ACE Engineering Academy

FACULTY **WAHEED UL HAQ**

Response of forced Vibration.

$$x(t) = X_o \cdot e^{-\zeta\omega_n t} \sin(\omega_d t + \phi) + \frac{F}{\left[(K - M\omega^2)^2 + (C\omega)^2\right]^{0.5}} \sin(\omega t - \phi)$$

as $t \to \infty$, $e^{-\zeta\omega_n t} \to 0$

$\sin(\omega t - \phi)$: ϕ → Phase Angle.

$R\cos\phi = K - M\omega^2$, $R \cdot \sin\phi = C\omega$.

$$\frac{C\omega}{K} = \frac{C\omega}{2\sqrt{MK}} \cdot \frac{\sqrt{M}}{\sqrt{K}} \times 2$$

$$= 2 \left(\frac{C}{2\sqrt{MK}}\right)\left(\frac{\omega}{\omega_n}\right)$$

$$= 2\zeta r.$$

$$\tan\phi = \frac{C\omega}{K - M\omega^2}$$

$$\tan\phi = \frac{\frac{C\omega}{K}}{1 - \frac{M}{K}\cdot\omega^2} = \frac{2\zeta\left(\frac{\omega}{\omega_n}\right)}{1 - \left(\frac{\omega}{\omega_n}\right)^2} = \frac{2\zeta r}{1 - r^2}$$

Phase Angle vs. r Graph.

$0 < r < 1$ ⇝ $\tan\phi = \dfrac{2\zeta r}{1 - r^2}$ $\quad\boxed{\tan\phi \propto \zeta}$

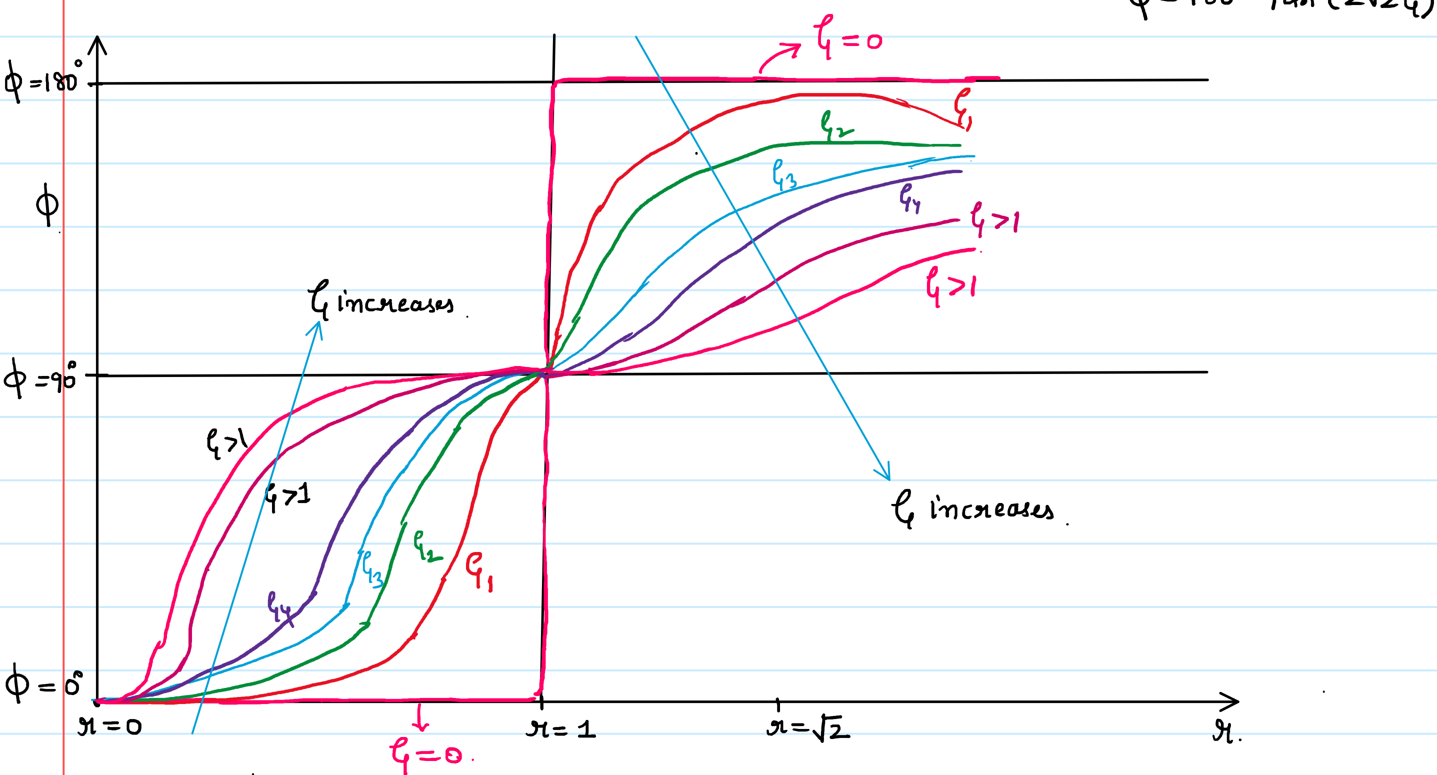
$1 < r < \sqrt{2}$

↳ $\phi = 180 - \tan^{-1}\left(\dfrac{2\zeta r}{1 - r^2}\right)$

As. $\zeta\uparrow$ $\;180 - \tan^{-1}\left(\dfrac{2\zeta r}{1 - r^2}\right)\downarrow$ $\;\phi\downarrow$

1. $r = 0$ $\quad \zeta = 0$ $\quad \phi = 0^\circ$
2. $r = 0$ $\quad \zeta \neq 0$ $\quad \phi = 0^\circ$
3. $r = 1$ $\quad \zeta = 0 / \zeta \neq 0$ $\quad \phi = 90^\circ$
4. $r = \sqrt{2}$ $\quad \zeta = 0$ $\quad \tan\phi = \dfrac{2 \times 0 \times \sqrt{2}}{1 - (\sqrt{2})^2} \Rightarrow \tan\phi = 0^-$ $\quad \phi = \tan^{-1}(0^-) = 180^\circ$
5. $r = \sqrt{2}$ $\quad \zeta \neq 0$ $\quad \tan\phi = \dfrac{2\zeta\cdot\sqrt{2}}{1 - (\sqrt{2})^2} \Rightarrow \tan\phi = -2\sqrt{2}\zeta \Rightarrow \phi = \tan^{-1}(-2\sqrt{2}\zeta)$

$$\phi = 180 - \tan^{-1}(2\sqrt{2}\zeta)$$



FACULTY **WAHEED UL HAQ**

Displacement $x(t) = A \cdot \sin(\omega t - \phi)$

$$A = \frac{F}{\left[(K - M\omega^2)^2 + (C\omega)^2\right]^{0.5}}$$

$$A = \frac{x_{static}}{\left[(1 - r^2)^2 + (2\xi r)^2\right]^{0.5}}$$

Velocity $\dot{x}(t) = A \cdot \omega \cdot \cos(\omega t - \phi) = A\omega \cdot \sin\left(\frac{\pi}{2} + (\omega t - \phi)\right)$

Acceleration $\ddot{x}(t) = -A\omega^2 \sin(\omega t - \phi) = A\omega^2 \sin(\pi + \omega t - \phi)$

$M\ddot{x} + C\dot{x} + Kx = F(t)$ $\qquad F(t) = F\sin(\omega t)$

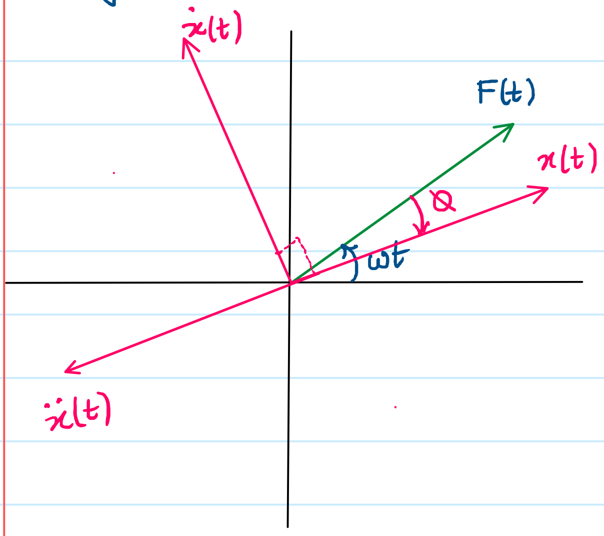
$M \cdot (-A\omega^2 \sin(\omega t - \phi)) + C \cdot (A\omega \cdot \cos(\omega t - \phi)) + K \cdot A\sin(\omega t - \phi) = F\sin(\omega t)$

$F\sin(\omega t) + MA\omega^2 \sin(\omega t - \phi) - CA\omega\cos(\omega t - \phi) - KA\sin(\omega t - \phi) = 0$

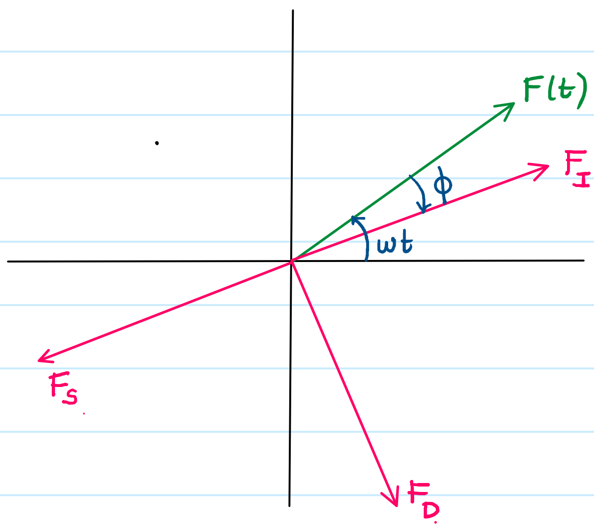
$F\sin(\omega t) + MA\omega^2 \sin(\omega t - \phi) + CA\omega \cdot \sin\left(\frac{3\pi}{2} + \omega t - \phi\right) + KA \cdot \sin(\pi + \omega t - \phi) = 0$

$\vec{F}_{ext} + \vec{F}_I + \vec{F}_D + \vec{F}_S = 0$

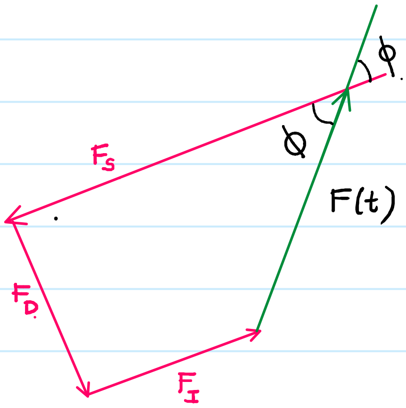
Phase Angle b/w $\vec{F}_{ext}$, displacement velocity & Acceleration



Phase Angle b/w $\vec{F}_{ext}$, F_I, $\vec{F}_D$, $\vec{F}_S$



ϕ – Angle subtended b/w $x(t)$ [Displacement of vibration] (o/p variable) and $F_{external}$ (i/p variable). [External force / Excitation]

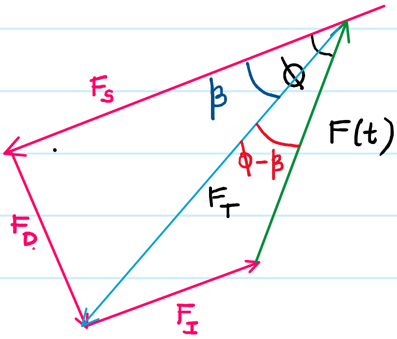


$$\tan\phi = \frac{F_D}{F_S - F_I}$$

$$\tan\phi = \frac{CA\omega}{KA - MA\omega^2}$$

$$\tan\phi = \frac{C\omega}{K - M\omega^2} = \frac{2\xi r}{1 - r^2}$$

F_T – Force transmitted to the foundation.



$$F_T = \sqrt{F_S^2 + F_D^2}$$

$$F_T = \sqrt{(KA)^2 + (CA\omega)^2}$$

$$F_T = \sqrt{K^2 + (C\omega)^2} \cdot A$$

$$A = \frac{F}{\sqrt{(K - M\omega^2)^2 + (C\omega)^2}}$$

$$F_T = \sqrt{K^2 + (C\omega)^2} \cdot \frac{F}{\sqrt{(K - M\omega^2)^2 + (C\omega)^2}}$$

$$\tan\beta = \frac{F_D}{F_S} = \frac{C\omega}{K} = 2\xi r$$

$(\phi - \beta)$ = Angle b/w. force transmitted to foundation and applied force.

$$\phi - \beta = \tan^{-1}\left(\frac{2\xi r}{1 - r^2}\right) - \tan^{-1}(2\xi r)$$

$$\boxed{\epsilon = \frac{F_T}{F} = \sqrt{\frac{K^2 + (C\omega)^2}{(K - M\omega^2)^2 + (C\omega)^2}}}$$

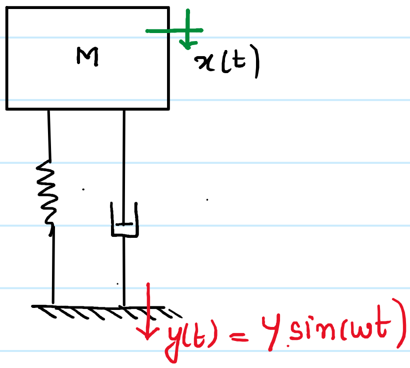
ϵ – Transmissibility Ratio

$$\epsilon = \frac{F_T}{F} = \frac{\text{force transmitted to foundation}}{\text{force applied}}$$

$$\epsilon = \frac{F_T}{F} = \sqrt{\frac{K^2 + (C\omega)^2}{(K - M\omega^2)^2 + (C\omega)^2}} = \sqrt{\frac{\frac{K^2 + (C\omega)^2}{K^2}}{\frac{(K - M\omega^2)^2 + (C\omega)^2}{K^2}}} = \sqrt{\frac{1 + \left(\frac{C\omega}{K}\right)^2}{\left(1 - \frac{M}{K}\cdot\omega^2\right)^2 + \left(\frac{C\omega}{K}\right)^2}}$$

$$\boxed{\epsilon = \sqrt{\frac{1 + (2\xi r)^2}{(1 - r^2)^2 + (2\xi r)^2}}}$$

Base Excitation



$$M\ddot{x} + c(\dot{x}-\dot{y}) + k.(x-y) = 0$$

$$M\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$$

$$M\ddot{x} + c\dot{x} + kx = c.Y.\omega\cos(\omega t) + k.Y.\sin(\omega t)$$

$$KY = R\cos\alpha \qquad CY\omega = R\sin\alpha$$

$$M\ddot{x} + c\dot{x} + kx = R\sin(\omega t + \alpha)$$

Displacement

$$x(t) = \frac{R}{\sqrt{(K-M\omega^2)^2 + (c\omega)^2}}\sin(\omega t + \alpha - \phi_1) \qquad R = \sqrt{K^2 + (c\omega)^2}\,.\,Y$$

$$Y = \frac{R}{\sqrt{K^2 + (c\omega)^2}}$$

$$x(t) = X.\sin(\omega t + \alpha - \phi_1)$$

$$\frac{X}{Y} = \frac{\dfrac{R}{\sqrt{(K-M\omega^2)^2 + (c\omega)^2}}}{\dfrac{R}{\sqrt{K^2 + (c\omega)^2}}} = \sqrt{\frac{K^2 + (c\omega)^2}{(K-M\omega^2)^2 + (c\omega)^2}} = \sqrt{\frac{1 + (2\xi r)^2}{(1-r^2)^2 + (2\xi r)^2}}$$

X – Amplitude of vibration for mass 'm'

Y – Amplitude of displacement given at the base.

Transmissibility Ratio vs r Graph

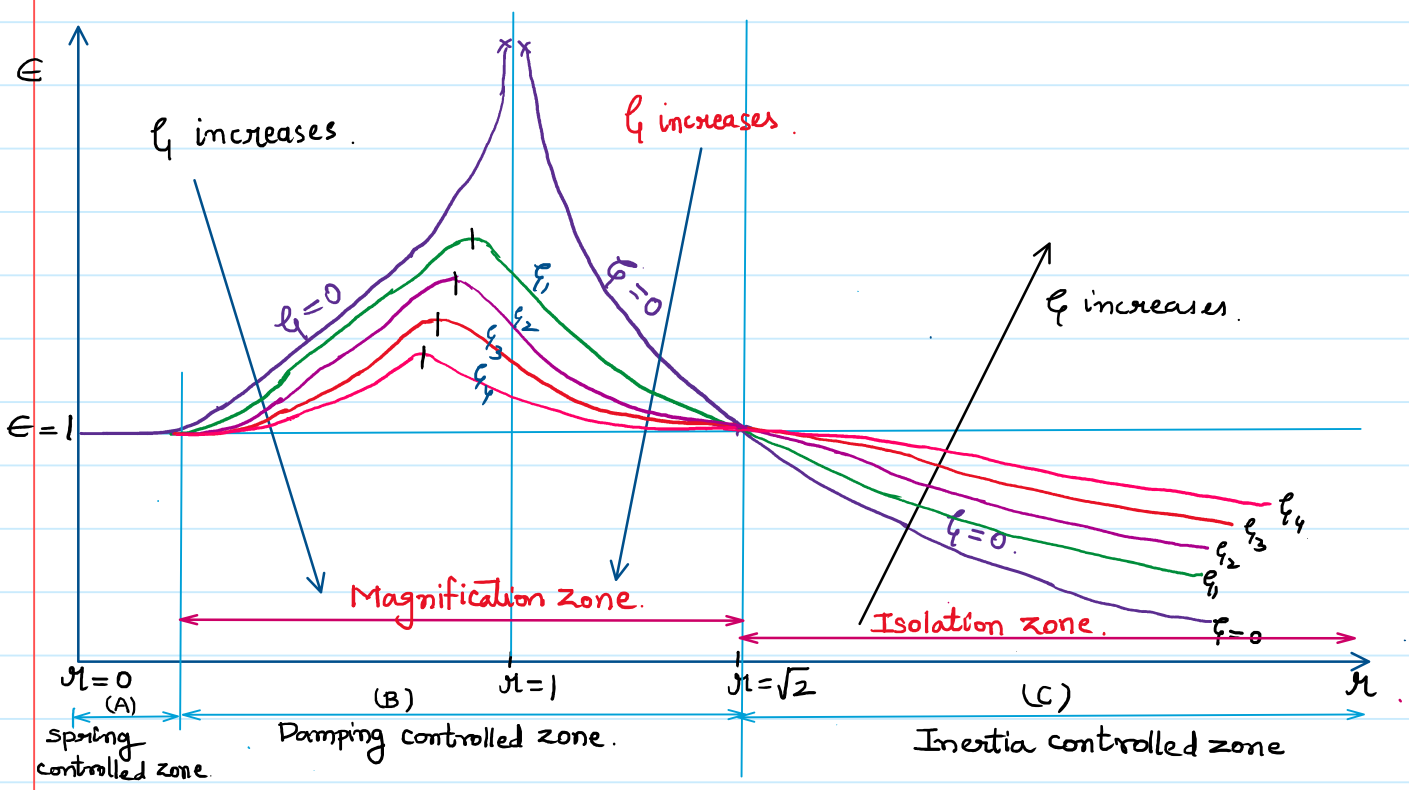
$$\epsilon = \frac{X}{Y} = \frac{F_T}{F} = \sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}}$$

$\zeta < 1 \longrightarrow$ underdamped system.

1. $r=0 \quad \zeta=0 \quad \epsilon=1$
2. $r=0 \quad \zeta\neq0 \quad \epsilon=1.$
3. $r=1 \quad \zeta=0 \quad \epsilon=\infty$
4. $r=1 \quad \zeta\neq0 \quad \epsilon=\frac{\sqrt{1+4\zeta^2}}{2\zeta}$

$\boxed{\epsilon \propto \frac{1}{\zeta}.}$

5. $r=\sqrt{2} \quad \zeta \doteq 0 \quad \epsilon=1$
6. $r=\sqrt{2} \quad \zeta\neq0 \quad \epsilon=1.$



$r <<< 1 \quad . \quad r^2 \to 0 \qquad (1-r^2)^2+(2\zeta r)^2 \longrightarrow 1, \qquad 1+(2\zeta r)^2 \longrightarrow 1.$

$\epsilon \to 1. \Rightarrow X=Y, \quad F_T = F$

$r<<<1 \qquad \frac{\omega}{\omega_n} <<<1 \qquad \omega_n >>> \omega \qquad \sqrt{\frac{K}{M}} >>> \omega. \rightarrow$ $K \uparrow\uparrow\uparrow$ ✓ $M \downarrow\downarrow\downarrow$ ✗.

lower excitation frequency.

(A) – Spring controlled zone. $\quad X \approx Y \quad F_T \approx F \quad$ Sesimograph.

$(r<1, \xi<1)$, $1+(2\xi r)^2 \longrightarrow 1$, $(1-r^2)^2+(2\xi r)^2$ = Decimal value.

$\epsilon > 1 \longrightarrow \quad \epsilon \propto \frac{1}{\xi}$

$X > Y, \; F_T > F$

$(0<r<1) \longrightarrow \quad \epsilon \propto \frac{1}{\xi}$

$(1<r<\sqrt{2}) \quad \epsilon = \sqrt{\dfrac{1+(2\xi r)^2}{(1-r^2)^2+(2\xi r)^2}} \rightarrow \epsilon \propto \frac{1}{\xi}$

Damper is supressing the vibration Amplitude or Transmissibility Ratio.

$\sqrt{2} < r <$ large finite value

$r\uparrow \quad (1-r^2)^2 \uparrow\uparrow\uparrow \quad (1-r^2)^2+(2\xi r)^2 \longrightarrow (1-r^2)^2 \quad \epsilon \rightarrow \dfrac{\sqrt{1+(2\xi r)^2}}{(1-r^2)}$

$\boxed{\epsilon \propto \xi}$

$\frac{\omega}{\omega_n} >> 1 \quad \omega_n << \omega. \quad \frac{k}{M} << \omega.$

K is decreasing ✗

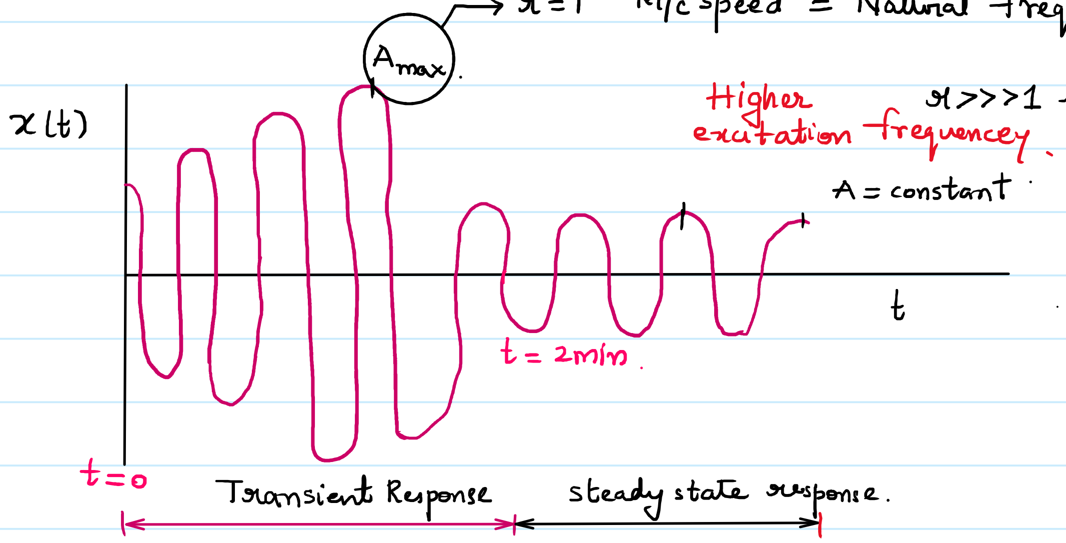
M is increasing ✓

As $r\uparrow\uparrow \quad (1-r^2)^2 \uparrow\uparrow\uparrow\uparrow \quad \epsilon \rightarrow 0$

$r=1 \quad A = Max. \quad \epsilon = Max. \longrightarrow$ dangerous situation.

$\longrightarrow$ Resonance.

$\rightarrow r=1$ M/c speed = Natural frequeny of system.



Higher excitation frequencey.

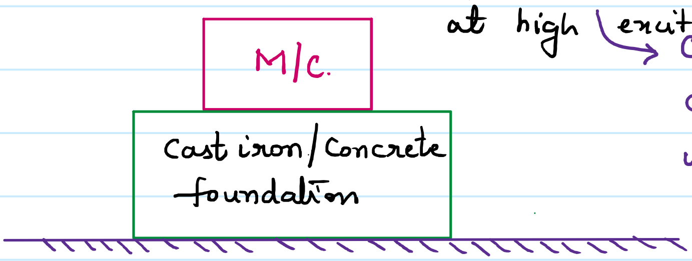
$r>>>1 \rightsquigarrow \epsilon \rightarrow 0 \quad F_T \rightarrow 0$

A = constant.

At $r >>> 1$ (Higher excitation frequency)

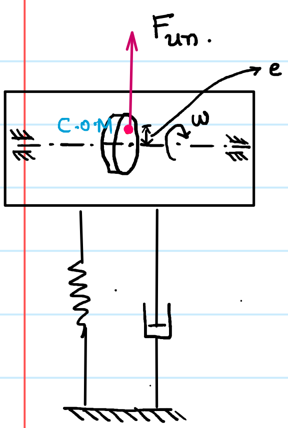
$M \rightarrow$ inertia controlled zone.

M/c can be operated at high excitation frequency



Combined centre of mass of machine and foundation will shift towards the ground. Isolation occurs.

Excitation of System due to unbalanced Rotating Mass



$F_{un} = me\omega^2 \sin(\omega t)$

m – Mass of rotor

M – Mass of system.

e – eccentric b/w C.O.M. and axis of rotation

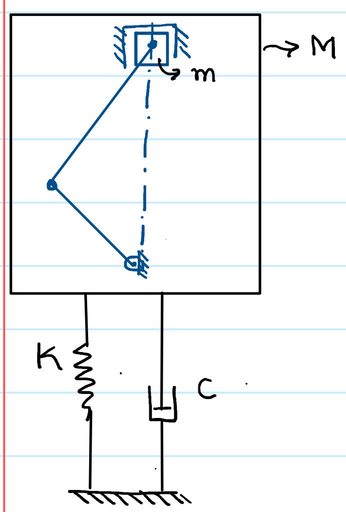
ω – excitation frequency / speed of rotor

E.O.M.

$M\ddot{x} + c\dot{x} + kx = me\omega^2 \sin(\omega t)$

$$\text{Amplitude of forced vibration} = \frac{me\omega^2}{\sqrt{(k - M\omega^2)^2 + (c\omega)^2}}$$

Excitation due to reciprocating mass.



M – mass of system, m – mass of piston,

ω – Excitation frequency / crank speed. r – crank lenght.

$F_{piston} = m_p \cdot a_p = mr\omega^2 \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$

$M\ddot{x} + c\dot{x} + kx = mr\omega^2 \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$

$n > 1$

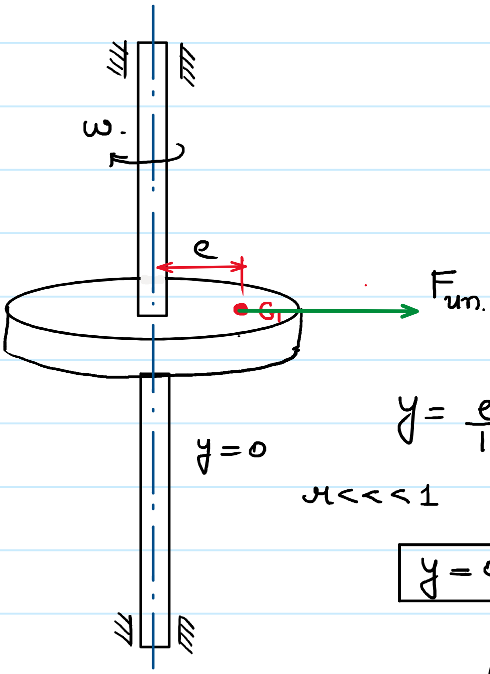
$M\ddot{x} + c\dot{x} + kx = mr\omega^2 \cos(\omega t)$

Amplitude of forced vibration

$$A = \frac{mr\omega^2}{\sqrt{(k - M\omega^2)^2 + (c\omega)^2}}$$

FACULTY WAHEED UL HAQ

Whirling of Shafts



$$F_{un} = me\omega^2$$

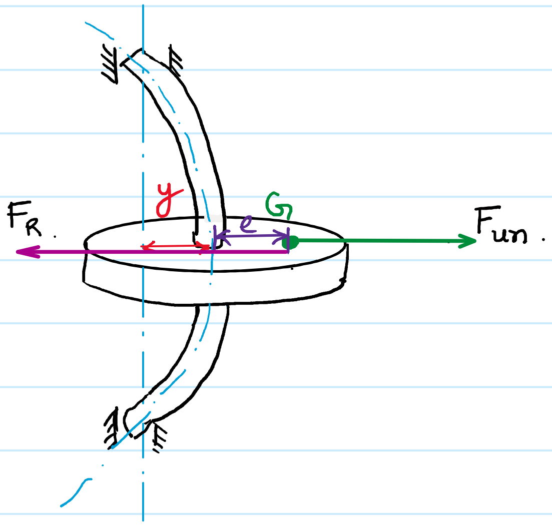
m – mass of rotor
e – eccentricity b/w C.O.M. of rotor and axis of rotation.
ω – speed of rotation.
k – stiffness of shaft.

$$y = \frac{er^2}{1-r^2}$$

$r<<1 \quad r^2 \to 0$

$$\boxed{y=0}$$

y – Deflection in shaft due to F_{un} / Radial of whirl / Amplitude of vibration.



$$F_{un} = m\cdot(y+e)\,\omega^2$$

$$F_R = ky$$

$$F_R = F_{un}$$

$$ky = m\cdot(y+e)\,\omega^2$$

$$\frac{k}{m}\cdot y = \omega^2\cdot y + \omega^2\cdot e$$

$$\omega_n^2\cdot y = \omega^2\cdot y + \omega^2\cdot e$$

$$(\omega_n^2-\omega^2)\cdot y = \omega^2\cdot e$$

Direction y and F_{un} is in same sense / same phase.

$r = \frac{\omega}{\omega_n}$

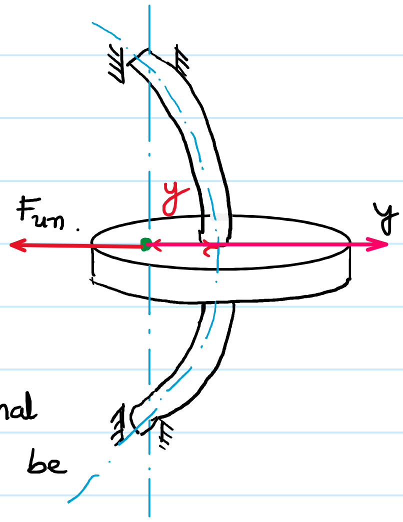
$$y = \frac{e\cdot\omega^2}{\omega_n^2-\omega^2} = \frac{e}{\left(\frac{\omega_n}{\omega}\right)^2-1} = \frac{e}{\frac{1}{r^2}-1}$$

$$y = \frac{er^2}{1-r^2}$$

$y = \frac{er^2}{1-r^2}$ if $r \to 1$. $y \to \infty$ resonance occurs.

$r = 1$ $\omega = \omega_n \longrightarrow$ Whirling speed – The speed at which the shaft will execute voilent oscillation is called whirling speed or critical speed.

if $r > 1$. $1 - r^2 = -ve$ value. $y = -ve$.



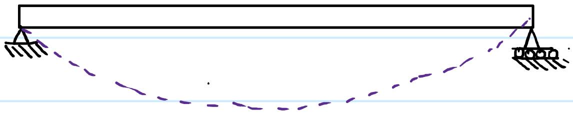
$$y = -\frac{er^2}{(1-r^2)}$$

F_{un} and y are in opposite phase.

if ω is above resonant frequency then External force and displacement of forced vibration will be in opposite phase.

$r >>> 1$ $1 - r^2 \approx -r^2$ $y = \frac{er^2}{1-r^2} \to y = \frac{er^2}{-r^2}$ $y = -e$

At very high excitation frequencies the C.O.M. of rotor will coincide with shaft axis.



Deflection. $EI.\frac{d^2y}{dx^2} = M_{x-x}$.

$$EI.\frac{d^3y}{dx^3} = \frac{dM_{x-x}}{dx} = S.F_{x-x}.$$

$$EI.\frac{d^4y}{dx^4} = \frac{d(S.F)_{x-x}}{dx} = w(x) = \text{Rate of Loading.}$$

Boundary cond. $x=0 \quad y=0, \qquad x=0 \quad M_{x-x}=0 \Rightarrow \frac{d^2y}{dx^2}=0$

$x=L \quad y=0 \qquad x=L \quad M=0 \Rightarrow \frac{d^2y}{dx^2}=0$

$$EI.\frac{d^4y}{dx^4} - w(x) = 0 \qquad w(x) = m.y.\omega^2$$

m – mass per unit lenght of shaft.

y – displacement due transverse vibration.

$$\frac{d^4y}{dx^4} - \underbrace{\frac{m.\omega^2}{EI}}_{\lambda^4}.y = 0$$

$$(D^4-\lambda^4)y = 0$$

$$(D^2+\lambda^2)(D^2-\lambda^2).y = 0 \Rightarrow D = \pm i\lambda, \pm\lambda$$

C.F.

$$y = A\sin(\lambda x) + B\cos(\lambda x) + C\sinh(\lambda x) + D.\cosh(\lambda x)$$

(i) $x=0, \; y=0$ (ii) $x=0 \quad M=0 \quad \frac{d^2y}{dx^2}=0$

(iii) $x=L \quad y=0$ (iv) $x=L \quad M=0 \quad \frac{d^2y}{dx^2}=0$

After. putting B.C. in C.F.

$$A\sin(\lambda L) = 0 \Rightarrow \lambda L = 0, \pi, 2\pi, 3\pi, \ldots n\pi$$

$$\lambda^4 = \frac{m\omega^2}{EI} \Rightarrow \left[\frac{m\omega^2}{EI}\right]^{1/4}.L = \pi, 2\pi, 3\pi, \ldots n\pi.$$

$$\left[\frac{m\omega^2}{EI}\right]^{1/4} \cdot L = \pi, 2\pi, 3\pi, \ldots\ldots n\pi$$

$$\frac{m\omega^2}{EI} \cdot L^4 = \pi^4, (2\pi)^4, (3\pi)^4 \ldots\ldots (n\pi)^4$$

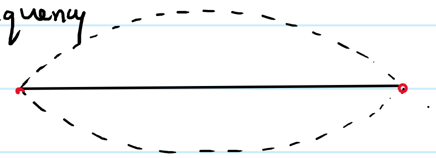
$$\omega^2 = \pi^4 \cdot \frac{EI}{mL^4}, \quad (2\pi)^4 \cdot \frac{EI}{mL^4}, \quad \ldots\ldots \quad (n\pi)^4 \cdot \frac{EI}{mL^4}$$

$$\omega = \underbrace{\pi^2\sqrt{\frac{EI}{mL^4}}}_{\omega_1}, \quad (2\pi)^2\sqrt{\frac{EI}{mL^4}}, \quad \ldots\ldots \quad (n\pi)^2\sqrt{\frac{EI}{mL^4}}$$

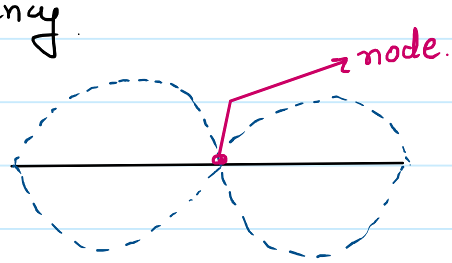
$$\omega = \omega_1, \quad 2^2 \cdot \omega_1, \quad 3^2\omega_1, \quad \ldots\ldots \quad n^2 \cdot \omega_1$$
$$\omega_1, \quad \omega_2, \quad \omega_3, \quad \ldots\ldots$$

Mode shapes

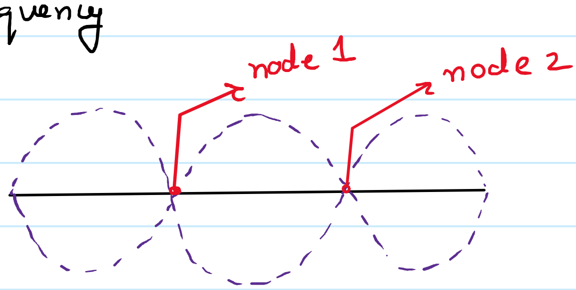
At ω_1 frequency



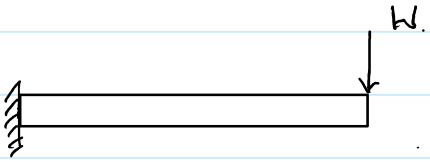
At ω_2. frequency



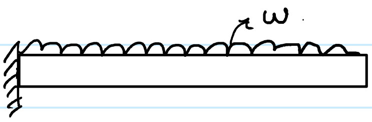
At ω_3 frequency



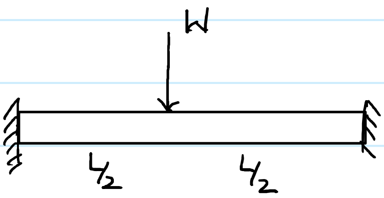
Deflection of beam.



$$\delta = \frac{WL^3}{3EI}$$



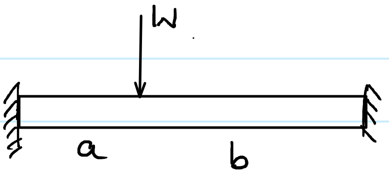
$$\delta = \frac{wL^4}{8EI}.$$



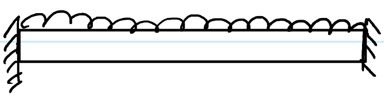
$\rightarrow a = L/2$
$b = L/2$

$$\delta = \frac{W\cdot(L/2)^3\cdot(L/2)^3}{3EIL^3} = \frac{WL^3}{192EI} = \frac{1}{4}\left(\frac{WL^3}{48EI}\right)$$

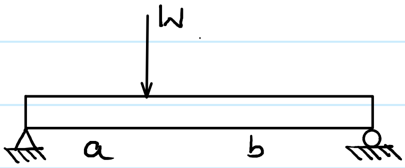
$$\delta_{f.B} = 1/4 \cdot \delta_{S.S.B}.$$



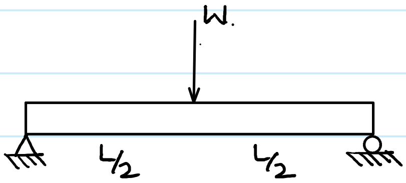
$$\delta = \frac{Wa^3\cdot b^3}{3EIL^3}$$



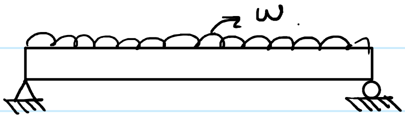
$$\delta = \frac{wL^4}{384EI}.$$



$$\delta = \frac{Wa^2\cdot b^2}{3EIL}$$



$$\delta = \frac{W\cdot(L/2)^2\cdot(L/2)^2}{3EIL} = \frac{WL^3}{48EI}.$$



$$\delta = \frac{5wL^4}{384EI}.$$

FACULTY WAHEED UL HAQ

29. In a spring mass damper system excited by a harmonic force with a damping of 10%. The amplitude at resonance is found to be 10 cm. What is the amplitude at half the resonance frequency. (GATE-16)

(a) 9.81 cm (b) 4.9 cm
(c) 7.35 cm $\boxed{\text{(d) 2.64 cm}}$

29Q. $\zeta = 10\% = 0.1$

$A = 10\,cm$ @ $r = 1$.

@ $r = 0.5$ $A = ?$

ACE Engineering Academy

FACULTY **WAHEED UL HAQ**

@ resonance. $\dfrac{A}{x_{static}} = \dfrac{1}{2\zeta} \Rightarrow \dfrac{10}{x_{static}} = \dfrac{1}{2\times 0.1}$

$x_{static} = 2\,cm$.

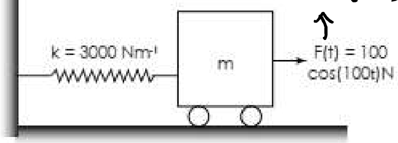
Amplitude @ $r = 0.5$

$$A = \frac{x_{static}}{\left[(1-r^2)^2 + (2\zeta r)^2\right]^{0.5}}$$

$$A = \frac{2}{\left[(1-0.5^2)^2 + (2\times 0.1\times 0.5)^2\right]^{0.5}} = 2.64.$$

30. A mass m attached to a spring is subjected to a harmonic force as shown in figure. The amplitude of the forced motion is observed to be 50 mm. The value of m (in kg) is (GATE-10)

$F(t) = F\sin(\omega t)$



$\boxed{\text{(a) 0.1}}$ (b) 1.0
(c) 0.3 $\boxed{\text{(d) 0.5}}$

30Q. $A = 50\,mm$.

$$A = \frac{F}{\left[(k-M\omega^2)^2 + (c\omega)^2\right]^{0.5}}$$

$m = ?$ $k = 3000\,N/m$.

$F = 100\,N$ $\omega = 100\,rad/s$.

$C = 0$

$$A = \left|\frac{F}{\left[(k-M\omega^2)^2 + (c\omega)^2\right]^{0.5}}\right| \qquad A = \frac{\pm F}{k - M\omega^2}$$

$$50\times 10^{-3} = \frac{\pm 100}{3000 - M(100)^2} \qquad M = 0.1\,kg,\ 0.5\,kg$$

Common data for 31 & 32

In a spring mass damper system k = 100 N/m and m = 1 kg under free vibrations the amplitude decays to half the initial value in every oscillation

31. The damping coefficient of the system is
(a) 2.19 N-sec /m (b) 3.29 N-sec/cm
(c) 5.15 N-sec/m (d) 3.45 N-sec/m

32. When a force F is applied statically it produces a deflection of 3 mm when the same force is applied with a frequency of 20 rad/sec the steady state amplitude is
(a) 1 mm along F $\boxed{\text{(b) 1 mm opposite to F}}$
(c) 3 mm along F (d) 0.5 mm opposite to F

31, 32Q.

$k = 100\,N/m$ $x_1 = 0.5\cdot x_0$

$m = 1\,kg$. $C = ?$

Logarithmic decrement

$$\delta = \ln\left(\frac{x_0}{x_1}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

$$\ln\left(\frac{1}{0.5}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \qquad \zeta = 0.1096$$

$$C = \zeta\cdot 2\sqrt{MK} = 0.1096\times 2\times\sqrt{1\times 100} = 2.19\,\frac{N\text{-}s}{m}.$$

$x_{static} = 3\,mm$.

$\omega = 20\,rad/s$

$$\omega_n = \sqrt{\frac{K}{M}} = \sqrt{\frac{100}{1}} = 10\,rad/s.$$

$$r = \frac{\omega}{\omega_n} = \frac{20}{10} = 2$$

$$A = \frac{x_{static}}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \qquad A = \frac{3}{\left[(1-2^2)^2 + (2\times 0.1\times 2)^2\right]^{0.5}}$$

$A = 1\,mm$.

@ $r > 1$ Applied Force F and displacement $x(t)$ are in opposite phase.

FACULTY **WAHEED UL HAQ**

33. In a spring mass dampers system the magnification factor at resonance is found to be 20. The damping ratio is
(a) 0.2 (b) 0.25
(c) 0.025 (d) 0.05

33Q. @ $r=1$ resonance.

$$D.M.F = \frac{1}{2\zeta} = 20.$$

$$\zeta = \frac{1}{2\times 20} = 1/40 = 0.025$$

FACULTY **WAHEED UL HAQ**

34. A machine of 100 kg mass has a 20kg rotor with 0.5mm eccentrically. The mounting springs have stiffness 85kN/m and damping is negligible. If the operating speed is 20 π rad/s and the unit is constrained to move vertically, the dynamic amplitude of the machine will be
(a) 0.470×10^{-4} (b) 1.000×10^{-4} m
(c) 1.270×10^{-4} m (d) 2.540×10^{-4} m

$$A = \frac{me\omega^2}{\sqrt{(K-M\omega^2)^2 + (c\omega)^2}}$$

$M = 100\,kg$ $m = 20\,kg$ $e = 0.5\,mm.$

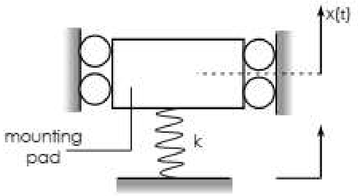
$K = 85000\,N/m.$ $C = 0$ $\omega = 20\pi$ rad/s

$A = ?$

$$A = \pm\frac{20\times 0.5\times 10^{-3}\times(20\pi)^2}{85000 - 100(20\pi)^2}$$

$$A = 1.270\times 10^{-4}\,m.$$

35. A proper base isolation is to be designed for mounting a sensitive instrument as shown in Fig. At the point of mounting, the base vibration due to other disturbances is indicated in the figure. If the permitted absolute displacement amplitude on the rigid mounting pad is 0.01 mm, find the stiffness of the spring. Assume that the total mass of the mounting pad and the instrument is 50 kg.
(GATE–99)



y(t) = 0.2 Sin(200 πt) mm

$K = ?$ $M = 50\,kg.$ $C = 0$ $X = 0.01$

$Y = 0.2$

$\omega = 200\pi$ rad/s

$$\frac{X}{Y} = \pm\left[\frac{K^2 + (c\omega)^2}{(K-M\omega^2)^2 + (c\omega)^2}\right]^{0.5}$$

$$\frac{X}{Y} = \frac{K}{K - M\omega^2}$$

$$\frac{0.01}{0.2} = \frac{\pm K}{K - 50(200\pi)^2}$$

$$K = 939.96\,kN/m.$$

36. A single degree of freedom system is specified by the following differential equation
$5\ddot{x} + 20\dot{x} + 80x = 8\cos 4t$,
Match the following
Group A
P. Frequency of Oscillations — 4 rad/s.
Q. Amplitude of Oscillations — 0.1
R. Damping ratio — 0.5
S. Magnification factor — 1.
Group B
1. 0.8 2. 0.5
3. 4 4. 1 5. 0.1
Codes :
(a) A–4, B–3, C–2, D–1
(b) A–3, B–5, C–2, D–4
(c) A–3, B–1, C–5, D–2
(d) A–3, B–4, C–2, D–1

$M = 5\,kg$ $C = 20\,\frac{N\text{-}s}{m}.$ $K = 80\,N/m.$

$F = 8N.$ $\omega = 4$ rad/s. $\zeta = \frac{c}{2\sqrt{MK}} = \frac{20}{2\sqrt{5\times 80}}$

$$\omega_n = \sqrt{\frac{K}{M}} = \sqrt{\frac{80}{5}} = 4 \text{ rad/s.}$$

@ $r=1$ $D.M.F = \frac{A}{x_{static}} = \frac{1}{2\zeta} = \frac{1}{2\times 0.5} = 1.$

$$A = \frac{x_{static}}{2\zeta} = \frac{F/K}{2\times 0.5} = \frac{8/80}{1} = 0.1$$

FACULTY **WAHEED UL HAQ**

ACE Engineering Academy

FACULTY **WAHEED UL HAQ**

37. A machine of 250 kg mass is supported on springs of total stiffness 100 kN/m. Machine has an unbalanced rotating force of 350 N at speed of 3600 rpm. Assuming a damping factor of 0.15, the value of transmissibility ratio is (GATE-06)

(a) 0.0531 (b) 0.9922
(c) 0.0162 (d) 0.0028

37Q. $M = 250\,kg$ $k = 100\,kN/m$

$F = 350\,N$ $N = 3600\,rpm$

$\zeta = 0.15$ $\epsilon = ?$

$$\epsilon = \sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}}$$

$$r = \frac{\omega}{\omega_n} = \frac{\frac{2\pi N}{60}}{\sqrt{\frac{K}{M}}} = \frac{\frac{2\pi\times3600}{60}}{\sqrt{\frac{100\times10^3}{250}}} = 6\pi >> 1.$$

$\epsilon << 1.$

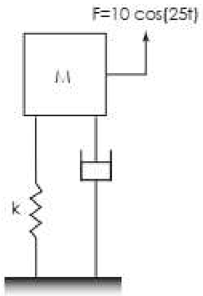
$$\epsilon = \sqrt{\frac{1+(2\times0.15\times6\pi)^2}{[1-(6\pi)^2]^2+(2\times0.15\times6\pi)^2}} = 0.0162$$

Efficiency of isolation $\eta = 1-\epsilon$

$= 1-0.0162$

$= 98.38\%$

38. A mass-spring-dashpot system with mass m=10 kg, spring constant k = 6250 N/m is excited by a harmonic excitation of 10 cos (25t) N. At the steady state, the vibration amplitude of the mass is 40mm. The damping coefficient (c, in N.s/m) of the dashpot is _______ (GATE-14)



38Q. $m = 10\,kg$

$k = 6250\,N/m$.

$F(t) = 10\cos(25t)$

$A = 40\,mm$. $C = ?$

$$A = \frac{F}{\sqrt{(k-m\omega^2)^2+(c\omega)^2}}$$

$$\Rightarrow 40\times10^{-3} = \frac{10}{[(6250-10(25)^2)^2+(c\times25)^2]^{0.5}} \Rightarrow C = 10\,N\text{-}s/m.$$

39. In vibration isolation, which one of the following statements is NOT correct regarding Transmissibility (T)? (GATE-14)

(a) T is nearly unity at small excitation frequencies → $\epsilon \approx 1$ @ $r <<< 1$ ✓

(b) T can be always reduced by using higher damping at ~~any~~ lower excitation frequency → $0 < r < \sqrt{2} \rightarrow \epsilon \propto 1/\zeta$ after $r > \sqrt{2}$ $\boxed{\epsilon \propto \zeta}$

(c) T is unity at the frequency ratio of $\sqrt{2}$ → $\epsilon = 1$ @ $r = \sqrt{2}$ $\zeta = 0 / \zeta \neq 0$ ✓

(d) T is infinity at resonance for undamped systems → $\epsilon = \infty$ @ $r = 1$ $\zeta = 0$ ✓

after $r > \sqrt{2}$ damping behaviour is determ-ental.

40. There are four samples P, Q, R and S with natural frequencies 64, 96, 128 and 256 Hz, respectively. They are mounted on test setups for conducting vibration experiments. If a loud pure note of frequency 144 Hz is produced by some instrument, → resonance which of the samples will show the most perceptible induced vibration ? (GATE–05)

(a) P (b) Q $\boxed{\text{(c) R}}$ (d) S

128 Hz is closer to 144Hz.

so this sample will produced loud note.

FACULTY **WAHEED UL HAQ**

41. A single degree freedom spring mass is subjected to a sinusoidal force of 10 N amplitude and frequency ω along the axis of the spring. The stiffness of the spring is 150N/m, damping factor is 0.2 and undamped natural frequency is 10 ω. At steady state, the amplitude of vibration (in m) is approximately **(GATE –15)**

(a) 0.05 (b) 0.07
(c) 0.70 (d) 0.90

$F = 10N \quad K = 150 N/m.$

$\xi = 0.2 \quad \omega_n = 10\omega.$

$r = \frac{\omega}{\omega_n} = 0.1 \qquad x_{static} = F/K = 10/150.$

$$A = \frac{x_{static}}{[(1-r^2)^2 + (2\xi r)^2]^{0.5}} = \frac{10/150}{[(1-10^2)^2 + (2\times 0.2\times 10)^2]^{0.5}}$$

$A = 0.7 \, m.$

42. A precision instrument package (m=1kg) needs to be mounted on a surface vibrating at 60 Hz. It is desired that only 5% of the base surface vibration amplitude be transmitted to the instrument. Assume that the isolation is designed with its natural frequency significantly lesser than 60Hz, so that the effect of damping may be ignored. The stiffness (in N/m) of the required mounting pad is ________ **(GATE-15)**

Base excitation

$M = 1 \, kg$

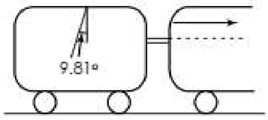
$\omega = 2\pi f = 2\pi \times 60 = 120\pi \; Hz.$

$$\epsilon = 5\% = 0.05 = \frac{\pm K}{K - M\omega^2} \Rightarrow 0.05 = \frac{\pm K}{K - 1(120\pi)^2}$$

$K =$

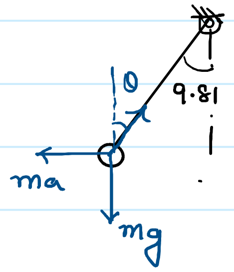
FACULTY WAHEED UL HAQ

43. A pendulum of mass 50 gm and length 500 mm is suspended from the roof of a moving rail coach. If the pendulum is in equilibrium at the position shown, the acceleration of the train at that instant is



(a) 0.981 m/sec²　　(b) 9.81 m/sec²
(c) 1.69 m/sec²　　(d) 1.86 m/sec²

$m = 50\,gm \quad l = 500\,mm$



FACULTY **WAHEED UL HAQ**

$\Sigma F_x = 0$

$T\sin\theta = ma.$

$T\cos\theta = mg$

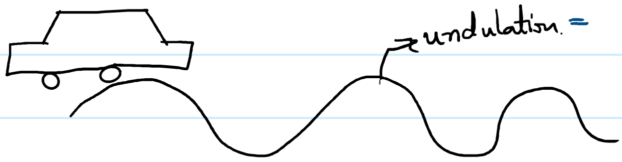
$\tan\theta = a/g \Rightarrow a = g\cdot\tan\theta$

$a = 9.81\cdot\tan 9.81.$

$= 1.69\, m/s^2$

44. When a vehicle travels on a rough road whose undulations can be assumed to be sinusoidal, the resonant conditions of the base excited vibrations are determined by the

(a) mass of the vehicle, stiffness of the suspension spring, speed of the vehicle, wavelength of the roughness curve
(b) speed of the vehicle only
(c) speed of the vehicle and the stiffness of the suspension spring
(d) amplitude of the undulations



45. A shaft has an attached disc at the center of its length. The disc has its center of gravity located at a distance of 2 mm from the axis of the shaft. When the shaft is allowed to vibrate in its natural bow-shaped mode, it has a frequency of vibration of 10 rad/sec. When the shaft is rotated a 300 revolutions per minute, it will whirl with a radius of

(a) 2 mm　　(b) 2.25 mm
(c) 2.50 mm　　(d) 3.00 mm

whirling of shaft.

$e = 2\,mm.$

$\omega_n = 10\, rad/s.$

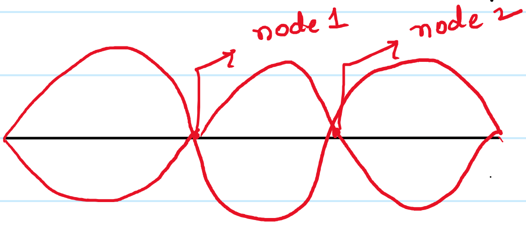
$\omega = \frac{2\pi \times 300}{60} = 10\pi\ rad/s$

$r = \frac{\omega}{\omega_n} = \frac{10\pi}{10} = \pi$

$y = \frac{e r^2}{1-r^2} = \frac{2\cdot(\pi)^2}{1-\pi^2} = -2.25\,mm.$

46. If two nodes are observed at a frequency of 1800 rpm during whirling of a simply supported long slender rotating shaft, the first critical speed of the shaft in rpm is (GATE-13)

(a) 200　　(b) 450
(c) 600　　(d) 900



$\omega_3 = 3^2\cdot\omega_1 = 1800$

$\omega_1 = \frac{1800}{9}$

$\omega_1 = 200\,rpm.$

47. The rotor shaft of a large electric motor supported between short bearings at both the ends shows a deflection of 1.8 mm in the middle of the rotor. Assuming the rotor to be perfectly balanced and supported at knife edges at both the ends, the likely critical speed (in rpm) of the shaft is (GATE-09)

(a) 350　　(b) 705
(c) 2810　　(d) 4430

$\delta = 1.8\,mm.$

$\omega_n = ?$

$\omega_n = \sqrt{\frac{K}{M}} = \sqrt{\frac{Mg/\delta_{static}}{M}}$

$\omega_n = \sqrt{\frac{g}{\delta_{static}}} \qquad \omega_n = \sqrt{\frac{9.81}{1.8\times10^{-3}}} = 73.82\, rad/s.$

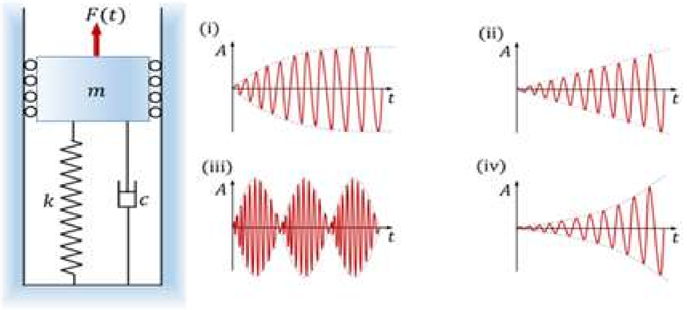
$N_{cr} = \frac{\omega_n \times 60}{2\pi} = 705\ rpm.$

FACULTY **WAHEED UL HAQ**

G-2022

FACULTY **WAHEED UL HAQ**

Q.40 A spring mass damper system (mass m, stiffness k, and damping coefficient c) excited by a force $F(t) = B\sin\omega t$, where B, ω and t are the amplitude, frequency and time, respectively, is shown in the figure. Four different responses of the system (marked as (i) to (iv)) are shown just to the right of the system figure. In the figures of the responses, A is the amplitude of response shown in red color and the dashed lines indicate its envelope. The responses represent only the qualitative trend and those are not drawn to any specific scale.



Four different parameter and forcing conditions are mentioned below.

(P) $c > 0$ and $\omega = \sqrt{k/m}$ (Q) $c < 0$ and $\omega \neq 0$

(R) $c = 0$ and $\omega = \sqrt{k/m}$ (S) $c = 0$ and $\omega \cong \sqrt{k/m}$

Which one of the following options gives correct match (indicated by arrow →) of the parameter and forcing conditions to the responses?

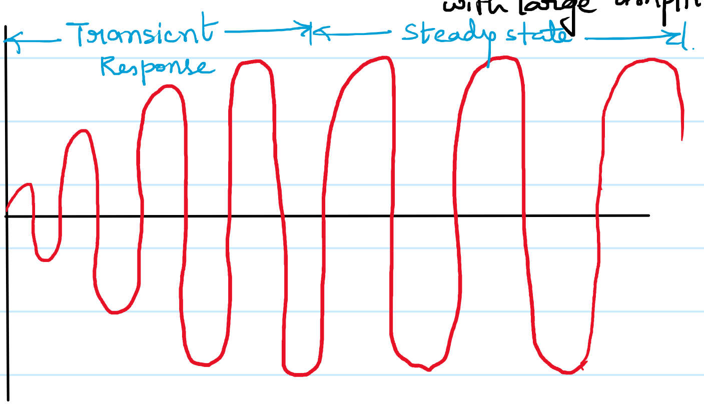
(A)	$\boxed{(P) \to (i)}$,	(Q) → (iii),	(R) → (iv),	(S) → (ii)
(B)	(P) → (ii),	(Q) → (iii),	(R) → (iv),	(S) → (i)
(C)	$\boxed{(P) \to (i)}$,	$\boxed{(Q) \to (iv)}$,	(R) → (ii),	(S) → (iii)
(D)	(P) → (iii),	(Q) → (iv),	(R) → (ii),	(S) → (i)

Response of forced vibration

$$x(t) = X_0 e^{-\xi\omega_n t}\sin(\omega_d t + \phi) + \frac{x_{static}}{[(1-r^2)^2 + (2\xi r)^2]^{0.5}}\sin(\omega t - \phi)$$

Cond. (P) – $c > 0$ $\omega = \sqrt{\frac{k}{M}}$ (Resonance) $r = 1$

$$x(t) = \underbrace{X_0 \cdot e^{-\xi\omega_n t}\sin(\omega_d t + \phi)}_{\text{as } t \to \infty,\ e^{-\xi\omega_n t} \to 0} + \underbrace{\frac{x_{static}}{2\xi}\sin(\omega_n t - \phi)}_{\text{steady state with large amplitude}}$$



FACULTY **WAHEED UL HAQ**

ACE Engineering Academy

FACULTY WAHEED UL HAQ

Q- $c < 0 \quad \omega \neq 0 \quad \xi = -ve.$

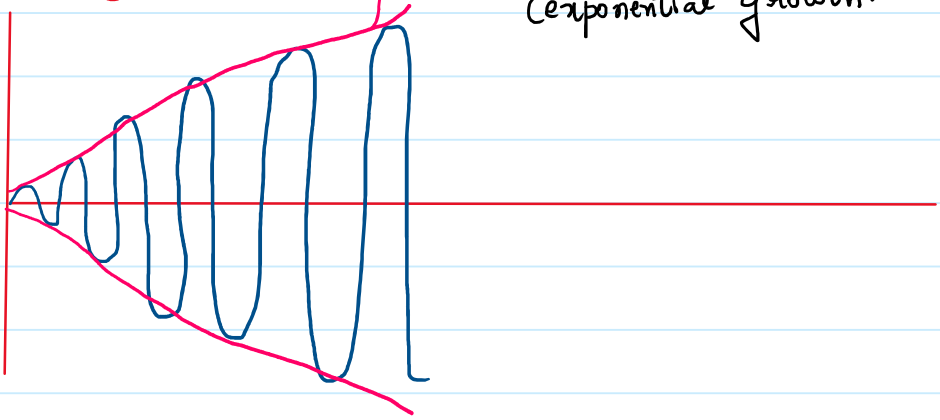
$$x(t) = X_0 \cdot e^{+\xi\omega_n t} \sin(\omega_d t + \phi) + \frac{x_{static} \sin(\omega t - \phi)}{[(1-r^2)^2 + (2\xi r)^2]^{0.5}}$$

as $t \to \infty$

$e^{\xi\omega_n t} \to \infty$

$e^{\xi\omega_n t}$

exponential growth.



(R) $C = 0 \quad \omega = \sqrt{\frac{K}{M}} \quad r = 1 \quad \xi = 0$

$$x(t) = X_0 e^{-\xi\omega_n t} \sin(\omega_d t + \phi) + \frac{x_{static}}{\left[\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2\right]^{0.5}} \sin(\omega t - \phi)$$

1. $\quad \omega_n$.

$\frac{x_{static}}{0}$

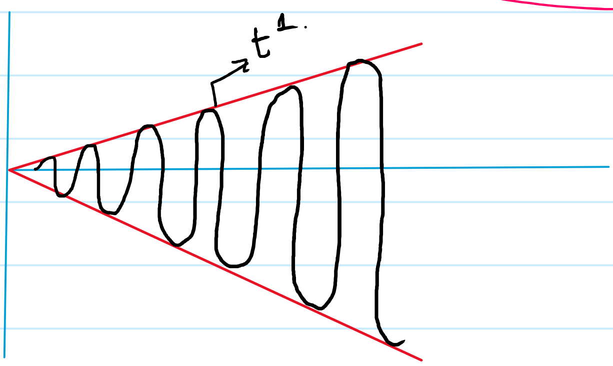
0

$$x(t) = X_0 \cdot \sin(\omega_n t + \phi) + \frac{\frac{d}{d\omega}(x_{static}) \sin(\omega t - \phi)}{\frac{d}{d\omega}\left[\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)\right]}$$

$$x(t) = X_0 \cdot \sin(\omega_n t + \phi) + \frac{x_{static} \cdot \cos(\omega t - \phi) \cdot t}{\left(0 - \frac{2\omega}{\omega_n}\right)}$$

$$x(t) = X_0 \cdot \sin(\omega_n t - \phi) - \frac{x_{static}\, \omega_n \cdot t^1}{2} \cos(\omega_n t - \phi)$$

linear variation wrt time.

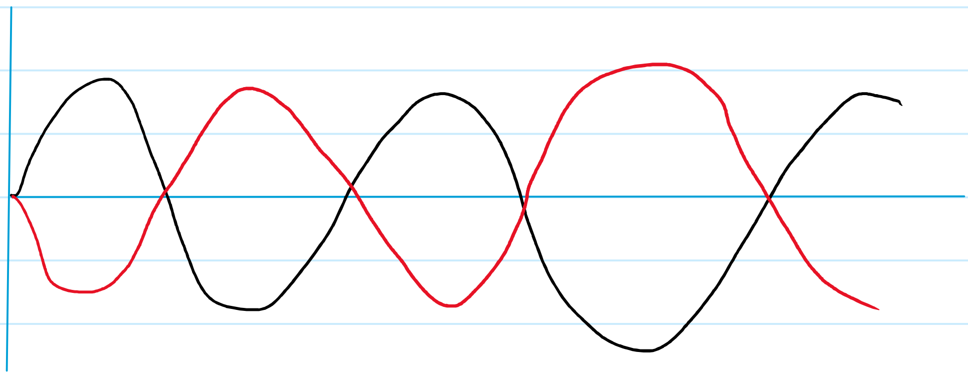


FACULTY WAHEED UL HAQ

$C = 0 \qquad \omega \cong \sqrt{\frac{K}{M}}$

$$x(t) = X_0 e^{-\underset{\downarrow\, 1}{\xi}\omega_n t} \sin(\underset{\downarrow\, \omega_n}{\omega_d} t + \phi) + \frac{x_{static}}{[(1-r^2)^2 + (2\underset{\downarrow}{\xi} r)^2]^{0.5}} \sin(\omega t - \phi)$$

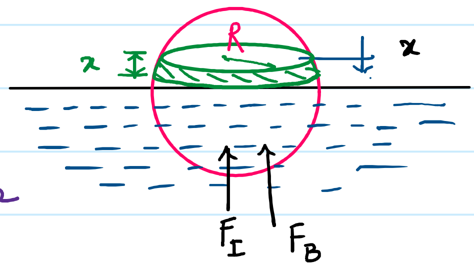
$$x(t) = X_0 \cdot \sin(\omega_n t + \phi) \pm \frac{x_{static}}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \cdot \sin(\omega t - \phi)$$



Q-19;

A hollow sphere of radius R_o is 50% immersed in the water. Natural frequency of undamped vibration is ____________ for small displacement.

ρ is the density of water.



Sol: Hollow sphere attain equilibrium after 50% of its volume is immersed.

Volume of water displaced = 50% volume of sphere $= 0.5 \times \frac{4}{3}\pi \cdot R_o^3 = \frac{2}{3}\pi R_o^3$

Mass of water displaced = Mass of 50% of sphere.

Mass of water displaced $= (\rho \cdot \times 50\%$ of $V_{sphere})$

Mass of sphere inside = Mass of water displaced.

$= \rho_w \cdot \frac{2}{3}\pi R_o^3$

Bouyancy force

Weight of sphere inside water after giving intial displacement (x)

$= \rho_w \cdot g \cdot \pi R_o^2 x.$