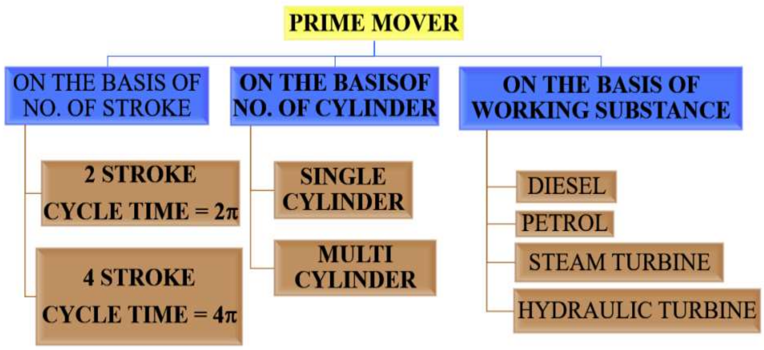
FLYWHEEL IN CASE OF PRIME MOVER:



NOTE: There is minimum fluctuation in hydraulic turbine therefore they do not required flywheel.
NOTE: In multicylinder engine the turning moment diagram is more uniform than single cylinder engine therefore multicylinder engines requires lesser inertia flywheel.

T_{supply} , T_{load}

Type. Pr.- 1. $T_{supply} = f(\theta)$ – (variable)
$T_{load} = $ constant.

I.C Engine.

Type. Pr. 2. $T_{supply} = $ constant.
$T_{load} = f(\theta)$ (variable)

Punching Machine.

Type Pr – 3. $\left.\begin{array}{l} T_{supply} = f(\theta) \\ T_{load} = f(\theta) \end{array}\right\}$ subjective.

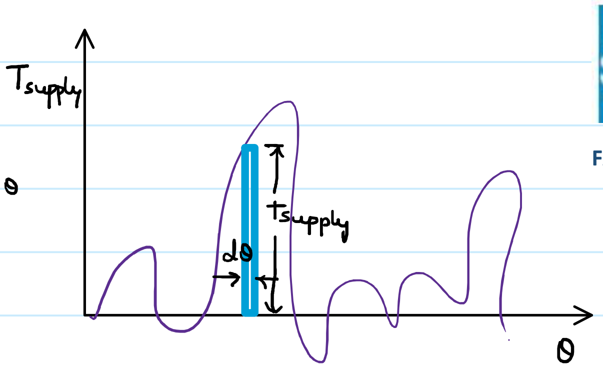
RESISTING TORQUE OR LOAD TORQUE:
Every prime mover will drive some machine the torque exert by machine on the prime mover is known as "Resisting Torque".

- The net area of T_{Load} v/s θ diagram represent the energy required per cycle.
- If $T_{Load} = f(\theta)$ then,
  The net energy required per cycle = net area of T_{Load} v/s θ diagram.
- If T_{Load} = Constant then,
  Net energy required per cycle = T_{load} x Cycle time

$T_{supply} = f(\theta)$.

Energy supplied.

$$E_{supplied} = \int_0^{cycle\ time} T_{supply} \cdot d\theta$$



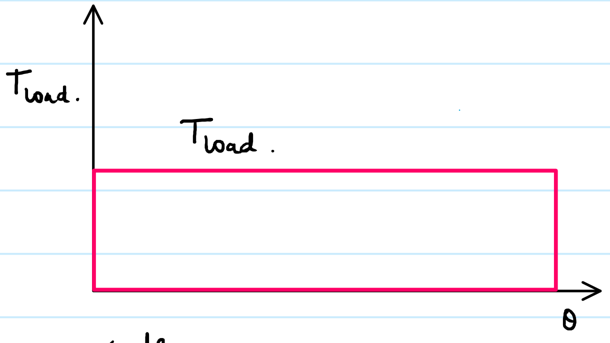
$$E_{supply} = T_{supply} \cdot \int_0^{cycle} d\theta$$

T_{supply} = constant

$T_{load} = const.$

Energy required.

$$= \int_0^{cycle\ time} T_{load} \cdot d\theta.$$



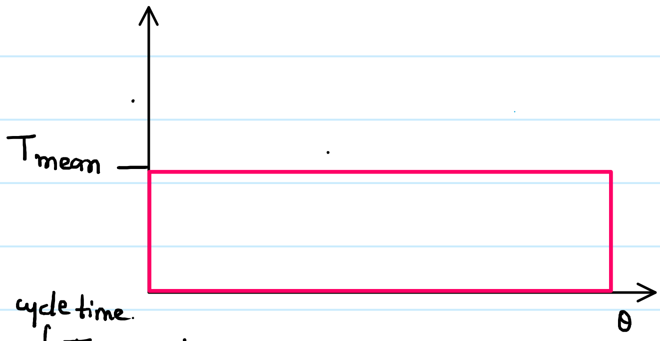
$$E_{required} / E_{load} = T_{load} \cdot \int_0^{cycle} d\theta. \quad \rightsquigarrow T_{load} = constant.$$

$$= \int_0^{cycle\ time} T_{load} \cdot d\theta. \quad \rightsquigarrow T_{load} = f(\theta)$$

AVERAGE OR MEAN TORQUE:

- It is an imaginary constant torque which when act on the crank shaft will be able to develop same amount of energy that is being developed by actual supply torque.

Net Energy Developed/cycle = Net Area of T_{mean} v/s θ diagram in one cycle



$$E_{supply} = \int_0^{cycle\ time} T_{mean} \cdot d\theta.$$

$$E_{supply} = T_{mean} \cdot \int_0^{cycle\ time} d\theta = \int_0^{cycle\ time} T_{supply} \cdot d\theta.$$

$$T_{mean} = \frac{1}{cycle\ time} \cdot \int_0^{cycle\ time} T_{supply} \cdot d\theta.$$

Conservation of Energy.

$$E_{supply} = E_{Load}$$

$$\int_0^{cycle\ time} T_{supply}\, d\theta = \int_0^{cycle\ time} T_{load}\, d\theta$$

T_{load} = constant.

$T_{supply} = f(\theta)$.

$$T_{mean} \cdot \int_0^{cycle\ time} d\theta = T_{load} \cdot \int_0^{cycle\ time} d\theta$$

$$T_{mean} = T_{load}$$

Fluctuation Torque

$$T_{fluc.} = T_{supply} - T_{load} > 0$$

$$T_{fluct.} = T_{load} - T_{supply}$$

$$T_{fly.} / T_{fluc.} = T_{supply} - T_{load} > 0$$

$$I_{fly} \cdot \alpha_{fly} = T_{supply} - T_{load} > 0 \longrightarrow T_{supply} > T_{load}$$

$\alpha_{fly} > 0$ (+ve value)

$$T_{fly.} / T_{fluc.} = T_{supply} - T_{load} < 0 \longrightarrow T_{supply} < T_{load}$$

$\alpha_{fly} < 0$ (−ve value)

$$T_{fly} = T_{supply} - T_{load} = 0 \longrightarrow T_{supply} = T_{load}$$

$\alpha_{fly} = 0$. ω_{fly} = constant.

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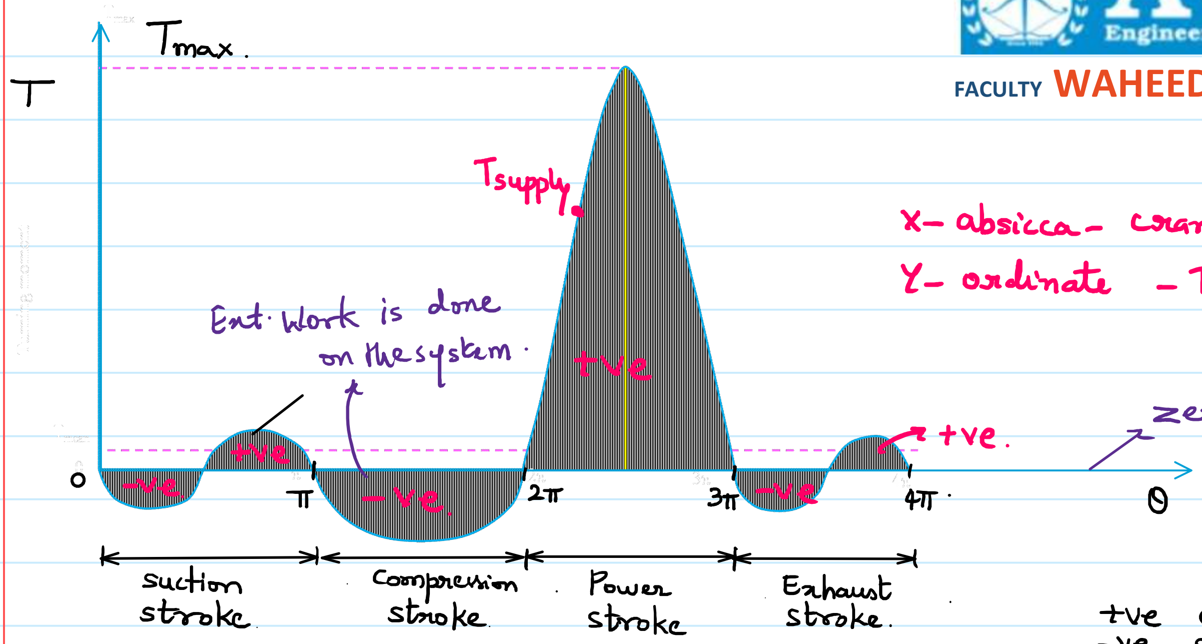
T - θ Diagram for 4 -stroke single cylinder single acting Engine



x – absicca – crank angle.
y – ordinate – T_{supply}.

+ve 0 to 90°
-ve 90 to 180

Suction stroke | Exhaust stroke. $F_P = F_G \pm F_I$. $F_I = mr\omega^2 \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$

In the suction stroke inertia force opposes the motion for some part of stroke and inertia force supports the motion for some part of stroke.

In the Exhaust-stroke inertia force opposes the motion for some part of stroke and inertia force supports the motion for some part of stroke.

Compression stroke – $F_P = F_G \pm F_I$

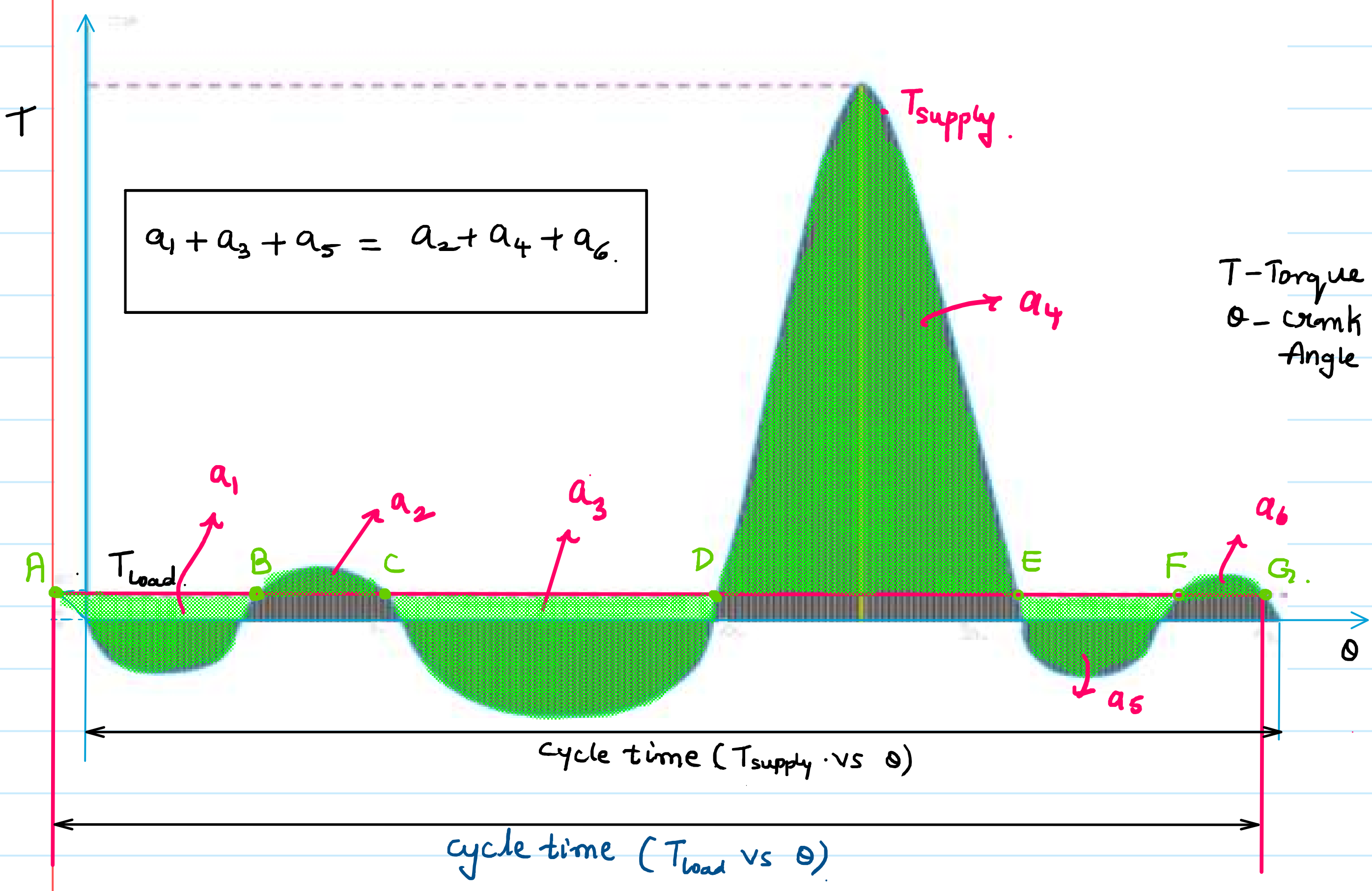
= External work is done on air/air-fuel. mixture.

Expansion stroke. $F_p = F_G \pm F_I$.

$F_G >>> F_I$.

$F_p \approx F_G$.

Work is done by the system. (+ve W.D)
Torque reaches the peak value in the Power stroke.



@ points A, B, C, ... G. $T_{supply} = T_{load} \Rightarrow T_{fluct.} / T_{fly.} = 0$

$$\alpha_{fly} = 0$$

Flywheel is isolated from source & Load.

point A, B, C, ... G are called isolation points

Energy @ A $= E = \frac{1}{2} \cdot I \cdot \omega_A^2$

Energy @ B $= E - a_1 = \frac{1}{2} \cdot I \cdot \omega_B^2$

Energy @ C $= E - a_1 + a_2 = \frac{1}{2} \cdot I \cdot \omega_C^2$

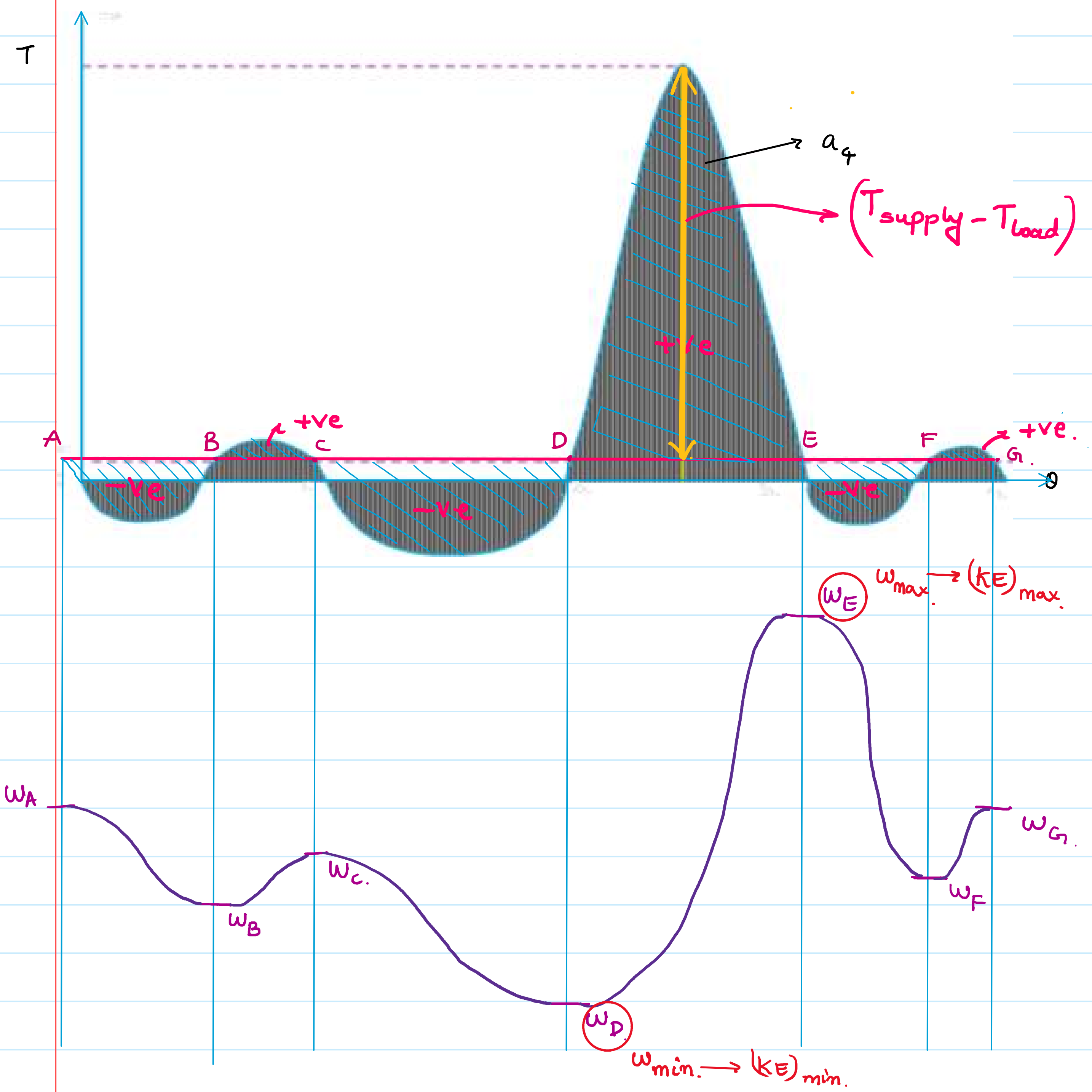
Energy @ D $= E - a_1 + a_2 - a_3 = \frac{1}{2} \cdot I \cdot \omega_D^2$ → KE_{min}.

Energy @ E $= E - a_1 + a_2 - a_3 + a_4 = \frac{1}{2} \cdot I \cdot \omega_E^2$ → KE_{max}

Energy @ F $= E - a_1 + a_2 - a_3 + a_4 - a_5 = \frac{1}{2} \cdot I \cdot \omega_F^2$

Energy @ G $= E - a_1 + a_2 - a_3 + a_4 - a_5 + a_6 = E = \frac{1}{2} \cdot I \cdot \omega_G^2$

T vs θ, ω vs θ.



Max. fluctuation in Energy / Kinetic Energy.

$$(\Delta KE)_{max} = KE_{max.} - KE_{min.}$$

$$(\Delta KE)_{max} = \frac{1}{2} \cdot I \cdot \omega_{max}^2 - \frac{1}{2} \cdot I \omega_{min}^2 = \frac{I}{2} \cdot (\omega_{max}^2 - \omega_{min}^2)$$

$$(\Delta KE)_{max} = I \cdot \frac{(\omega_{max} + \omega_{min})}{2} (\omega_{max} - \omega_{min})$$

$$(\Delta KE)_{max} = I.\omega_{avg}^2 . \frac{\Delta\omega}{\omega_{avg}} = I.\omega_{avg}^2 . C_s.$$

Max. fluctuation in KE.

$$(\Delta KE)_{max} = KE_{max} - KE_{min}.$$
$$= (E - a_1 + a_2 - a_3 + a_4) - (E - a_1 + a_2 - a_3)$$
$$= a_4.$$
$$= \int_{\theta_D}^{\theta_E} (T_{supply} - T_{load})\, d\theta$$

@ θ_D – $\omega = Min$, @ θ_E – $\omega = Max.$

Coefficient of fluctuation in speed.

$$C_s = \frac{\text{Range of speed}}{\text{Averagespeed / Mean speed}}.$$

Coefficient of fluctuation in Energy.

$$C_E = \frac{\text{Max. fluctuation in Energy}}{\text{Energy supplied for one cycle}}.$$

$$C_E = \frac{(\Delta KE)_{max}}{E_{supplied}} = \frac{I.\omega_{avg}^2 C_s}{\int_0^{\text{cycle time}} T_{supply}.d\theta} = \frac{\int_{\theta_1}^{\theta_2} (T_s - T_L).d\theta}{\int_0^{\text{cycle time}} T_s.d\theta}.$$

T_s – Supply Torque T_L = Load torque.

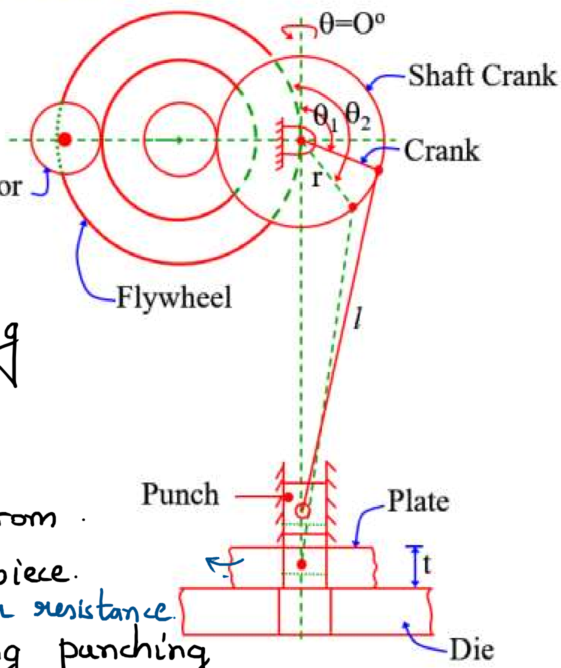
Stresses in flywheel.

$$\sigma = \rho V^2$$

σ – stress in flywheel.
ρ – density of flywheel material
V – peripheral velocity

FLYWHEEL IN PUNCHING PRESS:

ASSUMPTION: Tool dimension are negligible.



$\theta = 0^\circ$ Tool is at top dead centre.

$\theta = \theta_1$ Tool comes in contact with work piece.

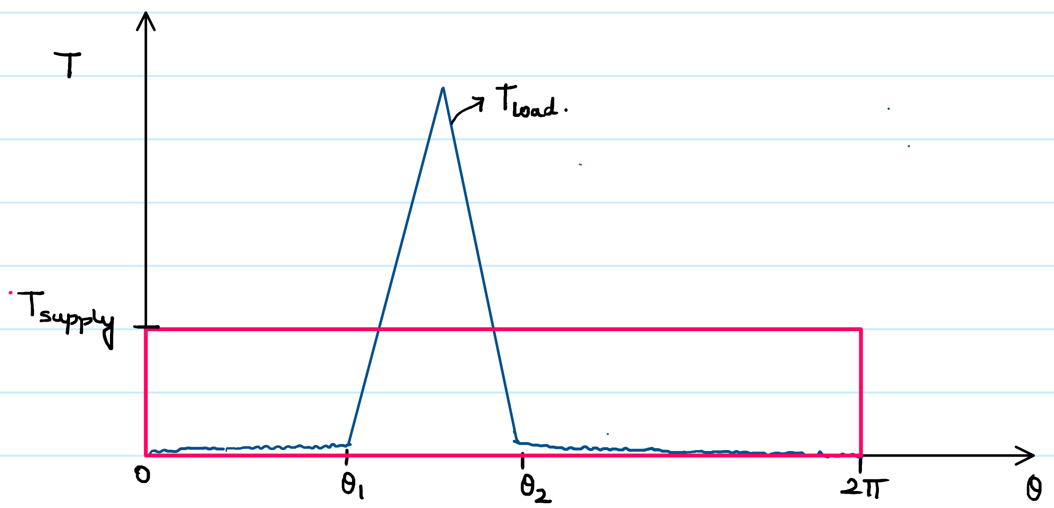
$\theta = \theta_2$ Tool completes the punching operation.

$0 < \theta < \theta_1 \rightarrow$ Tool is moving from T.D.C to workpiece.
Tool is overcoming air resistance.

$\theta_1 < \theta < \theta_2 \rightarrow$ Tool is performing punching operation.
Tool is overcoming material resistance.

$\theta_2 < \theta < 2\pi \rightarrow$ Tool moves back to the top dead centre.
Tool is overcoming air resistance.

T_{supply} = constant.
$T_{load} = f(\theta)$



T - θ Diagram for Punching Press

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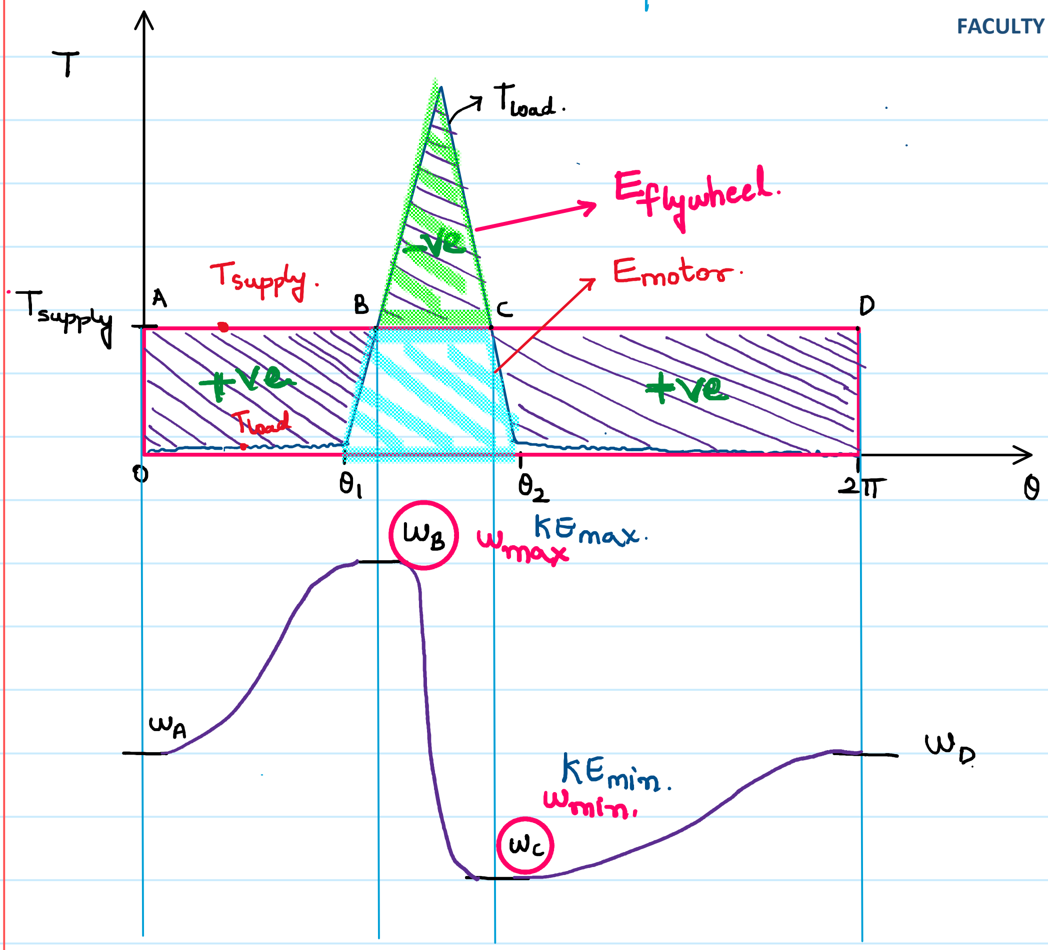
T vs θ , ω vs θ Graph.



@ points B, C
$T_{load} = T_{supply}$.
B, C are called isolation points.

Energy at A $= E = \frac{1}{2} \cdot I \cdot \omega_A^2$

Energy at B $= E + a_1 = \frac{1}{2} I \omega_B^2 \rightsquigarrow KE_{max}$

Energy at C $= E + a_1 - a_2 = \frac{1}{2} \cdot I \omega_C^2 \rightsquigarrow KE_{min}$.

Energy at D $= E + a_1 - a_2 + a_3 = E = \frac{1}{2} \cdot I \omega_D^2$

Max. fluctuation in Energy $= (KE)_{max} - (KE)_{min}$.

$= (E + a_1) - (E + a_1 - a_2)$

Energy supplied by flywheel $= (\Delta KE)_{max.} = a_2$

Energy for one cycle = Energy for cutting stroke. + Energy for idle stroke.

$E_{cycle} = E_{cutting} + E_{idle}$

0

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$$E_{cycle} = E_{cutting}$$

$$E_{cutting} = E_{motor} + E_{flywheel}.$$

$$E_{cycle} = E_{cycle}\cdot\left(\frac{\theta_2-\theta_1}{2\pi}\right) + (\Delta KE)_{max.}$$

$E_{motor} \longrightarrow (\theta_2-\theta_1)$

$E_{cycle} \longrightarrow 2\pi$

$$E_{motor} = E_{cycle}\cdot\frac{(\theta_2-\theta_1)}{2\pi}$$

$$(\Delta KE)_{max} = E_{cycle}\left[1 - \frac{(\theta_2-\theta_1)}{2\pi}\right]$$

$$(\Delta KE)_{max} = E_{cycle}\left[1 - \frac{t_{cutting}}{t_{cycle}}\right].$$

$$(\Delta KE)_{max} = E_{cycle}\left[1 - \frac{\text{thickness of plate}}{2\times\text{stroke lenght}}\right]$$

01. A four-stroke single cylinder I.C engine develops 80 kW at 300 rpm. The fluctuation of energy can be assumed to be 0.9 times the energy developed per cycle. If the coefficient of fluctuation of speed is not to exceed 2 percent and the maximum centrifugal stress in the rim of the flywheel is limited to 6MN/m^2, estimates the mean diameter of the rim and the moment of inertia of the flywheel. The density of flywheel material is 7500 kg/m^3.

$P = 80\,kW$ @ $300\,rpm$.

$C_E = 0.9,\quad C_s = 2\%$

$\sigma = 6\,MPa = 6\times10^6\,N/m^2$

$D = ?\quad I = ?\quad \rho = 7500\,kg/m^3$.

stresses in flywheel. $\sigma = \rho v^2$

$$\sigma = \rho\cdot\left(\frac{\pi DN}{60}\right)^2$$

$$6\times10^6 = 7500\left(\frac{\pi D\times300}{60}\right)^2$$

$$P = \frac{2\pi\cdot T_{mean}\cdot N_{mean}}{60{,}000}\ kW.$$

$$T_{mean} = \frac{60{,}000\times80}{2\pi\times300}\ N\text{-}m. \Rightarrow T_{mean} = 2546.47\,N\text{-}m,\quad D = 1.8\,mts.$$

$$E_{supply} = T_{mean}\times\text{cycle time} = 2546.47\times4\pi \cong 32\,kN\text{-}m.$$

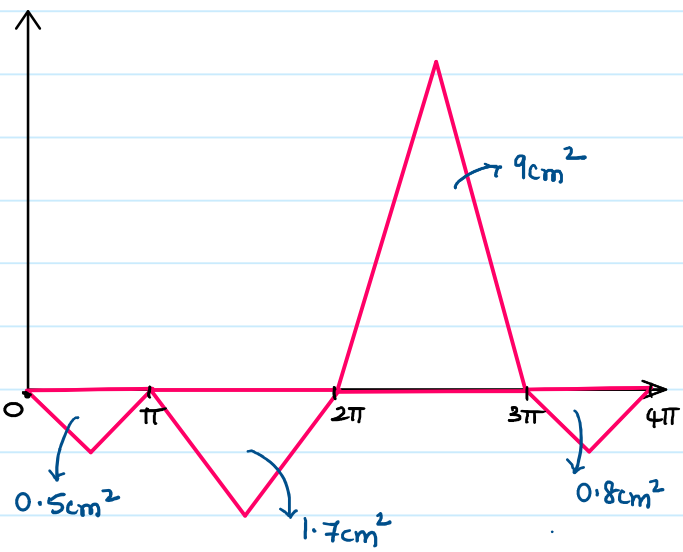
$$(\Delta KE)_{max} = C_E\cdot E_{supply} = 0.9\times32 = 28.8\,kN\text{-}m.$$

$$(\Delta KE)_{max} = I\omega_{avg}^2\cdot C_s \Rightarrow 28.8\times10^3 = I\cdot\left(\frac{2\pi\times300}{60}\right)^2\times0.02$$

$$I = 1459.02\,kg\text{-}m^2$$

02. The TMD for a four stroke engine may be assumed for the sake of simplicity to be represented by four triangles in each stroke. The area of these triangles are expansion stroke 9cm², exhaust stroke 0.8cm² (–ve), suction stroke 0.5cm²(–ve) and compression stroke 1.7cm²(–ve). Where 1cm² represents 1400 J of work. Assuming constant resistance determine the MI of the flywheel to keep the speed between 98 rpm and 102 rpm.

1cm – 1400J.



Sol $E_{supplied} = \int_0^{cycle\ time} T_s \cdot d\theta = \sum_{i=1}^{n} a_i$

$E_{supplied} = -a_1 - a_2 + a_3 - a_4$

$= -0.5 - 1.7 + 9 - 0.8$

$= 6cm^2.$

$= 6 \times 1400 = 8400 J.$

$T_{load} \times \text{cycle time} = 8400.$

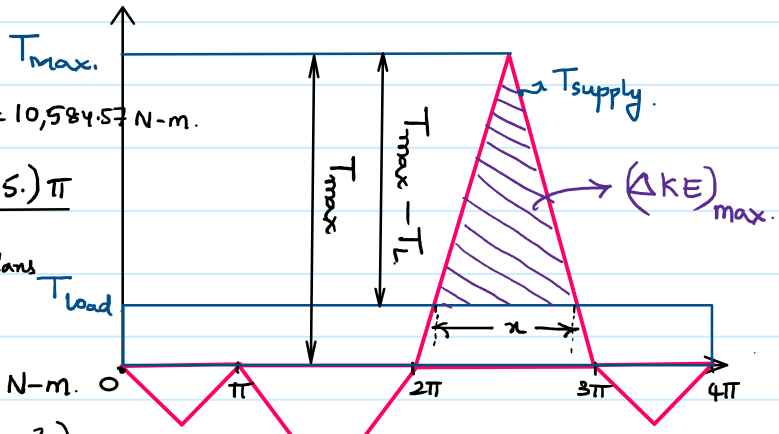
$T_{load} = 8400/4\pi = 668.45$ N-m.

$(\Delta KE)_{max} = \frac{1}{2}.(T_{max} - T_L) \times x.$

$= \frac{1}{2}.(8021.4 - 668.45) \times 2.879 = 10,584.57$ N-m.

$\frac{T_{max}}{\pi} = \frac{T_{max} - T_L}{x} \Rightarrow x = \frac{(8021.4 - 668.45)\pi}{8021.4}$

$x = 2.879$ radians



$A_{Expansion} = \frac{1}{2}.T_{max}.\pi.$

$9 \times 1400 = \frac{1}{2}.T_{max}.\pi \Rightarrow T_{max} = 8021.4$ N-m.

$(\Delta KE)_{max} = \frac{1}{2}.I.(\omega_{max}^2 - \omega_{min}^2)$

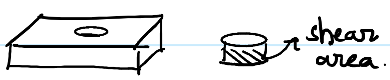
$\Rightarrow 10,584.57 = \frac{1}{2}.I.\left(\frac{2\pi}{60}\right)^2 [102^2 - 98^2] \Rightarrow I = 2413.\ kg\text{-}m^2$

03. A machine punching 40 mm diameter holes 30 mm thick plate requires 7 Nm of energy per mm² of shear area. To punch has a stroke of 100 mm and it takes 10 sec to complete one cycle. The mean speed of the flywheel is 25 m/sec, and the fluctuation of speed should not exceed 3% of the mean speed. Assuming that the motor supplies energy to the machine at a uniform rate, determine the power of the motor and the mass of the flywheel required.

Sol $d = 40mm.$ $t_{cycle} = 10sec$ stroke = 100mm

$t = 30mm.$ $V = 25 m/s.$

7N-m of Energy is required to shear 1mm² of area. Energy = $\frac{7N\text{-}m}{mm^2}$



$C_s = 3\%$ T_{supply} = constant.

Power = ? $m = ?$

$E_{cycle} = 7\pi \times dt = 7 \times \pi \times 40 \times 30 = 26,389.3$ N-m.

$\text{Power} = \frac{E_{cycle}}{t_{cycle}} = \frac{26,389.3}{10} = 2638.93 \frac{N\text{-}m}{sec} = 2.63 kW.$

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$$(\Delta KE)_{max} = E_{cycle}\left[1 - \frac{\text{thickness of Plate}}{2\times \text{Stroke}}\right] = mV^2 \cdot C_s$$

$$= 26{,}389.3\left[1 - \frac{30}{2\times100}\right] = m\times 25^2 \times 0.03$$

$$m = 1196.25 \text{ Kg}.$$

04. A machine tool performs an operation intermittently. It is driven continuously by a motor. Each operation takes 8 seconds and five operations are done per minute. The machine is fitted with a flywheel having a mass of 200 kg with a mean radius of gyration of 400 mm. When operation is being performed, the speed drops from the normal speed of 400 rpm and 250 rpm. Determine the power of the motor required.

$t_{cutting} = 8$ seconds.

No. of operation in a minute = 5.

$$t_{cycle} = \frac{60}{5} = 12 \text{ secs}.$$

$$I = mk^2$$

$$(\Delta KE)_{max} = E_{cycle}\left[1 - \frac{t_{cutting}}{t_{cycle}}\right]$$

$$\frac{1}{2} I (\omega_{max}^2 - \omega_{min}^2) = E_{cycle}\left[1 - \frac{t_{cutting}}{t_{cycle}}\right]$$

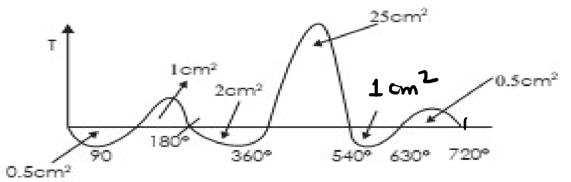
$$\frac{1}{2}\times 200\times 0.4^2 \left(\frac{2\pi}{60}\right)^2 [400^2 - 250^2] = E_{cycle}\left[1 - \frac{8}{12}\right]$$

$$E_{cycle} = 51322 \text{ N-m}.$$

$$\text{Power} = \frac{E_{cycle}}{t_{cycle}} = \frac{51322}{12} = 4.276 \text{ kW}.$$

05. Consider the following statements regarding the turning moment diagram of a reciprocating engine shown in the below figure.

(Scale 1 cm² = 100 N.m)



1. It is a four-stroke IC engine. — cycle time = $4\pi = 720°$.
2. The compression stroke is 0° to 180° 180 to 360°
3. Mean turning moment $T_m = 580/\pi$ N.m. → suction.
4. It is a ~~multi~~ single-cylinder engine.

Which of these statements are correct?

(a) 1, 2 and 3
(b) 1, 2 and 4
(c) 2, 3 and 4
(d) 1 and 3

$$T_{mean} = \frac{1}{\text{cycle time}} \int T_{supply}\, d\theta$$

area of T–θ diagram

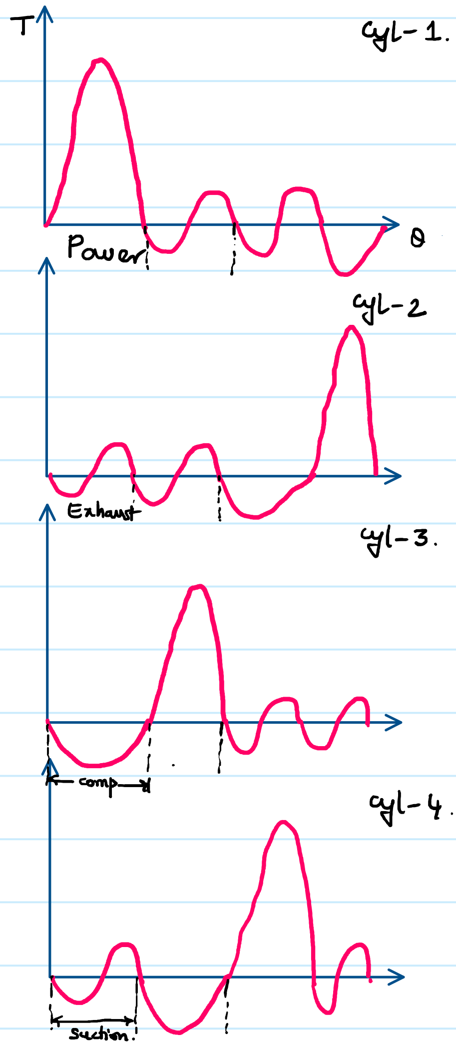
$$T_{mean} = \frac{(-0.5 + 1 - 2 + 25 - 1 + 0.5)\times 100}{4\pi}.$$

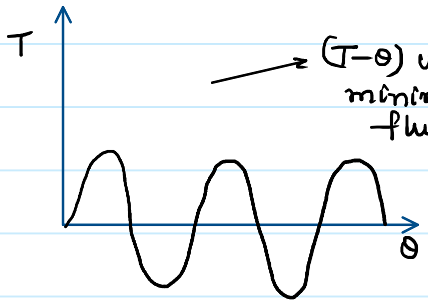
$$T_{mean} = \frac{23\times100}{4\pi}$$

$$T_{mean} = \frac{575}{\pi}.$$

T-θ diagram for Multi-Cylinder Engine

Firing order – 1342.





(T–θ) with minimum fluctuation.

$$(T_{fluct})_{single\ cylinder} > (T_{fluct})_{Multi-cylinder}$$

$$(I_{fly})_{single\ cylinder} > (I_{fly})_{Multi-cylinder}.$$

Energy @ A = E

Energy @ B = E+60

Energy @ C = E+60−40 = E+20

Energy @ D = E+60−40+80 = E+100

Energy @ E = E+100−100 = E.

Energy @ F = E+60

Energy @ G = E+60−60 = E

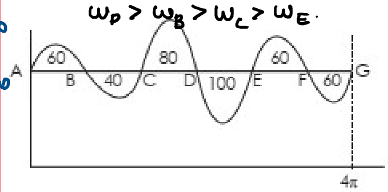
06. Refer to the turning moment diagram for a multi-cylinder engine the areas are indicated in mm2. If P, Q, R, S represent the velocity of flywheel at B, C, D and E respectively i.e.,

P. @ Point B
Q. @ Point C
R. @ Point D
S. @ Point E

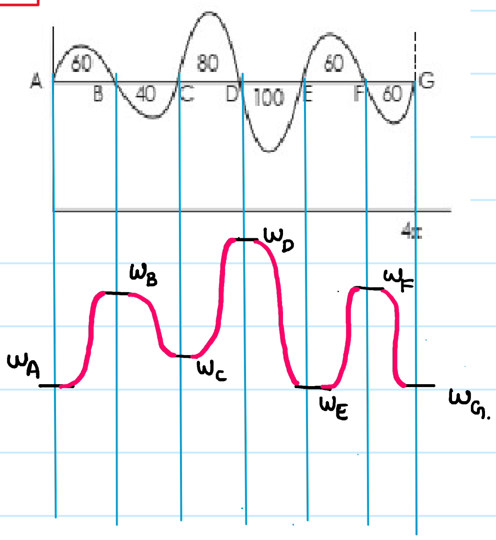
Then the appropriate relation is

R > P > Q > S.

$\omega_D > \omega_B > \omega_C > \omega_E$.



(a) R > P > S > Q (b) P > R > S > Q
(c) R > P > Q > S (d) S > P > R > Q



$$\omega_D > \omega_B = \omega_F > \omega_C > \omega_A = \omega_E = \omega_G$$

07. A disk type flywheel has a mass 'm' radius 'R' and width 'W'. When it is attached to an IC engine limits the coefficient of fluctuation of speed to 4%. If the radius is doubled and the mass is kept same by reducing the width the coefficient of fluctuation of speed will be

(a) unchanged
(b) will be reduced to 1%
(c) will be reduced to 2%
(d) none

mass – m.
Radius – R.
Width – w.
$\Big\}\ C_s = 4\%$

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If Radius is doubled.
m = constant.
$C_s = ?$. $\omega = const$.

$(\Delta KE)_{max} = constant$.
$= I\omega^2 . C_s = constant$.

$I . \propto 1/C_s$.

$I_1 C_{s_1} = I_2 . C_{s_2}$

$(m.R^2).C_{s_1} = m.(2R)^2.C_{s_2} \Rightarrow C_{s_2} = \frac{C_{s_1}}{4} = 1\%$

08. A flywheel connected to an IC Engine keeps the fluctuation of speed with in ± 2% of the mean speed, which is 600rpm. The fluctuation of energy is 5000 joules. If another flywheel of the same size is attached, the coefficient of fluctuation of speed will be

(a) 0.01 (b) 0.02
(c) 0.04 (d) remain same

I is doubled.

$C_s = \pm 2\% = 4\%$.
$N = 600 rpm$.
$(\Delta KE)_{max} = 5000 J$.

$I_1 C_{s_1} = I_2 . C_{s_2}$
$I_1\ 4\% = 2.I_1 . C_{s_2}$
$C_{s_2} = 2\% = \pm 1\% = 0.02$

09. A punching machine is driven by a 2 kW electric motor. The punching stroke consists of 1/4th the cycle time and consumes 3/4th of the energy per cycle the remaining 1/4th energy is consumed during the 3/4th of the cycle time. Express the fluctuation of energy as a fraction of energy required per cycle.

(a) 0.5 (b) 0.25
(c) 0.75 (d) 1

C_E

Power = 2kW.
$t_{cycle} = T$.
$E_{cycle} = P.x\,t_{cycle} = 2.T$ N-m.

Stroke.	Energy.	Time.
Cutting	$3/4 . E_{cycle} = 3/4 (2T)$	$1/4 . t_{cycle} = T/4$.
Idle.	$1/4 . E_{cycle} = 1/4 . (2T)$	$3/4\ t_{cycle} . = 3T/4$.

$E_{motor} = E_{cycle} . \left(\frac{\theta_2 - \theta_1}{2\pi}\right)$

$E_{cycle} = E_{cutting} + E_{idle}$.

$E_{motor} = E_{cycle} . \left(\frac{t_{cutting}}{t_{cycle}}\right)$

$E_{cutting} = E_{motor} + E_{flywheel}$.

$3/4 . (E_{cycle}) = E_{cycle} . \left(\frac{T/4}{T}\right) + (\Delta KE)_{max}$

$(\Delta KE)_{max} = \frac{E_{cycle}}{2}$.

$C_E = \frac{(\Delta KE)_{max}}{E_{cycle}} = \frac{E_{cycle}/2}{E_{cycle}} = 0.5$.

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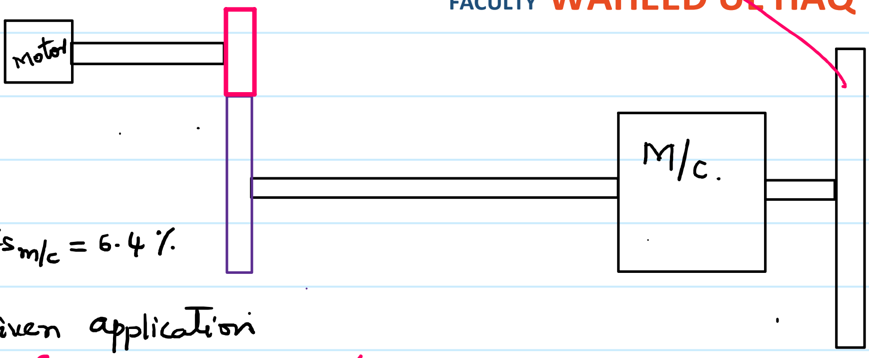
10. A punching machine is driven by an electric motor through a gear train with a reduction ratio of 4. A flywheel mounted on the machine shaft keeps the speed fluctuation with in ±3.2% of the mean speed. A naive designer suggested shifting the flywheel onto the motor shafts. What will be the change in coefficient of fluctuation of speed?
(a) the punching machine will not work
(b) the fluctuation of speed will be increase to ± 12.8%
(c) the fluctuation of speed will be decreased to ± 0.2%
(d) the fluctuation of speed will be decreased to ± 0.8%

Flywheel shifted to motor side.



Sol: $G = 4.$ $\left.\begin{aligned} C_s &= \pm 3.2\% \\ &= 6.4\% \end{aligned}\right\} C_{s_{m/c}} = 6.4\%$

$(\Delta KE)_{max}$ = const for given application

$$(\Delta KE)_{max} = \cancel{I}\cdot\omega_{m/c}^2 \cdot C_{s_{m/c}} = \cancel{I}\cdot \omega_{motor}^2 \cdot C_{s_m}.$$

$$C_{s_m} = \frac{\omega_{m/c}^2}{\omega_m^2}\cdot C_{s_{m/c}} = \left(\frac{1}{G}\right)^2 \cdot C_{s_{m/c}} = \frac{1}{4^2}\times 6.4\%$$

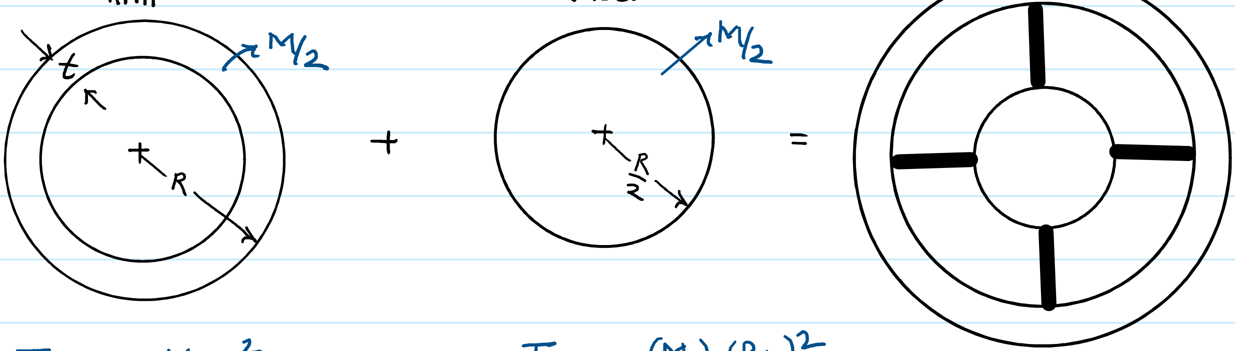
$$= 0.4\% = \pm 0.2\%$$

11. Consider a flywheel whose mass M is distributed almost equally between a heavy, $\boxed{\text{ring-like rim}}$ of radius R and a concentric disk-like feature of radius R/2. Other parts of the flywheel, such as spokes, etc, have negligible mass. The best approximation for α, if the moment of inertia of the fly wheel about its axis of rotation is expressed as αMR², is ______ .
(GATE-14)

$$I = I_{Rim} + I_{Disc}.$$

$\boxed{R >>>> t}$

Rim Disc.



$$I_{Rim} = \frac{M}{2}\cdot R^2 \qquad I_{Disc} = \frac{\left(\frac{M}{2}\right)\cdot\left(\frac{R}{2}\right)^2}{2}$$

$$I_{fly} = \frac{MR^2}{2} + \frac{MR^2}{16} = 0.5625\cdot MR^2.$$

12. The torque (in N-m) exerted on the crank shaft of a two stroke engine can be described as T=10000+1000sin2θ−1200 cos2θ, where θ is the crank angle as measured from inner dead center position. Assuming the resisting torque to be constant, the power (in kW) developed by the engine at 100 rpm is ______. (GATE –15)

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$T = \boxed{10,000} + 1000\sin 2\theta - 1200\cos 2\theta$ → T_{mean}.

T_{load} = constant Power @ $\boxed{100 \text{ rpm}}$ = ? → N_{mean}.

$$P = \frac{2\pi \cdot N_{mean} \cdot T_{mean}}{60,000} \text{ kW.} \qquad T_{mean} = \frac{1}{\text{cycle time}} \int_0^{\text{cycle time}} T_{supply} \cdot d\theta$$

$$T_{mean} = \frac{1}{2\pi} \cdot \int_0^{2\pi} (10,000 + 1000\sin 2\theta - 1200\cos 2\theta)\, d\theta$$

$$T_{mean} = \frac{1}{2\pi} \left[10,000[\theta]_0^{2\pi} + 1000\left[-\frac{\cos 2\theta}{2}\right]_0^{2\pi} - 1200\left[\frac{\sin 2\theta}{2}\right]_0^{2\pi} \right]$$

0 0

$$T_{mean} = 10,000 \text{ N-m.}$$

$$P = \frac{2\pi \times 10,000 \times 100}{60,000} = 104.71 \text{ kW.}$$

Max. fluction in Energy.

T_{load} = const ⇒ $T_{load} = T_{mean}$.

Isolation points.

@ $\theta = \theta_1$, ω = min. $T_s = T_L$

@ $\theta = \theta_2$, ω = Max. $T_s = T_L$

$$(\Delta KE)_{max} = \int_{\theta_1}^{\theta_2} (T_s - T_L)\, d\theta$$

$$T_s = T_L$$

$$10,000 + 1000\sin 2\theta - 1200\cos 2\theta = 10,000.$$

$$\tan 2\theta = 1.2.$$

$\boxed{T_s = 10000 + 1000\sin 2\theta}$.

$$2\theta = \tan^{-1}(1.2)$$

$$2\theta = 50.19, \quad 180 + \tan^{-1}(1.2)$$

$$= 50.19, \quad 230.19^\circ$$

$$\theta = 25.09, \quad 115.09^\circ$$

$$(\Delta KE)_{max} = \int_{25.09}^{115.09} (1000\sin 2\theta - 1200\cos 2\theta)\, d\theta = 1000\left[-\frac{\cos(2\theta)}{2}\right]_{25.09}^{115.09} - 1200\left[\frac{\sin 2\theta}{2}\right]_{25.09}^{115.09}$$

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Angular Acceleration of flywheel.

$$T_{fluct} / T_{fly} = T_S - T_L = I_{fly} \cdot \alpha_{fly}.$$

$$I_{fly} \cdot \alpha_{fly} = 10{,}000 + 1000\sin 2\theta - 1200\cos 2\theta - 10{,}000.$$

$$I_{fly} \cdot \alpha_{fly} = 1000\sin 2\theta - 1200\cos 2\theta.$$

$\alpha = f(\theta)$ for α = Max. or Min. $\frac{d\alpha}{d\theta} = 0 \Rightarrow \frac{d}{d\theta}(T_S - T_L) = 0$

$$I_{fly} \cdot \alpha_{fly} = A\sin(n\theta) \pm B\cdot\cos(n\theta)$$

$$I_{fly}(\alpha_{fly})_{max,\,min} = \pm\sqrt{A^2 + B^2} = \pm\sqrt{1000^2 + 1200^2}$$

$$\frac{d}{d\theta}(1000\sin 2\theta - 1200\cos 2\theta) = 0 \Rightarrow 1000\cos(2\theta) \times 2 - (-1200\sin(2\theta) \times 2) = 0$$

$$\tan 2\theta = -\frac{1000}{1200}.$$

$$2\theta = 180 - \tan^{-1}\left(\frac{1000}{1200}\right),\ 360 - \tan^{-1}\left(\frac{1000}{1200}\right)$$

$$2\theta = 140.19,\ 320.2$$

$$\theta = 70.09,\ 160.1$$

GOVERNORS:

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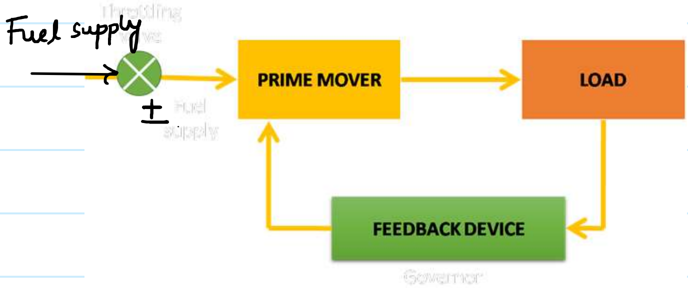
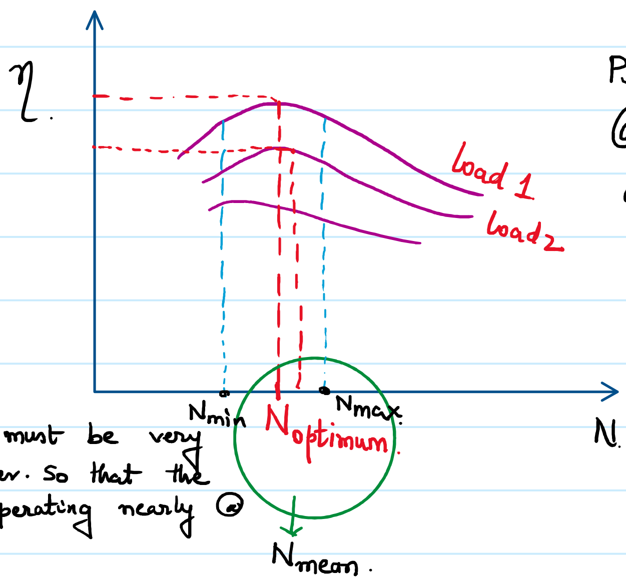
- Governor is a feedback device which regulate the fuel supply, whenever there is variation in output load.



FLYWHEEL	GOVERNORS
1. It is a reservoir of energy.	1. It is a feedback device.(Mechanism)
2. Flywheel has no control over the mean speed.	2. It can change the mean speed of prime mover.
3. It cannot change the fuel supply.	3. It can change the fuel supply (it can change well as quantity of fuel supply).
4. It control the fluctuation with in the cycle (Intra cycle fluctuation).	4. It controls the fluctuation between two consecutive cycle(inter cycle fluctuation)
5. Flywheel is continuous working device.	5. It is a intermittent working device.
6. If supply and load are uniform then flywheel is not required.	6. It is a compulsory device for all the prime movers.

Governor will work whenever there is correction in fuel supply.

Efficiency vs speed.



Prime mover must be operated @ $N_{optimum}$. So that $\eta = Max$ can be obtained.

Due to fluctuation in load there will be fluctuation in speed.

N_{max} and N_{min} must be very close to each other. So that the Prime mover is operating nearly @ $N_{optimum}$.

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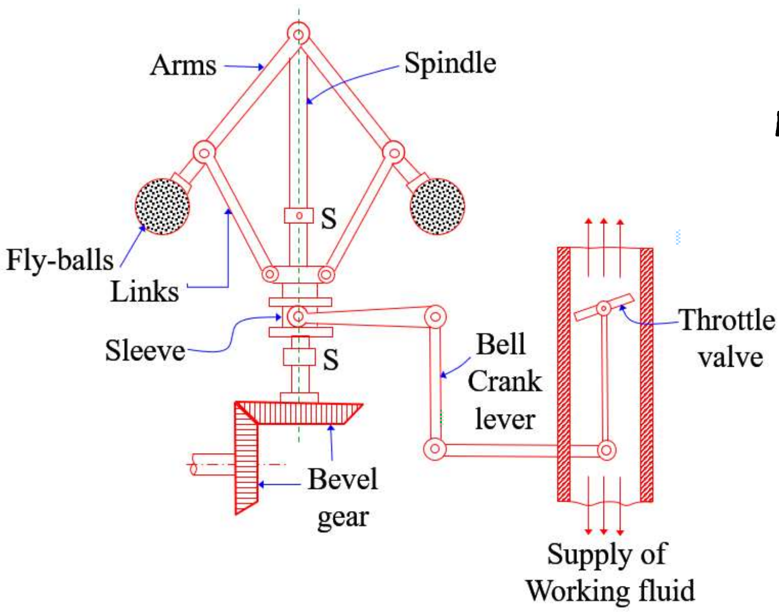
Classification of Governors

Governor
- centrifugal
- Inertia

- CENTRIFUGAL GOVERNORS
  - PENDULUM TYPE
    - WATT GOVERNOR
  - LOADED TYPE
    - GRAVITY CONTROLLED
      - PORTER GOVERNOR
      - PROELL GOVERNOR
    - SPRING CONTROLLED
      - HARTNELL GOVERNOR
      - WELSON GOVERNOR
      - HARTUNG GOVERNOR
      - PICKRING GOVERNOR

CENTRIFUGAL GOVERNOR:



Load ↓ Speed of Prime mover ↑
Sleeve will move upwards.
Throttle opening decreases.
Fuel supply decreases.

Load ↑ speed of Prime mover ↓ sleeve will move downwards. Throttle opening decreases. Fuel supply increases.