)} = \frac{-20}{24}\times\frac{32}{80} = -\frac{1}{3}$$

$$\Rightarrow \omega_5 = -140\text{ rpm} = 140\text{ rpm (CCW)}$$

11. Ans: (b)

Sol: Given $T_P = 20$, $T_Q = 40$,
$T_R = 15$, $T_S = 20$

Dia of Q = 2 × Dia of R

$m_Q.T_Q = 2m_R.T_R$

Given, module of R = m_R = 2 mm

$\Rightarrow m_Q = 2\ m_R$;

$$\frac{T_R}{T_Q} = 2\times2\times\frac{15}{40} = 1.5\text{ mm}$$

$m_P = m_Q = 2$ mm ;

$m_S = m_R = 1.5$ mm

$$\text{Radius} = \text{module}\times\frac{\text{No.of teeth}}{2}$$

Centre distance between P and S is given by

$R_P + R_Q + R_R + R_T$

$$= m_P\frac{T_P}{2} + m_Q\frac{T_Q}{2} + m_R\frac{T_R}{2} + m_S\frac{T_S}{2}$$

$$= 1.5\left[\frac{40+20}{2}\right] + 2\left[\frac{15+20}{2}\right]$$

$$= 45 + 35 = 80\text{ mm}$$

12. Ans: (a)

Sol: $$\frac{N_5}{N_2} = \frac{-T_2}{T_3}\times\frac{-T_4}{T_5} = \frac{20}{40}\times\frac{15}{30} = \frac{1}{4}$$

$$N_5 = \frac{N_2}{4} = \frac{1200}{4} = 300\text{ rpm in the same direction}$$

as that of gear 2 i.e, CCW

13. Ans: (a)

Sol: $T_{min} = \dfrac{2A_P}{\sqrt{1+G(G+2)\sin^2\phi}-1}$

Gear ratio $= \dfrac{1}{\text{Speed ratio}}$

Hence as speed ratio increases gear ratio G decreases from expression of T_{min}, as G decrease T_{min} increases.

14. Ans: (b)

Sol: Velocity at point of contact $= \omega_1 r_1 = \omega_2 r_2$ ....(1)

Considering pinion and gear as a system net force is zero on system so, from Newton's third law, $F_1 = F_2$

Rewrite equation (1)

$$r_1\dot{\theta}_1 = r_2\dot{\theta}_2 \qquad \text{........(2)}$$

Differentiating above equation with respect to Time 't'.

$$r_1\ddot{\theta}_1 = r_2\ddot{\theta}_2 \qquad \text{.........(3)}$$

Consider gear as a system

$$F_2 \times r_2 = I_2\ddot{\theta}_2$$

$$F_2 = \frac{I_2}{r_2} \times \frac{r_1\ddot{\theta}}{r_2} = I_2\frac{r_1}{r_2^2}\ddot{\theta}_1$$

Consider Pinion as a system

Net torque on pinion $(\tau_{ext}) = F_1 \times r_1 - \tau_1$

From Newton's second law

$$\tau_{ext} = -I_1\ddot{\theta}_1 \quad \text{(take a anticlockwise as positive)}$$

$$\tau_1 = -I_1\ddot{\theta}_1 + F_1 r_1$$

Put F_1 value in above equation $= I_1\ddot{\theta}_1 + I_2\left(\dfrac{r_1\ddot{\theta}_1}{r_2^2}\right) \times r_1$

$$\tau_1 = \left[I_1 + I_2\left(\frac{r_1}{r_2}\right)^2\right]\ddot{\theta}_1$$

15. Ans: (c)

Sol: Given

$T_2 = 60$, $\quad N_2 = 0$,

$T_3 = 20$, $\quad T_4 = 100$,

$N_4 = 100$ rpm (ccw +ve)

Relative velocity equation

$$\frac{N_4 - N_a}{N_2 - N_a} = -\frac{T_2}{T_4}$$

$$\Rightarrow \frac{100 - N_a}{0 - N_a} = \frac{-60}{100}$$

$$1.6\,N_a = 100$$

$$\Rightarrow N_a = \frac{100}{1.6} = 62.5 \text{ rpm (ccw)}$$

16. Ans: 3

Sol: $N_R = 0$

$N_S = 100$ rpm (ccw)

$\dfrac{N_P}{N_a} = ?$

$$\frac{N_S - N_a}{N_R - N_a} = -\frac{T_R}{T_S}$$

$$\frac{100 - N_a}{0 - N_a} = -\frac{100}{50}$$

$$N_a = +\frac{100}{3} \text{ rpm (ccw)}$$

$$\frac{N_S - N_a}{N_P - N_a} = -\frac{T_P}{T_S}$$

$$\frac{100 - N_a}{N_P - N_a} = -\frac{25}{50}$$

$\therefore N_P = 3\,N_a - 200$

$= 3 \times \dfrac{100}{3} - 200$

$= -100$ rpm

$= 100$ rpm (cw)

$$\left|\frac{N_P}{N_a}\right| = 3$$

17. Ans: 120

Sol: $N_P = 400$ rpm, $\quad N_s = ?$

$$\therefore \frac{N_P}{N_s} = \frac{T_s}{T_P}$$

$$\therefore \frac{400}{N_s} = \frac{80}{24}$$

$\therefore N_s = 120$ rpm

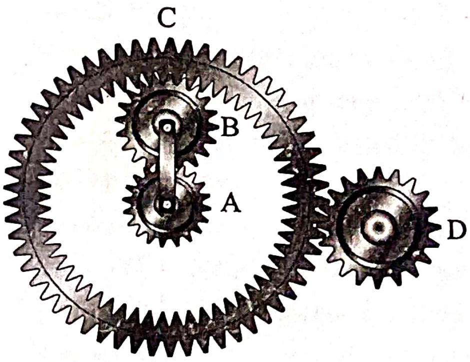
18. Ans: (c)

Sol:

Gear	No. of Teeth
A	20
B	30
C	80 (inner), 100 (outer)
D	20

$\omega_A = 300$ rpm (CW)

When carrier arm A and B fixed, the shown gear train becomes simple gear train.



$N_A = +300$ rpm(CW)

$N_D = ?$

∴ A Drives B

B Drives C

C Drives A.

$$\therefore \frac{N_A}{N_D} = +\frac{Z_B \times Z_C \times Z_D}{Z_A \times Z_B \times Z_C} = \frac{Z_D \times Z_C(\text{inner})}{Z_A \times Z_C(\text{outer})}$$

$$\therefore \frac{300}{N_D} = +\frac{Z_C \times Z_D}{Z_A \times Z_C} = +\frac{80 \times 20}{20 \times 100} = \frac{4}{5}$$

$\therefore N_D = +375$ rpm (CW)

19. Ans: (a)

Sol:

Condition	Arm	T_s (Sun)	T_P (Planet)	T_R (Ring)
let arm is fixed and sun gear is rotating with +x rpm	0	+x	$-\frac{T_S}{T_P}x$	$+\frac{T_P}{T_R}\left(-\frac{T_S}{T_R}\right)x$
Let arm is rotating with y rpm	y	y+x	$y - x\frac{T_S}{T_P}$	$y - x\frac{T_S}{T_P}$

$T_P = T_S$ (given)

And $R_R = R_S + 2R_P$ $\quad [\ m_R = m_S = m_P\ ]$

$\Rightarrow T_R = T_S + 2T_P$

$\Rightarrow T_R = 3T_P$

$\omega_s = y + X$

$$\omega_R = y - x\frac{T_s}{T_R}$$

$$= y - x\frac{T_S}{T_R} = y - \frac{x}{3}$$

$$\omega_R = y - \frac{x}{3} \Rightarrow y - \omega_R = \frac{x}{3}$$

$$\omega_S = y + 3y - 3\omega_R$$

$$= 4y - 3\omega_R$$

$$y = \frac{\omega_s}{4} + \frac{3\omega_R}{4}$$

20. Ans: (d)

Sol: Let, the CW direction of motion is taken as positive and counter clockwise as negative.

So, speed of pulley 2

$$\frac{N_2}{N_1} = \frac{D_1}{D_2}$$

$$\Rightarrow \frac{N_2}{2500} = \frac{150}{250} \Rightarrow N_2 = 1500 \text{ rpm (CCW)}$$

$$\frac{N_2}{N_7} = \frac{N_2}{N_3} \times \frac{N_3}{N_4} \times \frac{N_4}{N_5} \times \frac{N_5}{N_6} \times \frac{N_6}{N_7}$$

Here, $N_3 = N_4$, $N_6 = N_7$

$$\frac{N_2}{N_7} = \frac{Z_3}{Z_2} \times \frac{Z_5}{Z_4} \times \frac{Z_6}{Z_5}$$

$$\Rightarrow -\frac{1500}{N_7} = (-)\frac{44}{18} \times (-)\frac{33}{15} \times (-)\frac{36}{33}$$

$\Rightarrow \quad N_7 = 258.88$ rpm (CW)

21. Ans: 600

Sol: Speed of gear 1, $N_1 = 0$,

Teeth of gear 1, $z_1 = ?$

Speed of gear 3, $N_3 = 900$ rpm,

Teeth of gear 2, $z_2 = 20$

Speed of Arm, $N_{arm} = ?$,

Teeth of gear 3, $z_3 = 80$

Teeth of gear 2 $\Rightarrow z_3 = z_1 + 2z_2$

$80 = z_1 + 2(20)$

$z_1 = 40$

S. No	Condition of motion	Speed of arm	Speed of gear 1	Speed of gear 2	Speed of gear 3
1	Arm is fixed and gear 1 with +x rev	0	+1	$\frac{-z_1}{z_2}.(1)$	$\frac{-z_1}{z_3}.(1)$
2	Arm is fixed and gear 1 with +x rev	0	+x	$\frac{-z_1}{z_2}.(x)$	$\frac{-z_1}{z_3}.(x)$
3	Arm with +y rev	+y	y	+y	+y
4	Total	y	x+y	$y-x\left(\frac{z_1}{z_2}\right)$	$y-\frac{Z_1}{Z_3}(x)$

$N_1 = x + y = 0$

$\Rightarrow x = -y$

$N_3 = y - \frac{z_1}{z_3}(x) = 900$

$= y - \frac{40}{80}(x) = 900$

$\Rightarrow y - \frac{40}{80}(-y) = 900$

$1.5y = 900$

$y = \frac{900}{1.5} = 600 \text{ rpm}$

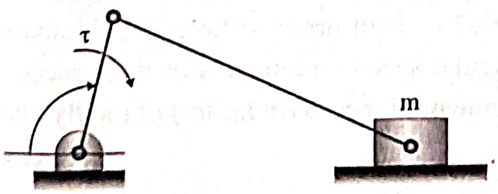
Chapter 4 Flywheels

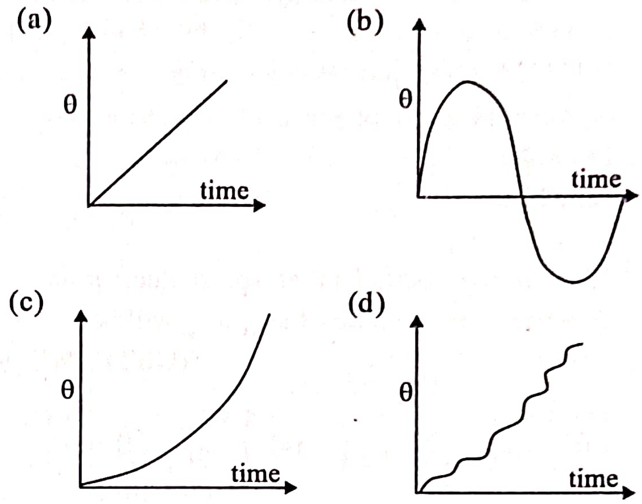
One Mark Questions

01. Which of the following statement is correct? **(GATE–ME–01)**
   (a) Flywheel reduces speed fluctuations during a cycle for a constant load, but flywheel does not control the mean speed of the engine if the load changes.
   (b) Flywheel does not reduce speed fluctuations during a cycle for a constant load, but flywheel does control the mean speed of the engine if the load changes
   (c) Governor controls speed fluctuations during a cycle for a constant load, but governor does not control the mean speed of the engine if the load changes.
   (d) Governor controls speed fluctuations during a cycle for a constant load, and governor also controls the mean speed of the engine if the load changes.

02. A circular solid disc of uniform thickness 20 mm, radius 200 mm and mass 20 kg, is used as a fly wheel. If it rotates at 600 rpm, the kinetic energy of the flywheel, in joules is **(GATE–ME-12)**
(a) 395 (b) 790 (c) 1580 (d) 3160

03. Consider a slider crank mechanism with nonzero masses and inertia. A constant torque τ is applied on the crank as shown in the figure. Which of the following plots best resembles variation of crank angle, θ versus time **(GATE –15 –Set 1)**





04. A flywheel in the form of a solid circular disc of radius 100 mm and uniform thickness has a mass of 10 kg. If it is rotating at a uniform angular velocity of 20 rad/s, the kinetic energy (in J) of the flywheel is ______ **(PI_GATE – 18)**

05. A flywheel is attached to an engine to keep its rotational speed between 100 rad/s and 110 rad/s. If the energy fluctuation in the flywheel between these two speeds is 1.05 kJ then the moment of inertia of the flywheel is ______ kg.m^2 (round off to 2 decimal places). **(GATE–20–SET–1)**

Two Marks Questions

01. A flywheel of moment of inertia 9.8 kg m^2 fluctuates by 30 rpm for a fluctuation in energy of 1936 Joules. The mean speed of the flywheel is (in rpm) **(GATE–ME-98)**
(a) 600 (b) 900 (c) 968 (d) 29470

02. For a certain engine having an average speed of 1200 rpm, a flywheel approximated as a solid disc, is required for keeping the fluctuation of speed within 2% about the average speed. The fluctuation of kinetic energy per cycle is found to be 2 kJ. What is the least possible mass of the flywheel if its diameter is not to exceed 1m? (GATE–ME-03)
(a) 40 kg (b) 51 kg
(c) 62 kg (d) 73 kg

03. If C_f is the coefficient of speed fluctuation of a flywheel then the ratio of $\omega_{max}/\omega_{min}$ will be (GATE–ME-06)

(a) $\frac{1-2C_f}{1+2C_f}$ (b) $\frac{2-C_f}{2+C_f}$ (c) $\frac{1+2C_f}{1-2C_f}$ (d) $\frac{2+C_f}{2-C_f}$

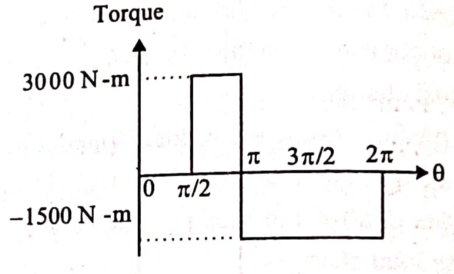
04. The speed of an engine varies from 210 rad/s to 190 rad/s. During cycle the change in kinetic energy is found to be 400 Nm. The inertia of the flywheel in kgm^2 is (GATE–ME-07)
(a) 0.10 (b) 0.20 (c) 0.30 (d) 0.40

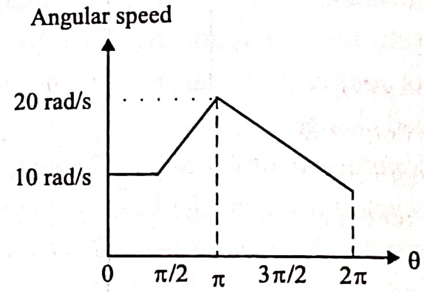
05. A flywheel connected to a punching machine has to supply energy of 400 Nm while running at a mean angular speed of 20 rad/s. If the total fluctuation of speed is not to exceed ± 2%, the mass moment of inertia of the flywheel in kg-m^2 is (GATE–ME-13)
(a) 25 (b) 50 (c) 100 (d) 125

06. Consider a flywheel whose mass M is distributed almost equally between a heavy, ring-like rim of radius R and a concentric disk-like feature of radius R/2. Other parts of the flywheel, such as spokes, etc, have negligible mass. The best approximation for α, if the moment of inertia of the flywheel about its axis of rotation is expressed as αMR^2, is ______ (GATE–ME-14)

07. Maximum fluctuation of kinetic energy in an engine has been calculated to be 2600 J. Assuming that the engine runs at an average speed of 200 rpm, the polar mass moment of inertia (in kg.m^2) of a flywheel to keep the speed fluctuation within ±0.5% of the average speed is ______ (GATE–ME-14)

08. Torque and angular speed data over one cycle for a shaft carrying a flywheel are shown in the figures. The moment of inertia (in kg.m^2) of the flywheel is ______ (GATE–ME-14)

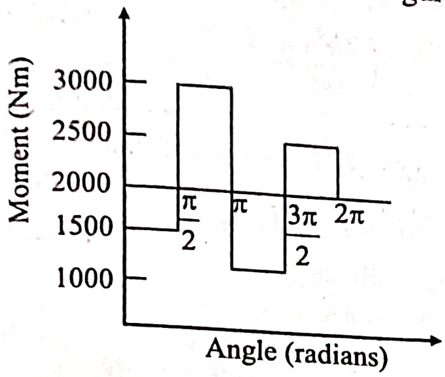




09. The torque (in N-m) exerted on the crank shaft of a two stroke engine can be described as T = 10000 + 1000sin2θ – 1200 cos2θ, where θ is the crank angle as measured from inner dead center position. Assuming the resisting torque to be constant, the power (in kW) developed by the engine at 100 rpm is ______ (GATE –15 –Set 3)

10. An engine, connected with a flywheel, is designed to operate at an average angular speed of 800 rpm. During operation of the engine, the maximum change in kinetic energy in a cycle is found to be 6240 J. In order to keep the fluctuation of the angular speed within ±1% of its average value, the moment of inertia (in kg-m^2) of the flywheel should be ______. (GATE - PI-16)

11. The turning moment diagram of a flywheel fitted to a fictitious engine is shown in the figure.



The mean turning moment is 2000 Nm. The average engine speed is 1000 rpm. For fluctuation in the speed to be within ±2% of the average speed, the mass moment of inertia of the flywheel is ______kg. m².

(GATE–20–SET–2)

12. A flywheel is to be used in an IC engine to limit fluctuation of angular speed. The average of the maximum and the minimum angular speed is 500 RPM, and the maximum fluctuation of energy is 10,000 N-m. Neglecting rotary inertia of any other components, the moment of inertia of the flywheel about its axis of rotation required to limit the maximum fluctuation of speed to 30 RPM (rounded off to one decimal place) in kg-m² is ______.

(GATE-PI-20)

13. The torque provided by an engine is given by $T(\theta) = 12000 + 2500 \sin(2\theta)$ N.m, where θ is the angle turned by the crank from inner dead center. The mean speed of the engine is 200 rpm and it drives a machine that provides a constant resisting torque. If variation of the speed from the mean speed is not to exceed ±0.5%, the minimum mass moment of inertia of the flywheel should be ______ kg.m². (round off to the nearest integer).

(GATE–21_SET–2)

KEY & Detailed Solutions

ONE MARK QUESTIONS				
01. (a)	02. (b)	03. (d)	04. 10	05. 1
TWO MARKS QUESTIONS				
01. (a)	02. (b)	03. (d)	04. (a)	05. (a)
06. (*)	07. 592.73 kgm²		08. 31.42	
09. 104.32		10. 44.45		
11. 3.582		12. 60.7		13. 570

One Mark Solutions

01. Ans: (a)

Sol: Flywheel reduces speed fluctuations during a cycle for constant load, but flywheel does not control the mean speed of the engine if the load changes.

02. Ans: (b)

Sol: $E = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{mr^2}{2}\right)(\omega)^2$

$$= \frac{1}{2}\left(\frac{1}{2}\times 20 \times 0.2^2\right)\left(\frac{2\times\pi\times 600}{60}\right)^2$$

$= 790$ N-m

03. Ans: (d)

Sol: $I\frac{d^2\theta}{dt^2} = T + f(\sin\theta, \cos\theta)$

Where 'T' is applied torque, f is inertia torque which is function of sinθ & cosθ

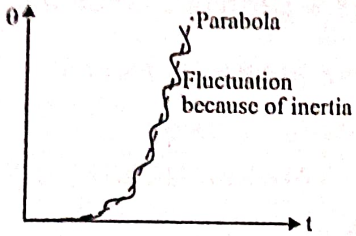
$$\frac{d\theta}{dt} = \frac{T}{I}t + f'(\sin\theta, \cos\theta) + c_1$$

$$\theta = \frac{T}{I}t^2 + c_1 t + f''(\sin\theta, \cos\theta)$$

θ is fluctuating on parabola

and at $t = 0,\ \theta = 0,$

$\dot{\theta}(\text{slope}) = 0$ (because it starts from rest)



04. Ans: 10

Sol: Given flywheel is a solid circular disc.

R = 100 mm, m = 10 kg, ω = 20 r/s

$$KE = \frac{1}{2}I\omega^2 \quad \text{But } I = \frac{1}{2}mR^2$$

$$\therefore\ KE = \frac{1}{2}\times\frac{1}{2}\times 10\times 0.1^2\times(20)^2$$

$$= 10\text{ N-m or } 10\text{ J}$$

KE = 10 J

05. Ans: 1

Sol: $(\Delta K.E)_{max} = \frac{1}{2}I(\omega_{max}^2 - \omega_{min}^2)$

$$1.05\times10^3 = \frac{1}{2}I(110^2 - 100^2)$$

$$I = \frac{2\times1.05\times10^3}{110^2-100^2} = 1\text{ kg-m}^2$$

Two Marks Solutions

01. Ans: (a)

Sol: Max fluctuation of K.E

ΔE = max K.E – min K.E

$$= \frac{1}{2}I(\omega_1)^2 - \frac{1}{2}I(\omega_2)^2 = \frac{1}{2}I(\omega_1^2-\omega_2^2)$$

$$= \frac{1}{2}I(\omega_1+\omega_2)(\omega_1-\omega_2) = I\omega(\omega_1-\omega_2)$$

$$\therefore\ \left(\frac{\omega_1+\omega_2}{2}\right) = \omega$$

(ω is mean angular velocity)

$$= I\omega^2\frac{(\omega_1-\omega_2)}{\omega} = I\omega^2 C_S$$

C_S = coefficient of fluctuation of speed

$\Delta E = I.\omega.(\omega_1 - \omega_2)$

$$= I\frac{2\pi N}{60}\left(\frac{2\pi N_1}{60} - \frac{2\pi N_2}{60}\right)$$

$$= \frac{4\pi^2}{3600}\times I\times N\times(N_1-N_2)$$

Given, ΔE = 1936 N-m

$(N_1 - N_2) = 30$ rpm

I = 9.8 Kg-m²

$$\therefore\ 1936 = \frac{4\times\pi^2}{3600}\times 9.8\times N\times 30$$

$\therefore\ N \simeq 600$ rpm

02. Ans: (b)

Sol: $C_S = 0.02$

$\Delta E = I\omega^2 C_S$

$$\omega = \frac{2\pi\times1200}{60} = 40\pi = 125.6\text{ rad/sec}$$

$$I = \frac{1}{2}mr^2 = \frac{1}{2}\times m\times 0.5^2 = 0.125\text{ m}$$

$\Rightarrow 2000 = 0.125\text{ m}\times(125.6)^2\times 0.02$

$\Rightarrow m = 51$ kg.

03. Ans: (d)

Sol: $C_f = \frac{(\omega_{max}-\omega_{min})}{\omega}$; $\omega = \frac{\omega_{max}+\omega_{min}}{2}$

$$C_f = \frac{2(\omega_{max}-\omega_{min})}{\omega_{max}+\omega_{min}};$$

$$\therefore\ \frac{2+C_f}{2-C_f} = \frac{\omega_{max}}{\omega_{min}}$$

04. Ans: (a)

Sol: $\Delta E = \frac{1}{2}I\omega_1^2 - \frac{1}{2}I\omega_2^2$

$$\Delta E = \frac{1}{2}I(\omega_1^2-\omega_2^2)$$

ΔE = 400 Nm, $\omega_1 = 210$ rad/sec,

$\omega_2 = 190$ rad/sec $\quad I = 0.1$ kgm²

05. Ans: (a)

Sol: Given $\Delta E = 400$ N-m;

$\omega = 20$ rad/sec;

$C_s = 0.04$

We know $\Delta E = I\omega^2 C_s$

$$I = \frac{\Delta E}{\omega^2 C_s} = 25 \text{ kg-m}^2$$

06.

Sol: The flywheel is considered as two parts $\frac{M}{2}$ as rim type with Radius R and $\frac{M}{2}$ as disk type with Radius $\frac{R}{2}$

$$I_{Rim} = \frac{M}{2}R^2,$$

$$I_{disk} = \frac{1}{2} \times \frac{M}{2} \times \left(\frac{R}{2}\right)^2 = \frac{MR^2}{16}$$

$$I = \frac{MR^2}{2} + \frac{MR^2}{16} = \frac{9}{16}MR^2 = 0.5625\ MR^2$$

$\therefore \alpha = 0.5625$

07. Ans: 592.73 kgm²

Sol: $\Delta E = 2600$ J, $N = 200$ rpm

$$\Rightarrow \omega = \frac{400 \times \pi}{60} \text{ rad/sec}$$

$C_s = 0.01 \quad (\pm 0.5\% = 1\%)$

$\Delta E = I\omega^2 C_s$

$$\Rightarrow I = \frac{\Delta E}{\omega^2 C_s} = \frac{2600 \times 60^2}{(400 \times \pi)^2 \times 0.01} = 592.73 \text{ kgm}^2$$

08. Ans: 31.42

Sol: From the T - θ diagram energy is Stored into flywheel during $\frac{\pi}{2}$ to π

$$\Delta E = \frac{\pi}{2} \times 3000 \text{ N-m} \quad [\because \text{ mean torque is zero}]$$

From ω - θ diagram $\omega_{max} = 20$ rad/sec

$\omega_{min} = 10$ rad/sec

$$\Delta E = \frac{1}{2}I(\omega_{max}^2 - \omega_{min}^2) \Rightarrow I = \frac{2\Delta E}{\omega_{max}^2 - \omega_{min}^2}$$

$$I = \frac{2 \times \frac{\pi}{2} \times 3000}{20^2 - 10^2} = 31.42 \text{ kg-m}^2$$

09. Ans: 104.32

Sol: N = 100 rpm

$$T_{mean} = \frac{1}{\pi}\int_0^{\pi} T d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi}(10000 + 1000\sin 2\theta - 1200\cos 2\theta)d\theta$$

$$= \frac{1}{\pi}[10000\theta - 500\cos 2\theta - 600\sin 2\theta]_0^{\pi}$$

$= 10000$ Nm

$$\text{Power} = \frac{2\pi NT}{60}$$

$$= \frac{2 \times \pi \times 100 \times 10000}{60} = 104719.75 \text{ W}$$

$P = 104.719$ kW

10. Ans: (44.45)

Sol: Given, mean speed N = 800 rpm

$$\omega = \frac{2\pi N}{60} \text{ rad/sec}$$

Fluctuation of energy, $\Delta E = 6240$ J

Fluctuation of speed = ± 1%

Moment of inertia, I = ?

We know

$\Delta E = I\,\omega^2 C_s$

$$I = \frac{\Delta E}{\omega^2 C_s} = \frac{6240}{\left(\frac{2\pi N}{60}\right)^2 \times 0.02} = 44.45 \text{ kg-m}^2$$

11. Ans: 3.582

Sol: Given:

Mean torque, $T_m = 2000$ N-m

N = 1000 rpm

Maximum fluctuation in speed = ± 2 %

I = ?

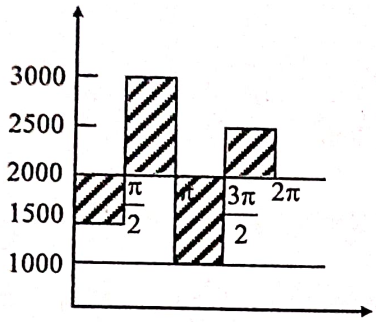
$\therefore C_s = \pm 2\,\% = 4\,\% = 0.04$

$N = 1000$ rpm

$\therefore\ \omega = \dfrac{2\pi(1000)}{60} = 104.7\text{rad/s}$

$T_m = 2000$ N-m

Turning moment diagram



The Hatched Area show the excess/ defect area above and below mean torque line.

Convert the diagram will give

$A_1 = \dfrac{\pi}{2} \times 500 = 250\,\pi$

$A_2 = 1000 \times \dfrac{\pi}{2} = 500\pi$

$A_3 = 1000 \times \dfrac{\pi}{2} = 500\pi$

$A_4 = 500 \times \dfrac{\pi}{2} = 250\pi$

Assuming energy at point ABE

$\therefore\ E_A = E$

$E_B = E - 250\,\pi$

$E_C = E - 250\,\pi + 500\pi = E + 250\pi$

$E_D = E + 250\,\pi + 250\pi = E$

$E_{max} = E + 250\,\pi$

$E_{max} = E - 250\,\pi$

$\therefore\ \Delta E = E + 250\,\pi - (E - 250\pi)$

$\Delta E = 500\,\pi$ N–m

$\Delta E = I\,\omega^2 C_s$

$500\pi = I \times (104.7)^2 \times 0.04$

$\therefore\ I = 3.582$ kg-m²

12. Ans: 60.7

Sol: Given,

Average speed = 500 rpm = N

Maximum fluctuation of energy = 10000 N-m = ΔE

Maximum fluctuation of speed = 30 rpm = ΔN

Mass moment of inertia = I = ?

Flywheel governing equation is

$\Delta E = I\omega^2 C_s = I\omega\,\Delta\omega \Rightarrow \Delta E = 10000$ N-m

$N = 500$ rpm

$\therefore \omega = \dfrac{2\pi(500)}{60} = 52.35$ rad/s

$\therefore\ \Delta N = 30$ rpm

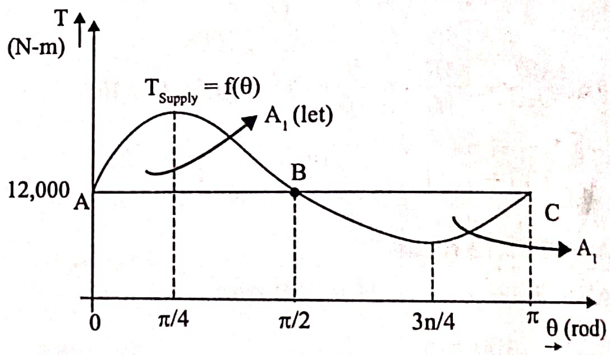
$\Delta\omega = \dfrac{2\pi(30)}{60} = \pi$ rad/s

$\therefore 1000 = I \times 52.35 \times \pi$

$\therefore\ I = \dfrac{10000}{52.35 \times \pi} = 60.69\ \text{kg.m}^2$

13. Ans: (570)

Sol:



$T(\theta) = 12000 + 2500 \sin 2\theta$ N-m

$N_m = 200$ rpm

$C_s = \pm 0.5$ %

$I = ?$

$C_s = 1\ \% = 0.01$

$T_{resisting}$ = constant (given)

Therefore, $T_{resisting} = T_{mean} = 12000$ N-m

Let,

Energy of flywheel at point A = E_A

Energy of flywheel at point B = E_B

$= E_A + A_1 = E_{max}$

Energy of flywheel at point C = E_C

$= E_A + A_1 - A_1$

$= E_A = E_{min}$

$$(\Delta E)_{max} = E_{max} - E_{min}$$
$$= E_A - E_C$$
$$= A_1$$

$$A_1 = \int_{\theta_A}^{\theta_B} (T_{supply} - T_{mean})\, d\theta$$

$$(\Delta E)_{max} = \int_0^{\pi/2} \{(12000 + 2500 \sin 2\theta) - 12000\}\, d\theta$$

$$= \int_0^{\pi/2} 2500 \sin 2\theta.\, d\theta$$

$$= 2500 \int_0^{\pi/2} \sin 2\theta.\, d\theta$$

$$= 2500\left[\frac{-\cos 2\theta}{2}\right]_0^{\pi/2}$$

$$= 1250[-\cos\pi + \cos 0]$$

$$= 1250\,[-(-1) + 1] = 2500\ \text{J}$$

$$(\Delta E)_{max} = I\,\omega_m^2\, C_s$$

$$\Rightarrow 2500 = I \times \left(\frac{2\pi \times 200}{60}\right)^2 \times 0.01$$

$$I = 569.93\ \text{kg-m}^2$$