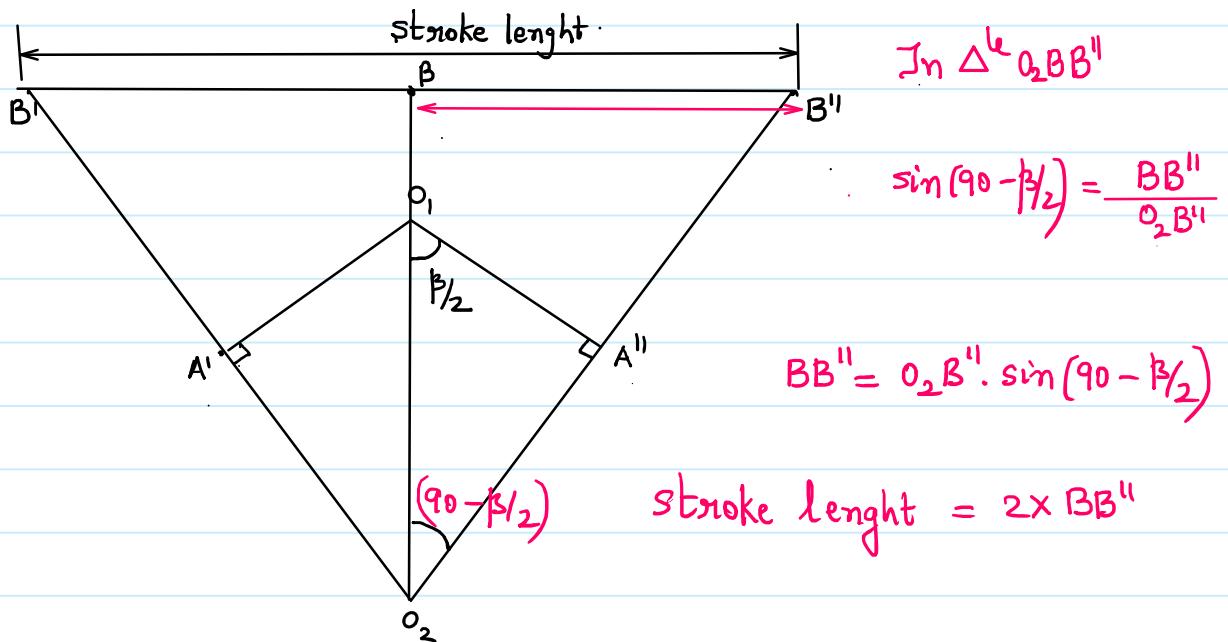


In $\triangle O_1O_2A''$

$$\cos \beta_{2/2} = \frac{O_1A''}{O_1O_2}$$

$$\cos \beta_{2/2} = \frac{\text{length of crank}}{\text{length of fixed link}}$$



$$\text{stroke length} = 2 \times O_2B'' \times \cos(\beta_{2/2})$$

$$= \frac{2 \times \text{length of slotted bar} \times \text{length of crank}}{\text{length of fixed link}}$$

Observations

- ★ The stroke length in Whitworth Quick Return Mechanism is dependent on **extension of slotted bar** diametrically opposite to the slider.
- ★ In the Crank Slotted lever Mechanism the stroke length is dependent on **length of slotted bar**, **length of Crank** and **length of Fixed link**.

case. If Q.R.R. = 2 or Q.R.R. = $\frac{1}{2}$

$$Q.R.R. = \frac{\theta_{cutting}}{\theta_{Return}} = 2 \quad \checkmark$$

$$Q.R.R. = \frac{1}{2} = \frac{\theta_{Return}}{\theta_{cutting}}$$

$$Q.R.R. = \frac{360 - \beta}{\beta} = 2 \Rightarrow \beta = 120^\circ$$

Whitworth mechanism. $\cos(\beta/2) = \frac{\text{length of fixed link} / L_{min.}}{\text{length of crank.}}$

$$\cos\left(\frac{120}{2}\right) = \frac{1}{2} = \frac{\text{length of fixed link} / L_{min.}}{\text{length of crank.}} = \frac{1}{Q.R.R.}$$

Crank slotted lever mechanism.

$$\cos\left(\frac{\beta}{2}\right) = \frac{\text{length of crank} / L_{min.}}{\text{length of fixed link.}}$$

$$\cos\left(\frac{120}{2}\right) = \frac{1}{2} = \frac{\text{length of crank} / L_{min.}}{\text{length of fixed link.}} = \frac{1}{Q.R.R.}$$

stroke length

Whitworth Mechanism

stroke length = $2 \times \text{extension of slotted bar}$
diametrically opposite to link 3 / slider



Crank slotted lever Mechanism

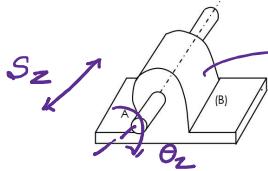
stroke = $2 \times \text{length of slotted bar} \times \text{length of crank}$

$$\text{If } Q.R.R. = 2 \quad \frac{1}{2} = \frac{\text{length of crank} / L_{min.}}{\text{length of fixed link.}} \quad \text{length of fixed link.}$$

stroke length = $2 \times \text{length of slotted bar} \times \frac{1}{2}$

stroke length = length of slotted bar.

01. A round bar A passes through the cylindrical hole in B as shown in the given figure. Which one of the following statements is correct in this regard?



→ cylindrical pair

$$D.o.F = 2$$

- (a) The two links shown form a kinematic pair
 - (b) The pair is completely constrained
 - (c) The pair has incomplete constraint
 - (d) The pair is successfully constrained

— / Lower pair.

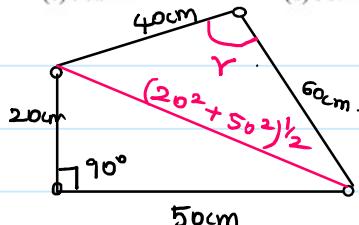
03. For a four-bar linkage in **toggle position**, the value of mechanical advantage is

Statement for Common data Q 04 & 05

In a crank rocker mechanism the lengths of the fixed link, crank, coupler and rocker are respectively 50, 20, 40 and 60 cm.

$$\rightarrow \phi = 0^\circ, 180^\circ$$

04. Extreme positions of rocker measured with the fixed link are
 (a) 60° & 134.43° (b) 60° & 13.77°
 (c) 134.43° & 13.77° (d) 65.37° & 18.19°



Statement for Common data Q. 06 & Q. 07

In an offset slider crank mechanism the length of crank is 20 cm and that of the connecting rod is 40 cm. The offset between the line of stroke and the centre of crank is 10 cm

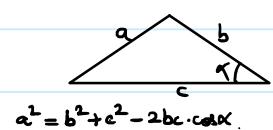
07. The quick return ratio is
(a) 1.0
(c) 1.12

$$\sin \theta_2 = \frac{e}{l-r} \Rightarrow \theta_1 = \sin^{-1} \left(\frac{10}{40-20} \right) = 30^\circ$$

$$\sin\theta_1 = \frac{e}{l+r} \Rightarrow \theta_1 = \sin^{-1}\left(\frac{10}{40+20}\right) = 9.594$$

$$\begin{aligned}
 \text{Stroke length} &= \sqrt{(l+x)^2 - e^2} - \sqrt{(l-x)^2 - e^2} \\
 &= \sqrt{(40+20)^2 - 10^2} - \sqrt{(40-20)^2 - 10^2} \\
 &= 41.8 \text{ cm.}
 \end{aligned}$$

$$\begin{aligned} Q.R.R. &= \frac{180 + (\theta_2 - \theta_1)}{180 - (\theta_2 - \theta_1)} \\ Q.R.R. &= \frac{180 + (30 - 9.594)}{180 - (30 - 9.594)} \\ Q.R.R. &= 1.25 \end{aligned}$$



08. In a quick return mechanism of the crank and slotted lever type the length of the crank is 20cm and that of the fixed link is also 40cm. The quick return ratio is



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length of crank = 20 cm. 7 Ratio 1:2

length of fixed link = 40cm.

7 Ratio 1:2

$$Q \cdot R \cdot R' = 2$$

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It is the study of parameters that describe the relative motion between the connected bodies without considering the effect of force/Torque acting on the body/link.

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Mass or inertia of the link is not considered.

1. Velocity Analysis
2. Acceleration Analysis. (Vector Alegbra)

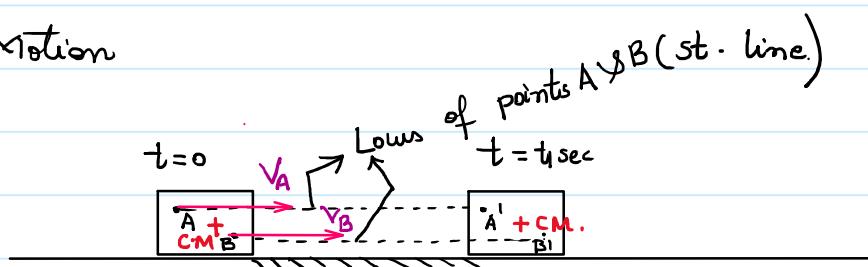
Types of Tensors.

1. Zero Order Tensor — Quantities which has only magnitude/ scalers.
Ex:- time, length, Mass
2. First order Tensor — Quantities which has magnitude & direction/ vectors.
Ex:- Force, Velocity.
3. Second Order Tensor — Quantities which has magnitude, direction and plane of reference is called Tensor
Ex:- stress, strain, Moment of Inertia.

Types of Motion

1. Rectilinear Motion
2. Curvilinear Motion
3. Angular Motion
4. General Plane Motion/Rolling Motion

Rectilinear Motion



$$x_A = x_B = x_{cm}.$$

$$V_A = V_B = V_{cm}.$$

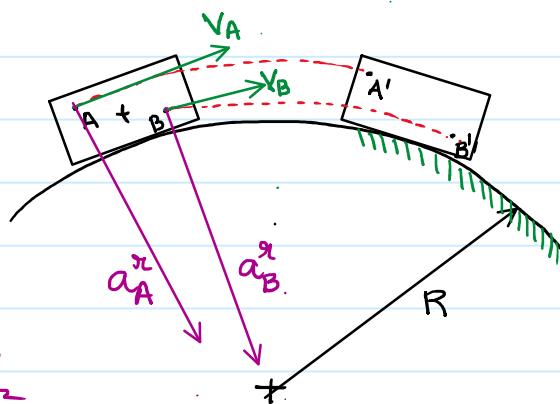
$$a_A = a_B = a_{cm}.$$

$$a_{cm}^x = \frac{V_{cm}^2}{R}$$

$$\text{As } R \rightarrow \infty \quad a_{cm}^x \rightarrow 0$$

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2. Curvilinear Motion



Radial acceleration

$$a_A^r = a_B^r = a_{cm}^r$$

$$V_A = V_B = V_{cm}$$

$$a_A = a_B = a_{cm}$$

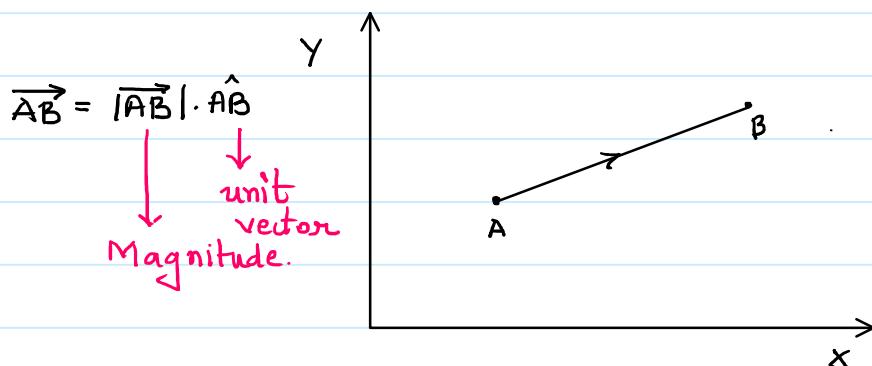
$$a_A^r = \frac{V_A^2}{R}$$

$$a_B^r = \frac{V_B^2}{R}$$

$$a_{cm}^r = \frac{V_{cm}^2}{R}$$

Locus of points A & B
is arc of circle.

Vectors

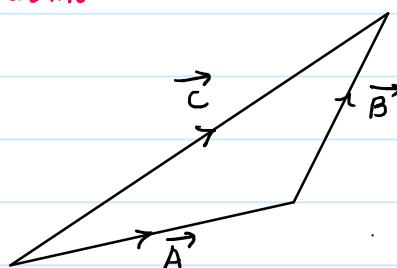


Addition of Vectors

$$\vec{A} + \vec{B} = \vec{C}$$

Head
Tail

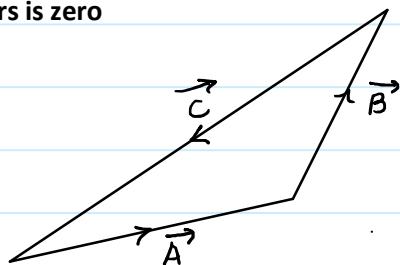
Resultant



In a closed polygon the summation of all vectors is zero

$$\vec{A} + \vec{B} + (-\vec{C}) = 0$$

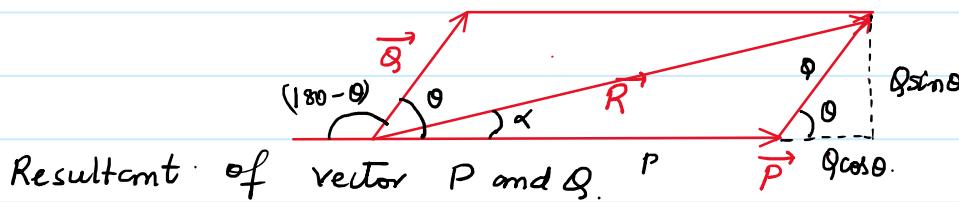
↓
Equilibrant



$$\vec{F} = \vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E}$$

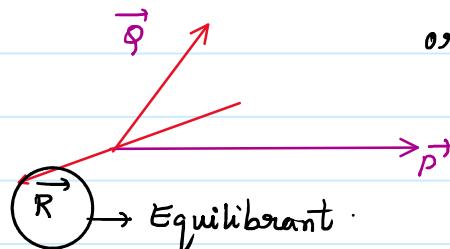
Tail
Head.

Parallelogram law of Vectors



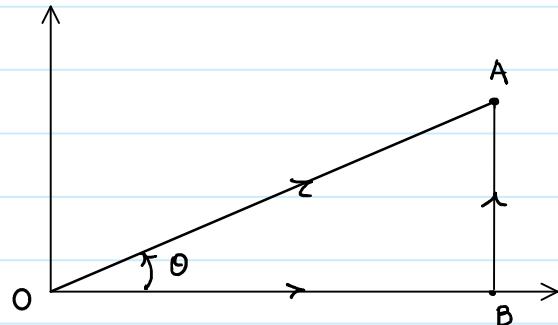
$$R^2 = P^2 + Q^2 + 2PQ \cos \theta$$

$$\tan \alpha = \frac{Q \sin \theta}{P + Q \cos \theta}$$



orientation of vectors
when a body is in
equilibrium.

Resolution of Vector

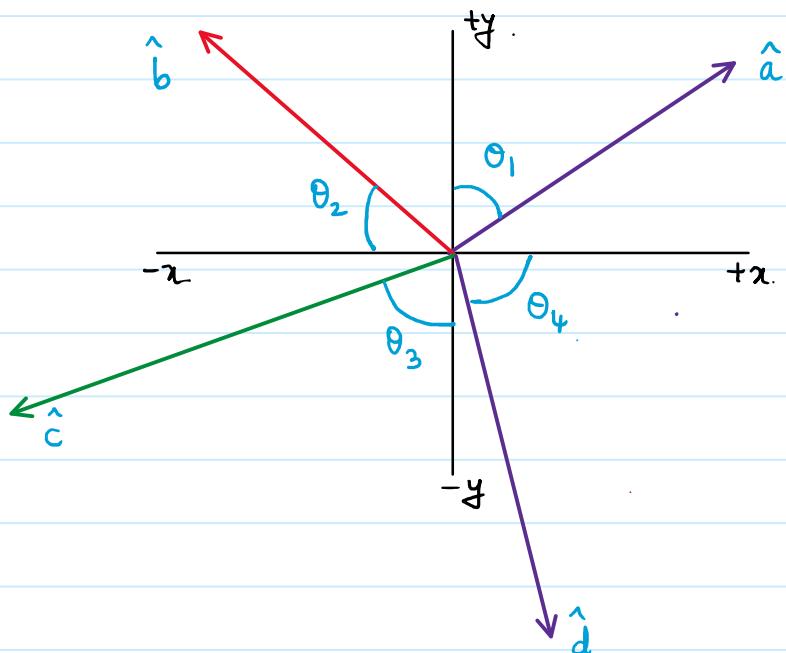


$$\vec{OA} = \vec{OB} + \vec{AB}$$

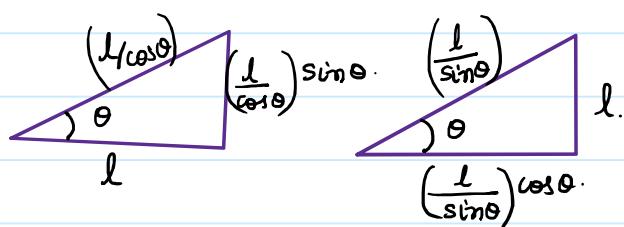
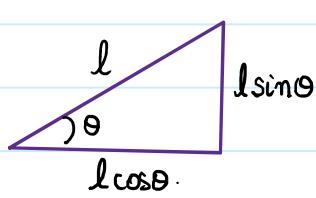
$$|\vec{OA}| \cdot \hat{\vec{OA}} = |\vec{OB}| \cdot \hat{\vec{OB}} + |\vec{AB}| \cdot \hat{\vec{AB}}$$

$$\hat{\vec{OA}} = \frac{|\vec{OB}|}{|\vec{OA}|} \cdot \hat{\vec{OB}} + \frac{|\vec{AB}|}{|\vec{OA}|} \cdot \hat{\vec{AB}}$$

$$\hat{\vec{OA}} = \cos \theta \hat{i} + \sin \theta \hat{j}$$



$$\begin{aligned}\hat{a} &= \sin\theta_1 \hat{i} + \cos\theta_1 \hat{j} \\ \hat{b} &= -\cos\theta_2 \hat{i} + \sin\theta_2 \hat{j} \\ \hat{c} &= -\sin\theta_3 \hat{i} - \cos\theta_3 \hat{j} \\ \hat{d} &= \cos\theta_4 \hat{i} - \sin\theta_4 \hat{j}\end{aligned}$$



Dot product

$$\begin{aligned}\hat{i} \cdot \hat{i} &= \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1 \\ \hat{i} \cdot \hat{j} &= \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0\end{aligned}$$

$$\cos\theta = \frac{\hat{a} \cdot \hat{b}}{|\hat{a}| |\hat{b}|}$$

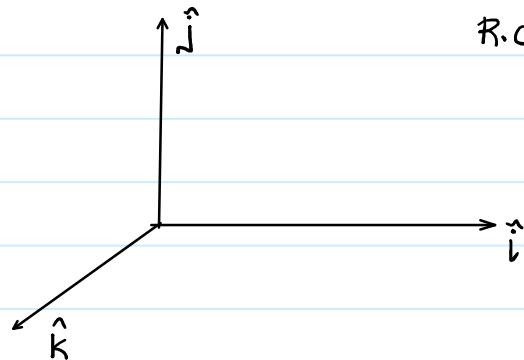
Cross Product

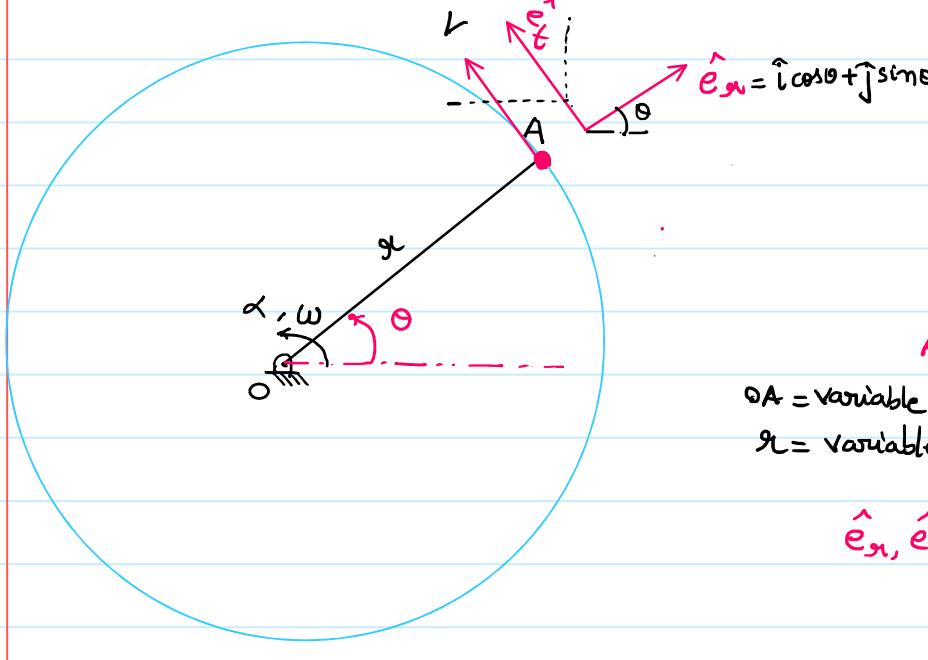
$$\hat{i} \times \hat{j} = \hat{k}$$

$$\hat{j} \times \hat{k} = \hat{i}$$

$$-\hat{i} \times \hat{j} = -\hat{k}$$

$$-\hat{i} \times -\hat{k} = -\hat{j}$$



ω - Angular Velocity α - Angular Acceleration

A - point A is having

 θA = variable \leftarrow relative motion r_A = variable along the rod. \hat{e}_r, \hat{e}_t - unit vectorsdescribing radial and
tangential direction. $OA = r = \text{variable}$.

$$\hat{OA} = \hat{i} \cos\theta + \hat{j} \sin\theta$$

$$\vec{OA} = |OA| \cdot \hat{OA}$$

$$\vec{r} = r [\hat{i} \cos\theta + \hat{j} \sin\theta]$$

Velocity $\vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt} [\hat{i} \cos\theta + \hat{j} \sin\theta] + r \cdot \frac{d}{dt} [\hat{i} \cos\theta + \hat{j} \sin\theta]$

$$\vec{v} = \dot{r} \hat{e}_r + r \cdot \underbrace{[\hat{-i} \sin\theta + \hat{j} \cos\theta]}_{\hat{e}_t} \cdot \frac{d\theta}{dt} \rightarrow \dot{\theta}$$

$$\boxed{\vec{v} = \dot{r} \hat{e}_r + r \dot{\theta} \hat{e}_t}$$

 \dot{r} - Rate of change in position of point A along Rod. $\dot{\theta}$ - Angular Velocity of Rod.

if $r = \text{constant}$ $\dot{r} = 0$ $\boxed{\vec{v} = \dot{r} \cdot \dot{\theta} \hat{e}_t = r \omega \cdot (\hat{e}_t)}$

$$\vec{v} = \dot{r} \hat{e}_r + r \dot{\theta} (\hat{e}_t)$$

$$\frac{uv' + vu'}{uv} \quad \frac{uv \cdot w' + u'vw + uw \cdot v'}{uvw}$$

Acceleration $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt} [\dot{r} \hat{e}_r] + \frac{d}{dt} [r \dot{\theta} \hat{e}_t]$

$$\vec{a} = \ddot{r} \hat{e}_r + \dot{r} \frac{d}{dt} [\hat{e}_r] + \frac{d\dot{r}}{dt} \cdot \dot{\theta} \cdot \hat{e}_t + r \cdot \frac{d\theta}{dt} \cdot \hat{e}_t + r \cdot \dot{\theta} \cdot \frac{d}{dt} [\hat{e}_t]$$

$$\vec{a} = \ddot{\alpha} \hat{e}_n + \dot{\alpha} \frac{d}{dt} [\hat{e}_n] + \frac{d\omega}{dt} \cdot \dot{\theta} \cdot \hat{e}_t + \alpha \cdot \frac{d\theta}{dt} \cdot \hat{e}_t + \alpha \cdot \dot{\theta} \cdot \frac{d}{dt} [\hat{e}_t]$$

$$\vec{a} = \ddot{\alpha} [i \cos \theta + j \sin \theta] + \dot{\alpha} \frac{d}{dt} [i \cos \theta + j \sin \theta] + \dot{\alpha} \dot{\theta} [-i \sin \theta + j \cos \theta] + \alpha \cdot \ddot{\theta} [-i \sin \theta + j \cos \theta] + \alpha \dot{\theta} \frac{d}{dt} [-i \sin \theta + j \cos \theta]$$

$$\vec{a} = \ddot{\alpha} [i \cos \theta + j \sin \theta] + \dot{\alpha} \underbrace{[-i \sin \theta + j \cos \theta]}_{(\hat{e}_n)} \dot{\theta} + \dot{\alpha} \dot{\theta} \underbrace{[-i \sin \theta + j \cos \theta]}_{\hat{e}_t} + \alpha \ddot{\theta} \underbrace{[-i \sin \theta + j \cos \theta]}_{\hat{e}_t} + \alpha \dot{\theta}^2 \underbrace{[-i \cos \theta - j \sin \theta]}_{-\hat{e}_n}$$

$$\vec{a} = \ddot{\alpha} (\hat{e}_n) + \dot{\alpha} \dot{\theta} (\hat{e}_t) + \dot{\alpha} \dot{\theta} (\hat{e}_t) + \alpha \ddot{\theta} (\hat{e}_t) + \alpha \dot{\theta}^2 (-\hat{e}_n)$$

$$\vec{a} = \ddot{\alpha} (\hat{e}_n) + \alpha \dot{\theta}^2 (-\hat{e}_n) + 2\dot{\alpha} \dot{\theta} (\hat{e}_t) + \alpha \ddot{\theta} (\hat{e}_t)$$

Acceleration of point A
along the Rod.

Radial
acceleration
of rod OA.

Coriolis
acceleration.

Tangential Acceleration
of Rod OA.

$$\alpha = \text{constant}$$

$$\dot{\alpha} = 0 \quad \ddot{\alpha} = 0$$

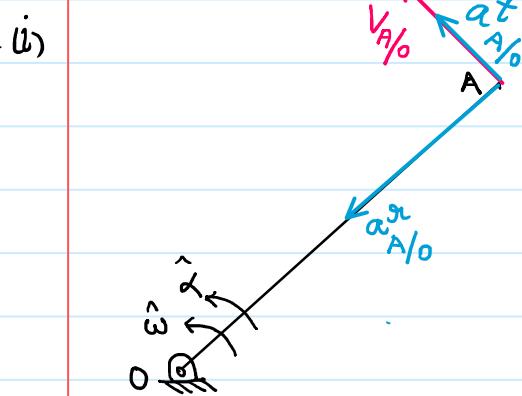
$$\vec{a} = \alpha \dot{\theta}^2 (-\hat{e}_n) + \alpha \ddot{\theta} (\hat{e}_t)$$

$$\vec{a} = \alpha \omega^2 (-\hat{e}_n) + \alpha \alpha \cdot (\hat{e}_t)$$

Velocity and Acceleration Analysis of a Single Link

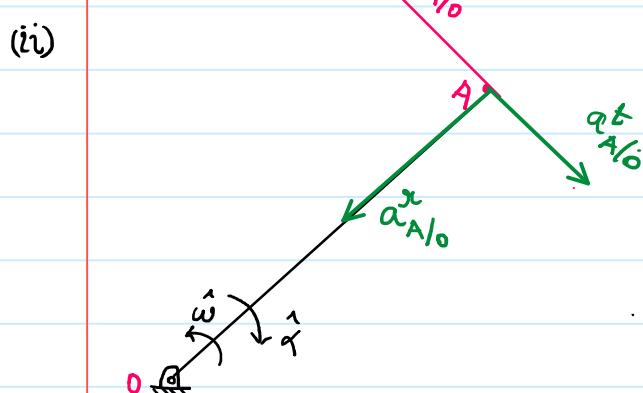
$$\hat{\omega} = \hat{\alpha} = +\hat{k}$$

Link is accelerating
 $\omega_{OA} = \text{constant}$

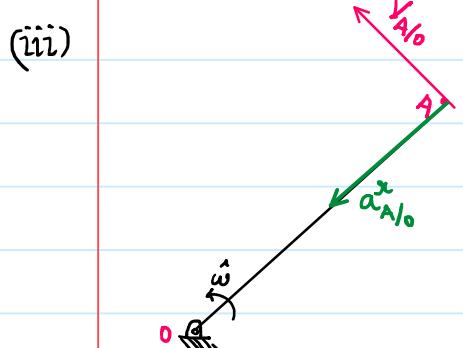


$V_{A/O}$
 Reference point
 Point of interest

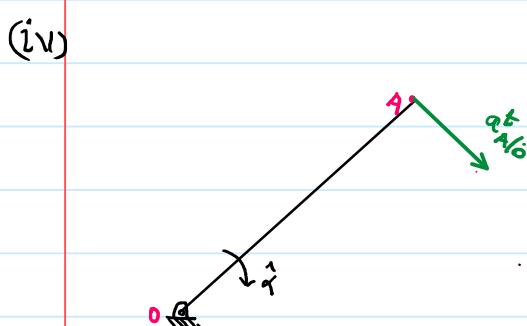
$a_{A/O}$
 Type of acceleration
 Radial / Tangent / Coriolis
 O/O
 Reference point
 Point of interest



$\hat{\omega} = +\hat{k}$, $\hat{\alpha} = -\hat{k}$
 Link is retarding.

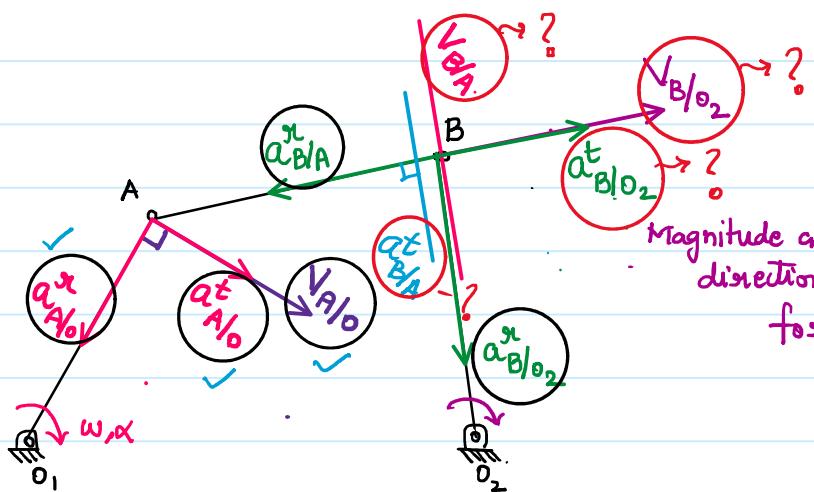


$\hat{\omega} = +\hat{k}$
 $\omega = \text{constant}$
 $\alpha = 0$
 $a_{A/O}^t = \omega A \cdot \alpha = 0$



$\omega = \text{constant}$, $\alpha \neq 0$
 $\hat{\alpha} = -\hat{k}$

Maximum Retardation.



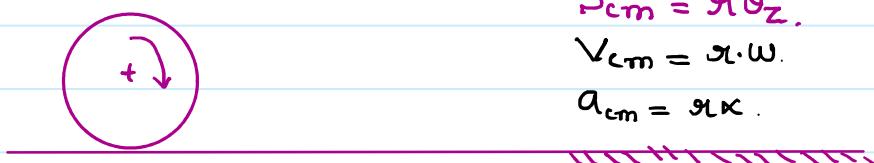
Magnitude and direction is unknown for \vec{v}

$$a_{B/A}^t, a_{B/o_2}^t, v_{B/A}, v_{B/o_2}$$

Rolling Motion

Velocity Analysis for Rolling motion

Rolling = Rotation + Translation.



$$S_{cm} = g_1 \theta_z.$$

$$V_{cm} = g \cdot w.$$

$$a_{cm} = gL \propto$$

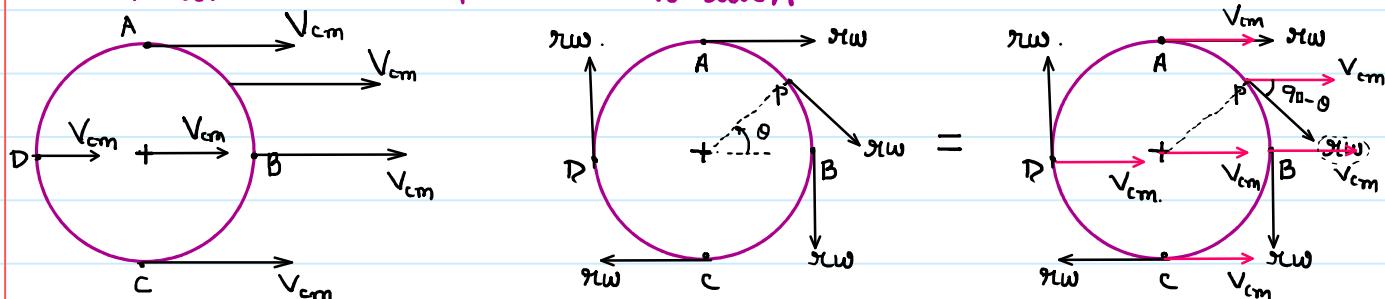
$$a^r = r\omega^2$$

$$a^t = \pi_{L\alpha}$$

Translation

十

Rotation



Velocities at points A, B, C, D, P.

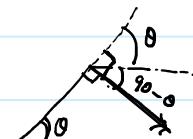
$$\vec{V_A} = V_{cm}(\hat{i}) + \pi w(\hat{i}) = 2V_{cm}(\hat{i})$$

$$\vec{V_p} = V_{cm}(\hat{i}) + \Re w \cdot [i \cos(\varphi_0 - \theta) - j \sin(\varphi_0 - \theta)] = V_{cm} \hat{i} + V_{cm} [i \sin \theta - j \cos \theta]$$

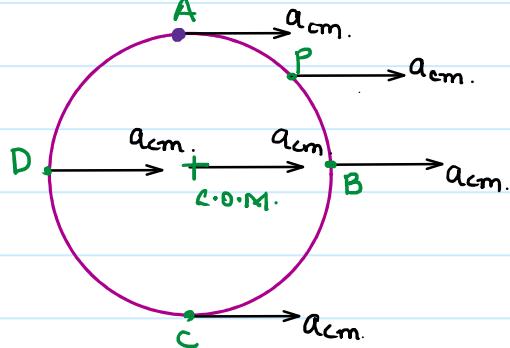
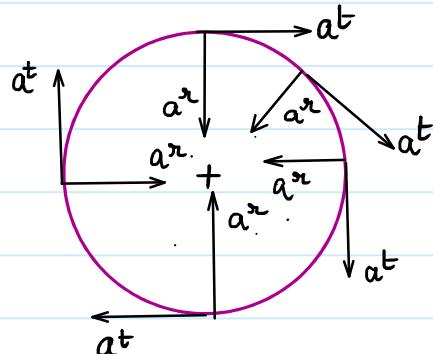
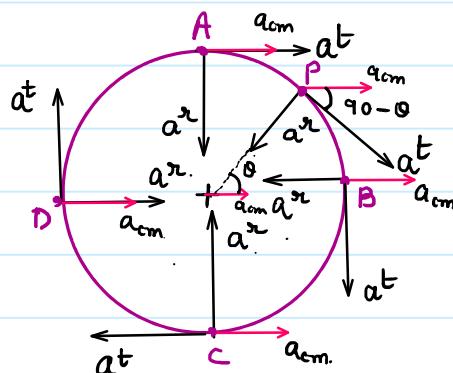
$$\vec{V}_B = V_{cm}(\vec{i}) + \omega(-\vec{j}) = V_{cm}[\vec{i}] - (\vec{j})$$

$$\vec{V}_c = V_{cm}(\hat{i}) + \omega(-\hat{i}) = 0$$

$$\vec{V}_D = V_{cm}(\hat{i}) + \omega \cdot (\hat{j}) = V_{cm}[\hat{i} + \hat{j}]$$



- ★ In pure rolling motion velocity at the point of contact is zero. About Point C the disc will undergo pure rotation.

Translation**+****Rotation**FACULTY **WAHEED UL HAQ****Rotation + Translation.**

Acceleration at points A, B, C, D & P.

$$\vec{a}_A = \vec{a}_{cm} + \vec{a}^t + \vec{a}^r = a_{cm}\hat{i} + \alpha\dot{\alpha}\hat{i} + \omega^2\hat{-j} \\ = \alpha\dot{\alpha}\hat{i} + \alpha\dot{\alpha}\hat{i} + \omega^2\hat{-j} = 2\alpha\dot{\alpha}\hat{i} + \omega^2\hat{-j}$$

$$\vec{a}_B = \vec{a}_{cm} + \vec{a}^t + \vec{a}^r = a_{cm}\hat{i} + \alpha\dot{\alpha}\hat{-j} + \omega^2\hat{-i} \\ = \alpha\dot{\alpha}\hat{i} + \omega^2\hat{-i} + \alpha\dot{\alpha}\hat{-j}$$

$$\vec{a}_C = \vec{a}_{cm} + \vec{a}^t + \vec{a}^r = \alpha\dot{\alpha}\hat{i} + \alpha\dot{\alpha}\hat{-i} + \omega^2\hat{j} = \omega^2\hat{j}$$

$$\vec{a}_D = \vec{a}_{cm} + \vec{a}^t + \vec{a}^r = \alpha\dot{\alpha}\hat{i} + \alpha\dot{\alpha}\hat{j} + \omega^2\hat{i}$$

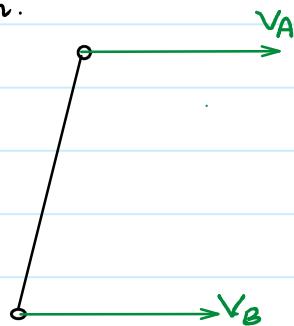
Acceleration at point P.

$$\vec{a}_P = \vec{a}_{cm} + \vec{a}^r + \vec{a}^t = \alpha\dot{\alpha}\hat{i} + \omega^2(-i\cos\theta - j\sin\theta) + \alpha\dot{\alpha}(i\cos(90-\theta) - j\sin(90-\theta)) \\ = \alpha\dot{\alpha}\hat{i} + \omega^2(-i\cos\theta - j\sin\theta) + \alpha\dot{\alpha}(i\sin\theta - j\cos\theta)$$

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Recognition of Motion

1. Translation.



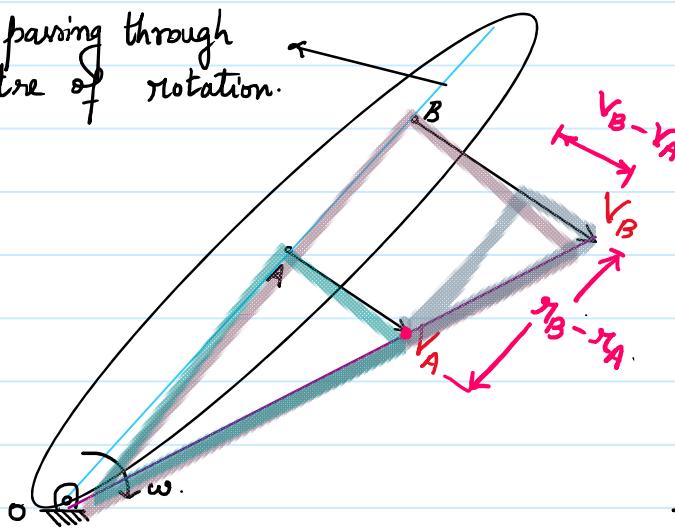
$$\begin{aligned} v_A &= v_B \\ a_A &= a_B \end{aligned} \quad \left. \begin{array}{l} \text{Translation.} \end{array} \right\}$$

$$\begin{aligned} v &= u + at \\ s &= ut + \frac{1}{2}at^2 \\ v^2 - u^2 &= 2as \end{aligned}$$

2. Angular Motion

oscillatory motion
circular motion

line passing through
centre of rotation.



$v_{B/A}$ - velocity of B wrt A

$$\frac{v_B}{OB} = \frac{v_A}{OA} = \frac{v_B - v_A}{r_B - r_A} = \frac{v_{B/A}}{r_{B/A}}$$

Point O is permanent center
of rotation.

It is having zero velocity
permanently!

- ★ Line passing through the centre of rotation will be always perpendicular to the Velocity Vectors at any point.
- ★ If there is velocity gradient across any dimension then the body will be subjected rotation

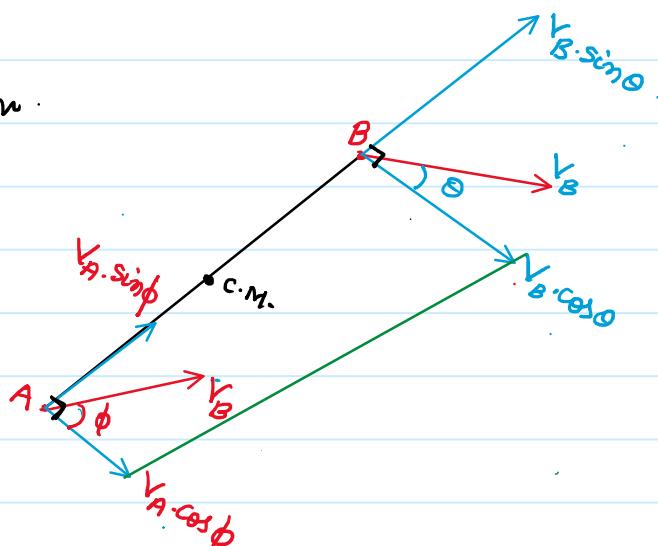
3. General Plane Motion

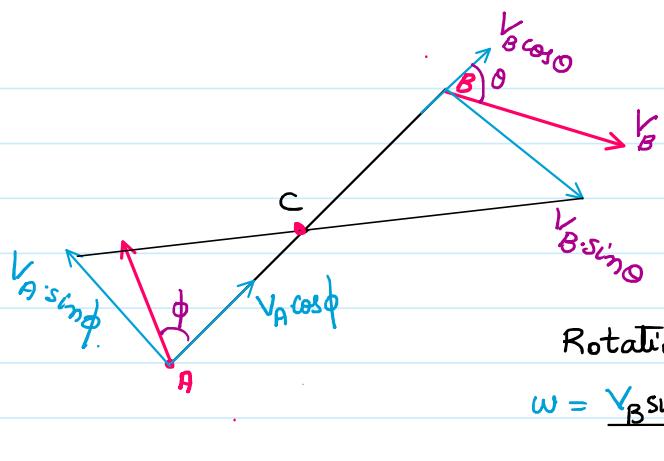
Rotation

$$\omega = \frac{v_B \cos \theta - v_A \cos \phi}{AB}$$

Translation:

$$v_A \sin \phi = v_B \sin \theta$$



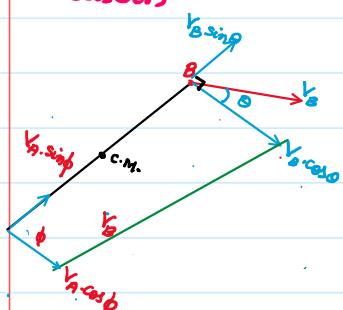


C-center of Rotation
(having only Translation velocity)

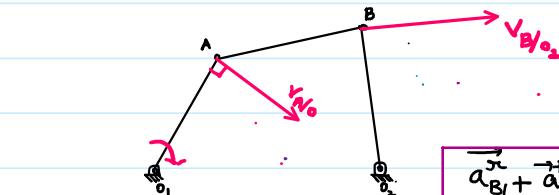
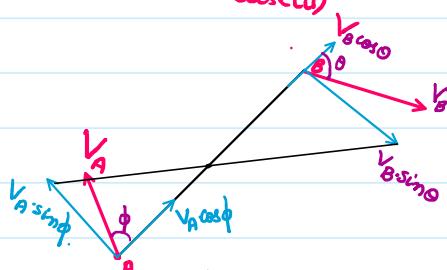
Translation

$$V_A \cos \phi = V_B \cos \phi$$

case(i)

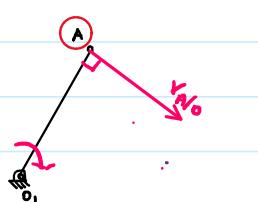


case(ii)



$$\begin{aligned} \vec{V}_B &= \vec{V}_A + \vec{V}_{B/A} \\ \vec{a}_B &= \vec{a}_A + \vec{a}_{B/A}^n + \vec{a}_{B/A}^t \end{aligned}$$

$$\vec{a}_{B/O_2} = \vec{a}_{A/O_1} + \vec{a}_{A/O_1}^t + \vec{a}_{B/A}^n + \vec{a}_{B/A}^t$$



General plane motion
Couple or link

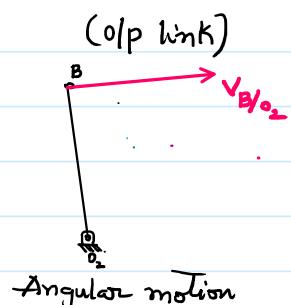
Angular motion
(clip link)

Absolute velocity and acceleration of point A

$$V_{A/O_1} = \omega_{O_1} A \cdot \omega_{O_1 A}$$

$$\vec{a}_{A/O_1} = \vec{a}_{A/O_1}^n + \vec{a}_{A/O_1}^t$$

$$\vec{a}_A = \omega_{O_1} A \cdot \omega_{O_1 A}^2 \cdot (-\hat{e}_n) + \omega_{O_1} A \cdot \alpha_{O_1 A} \cdot (\hat{e}_t)$$



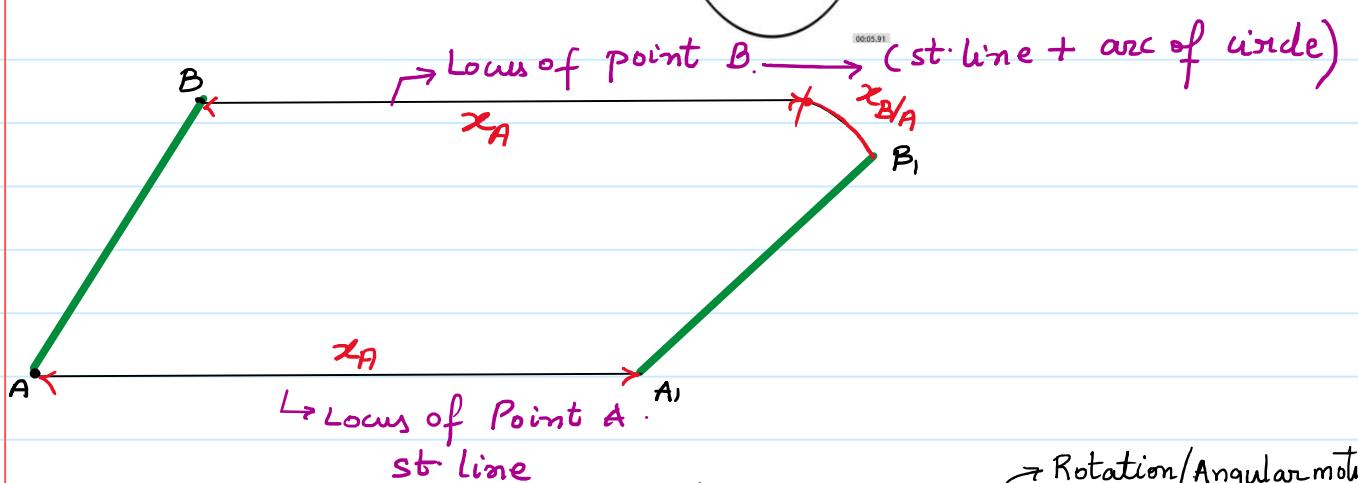
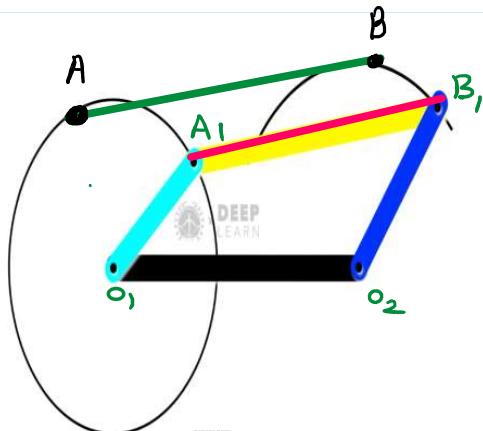
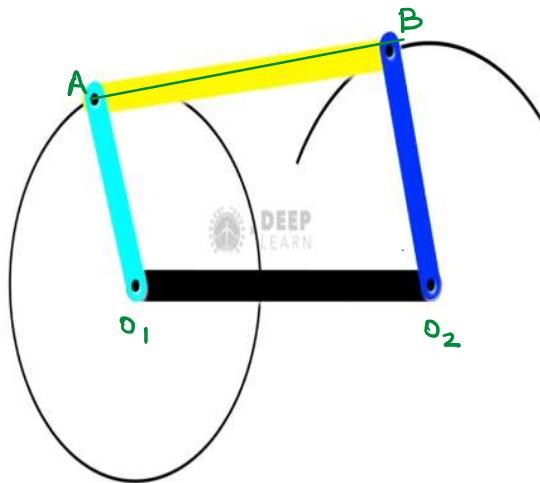
Angular motion

Absolute velocity and acceleration of point

$$V_{B/O_2} = \omega_{O_2} B \cdot \omega_{O_2 B}$$

$$\vec{a}_{B/O_2} = \vec{a}_{B/O_2}^n + \vec{a}_{B/O_2}^t$$

$$\vec{a}_B = \omega_{O_2} B \cdot \omega_{O_2 B}^2 \cdot (-\hat{e}_n) + \omega_{O_2} B \cdot \alpha_{O_2 B} \cdot (\hat{e}_t)$$



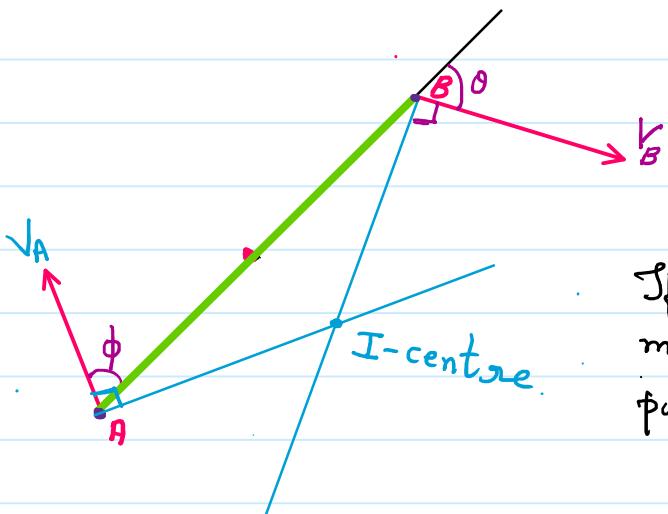
Absolute velocity of A /
Velocity of A wrt fixed point

Absolute velocity of point B /
Velocity of B wrt fixed point

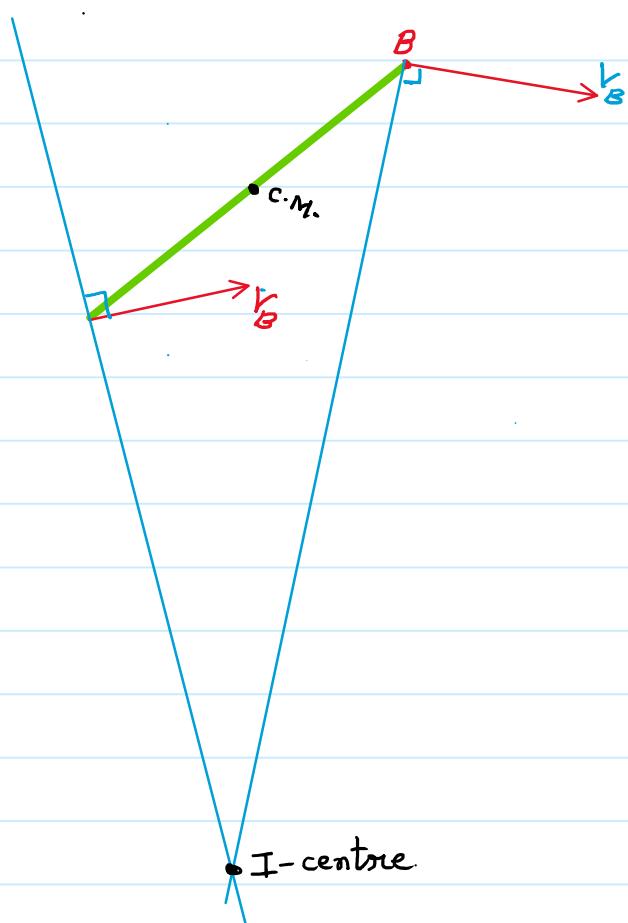
$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

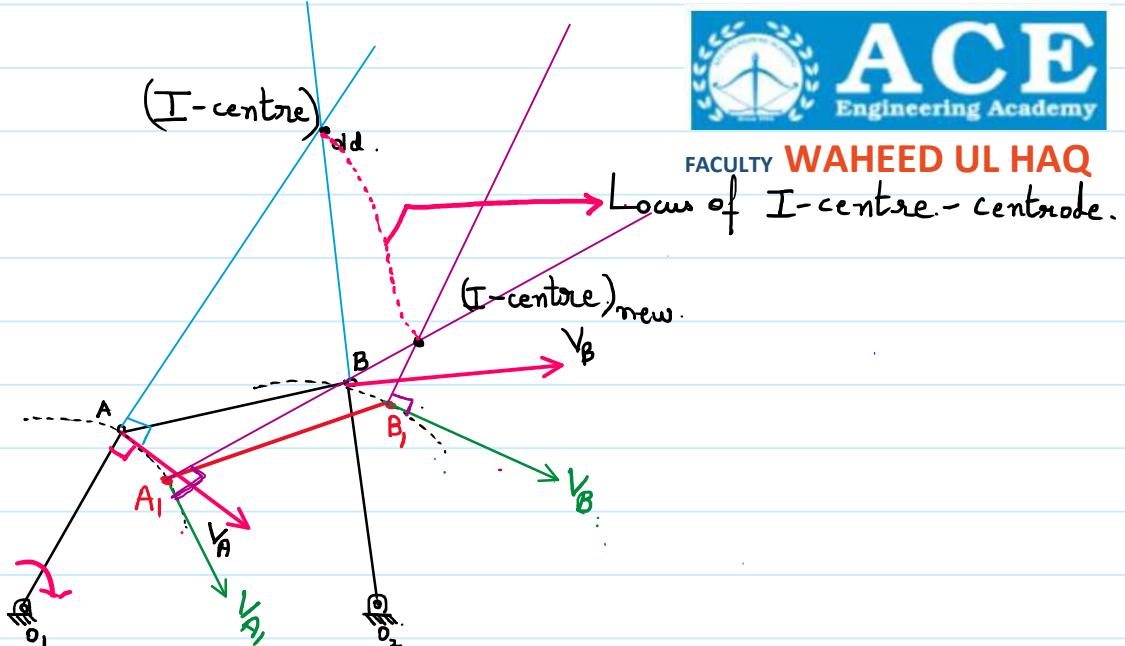
$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}^r + \vec{a}_{B/A}^t$$

Instantaneous Centre - It is a imaginary point in plane about which the link will undergo pure rotation.



If a body is in general plane motion it will undergo pure Rotation about the I-centre





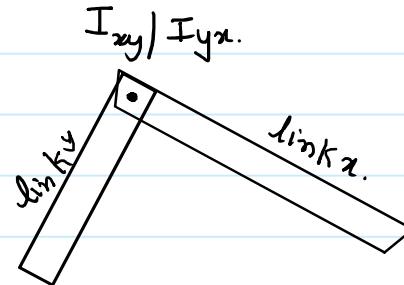
Line passing through the I-center is called I-axis of Rotations.
Locus of I-axis of Rotation is called as axode(plane).

No. of I-centres is given by $n_{C_2} = \frac{n(n-1)}{2}$
n - no. of links.

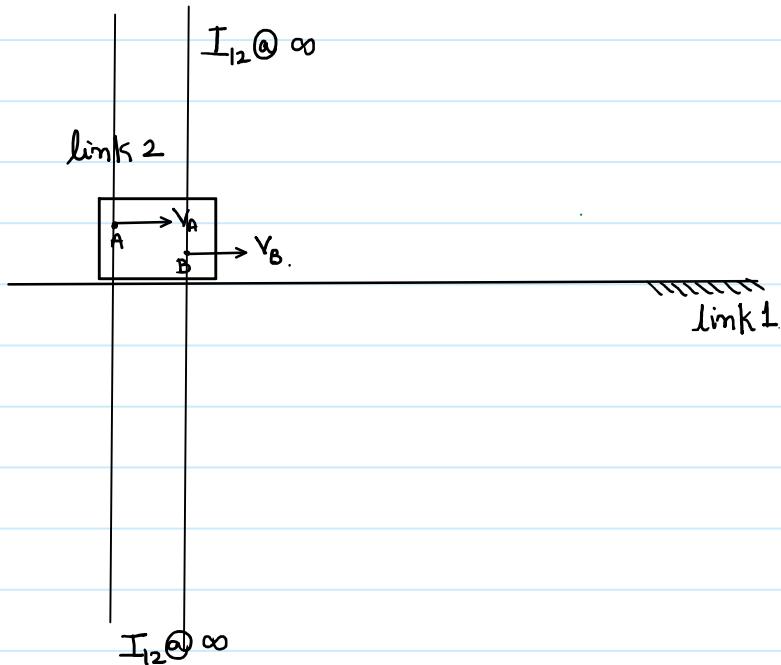
- ★ If the body/link is in pure rotation the I -centre will lie on the physical boundary of the body/link.
- ★ If the body/link is in translation the I -centre will lie at infinity in the plane of rotation.
- ★ If the body/link is in General Plane Motion the I-centre will lie anywhere in the plane of rotation.

Some of the Common I-centres

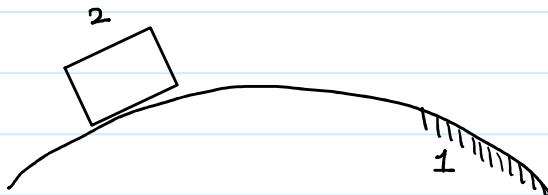
1. Revolute Joint



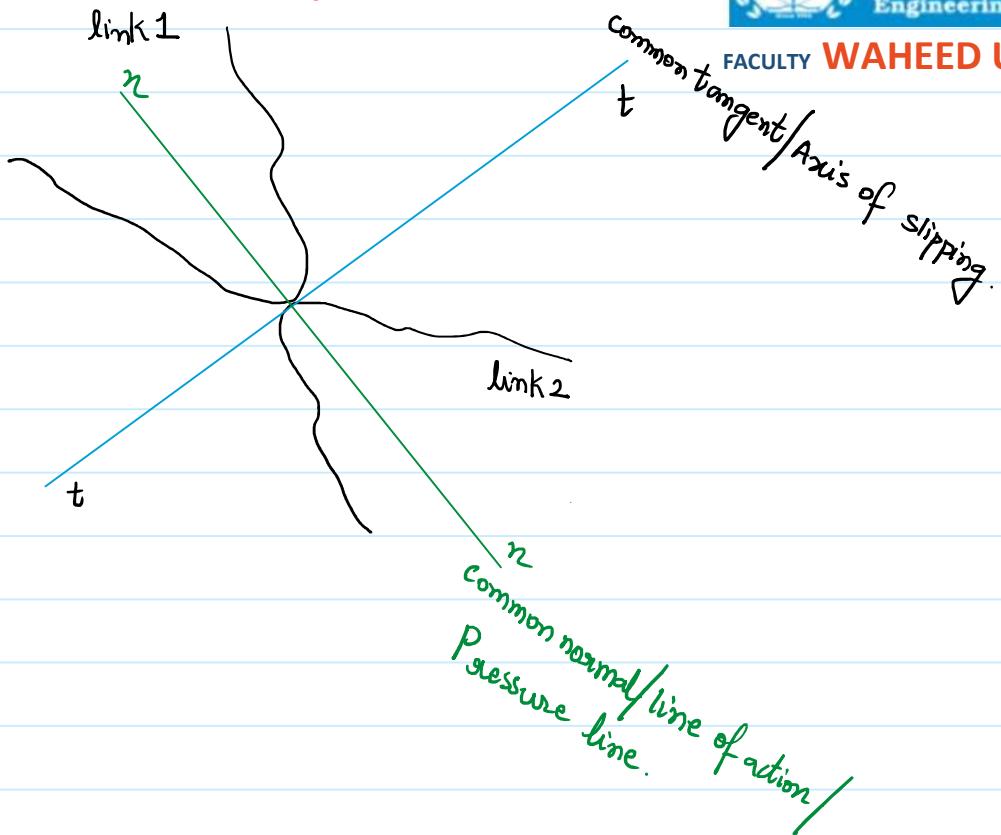
2. Prismatic Joint



3. Curvilinear Motion - centre of curvature becomes the I-centre.

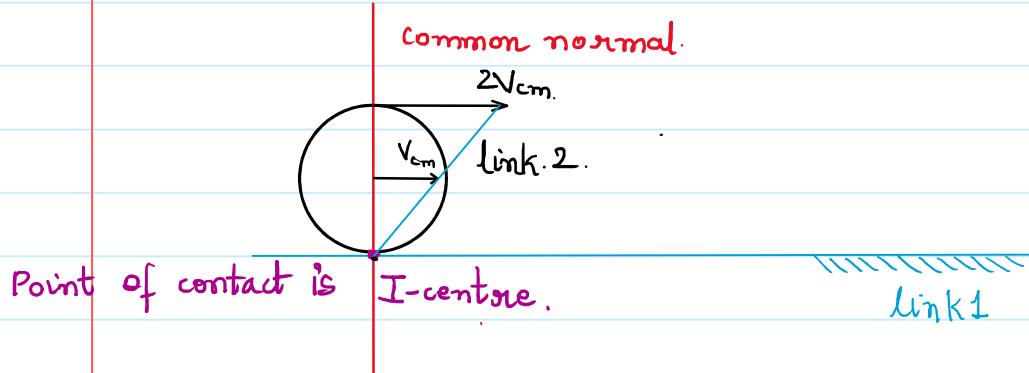


Higher Pair. - The I-centre lies somewhere on the common normal.



1. Pure Rolling Motion

$$S_{cm} = \pi \cdot \theta z$$



2. Pure Rotation

