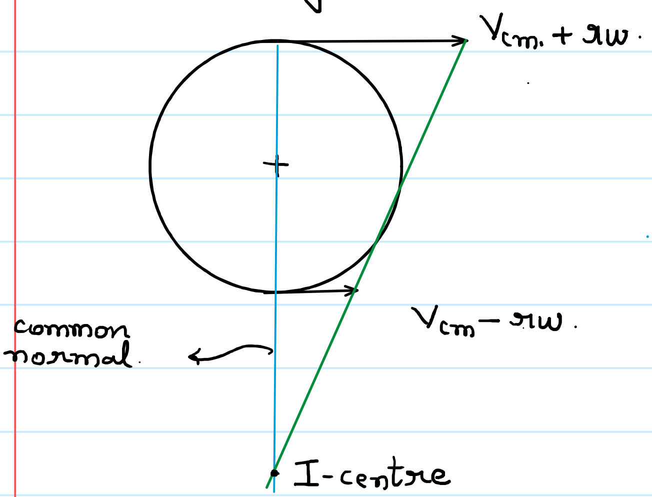
Rolling slipping

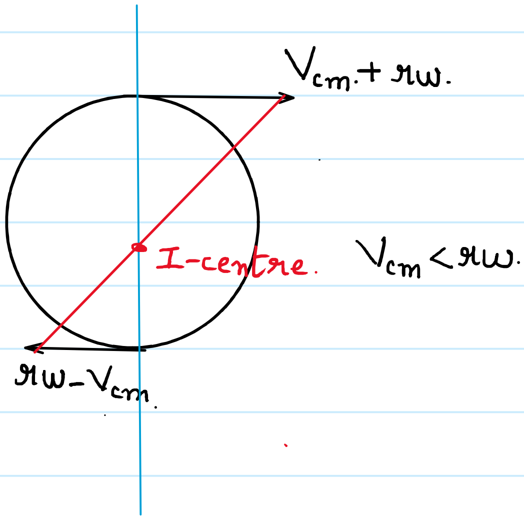
$$S_{cm} \neq r\theta_z$$

$S_{cm} > r\theta_z$ (Translation > Rotation) Forward slipping

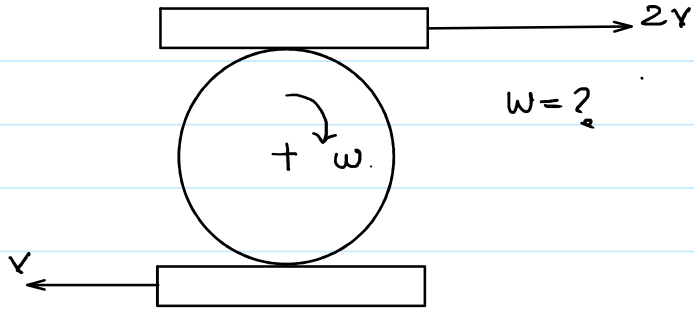
$S_{cm} < r\theta$ (Translation < Rotation) Backward slipping

Forward slipping

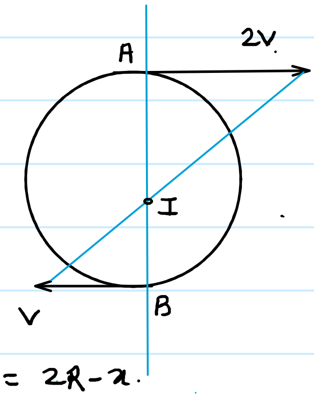




Gate - 2018/19.



$\omega = ?$



$IB = x.\quad IA = 2R - x.$

Relative velocity

$$\omega = \frac{V_A - V_B}{AB} = \frac{2V - (-V)}{2R} = \frac{3V}{2R}.$$

$$\frac{2V}{IA} = \frac{V}{IB} = \omega. \Rightarrow \frac{2V}{2R - x} = \frac{V}{x} = \omega.$$

$$\omega = \frac{3V}{2R}.$$

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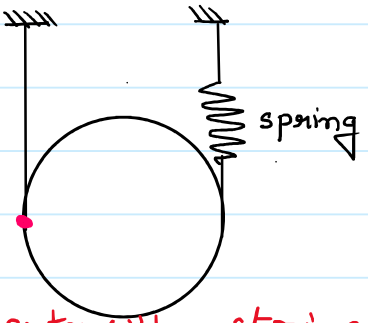
String and Pulley arrangement
(Rigid)



(Pure Rolling) I-centre

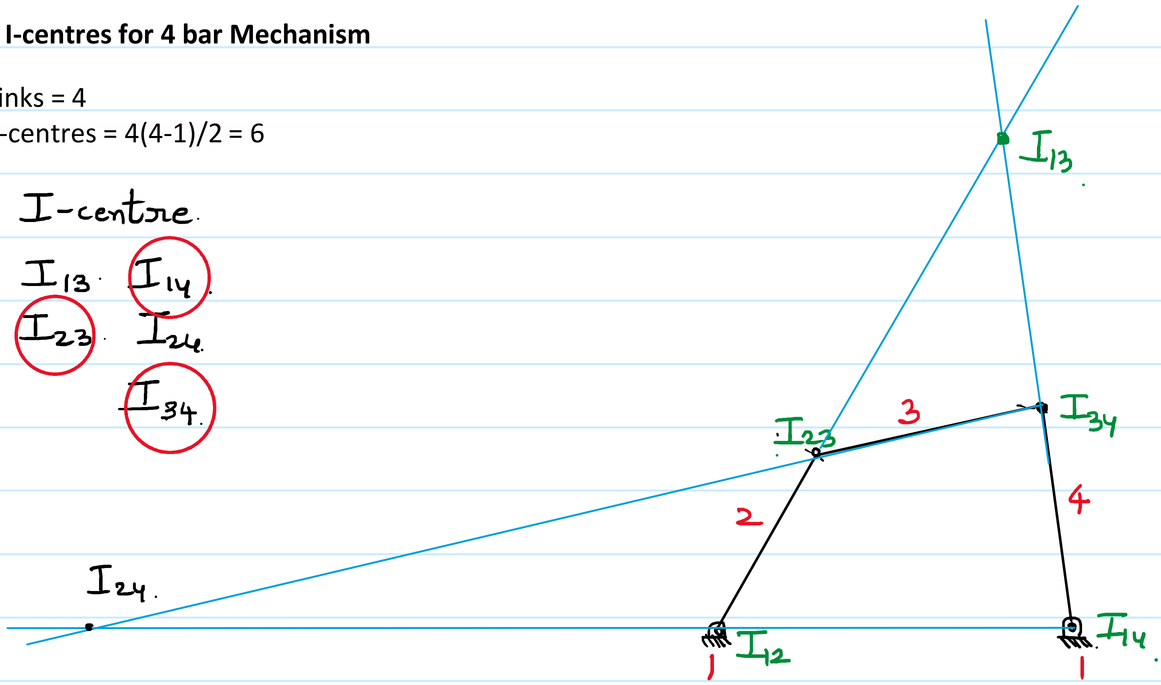
The point where inextensible string enters the pulley it can be assumed as a I-centre.

Locating the I-centres for 4 bar Mechanism

Number of Links = 4
Number of I -centres = 4(4-1)/2 = 6

Possible I-centre.

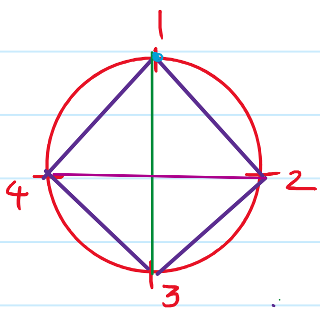
I_{12} I_{13}. I_{14}.
I_{23}. I_{24}.
I_{34}.



Arnhold Kennedy's Theorem - If the three bodies are connected directly or indirectly and there relative motion between them, then their I - centres will be colinear.

I_{24} must be colinear with I_{12}. & I_{14}.

I_{24} must be colinear with I_{23} & I_{34}.



I_{13} must be colinear with I_{12}. & I_{23}.

I_{13} must be colinear with I_{14} & I_{34}.

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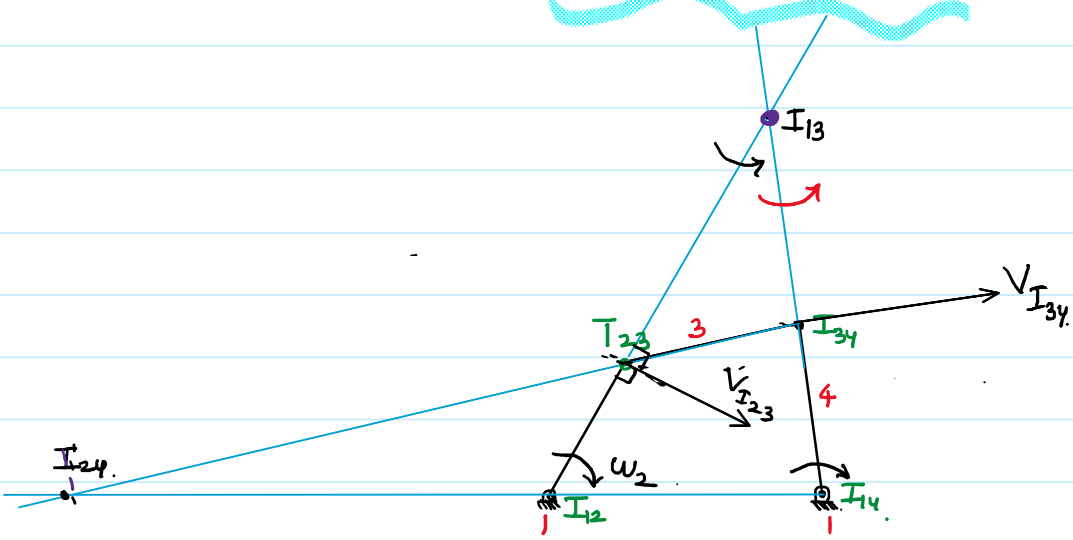
	Absolute I-centre	Relative I-centre
	Ex:- I_{12}, I_{13}, I_{14} (link 1) fixed link	Ex- I_{23}, I_{34}, I_{24}
1.	It is defined wrt fixed link.	It is defined on moving links.
2.	About this I-centre 1 link is assumed in rotation.	About this I-centre 2 links are assumed in rotation.
3.	It is used to predict the direction of angular velocity	It is used to calculate the magnitude of angular velocity.

Angular Velocity theorem - The velocity of an I-centre when it is assumed as a point on link 2 w.r.t link 1 **(fixed)** will be same when it is assumed as a point on link 3 w.r.t link 1 **(fixed).**



$$V_{I_{xy}} = I_{1x} I_{xy} \, \omega_x = I_{1y} I_{xy} \, \omega_y$$

$$V_{I_{23}} = I_{12} . I_{23} \, \omega_2 = I_{13} . I_{23} \, \omega_3$$

$$V_{I_{34}} = I_{13} I_{34} . \omega_3 = I_{14} . I_{34} . \omega_4$$

$$V_{I_{24}} = I_{12} I_{24} . \omega_2 = I_{14} I_{24} . \omega_4 .$$

★ If the relative I - centre lies between two absolute I - centres then the sense of rotation of adjacent will be in opposite sense.

★ If the relative I - centre lies on one side of two absolute I - centres then the sense of rotation of links will be same.

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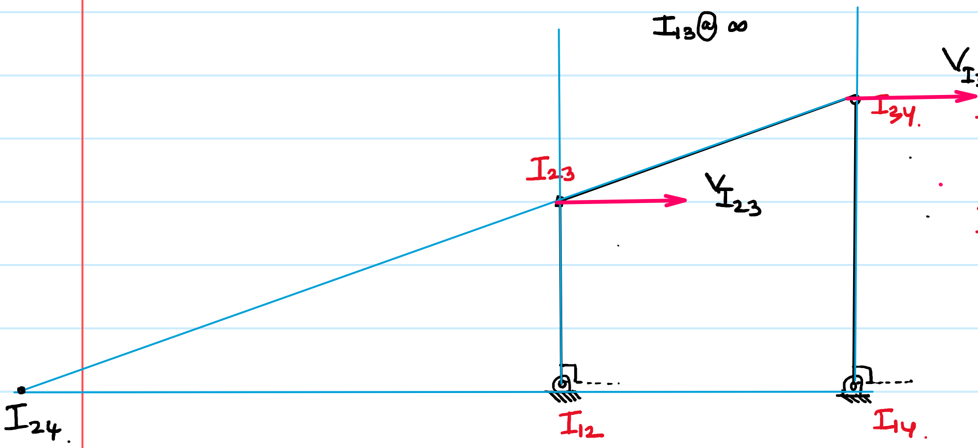
case(i) I/p and o/p links are //le.



I_{13} is colinear with I_{12} & I_{23}.

I_{13} is colinear with I_{14} & I_{34}

link 3 is in Translation.

$$V_{I_{23}} = I_{12} \cdot I_{23} \cdot \omega_2 = I_{13} I_{23} \cdot \omega_3 \quad \Rightarrow \quad \boxed{\omega_3 = 0}$$

∞

$\Delta^{le} I_{24} I_{14} I_{34}$ is similar to $\Delta^{le} I_{24} I_{12} I_{23}$.

$$\frac{I_{12} \cdot I_{24}}{I_{12} \cdot I_{23}} = \frac{I_{14} \cdot I_{24}}{I_{14} \cdot I_{34}}$$

$$\frac{I_{14} \cdot I_{24}}{I_{12} \cdot I_{24}} = \frac{I_{14} \cdot I_{34}}{I_{12} \cdot I_{23}}$$

$$V_{I_{24}} = I_{12} \cdot I_{24} \cdot \omega_2 = I_{14} I_{24} \cdot \omega_4$$

$$\frac{\omega_2}{\omega_4} = \frac{I_{14} \cdot I_{24}}{I_{12} \cdot I_{24}} = \frac{I_{14} \cdot I_{34}}{I_{12} \cdot I_{23}}$$

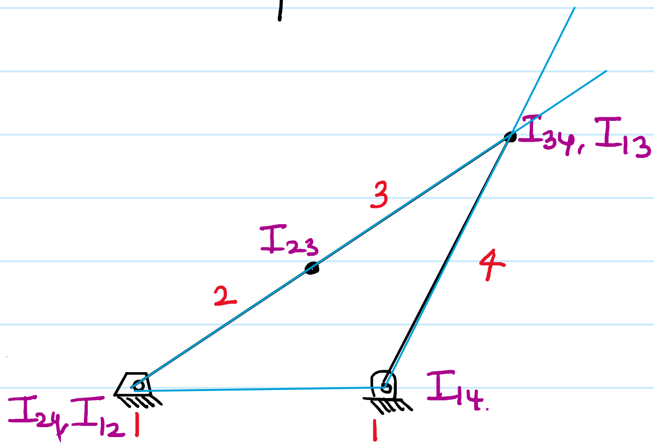
L_4

L_2

$$\frac{\omega_2}{\omega_4} = \frac{L_4}{L_2} \Rightarrow \omega_2 \cdot L_2 = \omega_4 \cdot L_4$$

case(ii) I/p link and coupler link are colinear.

$\omega_4 = 0$



$I_{13} / I_{12} \,\&\, I_{23}$

$I_{13} / I_{14} \,\&\, I_{34}$

$I_{24} / I_{12} \,\&\, I_{14}$

$I_{24} / I_{23} \,\&\, I_{34}$

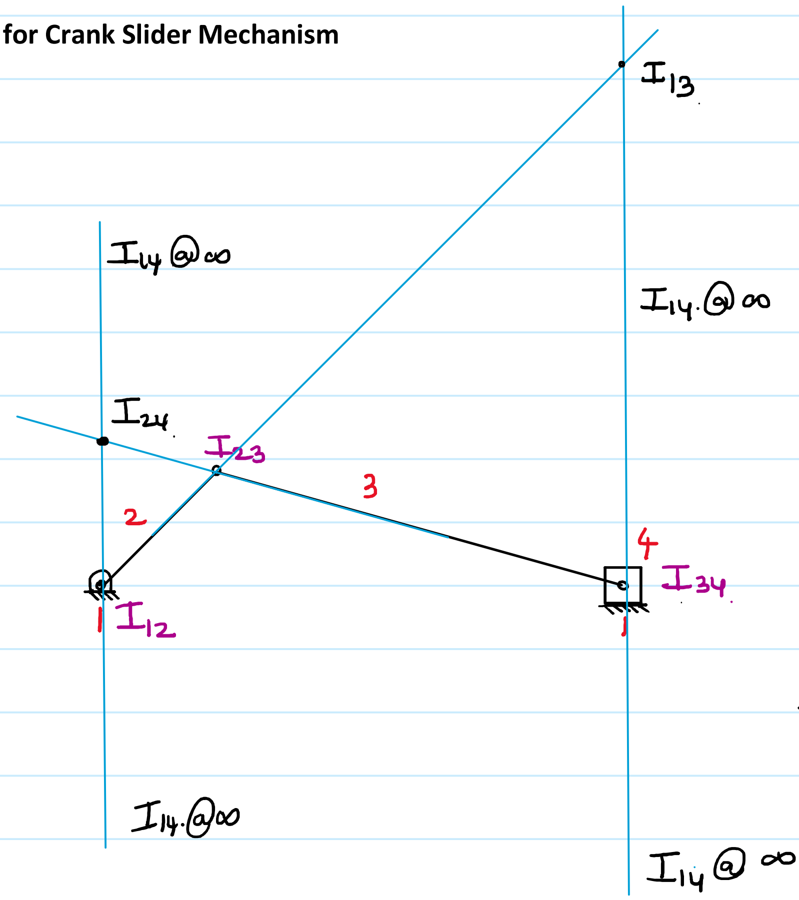
$$V_{I_{23}} = I_{12} I_{23} \cdot \omega_2 = I_{13} I_{23} \cdot \omega_3$$

$$= l_2 \cdot \omega_2 = l_3 \cdot \omega_3$$

$$V_{I_{34}} = \underbrace{I_{13} I_{34}}_{0} \cdot \omega_3 = I_{14} I_{34} \cdot \omega_4 \longrightarrow \omega_4 = 0$$

$$V_{I_{24}} = \underbrace{I_{12} I_{24}}_{0} \cdot \omega_2 = I_{14} I_{24} \cdot \omega_4 \longrightarrow \omega_4 = 0$$

Locating I - centres for Crank Slider Mechanism



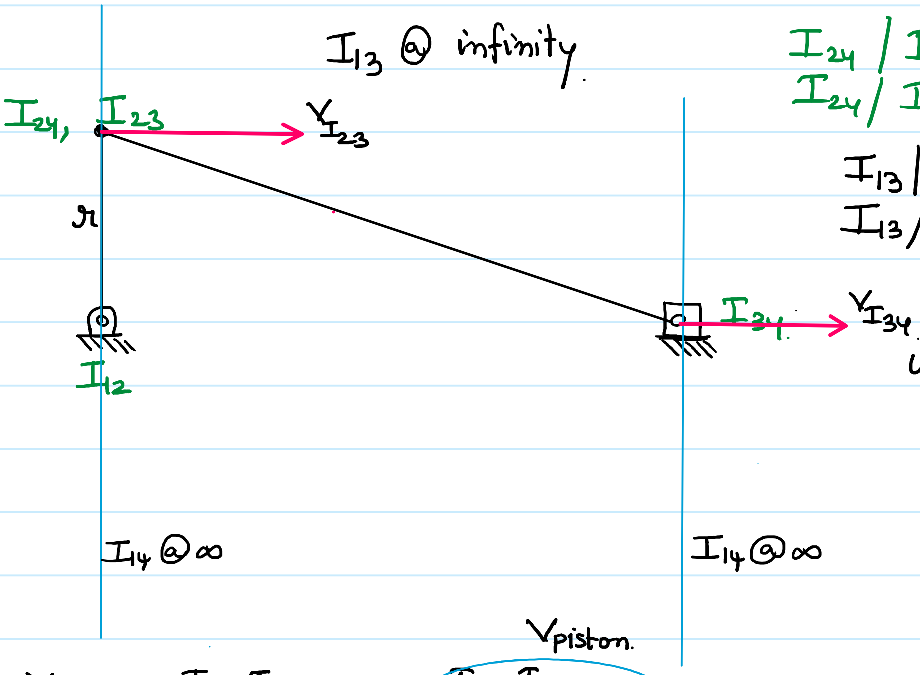
$I_{13} / I_{12} \,\&\, I_{23}$

$I_{13} / I_{14} \,\&\, I_{34}$

$I_{24} / I_{12} \,\&\, I_{14}$

$I_{24} / I_{23} \,\&\, I_{34}$

case(A) When the crank angle is 90°



I_{24} / I_{12} & I_{14}
I_{24} / I_{23} & I_{34}.

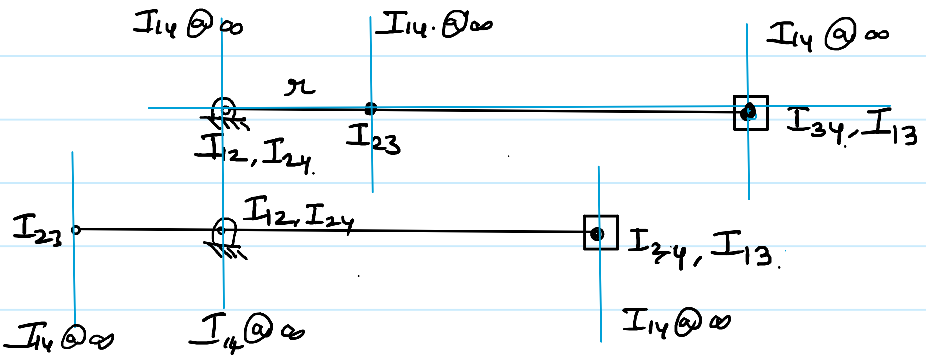
I_{13} | I_{12} & I_{23}
I_{13} / I_{14} & I_{34}

$\omega_3 / \omega_{conn.Rod} = 0$

$$V_{I_{24}} = \underbrace{I_{12} \cdot I_{24}}\cdot \omega_2 = I_{14} \cdot I_{24} \cdot \omega_4$$

$$\boxed{V_{piston} = r\omega.}$$

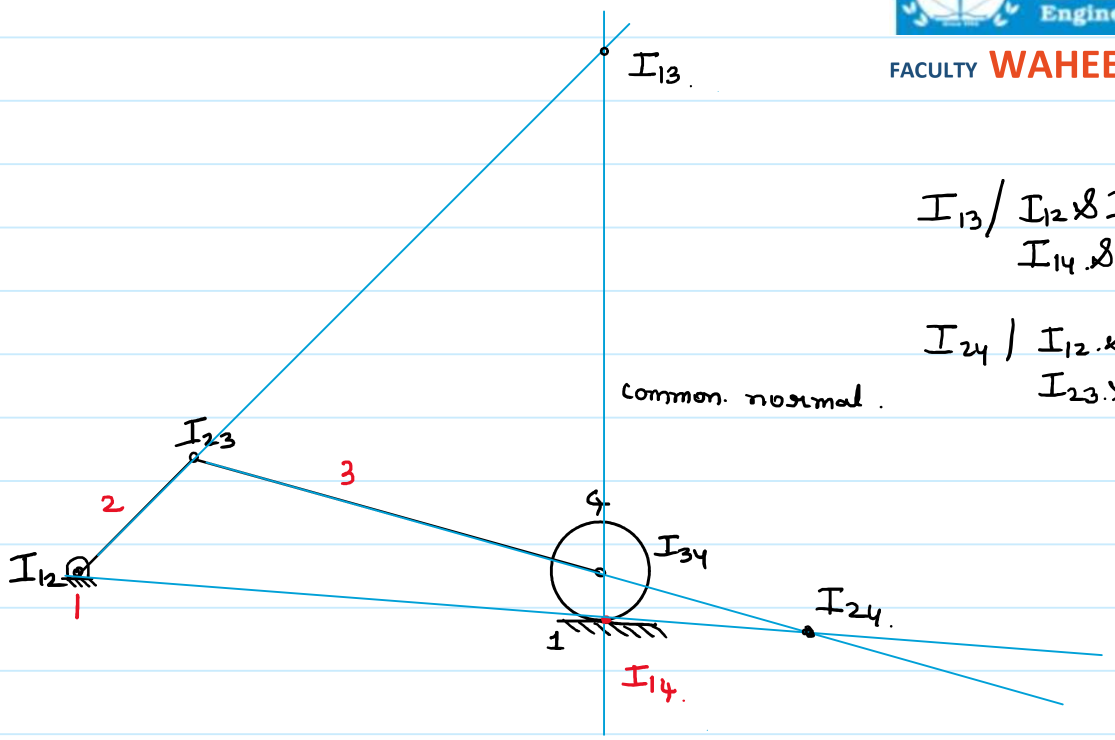
case(ii) When piston is at the extreme positions



$$V_{I_{23}} = I_{12} \cdot I_{23} \cdot \omega_2 = I_{13} \cdot I_{23} \cdot \omega_3$$
$$= r \cdot \omega_2 = l \cdot \omega_3$$
$$l_2 \omega_2 = l_3 \cdot \omega_3$$

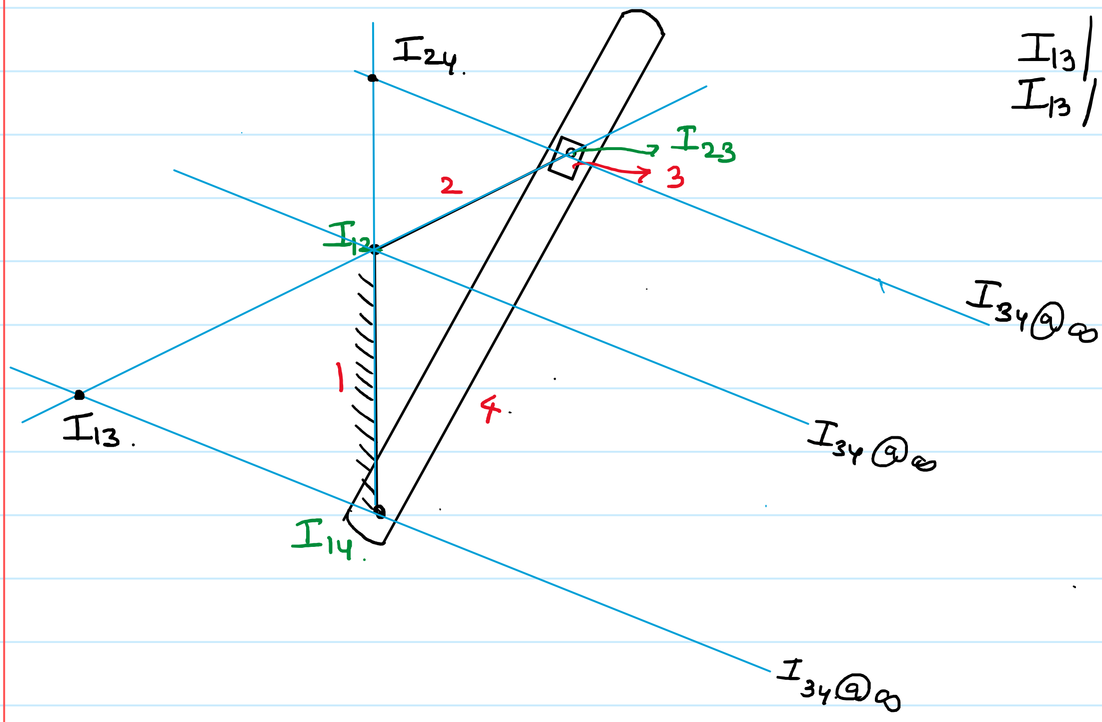
$$V_{I_{34}} = \cancel{I_{13} \cdot I_{34}}_{\to 0} \cdot \omega_3 = V_{piston}$$

$$V_{piston} = 0$$



I_{13} / I_{12} & I_{23}
I_{14} & I_{34}

I_{24} / I_{12} & I_{14}
I_{23} & I_{34}

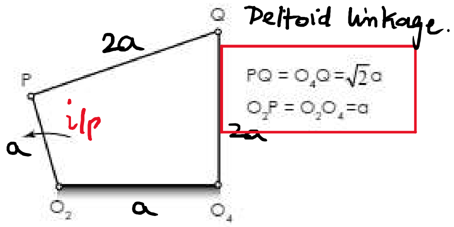


I_{13} / I_{12} & I_{23}
I_{13} / I_{14} & I_{34}

I_{24} / I_{12} & I_{14}
I_{24} / I_{23} & I_{34}

09. The input link O_2P of a four bar linkage is rotated at 2 rad/s counter clockwise direction as shown below. The angular velocity of the coupler PQ in rad/s, at an instant when $\boxed{\angle O_4O_2P = 180^\circ}$ is

(GATE-07)



(a) 4 (b) $2\sqrt{2}$

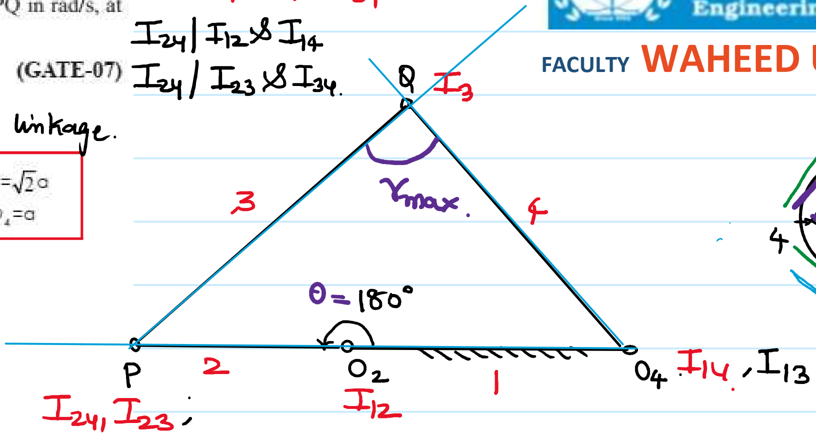
(c) 1 (d) $\frac{1}{\sqrt{2}}$

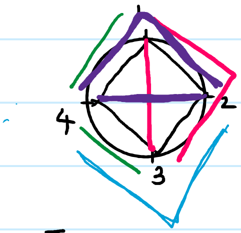
$I_{13} / I_{12}, I_{23}$

$I_{13} / I_{14}, I_{34}$

$I_{24} / I_{12} \,\&\, I_{14}$

$I_{24} / I_{23} \,\&\, I_{34}$.





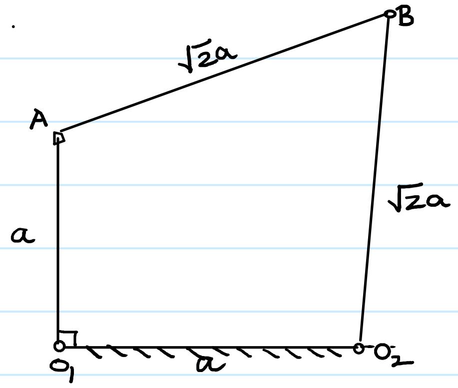
$$I_{12}.I_{23}.\omega_2 = I_{13}.I_{23}.\omega_3$$

$$a \times 2 = 2a \times \omega_3 \Rightarrow \omega_3 = 1\,rad/s.$$

$$I_{13}.I_{34}.\omega_3 = I_{14}.I_{34}.\omega_4$$

$$2a \times 1 = 2a \times \omega_4 \Rightarrow \omega_4 = 1\,rad/s.$$

Gate – 19/20.



$$\frac{\omega_4}{\omega_2} = ?$$

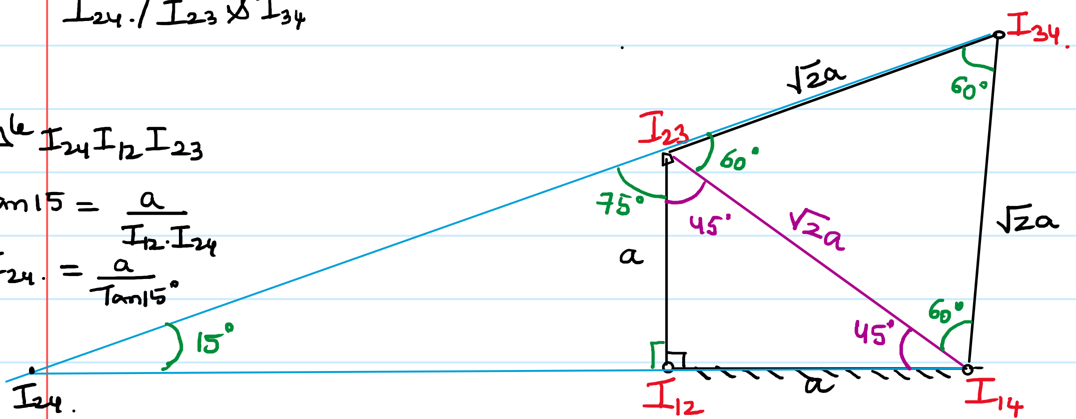
$I_{24} / I_{12} \,\&\, I_{14}$

$I_{24} / I_{23} \,\&\, I_{34}$

In Δ^{le} $I_{24}I_{12}I_{23}$

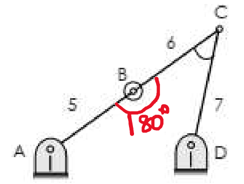
$$\tan 15 = \frac{a}{I_{12}.I_{24}}$$

$$I_{12}.I_{24} = \frac{a}{\tan 15^\circ}$$



$$\frac{\omega_4}{\omega_2} = \frac{I_{12}.I_{24}}{I_{14}.I_{24}} \Rightarrow \frac{\omega_4}{\omega_2} = \frac{a/\tan 15^\circ}{a + a/\tan 15^\circ} = \frac{1}{\tan 15^\circ + 1} = 0.788$$

11. In a four bar mechanism for the position shown the velocity of the crank is 1 rad/sec (cc). The angular velocity of the link DC is



~~(a) 0~~ (b) 6/7 CCW
(c) 5/7 CW (d) 7/5 CW

$\phi = 180^\circ \longrightarrow$ Toggle position.

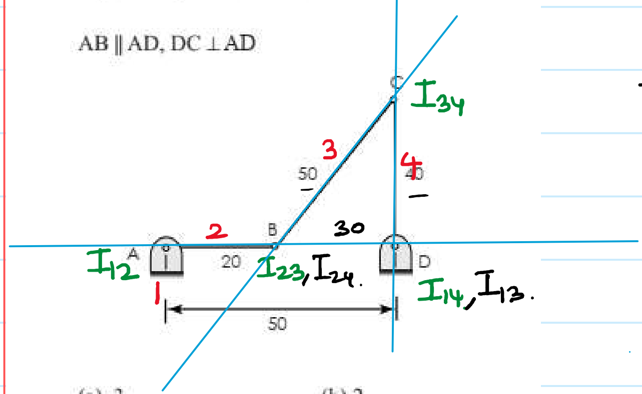
$\omega_{o/p.} = 0$

$l_2 . \omega_2 = l_3 . \omega_3 \Rightarrow \quad 5 \times 1 = 6 \times \omega_3$

$\omega_3 = 5/6$ rad/s.

24. Consider the four bar mechanism shown in figure. At the given position the output rocker is to be driven in clockwise direction at 2 rad/sec. Determine the required velocity of the input crank AB in rad/sec. The length of the links AB, BC, CD and DC are respectively 20 cm, 50 cm, 40 cm and 50 cm.

AB ∥ AD, DC ⊥ AD



(a) 3 (b) 2
(c) 5.5 (d) 12

$\omega_4 = 2$ rad/s. (C.W)

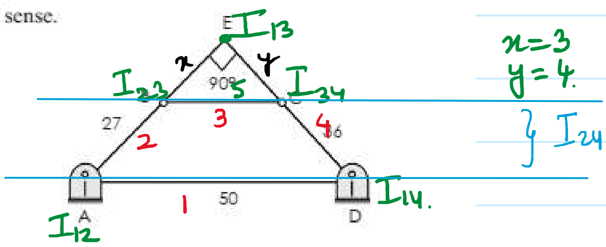
$\omega_2 = ?$

$$\frac{\omega_4}{\omega_2} = \frac{I_{12}.I_{24}}{I_{14}.I_{24}} \qquad \frac{\omega_4}{\omega_3} = \frac{I_{13}.I_{34}}{I_{14}.I_{34}}$$

$$\frac{2}{\omega_2} = \frac{20}{30} \qquad \frac{3}{\omega_3} = \frac{40}{40}$$

$\omega_2 = 3$ rad/s. (C.C.W) $\qquad \omega_3 = 1$ rad/s. (C.W)

The four bar mechanism is shown in figure at the configuration. If AB and DC are extended they meet at right angles and the lengths of the sides of the formed triangle AED are integers . The length of the links AB = 27cm, $\boxed{\text{BC = 5 cm}}$ CD = 36 cm and AD = 50 cm. The link AB has an instantaneous angular velocity of 1 rad/sec in CW sense.



$x = 3$
$y = 4$.
$\} \; I_{24} @ \infty$

$\Delta^{le} I_{12}.I_{13}.I_{14}$ & $\Delta^{le} I_{23}.I_{13}.I_{34}$ are similar

$$\frac{I_{23}.I_{34}}{I_{12}.I_{14}} = \frac{I_{23}.I_{13}}{I_{12}.I_{13}} = \frac{I_{13}.I_{34}}{I_{14}.I_{13}}.$$

$$\frac{5}{50} = \frac{x}{27+x} = \frac{y}{36+y}$$

$x = 3$

$y = 4$.

25. Angular velocity of two link BC is
(a) 3 rad/sec (b) 6 rad/sec
$\boxed{\text{(c) 9 rad/sec}}$ (d) zero

26. Angular velocity of link DC is
(a) 1 rad/sec (b) 1.6 rad/sec
(c) 2.4 rad/sec (d) zero

$V_{I_{23}} = I_{12}.I_{23}.\omega_2 = I_{13}.I_{23}.\omega_3$

$(27 \times 1) = 3 \times \omega_3$

$\omega_3 = 9$ rad/s.

$V_{I_{34}} = I_{14}.I_{34}.\omega_4 = I_{13}.I_{34}.\omega_3$

$36 \times \omega_4 = 4 \times 9 \Rightarrow \omega_4 = 1$ rad/s.

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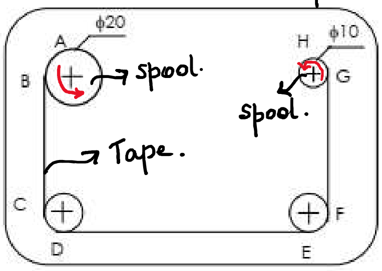
28. For the audio cassette mechanism shown in figure, where is the instantaneous center of rotation (point P) of the two spools ? $\rightarrow I_{24} = ?$

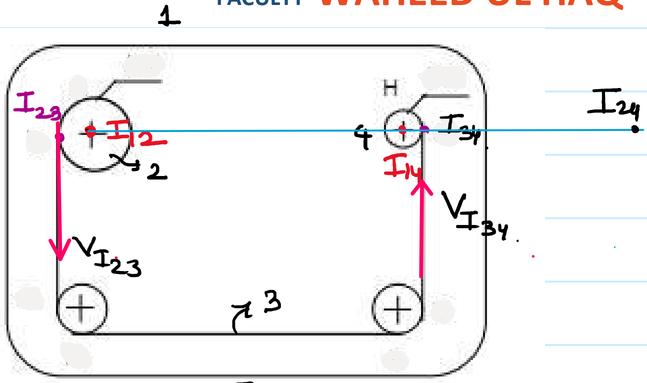


(a) Point ‘P’ lies to the left of both the spools but at infinity along the line joining ‘A’ and ‘H’

~~(b) Point ‘P’ lies in between the two spools on the line joining ‘A’ and ‘H’ such that PH = 2 AP.~~

(c) Point ‘P’ lines to the right of both the spools on the line joining ‘A’ and ‘H’ such that AH = HP

~~(d) Point ‘P’ lies at the intersection of the line joining ‘B’ and ‘C’ and the line joining ‘G’ and ‘F’~~



Pure Rolling

$$V_{I_{23}} = I_{12} I_{23} \cdot \omega_2$$

$$V_{I_{23}} = 10 \times \omega_2$$

$$V_{I_{34}} = I_{14} I_{34} \cdot \omega_4 = 5 \times \omega_4$$

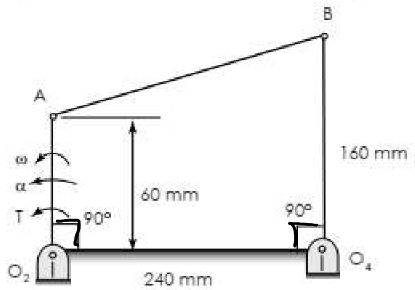
$$V_{I_{23}} = V_{I_{34}} \Rightarrow 10 \times \omega_2 = 5 \times \omega_4 \Rightarrow \frac{\omega_2}{\omega_4} = \frac{1}{2}$$

$$\frac{\omega_2}{\omega_4} = \frac{I_{14} \cdot I_{24}}{I_{12} \cdot I_{24}} = \frac{1}{2}$$

$$\frac{\omega_2}{\omega_4} = \frac{I_{14} \cdot I_{24}}{I_{12} I_{14} + I_{14} \cdot I_{24}} = 1/2$$

$$I_{12} \cdot I_{14} = I_{14} \cdot I_{24}$$

An instantaneous configuration of a four bar mechanism, whose plane is horizontal, is shown in the fig below.



$$\boxed{l_2 \cdot \omega_2 = l_4 \cdot \omega_4}$$

$$60 \times 8 = 160 \times \omega_4$$

$$\omega_4 = 3 \text{ rad/s}.$$

At this instant, the angular velocity and angular acceleration of link O_2A are $\omega = 8$ rad/s and $\boxed{\alpha = 0}$, $\rightarrow \omega = \text{constant}$. respectively, and the $\boxed{\text{driving torque } (\tau) \text{ is zero}}$. The link O_2A is balanced so that its center of mass falls at O_2

(GATE-05)

43. At the instant considered, what is the magnitude of the angular velocity of O_4B? $\omega_{O_4B} = ?$

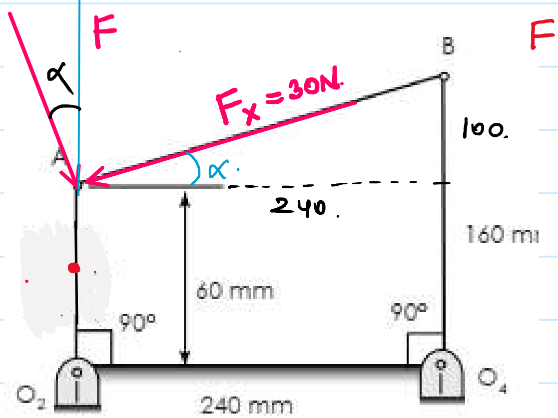
(a) 1 rad/s (b) 3 rad/s

(c) 8 rad/s (d) 64/3 rad/s

$$F_x = F \sin\alpha = 30$$

$$F = \frac{30}{\sin 22.61°}$$

$$F = 78 N.$$



44. At the same instant, if the component of the force at joint A along AB is 30N, then the magnitude of the joint reaction at O_2

(a) is zero (b) is 30 N

(c) is 78 N (d) cannot be determined

$$\tan\alpha = \frac{100}{240} \Rightarrow \alpha = 22.61°$$

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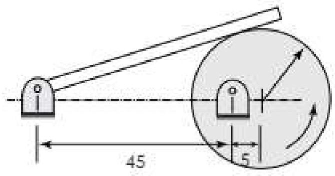
29. In the mechanism given below, if the angular velocity of the eccentric circular disc is 1 rad/s, the angular velocity (rad/s) of the follower link for the instant shown in the figure is (GATE-12)



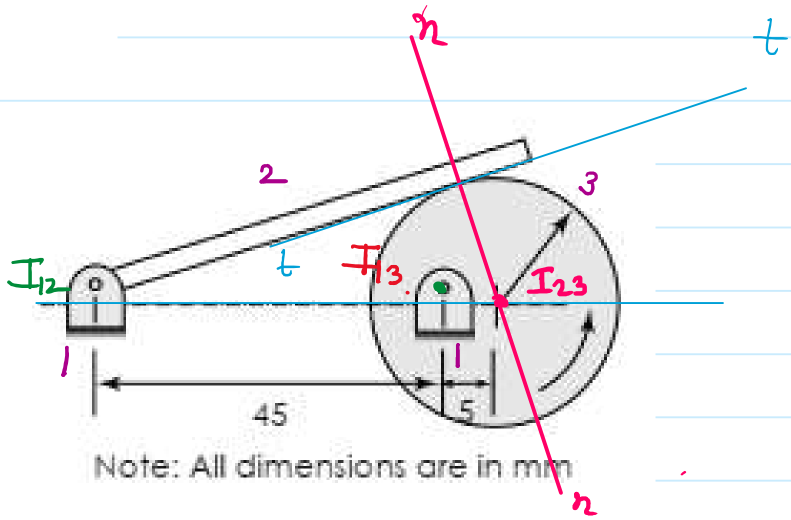
Note: All dimensions are in mm

(a) 0.05 (b) 0.1

(c) 5.0 (d) 10.0

$\omega_3 / \omega_{Disc} = 1 \, rad/s.$

$\omega_2 / \omega_{Rod} = ?$

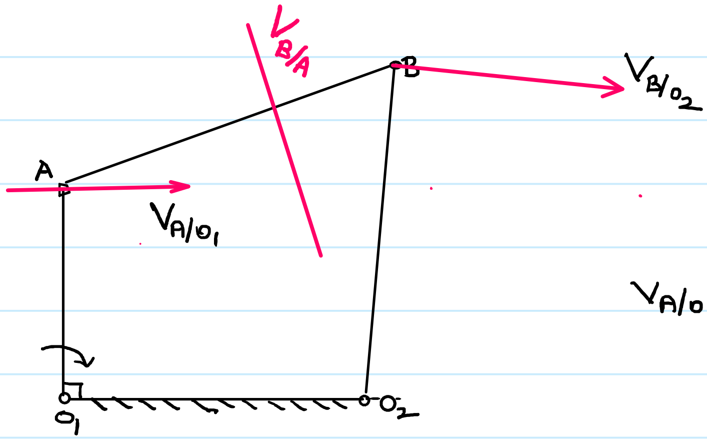


$$\frac{\omega_2}{\omega_3} = \frac{I_{13} \cdot I_{23}}{I_{12} \cdot I_{23}}$$

$$\frac{\omega_2}{1} = \frac{5}{50}$$

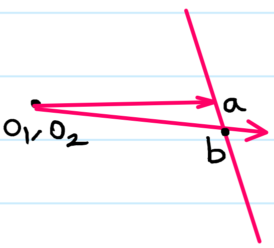
$$\omega_2 = 0.1 \, rad/s.$$

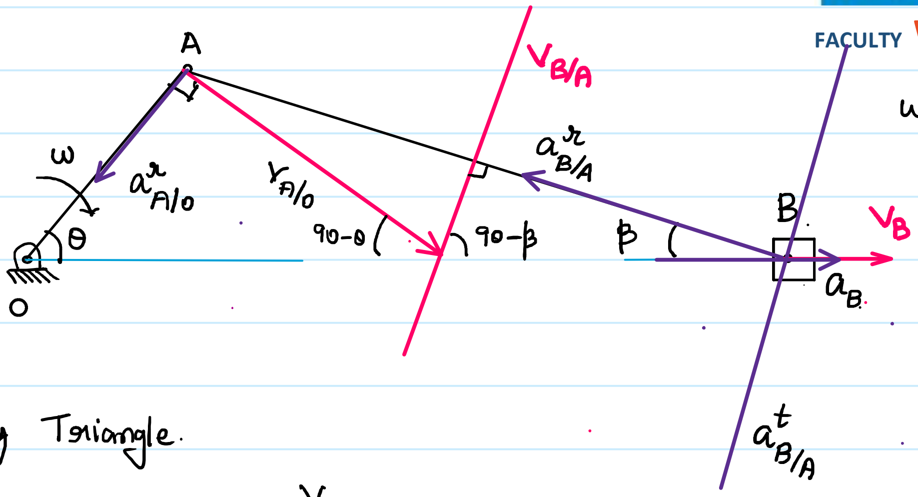
Velocity Analysis Using Relative Velocity Method (Graphical Method)



$V_{A/O_1} = O_1A \cdot \times \omega.$

Velocity Triangle.





$\omega_{crank} =$ constant.

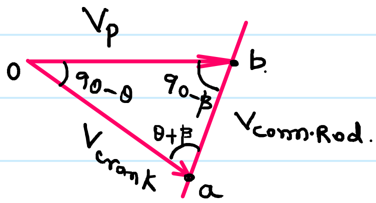
$\alpha_{crank} = 0$

$OA = r =$ crank lenght

$AB = l =$ connecting Rod lenght

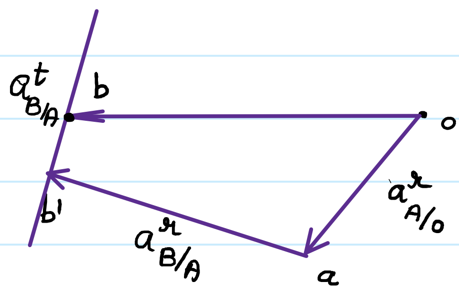
$OA \sin\theta = AB \sin\beta$

Velocity Triangle.



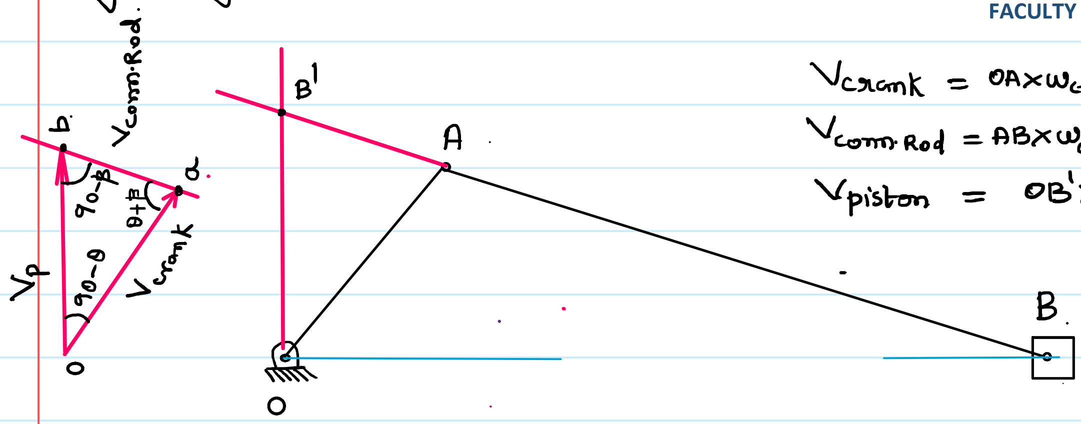
$$\frac{V_{piston}}{\sin(\theta+\beta)} = \frac{V_{crank}}{\sin(90-\beta)} = \frac{V_{conn.Rod}}{\sin(90-\theta)}$$

Acceleration Diagram.



Klein's Construction is the graphical method for performing velocity and acceleration analysis on the configuration diagram.

Velocity triangle.



$V_{crank} = OA \times \omega_{crank}$.

$V_{comm.Rod} = AB \times \omega_{comm.Rod} = AB' \times \omega_{crank}$.

$V_{piston} = OB' \times \omega_{crank}$.

1. Draw a line from the point O perpendicular to the line of stroke.
2. Extend the line of connecting rod on the other side such that it intersects at a point B'.

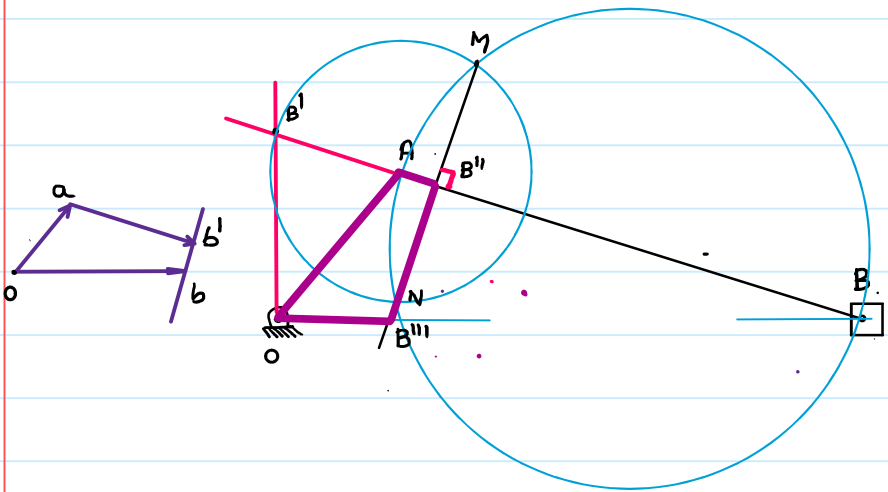
oa – $V_{crank} = OA \cdot \times \omega_{OA}$

ab – $V_{conn.Rod} = AB \cdot \times \omega_{AB}$.

ob – V_{piston}

$$\frac{\text{lenght of Klein Diag.}}{\text{lenght of Vel. Diag.}} = \frac{OA}{oa} = \frac{OB'}{ob} = \frac{AB'}{ab} = \frac{1}{\omega_{crank}}.$$

Velocity Triangle is scaled by a factor ω_{crank} when it is compared with the Klein diagram for Velocity.



$$\frac{OA}{oa} = \frac{AB''}{ab'} = \frac{B''B'''}{b'b} = \frac{OB'''}{ob} = \frac{1}{\omega_c^2}$$

$oa = a^r_{A/O} = OA \cdot \omega^2_{crank}$

$ab' = a^r_{B/A} = AB.\ \omega^2_{conn.Rod}$

$b'b = a^t_{B/A} = AB \cdot \alpha_{conn.Rod}$.

$ob = a_{piston}$

1. With A as centre and AB' as radius draw a circle.
2. With midpoint of AB as centre and AB as diameter draw another circle.
3. The two circles intersects at points M and N.
4. Join M , N and extend the line up till line of stroke of piston.

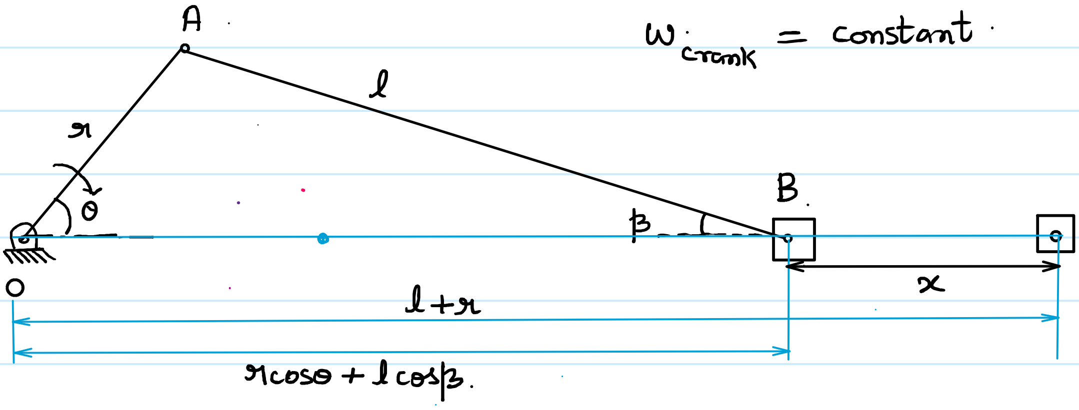
$a^r_{crank} = OA \times \omega^2_{crank}$

$a^r_{conn.Rod} = AB \times \omega^2_{connRod} = AB'' \times \omega^2_{crank}$

$a^t_{conn.Rod} = AB\alpha_{conn.Rod} = B''B''' \times \omega^2_{crank}$

$a_{piston} = OB''' \times \omega^2_{crank}$

$\omega_{crank} = \text{constant}$.



$OA \sin\theta = AB \sin\beta$.

$\boxed{r\sin\theta = l\sin\beta}$

$$\boxed{\sin\beta = \frac{\sin\theta}{(l/r)} = \frac{\sin\theta}{n}.} \Rightarrow \beta = f(\theta)$$

Analysis of Connecting Rod.

$$\frac{d\beta}{dt}\cos\beta = \cos\theta \cdot \frac{d\theta}{dt} \times \frac{1}{n}.$$

$$\omega_{conn.Rod.} = \frac{\omega_{crank}.\cos\theta}{n.\cos\beta.}$$

$$\cos\beta = \sqrt{1-\sin^2\beta}$$
$$= \sqrt{1-\left(\frac{\sin\theta}{n}\right)^2} = \frac{1}{n}\sqrt{n^2-\sin^2\theta}$$

$$\omega_{conn.Rod} = \frac{\omega_{crank}.\cos\theta}{n.\frac{1}{n}.\sqrt{n^2-\sin^2\theta}}$$

Angular Velocity of connecting Rod → $\boxed{\omega_{conn.Rod} = \frac{\omega_{crank}.\cos\theta}{\sqrt{n^2-\sin^2\theta}}}$

Diff. w.r.t. t.

$$\frac{d\omega_{conn.Rod}}{dt} = \alpha_{conn.Rod} = \frac{d}{dt}\left[\frac{\overset{\text{constant}}{\omega_{crank}}.\cos\theta}{\sqrt{n^2-\sin^2\theta}}\right]$$

$$\alpha_{conn.Rod} = \frac{\omega_{crank}.}{\sqrt{n^2-\sin^2\theta}}\frac{d}{dt}(\cos\theta) + \omega_{crank}.\cos\theta\,\frac{d}{dt}\left(\frac{1}{\sqrt{n^2-\sin^2\theta}}\right)$$

$n^2-\sin^2\theta \approx n^2$

$$\boxed{\alpha_{conn.Rod.} = \frac{\omega_{crank}^2.(-\sin\theta)}{n.}}$$

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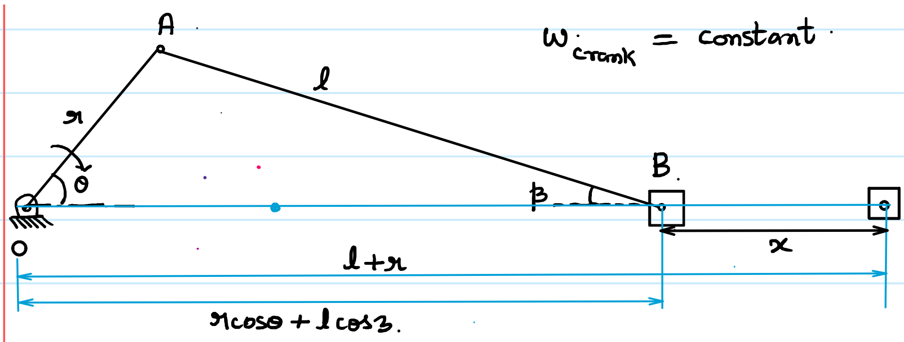

Piston displacement $x = (l+r) - (l\cos\beta + r\cos\theta)$

$$x = r\left[\frac{l}{r}+1\right] - \left(\frac{l}{r}\cdot\cos\beta + \cos\theta\right)$$

$$x = r\cdot\left[(n+1) - (n\cos\beta + \cos\theta)\right]$$

$$\boxed{x = r\cdot\left[(n+1) - \left(n\cdot\frac{1}{n}\sqrt{n^2-\sin^2\theta} + \cos\theta\right)\right] = r\left[(n+1) - \left(\sqrt{n^2-\sin^2\theta} + \cos\theta\right)\right]}$$

$$x = f(\theta)$$

Velocity of Piston $\frac{dx}{dt} = V_{piston} = r\left[0 - \left(\frac{1}{2\sqrt{n^2-\sin^2\theta}}(0 - 2\sin\theta\cos\theta) - \sin\theta\right)\right]\frac{d\theta}{dt}$

$$V_{piston} = r\cdot\omega\cdot\left[\sin\theta + \frac{\sin 2\theta}{2\cdot\sqrt{n^2-\sin^2\theta}}\right]$$

$n^2 - \sin^2\theta \simeq n^2$

$$\boxed{V_{piston} = r\omega\cdot\left[\sin\theta + \frac{\sin 2\theta}{2n}\right]}$$

Acceleration of Piston

$$a_{piston} = \frac{dV_p}{dt} = \frac{d}{dt}\left[r\omega\left[\sin\theta + \frac{\sin 2\theta}{2n}\right]\right]$$

$$a_{piston} = r\omega\left[\cos\theta + \frac{2\cos 2\theta}{2n}\right]\cdot\frac{d\theta}{dt}$$

$$a_{piston} = r\omega^2\cdot\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

Velocity of piston is maximum when $\left[\sin\theta + \frac{\sin 2\theta}{2n}\right] = 1$.

$(V_p)_{max} = r\omega$. $\boxed{\theta \le 90^\circ}$

Max.

Acceleration of piston @ $\theta = 0^\circ$ $a_p = r\omega^2\left[1 + \frac{1}{n}\right]$

Acceleration of piston @ $\theta = 90^\circ$ $a_p = r\omega^2\left[0 - \frac{1}{n}\right]$

Acceleration of Piston @ $\theta = 180^\circ$ $a_p = r\omega^2\left[-1 + 1/n\right]$

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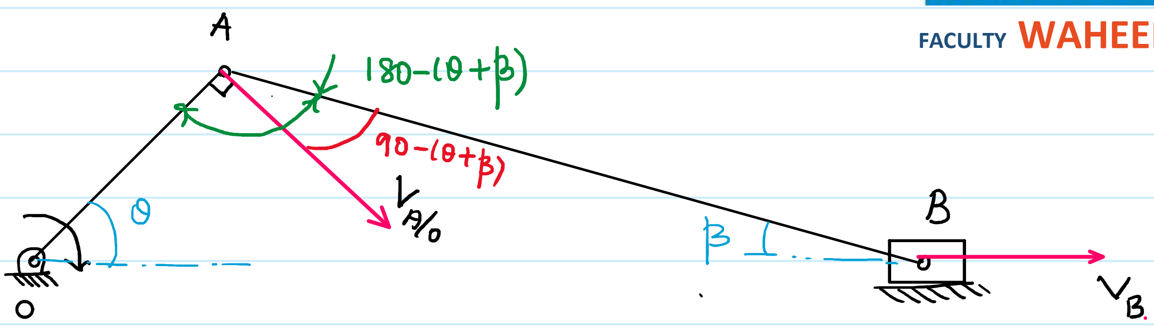
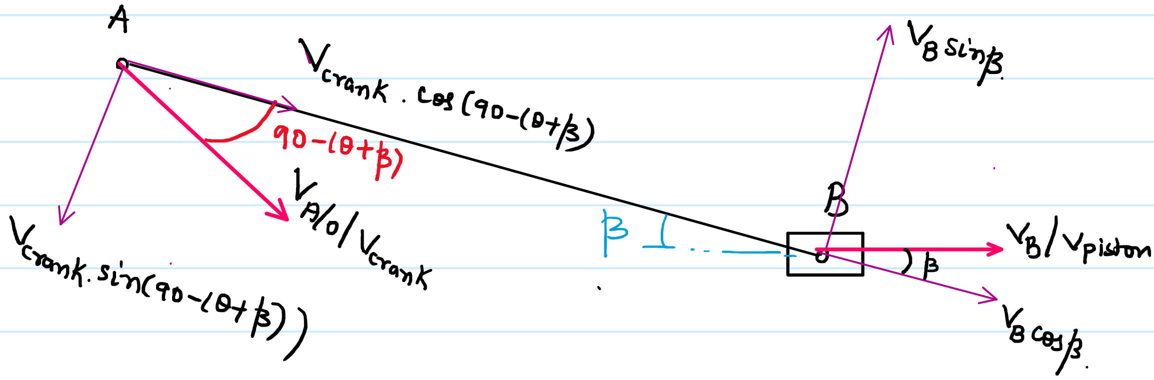


109 CRPQ.



Translation $V_{crank} \cdot \cos(90-(\theta+\beta)) = V_B \cos\beta$

Velocity of Piston/X-head. $V_B = \dfrac{V_{crank} \cdot \sin(\theta+\beta)}{\cos\beta}$

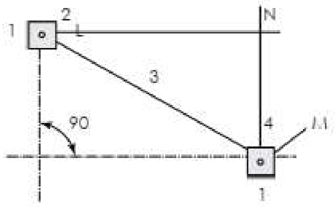
Rotation $\omega_{conn.Rod} = \dfrac{V_{crank} \cdot \sin(90-(\theta+\beta)) - (-V_B \cdot \sin\beta)}{AB}$

$$\omega_{conn.Rod} = \frac{V_{crank} \cdot \cos(\theta+\beta) + V_B \cdot \sin\beta}{AB}$$

$I_{13}/I_{12}\ \&\ I_{23},\quad I_{13}/I_{34}\ \&\ I_{14}$

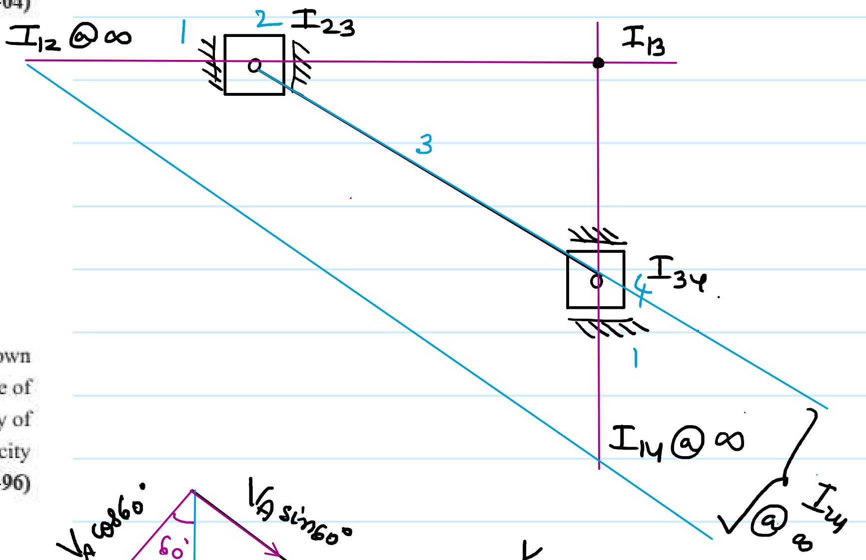
$I_{24}/I_{12}\ \&\ I_{14},\quad I_{24}/I_{23}\ \&\ I_{34}$

22. The figure below shows a planar mechanism with single degree of freedom. The instant center 24 for the given configuration is located at a position (GATE-04)

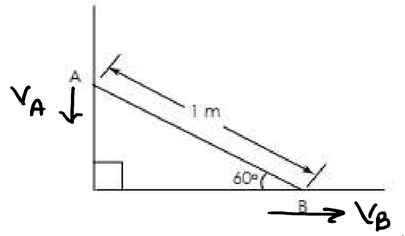


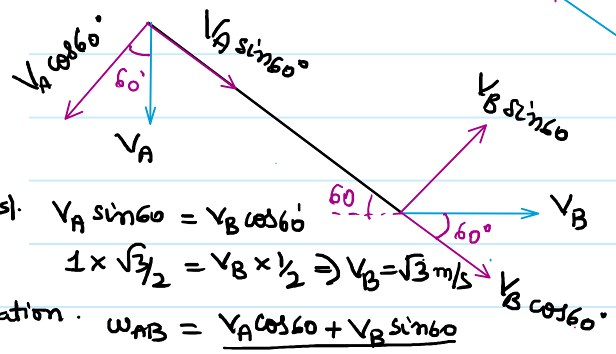
(a) L (b) M

(c) N (d) ∞



23. A rod of length 1 m is sliding in a corner as shown in fig. At an instant when the rod makes an angle of 60 degree with the horizontal plane, the velocity of point 'A' on the rod is 1 m/s. The angular velocity of the rod at this instant is (GATE-96)



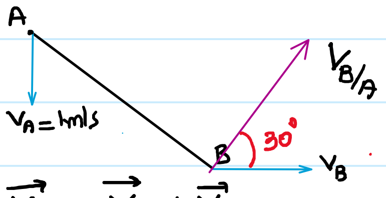


Trans. $V_A \sin 60 = V_B \cos 60$

$1 \times \sqrt{3}/2 = V_B \times 1/2 \Rightarrow V_B = \sqrt{3}$ m/s

Rotation. $\omega_{AB} = \dfrac{V_A \cos 60 + V_B \sin 60}{AB}$

$\omega_{AB} = \dfrac{(1 \times \frac{1}{2}) + (\sqrt{3} \times \frac{\sqrt{3}}{2})}{1} = 2$ rad/s



$\vec{V_B} = \vec{V_A} + \vec{V_{B/A}}$

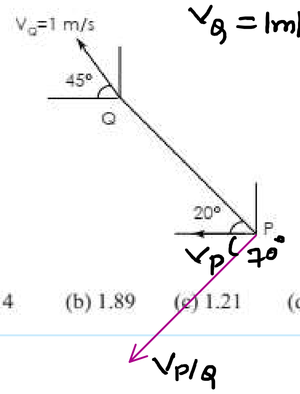
$V_B(\hat{i}) = 1(-\hat{j}) + AB \cdot \omega_{AB} \cdot (\hat{i}\cos 30 + \hat{j}\sin 30)$

i-coff. $V_B = AB \cdot \omega_{AB} \cdot \cos 30° = 1 \times 2 \times \sqrt{3}/2 = \sqrt{3}$ m/s

j-coff $0 = -1 + 1 \cdot \omega_{AB} \cdot \sin 30 \Rightarrow \omega_{AB} = 1/\sin 30 = +2$ rad/s

27. A rigid link PQ is 2 m long and oriented at 20° to the horizontal as shown in the figure. The magnitude and direction of velocity V_Q, and the direction of velocity V_p are given. The magnitude of V_p (in m/s) at this instant is (GATE-14)

$V_Q = 1$ m/s.



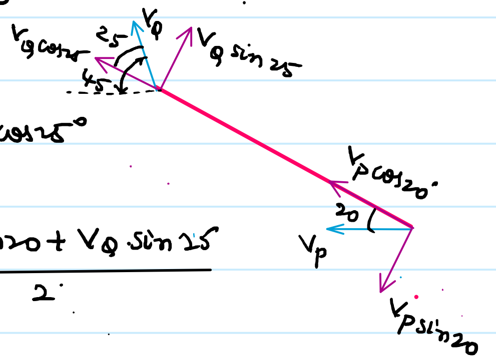
(a) 2.14 (b) 1.89 (c) 1.21 (d) 0.96

- Method 1

$V_P \cos 20° = V_Q \cos 25°$

Rotation

$\omega_{PQ} = \dfrac{V_P \cdot \sin 20 + V_Q \sin 25}{2}$



Method 2

$\vec{V_P} = \vec{V_Q} + \vec{V_{P/Q}}$

$V_P \cdot (-\hat{i}) = 1(-\hat{i}\cos 45° + \hat{j}\sin 45) + PQ \cdot \omega_{PQ} (-\hat{i}\cos 70° - \hat{j}\sin 70)$

$V_P(-\hat{i}) = 1(-\hat{i}\cos 45° + \hat{j}\sin 45) + 2 \cdot \omega_{PQ} \cdot (-\hat{i}\cos 70 - \hat{j}\sin 70)$

$V_P = 0.96$ m/s.

$\omega_{PQ} = 0.37$ rad/s.

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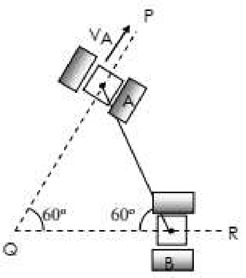
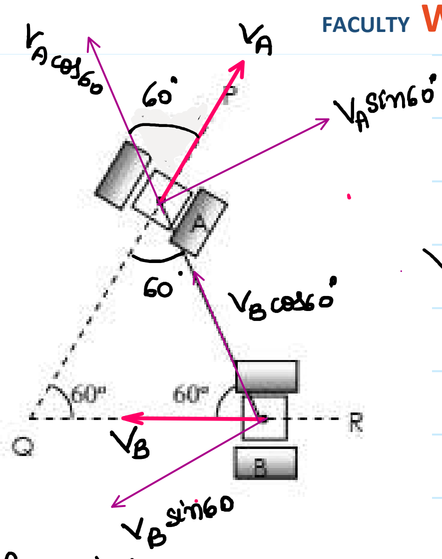
30. The rod AB, of length 1 m, shown in the figure is connected to two sliders at each end through pins. The sliders can slide along QP and QR. If the velocity V_A of the slider at A is 2 m/s, the velocity of the midpoint of the rod at this instant is ____ m/s.



(GATE – 16)



Translation

$V_A\cos60 = V_B\cos60°$

$V_A = V_B = 2\,m/s.$

$V_{midpoint} = V^{Trans.} = V_A\cdot\cos60° = 1\,m/s.$

$V_A\sin60° = V_B\cdot\sin60°$

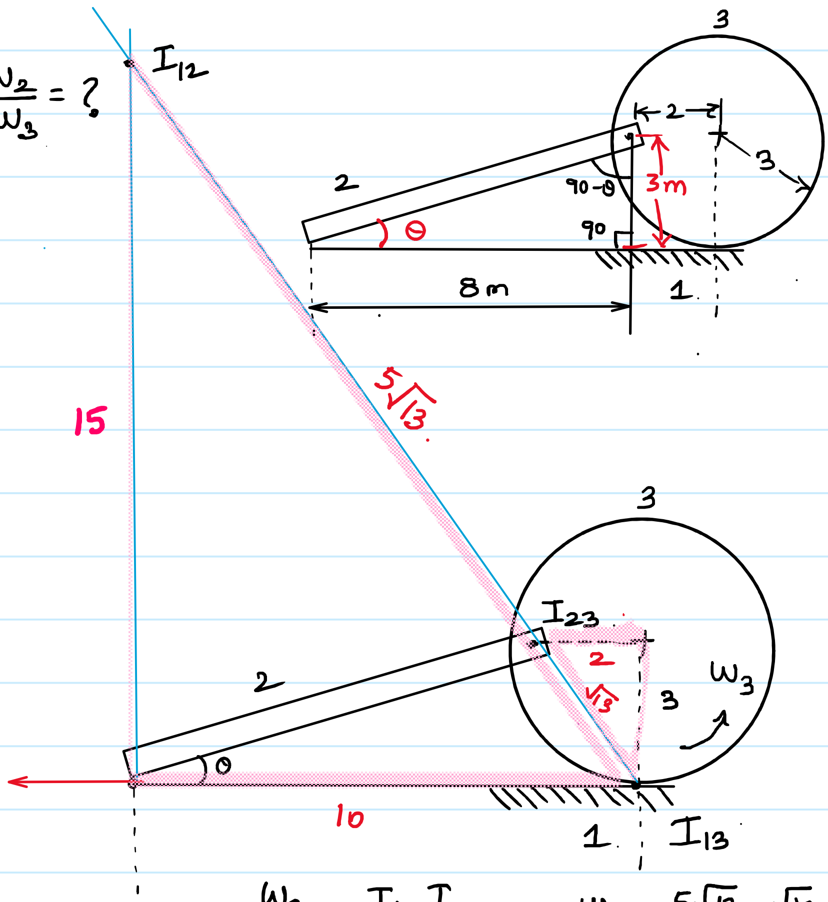
Mid point of AB is also centre of rotation of Rod AB

@ centre of rotation only Translational velocity is present.

@ this point velocity is minimum.

G-23.

$\frac{\omega_2}{\omega_3} = ?$



$\tan\theta = 3/8$

$\theta = 20.55°$

$V_{cm} = r\omega_3$

$V_{cm} = 3\cdot\omega_3.$

Resultant Velocity

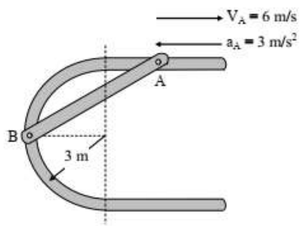
$= (V_{cm}^2 + r\omega)^{1/2}$

$= \sqrt{(3\omega_3)^2 + (2\omega_3)^2} = \sqrt{13}\,\omega_3.$

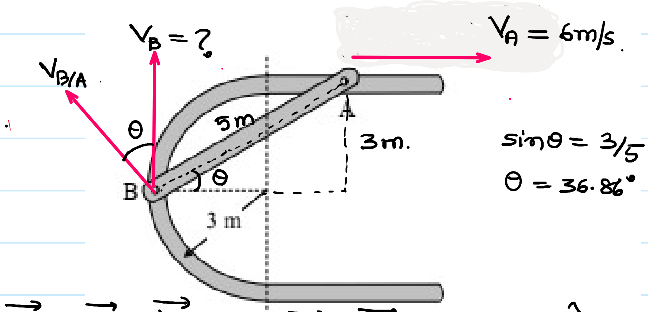
$$\frac{\omega_2}{\omega_3} = \frac{I_{13}\cdot I_{23}}{I_{12}\cdot I_{23}} \Rightarrow \frac{\omega_2}{\omega_3} = \frac{5\sqrt{13}-\sqrt{13}}{\sqrt{13}} = 4.$$

47. The rigid rod (AB) of length 5 m is confined to move along the path due to the pins at its ends. At the instant shown, point A has the motion shown below. Which of the following calculated results related to the above problem statement is/are correct ?

$V_B = ?$
$a_B = ?$



(a) The magnitude of velocity of point B at this instant is 8 m/s
(b) The magnitude of velocity of point B at this instant is 4 m/s
(c) The magnitude of acceleration of point B at this instant is 24.8 m/s²
(d) The magnitude of acceleration of point B at this instant is 21.4 m/s²



$\sin\theta = 3/5$
$\theta = 36.86°$

$$\vec{V_B} = \vec{V_A} + \vec{V_{B/A}} \Rightarrow \vec{V_B} = \vec{V_A} + AB\cdot\omega_{AB}\,(\hat{e_t})$$

$$V_B(\hat{j}) = 6(\hat{i}) + 5\,\omega_{AB}(-\hat{i}\sin 36.86 + \hat{j}\cos 36.86)$$

i-coff $\quad 0 = 6 - 5\omega_{AB}\cdot\sin 36.86$

$$\omega_{AB} = 2\,rad/s.$$

j-coff. $\quad V_B = 5\omega_{AB}\cos 36.86 = 5\times2\times0.8 = 8\,m/s.$

$$V_B = OB\cdot\omega_B \Rightarrow \omega_B = 8/3 = 2.667\,rad/s.$$

Acceleration Analysis.

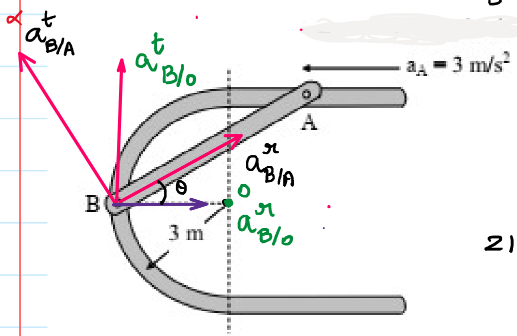
$$\vec{a_B} = \vec{a_A} + \vec{a_{B/A}}$$

$$a_B^r + a_B^t = 3(-\hat{i}) + \vec{a}^r_{B/A} + \vec{a}^t_{B/A}$$

$a_B^t = OB\cdot\alpha_{OB}$ $\qquad a^t_{B/A} = AB\cdot\alpha_{AB} = 5\alpha_{AB}$

$$a_B^r = \frac{V_B^2}{OB} = \frac{8^2}{3} = \frac{64}{3} = 21.33$$

$$a^r_{B/A} = AB\cdot\omega_{AB}^2 = 5\times2^2 = 20\,m/s^2$$



$$21.33(\hat{i}) + 3\alpha_{OB}(\hat{j}) = 3(-\hat{i}) + 20(\hat{i}\cos\theta + \hat{j}\sin\theta)$$
$$+ 5\alpha_{AB}\cdot(-\hat{i}\sin\theta + \hat{j}\cos\theta)$$

$$21.33(\hat{i}) + 3\alpha_{OB}(\hat{j}) = -3\hat{i} + 20\cos 36.86\,\hat{i} + 20\sin 36.86\,\hat{j} - 5\alpha_{AB}\sin 36.86 + 5\alpha_{AB}\cos 36.86$$

i-coff. $\quad 21.33 = -3 + 20\cos 36.86 - 5\alpha_{AB}\sin 36.86 \qquad \alpha_{AB} = -2.77\,rad/s^2$

$a^t_{B/A}$ must be in 4^{th} quadrant.

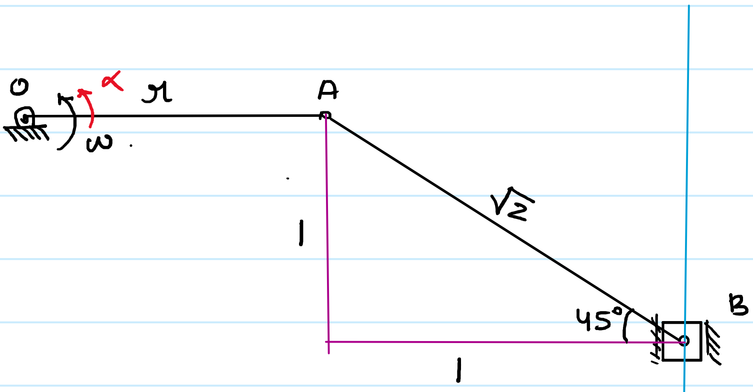
j-coff. $\quad 3\alpha_{OB} = 20\sin 36.86 + 5\alpha_{AB}\cos 36.86 \Rightarrow 3\alpha_{OB} = 20\sin 36.86 + 5(-2.77)\cos 36.86$

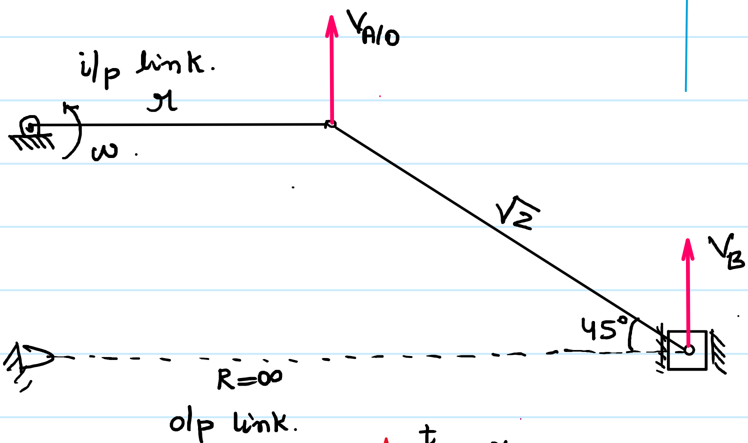
$$\alpha_{OB} = 0.305\,rad/s^2$$

$$a_B = \sqrt{(a_B^r)^2 + (a_B^t)^2} \Rightarrow a_B = \sqrt{(21.33)^2 + (3\times0.305)^2} = 21.34\,m/s^2$$

G-19

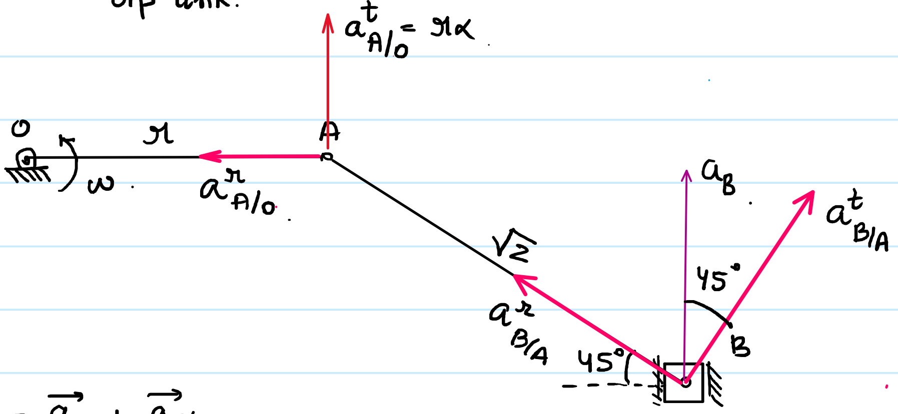
$a_{piston} = ?$ $\omega = \text{constant}$





$V_{A/O} = V_B = r\omega$

$V_{AB} = 0$



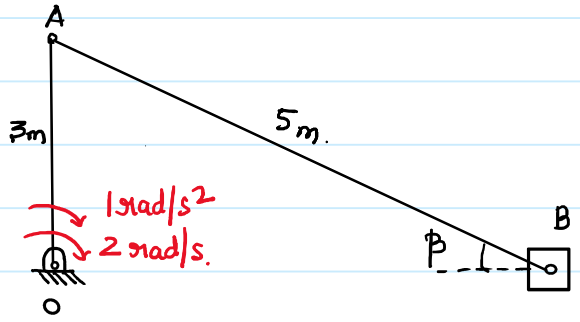
$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

$a_B(\hat{j}) = r\cdot\omega^2(-\hat{i}) + r\alpha(\hat{j}) + \vec{a}^{\,r}_{B/A} + \vec{a}^{\,t}_{B/A}$

$a_B(\hat{j}) = r\omega^2(-\hat{i}) + r\alpha(\hat{j})\ \ \sqrt{2}\cdot\omega_{AB}^2\cdot(\underbrace{\cancel{-\hat{i}}}_{0}\cos45° + \hat{j}\sin45) + \sqrt{2}\cdot\alpha_{AB}\cdot(\hat{i}\sin45° + \hat{j}\cos45°)$

$\hat{i}$ - coff $0 = -r\omega^2 + \sqrt{2}\cdot\alpha_{AB}\sin45$ $\alpha_{AB} = r\omega^2$.

$\hat{j}$ - coff $a_B = \sqrt{2}\cdot\alpha_{AB}\cdot\cos45° + r\alpha = \sqrt{2}\times r\omega^2\times\cos45° = r\omega^2 + r\alpha$.



$\theta = 90^\circ$

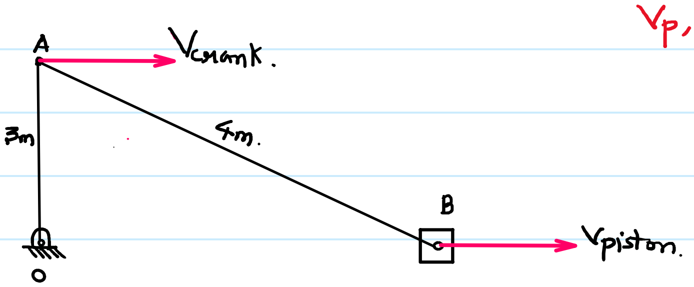
$\beta = \sin^{-1}(3/5)$

$\beta = 36.86^\circ$

$\omega_{conn.Rod}$, $\alpha_{Conn.Rod}$ = ?

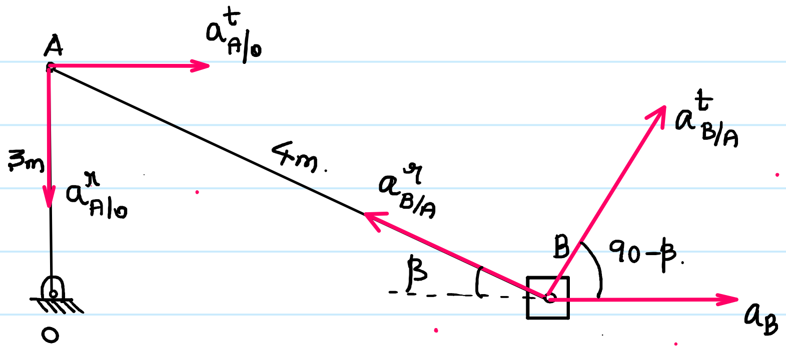
V_p, a_p = ?

Velocity Analysis.



$V_{piston} = r \cdot \omega = 3 \times 2 = 6\,m/s.$

$V_{conn.Rod} = 0 \Rightarrow \omega_{conn.Rod} = 0$



$a^r_{A/o} = OA \cdot \omega^2_{OA}$

$= 3 \times 2^2 = 12\,m/s^2$

$a^t_{A/o} = OA \cdot \alpha.$

$= 3 \times 1 = 3\,m/s^2$

$a^r_{B/A} = AB \cdot \cancel{\omega^2_{AB}} = 0$

$a^t_{B/A} = AB \cdot \alpha_{AB} = 5\alpha_{AB}$

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$$a_B(\hat{i}) = \vec{a}^{\,r}_{A/o} + \vec{a}^{\,t}_{A/o} + \cancel{\vec{a}^{\,r}_{B/A}} + a^t_{B/A}$$

$$a_B(\hat{i}) = 12(-\hat{j}) + 3(\hat{i}) + 5\alpha_{AB}(\hat{i}\cos(90-\beta) + \hat{j}\sin(90-\beta))$$

$$a_B(\hat{i}) = -12\hat{j} + 3\hat{i} + 5\alpha_{AB}.(\hat{i}\sin 36.86 + \hat{j}\cos 36.86)$$

j-coff. $\quad 0 = -12 + 5\alpha_{AB}.\cos 36.86$

$$\alpha_{AB} = 3\,rad/s^2$$

i-coff. $\quad a_B = 3 + 5\alpha_{AB}.\sin 36.86 \Rightarrow a_B = 3 + 5(3) \times 0.6$

$$a_B = 12\,m/s^2$$