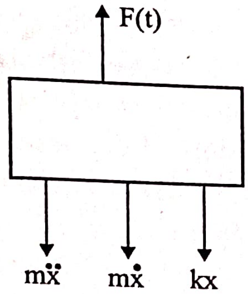
$\omega_d > \omega_n$

$$\frac{(1)}{(2)} = \frac{\omega_p}{\omega_d} = \frac{\sqrt{1-2\zeta^2}}{\sqrt{1-\zeta^2}} < 1$$

$\Rightarrow \omega_p < \omega_d < \omega_n$

50. Ans: (c)

Sol: FBD of mass



$m\ddot{x} + c\dot{x} + kx = F(t)$

Solution of differential equation

x(t) = (C.F) + (P.I)

Considering condition (P)

If $c > 0$ and $\omega = \sqrt{k/m}$

For this condition the displacement (x) is given by

[Assume the system to be under damped]

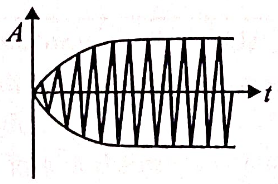
$x(t) = X_0 e^{-\zeta\omega_n t}\cos(\omega_d t - \phi) + X\cos(\omega t - \phi)$

Transient response + steady state response

As $t \to \infty$ the transient response decays to zero and only steady state response will remain and the amplitude will be maximum because of resonance.

$x(t) = X\cos(\omega t - \phi)$

For this condition the response curve will be



Considering condition (Q)

$c < 0$ and $\omega \neq 0$

The differential equation becomes

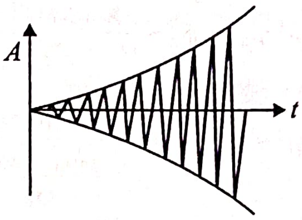
$m\ddot{x} - c\dot{x} + kx = F(t)$

Solution of above differential equation is

$x(t) = X_0 e^{+ct}\cos(\omega_d t - \phi) + X\cos(\omega t - \phi)$

As $t \to \infty$ the transient response approaches to ∞ and increases exponentially

The plot will be



Considering condition (R)

$c = 0, \quad \omega = \sqrt{\frac{k}{m}}$ (Resonance)

If $c = 0, \quad \xi = 0$

$$x(t) = X\sin(\omega_n t + \phi) + \frac{x_{static}\cos(\omega t + \phi)}{\left[\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2\right]^{0.5}}$$

If $\omega = \omega_n$ this term will $\frac{0}{0}$.

Applying L Hospital rule,

$$x(t) = X\sin(\omega_n t + \phi) + \frac{x_{static}\frac{d}{d\omega}(\cos(\omega t + \phi))}{\frac{d}{d\omega}\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)}$$

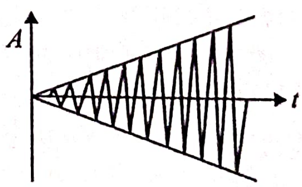
$$= X\sin(\omega_n t + \phi) + \frac{x_{static}(-\sin(\omega t + \phi))\,t}{\left(0-\left(\frac{2\omega}{\omega_n^2}\right)\right)}$$

$$= X\sin(\omega_n t + \phi) + \frac{x_{static}\,t(\sin(\omega t + \phi))}{\left(2\times\frac{\omega_n}{\omega_n^2}\right)}$$

$$= X\sin(\omega_n t + \phi) + \frac{x_{static}\,\omega_n t\ \sin(\omega t + \phi)}{2}$$

Response will be linearly increasing with t.

So the correct plot will be



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Considering condition (S)

$$c = 0,\ \ \omega \cong \sqrt{\frac{k}{m}},\ \Rightarrow c = 0,\ \xi = 0$$

$$x(t) = X\sin(\omega_n t + \phi) + \frac{x_{static}\sin(\omega t - \phi)}{\left[(1-r^2)^2 + (2\xi r)^2\right]^{0.5}}$$

$$x(t) = X\sin(\omega_n t + \phi) + \frac{x_{static}\sin(\omega t - \phi)}{\frac{(\omega_n^2 - \omega^2)}{\omega_n^2}}$$

$$x(t) = X\sin(\omega_n t + \phi) + \frac{x_{static}\,\omega_n^2\sin(\omega t - \phi)}{\omega_n^2 - \omega^2}$$

$$x(t) = X\sin(\omega_n t + \phi) + \frac{F\sin(\omega t - \phi)}{m\,(\omega_n^2 - \omega^2)}$$

$$x(t) = X\sin(\omega_n t + \phi) - \frac{F}{m}\frac{\sin(\omega t - \phi)}{(\omega^2 - \omega_n^2)}$$

If the force frequency is close to, but not exactly equal to, natural frequency of the system, a phenomenon is known as beating. In this kind of vibration the amplitude builds up and then diminishes in a regular pattern.

The correct plot will be