04. The 210 kg rotor of a turbojet aircraft engine has a radius of gyration of 220 mm and rotates ccw at 18,000 rpm as viewed from the front. If the aircraft is travelling at 1200 km/h and starts to execute an inside vertical loop of 3800 m radius, compute the gyroscopic moment 'M' transmitted to the airframe. What correction to the controls does the pilot have to make in order to remain in vertical plane?

$M = 210\,kg$

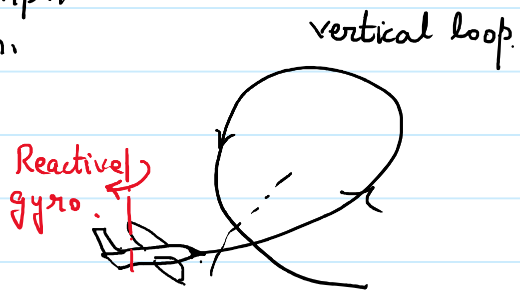
$k = 220\,mm.$

$N = 18000\,rpm.$ C.C.W from front.

$V = 1200\,kmph$

$R = 3800\,m.$

$M = ?$



$$M = I.\omega.\omega_p.$$
$$= mk^2.\left(\frac{2\pi N}{60}\right)\left(\frac{V}{R}\right)$$
$$= 210 \times 0.22^2 \times \left(\frac{2\pi \times 18000}{60}\right)\left(\frac{1200 \times 1000}{3600 \times 3800}\right)$$
$$= \underline{1680.58}\ N\text{-}m.$$

ω_{spin}

$\hat{\omega}_{spin} = \hat{i}$ $\quad \hat{\omega}_p = +\hat{k}$

Active gyro $= \hat{\omega}_p \times \hat{\omega}_s = \hat{k} \times \hat{i} = \hat{j}$

Reactive gyro $= \hat{\omega}_s \times \hat{\omega}_p = \hat{i} \times \hat{k} = -\hat{j}$

Reactive gyro will cause aeroplane to turn right side, so the correction in control is to turn the aeroplane towards left, and vertical loop can be completed.

05. A car is moving on a curved horizontal road of radius 100 m with a speed of 20 m/s. The rotating masses of the engine have an angular speed of 100 rad/s in clockwise direction when viewed from the front of the car. The combined moment of inertia of the rotating masses is 10 kg-m². The magnitude of the gyroscopic moment (in N-m) is_____.
(GATE – 16)

$R = 100\,m.$

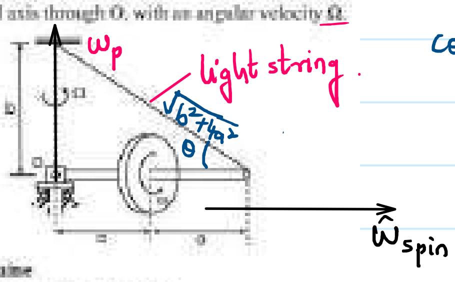
$V = 20\,m/s$

$\omega_s = 100\,rad/s$

$I = 10\,kg\text{-}m^2$

$$C = I.\omega_s.\omega_p$$
$$= 10 \times 100 \times \frac{20}{100} = \underline{200\,N\text{-}m}.$$

06. A thin disc of radius 'r' and mass 'm' is mounted on a light rod of length 2a which is freely hinged at O. The other end A of rod being supported by a light string. The disc spins with an angular velocity ω as shown and the whole assembly rotates about a vertical axis through O, with an angular velocity Ω.



$\sin\theta = \dfrac{b}{\sqrt{b^2+4a^2}}$

$\cos\theta = \dfrac{2a}{\sqrt{b^2+4a^2}}$

$\hat{\omega}_{spin} = \hat{i} \qquad \hat{\omega}_p = \hat{j}$

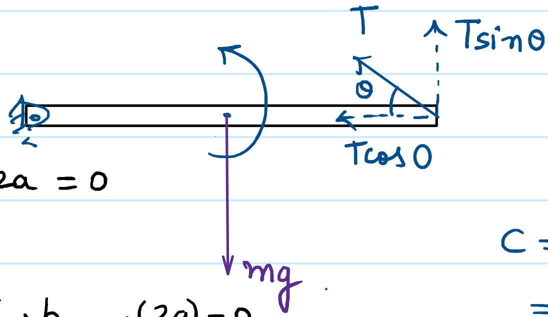
Active gyro. $= \hat{\omega}_p \times \hat{\omega}_s = \hat{j} \times \hat{i} = -\hat{k}$

Reactive gyro. $= \hat{\omega}_s \times \hat{\omega}_p = \hat{i} \times \hat{j} = \hat{k}$ C.C.W

Determine

(a) The tension in the string.

(b) What will be the string tension if the system rotates with a velocity Ω in the opposite directions ?



a)

$\Sigma M_{Hinge} = 0$

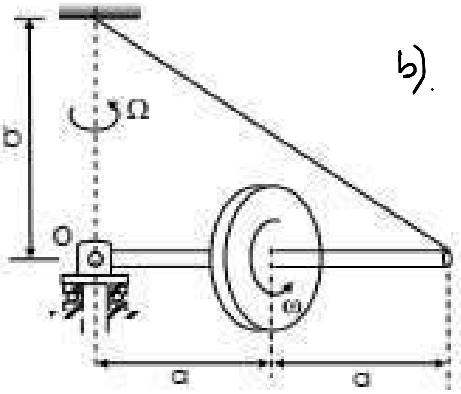
$-C + mg(a) - T\sin\theta \times 2a = 0$

$C = I \cdot \omega \cdot \omega_p$

$= \dfrac{mr^2}{2} \cdot \omega \cdot \Omega$

$-\dfrac{mr^2}{2} \cdot \omega \cdot \Omega + mg \cdot (a) - T \cdot \dfrac{b}{\sqrt{b^2+4a^2}} \cdot (2a) = 0$

$T = \left[mg\,a - \dfrac{mr^2 \cdot \omega\,\Omega}{2}\right] \times \dfrac{\sqrt{b^2+4a^2}}{2ab}$



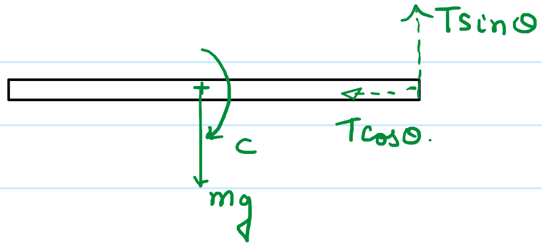
b).

$\hat{\omega}_{spin} = \hat{i}$

$\hat{\omega}_p = -\hat{j}$

Active gyro. $= \hat{\omega}_p \times \hat{\omega}_s = -\hat{j} \times \hat{i} = +\hat{k}$

Reactive gyro $= \hat{\omega}_s \times \hat{\omega}_p = \hat{i} \times -\hat{j} = -\hat{k}$ (C-W)



$T = \left[mg\,a + \dfrac{mr^2 \cdot \omega\,\Omega}{2}\right] \dfrac{\sqrt{b^2+4a^2}}{2ab}$

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Cam and Follower

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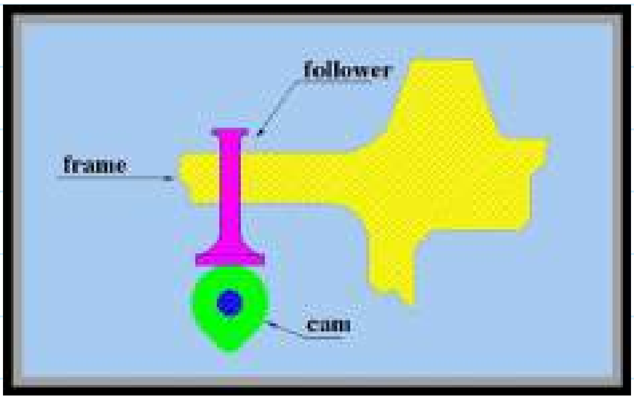
- A Cam and follower mechanism consist of
  1. Cam –
  2. Follower–
  3. Frame –

DoF = 1

- This mechanism is an example of Higher Pair, Force Closed, Constrained Pair , Open Pair.

- In it the cam is main driving element and follower follows its motion. driven.
  Generally the Cam rotates and follower translates or oscillates.



y – follower displacement
θ – Cam Angular displace-ment.

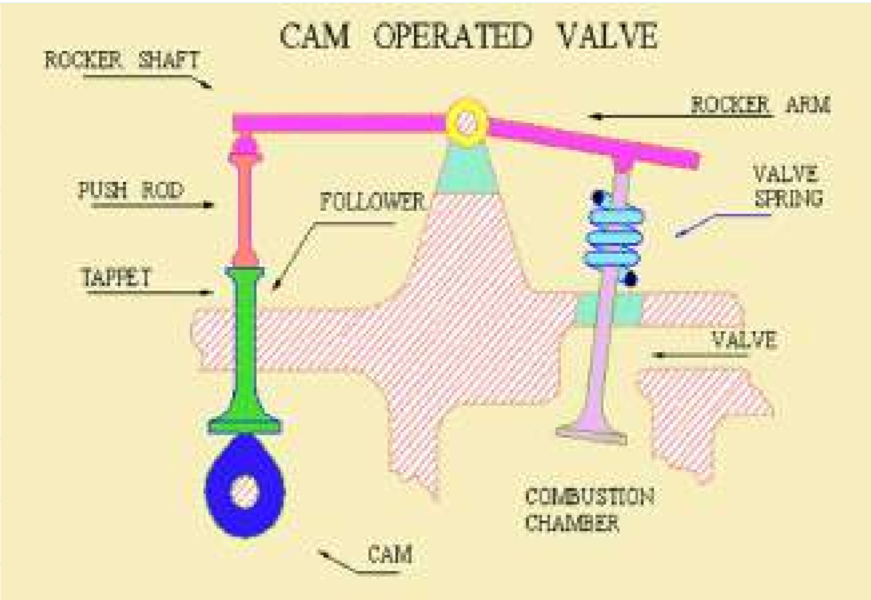
Type of motion impart to the follower is dependent on nature of cam profile

- Cam and Follower mechanism is an exact function generator mechanism.

$y = f(\theta)$

- For ex. In the case of moving the valves of an automobile, First in IC Engine the valves have to be kept open then keep it open then close it and keep it closed.

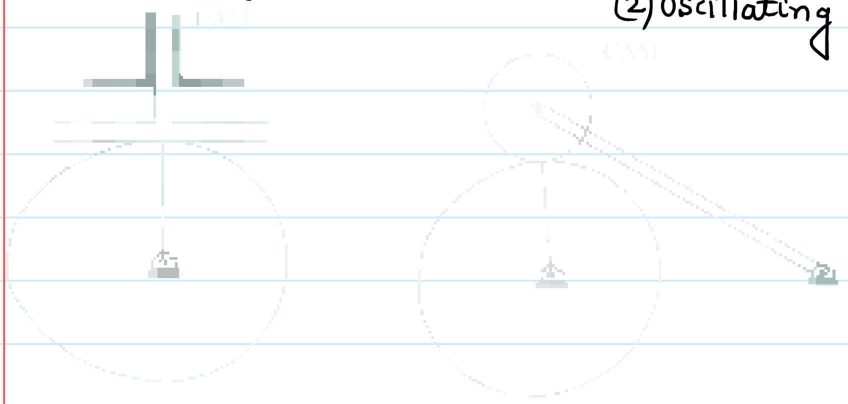
- All these timing operations can be easily incorporated by having Cam and Follower mechanism.



Classification of follower based on motion

(1) Reciprocating follower.

(2) oscillating follower.

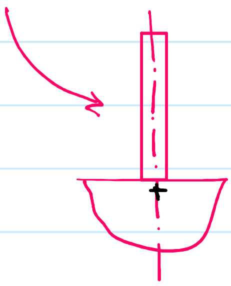


Applications of Cam and Follower

- Cam and follower are widely used for operating inlet and exhaust valve of IC Engine
- In Wall Clock
- In feed mechanism of Automatic Lathe Machine
- In Paper Cutting Machine
- In Weaving Textile Machine
- In Food processing machines

Classification of Followers

- Knife edge follower
- Roller follower
- Flat face follower
- Spherical end follower or Mushroom follower



Knife edge Follower

- It is the simplest follower
- The contacting end is a sharp knife edge, it is called a knife edge follower
- In it a considerable side thrust exists between the follower and guide
- It causes infinitely large contacting stresses and results in high rate of wear, therefore it is of very little practical use

Roller Follower

- When the contacting end of the follower is a roller, it is called a roller follower.
- Since the rolling motion takes place between the contacting surfaces therefore the rate of wear is greatly reduced but the side thrust exists.
- In case of steep cam roller follower has a tendency to jam the cam, therefore it is not preferred in this situation.
  → Mechanism is getting locked.
- It is used in gas and oil engines.
- Roller followers are extensively used where more space is available such as in stationary gas and oil engines.

Flat face follower

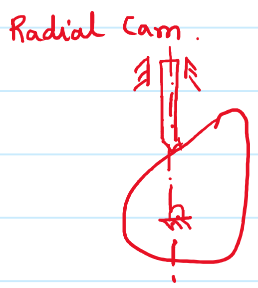
- When the contacting end is perfectly flat face normal to the stem of follower, it is called as flat face follower.
- The side thrust in it gets reduced w.r.t. roller follower.
- The relative motion is sliding in nature due to which there is more wear of cam surface in it.
- It causes high surface stresses.
- It is used where limited space is available such as in cams which operate the valves of automobile engines.

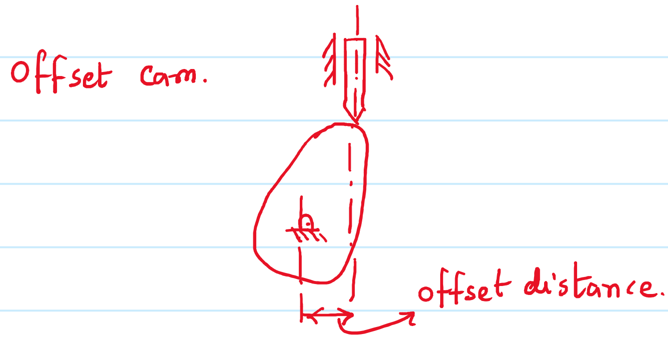
Spherical/Mushroom faced follower

- When the contacting end of the follower is of spherical shape, it is called spherical faced follower.
- The centre of spherical surface is provided on the centre line of follower.
- It is used for relatively steep cam and is useful where space may not be adequate.
- It minimises the surface stresses.
- It is used in automobile engines, aircraft engines.

Classification of Cam (On the basis of line of movement of the follower)

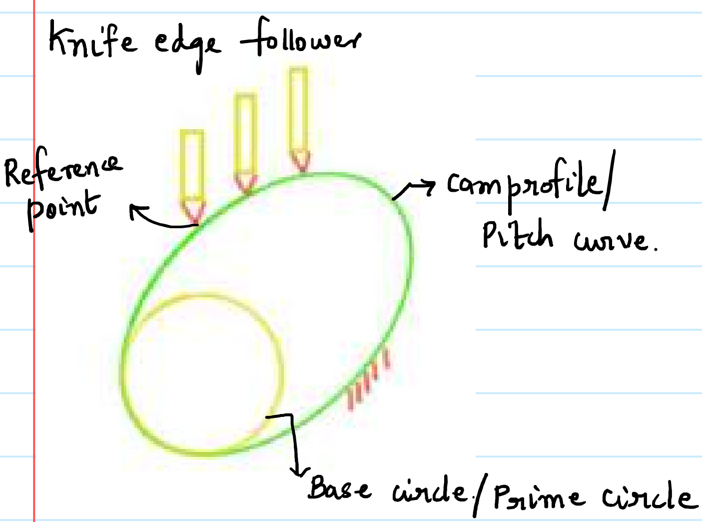
- Radial Cam - Line of stroke of follower pass through the centre of rotation of CAM
- Offset Cam - Line of stroke of follower does not pass through the centre of rotation of CAM. It is offset by some distance.

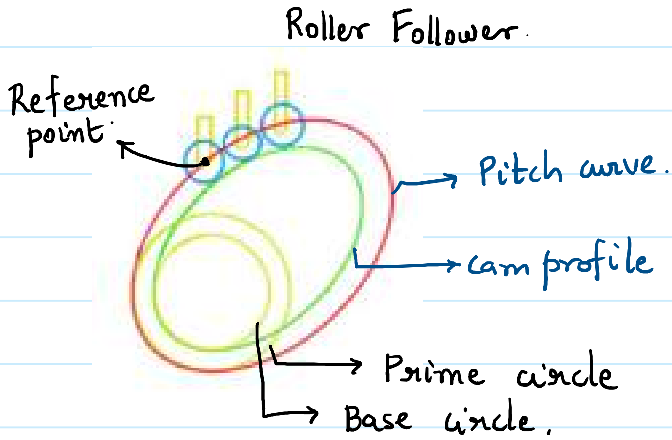




Cam and Follower Terminology:
In order to discuss the terminology we have to consider the inversion of mechanism that is we shall assume Cam as fixed element and follower as moving element.

- Base Circle - It is the smallest circle tangential to the cam profile
- Trace Point - It is the reference on the follower which along the cam as the cam rotates. For knife follower the trace point is point of contact and for roller follower the centre of roller is trace point.
- Pitch Curve - Locus of trace point is called pitch curve.
- Prime Circle - The circle which is tangential to pitch curve is called prime circle.
- Lift or Stroke - Distance between the extreme position of follower.
- Angle of ascent - Angular displacement of Cam for which follower moves from bottom most position to top most top most position.
- Angle of descent - Angular displacement of Cam for which follower moves from top most pos'n to bottom most pos'n
- Angle of dwell - Angular displacement of Cam for which follower does not move.





for knife edge. $R_{Roller} \rightarrow 0$.

$R_{prime} = R_{base} + R_{Roller}$.

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Pressure Angle / Deviation Angle

higher pair.

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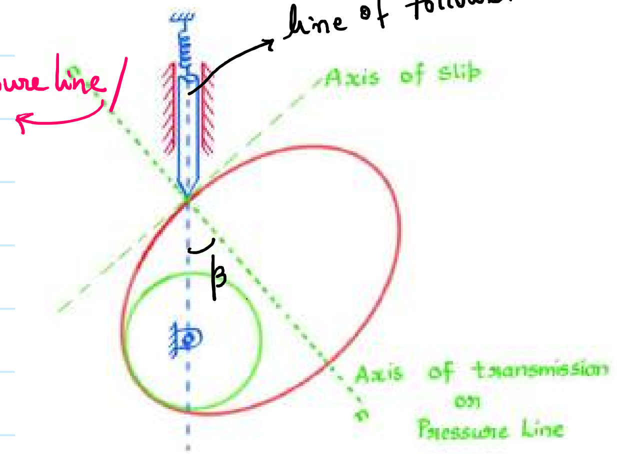
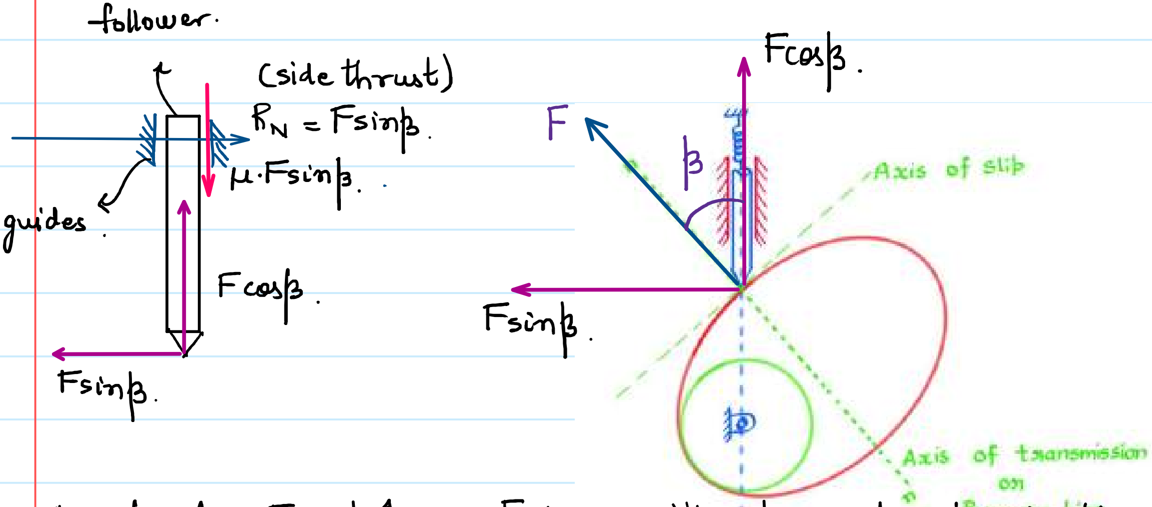
- It is the angle included between common normal to the pitch curve through trace point and the line of action of follower motion.
- It is compliment of transmission angle and it is an index of merit of the mechanism. $\beta + \gamma = 90°$

β – Pressure Angle.

- It measures the steepness of the cam profile.
- Pressure angle varies in magnitude at all instants of the follower motion. The point where it is maximum is known as Pitch Point.
- The locus of pitch point is known as Pitch Circle.



Force Angle of Cam. and follower.



As $\beta \uparrow$. $F\sin\beta \uparrow$, $F\sin\beta$ will try to bend the follower.
$\mu F\sin\beta \uparrow$. friction b/w follower and guides increases. Cam and follower mechanism get locked.
As. $\beta \uparrow$ $F\cos\beta \downarrow$. the contact b/w Cam and follower may lost.

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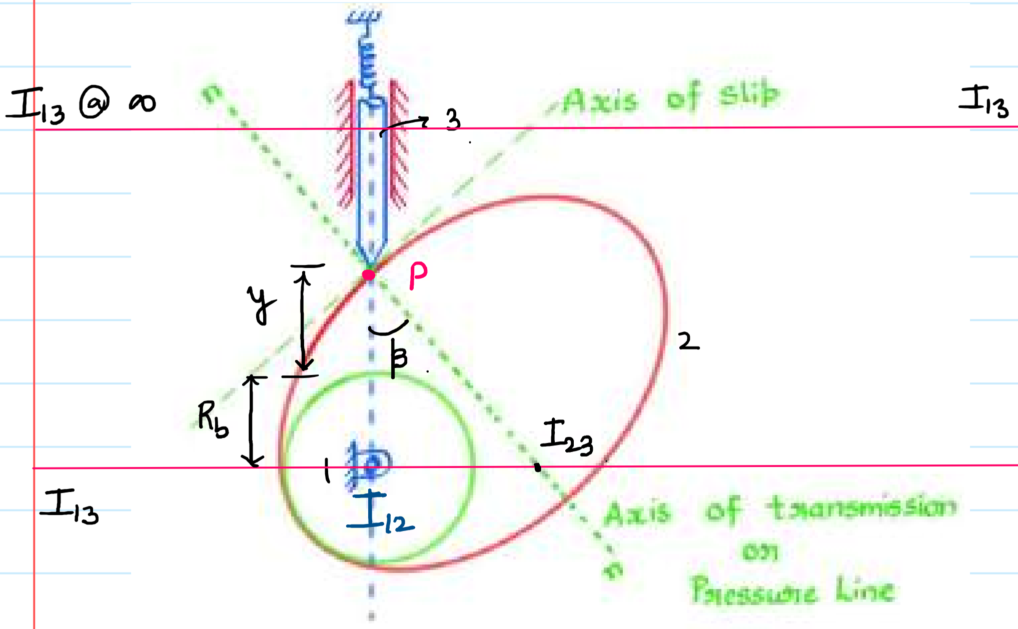

I_{23} must lie on common normal and it must be colinear with I_{12} & I_{13}.

In $\triangle^{le}$ $I_{12}.P.I_{23}$

$$\tan\beta = \frac{I_{12}.I_{23}}{(R_b+y)}$$

$$I_{12}.I_{23}.\omega_2 = I_{13}.I_{23}.\omega_3 \rightarrow V_{follower}$$

$$V_{follower} = I_{12}.I_{23}.\omega_2. \qquad \omega_2 = \frac{d\theta}{dt}.$$

$$V_{follower} = (R_b+y)\tan\beta . \frac{d\theta}{dt}. \qquad y = f(\theta)$$

$$V_f = \frac{dy}{dt} = \frac{dy}{d\theta}.\frac{d\theta}{dt}$$

$$V_f. = \cancel{\frac{d\theta}{dt}}.\frac{dy}{d\theta} = (R_b+y)\tan\beta . \cancel{\frac{d\theta}{dt}}. \qquad V_f. = \omega.\frac{dy}{d\theta}$$

$$\tan\beta = \frac{\frac{dy}{d\theta}}{(R_b+y)} = \frac{V_{follower}}{(R_b+y)\,\omega_{cam}}.$$

R_b - Radius of Base circle, y - follower displacement.

Effect of various parameters on Pressure Angle

1. Size of Cam

$$\tan\beta. = \frac{V_f}{(R_b+y)\omega} \Rightarrow \tan\beta \propto \frac{1}{R_b}.$$

As $R_b \uparrow$ $\tan\beta \downarrow$. $\beta \downarrow$.

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2. Type of follower

$$\tan\beta = \frac{V_f}{(R_b + R_{roller} + y)\cdot\omega_2}$$

$$\tan\beta \propto \frac{1}{R_{roller}}$$

$R_{Roller} \uparrow \ \tan\beta \downarrow \ \ \beta \downarrow$.

3. Velocity of follower

$$\boxed{\tan\beta \propto V_{follower}}$$

$V_f \uparrow \ \tan\beta \uparrow \ \ \beta \uparrow$.

$$\boxed{V_{cycloidal} = V_{uniform\ acc^n} > V_{S.H.M.} > V_{uniform\ velocity}}$$

Cycloidal cams are mostly preferred.

V_f is more in cycloidal cams. $\tan\beta \downarrow$, $R_b \uparrow$.

$R_b \uparrow$. These cams are large in size.

Cycloidal motion is found in High speed cams. The high speed cams are large in size.

4. **Effect of slope of displacement curve.**

$$\tan\beta = \frac{dy/d\theta}{(R_b + y)}$$

It signifies about the steepness of cam.

$y \rightarrow$ follower displacement

$\theta \rightarrow$ Angular displacement of cam.

Cam – 1. θ 30° $\longrightarrow$ y=40 mm.

Cam – 2 θ – 30° $\longrightarrow$ y-50mm. $\longrightarrow$ cam 2 is steeper compared with cam 1.

For small rotation of cam if the follower experience more displacement then the cam is steeper.

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5. Effect of Offset distance.

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$$\tan\beta = \frac{\left[\frac{dy}{d\theta} - e\right]}{\sqrt{R_b^2 - e^2} + y}$$

$$\tan\beta \propto \left[\frac{dy}{d\theta} - e\right]$$

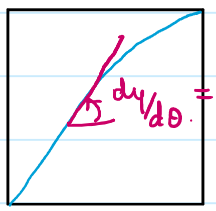
$R_b >>> e$. $\sqrt{R_b^2 - e^2} \approx R_b$.

As $e \uparrow$ $\left[\frac{dy}{d\theta} - e\right] \downarrow$ $\tan\beta \downarrow$. $\beta \downarrow$.

6. Effect of stroke

Forward stroke

y



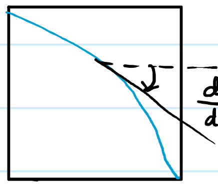
$dy/d\theta$ = c.c.w. (+ve)

θ

$$\tan\beta \propto \left[\frac{dy}{d\theta} - e\right].$$

Return stroke

y



$\frac{dy}{d\theta}$ = c.w. (−ve)

θ

$$\tan\beta \propto \left[-\frac{dy}{d\theta} - e\right]$$

$$(\beta)_{Return} > (\beta)_{forward}.$$

Physical derivates.

y − displacement of follower.

$y = f(\theta)$.

Velocity $\Rightarrow$ $v = \frac{dy}{dt} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dt} = \omega \cdot \frac{dy}{d\theta}$. $\rightarrow$ slope of displacement curve

$v = f(\theta)$

Acceleration $\Rightarrow$ $a = \frac{dv}{dt} = \frac{dv}{d\theta} \cdot \frac{d\theta}{dt} = \omega \cdot \frac{d}{d\theta}\left[\omega \cdot \frac{dy}{d\theta}\right]$

$= \omega^2 \cdot \frac{d^2y}{d\theta^2}$ $\rightarrow$ convex . or concave.

$a = f(\theta)$

Jerk $\Rightarrow$ $J = \frac{da}{dt} = \frac{da}{d\theta} \cdot \frac{d\theta}{dt} = \omega \cdot \frac{d}{d\theta}\left[\omega^2 \cdot \frac{d^2y}{d\theta^2}\right] = \omega^3 \cdot \frac{d^3y}{d\theta^3}$

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Types of motion

1. Uniform Velocity
2. Uniform Acceleration and Uniform Retardation
3. Simple Harmonic Motion
4. Cycloidal Motion

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h - stroke of follower.

θ_a - Angular disp. of cam.

1. Uniform Velocity.

$$V = \text{constant}$$

$$V = \omega \cdot \frac{dy}{d\theta} \qquad \omega = \text{constant}.$$

$$\frac{dy}{d\theta} = \text{constant}.$$

$$y = c\theta^1$$

$$y = h \quad \theta = \theta_a \quad c = \frac{h}{\theta_a}.$$

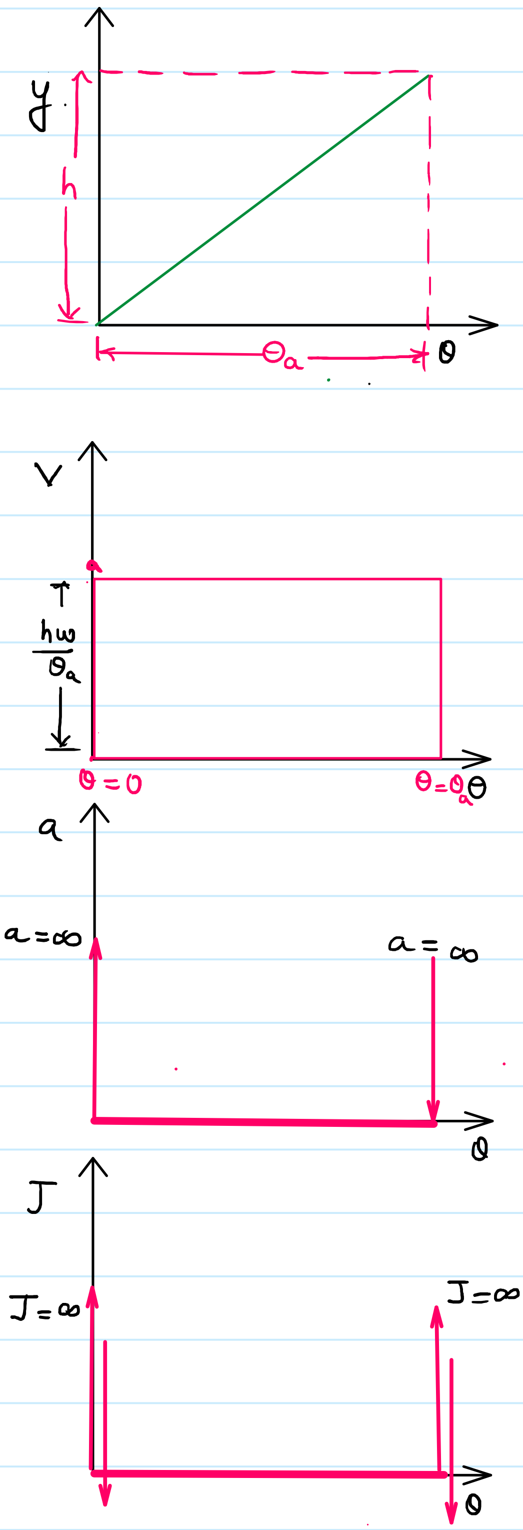
$$y = \frac{h}{\theta_a} \cdot \theta$$

Velocity $\quad V = \frac{dy}{dt} = \omega \cdot \frac{dy}{d\theta} = \frac{\omega h}{\theta_a} \theta^0.$

$$V = f(\theta^0) = \text{const}.$$

Acceleration $\quad a = \frac{dv}{dt} = 0$

Jerk $\quad J = \frac{da}{dt} = 0$



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2. Uniform Acceleration and Uniform Retardation

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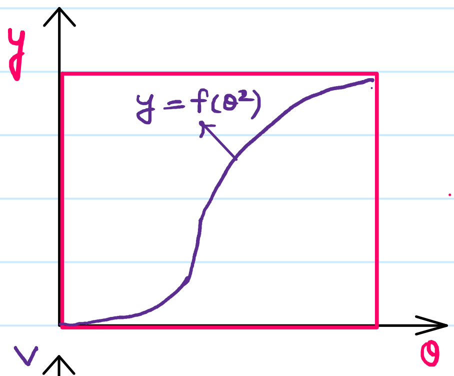
acceleration $a = \text{const}$

$$a = \omega^2 \cdot \frac{d^2y}{d\theta^2} \qquad \omega = \text{const}.$$

$$\frac{d^2y}{d\theta^2} = \text{const}.$$

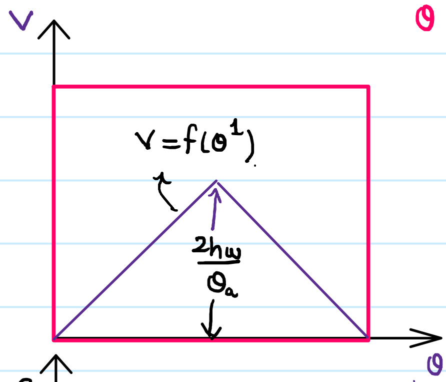
$$y = c_1\theta^2 + c_2\theta + c_3.$$



initial conditions. @ $\theta = 0 \quad y = 0 \Rightarrow c_3 = 0$

@ $\theta = 0 \quad V = 0$ or $\frac{dy}{d\theta} = 0 \Rightarrow c_2 = 0$

$$V = \omega.\frac{dy}{d\theta} \Rightarrow \frac{dy}{d\theta} = 0.$$

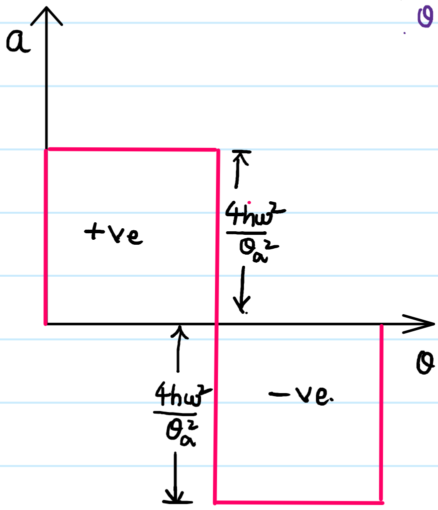


$$y = c_1\theta^2 \Rightarrow y = f(\theta^2)$$

$$\theta = \frac{\theta_a}{2},\ y = \frac{h}{2},\ V = \text{Max}.\quad c_1 = \frac{(h/2)}{\theta_a^2/4} = \frac{2h}{\theta_a^2}$$

Velocity of follower $\quad V = \omega.\frac{dy}{d\theta} = \omega.\frac{d}{d\theta}\left(\frac{2h}{\theta_a^2}\theta^2\right)$

$$V = \frac{4h\omega}{\theta_a^2}.\theta^1 \qquad V = f(\theta^1)$$

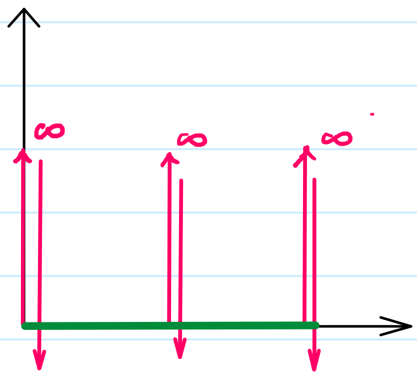


$$V_{max} = \frac{4h\omega}{\theta_a^2}\cdot\frac{\theta_a}{2}$$

$$V_{max} = \frac{2h\omega}{\theta_a}.$$

Acceleration $\quad a = \omega^2.\frac{d^2y}{d\theta^2} = \omega^2.\frac{d^2}{d\theta^2}\left(\frac{2h}{\theta_a^2}.\theta^2\right)$

$$a = \frac{4h\omega^2}{\theta_a^2}$$



Jerk. $\quad J = \frac{da}{dt} = 0.$

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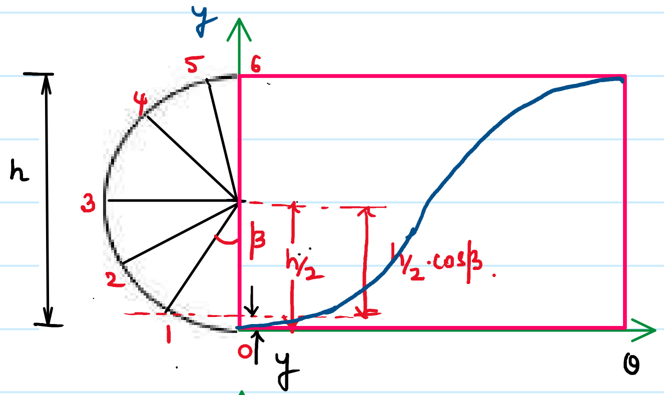
3. Simple Harmonic Motion

$$y = \frac{h}{2}\left[1-\cos\beta\right].$$

$\beta \to \pi \quad y \to h. \quad \theta \to \theta_a.$



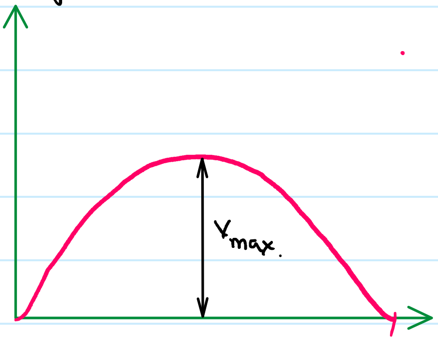
$\beta \to \pi$

$\theta \to \theta_a.$

$$\beta = \frac{\pi\theta}{\theta_a}$$

$$y = h/2.\left[1-\cos\frac{\pi\theta}{\theta_a}\right].$$

Velocity $\quad v = \omega.\dfrac{dy}{d\theta}$



$$v = \omega.\frac{d}{d\theta}\left[1-\cos\frac{\pi\theta}{\theta_a}\right]\times\frac{h}{2}$$

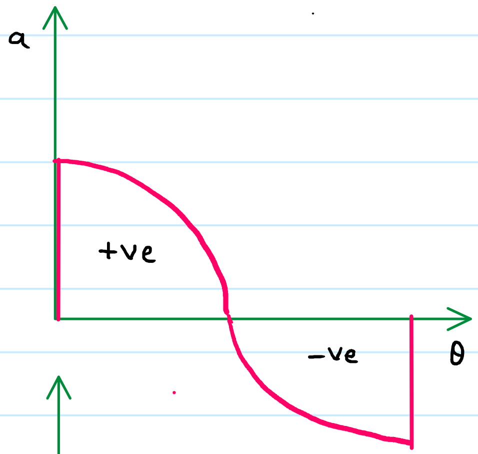
$$= \omega.\left[0-\left(-\sin\frac{\pi\theta}{\theta_a}\times\frac{\pi}{\theta_a}\right)\right]\times\frac{h}{2}$$

$\dfrac{dy}{d\theta}$

$$= \frac{\pi h.\omega}{2\theta_a}\sin\left(\frac{\pi\theta}{\theta_a}\right)$$

$$V_{max} = \frac{\pi h\omega}{2\theta_a}$$

Acceleration $\quad a = \omega^2.\dfrac{d^2y}{d\theta^2}$



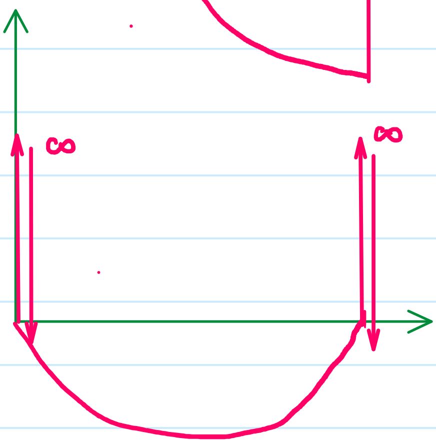
$$a = \omega^2.\frac{d}{d\theta}\left[\frac{\pi h}{2\theta_a}.\sin\left(\frac{\pi\theta}{\theta_a}\right)\right]$$

$$a = \frac{\omega^2.h}{2}.\cos\left(\frac{\pi\theta}{\theta_a}\right).\left(\frac{\pi}{\theta_a}\right)^2$$

$\dfrac{d^2y}{d\theta^2}$

$$a = \frac{\pi^2.h\omega^2}{2\theta_a^2}.\cos\left(\frac{\pi\theta}{\theta_a}\right).$$

$$a_{max} = \frac{\pi^2.h\omega^2}{2\theta_a^2}$$



Jerk. $\quad J = \dfrac{da}{dt} = \omega^3.\dfrac{d^3y}{d\theta^3}$

$$= \omega^3.\frac{d}{d\theta}\left[\frac{h}{2}.\cos\left(\frac{\pi\theta}{\theta_a}\right)\left(\frac{\pi}{\theta_a}\right)^2\right]$$

$$= \frac{\pi^3.h\omega^3}{2\theta_a^3}.\left[-\sin\left(\frac{\pi\theta}{\theta_a}\right)\right].$$

$$J_{max} = \frac{-\pi^3.h\omega^3}{2\theta_a^3}$$

4. Cycloidal Motion

Displacement of follower.

$$y = \frac{h}{\pi}\left[\frac{\pi\theta}{\theta_a} - \frac{1}{2}\cdot\sin\left(\frac{2\pi\theta}{\theta_a}\right)\right]$$

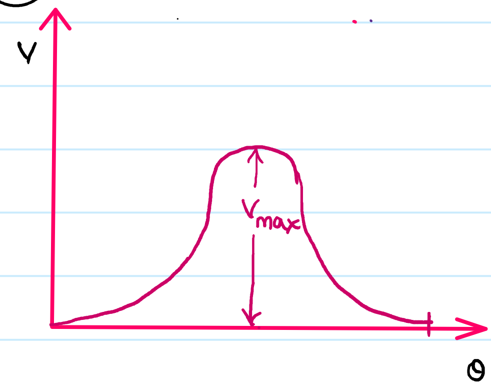
Radius of generating circle $= \frac{h}{2\pi}$.

Velocity of follower.

$$v = \omega\cdot\frac{dy}{d\theta} = \omega\cdot\frac{d}{d\theta}\left[\frac{h}{\pi}\left(\frac{\pi\theta}{\theta_a} - \frac{1}{2}\sin\left(\frac{2\pi\theta}{\theta_a}\right)\right)\right]$$

$$v = \underbrace{\frac{\omega\cdot h}{\pi}\left[\frac{\pi}{\theta_a} - \frac{1}{2}\left(\frac{2\pi}{\theta_a}\right)\cdot\cos\left(\frac{2\pi\theta}{\theta_a}\right)\right]}_{\frac{dy}{d\theta}}$$

$$v = \frac{h\omega}{\theta_a}\left[1 - \cos\left(\frac{2\pi\theta}{\theta_a}\right)\right]$$



θ	0°	$\frac{\theta_a}{4}$	$\frac{\theta_a}{2}$	$\frac{3\theta_a}{4}$	θ_a
v	0	$\frac{h\omega}{\theta_a}$	$\frac{2h\omega}{\theta_a}$	$\frac{h\omega}{\theta_a}$	0

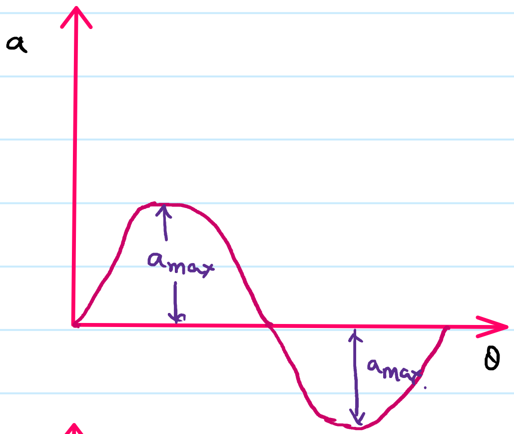
$\to V_{max}$.

Acceleration.

$$a = \omega^2\cdot\frac{d^2y}{d\theta^2} = \omega^2\cdot\frac{d}{d\theta}\left[\frac{h}{\theta_a}\left(1 - \cos\left(\frac{2\pi\theta}{\theta_a}\right)\right)\right]$$

$$= \underbrace{\frac{\omega^2 h}{\theta_a}\cdot\left[0 + \sin\left(\frac{2\pi\theta}{\theta_a}\right)\cdot\frac{2\pi}{\theta_a}\right]}_{\frac{d^2y}{d\theta^2}}$$

$$a = \frac{2\pi\cdot h\omega^2}{\theta_a^2}\cdot\sin\left(\frac{2\pi\theta}{\theta_a}\right)$$

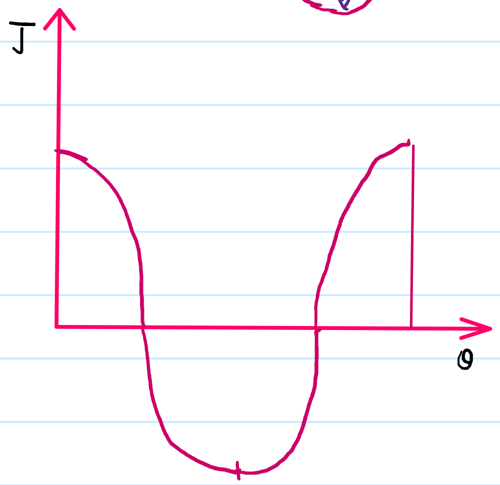


θ	0°	$\frac{\theta_a}{4}$	$\frac{\theta_a}{2}$	$\frac{3\theta_a}{4}$	θ_a
a	0	$\frac{2\pi h\omega^2}{\theta_a^2}$	0	$-\frac{2\pi h\omega^2}{\theta_a^2}$	0

Jerk.

$$J = \omega^3\cdot\frac{d^3y}{d\theta^3} = \omega^3\cdot\frac{d}{d\theta}\left[\frac{h}{\theta_a}\cdot\sin\left(\frac{2\pi\theta}{\theta_a}\right)\cdot\frac{2\pi}{\theta_a}\right]$$

$$J = \frac{4\pi^2\cdot h\omega^3}{\theta_a^3}\cdot\cos\left(\frac{2\pi\theta}{\theta_a}\right)$$



θ	0°	$\frac{\theta_a}{4}$	$\frac{\theta_a}{2}$	$\frac{3\theta_a}{4}$	θ_a
J	$\frac{4\pi^2\cdot h\omega^3}{\theta_a^3}$	0	$-\frac{4\pi^2 h\omega^3}{\theta_a^3}$	0	$\frac{4\pi^2\cdot h\omega^3}{\theta_a^3}$

In Cycloidal motion the Jerk is finite at all the instances of motion.
This type of motion is preferred for High speed Cams.

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S.No.	Motion	Velocity	Acceleration	Jerk.
1.	Uniform velocity.	$1\frac{h\omega}{\theta_a}$	$0\frac{h\omega^2}{\theta_a^2}$	$0\frac{h\omega^3}{\theta_a^3}$
2.	Uniform Acceleration.	$2\frac{h\omega}{\theta_a}$	$4\frac{h\omega^2}{\theta_a^2}$	$0\frac{h\omega^3}{\theta_a^3}$
3.	Simple Harmonic Motion	$\frac{\pi}{2}\frac{h\omega}{\theta_a}$	$\frac{\pi^2}{2}\frac{h\omega^2}{\theta_a^2}$	$\frac{\pi^3}{2}\frac{h\omega^3}{\theta_a^3}$
4.	Cycloidal Motion.	$2\frac{h\omega}{\theta_a}$	$2\pi\frac{h\omega^2}{\theta_a^2}$	$(2\pi)^2\frac{h\omega^3}{\theta_a^3}$

01. The pressure angle in a cam depends on
1. Follower type
2. Displacement profile
3. Angle of action of cam
4. Offset between cam centre and follower

(a) 2, 4
(b) 1, 2, 3
(c) 2, 3, 4
(d) 1, 2, 3, 4

$\tan\beta \propto \frac{1}{R_{Roller}}$ ✓

$\tan\beta \propto \frac{dy}{d\theta}$ ⇝ cam profile.

$\tan\beta. \propto \frac{1}{d\theta}$ ⇝ cam angle.

$\tan\beta \propto. \left[\frac{dy}{d\theta} - e\right]$.

02. In a radial cam follower system with same lift and angle of action the maximum velocity during rise with the following profiles
I. Uniform velocity profile
II. Simple Harmonic motion profile
III. Cycloidal motion profile

(a) I > II > III
(b) II > III > I
(c) III > I > II
(d) III > II > I

$V_{cycliod} = V_{uni.acc^n} > V_{S.H.M.} > V_{uniform\ velo.}$

III > II > I

03. The pressure angle of a cam during lift can be reduced by
(a) ~~Decreasing~~ increase the base circle radius ✗
(b) Increasing the angle of action
(c) ~~Reducing~~ increasing the roller radius
(d) By reducing the offset distance

$\tan\beta \propto \frac{1}{R_{base}}$.

$\tan\beta. \propto \frac{1}{d\theta}$ ⇝ cam angle / Angle of action.

$\tan\beta \propto \frac{1}{R_{Roller}}$

$\tan\beta \propto \left[\frac{dy}{d\theta} - e\right]$ As $e\uparrow$ $\left[\frac{dy}{d\theta} - e\right]\downarrow$ $\tan\beta\downarrow$.

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04. A cam follower rises by 4cm during 90° rotation of the cam with SHM. The cam is rotating at an uniform angular velocity of 2 rad/sec. The displacement, velocity and acceleration of the follower after 2/3 of the rotation of the cam during rise is

(a) 2cm, 8 cm/sec, 16 cm/sec²
(b) 3cm, 7 cm/sec, 16 cm/sec²
(c) 2cm, 8 cm/sec, 32 cm/sec²
(d) 2.67cm, 6 cm/sec, 23.86 cm/sec²

$h = 4cm.$

$\theta_a = 90^\circ = \pi/2.$

$\omega_{cam} = 2 rad/s.$

@ $\theta = 2/3 \cdot \theta_a.$

$y, v, a = ?$

Disp. $y = \frac{h}{2}\left[1 - \cos\frac{\pi\theta}{\theta_a}\right] \Rightarrow y = 4/2\left[1 - \cos\frac{2\pi}{3}\right] = 2[1-(-0.5)] = 3cm.$

Velocity. $v = \frac{\pi h \omega}{2\theta_a} \cdot \sin\left(\frac{\pi\theta}{\theta_a}\right) = \frac{\pi \times 4 \times 2}{2 \times \pi/2} \cdot \sin\left(\frac{2\pi}{3}\right) = 7cm/s.$

Acceleration $a = \frac{\pi^2 h \omega^2}{2\theta_a^2} \cdot \cos\left(\frac{\pi\theta}{\theta_a}\right) = \frac{\pi^2 \cdot (4) \cdot (2)^2}{2 \cdot (\pi/2)^2} \cdot \cos\left(2\pi/3\right) = 16 cm/s^2.$

05. The profile of a radial cam is specified as

x = 15 cosθ

and y = 10 + 5 sinθ

where θ is the angle of rotation of the cam and x, y are the Cartesian coordinates of the profile for a specific range. Determine the pressure angle at an angle θ = 30°. (Assume cam center as origin)

(a) 30° (b) 16° (c) 0° (d) 45°

$x = 15\cos\theta \qquad y = 10 + 5\sin\theta.$

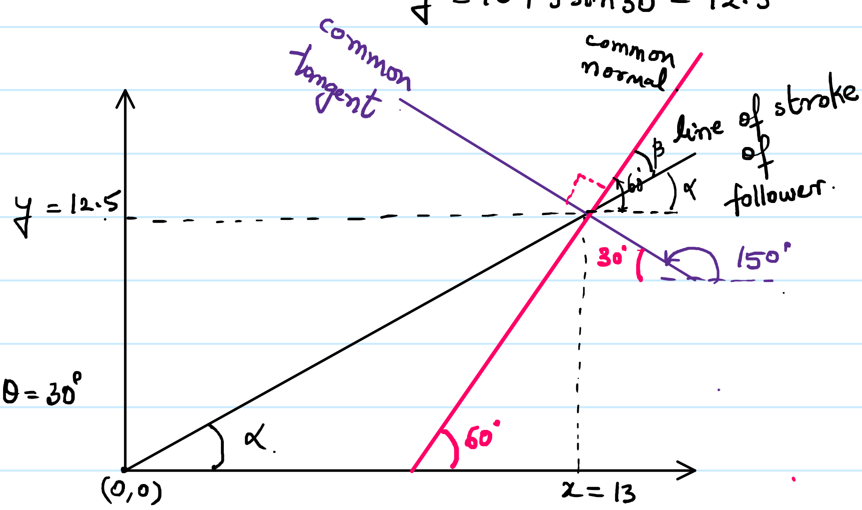
$\theta = 30^\circ.$

$x = 0 \qquad \theta = 30^\circ$

$y = 0 \qquad x = 15\cos 30^\circ = 12.99 \approx 13$

$y = 10 + 5\sin 30 = 12.5$

$\alpha = \tan^{-1}\left(\frac{12.5}{13}\right) = 43.87.$



common Tangent.

$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{0 + 5\cos\theta}{15(-\sin\theta)}$

slope of common tangent @ $\theta = 30^\circ$

$\tan\theta_1 = \frac{5\cos 30^\circ}{-15\sin 30^\circ}$

$\theta_1 = \tan^{-1}\left(-\frac{5 \times \sqrt{3}/2}{15 \times 1/2}\right)$

$\theta_1 = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = 150^\circ$ from $+x$-axis.

$\beta = 60^\circ - \alpha$

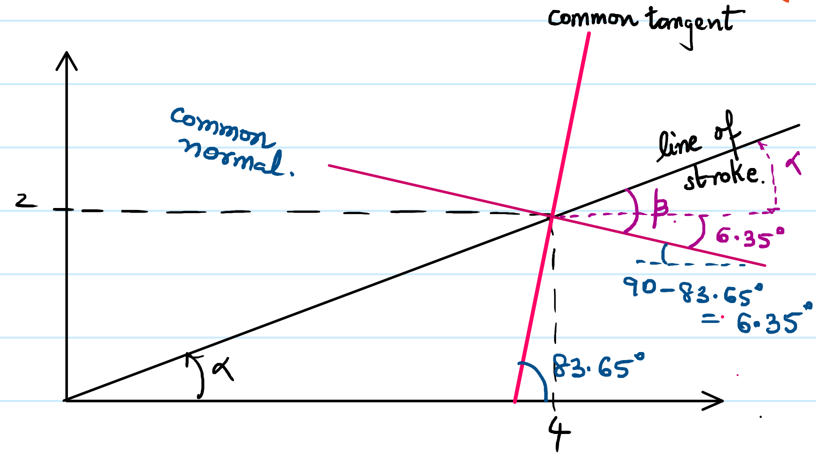
$= 60^\circ - 43.87^\circ$

$= 16.13^\circ$

06. A segment of a radial cam profile is given by the equation in Cartesian space
$y = 2x^2 - 7x - 2$ with the origin of the coordinate frame at the centre. For the knife edge follower determine the pressure angle at a location ($x = 4$, $y = 2$)
(a) 32.82° (b) 63.4°
(c) 83.7° (d) 58°

$y = 2x^2 - 7x - 2$

$\beta = ?$ @ $x = 4$ $y = 2$.



$\alpha = Tan^{-1}(2/4) = 26.56°$

Common Tangent.

$\dfrac{dy}{dx} = 4x - 7.$

Slope of common Tangent @ $x = 4.$ $y = 2$

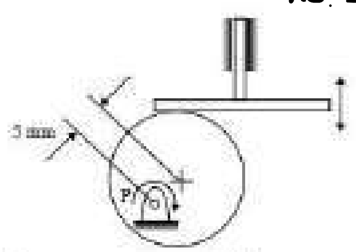
$Tan\theta_1 = 4(4) - 7 \Rightarrow Tan\theta_1 = 9.$ $\theta_1 = Tan^{-1}(9) = 83.65°$ with $+x$-axis.

Pressure Angle

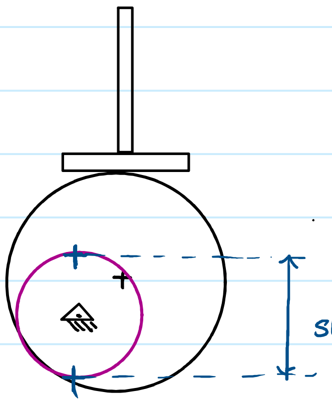
$\beta = \alpha + 6.35° = 26.56° + 6.35° = 33°.$

07. Consider a circular cam with a flat face follower as shown in the figure below. The cam is rotated in the plane of the paper about point P lying 5 mm away from its center. The radius of the cam is 20 mm. The distance (in mm) between the highest and the lowest positions of the flat face follower is
(GATE -16)

stroke. = ?

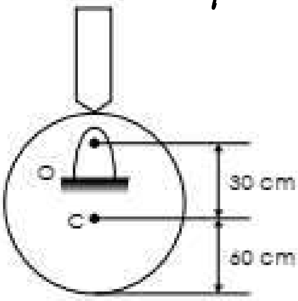


(a) 5 (b) 10
(c) 40 (d) 45

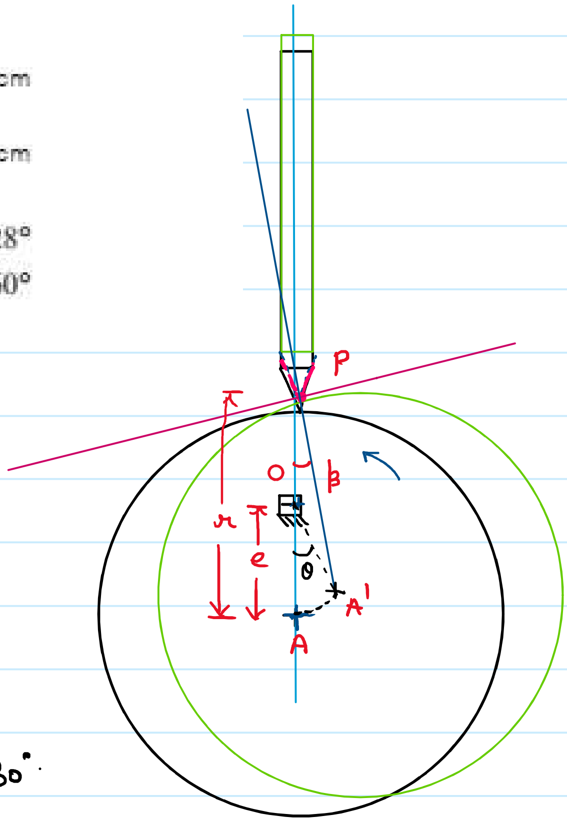


stroke = 2e.
= 2×5
= 10 mm.

08. For the eccentric circular cam and follower mechanism shown below for the pressure angle at pitch point is $\longrightarrow \beta = \text{Max.}$



(a) 30°
(b) 28°
(c) 45°
(d) 60°



$$AA' = r\sin\beta = e\sin\theta.$$

$$60\sin\beta = 30\sin\theta.$$

$$\beta = \text{Max.} \quad \sin\theta = 1$$

$$\sin\beta = 30/60.$$

$$\beta = \sin^{-1}(1/2) = 30^\circ.$$

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Flywheel

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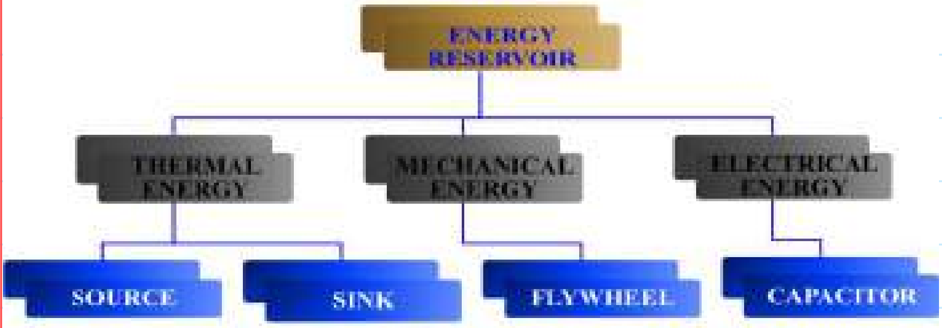


Flywheel is the energy reservior which stores the energy when there is excess in supply of energy and it supplies the same amount of energy when there is deficiency in energy.

Application of flywheel → Prime mover
Application of flywheel → Punching Press

Prime Mover – It is a device which is used to drive some m/c or external load.

Supply Torque – Torque exerted on the shaft of prime mover is called as supply Torque.

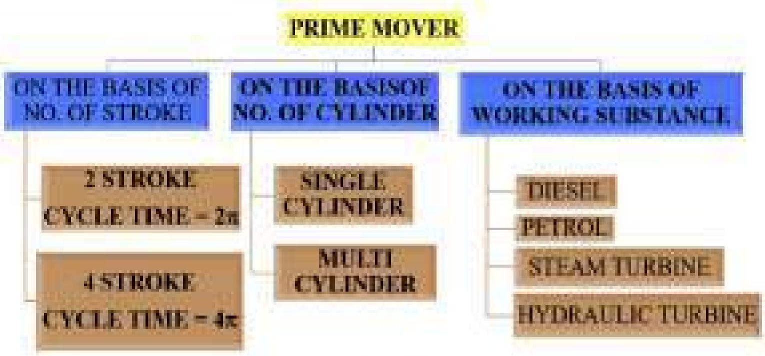
Torque on crank shaft $\Rightarrow$ $T = \dfrac{F_P \cdot \sin(\theta+\beta)}{\cos\beta} \times r$. $r \rightarrow$ constant.

$$F_P = F_G \pm F_I, \quad F_I \rightarrow m r \omega^2 \cdot \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

$$F_P = f(\theta) \qquad \beta = f(\theta)$$

$$T_{supply} = f(\theta) \longrightarrow \text{I.C. Engine}$$

FLYWHEEL IN CASE OF PRIME MOVER:



NOTE: There is minimum fluctuation in hydraulic turbine therefore they do not required flywheel.
NOTE: In multicylinder engine the turning moment diagram is more uniform than single cylinder engine therefore multicylinder engines requires lesser inertia flywheel.

T_{supply} , T_{load}

Type Pr.-1. $T_{supply} = f(\theta)$ – (variable) ✓
T_{load} = constant.

$\boxed{\text{I.C. Engine}}$

Type Pr. 2. T_{supply} = constant.
$T_{load} = f(\theta)$ (variable) ✓

$\boxed{\text{Punching Machine.}}$

Type Pr–3. $\left.\begin{array}{l} T_{supply} = f(\theta) \\ T_{load} = f(\theta) \end{array}\right\}$ subjective.

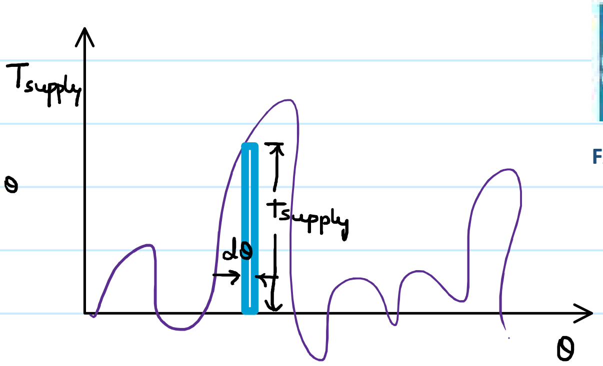
RESISTING TORQUE OR LOAD TORQUE:
Every prime mover will drive some machine the torque exert by machine on the prime mover is known as "Resisting Torque".

- The net area of T_{Load} v/s θ diagram represent the energy required per cycle.
- If $T_{Load} = f(\theta)$ then,
  The net energy required per cycle = net area of T_{Load} v/s θ diagram.
- If T_{Load} = Constant then,
  Net energy required per cycle = T_{load} x Cycle time

$T_{supply} = f(\theta)$

Energy supplied

$$E_{supplied} = \int_0^{cycle\ time} T_{supply} \cdot d\theta$$



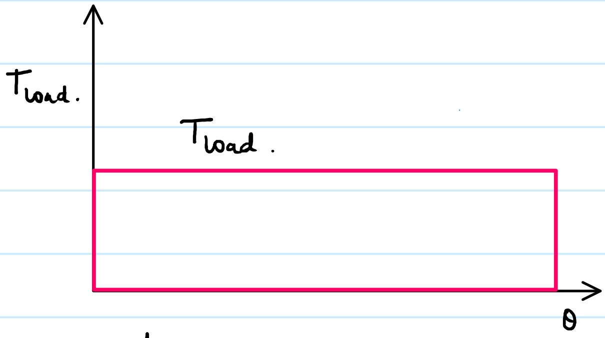
$$E_{supply} = T_{supply} \cdot \int_0^{cycle} d\theta$$

$T_{supply} = constant$

$T_{load} = const.$

Energy required

$$= \int_0^{cycle\ time} T_{load} \cdot d\theta$$



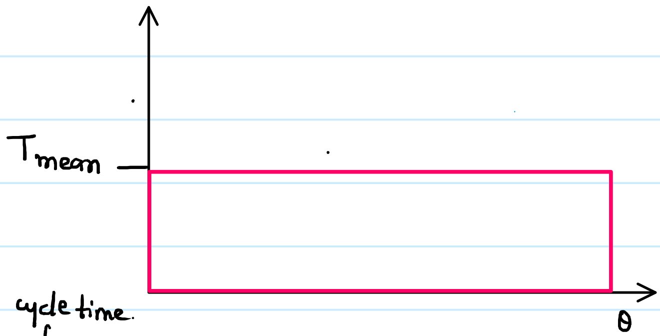
$$E_{required} / E_{load} = T_{load} \cdot \int_0^{cycle} d\theta \quad \rightsquigarrow \quad T_{load} = constant$$

$$= \int_0^{cycle\ time} T_{load} \cdot d\theta \quad \rightsquigarrow \quad T_{load} = f(\theta)$$

AVERAGE OR MEAN TORQUE:

- It is an imaginary constant torque which when act on the crank shaft will be able to develop same amount of energy that is being developed by actual supply torque.

Net Energy Developed/cycle = Net Area of T_{mean} v/s θ diagram in one cycle



$$E_{supply} = \int_0^{cycle\ time} T_{mean} \cdot d\theta$$

$$E_{supply} = T_{mean} \cdot \int_0^{cycle\ time} d\theta = \int_0^{cycle\ time} T_{supply} \cdot d\theta$$

$$T_{mean} = \frac{1}{cycle\ time} \cdot \int_0^{cycle\ time} T_{supply} \cdot d\theta$$

Conservation of Energy.

$$E_{supply} = E_{Load}$$

$$\int_0^{cycle\ time} T_{supply}\, d\theta = \int_0^{cycle\ time} T_{load}\, d\theta$$

T_{load} = constant.

$T_{supply} = f(\theta)$.

$$T_{mean} \cdot \int_0^{cycle\ time} d\theta = T_{load} \cdot \int_0^{cycle\ time} d\theta$$

$$T_{mean} = T_{load}$$

Fluctuation Torque

$$T_{fluc} = T_{supply} - T_{load} > 0$$

$$T_{fluct} = T_{load} - T_{supply}$$

$$T_{fly} / T_{fluc} = T_{supply} - T_{load} > 0$$

$$I_{fly} \cdot \alpha_{fly} = T_{supply} - T_{load} > 0 \longrightarrow T_{supply} > T_{load}$$

$\alpha_{fly} > 0$ (+ve value)

$$T_{fly} / T_{fluc} = T_{supply} - T_{load} < 0 \longrightarrow T_{supply} < T_{load}$$

$\alpha_{fly} < 0$ (−ve value)

$$T_{fly} = T_{supply} - T_{load} = 0 \longrightarrow T_{supply} = T_{load}$$

$\alpha_{fly} = 0$. ω_{fly} = constant.

T - θ Diagram for 4 -stroke single cylinder single acting Engine

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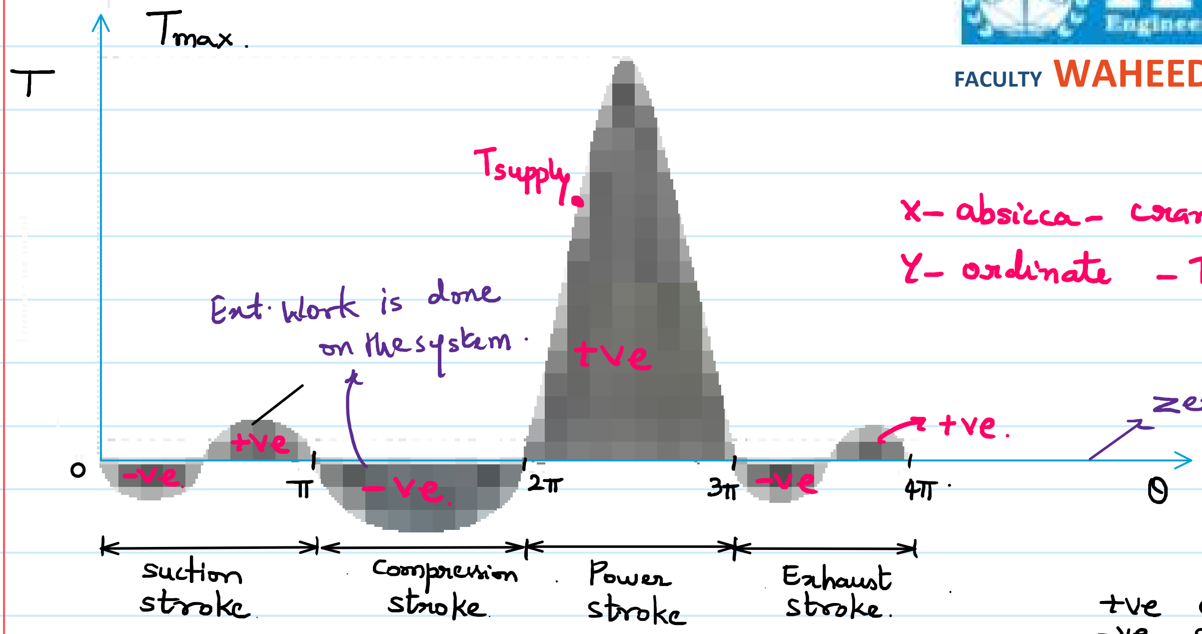

X – absicca – crank angle.
Y – ordinate – Tsupply.

+ve 0 to 90°
-ve 90 to 180

Suction stroke | Exhaust stroke. $F_P = F_G \pm F_I$. $F_I = m\,r\,\omega^2 \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$

In the suction stroke inertia force opposes the motion for some part of stroke and inertia force supports the motion for some part of stroke.

In the Exhaust stroke inertia force opposes the motion for some part of stroke and inertia force supports the motion for some part of stroke.

Compression stroke – $F_P = F_G \pm F_I$

= External Work is done on air/air-fuel mixture.

Expansion stroke. $F_P = F_G \pm F_I$.

$F_G >>> F_I$.

$F_P \approx F_G$.

Work is done by the system. (+ve W.D)

Torque reaches the peak value in the Power stroke.

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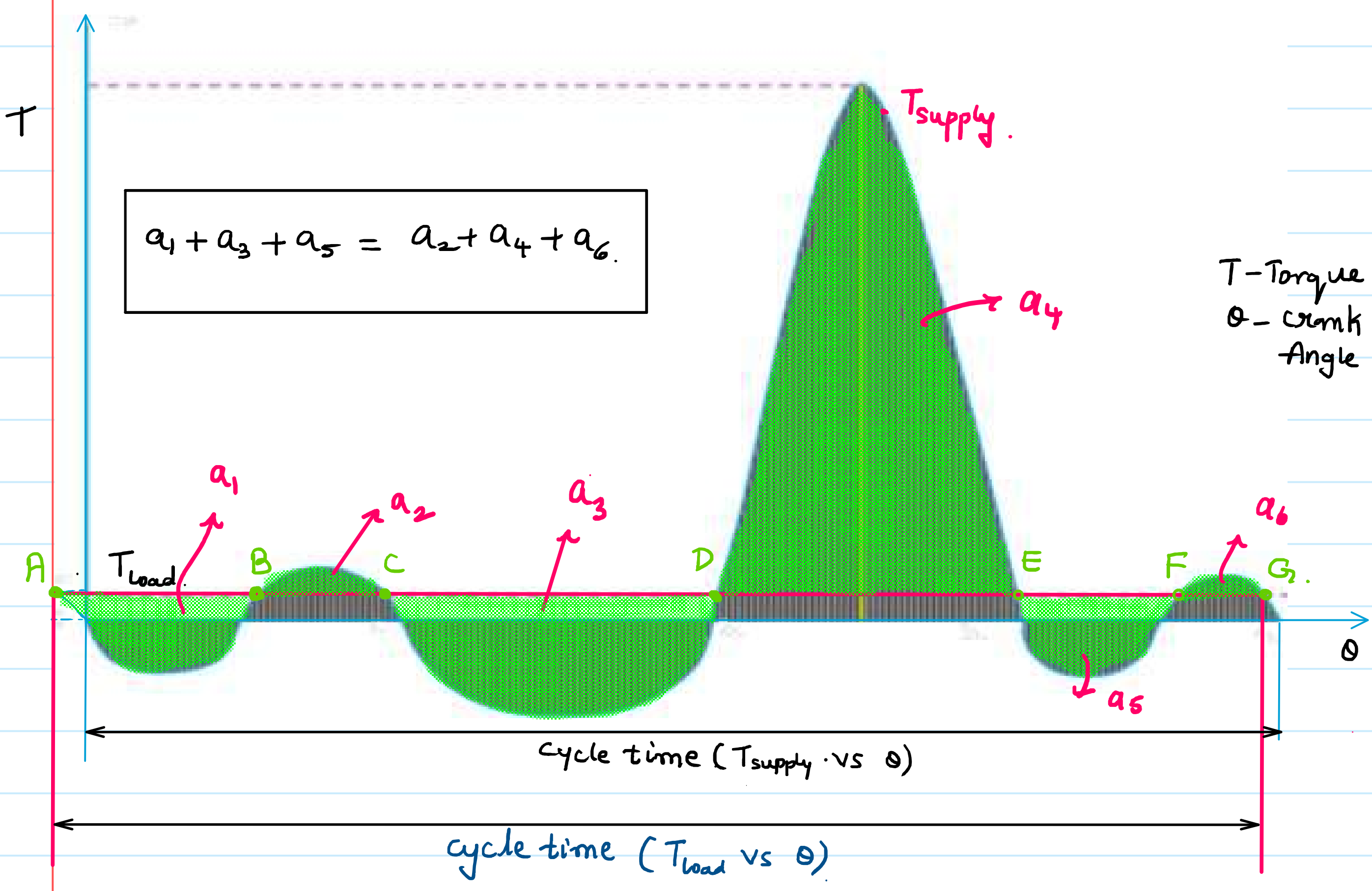

@ points A, B, C, ... G. $T_{supply} = T_{load} \Rightarrow T_{fluct.} / T_{fly.} = 0$

$$\alpha_{fly} = 0$$

Flywheel is isolated from source & Load.

point A, B, C, ... G are called isolation points

Energy @ A $= E = \frac{1}{2} \cdot I \cdot \omega_A^2$

Energy @ B $= E - a_1 = \frac{1}{2} \cdot I \cdot \omega_B^2$

Energy @ C $= E - a_1 + a_2 = \frac{1}{2} \cdot I \cdot \omega_C^2$

Energy @ D $= E - a_1 + a_2 - a_3 = \frac{1}{2} \cdot I \cdot \omega_D^2$ → KE_{min}

Energy @ E $= E - a_1 + a_2 - a_3 + a_4 = \frac{1}{2} \cdot I \cdot \omega_E^2$ → KE_{max}

Energy @ F $= E - a_1 + a_2 - a_3 + a_4 - a_5 = \frac{1}{2} \cdot I \cdot \omega_F^2$

Energy @ G $= E - a_1 + a_2 - a_3 + a_4 - a_5 + a_6 = E = \frac{1}{2} \cdot I \cdot \omega_G^2$

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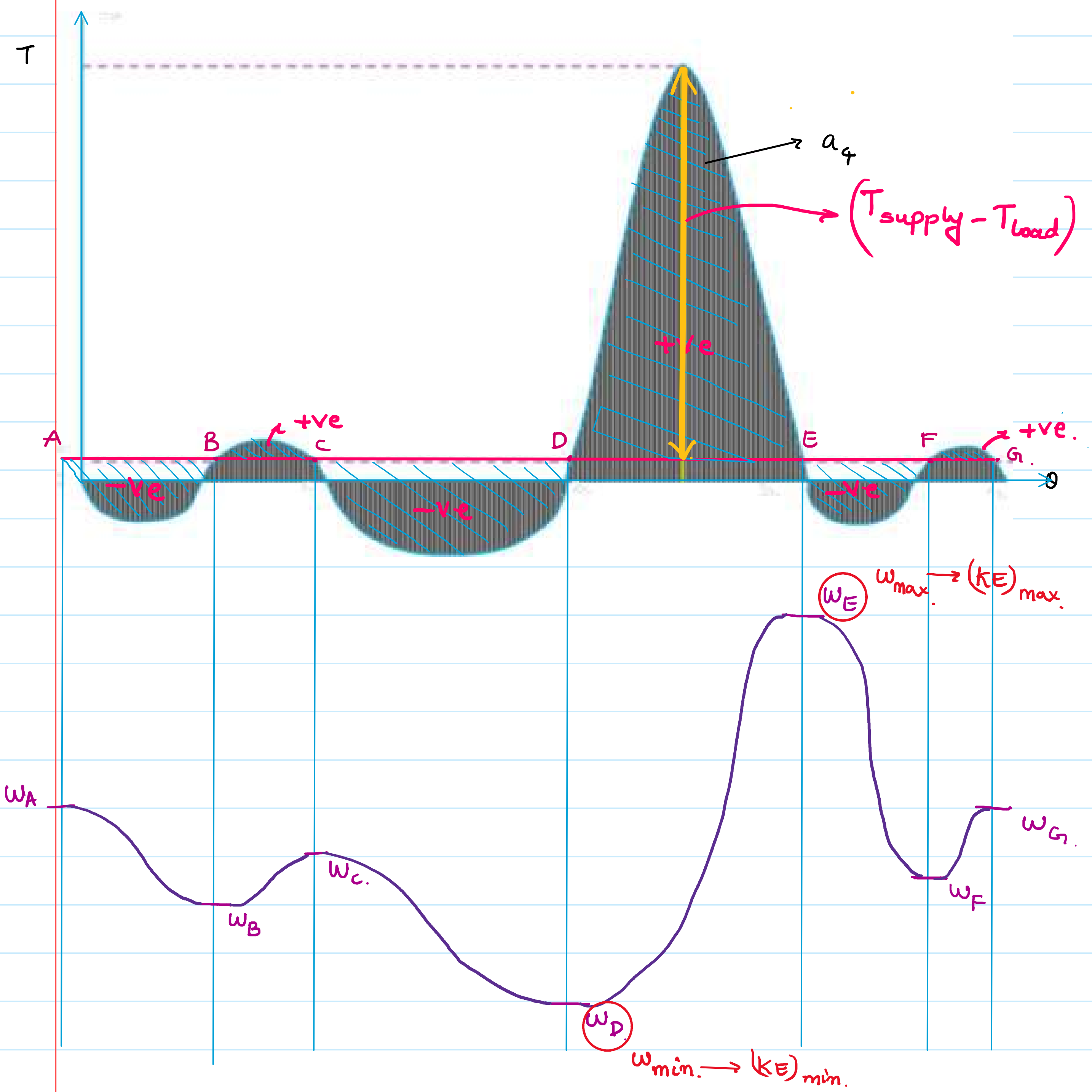
T vs θ, ω vs θ.



Max. fluctuation in Energy / Kinetic Energy.

$$(\Delta KE)_{max} = KE_{max} - KE_{min}.$$

$$(\Delta KE)_{max} = \frac{1}{2} \cdot I \cdot \omega_{max}^2 - \frac{1}{2} \cdot I \omega_{min}^2 = \frac{I}{2} \cdot (\omega_{max}^2 - \omega_{min}^2)$$

$$(\Delta KE)_{max} = I \cdot \frac{(\omega_{max} + \omega_{min})}{2} (\omega_{max} - \omega_{min})$$

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$$(\Delta KE)_{max} = I \cdot \omega_{avg}^2 \cdot \frac{\Delta \omega}{\omega_{avg}} = I \cdot \omega_{avg}^2 \cdot C_s .$$

Max. fluctuation in KE.

$$(\Delta KE)_{max} = KE_{max} - KE_{min}.$$
$$= (E - a_1 + a_2 - a_3 + a_4) - (E - a_1 + a_2 - a_3)$$
$$= a_4.$$
$$= \int_{\theta_D}^{\theta_E} (T_{supply} - T_{load})\, d\theta$$

@ θ_D – $\omega = Min$, @ θ_E – $\omega = Max.$

Coefficient of fluctuation in Speed.

$$C_s = \frac{\text{Range of speed}}{\text{Averagespeed / Mean speed}}.$$

Coefficient of fluctuation in Energy.

$$C_E = \frac{\text{Max. fluctuation in Energy}}{\text{Energy supplied for one cycle}}.$$

$$C_E = \frac{(\Delta KE)_{max}}{E_{supplied}} = \frac{I \cdot \omega_{avg}^2 \cdot C_s}{\text{cycle time} \int_0 T_{supply} \cdot d\theta} = \frac{\int_{\theta_1}^{\theta_2} (T_s - T_L) \cdot d\theta.}{\text{cycle time} \int_0 T_s \cdot d\theta.}$$

T_s – Supply Torque T_L = Load torque.

Stresses in flywheel.

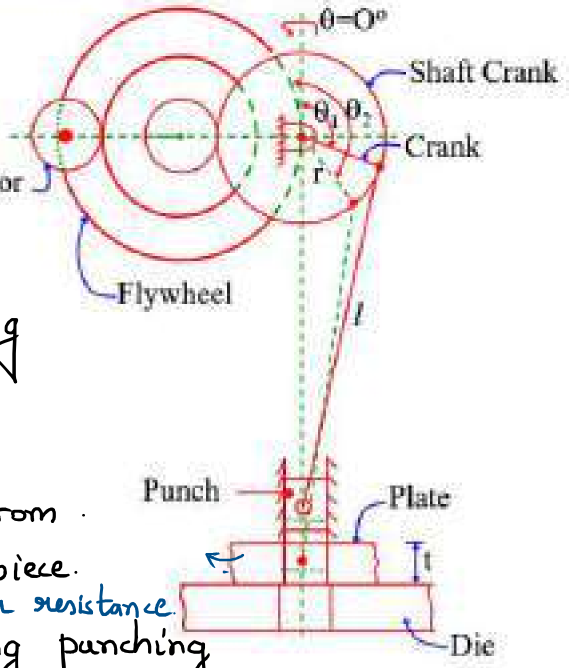
$$\sigma = \rho V^2$$

σ – stress in flywheel.
ρ – density of flywheel material
V – peripheral velocity

Punching Press

FLYWHEEL IN PUNCHING PRESS:

ASSUMPTION: Tool dimension are negligible.



$\theta = 0^\circ$ Tool is at top dead centre.

$\theta = \theta_1$ Tool comes in contact with work piece.

$\theta = \theta_2$ Tool completes the punching operation.

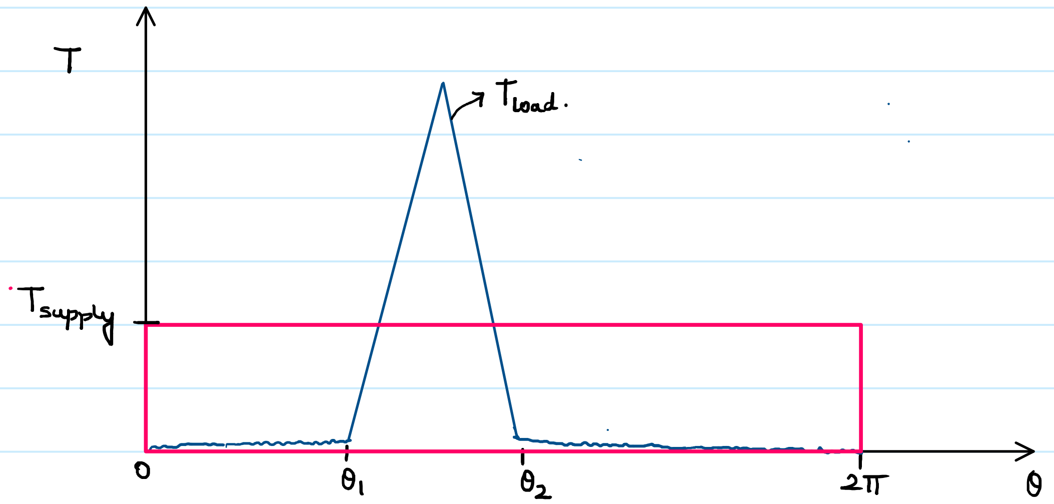
$0 < \theta < \theta_1 \rightarrow$ Tool is moving from T.D.C to workpiece.
Tool is overcoming air resistance.

$\theta_1 < \theta < \theta_2 \rightarrow$ Tool is performing punching operation.
Tool is overcoming material resistance.

$\theta_2 < \theta < 2\pi \rightarrow$ Tool moves back to the top dead centre.
Tool is overcoming air resistance.

T_{supply} = constant.

$T_{load} = f(\theta)$



T - θ Diagram for Punching Press

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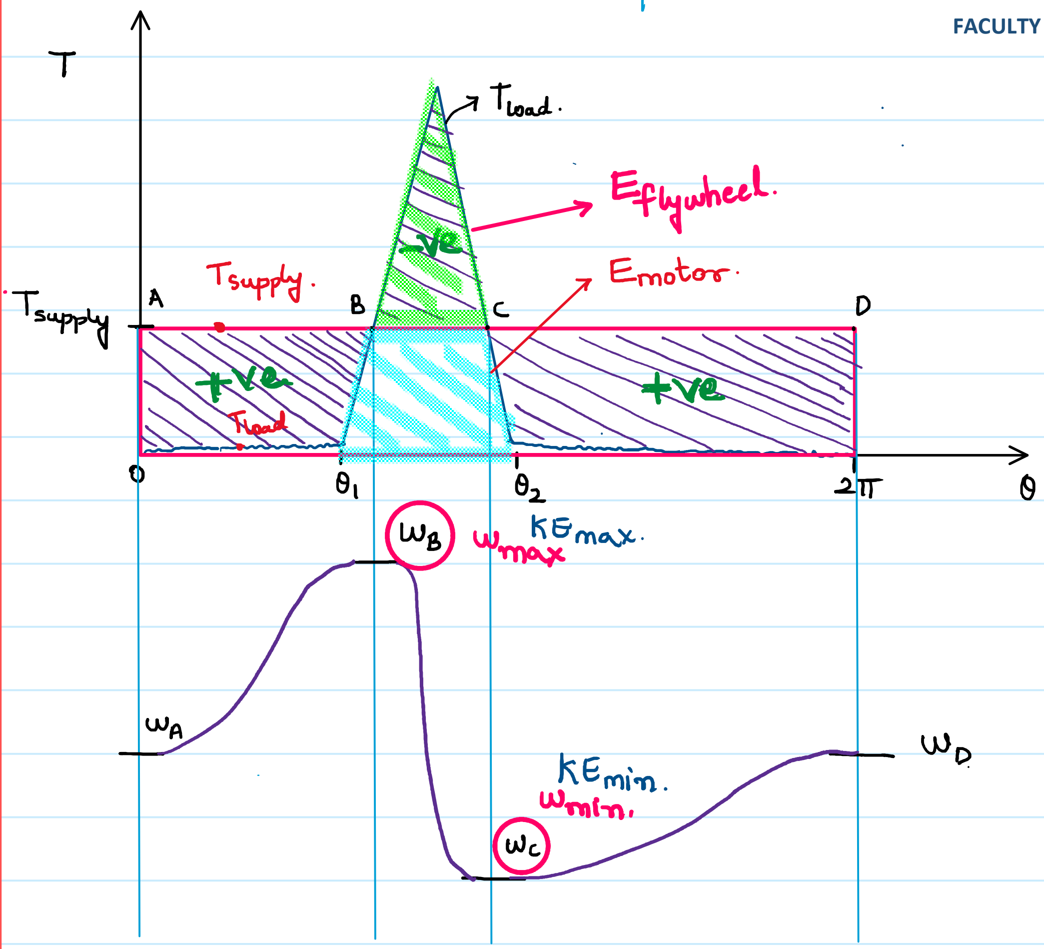
T vs θ , ω vs θ Graph.



@ points B, C
$T_{load} = T_{supply}$.
B, C are called isolation points.

Energy at A $= E = \frac{1}{2} \cdot I \cdot \omega_A^2$

Energy at B $= E + a_1 = \frac{1}{2} I \omega_B^2 \rightsquigarrow KE_{max}$

Energy at C $= E + a_1 - a_2 = \frac{1}{2} \cdot I\omega_C^2 \rightsquigarrow KE_{min}$.

Energy at D $= E + a_1 - a_2 + a_3 = E = \frac{1}{2} \cdot I\omega_D^2$

Max. fluctuation in Energy $= (KE)_{max} - (KE)_{min}$.
$= (E + a_1) - (E + a_1 - a_2)$

Energy supplied by flywheel $= (\Delta KE)_{max} = a_2$

Energy for one cycle = Energy for cutting stroke + Energy for idle stroke.

$E_{cycle} = E_{cutting} + E_{idle}$
0

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$$E_{cycle} = E_{cutting}$$

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$$E_{cutting} = E_{motor} + E_{flywheel}.$$

$$E_{cycle} = E_{cycle}\cdot\left(\frac{\theta_2-\theta_1}{2\pi}\right) + (\Delta KE)_{max}.$$

$E_{motor} \longrightarrow (\theta_2-\theta_1)$

$E_{cycle} \longrightarrow 2\pi$

$$E_{motor} = E_{cycle}\cdot\frac{(\theta_2-\theta_1)}{2\pi}$$

$$(\Delta KE)_{max} = E_{cycle}\left[1 - \frac{(\theta_2-\theta_1)}{2\pi}\right]$$

$$(\Delta KE)_{max} = E_{cycle}\left[1 - \frac{t_{cutting}}{t_{cycle}}\right]$$

$$(\Delta KE)_{max} = E_{cycle}\left[1 - \frac{\text{thickness of plate}}{2\times\text{stroke lenght}}\right]$$

01. A four-stroke single cylinder I.C engine develops 80 kW at 300 rpm. The fluctuation of energy can be assumed to be 0.9 times the energy developed per cycle. If the coefficient of fluctuation of speed is not to exceed 2 percent and the maximum centrifugal stress in the rim of the flywheel is limited to 6MN/m², estimates the mean diameter of the rim and the moment of inertia of the flywheel. The density of flywheel material is 7500 kg/m³.

$P = 80\,kW$ @ $300\,rpm$.

$C_E = 0.9$, $C_s = 2\%$

$\sigma = 6\,MPa = 6\times10^6\,N/m^2$

$D = ?$, $I = ?$, $\rho = 7500\,kg/m^3$.

stresses in flywheel. $\sigma = \rho v^2$

$$\sigma = \rho\cdot\left(\frac{\pi D N}{60}\right)^2$$

$$6\times10^6 = 7500\left(\frac{\pi D\times300}{60}\right)^2$$

$D = 1.8\,mts.$

$$P = \frac{2\pi\cdot T_{mean}\cdot N_{mean}}{60{,}000}\ kW.$$

$$T_{mean} = \frac{60{,}000\times80}{2\pi\times300}\ N\text{-}m. \Rightarrow T_{mean} = 2546.47\,N\text{-}m,$$

$E_{supply} = T_{mean}\times\text{cycle time} = 2546.47\times4\pi \cong 32\,kN\text{-}m.$

$(\Delta KE)_{max} = C_E\cdot E_{supply} = 0.9\times32 = 28.8\,kN\text{-}m.$

$$(\Delta KE)_{max} = I\omega_{avg}^2\cdot C_s \Rightarrow 28.8\times10^3 = I\cdot\left(\frac{2\pi\times300}{60}\right)^2\times0.02$$

$$I = 1459.02\,kg\text{-}m^2$$

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02. The TMD for a four stroke engine may be assumed for the sake of simplicity to be represented by four triangles in each stroke. The area of these triangles are expansion stroke 9cm², exhaust stroke 0.8cm² (−ve), suction stroke 0.5cm²(−ve) and compression stroke 1.7cm²(−ve). Where 1cm² represents 1400 J of work. Assuming constant resistance determine the MI of the flywheel to keep the speed between 98 rpm and 102 rpm.

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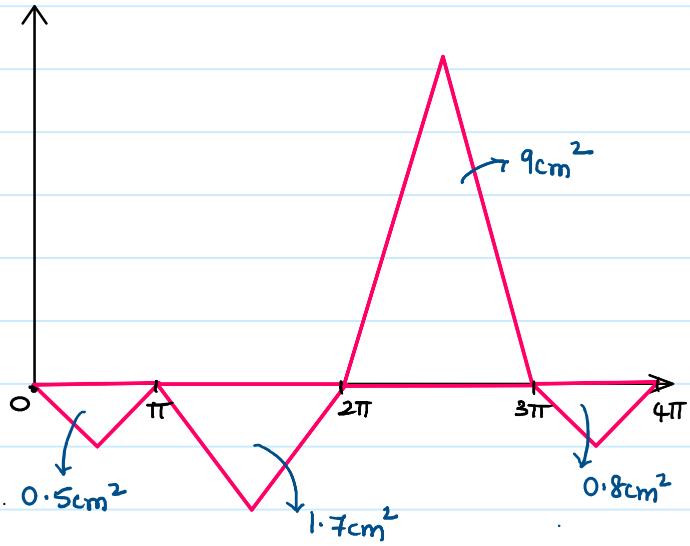
$1\,cm - 1400\,J.$



Sol $\quad E_{supplied} = \int_0^{\text{cycle time}} T_s \, d\theta = \sum_{i=1}^{n} a_i$

$E_{supplied} = -a_1 - a_2 + a_3 - a_4$

$= -0.5 - 1.7 + 9 - 0.8$

$= 6\,cm^2$

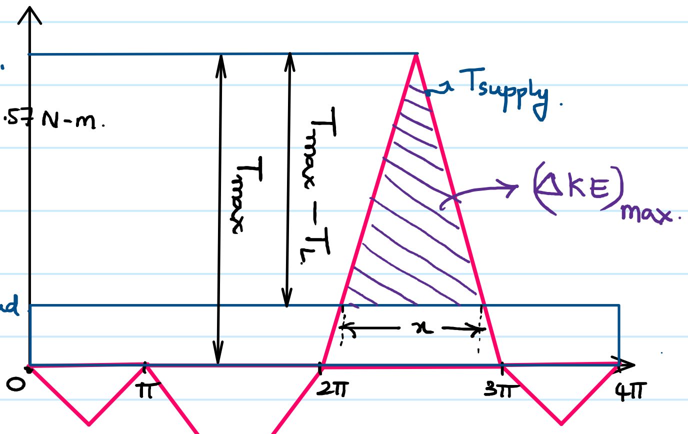
$= 6 \times 1400 = 8400\,J$

$T_{load} \times \text{cycle time} = 8400$

$T_{load} = 8400/4\pi = 668.45\,N\text{-}m.$

$(\Delta KE)_{max} = \frac{1}{2} \cdot (T_{max} - T_L) \times x.$

$= \frac{1}{2} \cdot (8021.4 - 668.45) \times 2.879 = 10{,}584.57\,N\text{-}m.$



$\frac{T_{max}}{\pi} = \frac{T_{max} - T_L}{x} \Rightarrow x = \frac{(8021.4 - 668.45)\pi}{8021.4}$

$x = 2.879$ radians

$A_{Expansion} = \frac{1}{2} \cdot T_{max} \cdot \pi$

$9 \times 1400 = \frac{1}{2} \cdot T_{max} \cdot \pi \Rightarrow T_{max} = 8021.4\,N\text{-}m.$

$(\Delta KE)_{max} = \frac{1}{2} \cdot I \cdot (\omega_{max}^2 - \omega_{min}^2)$

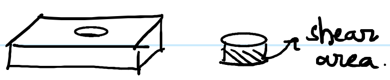
$\Rightarrow 10{,}584.57 = \frac{1}{2} \cdot I \cdot \left(\frac{2\pi}{60}\right)^2 [102^2 - 98^2] \Rightarrow I = 2413\,kg\text{-}m^2$

03. A machine punching 40 mm diameter holes 30 mm thick plate requires 7 Nm of energy per mm² of shear area. To punch has a stroke of 100 mm and it takes 10 sec to complete one cycle. The mean speed of the flywheel is 25 m/sec, and the fluctuation of speed should not exceed 3% of the mean speed. Assuming that the motor supplies energy to the machine at a uniform rate, determine the power of the motor and the mass of the flywheel required.

Sol $\quad d = 40\,mm. \quad t_{cycle} = 10\,sec \quad \text{stroke} = 100\,mm$

$t = 30\,mm. \quad V = 25\,m/s.$

7 N-m of Energy is required to shear 1 mm² of area. $\quad \text{Energy} = \frac{7\,N\text{-}m}{mm^2}$



$C_s = 3\%$ $\quad T_{supply} = \text{constant}.$

Power = ?. $\quad m = ?.$

$E_{cycle} = 7 \cdot \pi \times d t = 7 \times \pi \times 40 \times 30 = 26{,}389.3\,N\text{-}m.$

$\text{Power} = \frac{E_{cycle}}{t_{cycle}} = \frac{26{,}389.3}{10} = 2638.93\,\frac{N\text{-}m}{sec} = 2.63\,kW.$

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$$(\Delta KE)_{max} = E_{cycle}\left[1 - \frac{\text{thickness of Plate}}{2\times\text{Stroke}}\right] = mV^2 \cdot C_s$$

$$= 26{,}389.3 \cdot \left[1 - \frac{30}{2\times 100}\right] = m \times 25^2 \times 0.03$$

$$m = 1196.25 \text{ Kg}.$$

04. A machine tool performs an operation intermittently. It is driven continuously by a motor. Each operation takes 8 seconds and five operations are done per minute. The machine is fitted with a flywheel having a mass of 200 kg with a mean radius of gyration of 400 mm. When operation is being performed, the speed drops from the normal speed of 400 rpm and 250 rpm. Determine the power of the motor required.

$t_{cutting} = 8$ seconds.

No. of operation in a minute = 5.

$$t_{cycle} = \frac{60}{5} = 12 \text{ secs}.$$

$$I = mk^2$$

$$(\Delta KE)_{max} = E_{cycle}\left[1 - \frac{t_{cutting}}{t_{cycle}}\right]$$

$$\frac{1}{2} \cdot I \cdot (\omega_{max}^2 - \omega_{min}^2) = E_{cycle}\left[1 - \frac{t_{cutting}}{t_{cycle}}\right]$$

$$\frac{1}{2} \times 200 \times 0.4^2 \cdot \left(\frac{2\pi}{60}\right)^2 \left[400^2 - 250^2\right] = E_{cycle}\left[1 - \frac{8}{12}\right]$$

$$E_{cycle} = 51322 \text{ N-m}.$$

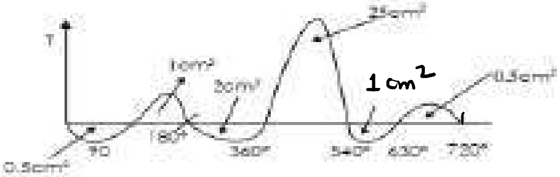
$$\text{Power} = \frac{E_{cycle}}{t_{cycle}}$$

$$= \frac{51322}{12}$$

$$= 4.276 \text{ kW}.$$

05. Consider the following statements regarding the turning moment diagram of a reciprocating engine shown in the below figure.

(Scale 1 cm² = 100 N.m)



1. It is a four-stroke IC engine. — cycle time $= 4\pi = 720°$.
2. The compression stroke is 0° to 180° (180 to 360°) → suction.
3. Mean turning moment $T_m = 580/\pi$ N.m.
4. It is a ~~multi~~ single cylinder engine.

Which of these statements are correct?

(a) 1, 2 and 3 (b) 1, 2 and 4

(c) 2, 3 and 4 (d) 1 and 3

$$T_{mean} = \frac{1}{\text{cycle time}} \int T_{supply} \cdot d\theta$$

area of T–θ diagram

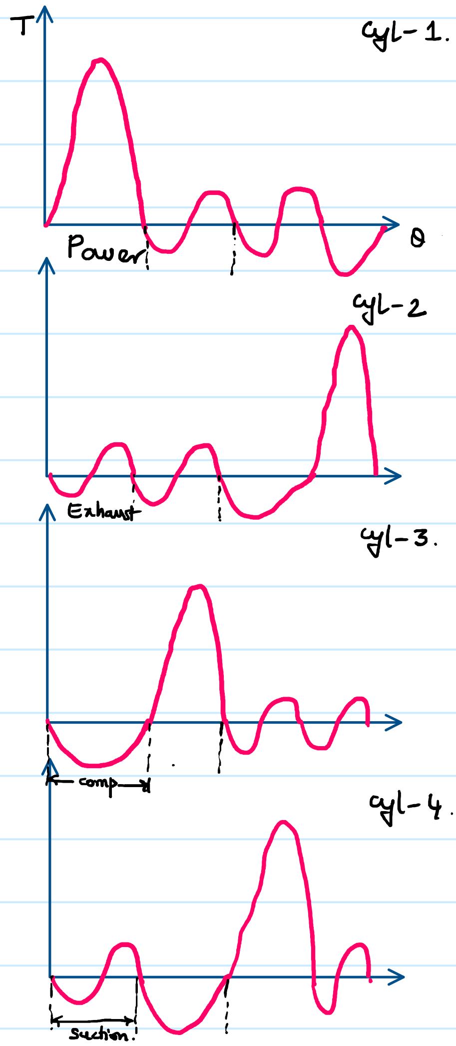
$$T_{mean} = \frac{(-0.5 + 1 - 2 + 25 - 1 + 0.5) \times 100}{4\pi}.$$

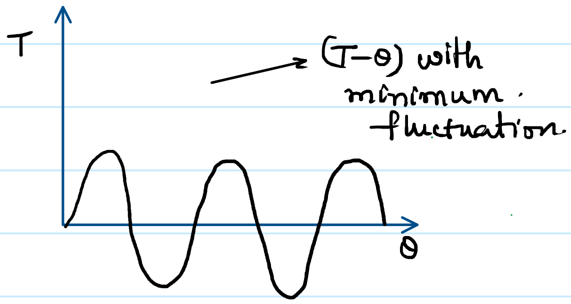
$$T_{mean} = \frac{23 \times 100}{4\pi}$$

$$T_{mean} = \frac{575}{\pi}.$$

T-θ. diagram for Multi-Cylinder Engine.

Firing order — 1342.





$(T_{fluct})_{single\ cylinder} > (T_{fluct})_{Multi-cylinder}$

$(I_{fly})_{single\ cylinder} > (I_{fly})_{Multi-cylinder}$.

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Energy @ A = E

Energy @ B = E+60

Energy @ C = E+60−40 = E+20

Energy @ D = E+60−40+80 = E+100

Energy @ E = E+100−100 = E.

Energy @ F = E+60

Energy @ G = E+60−60 = E

06. Refer to the turning moment diagram for a multi-cylinder engine the areas are indicated in mm2. If P, Q, R, S represent the velocity of flywheel at B, C, D and E respectively i.e.,

P. @ Point B

Q. @ Point C

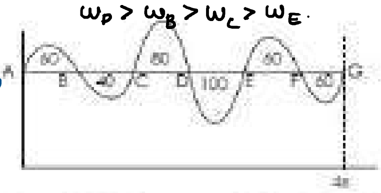
R. @ Point D

S. @ Point E

Then the appropriate relation is

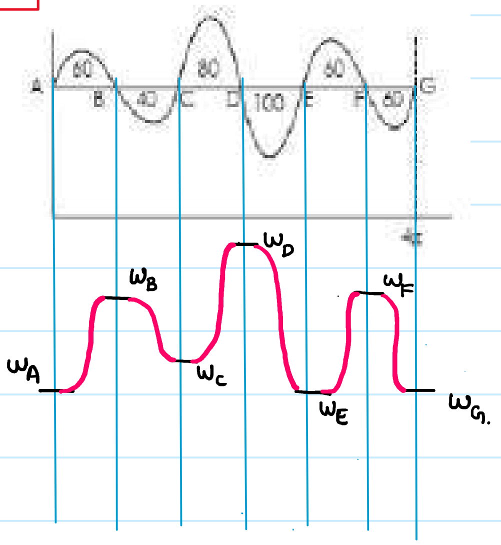
R > P > Q > S.

$\omega_D > \omega_B > \omega_C > \omega_E$.



(a) R > P > S > Q (b) P > R > S > Q

(c) R > P > Q > S (d) S > P > R > Q



$\omega_D > \omega_B = \omega_F > \omega_C > \omega_A = \omega_E = \omega_G$

07. A disk type flywheel has a mass 'm' radius 'R' and width 'W'. When it is attached to an IC engine limits the coefficient of fluctuation of speed to 4%. If the radius is doubled and the mass is kept same by reducing the width the coefficient of fluctuation of speed will be

(a) unchanged
(b) will be reduced to 1% ✓
(c) will be reduced to 2%
(d) none

mass – m.
Radius – R.
Width – w.
$\}$ $C_s = 4\%$

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If Radius is doubled.

m = constant.

$C_s = ?$. ω = const.

$(\Delta KE)_{max}$ = constant.

$= I\omega^2 . C_s$ = constant.

$I . \propto 1/C_s$.

$I_1 C_{s_1} = I_2 . C_{s_2}$

$(m.R^2) . C_{s_1} = m.(2R)^2 . C_{s_2} \Rightarrow C_{s_2} = \frac{C_{s_1}}{4} = 1\%$

08. A flywheel connected to an IC Engine keeps the fluctuation of speed with in ± 2% of the mean speed, which is 600rpm. The fluctuation of energy is 5000 joules. If another flywheel of the same size is attached, the coefficient of fluctuation of speed will be

(a) 0.01
(b) 0.02 ✓
(c) 0.04
(d) remain same

I is doubled.

$C_s = \pm 2\% = 4\%$.

N = 600 rpm.

$(\Delta KE)_{max} = 5000$ J.

$I_1 C_{s_1} = I_2 . C_{s_2}$

$I_1 \, 4\% = 2 . I_1 . C_{s_2}$

$C_{s_2} = 2\% = \pm 1\% = 0.02$

09. A punching machine is driven by a 2 kW electric motor. The punching stroke consists of 1/4th the cycle time and consumes 3/4th of the energy per cycle the remaining 1/4th energy is consumed during the 3/4th of the cycle time. Express the fluctuation of energy as a fraction of energy required per cycle.

(a) 0.5 ✓
(b) 0.25
(c) 0.75
(d) 1

C_E

Power = 2 kW.

t_{cycle} = T.

$E_{cycle} = P. \times t_{cycle} = 2.T$ N-m.

Stroke.	Energy.	Time.
Cutting	$3/4 . E_{cycle} = 3/4 (2T)$	$1/4 . t_{cycle} = T/4$
Idle.	$1/4 . E_{cycle} = 1/4 . (2T)$	$3/4 \, t_{cycle} . = 3T/4$

$E_{motor} = E_{cycle} . \left(\frac{\theta_2 - \theta_1}{2\pi}\right)$

$E_{cycle} = E_{cutting} + E_{idle}$.

$E_{motor} = E_{cycle} . \left(\frac{t_{cutting}}{t_{cycle}}\right)$

$E_{cutting} = E_{motor} + E_{flywheel}$.

$3/4 . (E_{cycle}) = E_{cycle} . \left(\frac{T/4}{T}\right) + (\Delta KE)_{max}$

$(\Delta KE)_{max} = \frac{E_{cycle}}{2}$.

$C_E = \frac{(\Delta KE)_{max}}{E_{cycle}} = \frac{E_{cycle}/2}{E_{cycle}} = 0.5$.

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10. A punching machine is driven by an electric motor through a gear train with a reduction ratio of 4. A flywheel mounted on the machine shaft keeps the speed fluctuation with in ±3.2% of the mean speed. A naive designer suggested shifting the flywheel onto the motor shafts. What will be the change in coefficient of fluctuation of speed?
(a) the punching machine will not work
(b) the fluctuation of speed will be increase to ± 12.8%
(c) the fluctuation of speed will be decreased to ± 0.2%
(d) the fluctuation of speed will be decreased to ± 0.8%

Flywheel shifted to motor side.

Sol $G = 4.$ $\quad C_s = \pm 3.2\% = 6.4\% \Big\} \; C_{s_{m/c}} = 6.4\%$

$(\Delta KE)_{max}$ = const for given application

$$(\Delta KE)_{max} = \cancel{I} \cdot \omega^2_{m/c} \cdot C_{s_{m/c}} = \cancel{I} \cdot \omega^2_{motor} \cdot C_{s_m}.$$

$$C_{s_m} = \frac{\omega^2_{m/c}}{\omega^2_m} \cdot C_{s_{m/c}} = \left(\frac{1}{G}\right)^2 \cdot C_{s_{m/c}} = \frac{1}{4^2} \times 6.4\%$$

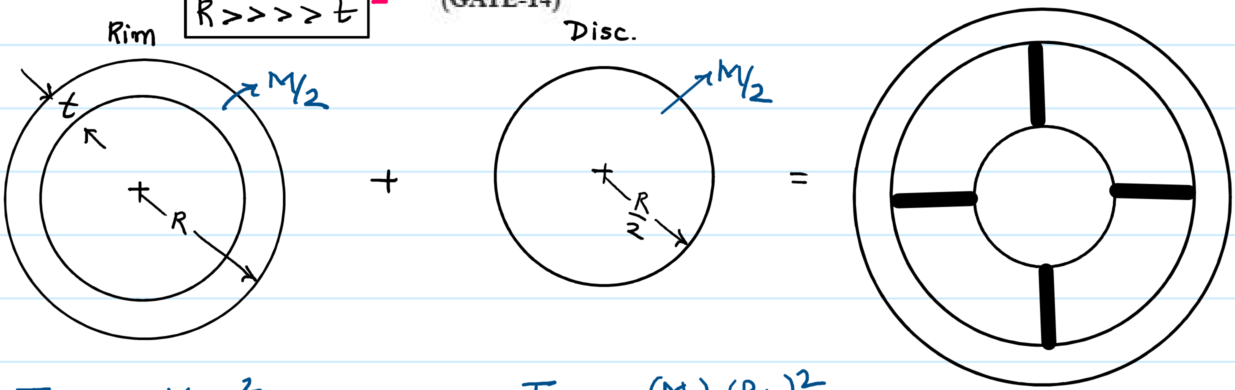
$$= 0.4\% = \pm 0.2\%$$

11. Consider a flywheel whose mass M is distributed almost equally between a heavy, $\boxed{\text{ring-like rim}}$ of radius R and a concentric disk-like feature of radius R/2. Other parts of the flywheel, such as spokes, etc, have negligible mass. The best approximation for α, if the moment of inertia of the fly wheel about its axis of rotation is expressed as αMR², is ______.

(GATE-14)

$$I = I_{Rim} + I_{Disc}.$$

$\boxed{R >>>> t}$



$$I_{Rim} = \frac{M}{2} \cdot R^2 \qquad I_{Disc} = \frac{\left(\frac{M}{2}\right) \cdot \left(\frac{R}{2}\right)^2}{2}$$

$$I_{fly} = \frac{MR^2}{2} + \frac{MR^2}{16} = 0.5625 \cdot MR^2.$$

12. The torque (in N-m) exerted on the crank shaft of a two stroke engine can be described as $T = 10000 + 1000\sin 2\theta - 1200\cos 2\theta$, where θ is the crank angle as measured from inner dead center position. Assuming the resisting torque to be constant, the power (in kW) developed by the engine at 100 rpm is ______. (GATE –15)

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$$T = \underbrace{\boxed{10{,}000}}_{T_{mean}} + 1000\sin 2\theta - 1200\cos 2\theta$$

T_{Load} = constant Power @ $\underbrace{\boxed{100 \text{ rpm}}}_{N_{mean}}$ = ?

$$P = \frac{2\pi \cdot N_{mean} \cdot T_{mean}}{60{,}000} \text{ kW.} \qquad T_{mean} = \frac{1}{\text{cycle time}} \int_0^{\text{cycle time}} T_{supply} \cdot d\theta$$

$$T_{mean} = \frac{1}{2\pi} \cdot \int_0^{2\pi} (10{,}000 + 1000\sin 2\theta - 1200\cos 2\theta)\, d\theta$$

$$T_{mean} = \frac{1}{2\pi} \left[10{,}000[\theta]_0^{2\pi} + 1000\left[\frac{-\cos 2\theta}{2}\right]_0^{2\pi} - 1200\left[\frac{\sin 2\theta}{2}\right]_0^{2\pi} \right]$$

0 0

$$T_{mean} = 10{,}000 \text{ N-m.}$$

$$P = \frac{2\pi \times 10{,}000 \times 100}{60{,}000} = 104.71 \text{ kW.}$$

Max. fluction in Energy.

T_{Load} = const $\Rightarrow$ $T_{Load} = T_{mean}$.

Isolation. points.

@ $\theta = \theta_1$, ω = min. $T_s = T_L$

@ $\theta = \theta_2$, ω = Max. $T_s = T_L$

$$(\Delta KE)_{max} = \int_{\theta_1}^{\theta_2} (T_s - T_L)\, d\theta.$$

$$T_s = T_L$$

$$10{,}000 + 1000\sin 2\theta - 1200\cos 2\theta = 10{,}000.$$

$$\boxed{T_s = 10{,}000 + 1000\sin 2\theta.}$$

$$\tan 2\theta = 1.2.$$

$$2\theta = \tan^{-1}(1.2)$$

$$2\theta = 50.19, \quad 180 + \tan^{-1}(1.2)$$

$$= 50.19, \quad 230.19^\circ$$

$$\theta = 25.09, \quad 115.09^\circ$$

$$(\Delta KE)_{max} = \int_{25.09}^{115.09} (1000\sin 2\theta - 1200\cos 2\theta)\, d\theta = 1000\left[-\frac{\cos(2\theta)}{2}\right]_{25.09}^{115.09} - 1200\left[\frac{\sin 2\theta}{2}\right]_{25.09}^{115.09}$$

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Angular Acceleration of flywheel

$$T_{fluct} / T_{fly} = T_S - T_L = I_{fly} \cdot \alpha_{fly}.$$

$$I_{fly} \cdot \alpha_{fly} = \cancel{10,000} + 1000\sin 2\theta - 1200\cos 2\theta - \cancel{10,000}.$$

$$I_{fly} \cdot \alpha_{fly} = 1000 \sin 2\theta - 1200 \cos 2\theta.$$

$\alpha = f(\theta)$ for $\alpha = $ Max. or Min. $\quad \frac{d\alpha}{d\theta} = 0 \Rightarrow \frac{d}{d\theta}(T_s - T_L) = 0$

$$I_{fly} \cdot \alpha_{fly} = A\sin(n\theta) \pm B \cdot \cos(n\theta)$$

$$I_{fly}\left(\alpha_{fly}\right)_{max,\, min} = \pm\sqrt{A^2 + B^2} \qquad = \pm\sqrt{1000^2 + 1200^2}$$

$$\frac{d}{d\theta}(1000\sin 2\theta - 1200\cos 2\theta) = 0 \Rightarrow 1000\cos(2\theta) \times 2 - (-1200\sin(2\theta) \times 2) = 0$$

$$\tan 2\theta = -\frac{1000}{1200}.$$

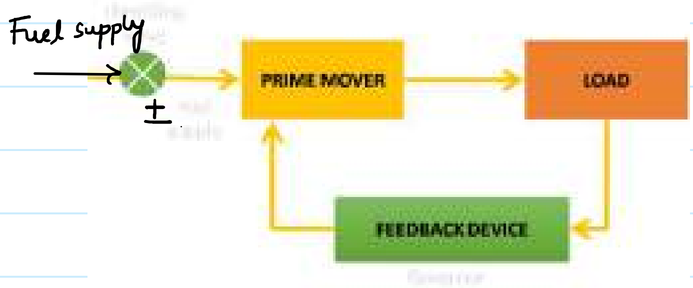
$$2\theta = 180 - \tan^{-1}\left(\frac{1000}{1200}\right),\ 360 - \tan^{-1}\left(\frac{1000}{1200}\right)$$

$$2\theta = 140.19,\quad 320.2$$

$$\theta = 70.09,\quad 160.1$$

GOVERNORS:

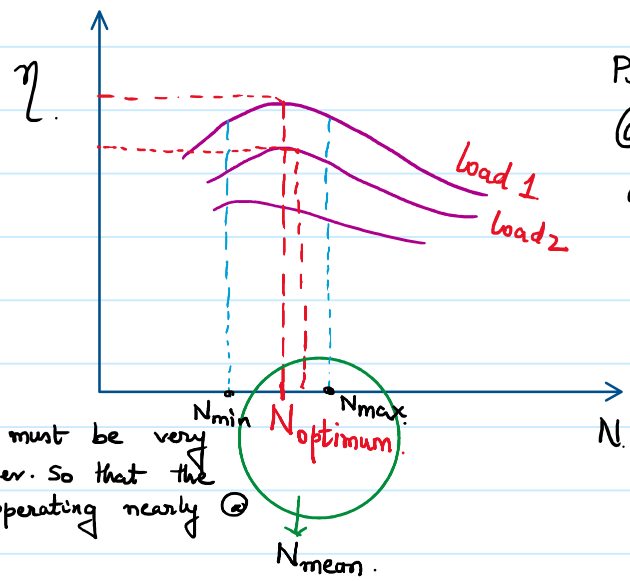
- Governor is a feedback device which regulate the fuel supply, whenever there is variation in output load.



FLYWHEEL	GOVERNORS
1. It is a reservoir of energy.	1. It is a feedback device. (Mechanism)
2. Flywheel has no control over the mean speed.	2. It can change the mean speed of prime mover.
3. It cannot change the fuel supply.	3. It can change the fuel supply (it can change well as quantity of fuel supply).
4. It control the fluctuation with in the cycle (Intra cycle fluctuation).	4. It controls the fluctuation between two consecutive cycle(inter cycle fluctuation)
5. Flywheel is continuous working device.	5. It is a intermittent working device.
6. If supply and load are uniform then flywheel is not required.	6. It is a compulsory device for all the prime movers.

Governor will work whenever there is correction in fuel supply.

Efficiency vs speed.



Prime mover must be operated @ $N_{optimum}$. So that $\eta = Max$. can be obtained.

Due to fluctuation in load there will be fluctuation in speed.

N_{max} and N_{min} must be very close to each other. So that the Prime mover is operating nearly @ $N_{optimum}$.

Classification of Governors

ACE Engineering Academy

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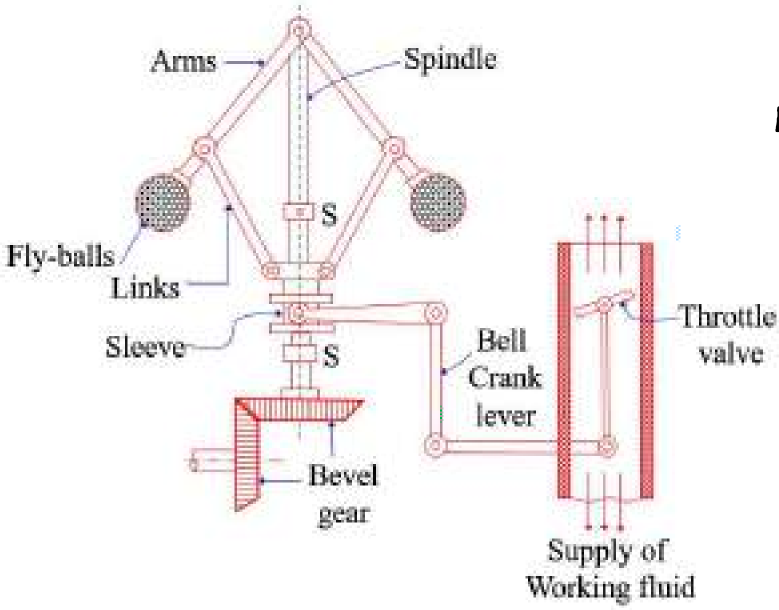
Governor

centrifugal ←→ Inertia

CENTRIFUGAL GOVERNOR:



Load ↓ Speed of Prime mover ↑
sleeve will move upwards.
Throttle opening decreases.
Fuel supply decreases.

Load ↑ speed of Prime mover ↓ sleeve will move downwards. Throttle opening decreases. Fuel supply increases.

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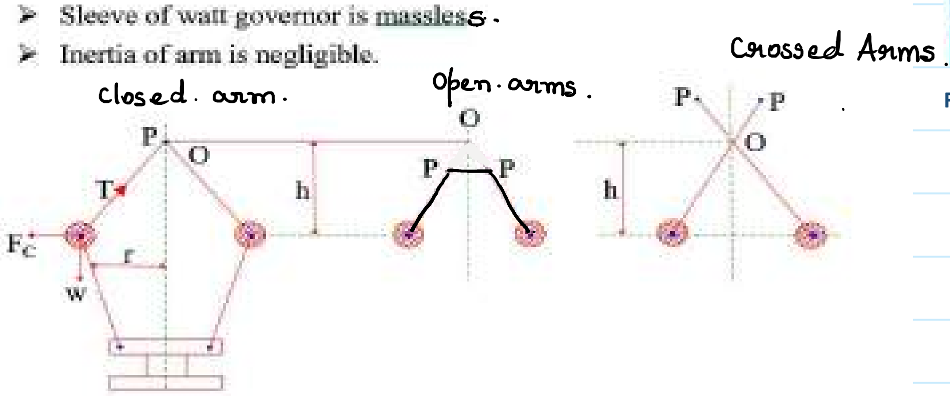
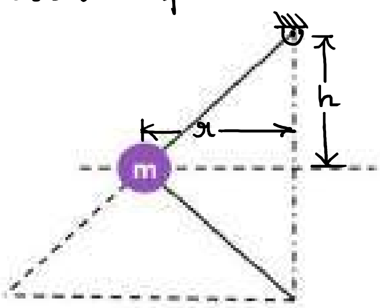
ANALYSIS OF WATT GOVERNOR:

- Sleeve of watt governor is massless.
- Inertia of arm is negligible.



HEIGHT OF GOVERNOR (h): It is the distance between the plane containing governor balls to the point where upper arms are intersecting with the governor axis either by their own or extended.

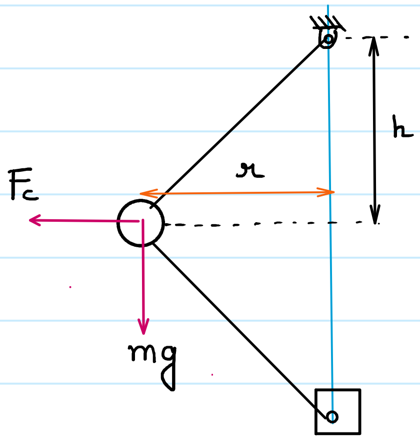
r = radius of rotation of flyballs.



Watt governor is used for low speed Prime movers.

LIMITATIONS: The variation of 'h' was appreciable for low value of 'N', for high speed the variation in 'h' is very small due to which the governor become insensitive corresponding to high speed.

- The working range of watt governor is very less.



$$\Sigma M_{Hinge} = 0$$

$$F_c \times h = mg \times r.$$

$$mr\omega^2 \times h = mg \times r.$$

$$\omega^2 = g/h.$$

$$\boxed{h = g/\omega^2}$$

$$h = \frac{9.81}{\left(\frac{2\pi}{60}\right)^2 \cdot N^2} = \frac{895}{N^2} \text{ mts.}$$

$h \propto \frac{1}{\omega^2}$. as $\omega \uparrow$ $\omega^2 \uparrow\uparrow$ $h \downarrow\downarrow$.

N-speed of governor. h – height of governor

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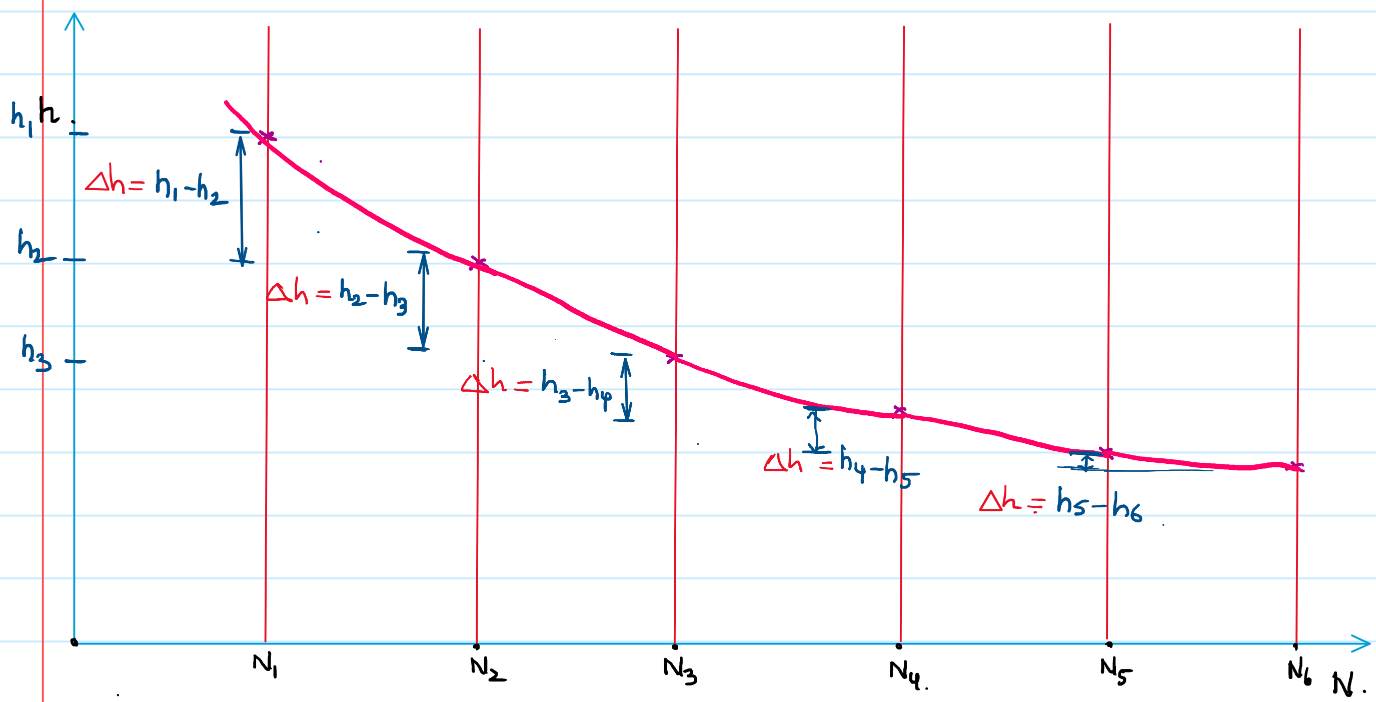


As $N \uparrow$ $\Delta h \downarrow$ for N_7 $\Delta h \to 0$ $h_6 - h_7 = 0 \Rightarrow h_6 = h_7$.

There is no sleeve displacement for speed N_7 and governor becomes insensitive at higher speeds.

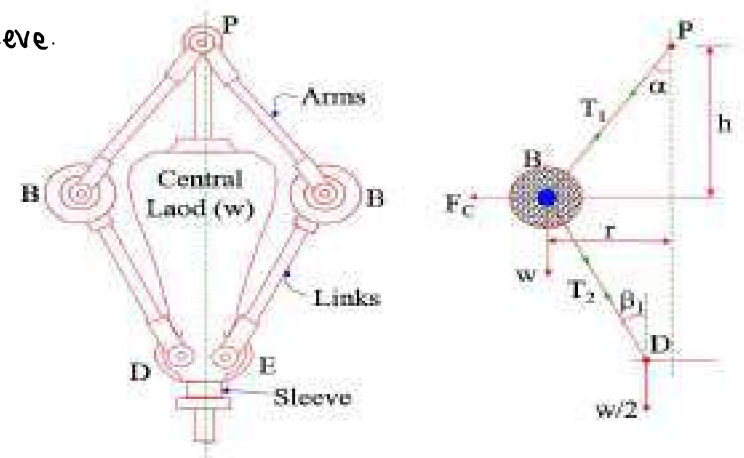
Hence Watt governor is used only for low speed prime mover.

Porter Governor → Dead wt. governor / Gravity controlled governor.

m – Mass of flyball.

M – Mass of sleeve.

ANALYSIS OF PORTER GOVERNOR:



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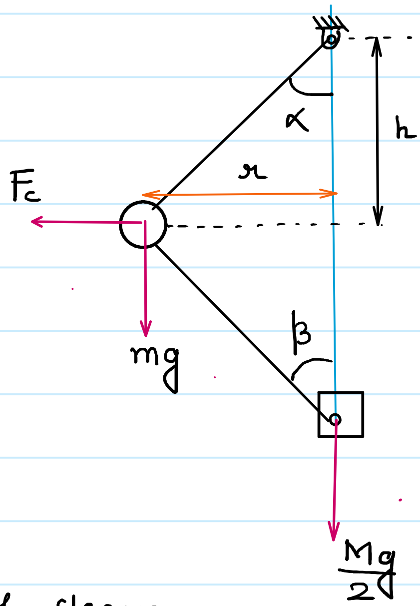
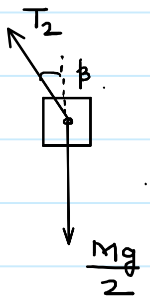


$$\boxed{\tan\alpha = \frac{r}{h}.}$$

F.B.D. of sleeve.

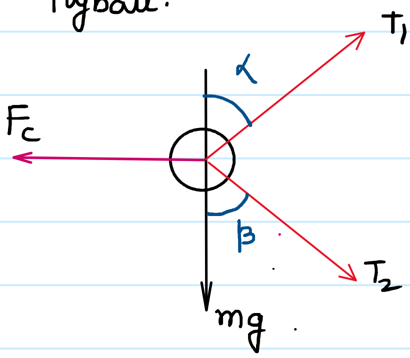


T_2 – Tension in lower arm.

$$\Sigma F_y = 0$$

$$T_2 \cdot \cos\beta = \frac{Mg}{2} \Rightarrow T_2 = \frac{Mg}{2\cos\beta}.$$

F.B.D. of flyball.



$$\Sigma F_x = 0$$

$$T_1 \sin\alpha + T_2 \sin\beta = F_c.$$

$$T_1 \sin\alpha = F_c - T_2 \cdot \sin\beta.$$

$$T_1 \sin\alpha = F_c - \frac{Mg}{2} \cdot \tan\beta. \rightarrow (A)$$

$$\Sigma F_y = 0$$

$$T_1 \cos\alpha = mg + T_2 \cdot \cos\beta.$$

$$T_1 \cos\alpha = mg + \frac{Mg}{2} \rightarrow (B)$$

$$\frac{A}{B} = \tan\alpha = \frac{F_c - \frac{Mg}{2}\tan\beta}{mg + \frac{Mg}{2}}.$$

$$mg \cdot \tan\alpha + \frac{Mg}{2} \cdot \tan\alpha = mr\omega^2 - \frac{Mg}{2} \cdot \tan\beta.$$

$$mr\omega^2 = \tan\alpha \cdot \left[mg + \frac{Mg}{2} \cdot \left(1 + \frac{\tan\beta}{\tan\alpha}\right)\right]$$

$$m\cancel{r}\omega^2 = \frac{\cancel{r}}{h} \cdot \left[mg + \frac{Mg}{2}(1+q)\right]. \Rightarrow h = \frac{mg + \frac{Mg}{2}(1+q)}{m\omega^2}$$

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Considering friction at sleeve.

$$h = \frac{\left[mg + \frac{(Mg \pm f)}{2}(1+q)\right]}{m\omega^2}$$

$f = 0$ when the sleeve is not moving. $\omega =$ constant.
$f = -ve$ when the sleeve is moving downwards ω decreases.
$f = +ve$ when the sleeve is moving upwards. ω increases.

$f = 0$, upper and lower arms are of equal lenght. $\alpha = \beta$, $q = 1$

$$h = \frac{mg + Mg}{m \cdot \omega^2} \Rightarrow h = \left(\frac{m+M}{m}\right) \cdot \frac{g}{\omega^2}$$

$$\omega^2 = \left(\frac{m+M}{m}\right) \cdot g/h.$$

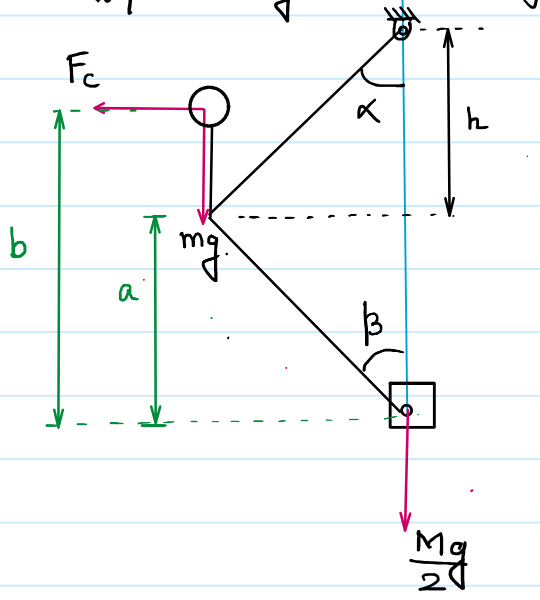
$$\omega^2_{porter} = \left(\frac{m+M}{m}\right) \cdot \omega^2_{watt}$$

$$\frac{\omega^2_{porter}}{\omega^2_{watt}} = \left(\frac{m+M}{m}\right) > 1.$$

$(M > m)$

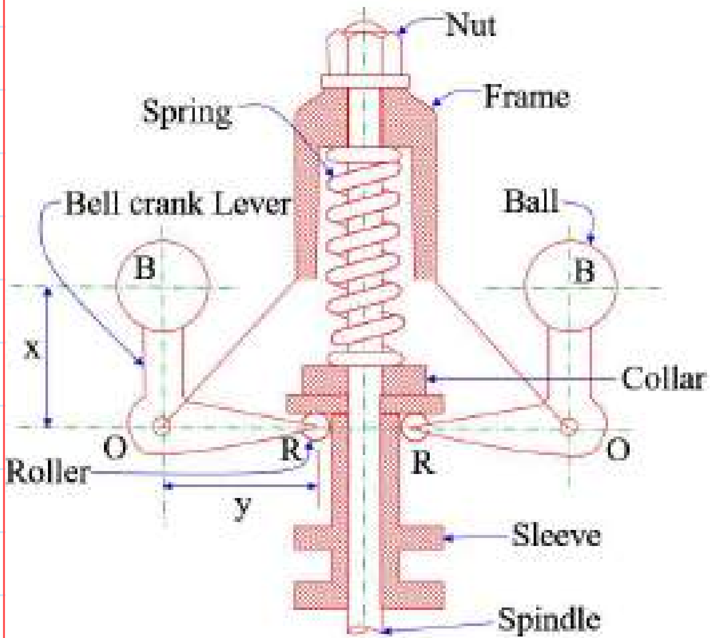
Porter governor can be used for high speed Prime movers.

Proell governor – Dead wt / Gravity Controlled governor.



$$h = \left(\frac{a}{b}\right)\left[\frac{mg + \frac{(Mg \pm f)}{2}(1+q)}{m\omega^2}\right]$$

HARTNELL GOVERNOR / Spring load Governor.



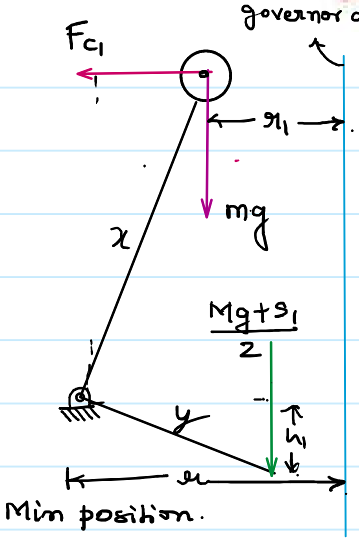
r_1, r_2 radii of rotation for $\omega_{min}, \omega_{max}$

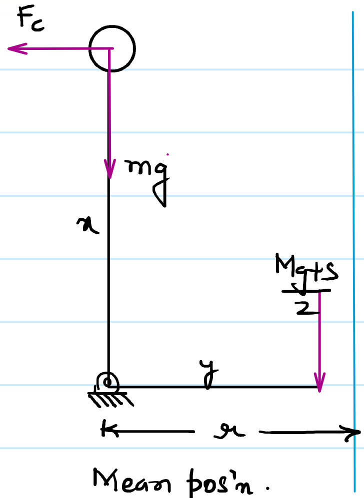
S_1, S, S_2 – Spring forces @ min pos'n, mean pos'n, max. pos'n.

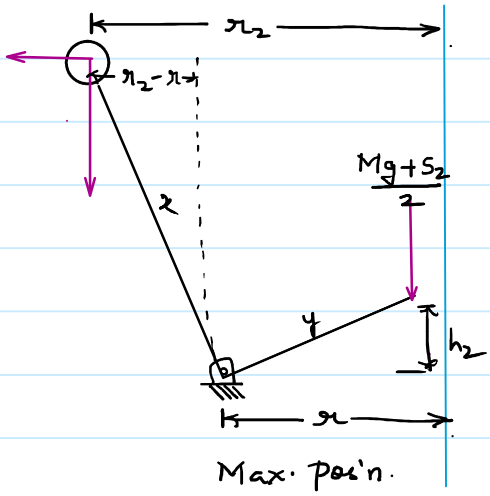
x – lenght of ball arm.

y – lenght of sleeve arm.

m, M – masses of flyball & sleeve.







$\Sigma M_{Hinge} = 0$

$F_{c_1} \times x - mg(r - r_1) = \frac{Mg+S_1}{2} \cdot y$

0

$F_{c_1} \cdot x = \frac{Mg+S_1}{2} \cdot y$

$M = 0 \Rightarrow F_{c_1} \cdot x = \frac{S_1}{2} \cdot y \rightarrow (A)$

$\Sigma M_{Hinge} = 0$

$F_c x. = \frac{Mg+S}{2} \cdot y$

$M = 0$

$F_c \cdot x = \frac{S}{2} \cdot y$

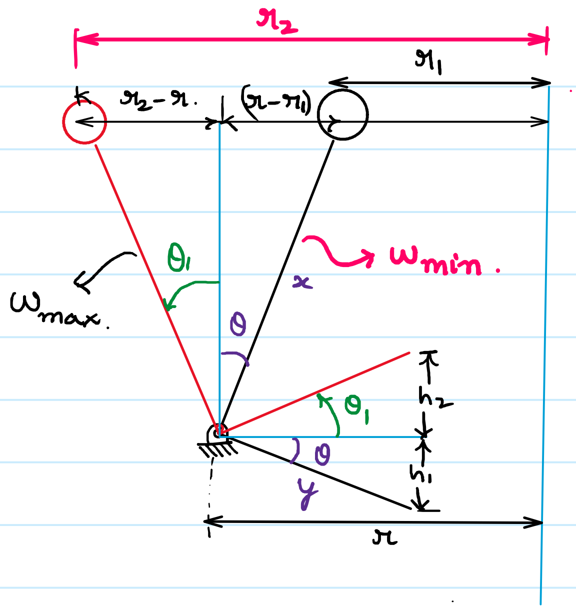
$\Sigma M_{Hinge} = 0.$

$F_{c_2} \cdot x + mg(r_2 - r) = \left(\frac{Mg+S_2}{2}\right) y$

$F_{c_2} \cdot x = \frac{S_2}{2} \cdot y \rightarrow (B)$

$$(B-A) = (F_{c_2} - F_{c_1})x = \left(\frac{S_2 - S_1}{2}\right) \cdot y$$

Neglecting the effect of obliquity of arms., Neglecting the effect of moment due to revolving mass.



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sleeve displacement $= h_1 + h_2$

$$\frac{h_1}{y} = \frac{(r - r_1)}{x}$$

$$h_1 = \frac{(r - r_1) \cdot y}{x}$$

$$\frac{h_2}{y} = \frac{(r_2 - r)}{x} \Rightarrow h_2 = \frac{(r_2 - r) \cdot y}{x}$$

sleeve displacement $h = h_1 + h_2 = \frac{y}{x}(r - r_1) + \frac{y}{x}(r_2 - r)$

$$h = \frac{y}{x} \cdot (r_2 - r_1)$$

Spring force $= S_1 = k h_1$

Spring force $= S_2 = k \cdot h_2$

$$(F_{c_2} - F_{c_1}) x = \frac{(S_2 - S_1)}{2} \cdot y$$

$$(F_{c_2} - F_{c_1}) x = \frac{k}{2} \cdot (h_2 - (-h_1)) y \Rightarrow (F_{c_2} - F_{c_1}) x = \frac{k}{2} \cdot (h_1 + h_2) \cdot y$$

$$(F_{c_2} - F_{c_1}) \cdot x = \frac{k}{2} \cdot \frac{y}{x} \cdot (r_2 - r_1) \cdot y$$

$$k = \frac{2(F_{c_2} - F_{c_1})}{(r_2 - r_1)} \cdot \left(\frac{x}{y}\right)^2$$

$x, y =$ constant

$k =$ const

$$\boxed{\frac{F_{c_2} - F_{c_1}}{r_2 - r_1} = \frac{F_c - F_{c_1}}{r - r_1}}$$

Terminology of Governors

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Centrifugal Force - The force which has tendency to move the flyballs away from the governor axis is called centrifugal force.

$F_c = mr\omega^2$

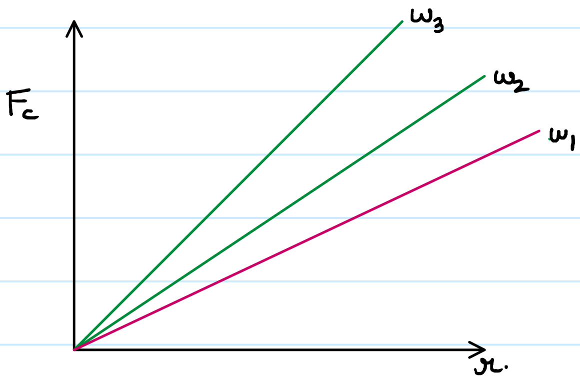
$\hookrightarrow F_c \propto r^1$

$\frac{F_c}{r} = m\omega^2$

$m = const.$

$\frac{F_c}{r} = \tan\theta = m\omega^2$

$\tan\theta \propto \omega^2$

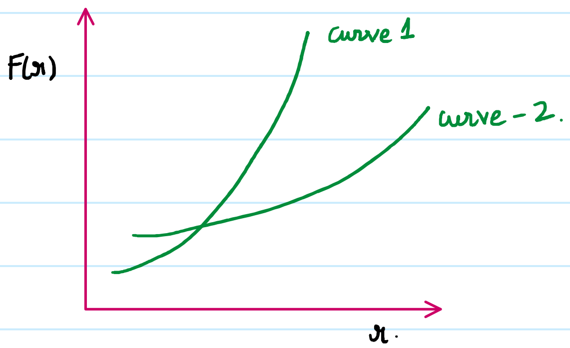


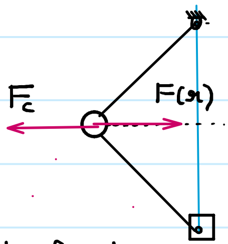
$\omega_3 > \omega_2 > \omega_1$

Controlling Force - The force which has the tendency to move the flyballs towards the governor axis is called as Controlling force.

Controlling Force in case of dead weight governor is the resultant of Weight of flyball, sleeve, friction at sleeve. **Major Component of Controlling force is from the Weight of the sleeve only.**

Controlling Force curve for Dead weight governor (Porter & Proell)



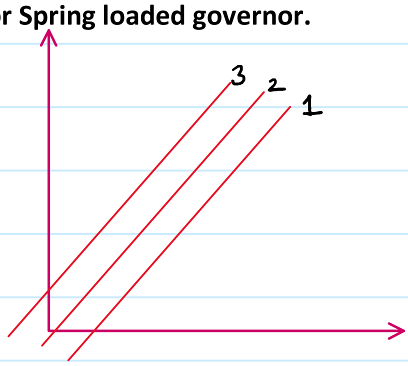


F_c – centrifugal force

F(r) – controlling force.

Controlling force in case of spring controlled governor is the resultant of Weight of flyball, sleeve, Spring and Friction at sleeve. **The major component of controlling force is coming from spring only.**

Controlling Force curve for Spring loaded governor.



$C.F_1 = ar - b$

$CF_2 = ar.$

$CF_3 = ar + b$

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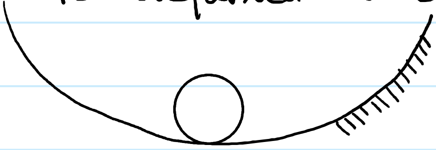
Types of Equilibrium

1. Stable equilibrium. – system will come back to initial equilibrium position after distrubing.
No external work is required to bring back to intial equilibrium pos'n.

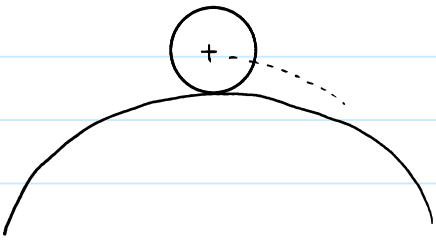


2. Neutral Equilibrium – System will move to new equilibrium pos'n. when it is distrubed.

External Work is required.

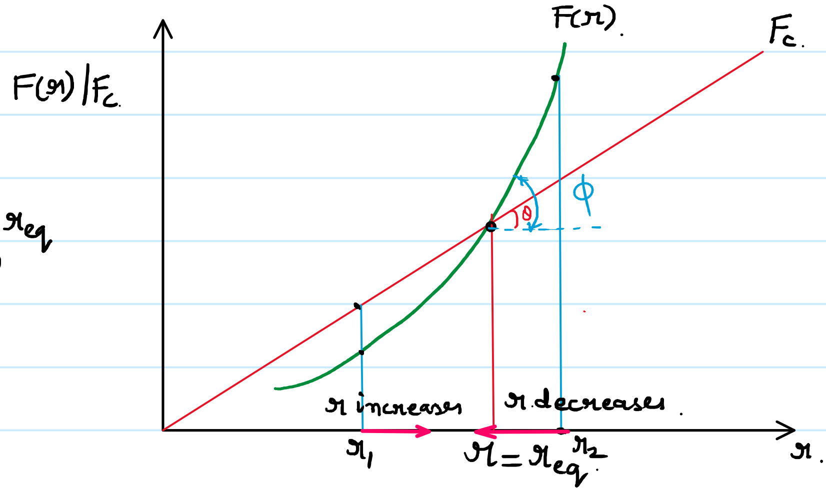
3. Unstable equilibrium. – System continues to undergo the change in state after its is distrubed from the equilibrium.

External Work is required.



$$(W.D)_{\text{unstable equilibrium}} > (W.D)_{\text{Neutral equilibrium}}$$

Stability Analysis of Dead wt. governor.



@ $r = r_{eq}$
$\phi > \theta$

@ $r = r_{eq}$ $F_c = F(r)$.

@ $r = r_1$ $F_c > F(r)$

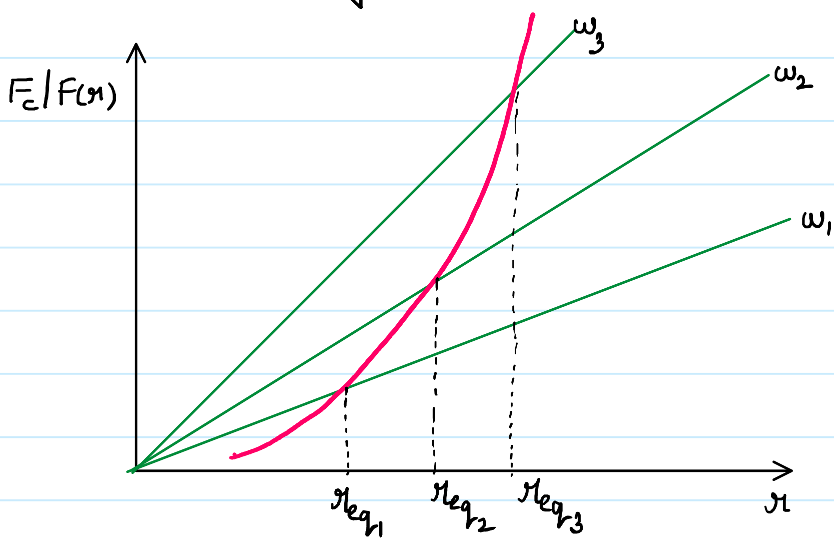
F_c increases r.

@ $r = r_2$ $F(r) > F_c$.

$F(r)$ decreases r.

F_c, $F(r)$ will push the governor to $r = r_{eq}$. Hence the given governor is stable.

ϕ – Slope of controlling curve, θ – slope of F_c vs r graph.

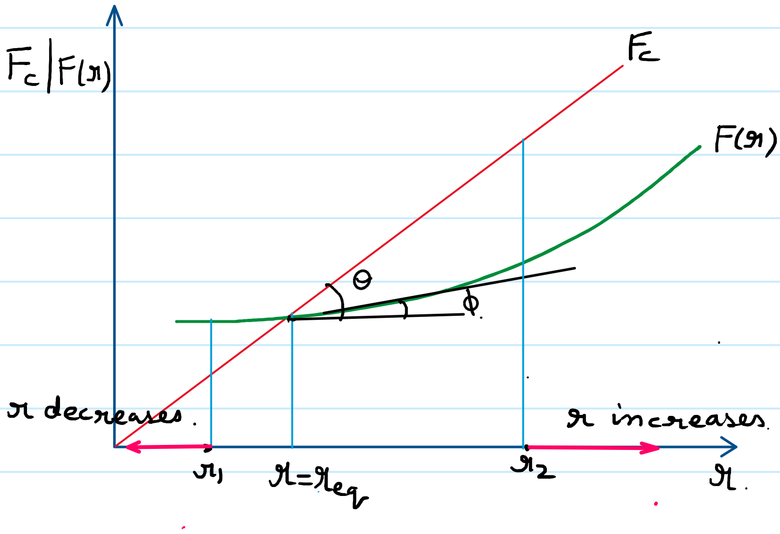


r_{eq} – equilibrium radius.

As $r_{eq}\uparrow$ $\omega\uparrow$ sleeve will move upward, opening of throttle valve will decrease., Fuel supply decreases.

As $r_{eq}\downarrow$ $\omega\downarrow$ sleeve will move downward, opening of throttle valve will increase., Fuel supply increases.

Hence the governor is said to be stable.



@ $r = r_{eq}$ $F_c = F(r)$

$\theta > \phi$

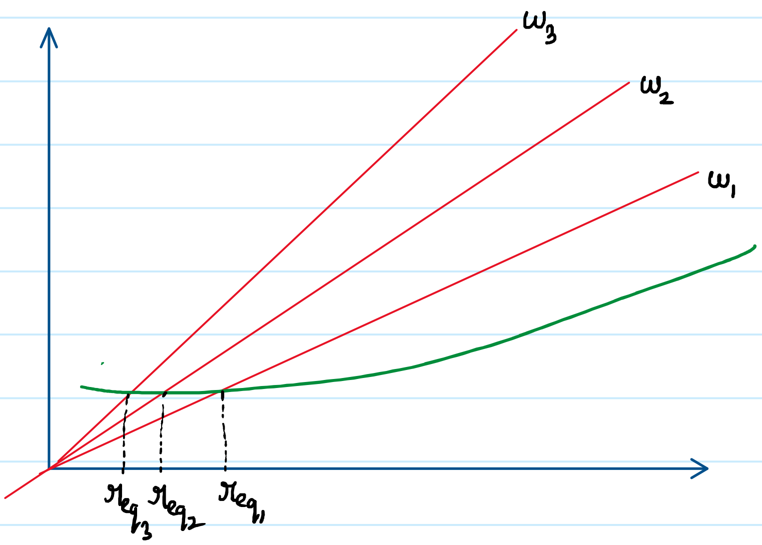
@ $r = r_1$ $F(r) > F_c$

$F(r)$ decreases r.

@ $r = r_2$ $F_c > F(r)$

F_c increases r.

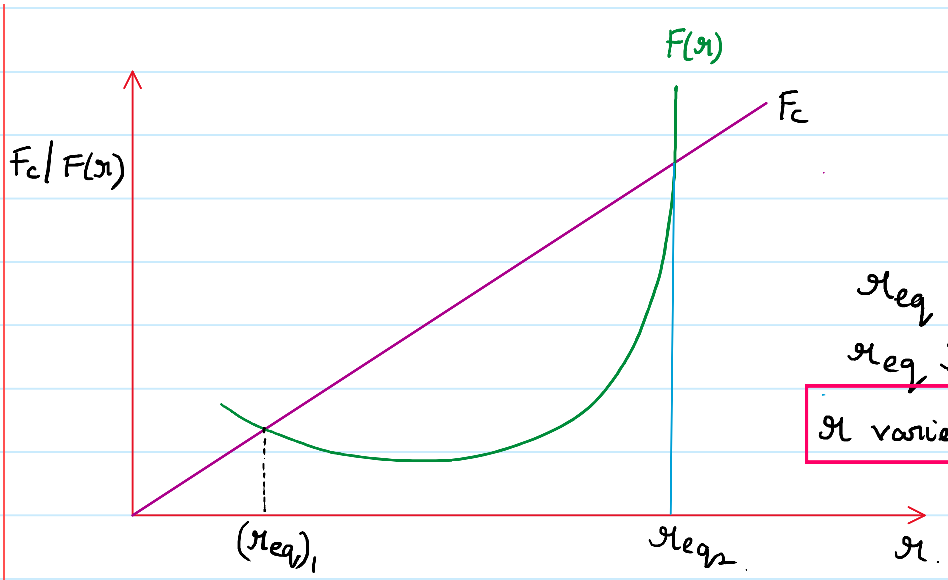
F_c, $F(r)$ will deviate the governor from $r = r_{eq}$.
Hence the governor is unstable.



As $r_{eq} \uparrow$ $\omega \downarrow$ sleeve will move upwards, throttle valve opening decrease, Fuel supply decreases.

As $r_{eq} \downarrow$ $\omega \uparrow$ sleeve will move downward, throttle valve opening increases, Fuel supply increases.

Governor is said to be unstable.



$r_{eq} \uparrow$ ω = constant.

$r_{eq} \downarrow$ ω = constant.

$\boxed{r \text{ varies} \quad \omega = \text{constant}}$

↳ Isochronism.

@ $r = r_{eq_1}$ $F_c = F(r)$ sleeve is closer to B.D.C., opening of throttle more, Fuel Supply increases.

@ $r = r_{eq_2}$ $F_c = F(r)$, sleeve is closer to T.D.C, opening of throttle less, Fuel supply decreases.

ω = constant

$$\omega^2 = \left[\frac{mg + \frac{Mg}{2}(1+q)}{mh}\right] \qquad m, M, g \longrightarrow \text{constant.}$$

$$\tan\alpha = \frac{r}{h} \Rightarrow h \propto r. \qquad \omega^2 \propto \frac{1}{h}, \quad h \propto r. \Rightarrow \boxed{\omega^2 \propto \frac{1}{r}}$$

Isochronism is not possible in case of Dead Wt. Governor.

For a stable governor there must be unique value of speed for a given radius of rotation of flyballs.

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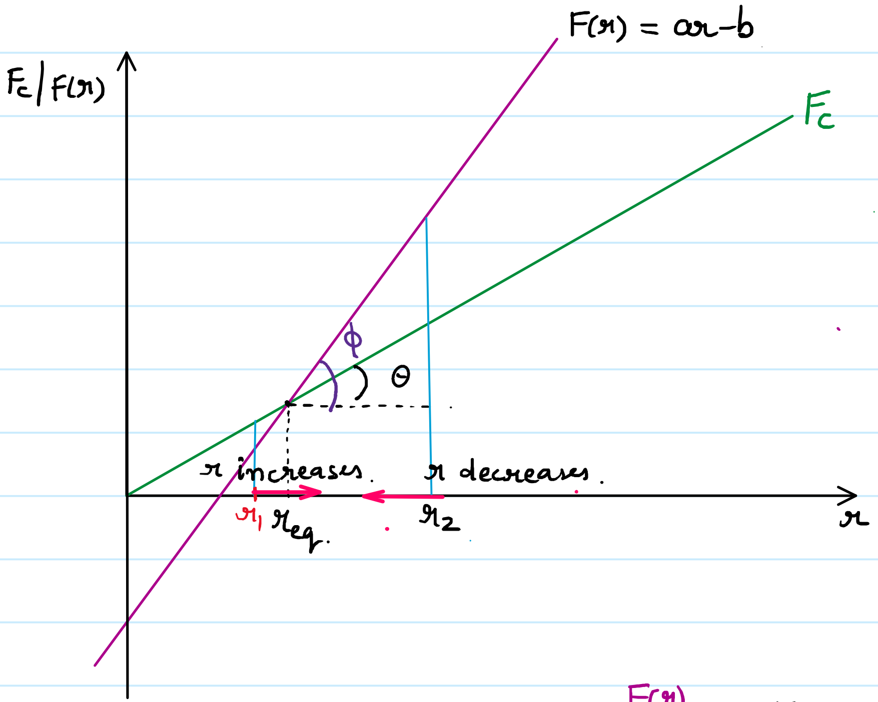


@ $r = r_{eq}$ $\quad F_c = F(r)$

$\phi > \theta$

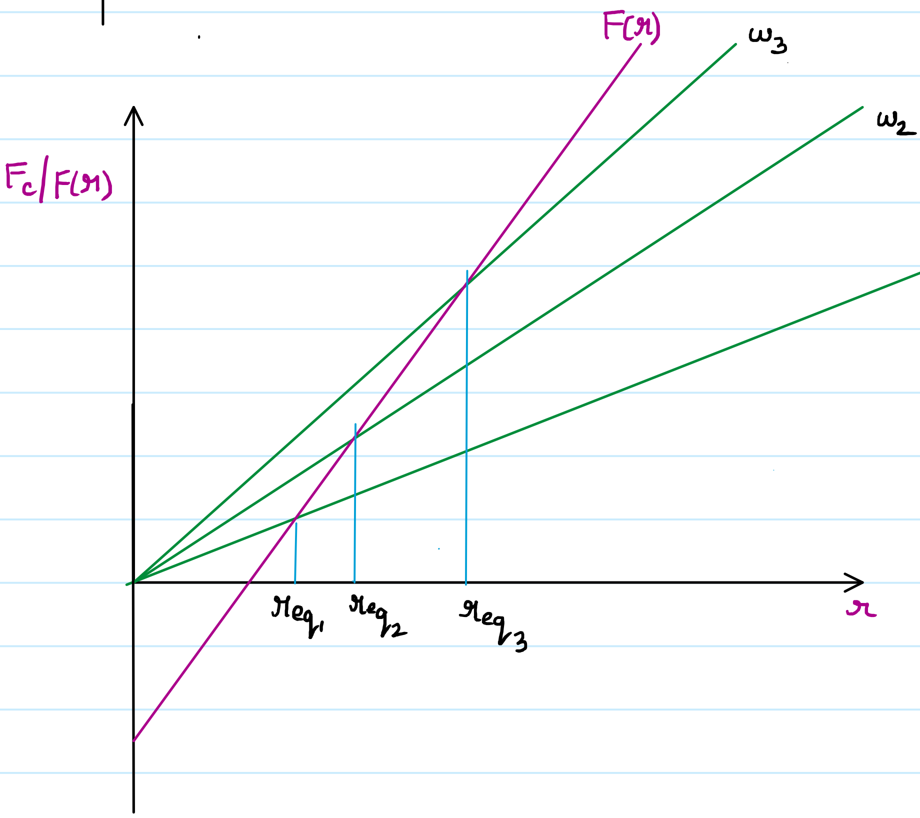
@ $r = r_1$, $\quad F_c > F(r)$

F_c increases r.

@ $r = r_2$ $\quad F(r) > F_c$

$F(r)$. decreases r.

$F(r) = ar - b$



As r_{eq} increases $\quad \omega$ increases. $\quad$ Sleeve upward fuel supply $\downarrow$.

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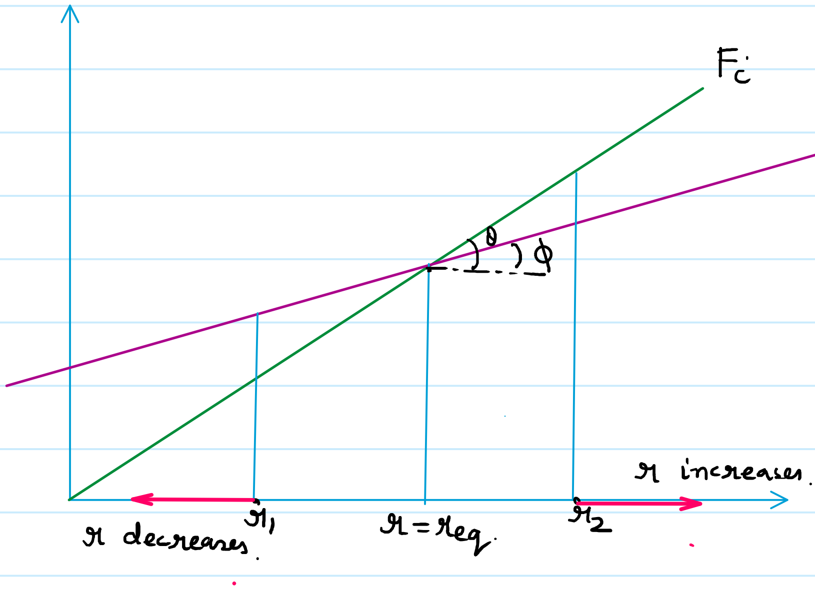


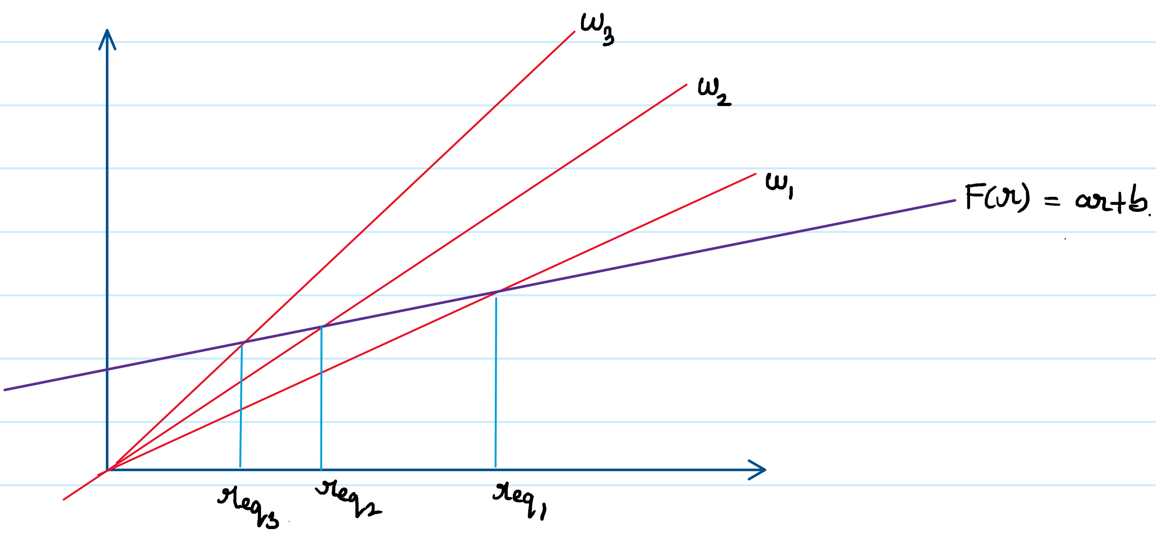
$F(r) = ar + b$

@ $r = r_{eq}$. $F_c = F$.
$\theta > \phi$.

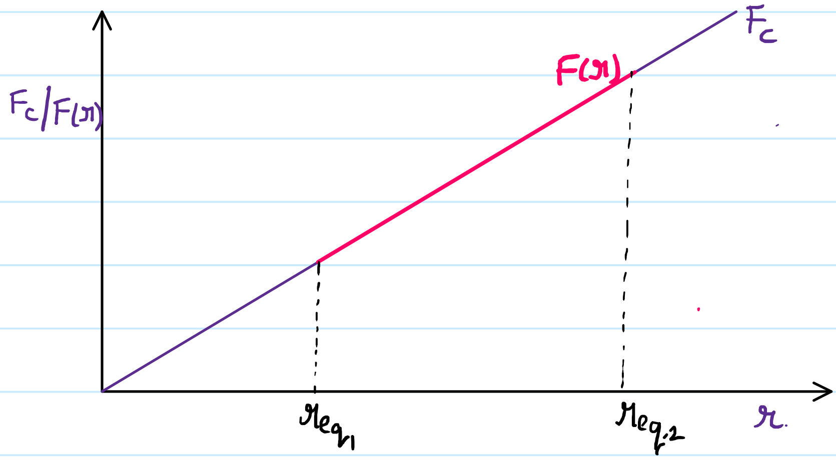
@ $r = r_1$ $F(r) > F_c$
$F(r)$ decreases r

@ $r = r_2$ $F_c > F(r)$
F_c increases r

Governor is unstable.



As $r_{eq} \uparrow$ $\omega \downarrow$, $r_{eq} \downarrow$. $\omega \uparrow$. Hence the governor is unstable.



Isochronous governor.

@ $r = r_{eq_1}, r_{eq_2}$

$F_c = F(r) \quad \theta = \phi$

$\omega = \text{constant}$

$F(r) = ar$.

For Hartnell governor.

$$F_{c_1} \times x = \frac{Mg + S_1}{2} \cdot y$$

$$F_{c_2} \times x = \frac{Mg + S_2}{2} y$$

$$\frac{\cancel{m} \cdot r_1 \cdot \cancel{\omega_1^2} \cdot \cancel{x}}{\cancel{m} \cdot r_2 \cdot \cancel{\omega_2^2} \cdot \cancel{x}} = \frac{\frac{Mg+S_1}{\cancel{2}} \cdot \cancel{y}}{\frac{Mg+S_2}{\cancel{2}} \cdot \cancel{y}} \Rightarrow \frac{r_1}{r_2} = \frac{Mg+S_1}{Mg+S_2}$$

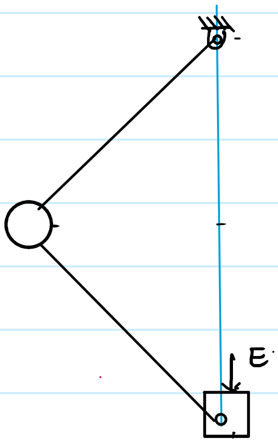
Condition for isochronism for Hartnell governor $\rightarrow$ $\boxed{\frac{r_1}{r_2} = \frac{S_1}{S_2}}$

Sensitivity of the Governor - It the ability of the governor to sense the change in given load and alter the fuel supply accordingly.

$$\boxed{\text{Sensitivity} = \frac{N_{max} - N_{min}}{\left(\frac{N_{max} + N_{min}}{2}\right)}}$$ $\rightarrow$ Governor coupled to Prime mover.

$$\boxed{\text{Sensitivity} = \frac{\left(\frac{N_{max} + N_{min}}{2}\right)}{(N_{max} - N_{min})}}$$ $\rightarrow$ Governor is considered as individual mechanism.

Effort of the Governor - It is the average force exerted on the sleeve for The percentage change in speed.



$$\omega^2 = \frac{mg + \frac{Mg}{2}(1+q)}{mh} \rightarrow (1) \qquad \omega'^2 = \left[\frac{mg + \frac{Mg}{2}(1+q)}{mh'}\right]$$

$$\omega' = \omega(1+c) \qquad c \rightarrow \text{\% change in the speed.}$$

$$\omega'^2 = \left[\frac{mg + \left(\frac{Mg+E}{2}\right)(1+q)}{mh}\right] \longrightarrow (2)$$

$$\frac{1}{2} = \frac{\cancel{\omega^2}}{(1+c)^2 \cdot \cancel{\omega^2}} = \frac{\left[mg + \frac{Mg}{2}(1+q)\right] \times \cancel{\frac{1}{mh}}}{\left[mg + \frac{Mg+E}{2}(1+q)\right] \times \cancel{\frac{1}{mh}}}$$

$$\frac{1}{(1+c)^2} = \frac{2mg + Mg(1+q)}{2mg + (Mg+E)(1+q)} \qquad 1 + \underset{0}{\cancel{c^2}} + 2c \approx 1 + 2c$$

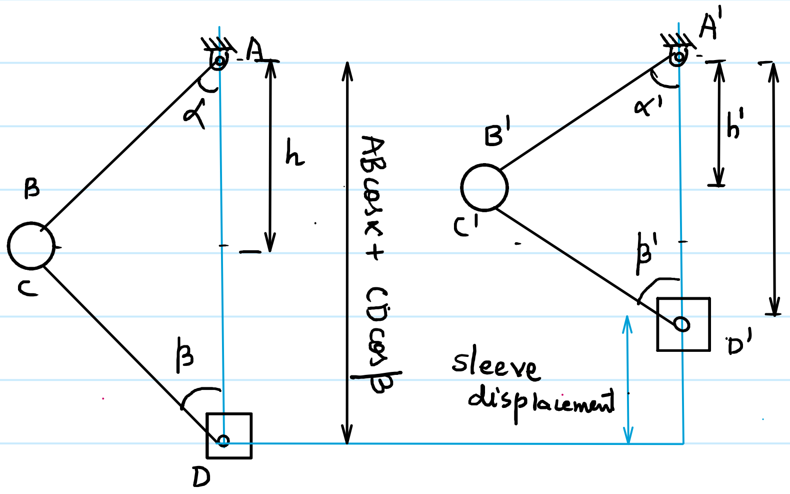
$$2mg + Mg(1+q) + E.(1+q) = (1+2c) \cdot \left[2mg + Mg.(1+q)\right]$$

$$\cancel{2mg} + \cancel{Mg(1+q)} + E.(1+q) = \left[\cancel{2mg} + \cancel{Mg(1+q)}\right] + 2c\left[2mg + Mg.(1+q)\right]$$

$$\text{Effort} \longrightarrow \frac{E}{2} = \frac{2c.\left[2mg + Mg.(1+q)\right]}{2 \cdot (1+q)} \Rightarrow \frac{E}{2} = \frac{c \cdot \left[2mg + Mg(1+q)\right]}{(1+q)}$$

$$q = 1 \qquad E/2 = c \cdot [m+M]g.$$

Sleeve Displacement - It is the change in the position of sleeve for a percentage change in speed.



$A'B'\cos\alpha' + C'D'\cos\beta'$

$AB = A'B'$

$CD = C'D'$

$\alpha = \beta \Rightarrow AB = CD = A'B' = C'D'$

$$\text{Sleeve displacement} = (AB\cos\alpha + CD\cdot\cos\beta) - (A'B'\cos\alpha' + C'D'\cos\beta')$$

$$= 2h - 2h' = 2\cdot(h - h')$$

$$h = \frac{g}{\omega^2} \qquad h' = \frac{g}{\omega'^2} \Rightarrow h' = \frac{g}{\omega^2(1+c)^2}$$

$$\text{sleeve displacement} = 2h\cdot\left[1 - \frac{h'}{h}\right] = 2h\cdot\left[1 - \frac{g/\omega^2(1+c)^2}{g/\omega^2}\right]$$

$$\text{sleeve displacement} = 2h\cdot\left[1 - \frac{1}{(1+c)^2}\right] = 2h\left[1 - \frac{1}{1+2c}\right]$$

$c^2 \approx 0$

$$= \frac{4hc}{(1+2c)}$$

Power of the Governor - It is the work done on the sleeve for a percentage change in speed.

$$\text{Work done} = \text{Effort} \times \text{sleeve displacement}$$

$$\text{Work done} = \frac{E}{2} \times 2\cdot(h - h') = c\frac{[2mg + Mg(1+q)]}{(1+q)} \times \frac{4hc}{1+2c}$$

$q = 1$ $\quad$ $\text{Work done} = [m + M]g\cdot h\cdot\left(\frac{4hc^2}{1+2c}\right)$

for Watt governor $M = 0$.

Hunting - A highly sensitive governor produces large sleeve displacement
For a small change in speed. Due to the variation in load there will be changes in speed this causes the rapid displacement of sleeve.
This phenomenon is called as Hunting. Upper & lower arms , sleeve experiences violent oscillations.

At Hunting governor is at resonance.
If the frequency of fluctuation in speed coincides with the natural frequency of the governor then resonance will occur.

Coefficient of Detention/Coefficient of Insensitiveness - Due to the presence friction the governor becomes insensitive over a range of speed. In order to overcome the friction the governor is required to undergo additional change in speed.

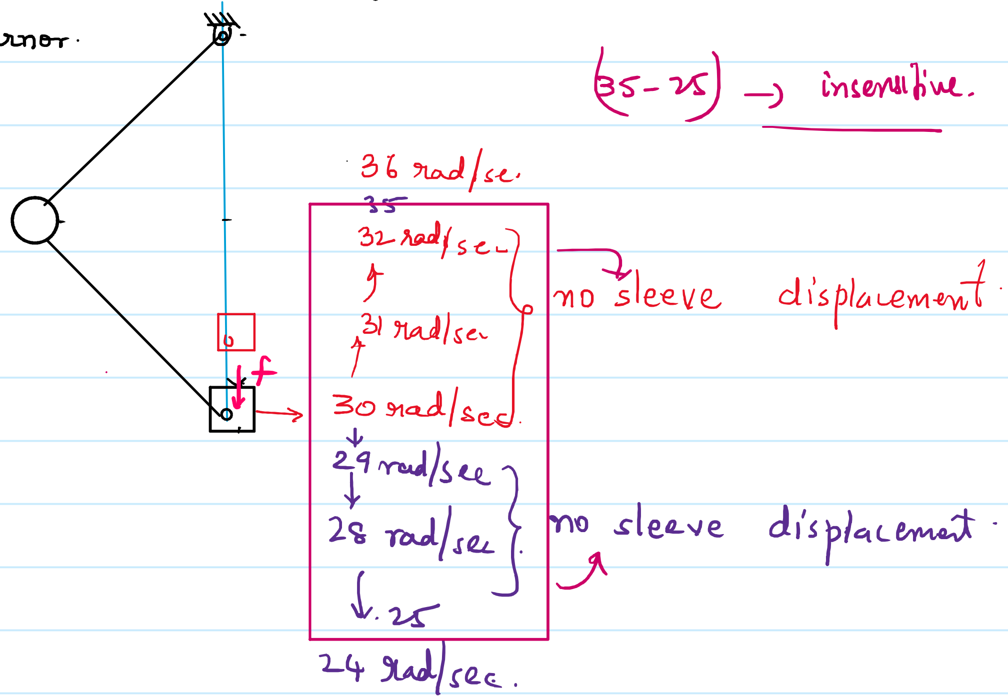
$$\text{Coefficient of Dentention} = \frac{\omega' - \omega''}{\left(\frac{\omega' + \omega''}{2}\right)} = \frac{2f}{(m+M)g}$$

ω' – Speed of governor for which there is no sleeve displacement.

ω'' – Speed of governor for which there is no sleeve displacement.

f – friction force , m – mass of flyball M – Mass of sleeve.

$M = 0$ for Watt governor.



01. For a governor running at constant speed, what is the value of the force acting on the sleeve ?
(a) Zero
(b) Variable depending upon the load
(c) Maximum
(d) Minimum

$\omega = \text{constant} \Rightarrow C = 0.$

$\text{Effort} = ?$

$C = 0 \rightarrow \text{Effort} = 0.$

02. When a porter governor is running at equilibrium speed the friction at the sleeve will be
(a) in the upward directions
(b) in the downward directions
(c) perpendicular to the spindle
(d) zero

equilibrium speed $\rightarrow \omega = \text{constant}$

$\hookrightarrow$ sleeve is neither moving up or downward.

friction = 0.

03. In a porter governor all the links are connected on the axis of the spindle and are of same length, and are inclined to the axis by the same angle. The radius of rotation is $\sqrt{3}$ times the height of the governor which is 20 cm, sleeve mass is 10 kg and the ball mass is 2 kg each, the equilibrium speed is
(a) 17.15 rad/sec
(b) 7.67 rad/sec
(c) 12.05 rad/sec
(d) 14.15 rad/sec

$\alpha = \beta \Rightarrow q = 1.$

$r = \sqrt{3} \cdot h.$

$h = 20\text{cm}.$

$M = 10\text{ kg}$

$m = 2\text{kg}$

$$\omega^2 = \left(\frac{m+M}{m}\right) \cdot g/h.$$

$$\omega^2 = \left(\frac{10+2}{2}\right) \times \frac{9.81}{0.2}$$

$$\omega = 17.15 \text{ rad/s.}$$

04. In a Hartnell governor the ball arm and sleeve arm are of equal length. The sleeve mass is negligible and the ball mass is 1kg. At a ball radius of 25cm. The ball arm is vertical and the equilibrium speed is 20 rad/sec. If the spring stiffness is 200 N/cm the initial compression in the spring at this position is
(a) 1 cm (b) 0.5 cm
(c) 2 cm (d) 0.25 cm

$x = y$

$m = 1\text{kg}$

$r = 25\text{cm}.$

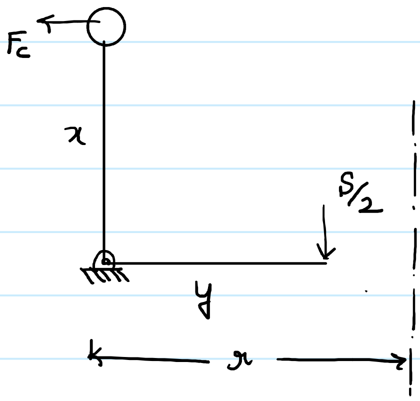
$\omega = 20 \text{ rad/s}.$

$K = 200\text{N/cm}$

$\delta_{\text{intial}} = ?.$

For Hartnell Governor.

$$F_c \times x = \frac{S}{2} \times y.$$



$$S = 2F_c.$$

$$K \cdot \delta_{\text{intial}} = 2 \times m r \omega^2$$

$$200 \times \delta_{\text{intial}} = 2 \times 1 \times 0.25 \times (20)^2$$

$$\delta_{\text{intial}} = 1\text{cm}.$$

05. In a Hartnell governor the sleeve arm and ball arm are equal and half the radius of rotation in the mean position. The mean speed is 20 rad/sec; and the fulcrum is at a distance of 40 cm from the axis of rotation. The sleeve mass is negligible. If the ball mass is 1 kg each. The spring force at the mean position is
(a) 320 N
(b) 160 N
(c) 640 N
(d) None of the above

$x = y \quad \omega = 20\,rad/s.$

$r = 40cm. \; , m = 1kg$

$\Sigma M_{Hinge} = 0$

$F_c \times x = \frac{S}{2} \times y.$

$S = 2F_c \Rightarrow S = 2mr\omega^2 = 2\times1\times0{\cdot}4\times20^2$

$S = 320N.$

06. The sensitiveness of the governor is equal to
(a) $\frac{N_1+N_2}{2N_1N_2}$ (b) $\frac{N_1-N_2}{2N_1N_2}$
(c) $\frac{2(N_1-N_2)}{N_1+N_2}$ (d) $\frac{2(N_1+N_2)}{N_1-N_2}$

Where, N_1 = Maximum equilibrium speed
N_2 = Minimum equilibrium speed and
$\frac{N_1+N_2}{2}$ = Mean equilibrium speed

$$\text{Sensitivity} = \frac{\text{Range of Speed}}{\text{Mean Speed}} = \frac{N_{max}-N_{min}}{\left(\frac{N_{max}+N_{min}}{2}\right)}$$

Sensitivity of isochronous governor is infinite.

07. A governor is sensitive and stable, supplied by a reliable vendor. But when connected to a steam engine it has become insensitive. The possible reason could be
(a) steam engine requires isochronous governor
(b) the power of the governor could be too low to actuate the throttle linkage
(c) due to hunting it might have lost sensitivity
(d) either (a) or (c)

→ no sleeve displacement.

→ $\omega = const.$ for all radii

→ Highly sensitive governor.

08. The control force curve for a spring-loaded governor is a straight line. At a radius of 50cm the control force is 600N and at 60cm it is 700N. Assuming that the ball arm and sleeve arm are of equal length, identify the nature of the governor and also indicate how it can be made isochronous.
(a) Unstable, reduce the initial compression of the spring by 100 N
(b) Stable, reduce the initial compression of the spring by100 N
(c) Unstable, increase the initial compression of the spring by 100 N
(d) Stable, increase the initial compression of the spring by 100 N

$F(r) = ar + b$

@ $r = 50cm. \; F(r) = 600N.$

@ $r = 60cm \; F(r) = 700N.$

$600 = a\cdot(0{\cdot}5) + b.$

$700 = a\cdot(0{\cdot}6) + b$

$a = 100/0{\cdot}1 = 1000r.$

$b = 100.$

$F(r) = 1000r + 100. \longrightarrow$ Unstable governor.

$F(r) = 1000r \longrightarrow$ Isochronous governor.

09. By increasing the dead weight in a Porter Governor the following does not happen
(a) It will become less sensitive
(b) The equilibrium speed will increase
(c) The stability will increase
(d) None of the above

$$\omega^2 = \left[mg + \frac{Mg}{2}(1+q)\right]\cdot\frac{1}{mh}.$$

a, b, c → all happens.

$\omega^2 \propto M.$

$M\uparrow$ stability $\uparrow$ sensitivity $\downarrow$

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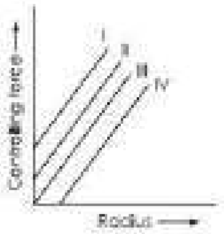
10. The plots of controlling force versus radii of rotation of the balls of spring controlled governors are shown in the given diagram. A stable governor is characterized by the curve labeled.



(a) I (b) II
(c) III (d) IV

I, II — $ar+b$. unstable governor
III → ar. → Isochronous governor.
IV → $ar-b$. → stable governor.

12. A spring controlled governor is found unstable. It can be made stable by
(a) increasing the spring stiffness ✗
(b) decreasing the spring stiffness ✓
(c) increasing the ball weight
(d) decreasing the ball weight

$F(r) = ar+b$ unstable
↓ decreasing spring stiffness
$F(r) = ar-b$ stable.

13. For a given fractional change of speed, if the displacement of the sleeve is high, then the governor is said to be
(a) Hunting
(b) Isochronous
(c) Sensitive
(d) Stable

14. Effect of friction at the sleeve of a centrifugal governor is to make it
(a) More sensitive
(b) More stable
(c) Insensitive over a small range of speed
(d) Unstable

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15. In a centrifugal governor, the controlling force is observed to be 14 N when the radius of rotation is 2 cm and 38 N when the radius of rotation is 6 cm, the governor:
(a) is a stable governor
(b) is an unstable governor
(c) is an isochronous governor
(d) cannot be said of what type with the given data

$F(r) = 14N \quad r = 2cm.$
$F(r) = 38N \quad r = 6cm.$

$F(r) = ar + b. \quad = \boxed{600r + 2}$ → unstable governor.
$14 = a\cdot(0.02) + b.$
$38 = a(0.06) + b$

$a = 600$
$b = 2$

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16. The controlling force in a spring controlled governor is 1500 N when the radius of rotation of the balls is 200 mm and 887.5 N when it is 130 mm. The mass of each ball is 8 kg. If the controlling force curve is a straight line, determine the controlling force and the speed of rotation when the radius of rotation is 150 mm. Also find the increase in the initial tension so that the governor is isochronous. What will be the isochronous speed?

$F(r) = 1500N. \quad r = 200mm. \quad m = 8kg$
$F(r) = 887.5 \quad r = 130mm.$

$r = 150mm. \quad F(r) = ? \quad \omega = ?$
intial tension. for isochronous governor = ?
$\omega_{isochronous} = ?$

MSQ

17. Consider the following statements in case of a governor and which of them is/are CORRECT ?
(a) A governor is said to be unstable if the radius of rotation falls as the speed increases. $r_{eq} \uparrow \omega \downarrow$ unstable
(b) Spring controlled governors never become isochronous.
(c) By increasing the initial compression of the spring the mean speed can be reduced.
(d) Isochronisms for a centrifugal governor can be achieved only at the expense of its stability.

$F(r) = ar - b \rightarrow ar.$

Gravity controlled governor never becomes isochronous.

(c). $F_c \times r = \frac{S \times y}{2}$
$mr\omega^2 = k.\delta_{intial}.$
$\omega^2 \propto \delta_{intial}.$

$F(r) = ar + b.$
$1500 = a(0.2) + b \qquad a = 8750$
$887.5 = a.(0.13) + b. \qquad b = -250.$

$F(r) = 8750r - 250.$
@ $r = 150mm.$

$F(r) = 8750(0.15) - 250$
$= 1062.5 = mr\omega^2$
$= 1062.5 = 8 \times 0.15 \times \omega^2$
$\omega = \sqrt{\frac{1062.5}{1.2}} = 29.756 \ rad/s.$

$F(r) = 8750r - 250$
↳ if intial tension is increase by 250N it becomes isochronous.

$F(r) = 8750\cancel{r} = m\cancel{r}.\omega^2_{isochronous} \qquad \omega_{isochronous.} = \sqrt{\frac{8750}{8}}$
$= 33.07 \ rad/s.$

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Comparison of Stable Governors

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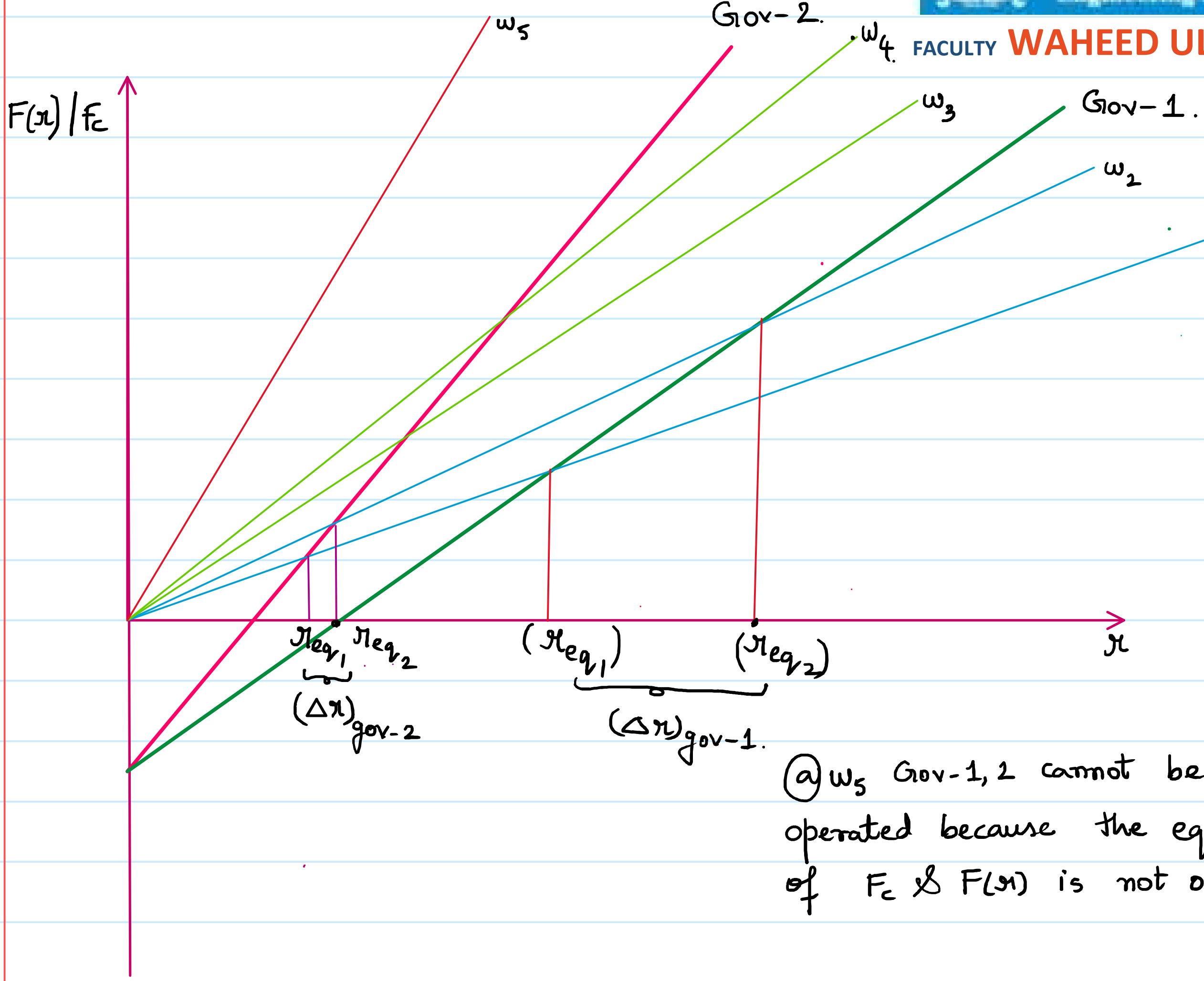

@ ω_5 Gov-1, 2 cannot be operated because the equilibrium of F_c & $F(r)$ is not obtained.

Range of speed. Gov-1 $\longrightarrow$ ω_1 to ω_2. (Low speed governor)

Gov-2 $\longrightarrow$ ω_1 to ω_4 (High speed governor).

$$\boxed{\left(\frac{dF(r)}{dr}\right)_{gov-2} > \left(\frac{dF(r)}{dr}\right)_{gov-1}}$$

$$(\Delta r)_{gov-1} > (\Delta r)_{gov-2}$$

Gov-1 produced more change in radius when speed increases from ω_1 to ω_2 $\rightarrow$ Gov-1 produces more sleeve displacement.

Gov-1 is more sensitive then Gov-2.

Gov-2 is more stable then Gov-1

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11. Hartnell governors are specified by their C.F. curves

CF = 50 r – 2000 for governor 1] $\|^{le}$ line.

CF = 50 r – 1000 for governor 2

CF = 100 r – 1000 for governor 3

where CF in Newtons and r is in cm.

Identify the correct statements about them

1) Governor 1 is useful for very low speeds where as Governor 2 is useful at high speeds and 3 at still higher speeds. ✓

2) The minimum radius at which the governor 1 is active is greater than 40 cm where as it is greater than 20 cm for governor 2. ✓

3) In the range of 50 cm to 60 cm radii of rotation. The Governor 3 is less sensitive than governor 2. ✓

4) At any radius above 20 cm governor 3 has higher speed compared to governor 2 ✓

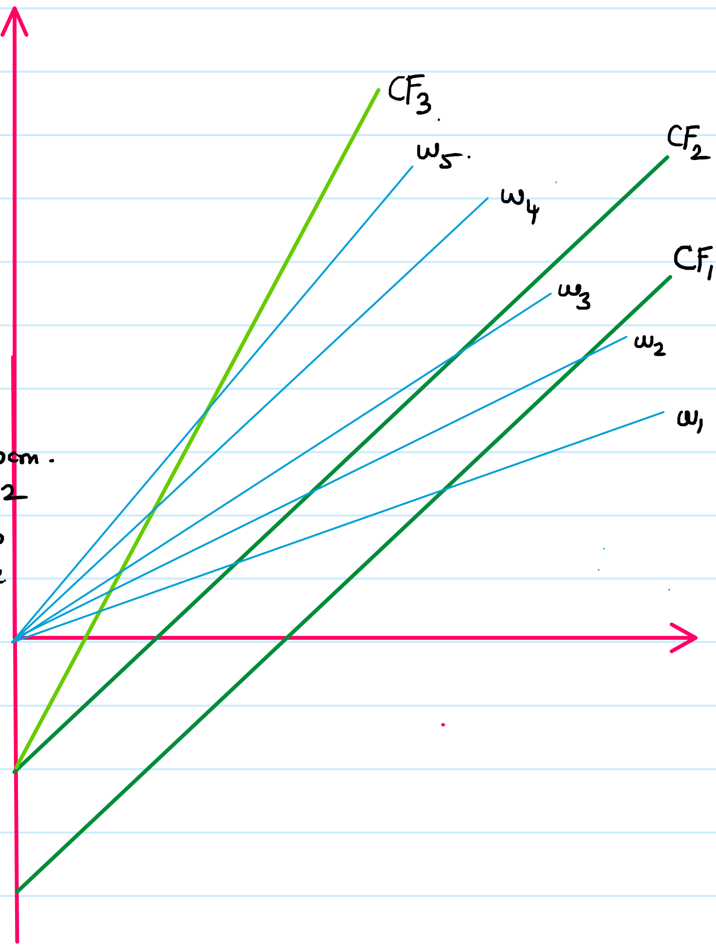
(a) all are correct

(b) 2, 3, 4 are correct

(c) 1, 3, 4 are correct

(d) 1, 2, 3 are correct

@ $r = 10$cm. Gov-3 is active., @ $r = 20$cm. Gov-3. is operating, Gov-2 is about to get active. Gov-1 is not active.



	Range of speed
Gov-1	ω_1 to ω_2
Gov-2	ω_1 to ω_3
Gov-3.	ω_1 to ω_5

$$CF_1 = 50r - 2000 \geq 0$$

$$(r_{min})_{gov-1} \geq \frac{2000}{50} = 40\text{cm.}$$

$\longrightarrow$ Gov-1 becomes active. $r \geq 40$cm.

$$CF_2 = 50r - 1000 \geq 0$$

$$(r_{min})_{gov-2} \geq 20\text{cm.}$$ $\longrightarrow$ Gov-2 become active $r \geq 20$cm.

$$CF_3 = 100r - 1000 \geq 0$$

$$(r_{min})_{gov-3} \geq 10\text{cm.}$$ $\longrightarrow$ Gov-3 becomes active $r \geq 10$cm.

$$\left(\frac{dF(r)}{dr}\right)_{gov-3} > \left(\frac{dF(r)}{dr}\right)_{gov-2,1}$$

Gov-3 is more stable and less sensitive then Gov-2 & Gov-1.

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GEARS:
Gears are the example of higher pair mechanism they are used to transfer constant velocity from one shaft to another shaft.

Gear: The wheel which is larger in size is known as gear. Due to its bigger size mass moment of inertia is more, therefore it is preferred as **driven (o/p)** element.

Pinion: The wheel which is smaller in size is known as pinion. Due to smaller size it's mass moment of inertia is less and hence preferred for **driver(i /p)** element.

Classification of Gears:

Gears may be classified according to the relative position of the axes of shafts to be connected. It may be

1. Parallel axes
   a. Spur Gear
   b. Helical Gear
   c. Double Helical / Herringbone Gear

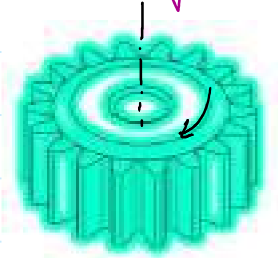
2. Intersecting axes
   a. Straight Bevel Gear
   b. Spiral Bevel Gear
   c. Zerol Bevel Gear
   d. Crown Gear

3. Neither parallel nor intersecting i.e. skew axes.
   a. Hypoid Gear
   b. Worm and Wheel
   c. Crossed Helical Gears

Gears for connecting parallel shafts

1. Spur gear → Teeth are cut on the cylinderical surface.

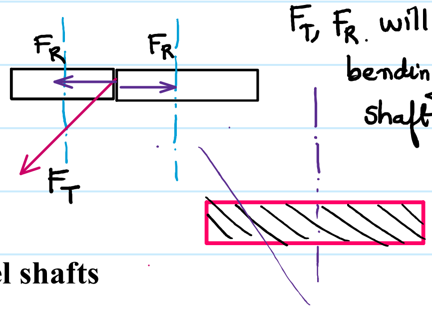
- Straight Spur gear is the simplest form of gears having teeth parallel to the gear axis.
- The contact of two teeth takes place over the entire width along a line parallel to the axes of rotation. → sudden type of contact b/w. teeth. cause shock loading.
- As gear rotate, the line of contact goes on shifting parallel to the shaft.
- Spur gears generate noise in high speed applications due to sudden contact over the entire face width between two meshing teeth. → shock loading.



- Spur gears are cheapest.

Use:

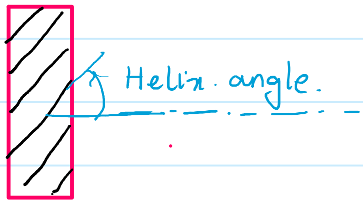
- Automobiles.



F_T, F_R will cause. bending of shaft.

F_R – Radial force.

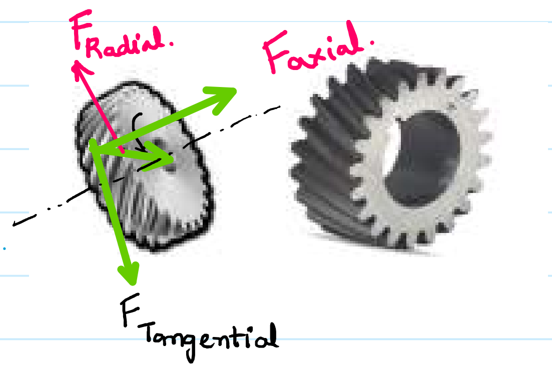
F_T – Tangential force.

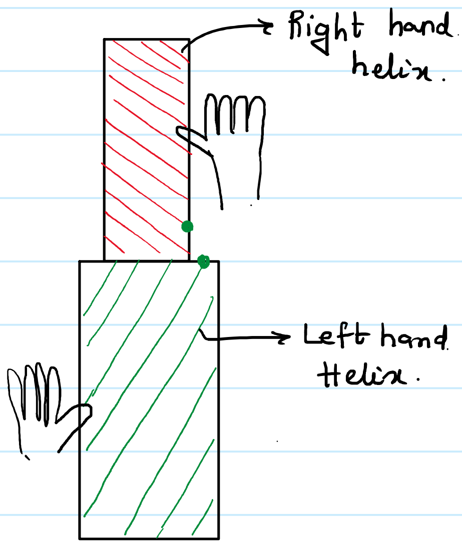


Gears for connecting parallel shafts

2. Helical Gears:

- In helical gears teeth are part of helix instead of straight across the gear parallel to the axis.
- The mating gears will have same helix angle but in opposite direction for proper mating.
- As the gear rotates, the contact shifts along the line of contact in involute helicoid across the teeth.





Gears for connecting parallel shafts

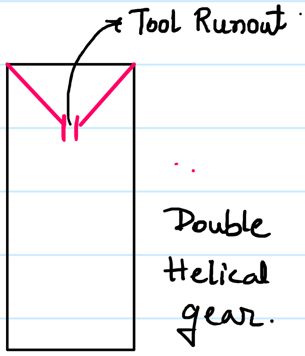
2. Helical Gears:

- There is gradual engagement at the beginning of any individual tooth, starting at the point on leading edge and progressing across the face of the gear as it rotates. → Reduces the effects of shock loading.
- This results in reduced dynamic effects and less noise.
- The inclined teeth develops thrust loads and bending couples, which are not present with spur gear.

Gears for connecting parallel shafts
3. Herringbone Gears:

- Single Helical gear will produce an axial thrust on the shaft bearing due to the inclination of the teeth it can be eliminated by employing double helical gears with two sets of teeth back to back, each set cut to opposite hand.

Herring bone.



Gears for connecting parallel shafts
3. Herringbone Gears:

- Herringbone gears are also known as Double Helical Gears.
- Herringbone gears are made of two helical gears with opposite helix angles, which can be up to 45 degrees.
- Helical gears use to obtain herringbone have **same module, number of teeth and pitch circle diameter,** but with teeth having **opposite hand of helix.**
- In Double helical gear there is a groove known as tool run out where as in Herringbone gear this groove is not present.

Use:

- High power applications such as ship drives and large turbines.

Gears for connecting parallel shafts

4. Rack and Pinion:

- In these gears the spur rack can be considered to be spur gear of infinite pitch radius with its axis of rotation placed at infinity parallel to that of pinion.
- The pinion rotates while the rack translates.

Gear of infinite Radius of Pitch circle.

Gears for Intersecting Shafts:

1. Straight Bevel Gears: – Teeth are cut on the conical surface

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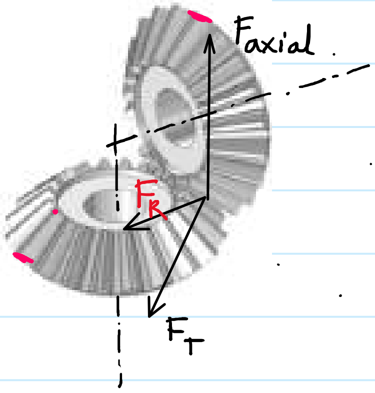
- Straight bevel gears are provided with straight teeth, radial to the point of intersection of the shaft axes and vary in cross section through the length inside generator of the cone.

C/s of teeth is uniform in case of spur, Helical gear.

- Straight Bevel Gears can be seen as modified version of straight spur gears in which teeth are made in conical direction instead of parallel to axis.

- Produce noise in high speed applications.

$F_{Tangential}$, F_{Radial}, F_{axial} are all present.



Gears for Intersecting Shafts:

2. Spiral Bevel Gears: → Teeth are cut on conical surface.

- Bevel gears are made with their teeth are inclined at an angle to face of the bevel and forms a circular arc.

- Spiral gears are also known as helical bevels.

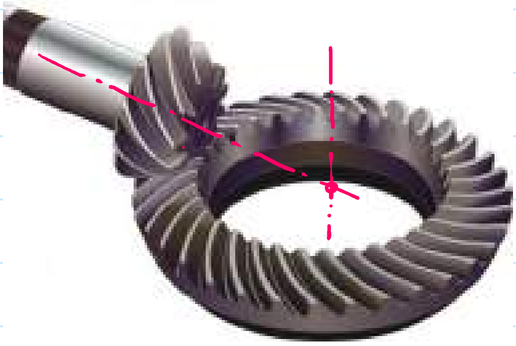
- It gives the same advantage of helical teeth.

- Spiral bevels are difficult to design and costly to manufacture.

- They have smooth teeth engagement which results in quite application even at high speeds.

- Spiral bevel gears have better strength so use for high power transmission applications.

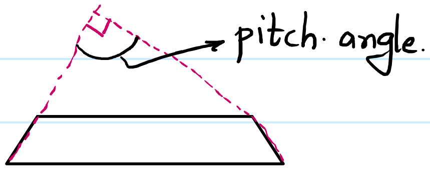
Note: When the axes are at right angles the larger gear is called a crown wheel and the smaller pinion.



Gears for Intersecting Shafts:
2. Spiral Bevel Gears – Types:

(a) Mitre gears: $\boxed{V.R. = 1.}$

- Two identical bevel gears mounted on shafts, which are intersecting at right angles. (Modifying the direction of angular velocity without changing magnitude)
- The pinion and gear have same dimensions namely addendum, dedendum, pitch circle diameter, number of teeth and module.
- The pinion and gear rotate at same speed.



Gears for Intersecting Shafts:
2. Spiral Bevel Gears – Types:

(b) Crown gear:

- When one of the gear has pitch angle 90° then that gear is called crown gear.
- Such gears are mounted on shafts which are intersecting at an angle that is more than 90°.

(c) Hypoid gears:

- Similar to spiral bevel gears that are mounted on shafts which are non-parallel, non-intersecting.
- Hypoid gears are based upon pitch surfaces which are hyperboloids of revolution.

Use:

- Automobile differentials.

Gears for Intersecting Shafts:

2. Spiral Bevel Gears – Types:

(d) Zerol gear:

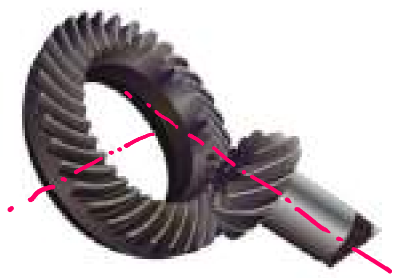
- Spiral bevel gears with zero spiral angle.
- These gears theoretically give more gradual contact and a slightly larger contact ratio.

NOTE: Bevel gears are always designed in pairs.

Gears for Skew Shafts:

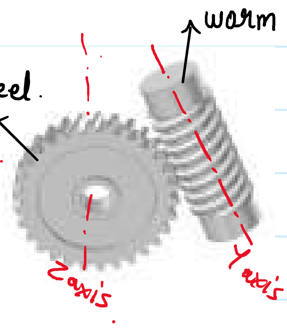
1. Hypoid Gears:

- The Hypoid Gears are made of the frusta of hyperboloids of revolution.
- Two matching hypoid gears are made by revolving the same line of contact, these gears are not interchangeable. → Mfg in pairs.
- Similar to bevel gears except that the shafts are offset and non-intersecting.



Gears for Skew Shafts:
2. Worm Gears:

- A special form of skew gears which has line contact is the worm and wheel pair.
- Usually, though not necessarily, the axes are at right angles.
- Teeth on worm gear are cut continuously like the threads on a screw.
- Worm resembles a screw.
- Direction of rotation of the worm gear (or worm wheel) depends upon the direction of rotation of the worm.



Reduction Ratio of 20:1 to 100:1 can obtained in single stage

Advantages of Worm gear drives:
(i) High speed reduction such as 100:1 can be obtained with a single pair of worm gears.
(ii) Compact with small overall dimensions.
(iii) Smooth and silent operation.

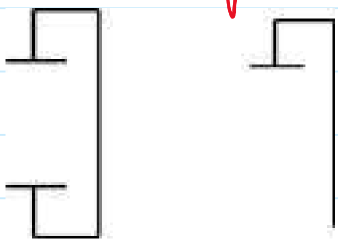
$\eta \leq 50\%$

4. Self-locking provision can be made where the motion is transmitted only from worm to worm wheel, this is advantageous in applications like cranes and lifting devices.

Drawbacks of Worm gear drives:
(i) Efficiency is low compared to other gear drives.
(ii) Worm wheel is generally made of phosphor bronze, which increases the cost.

Ring gear or Annulus: The gear which is having teeth in inner side is known as ring gear and annulus.

Representation of Internal gear.



Performance Parmeter.

→ Gear Ratio - G = $\dfrac{\text{No of teeth on the wheel (driven element)}}{\text{No of teeth on Pinion(driving element)}}$ $G > 1$.

$$G = \frac{\omega_{pinion}}{\omega_{Gear}} > 1$$

→ Velocity Ratio $V.R/\alpha = \dfrac{\text{No. of teeth on driving element}}{\text{No. of teeth on driven element}}$

$$\alpha < 1.$$

$$\boxed{G = \frac{1}{\alpha}}$$

→ Train value.

$$T.V. = \frac{\text{Product of No. of teeth on driving element}}{\text{Product of No. of teeth on driven element}}$$

→ Speed Ratio $= \dfrac{1}{\text{Train Value}}$.

Gear Terminology:

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Important Circle:

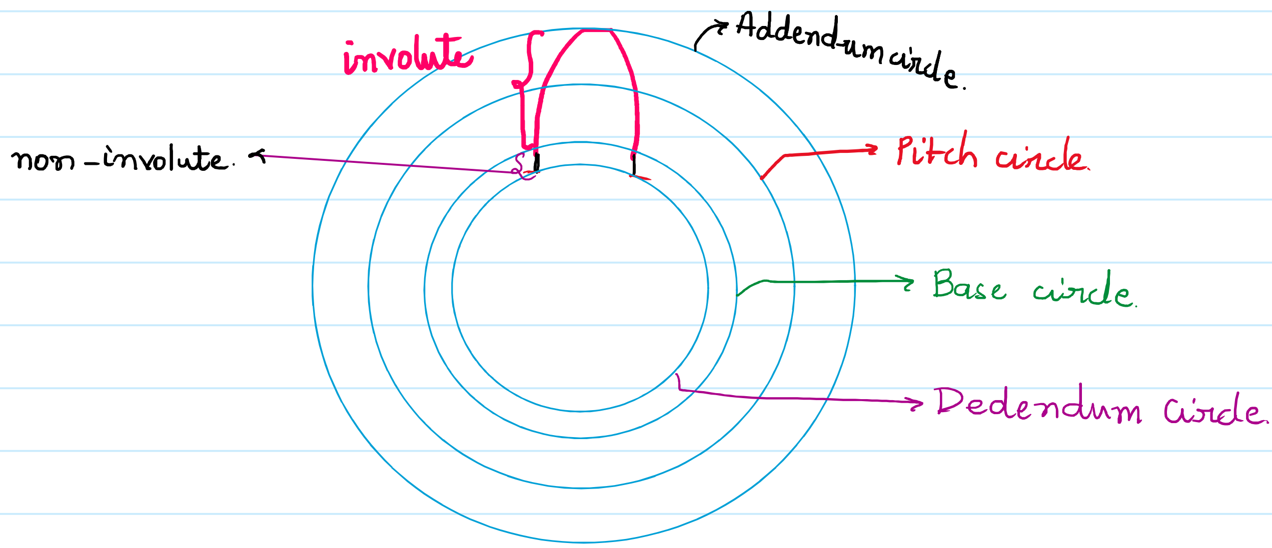
(a)**Pitch Circle**: It denotes the size of the gears, all the important directions such as tooth thickness, tooth space, arc of contact etc. are measured on the pitch circle circumference. Pitch circle corresponds to Pure Rolling action.

- It is an imaginary circle i.e., it's radius can be changed.
- Two gears which are in mesh can be represented by their pitch circle and the point of the contact of the pitch circle is known as pitch point.

(b) **Base Circle**: It is a real circle i.e., it's radius cannot be changed. It is the circle from where the involute profile begins.

(c) **Addendum Circle**: The circle which passes through top of the gear teeth is known as addendum circle.

(d) **Dedendum** Circle: Circle from where gear tooth begins.



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Units of Gears:

(a) Circular Pitch (p_c): It is the distance between two similar points an adjacent tooth measured along pitch circle circumference.

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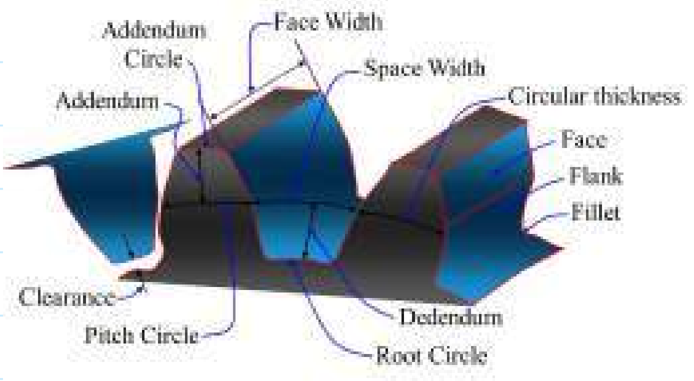
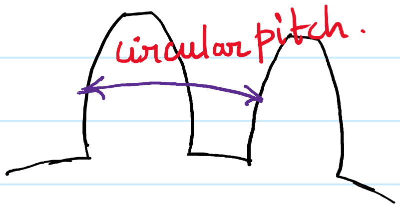


$$p_c = \pi m. \quad = \frac{\pi D}{T}.$$



module – $m = \frac{\text{Diameter of pitch circle}}{\text{No. of teeth}}$.

module signifies the thickness of teeth.

Diametrical pitch – $p_D = \frac{T}{D}$.

No. of teeth per inch of diameter.

F.P.S.

Product of Diametrical Pitch and circular pitch.

$$p_D \times p_c = \frac{T}{D} \cdot \frac{\pi D}{T} = \pi.$$

Product of Diametrical pitch and Module.

$$p_D \times m = T/D \cdot \times D/T = 1.$$

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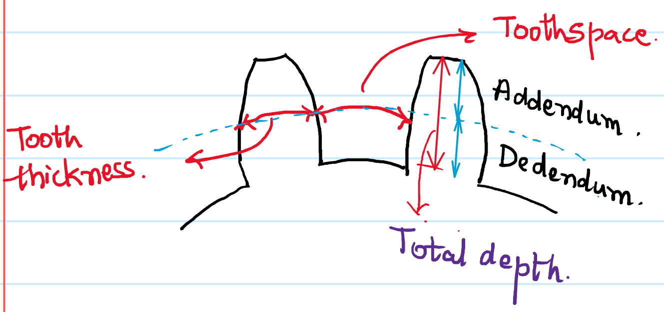
Important Dimensions:

Set (A): Dimensions which are visible in a single gear (without meshing).

1. **Tooth thickness**: the thickness of the tooth measured along pitch circle circumference.

2. **Tooth Space**: The distance between adjacent tooth is known as tooth space, during meshing tooth thickness of the gear will entre in to tooth space of pinion and vice-versa.

3. **Dedendum**: The radial distance between pitch circle and dedendum circle is known as addendum.



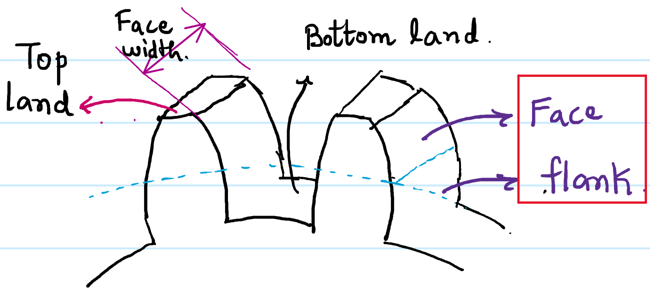
circular pitch = (Tooth space + Tooth thickness)

Total depth = Addendum + Dedendum.

Dedendum = 1.125 m.

4. **Addendum** is the radial height above the pitch circle

	Addendum.
Full depth.	1 module.
Stub.	(0.85 to 0.95) module.



Face - It is the portion of tooth above the pitch circle along the axis of gear.

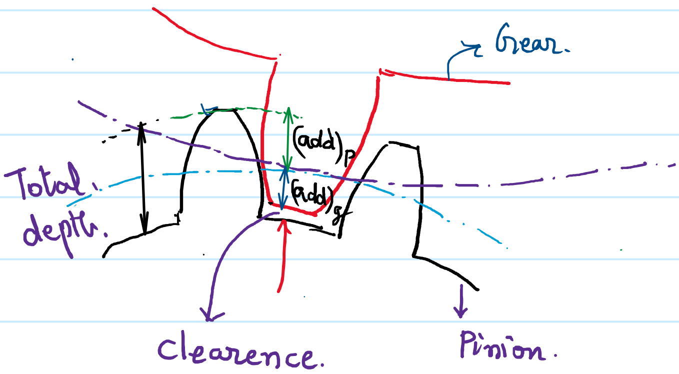
Flank - It is the portion of tooth below the pitch circle along the axis of gear.

Set (B):

01. **Working Depth** : Summation (addition) of addendum of gear and addendum of pinion is known as working depth.

02. **Clearance**: The distance between addendum circle of gear and dedendum circle of pinion is known as clearance.

- If clearance is absent then it will result in **interference.**



Working depth $= (add)_{pinion} + (add)_{gear}$

$(add)_p$ – addendum of pinion

$(add)_g$ – addendum of gear

$$\text{clearence} = \text{Total depth} - \text{Working depth}$$

$$= (\text{Addendum} + \text{Dedendum})_{pinion} - \left((\text{Addendum})_{pinion} + (\text{Addendum})_{gear}\right)$$

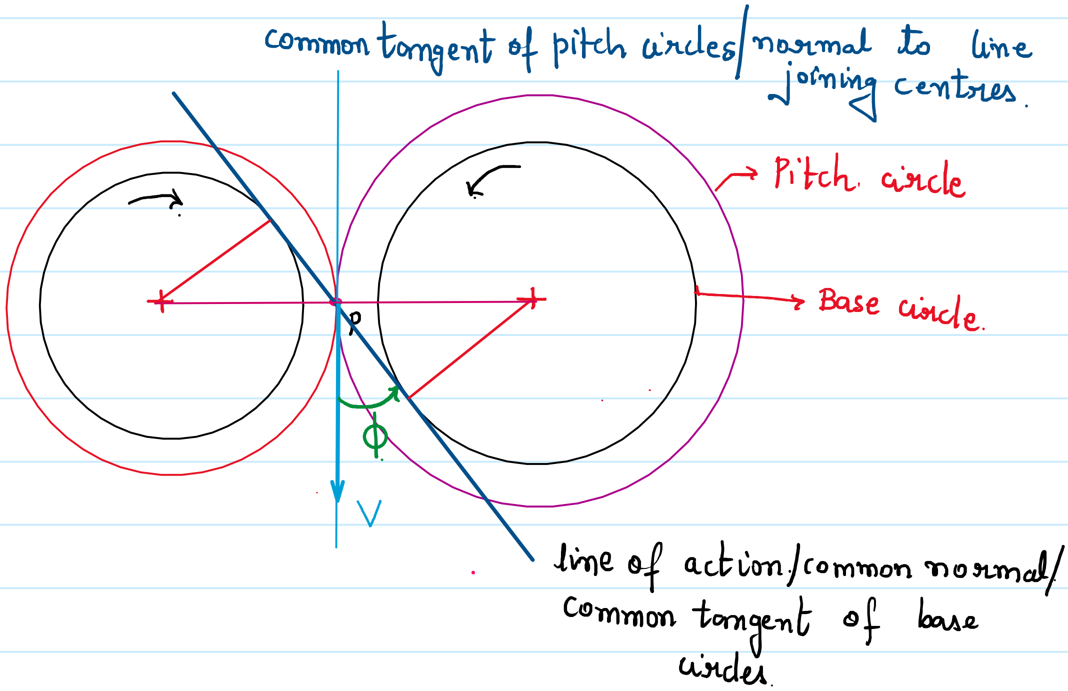
$$= (\text{Dedendum})_{pinion} - (\text{Addendum})_{gear}$$

3. **Backlash**:

The amount by which the tooth space of a gear is larger than tooth thickness of mating element is known as backlash.

- If backlash is absent then **rubbing** will take place in both the surface which will result in **more friction, more wear** , due to rubbing there will be more **heat generation** and as thermal expansion is prevented as no space is available it will further result in **thermal stress and strain concentration** due to which the tooth become week.
- If backlash increases there will be more noise and more vibrations.

1. **Pressure Angle** (f):
   It is the angle between **common normal** and **velocity vector** at pitch point **rotated** in a direction similar to **driven element**.



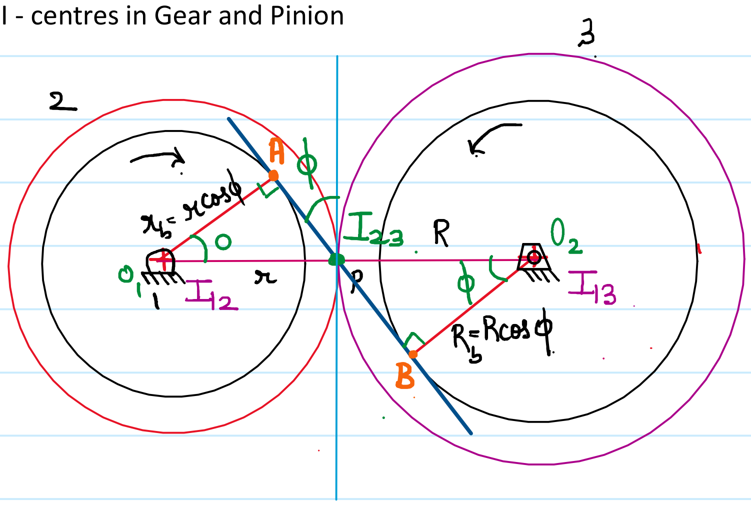
P– Pitch point.

Pressure Angle - It is the angle subtended between the common normal/common tangent of base circles and normal to line joining centres/common tangent of pitch circle measured at the pitch point.

- When a straight line rolls without slipping on a circle locus of any point on it will be **involute**, the circle is known as base circle and the line which generates involute is known as **common normal**.
- **For constant velocity ratio there must be meshing between conjugate profile therefore the common normal should be same St. line.**
- The entire action during meshing such as **beginning of engagement, ending of engagement, force transmission** or everything will take place along the **common normal**, therefore it is also called as line of action.
- The angle between common normal and common tangent to both the pitch point circles at pitch point is known as pressure angle.

Law of Gearing

Locating I - centres in Gear and Pinion



I_{23} must lie on common normal and colinear with I_{12} & I_{13}.

$O_1P - r -$ Pitch circle radius of Pinion.

$O_2P - R -$ Pitch circle radius of gear.

$AP = r\sin\phi$

$BP = R\sin\phi$

$$V_{I_{23}} = I_{12}I_{23}\cdot\omega_2 = I_{13}\cdot I_{23}\cdot\omega_3$$

$$\frac{\omega_2}{\omega_3} = \frac{I_{13}\cdot I_{23}}{I_{12}\cdot I_{23}} = \frac{R}{r} \neq \text{constant}.$$

$\Delta^{le}\ I_{12}\cdot A\, I_{23}$ and $\Delta^{le}\ I_{13}\cdot B\cdot I_{23}$ are similar Δ^{les}.

$$\frac{I_{13}\cdot I_{23}}{I_{12}\cdot I_{23}} = \frac{I_{13}\cdot B}{I_{12}\cdot A} = \frac{R_b}{r_b} = \frac{R\cos\phi}{r\cos\phi}$$

$$\frac{\omega_2}{\omega_3} = \frac{R_b}{r_b} = \frac{R\cos\phi}{r\cos\phi} = \text{constant}.$$

Law of Gearing:

- The ratio of first kinematic ratio .i.e., angular velocity must be constant.
- **Since the pitch circle is an imaginary circle that is radius can be changed therefore from here we cannot conclude that angular velocity will be constant.**
- The pitch point 'P' will be fixed point and it will divide the common normal in a constant ratio.
- All the elements satisfying law of gearing are known as gears presence of teeth is not a necessary condition.
- To transfer the relative motion at constant ratio the common normal must always pass through the pitch point.

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Case(i)

Pinion –

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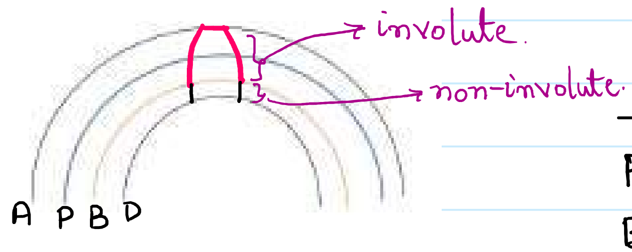


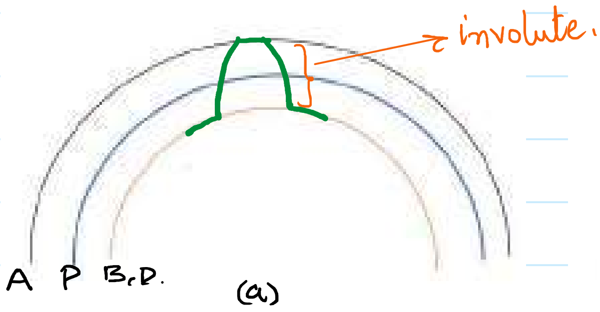
A – Addendum circle.

P – Pitch circle

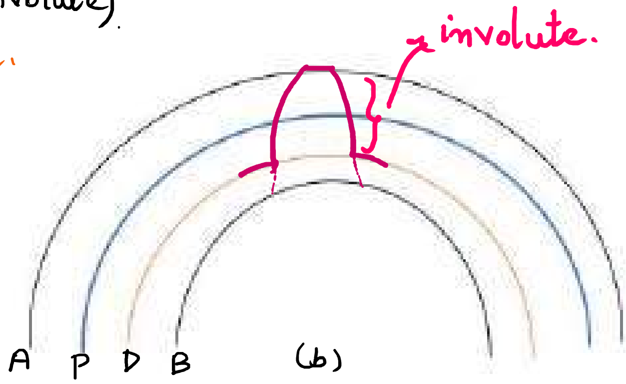
B – Base circle

D – Dedendum circle.

Gear.– $R_{Dedendum} \geq R_{base}$ → (Profile of Gear tooth is completely involute).



$R_{Base} = R_{Dedendum}$.



$R_{dedendum} > R_{base}$.

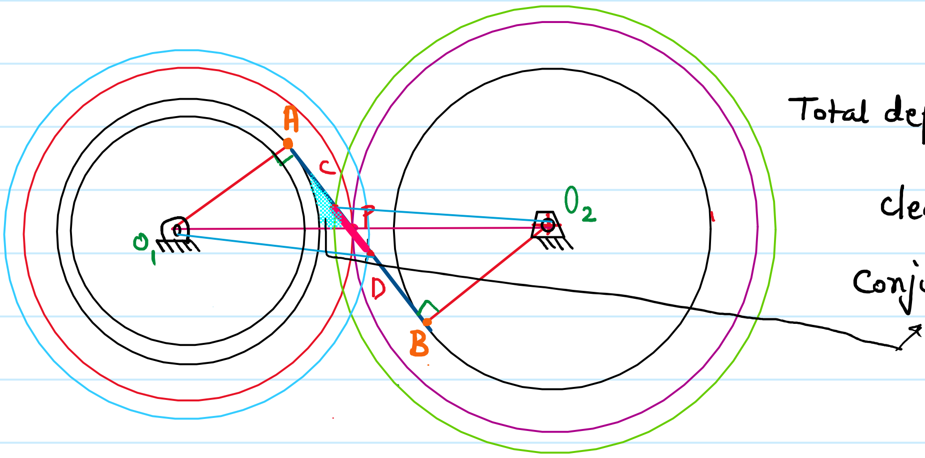
A, B are called points of tangency.

$O_1P - r$ $\quad$ $O_1A - r\cos\phi$

$O_2P - R$ $\quad$ $O_2B - R\cos\phi$

$AP - r\sin\phi$ $\quad$ $BP - R\sin\phi$

case(ii)



Total depth > Working depth

clearence ≠ 0

Conjugate action occurs.

@ point C, D addendum of gear, pinion cut the line of actions.

AB – Max. lenght of line of action / Path of contact

CD – Actual lenght of line of action / Path of contact.

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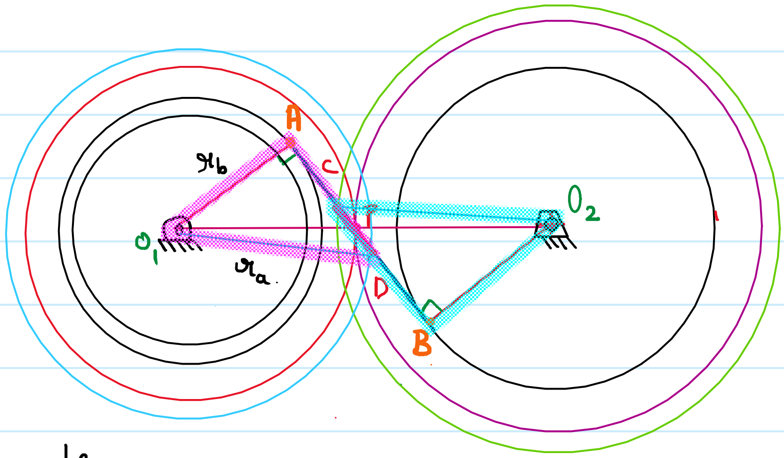

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In Δ^{le} O_1AD

$$AD = \sqrt{O_1D^2 - O_1A^2} = \sqrt{r_A^2 - r_b^2} = \sqrt{r_A^2 - (r\cos\phi)^2}$$

In Δ^{le} O_2BC

$$BC = \sqrt{O_2C^2 - O_2B^2} = \sqrt{R_A^2 - R_b^2} = \sqrt{R_A^2 - (R\cos\phi)^2}$$

$$PD = AD - AP = \sqrt{r_A^2 - r_b^2} - (r\sin\phi)$$

$$CP = BC - BD = \sqrt{R_A^2 - R_b^2} - (R\sin\phi)$$

Actual lenght of Path of contact $= CP + PD$

$$= \underbrace{\sqrt{R_A^2 - R_b^2} - (R\sin\phi)}_{\text{Actual lenght of Path of Approach}} + \underbrace{\sqrt{r_A^2 - r_b^2} - (r\sin\phi)}_{\text{Actual lenght of Path of Recess}}$$

Max. lenght of Path of Contact $= AP + BP$.

$$= \underbrace{r\sin\phi}_{\text{Max. lenght of Path of Approach}} + \underbrace{R\sin\phi}_{\text{Max. lenght of Path of recess}}$$

Observations

Max. Length of Path of Approach is dependent on Pinion dimensions.
Actual Length of Path of Approach is dependent on Gear dimensions.
Max. length of Path of recess is dependent on Gear dimensions.
Actual length of Path of recess is dependent on Pinion dimensions.

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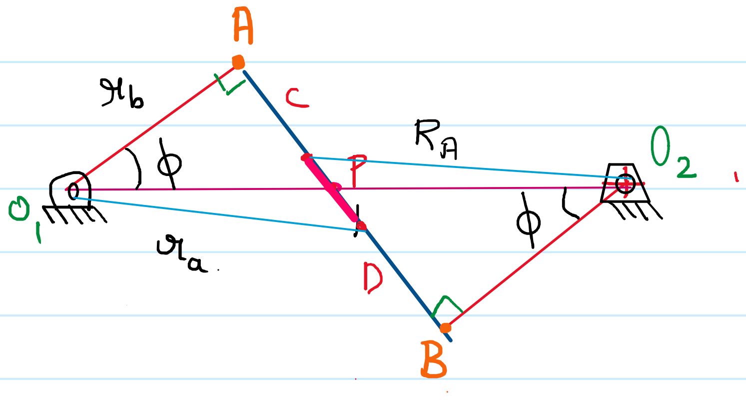
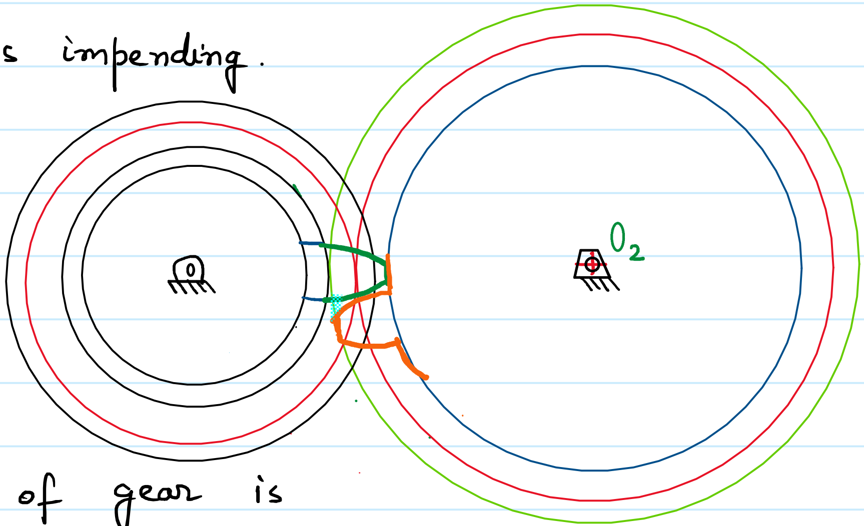


case (iii)

Interference is impending.

clearence ≠ o.

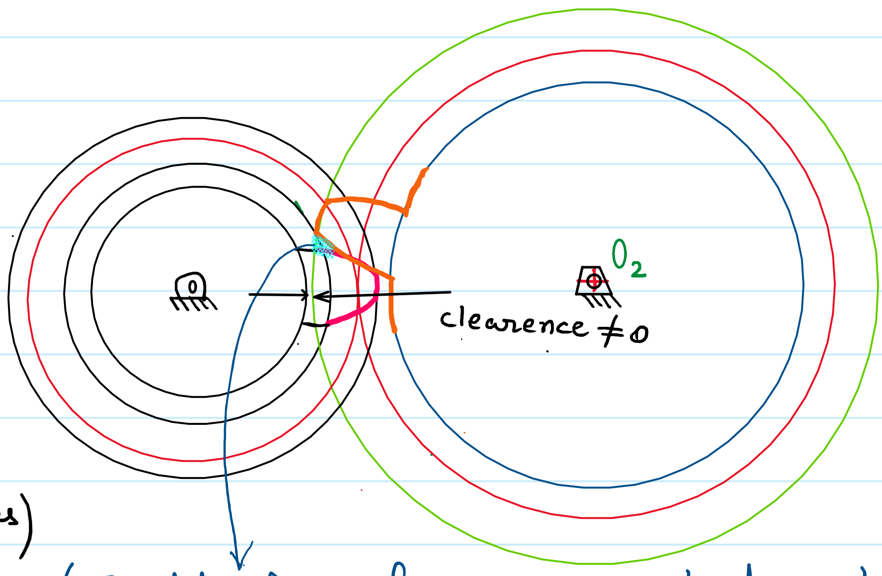


Involute face of gear is tending to get in action with non-involute flank of pinion.

Case (iv)

clearence ≠ o.

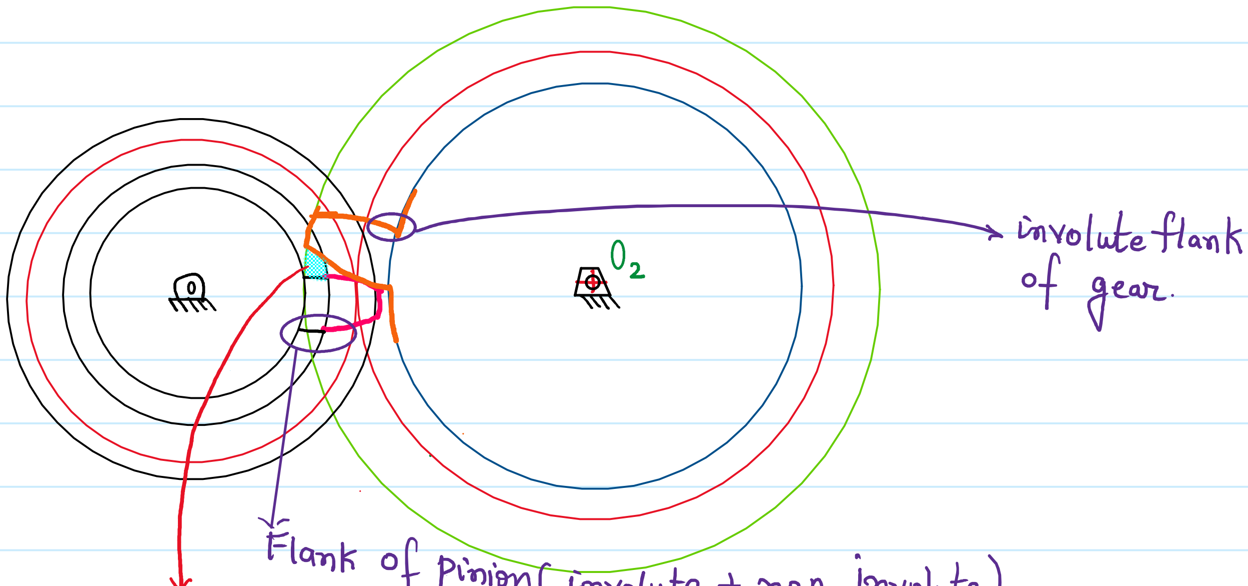
Total depth > Working depth



(Interference occurs)

Non-conjugate action - (Involute face of gear is tending to get in action with non-involute flank of pinion) V.R = constant.

Case (V)



Involute face of gear is about to get in action with non-involute flank of pinion. (non-conjugate action, V.R $\neq$ constant)

Clearance = 0

Total depth = Working depth.

Interference occurs.

INTERFERENCE:

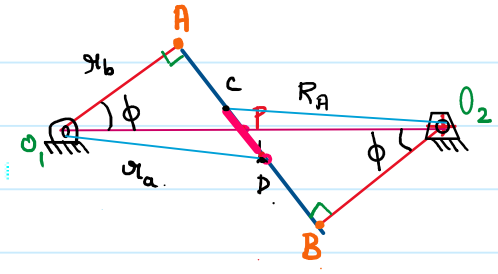
- Whenever non-conjugate meshing (meshing of involute profile with non-involute profile) will take place gears will join in each other and they will not be able to transfer the velocity ratio.
- Interference should be avoided.

Necessary condition for Interference:

- The presence of **non-involute profile is a necessary condition** for interference but, it is **not the sufficient one**.

Sufficient and necessary condition for Interference:

- If addendum of gear penetrate into the base circle of pinion or crosses the point of tangent to the base circle, it will result in interference.
- If the clearance is present interference may or may not occur, but if clearance is absent interference will take place certainly.
- If working depth is equal to full depth interference will take place and if working depth is less than full depth than interference may or may not occur.



- The **actual path of approach** will be decided by **gear dimensions** since gear will begin the engagement, whereas **pinion** will **end the engagement** so **actual path of recess** will be decided **by pinion.**

- The **maximum path of approach** will be decided by **pinion** and **maximum path of recess** will be decided by **gears**.

- The point of beginning of engagement 'C' has more possibility to cross point of tangency of the base circle of pinion (point A) therefore there will be more chances of interference at the **flank of pinion** than that of gear.

- Clearance is measured on the pinion side.

- If two wheels are in mesh having same size of addendum than the wheel which is driven will interfere first.

- In case of gear and pinion **gear will always interference first**.

- If two wheels are in mesh having different size of addendum than the **wheel having larger addendum will interfere** first.

- Interference occurs at the beginning of engagement.

Method to avoid interference

1. Increasing Centre distance
2. Undercutting
3. Stubbing
4. Using cycloidal teeth
5. By selecting the no. of teeth properly on pinion and Gear

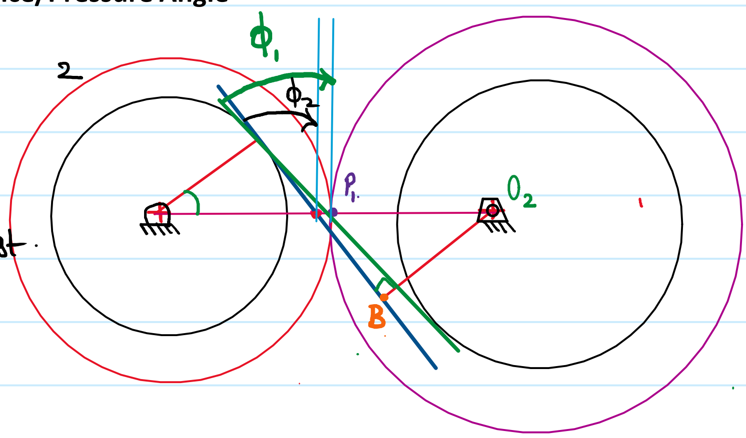
1. Increasing Centre distance/Pressure Angle

$r_b = \text{const.}$

$r_b = r\cos\phi$

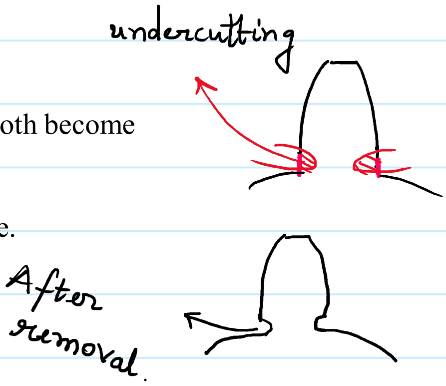
$r \uparrow \ \cos\phi \downarrow \quad r_b = \text{const.}$

$\cos\phi \downarrow \ \phi \uparrow$.

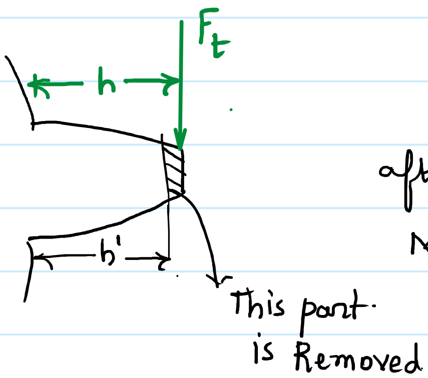


2. Under cutting:

- The process of removal of non-involute portion from the flank of pinion is known as under cutting.
- It may take place at the time of manufacturing.
- It may take place as a result of interference.
- It may done purposely to avoid the interference.
- Undercutting always result in stress concentration due to which the tooth become weaker and chances of failure will increase
- Therefore, undercutting is least preferred process to avoid interference.



3. Stubbing



$M = F_t \times h$.

after stubbing h decreases.

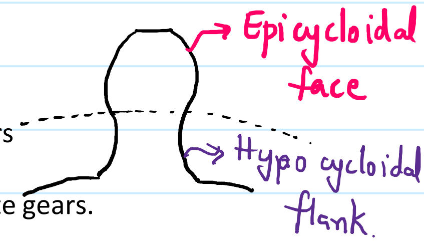
M decreases

$\sigma \propto M$

$M \downarrow$ $\sigma \downarrow$ strenght $\uparrow$.

Modification of addendum.

4. Cycloidal teeth



1. Exact centre distance is required for cycloidal teeth
2. Tooth will be consisting of epi cycloidal face and Hypocycloidal flank.
3. Epi cycloidal face is in action with the hypo cycloidal flank of mating gears
4. This gives conjugate action.
5. Pressure angle is constant throughout the engagement in case of involute gears.
6. Pressure angle varies in case of cycloidal teeth
7. It is maximum at the beginning of engagement, zero at pitch point and maximum at the End of engagement.
8. Constant velocity ratio is not maintained during transmission of relative motion

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5. Selecting the number of teeth properly on Gear and Pinion

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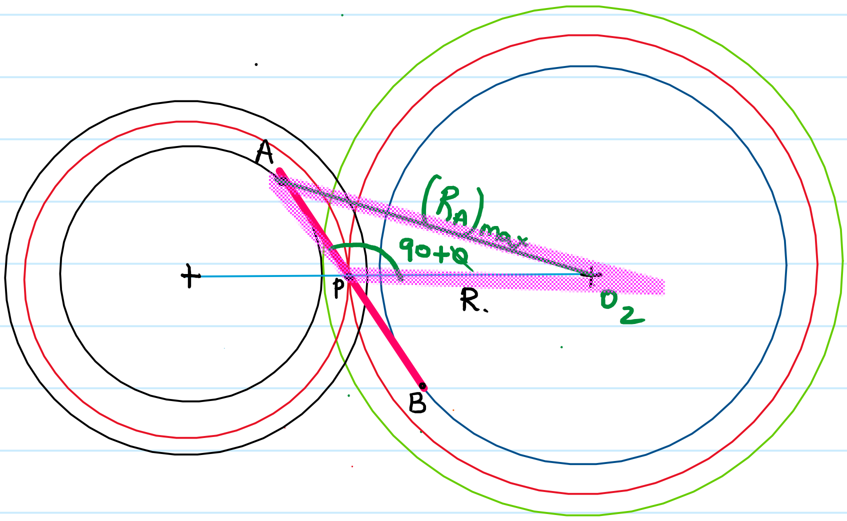


In $\Delta^{le}\, O_2PA$.

$$O_2A^2 = O_2P^2 + AP^2 - 2\cdot O_2P\cdot AP\cdot \cos\angle O_2PA$$

$$(R_A)^2_{max} = R^2 + (r\sin\phi)^2 - 2\cdot R\cdot r\sin\phi\cdot \cos(90+\phi)$$

$$(R_A)^2_{max} = R^2 + r^2\sin^2\phi - 2Rr\sin\phi\cdot(-\sin\phi)$$

$$(R_A)^2_{max} = R^2 + r^2\sin^2\phi + 2Rr\sin^2\phi$$

f_w – addendum coefficient of wheel.

Max. addendum $\leq (R_A)_{max} - R$.

$$(add)_{max} \leq \sqrt{R^2 + r^2\sin^2\phi + 2Rr\sin^2\phi} - R$$

$$(add)_{max} \leq R\left[\sqrt{1 + \left(\frac{r}{R}\right)^2\sin^2\phi + 2\left(\frac{r}{R}\right)\sin^2\phi} - 1\right]$$

$$\frac{r}{R} = d = \frac{1}{G} = \frac{Z_p}{Z_G}$$

$$Z_p = d\cdot Z_G$$

$$f_w \times m \leq \frac{m\cdot Z_G}{2}\left[\sqrt{1 + d(d+2)\sin^2\phi} - 1\right]$$

$$Z_G \geq \frac{2 f_w}{\sqrt{1+d(d+2)\sin^2\phi} - 1}$$

$$Z_p \geq \frac{2 f_w \times d}{\sqrt{1+d(d+2)\sin^2\phi} - 1}$$

Minimum no. of teeth on Pinion.

For Rack and Pinion Mechanism.

$$\lambda = \frac{r}{R} \quad \text{for Rack.} \quad R \to \infty \quad \lambda \to 0$$

$$z_p \geq \frac{2 \cdot f_w \times \lambda}{\sqrt{1 + \lambda(\lambda + 2)\sin^2\phi} - 1}$$

if $\lambda \to 0$ $\quad z_p \to \frac{0}{0}$

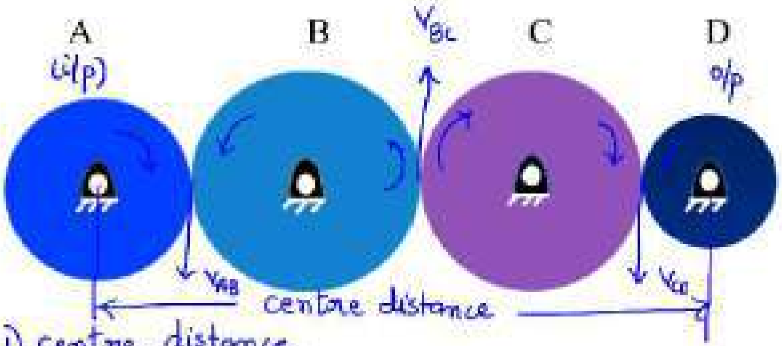
$$z_p = \frac{\frac{d}{d\lambda}(2 \cdot f_w \times \lambda)}{\frac{d}{d\lambda}\left(\sqrt{1 + \lambda(\lambda + 2)\sin^2\phi} - 1\right)}$$

$$z_p = \frac{2}{\sin^2\phi}$$

GEAR TRAIN:

- Gear Train
  - D.O.F = 1 — When two gears are mounted on same shaft
    - Reverted gear train (R.G.T)
    - Compound gear train (C.G.T)
  - D.O.F = 2 → Epicyclic Gear Train.
  - All gears are mounted on same plane and on different shaft. (S.G.T) → Simple Gear Train.

01. Simple Gear Train (S.G.T):



module
$m_A = m_B = m_C = m_D$

Pressure Angle
$\phi_A = \phi_B = \phi_C = \phi_D$

1) centre distance

2) Train value

for Pure Rolling
$V_{AB} = V_{BC} = V_{CD}$

Centre distance:

$$= r_A + 2r_B + 2r_C + r_D$$

$$= \frac{m_A \cdot Z_A}{2} + m_B \cdot Z_B + m_C \cdot Z_C + \frac{m_D \cdot Z_D}{2}$$

$V = r\omega$

$V = \frac{m \cdot Z}{2} \omega$

$\boxed{Z \propto 1/\omega}$

Train value
T.V

$$= \frac{\omega_{o/p}}{\omega_{i/p}} = \frac{\omega_D}{\omega_A} = \left(-\frac{\omega_B}{\omega_A}\right)\left(-\frac{\omega_C}{\omega_B}\right)\cdot\left(-\frac{\omega_D}{\omega_C}\right)$$

$$= \left(-\frac{Z_A}{Z_B}\right)\left(-\frac{Z_B}{Z_C}\right)\cdot\left(-\frac{Z_C}{Z_D}\right)$$

$$\frac{\omega_D}{\omega_A} = \frac{-Z_A}{Z_D}$$

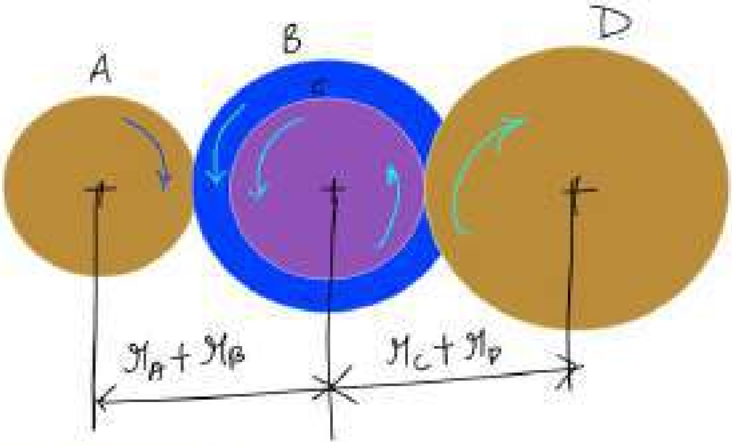
- Idler gears: The gear whose teeth does not appears in the final expression of train value are known as idler gear.
- They are use to fill the space between driven and driven element.
- Idler cannot effect the magnitude of train value but they certainly effect the direction of output element with the respect to input.

Gear B and C are called idler gears.

1. If odd no. of idlers are used direction of i/p and o/p gear is same.
2. In case of even no. of idler gear the direction of i/p and o/p is different.

02. Compound Gear Train (C.G.T):

- In this alternate gears are called driver – driven – driver – driven.



$\phi_A = \phi_B$

$m_A = m_B$

$\phi_C = \phi_D$

$m_C = m_D$

$m_B = m_C$ ✓

$m_B \neq m_C$ ✓

Centre distance:

$$= r_A + r_B + r_C + r_D$$

$$= \frac{m_A \cdot Z_A}{2} + \frac{m_B \cdot Z_B}{2} + \frac{m_C \cdot Z_C}{2} + \frac{m_D \cdot Z_D}{2}$$

$m_A = m_B$, $m_B = m_C$ / $m_B \neq m_C$

$m_C = m_D$,

Train value:

$$T.V. = \frac{\omega_{o/p}}{\omega_{i/p}} = \frac{\omega_D}{\omega_A} = \left(\frac{-\omega_B}{\omega_A}\right)\left(\frac{\omega_C}{\omega_B}\right)\left(\frac{-\omega_D}{\omega_C}\right)$$

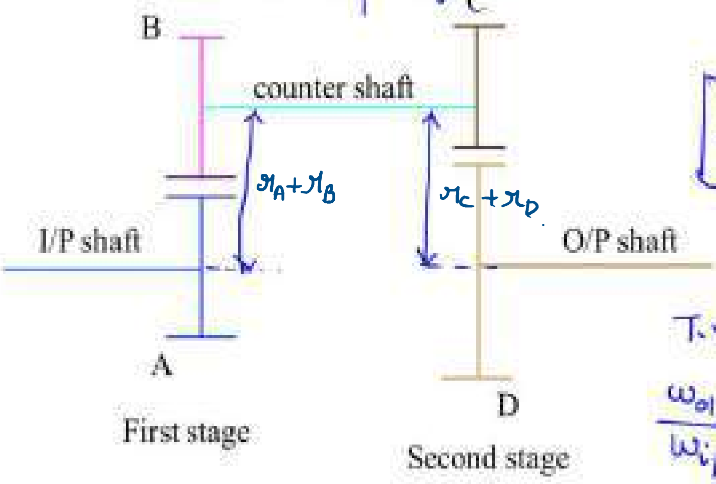
$$\frac{\omega_C}{\omega_B} \to 1$$

$$\boxed{\frac{\omega_D}{\omega_A} = \frac{Z_A \cdot Z_C}{Z_B \cdot Z_D}}$$

03. Reverted Gear Train (R.G.T): If the input and output shafts are co-axial then we use reverted gear train.

➢ EX. Gear box and Watches etc.

B - C are compound gear



Centre distance.

$$r_A + r_B = r_C + r_D$$

$$\boxed{\frac{m_A \cdot Z_A}{2} + \frac{m_B \cdot Z_B}{2} = \frac{m_C Z_C}{2} + \frac{m_D \cdot Z_D}{2}}$$

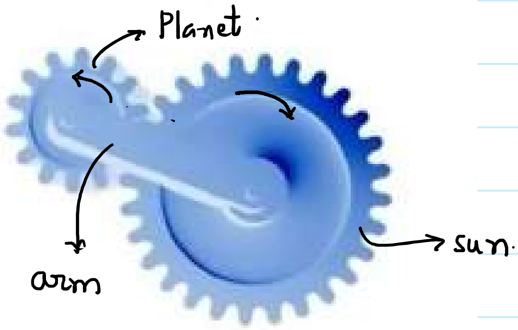
$m_A = m_B$
$\phi_A = \phi_B$
$m_C = m_D$
$\phi_C = \phi_D$
$m_B = m_C$ ✓
$m_B \neq m_C$ ✓

T.V.

$$\frac{\omega_{o/p}}{\omega_{i/p}} = \frac{\omega_D}{\omega_A} = \left(\frac{-\omega_B}{\omega_A}\right)\left(\frac{\omega_C}{\omega_B}\right)\left(\frac{\omega_D}{\omega_C}\right)$$

$$\boxed{\frac{\omega_D}{\omega_A} = \frac{-Z_A}{Z_B} \cdot \frac{-Z_C}{Z_D}}$$

Epicyclic Gear Train



$$\omega_s \cdot Z_s = \omega_p \cdot Z_p$$

$$\omega_p = -\frac{Z_s}{Z_p} \cdot (+1)$$

Advantages:

- They are compact in size.
- Both static and Dynamic forces are balanced if multiple planets are used.
- High torque ratio or velocity ratio can be achieved.
- Bi-directional output can be obtained from a single unidirectional input. DOF = 2
- It is used in automobile transmission system

SNo.	Condition of motion	Arm	Sun Gear	Planet Gear
1.	Arm is fixed and sun rotated with (+1 rev)	0	+1	$-\frac{Z_s}{Z_p}$
2.	Arm is fixed and sun rotated $+x$ rev	0	$+x$	$-x \cdot \frac{Z_s}{Z_p}$
3.	Arm rotated by (+y)	+y	+y	+y
4.	Total	y	$x+y$	$y - x \cdot \frac{Z_s}{Z_p}$

Speed of Sun $N_s = x + y$ $\Rightarrow$ $N_s = x + N_{arm} \longrightarrow$ (A)

Speed of Planet $N_p = y - x \cdot \frac{Z_s}{Z_p}$ $\qquad N_p = N_{arm} - x \cdot \frac{Z_s}{Z_p} \longrightarrow$ (B)

$\left(\frac{A}{B}\right)$ $$\boxed{\frac{N_s - N_{arm}}{N_p - N_{arm}} = -\frac{Z_p}{Z_s}}$$

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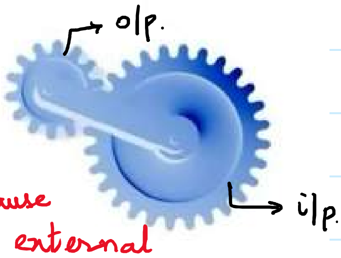
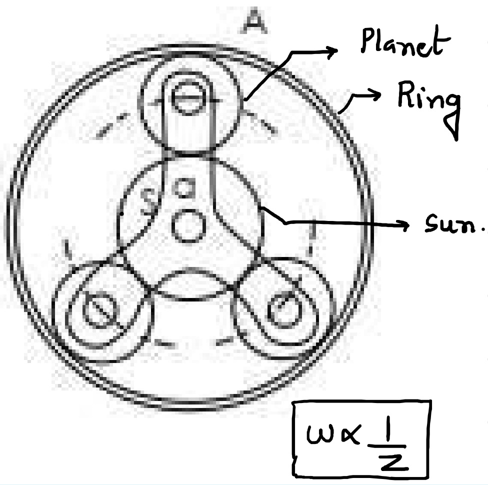
T.V.

$$\frac{\omega_{o/p}}{\omega_{i/p}} = \frac{N_p - N_{arm}}{N_S - N_{arm}} = \frac{-Z_S}{Z_P}$$

→ because of external gearing.



→ Sun – i/p. Ring – o/p.
Arm – i/p Planet – (coupler link)



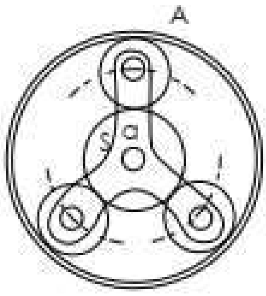
$\boxed{\omega \propto \frac{1}{Z}}$

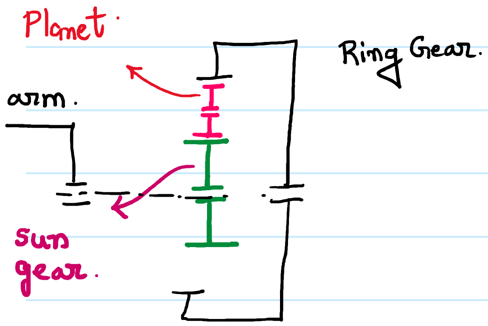
T.V.

$$\frac{\omega_{o/p}}{\omega_{i/p}} = \frac{\omega_R - \omega_{arm}}{\omega_S - \omega_{arm}} = \frac{\omega_P - \omega_{arm}}{\omega_S - \omega_{arm}} \cdot \frac{\omega_R - \omega_{arm}}{\omega_P - \omega_{arm}}$$

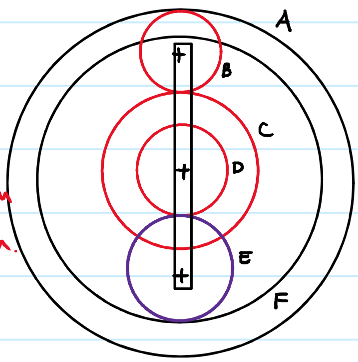
$$\frac{\omega_R - \omega_{arm}}{\omega_S - \omega_{arm}} = \left(\frac{-Z_S}{Z_P}\right)\left(+\frac{Z_P}{Z_R}\right)$$

$$\frac{\omega_R - \omega_{arm}}{\omega_S - \omega_{arm}} = \frac{-Z_S}{Z_R}.$$





→ A – i/p, F – o/p. C–D – compound gears.
Arm – i/p. $\omega_C = \omega_D$.



T.V.

$$\frac{\omega_{o/p}}{\omega_{i/p}} = \frac{\omega_F - \omega_{arm}}{\omega_A - \omega_{arm}} = \frac{\omega_B - \omega_{arm}}{\omega_A - \omega_{arm}} \cdot \frac{\omega_C - \omega_{arm}}{\omega_B - \omega_{arm}} \cdot \frac{\omega_D - \omega_{arm}}{\omega_C - \omega_{arm}} \cdot \frac{\omega_E - \omega_{arm}}{\omega_D - \omega_{arm}} \cdot \frac{\omega_F - \omega_{arm}}{\omega_E - \omega_{arm}}$$

1.

$$\frac{\omega_F - \omega_{arm}}{\omega_A - \omega_{arm}} = \left(+\frac{Z_A}{\cancel{Z_B}}\right)\left(-\frac{\cancel{Z_B}}{Z_C}\right)\left(-\frac{Z_D}{\cancel{Z_E}}\right)\left(+\frac{\cancel{Z_E}}{Z_F}\right)$$

Gear B and E are idler gears.

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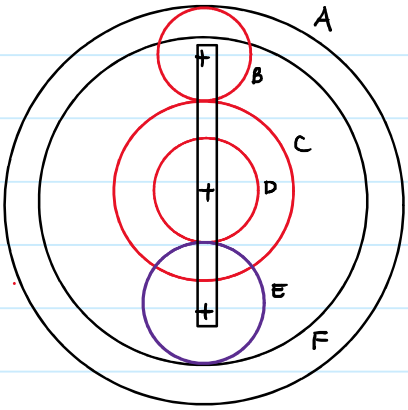

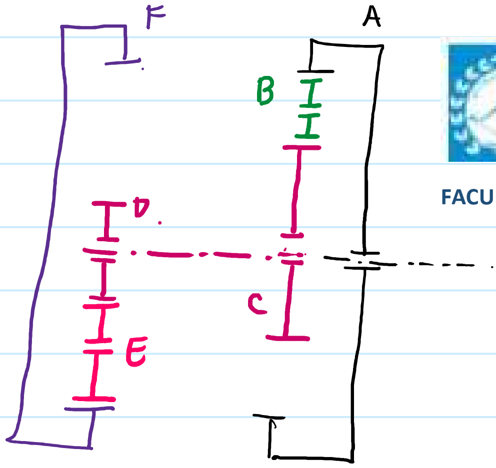

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Assumption.

1. Inertia of wheels and arms is ignored.

Torque Analysis.

$$\Sigma \text{Torque} = 0$$

$$T_{i/p} + T_{arm} + T_{o/p} = 0$$

$$T_s + T_{arm} + T_p = 0.$$

$$\Sigma \text{Power} = 0$$

$$P_{i/p} + P_{arm} + P_{o/p} = 0$$

$$T_s \cdot \omega_s + T_{arm} \cdot \omega_{arm} + T_p \cdot \omega_p = 0.$$

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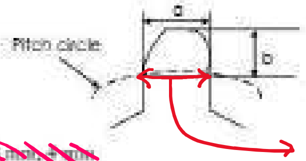
01. In gears, interference takes place when
(a) the tip of a tooth of a mating gear digs into the portion between base and root circles → Flank portion of pinion.
(b) gears do not move smoothly in the absence of lubrication → Result of interference
(c) pitch of the gear is not same
(d) gear teeth are undercut → Result of interference.

02. One tooth of a gear having 4 module and 32 teeth is shown in the figure.
Assume that the gear tooth and the corresponding tooth space make equal intercepts on the pitch circumference. The dimensions 'a' and 'b', respectively, are closest to (GATE– 08)



~~(a) 6.08 mm, 4 mm~~
(b) 6.48 mm, 4.2 mm
(c) 6.28 mm, 4.3 mm
(d) 6.28 mm, 4.1 mm

$m = 4\,mm \qquad Z = 32$

circular pitch = Tooth space + Tooth Thickness

tooth space = tooth thickness.

$\text{Tooth thickness} = \frac{\pi m}{2}$

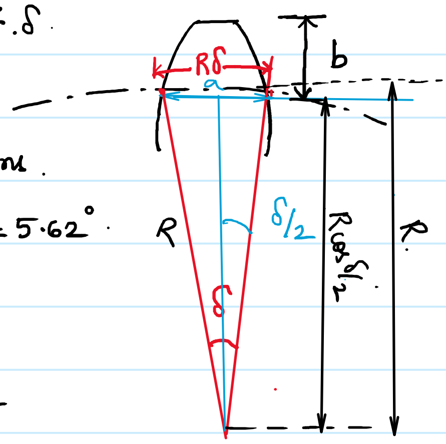
tooth thickness $= R\delta$.

$\frac{\pi m}{2} = \frac{m.Z}{2}.\delta$.

$\delta = \frac{\pi}{Z} = \frac{3.1414}{32}$

$\delta = 0.0981$ radians.

$= 0.0981 \times \frac{180}{\pi} = 5.62^\circ$.



$a = (R.\sin\delta/2) \times 2$

$= \frac{m.Z}{2} \sin\delta/2 . \times 2$

$= \frac{32\times4}{2} \times \sin\left(\frac{5.62}{2}\right) \times 2$

$= 6.28$

$b = add. + (R - R.\cos\delta/2)$

$= 4 + 64\left(1-\cos\left(\frac{5.62}{2}\right)\right)$

$= 4.077 \simeq 4.1\,mm$

03. An involute pinion and gear are in mesh. If both have the same size of addendum, then there will be an interference between the
$\boxed{\text{(a) tip of the gear tooth and flank of pinion}}$
(b) tip of the pinion and flank of gear
(c) flanks of both gear and pinion
(d) tips of both gear and pinion

04. For a pinion of 15 teeth, undercutting decreases (increases/decreases) with increase (increases/decreases) of pressure angle / centre distance.

05. For spur with gear ratio greater than one, the interference is most likely to occur near the
(a) pitch point
(b) point of beginning of contact
(c) point of end of contact
(d) root of the tooth

$G > 1.$

06. Consider the following specifications of gears A, B, C and D:

Gears	A	B	C	D
Number Of teeth	20	60	20	60
Pressure Angle	14.5°	14.5°	20°	14.5°
Module	1	3	3	1
Material	Steel	Brass	Brass	Steel

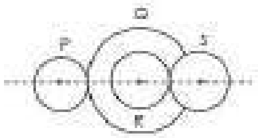
Which of these gears form the pair of spur gears to achieve a gear ratio of 3?
(a) A and B
(b) A and D
(c) B and C
(d) C and D

G.R. = 3.
A, B
$\downarrow$ $\downarrow$
20 60
$\phi_A = \phi_B$
$m_A \neq m_B$
cannot mesh.

A & D
$\downarrow$ $\downarrow$
20 60
$\phi_A = \phi_D$ ✓
$m_A = m_D$. ✓

Compound Gear Trains

07. A compound gear train with gears P,Q,R and S has number of teeth 20, 40,15 and 20, respectively. Gears Q and R are mounted on the same shaft as shown in the figure below. The diameter of the gear Q is twice that of the gear R. If the module of the gear R is 2 mm, the center distance in mm between gears P and S is (GATE-13)



(a) 40
(b) 80
(c) 120
(d) 160

$Z_P = 20$
$Z_Q = 40$
$Z_R = 15$
$Z_S = 20.$

Q – R $\rightarrow$ compound gear

$d_Q = 2 d_R \Rightarrow m_Q \cdot Z_Q = 2 \cdot m_R \cdot Z_R.$

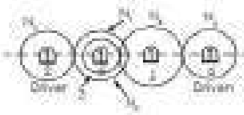
$m_R = 2\,mm.$
$m_S = 2\,mm.$
$m_Q = 1.5$
$m_P = 1.5\,mm.$

$m_Q = \frac{2 \times 2 \times 15}{40} = 1.5\,mm.$

centre distance $= r_P + r_Q + r_R + r_S.$

$= \frac{m_P \cdot Z_P}{2} + \frac{m_Q \cdot Z_Q}{2} + \frac{m_R \cdot Z_R}{2} + \frac{m_S \cdot Z_S}{2}$

08. A gear train is made up of five spur gears as shown in the figure. Gear 2 is driver and gear 6 is driven member. N_2, N_3, N_4, N_5 and N_6 represent number of teeth on gears 2,3,4,5 and 6 respectively. The gear(s) which act(s) as idler(s) is/are



(GATE –15)

(a) Only 3
(b) Only 4
(c) Only 5
(d) both 3 and 5

T.V.

$\frac{\omega_6}{\omega_2} = \left(-\frac{\omega_3}{\omega_2}\right)\left(+\frac{\omega_4}{\omega_3}\right)\left(\frac{\omega_5}{\omega_4}\right)\left(-\frac{\omega_6}{\omega_5}\right)$

$\frac{\omega_6}{\omega_2} = \left(-\frac{Z_2}{Z_3}\right) \cdot \left(-\frac{Z_4}{\cancel{Z_5}}\right) \cdot \left(-\frac{\cancel{Z_5}}{Z_6}\right)$

Gear 5 is idler wheel.

Reverted Gear Trains

Data for Q. 09 & 10 are given below.

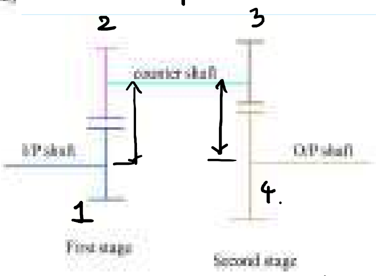
The overall gear ratio in a 2 stage speed reduction gear box (with all spur gears) is 12. The input and output shafts of the gear box are collinear. The countershafts which is parallel to the input and output shafts has a gear (Z_2 teeth) and pinion (Z_3 = 15 teeth) to mesh with pinion (Z_1 = 16 teeth) on the input shaft and gear (Z_4 teeth) on the output shaft respectively. It was decided to use a gear ratio of 4 with 3 module in the first stage and 4 module in the second stage. (GATE-03)

Reverted.

09. Z_2 and Z_4 are
(a) 64 and 45 ✓
(b) 45 and 64
(c) 48 and 60
(d) 60 and 48

10. The centre distance in the second stage is
(a) 90 mm
(b) 120 mm ✓
(c) 160 mm
(d) 240 mm

$G = 12 \Rightarrow G = G_1 G_2$
$G_1 = 4.$
$G_2 = G/G_1 = 12/4 = 3.$



Gear 1 – pinion. $Z_1 = 16$
Gear 2 – wheel. $Z_2 = ?$
Gear 3 – Pinion. $Z_3 = 15$
Gear 4 – wheel. $Z_4 = ?$

$m_1 = m_2 = 3\,mm.$
$m_3 = m_4 = 4\,mm.$

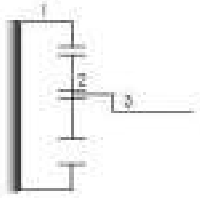
$G_1 = \frac{\omega_1}{\omega_2} = \frac{Z_2}{Z_1} = 4.$

$Z_2 = 4 \cdot Z_1 = 4 \times 16 = 64.$

$G_2 = \frac{\omega_3}{\omega_4} = \frac{Z_4}{Z_3} \Rightarrow Z_4 = 3 Z_3 = 3 \times 15 = 45$

centre distance $= r_1 + r_2 = \frac{m_1 Z_1}{2} + \frac{m_2 Z_2}{2} =$

11. A planetary gear train is shown in Fig. Internal gear (1) has 104 teeth and is held fixed and planet gear (2) has 96 teeth. How much does the planet gear rotate for sixty revolutions of the planet carrier (3) in clockwise direction ? (GATE-99)



Gear1 - internal. $z_1 = 104$
Gear-2 $\omega_1 = 0$
$z_2 = 96$

$$\frac{\omega_1 - \omega_{arm}}{\omega_2 - \omega_{arm}} = +\frac{z_2}{z_1} \Rightarrow \frac{0-60}{\omega_2 - 60} = +\frac{96}{104}$$

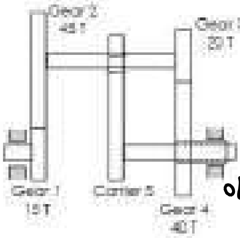
$\omega_2 = ?$ $\quad \omega_{arm} = 60$ rev. C.W.

$\omega_2 =$ ______ rev.

Data for Q. 12 & 13 are given below.

A planetary gear train has four gears and one carrier, angular velocities of the gears are ω_1, ω_2, ω_3 and ω_4 respectively. The carrier rotates with angular velocity ω_5. (GATE – 06)

arm



i/p

o/p.

2-3 compound gear.

T.V.

$$\frac{\omega_{o/p}}{\omega_{i/p}} = \frac{\omega_4 - \omega_{arm}}{\omega_1 - \omega_{arm}} = \frac{\omega_2 - \omega_{arm}}{\omega_1 - \omega_{arm}} \cdot \frac{\omega_3 - \omega_{arm}}{\omega_2 - \omega_{arm}} \cdot \frac{\omega_4 - \omega_{arm}}{\omega_3 - \omega_{arm}}$$

$$\frac{\omega_4 - \omega_{arm}}{\omega_1 - \omega_{arm}} = \left(-\frac{z_1}{z_2}\right)\left(-\frac{z_3}{z_4}\right)$$

$$\frac{\omega_4 - \omega_{arm}}{\omega_1 - \omega_{arm}} = \left(-\frac{15}{45}\right)\cdot\left(\frac{-20}{40}\right) = 1/6.$$

12. What is the relation between the angular velocities of Gear 1 and Gear 4 ?

(a) $\boxed{\dfrac{\omega_1 - \omega_5}{\omega_4 - \omega_5} = 6}$ ✓ (b) $\dfrac{\omega_4 - \omega_5}{\omega_1 - \omega_5} = 6$

(c) $\dfrac{\omega_1 - \omega_2}{\omega_4 - \omega_3} = \left[\dfrac{2}{3}\right]$ (d) $\dfrac{\omega_2 - \omega_5}{\omega_4 - \omega_5} = -\dfrac{8}{9}$

13. For $\omega_1 = 60$ rpm clockwise (cw) when looked from the left, what is the angular velocity of the carrier and its direction so that Gear 4 rotates in counter clockwise (ccw) direction at twice the angular velocity of Gear 1 when looked from the left?

(a) 130 rpm, cw
(b) 223 rpm, ccw
(c) 256 rpm, cw
(d) 156 rpm, ccw ✓

$\omega_1 = 60$ rpm.
$\omega_{arm} = ?$
$\omega_4 = -120$ rpm.

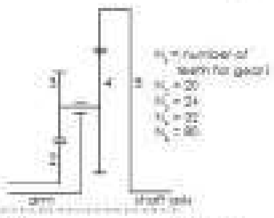
$$\frac{-120 - \omega_{arm}}{60 - \omega_{arm}} = 1/6.$$

$\omega_{arm} =$ ___.

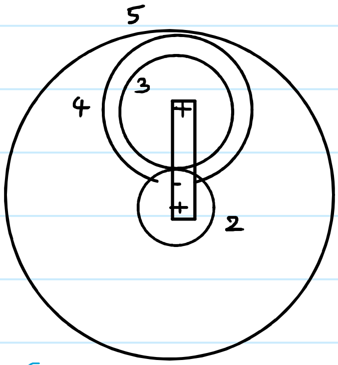
14. For the epicylic gear arrangement shown in the figure, $\omega_2 = 100$ rad/s clockwise (CW) and $\omega_{arm} = 80$ rad/s counter clockwise (CCW). The angular velocity ω_5 (in rad / s) is (GATE-10)

$\omega_2 = 100$ rad/s, $\omega_{arm} = -80$ rad/s

$\omega_5 = ?$



(a) 0 (b) 70 CW
(c) 140 CCW (d) 140 CW



$z_2 = 20$
$z_3 = 24$
$z_4 = 32$
$z_5 = 80$

$$\frac{\omega_5 - \omega_{arm}}{\omega_2 - \omega_{arm}} = \frac{\omega_3 - \omega_{arm}}{\omega_2 - \omega_{arm}} \cdot \frac{\omega_4 - \omega_{arm}}{\omega_3 - \omega_{arm}} \cdot \frac{\omega_5 - \omega_{arm}}{\omega_4 - \omega_{arm}}$$

1

$$= \left(\frac{-z_2}{z_3}\right) \cdot \left(+\frac{z_4}{z_5}\right)$$

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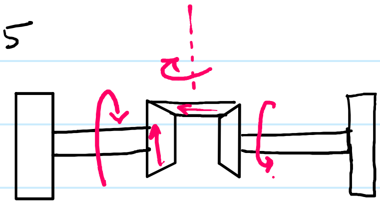
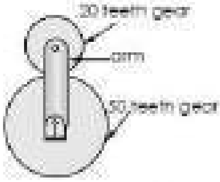
15. In an automobile service station, an automobile is in a lifted up position by means of a hydraulic jack. A person working in the service station gave a tap to one rear wheel and made it rotate by one revolution. The rotation of another rear wheel is
(a) zero
(b) also one revolution in the same direction
(c) also one revolution but in the opposite direction
(d) unpredictable

15



16. The number of degrees of freedom of the planetary gear train shown in the figure is (GATE –15)



(a) 0 (b) 1
(c) 2 (d) 3

16)

$\boxed{DoF = 2.}$

17. An Involute pinion has16 teeth and a module of 5 mm. What is the minimum value of the pressure angle so that the dedendum (with a standard value of one module) consists of purely an involute profile to eliminate the interference completely.
(a) 29° (b) 25°
(c) 20° (d) 32.5°

17) Possible only Wheels not pinion complete (for involute profile)

$R_{Dedendum} \geq R_{Base}.$

$R - \text{dedendum} \geq R \cdot \cos\phi.$

$40 - 5 \geq 40\cos\phi.$

$\phi \leq \cos^{-1}(35/40)$ $\quad \phi \leq 28.95°.$

ϕ increases $\Rightarrow \cos\phi \downarrow$. $R \uparrow$;

so $R_b =$ constant.

$m = 5 . \; z = 16$

$R = \frac{5 \times 16}{2} = 40.$

dedendum $= 1m.$

$\phi = 14.5°$

$\phi = 20°.$

18. ~~In a sun and planet gear train, the fixed sun gear has 40 teeth and planet gear has 20 teeth. The carrier rotates at 10 rad/s and transmits a torque of 5 N-m. The holding torque on sun gear is ______ N-m.~~

19. Train value of a gear train is
(a) equal to speed ratio
$\boxed{\text{(b) equal to reciprocal of speed ratio}}$
(c) always greater than one
(d) always less than one

$T.V. = \frac{1}{\text{Speed Ratio}}$

20. The three armed spider of given figure is driven at 180 rpm. The annulus A in the gear rotates at 300 rpm about the axis of the fixed wheel S which has 80 teeth. What will be the number of teeth required on wheel P?

(a) 15 (b) 25
(c) 30 (d) 20

$\omega_{arm} = 180$ rpm.

$\omega_{Ring} = 300$ rpm.

$\omega_s = 0.$

$Z_s = 80.$

$Z_p = 2_0.$

$R_R = R_s + 2 \cdot R_p.$

$\frac{m_R Z_R}{2} = \frac{m_s \cdot Z_s}{2} + m_p \cdot Z_p$

$Z_R = Z_s + 2Z_p$
$80 + 2Z_p$

T.V.

$$\frac{\omega_R - \omega_{arm}}{\omega_s - \omega_{arm}} = \frac{\omega_p - \omega_{arm}}{\omega_s - \omega_{arm}} \cdot \frac{\omega_R - \omega_{arm}}{\omega_p - \omega_{arm}}$$

$$\frac{\omega_R - \omega_{arm}}{\omega_s - \omega_{arm}} = \left(-\frac{Z_s}{\cancel{Z_p}}\right)\left(+\frac{\cancel{Z_p}}{Z_R}\right)$$

$$\frac{300 - 180}{0 - 180} = \frac{-80}{80 + 2Z_p} \Rightarrow Z_p = \_\_\_.$$

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21. Which of the following statement(s) is/are correct ?

(a) $\boxed{\text{Bevel gear is used for connecting two non-parallel or intersecting but coplanar shafts.}}$ ✓

(b) Spur gear is used for connecting two parallel and coplanar shafts with teeth parallel to the axis of the gear wheel. ✓

✓ (c) Mitre gear is used for connecting two shafts whose axes are mutually perpendicular to each other. } Mitre gear – identical gears are used.

$VR = 1$

(d) Helical gear is used for connecting two parallel and coplanar shafts with teeth inclined to the axis of the gear wheel. ✓

18. In a sun and planet gear train, the fixed sun gear has 40 teeth and planet gear has 20 teeth. The carrier rotates at 10 rad/s and transmits a torque of 5 N-m. The holding torque on sun gear is _____ N-m.

$\omega_s = 0$

$Z_s = 40$

$Z_p = 20.$

$\omega_{arm} = 10\ rad/s.$

$T_{arm} = 5\ N\text{-}m.$

$T_{sun/fixed} = ?$

T.V. $\dfrac{\omega_p - \omega_{arm}}{\omega_s - \omega_{arm}} = -\dfrac{Z_s}{Z_p}$

$\dfrac{\omega_p - 10}{0 - 10} = -\dfrac{40}{20} = -2$

$\omega_p = 30\ rad/s.$

$\Sigma Power = 0$

$P_s + P_p + P_{arm} = 0$

$T_s \cdot \omega_s + T_p \cdot \omega_p + T_{arm} \cdot \omega_{arm} = 0$

$T_p \cdot (30) + 5(10) = 0 \Rightarrow T_p = -50/30 = -1.67\ N\text{-}m.$

$\Sigma Torque.$

$T_s + T_p + T_{arm} = 0$

$T_s + (-1.67) + 5 = 0 \Rightarrow T_s = \underline{-3.33\ N\text{-}m}$

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What is BALANCING ?

- The process of either removing or reducing the unbalanced force or couple from a system is known as balancing.
- Balancing can be done either by adding the counter masses or by removing the extra masses present in the system.

Why Balancing is required ?

- Inertia force and inertia moment of a body are equal and opposite to the resultants of all external forces and moments.
- Accelerating parts of a machine generate inertia forces and moments, which are transmitted to the machine's frame or foundation.
- Variations in acceleration over time result in dynamic forces and moments on the foundation, leading to harmful vibrations and noise during operation.
- Balancing these inertia forces and moments is essential for increasing the life expectancy and ensuring smoother operation of the machine.

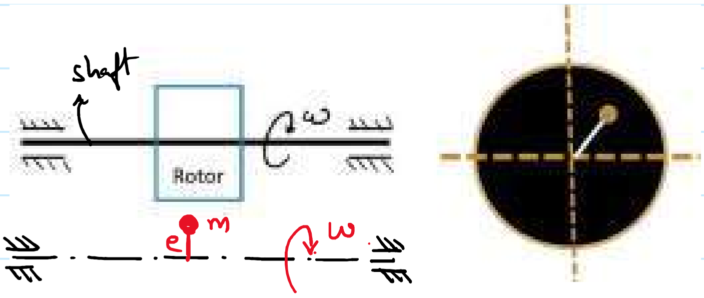
1. Types of balancing
    1. Static balancing.
    2. Dynamic balancing.

Aspects of Balancing problem

- The evaluation and analysis of inertia forces and moments
- The determination of convenient methods of balancing these quantities.

BALANCING OF ROTATING MASSES:
(A) STATIC BALANCING:

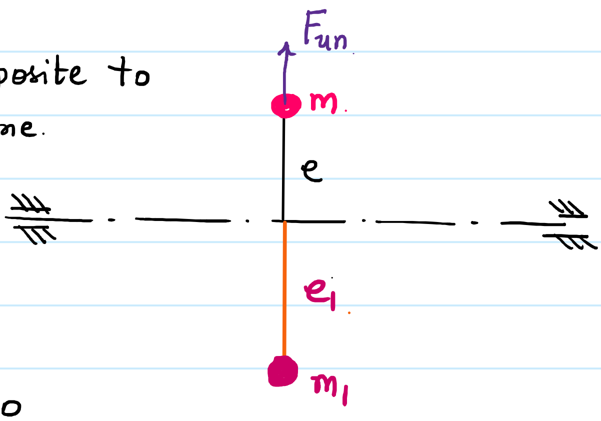
- A system is said to be statically balanced if there is no unbalanced force in the system.
- The centre of mass will lie on the axis of rotation.
- The force polygon will be completely closed.



METHOD(1): Adding the balancing masses in the same plane:

a) BALANCING BY SINGLE COUNTER MASS:

Mass is added diametrically opposite to unbalanced mass in same plane.
m = mass of rotor



- Force polygon will be closed
- C.O.M will lie on the axis of rotation

Σ☞X = 0
Σ☞Y = 0

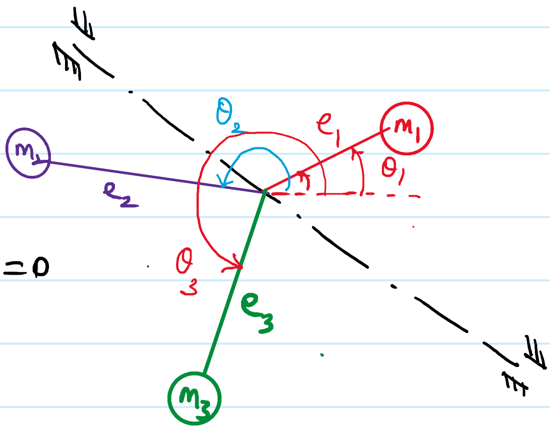
$$\Sigma F_y = 0$$
$$me\omega^2 - m_1 e_1 \omega^2 = 0$$
$$me = m_1 e_1$$

METHOD(2): Adding the balancing masses in the same plane:

b) BALANCING BY MORE THAN ONE COUNTER MASS:

- If the system is statically balanced then:

Σ☞X = 0
Σ☞Y = 0



$$\Sigma F_x = 0$$
$$m_1 e_1 \omega^2 \cos\theta_1 + m_2 e_2 \omega^2 \cos\theta_2 + m_3 e_3 \omega^2 \cos\theta_3 = 0$$
$$\sum_{i=1}^{n} m_i e_i \cos\theta_i = 0$$

$$\Sigma F_y = 0$$
$$m_1 e_1 \omega^2 \sin\theta_1 + m_2 e_2 \omega^2 \sin\theta_2 + m_3 e_3 \omega^2 \sin\theta_3 = 0$$
$$\sum_{i=1}^{n} m_i e_i \sin\theta_i = 0$$

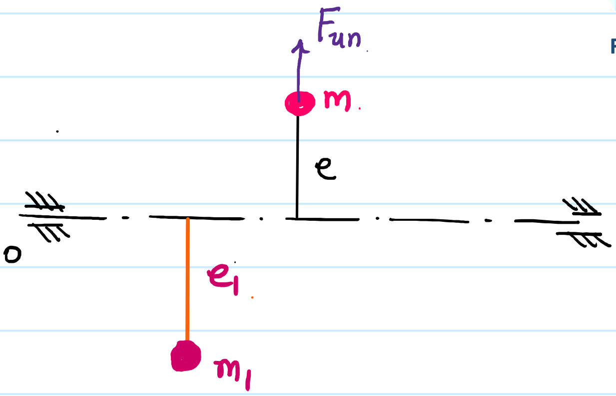
METHOD(2): Adding the balancing masses in the parallel plane:

a) BY ADDING A SINGLE BALANCING MASS:

$$\sum F_y = 0$$

$$m e \omega^2 - m_1 e_1 \omega^2 = 0$$

$$m e = m_1 e_1$$



b) STATIC BALANCING OF SYSTEM BY ADDING MORE THAN ONE MASS IN PARALLEL PLANE:

$$\sum F_x = 0$$

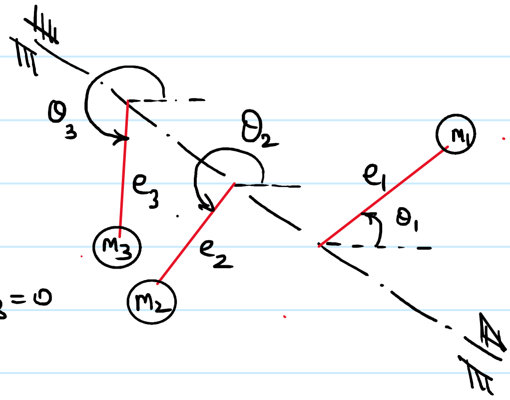
$$m_1 e_1 \omega^2 \cdot \cos\theta_1 + m_2 e_2 \cdot \omega^2 \cdot \cos\theta_2 + m_3 e_3 \cdot \omega^2 \cdot \cos\theta_3 = 0$$

$$\sum m_i e_i \cos\theta_i = 0$$

$$\sum F_y = 0$$

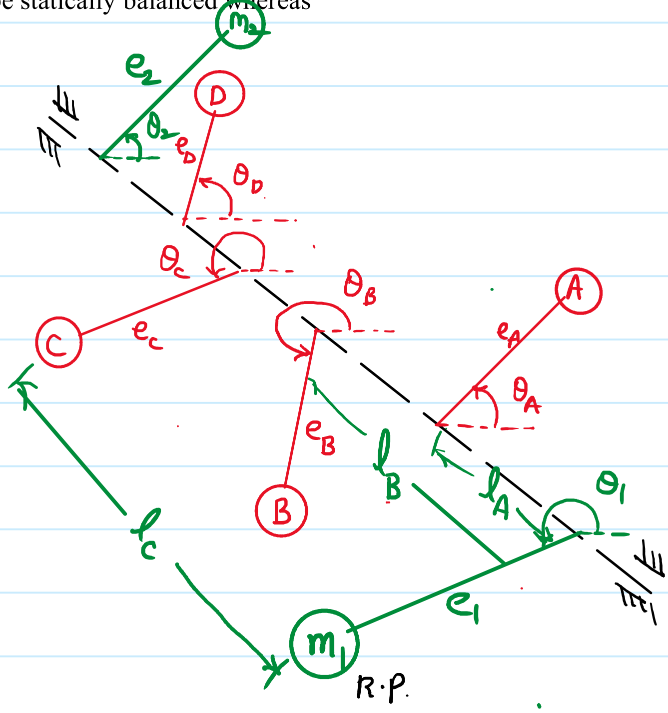
$$m_1 e_1 \omega^2 \cdot \sin\theta_1 + m_2 e_2 \cdot \omega^2 \cdot \sin\theta_2 + m_3 e_3 \cdot \omega^2 \cdot \sin\theta_3 = 0$$

$$\sum m_i e_i \sin\theta_i = 0$$



B) DYNAMIC BALANCING:

- A system is said to be dynamically balanced if the force polygon as well as couple polygon both are closed
  Since the force polygon is closed: $\Sigma F_x = 0 \quad \Sigma F_y = 0$
  Since the couple polygon is closed: $\Sigma M_x = 0 \quad \Sigma M_y = 0$
- A system which is dynamically balanced will also be statically balanced whereas reverse is not true.



m_A, m_B, m_C, m_D ----- are unbalanced masses.

e_A, e_B, e_C, e_D ------ are radial position of unbalanced masses

$\theta_A, \theta_B, \theta_C, \theta_D$ --- are angular pos'n. of unbalanced masses measured from +x-axis. in (c.c.w)

l_A, l_B, l_C, l_D ---- distance from the plane of mass m_1 to the planes m_A, m_B, m_C.....

m_1, m_2 – Balancing masses.
e_1, e_2 – Radial position of masses m_1, m_2.

$\Sigma F_x = 0 \quad m_1 e_1 \omega^2 \cos\theta_1 + m_A e_A \cdot \omega^2 \cdot \cos\theta_A + \text{------} \; m_2 e_2 \omega^2 \cos\theta_2 = 0$

$$\sum_{i=1}^{n} m_i e_i \cos\theta_i = 0$$

$\Sigma F_y = 0 \quad m_1 e_1 \omega^2 \sin\theta_1 + m_A e_A \omega^2 \sin\theta_A + \text{------} \; m_2 e_2 \omega^2 \sin\theta_2 = 0$

$$\sum_{i=1}^{n} m_i e_i \sin\theta_i = 0$$

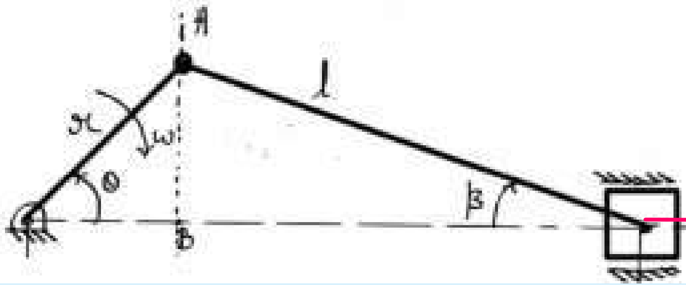
Moments about m_1 (mass) Plane

$\Sigma M_x = 0 \quad m_A \cdot e_A \cdot \omega^2 \cdot \cos\theta_A \cdot l_A + m_B e_B l_B \cdot \omega^2 \cos\theta_B + \text{----} \; m_2 e_2 l_2 \omega^2 \cos\theta_2 = 0$

$$\sum_{i=1}^{n} m_i e_i l_i \cos\theta_i = 0$$

$\Sigma M_y = 0 \quad m_A \cdot e_A \cdot l_A \, \omega^2 \sin\theta_A + \text{------} \; m_2 e_2 \cdot l_2 \cdot \omega^2 \sin\theta_2 = 0$

$$\sum_{i=1}^{n} m_i e_i l_i \sin\theta_i = 0$$



(along the line of stroke)
$F_{un} = m_p \cdot a_p$.

Mass of crank and connecting Rod is ignored.
ω_{crank} = constant.

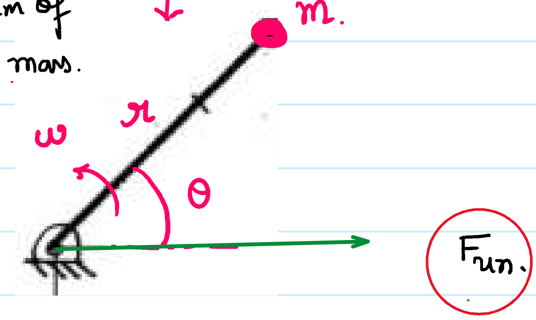
$$F_{un} = m \cdot r\omega^2 \left[\cos\theta + \frac{\cos 2\theta}{n}\right] = mr\omega^2 \cdot \cos\theta + \frac{mr\omega^2 \cdot \cos(2\theta)}{n}.$$

$n > 1$.

$$\boxed{F_{un} = mr\omega^2 \cdot \cos\theta}$$ → is converted into rotating unbalance by assuming mass @ crank pin.

F_{un}. is acting along the line of stroke.

F_{un} is represented in the form of a rotating mass.



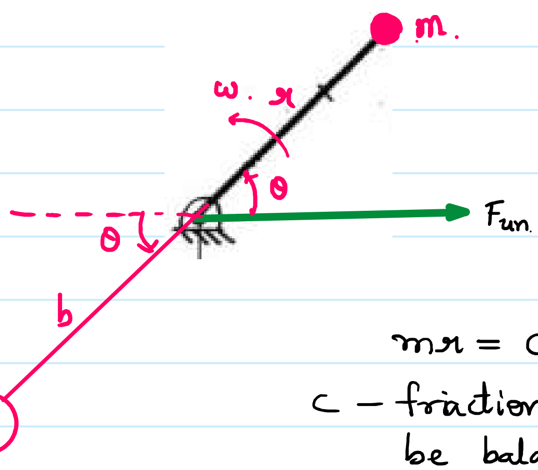
$\Sigma F_x = 0$.

$mr\omega^2 \cdot \cos\theta - Bb\omega^2 \cdot \cos\theta$

$(1-c) mr\omega^2 \cdot \cos\theta$

ΣF_y.

$Bb\omega^2 \cdot \sin\theta = cmr\omega^2 \cdot \sin\theta$.



$mr = C \cdot B \cdot b$.

c – fraction of mass to be balanced.

Resultant Unbalance force $= \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

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$$R_{un} = \sqrt{[(1-c)mr\omega^2\cdot\cos\theta]^2 + [cmr\omega^2\sin\theta]^2}$$

$$R_{un.} = f(\underbrace{c}_{\text{variable}}, \underbrace{m, r, \omega}_{\text{constant}}, \overset{\text{variable}}{\theta})$$

$R_{un} = $ Min. $\quad \frac{dR_{un}}{dc} = 0 \Rightarrow c = 0.5$

$$R_{min} = \frac{mr\omega^2}{2}$$

COMPLETE BALANCING:

- Initially the reciprocating mass is assumed at crank pin i.e, the problem is converted into rotating unbalance mass and thus location of balancing mass obtained.
- In a complete balancing the unbalanced force along the line of stroke become zero and the unbalanced force perpendicular to the line of stroke is $Bb\omega^2\sin(\theta)$, having maximum magnitude $Bb\omega^2$.
- Therefore in complete balancing only the direction of maximum unbalanced force has been changed. Whereas its magnitude is unaffected that's why complete balancing is not preferred for reciprocating masses.

$$F_{un} = \overset{F_p}{mr\omega^2\cdot\cos\theta} + \overset{F_s}{\frac{mr\omega^2}{n}\cdot\cos(2\theta)}$$

$$= mr\boxed{\omega}^2\cdot\cos(\boxed{\omega}t) + \frac{m\cdot r\cdot(\boxed{2\omega})^2\cdot\cos(\boxed{2\omega}t)}{4n}$$

F_p – Primary Unbalance force.

F_s – Secondary Unbalance force.

m – mass of piston.
r – crank lenght
ω – angular velocity of crank.

Force.	crank lenght	speed.	crank angle
Primary	r.	ω.	θ
Secondary	$\frac{r}{4n} = \frac{r}{4(\frac{l}{r})} = \frac{r^2}{4l}$	2ω.	2θ.

DIFFERENCE BETWEEN ROTATING AND RECIPROCATING BALANCE:

- The rotating unbalance force is always constant in magnitude but changes its direction continuously, whereas reciprocating unbalance force magnitude changes continuously but it always at along the line of stroke.
- In case of rotating unbalance the system can be balanced by either adding or removing the masses, where as in case of reciprocating unbalance removal of masses is not possible.

  $F_i = mrw^2 + mrw^2\cos(2\theta)/n$

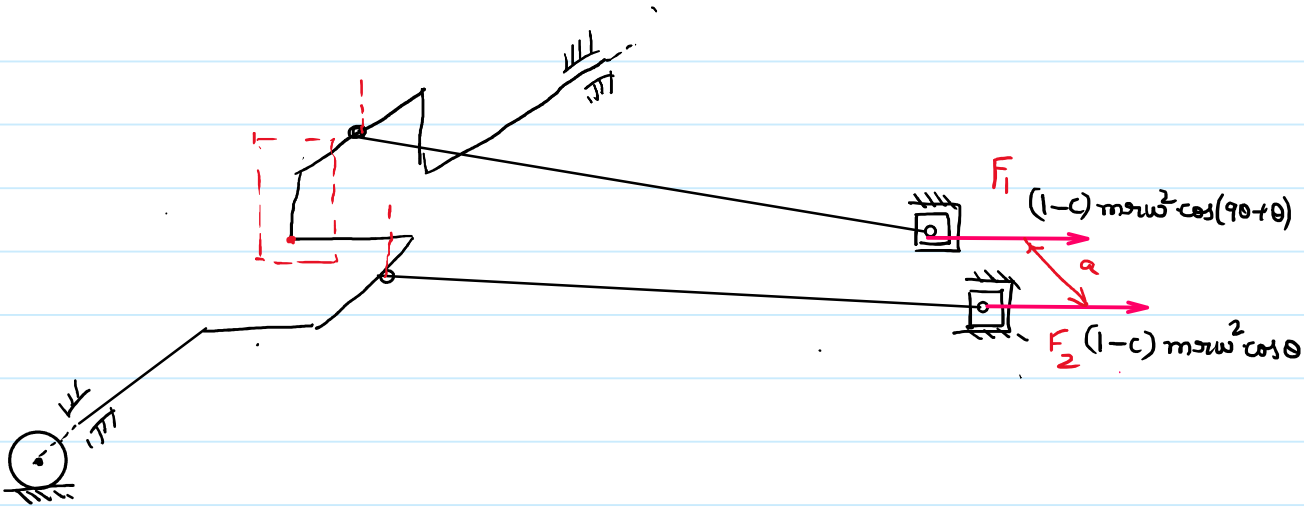
  $F_i = (F_i)_{primary} + (F_i)_{secondary}$
- Effect of Unbalanced cannot be eliminated for reciprocating that is why we opt for partial balancing.

EFFECT OF THE PARTIAL BALANCING OF LOCOMOTIVE:

- The partial balancing results in unbalanced force along the line of stroke (FH) it results in variation in tractive force along the line of stroke and swaying couple about centre line.
- Unbalanced force perpendicular to line of stroke (FV) due to which there is variation in pressure on the rails which results in hammering action which is known as "Hammer Blow".

SWAYING COUPLE:

- The two unbalanced force acting along the line of stroke of the two cylinders constitutes a couple between the centre plane is known as swaying couple.
- It tends to sway the engine alternately in clockwise and counter clockwise direction.

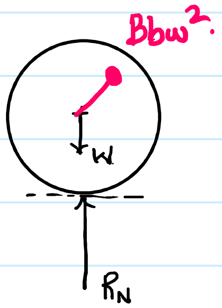


Traction force $= (1-c)\,mr\omega^2\cdot\cos\theta + (1-c)\,mr\omega^2\cdot\cos(90+\theta)$

$= (1-c)\,mr\omega^2\cdot[\cos\theta - \sin\theta] = \pm\sqrt{2}\,(1-c)\,mr\omega^2.$

Swaying Couple. $= (1-c)\,mr\omega^2\cdot\cos\theta \times a/2 - (1-c)\,mr\omega^2\cdot\cos(90+\theta)\times a/2.$

$= (1-c)\,mr\omega^2 \times a/2\cdot[\sin\theta + \cos\theta] = \dfrac{(1-c)\,mr\omega^2}{\sqrt{2}} \times a.$



$$R_N = W \pm Bb\omega^2 \cdot \sin\theta.$$

$\theta = 90^\circ$

$$R_N = W \pm Bb\omega^2.$$

$$R_N = W + Bb\omega^2 \longrightarrow \text{safe.}$$

$$R_N \geq 0$$

$$W - Bb\omega^2 \geq 0.$$

$$\omega \leq \sqrt{\frac{W}{Bb}}.$$

BALANCING OF LOCOMOTIVE:

- Most of the locomotive have two cylinder of same dimension which are placed at right angle to each other in order to have uniform turning moment diagram. There are 4 type of locomotive:

   1. INSIDE CYLINDER LOCOMOTIVE: If the cylinders are placed inside the two wheels it is known as inside cylinder locomotive.

   2. OUTSIDE CYLINDER LOCOMOTIVE: If the cylinders are placed outside the two wheels it is known as outside cylinder locomotive.

BALANCING OF LOCOMOTIVE:

   3. SINGLE OR UNCOUPLED LOCOMOTIVE: In this the effort is transmitted to one pair of wheel only.

   4. COUPLED LOCOMOTIVE: In it the driving wheel are connected to the leading or trailing wheels with the help of an outside coupling rod.

NOTE:

1. In the locomotive the value of n is very large therefore we can neglect the secondary inertia force.

2. In locomotive to balance the primary inertia force the balancing mass is kept on the wheels and partial balancing was done.

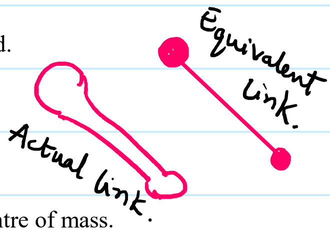
DYNAMICALLY EQUIVALENT LINK FOR CONNECTING ROD:

- The two member are equivalent for dynamic analysis purpose if the following three condition are satisfied.

   1. The location of centre of mass must be same.

   2. The total mass of the system must be same.

   3. The mass moment of inertia of actual and equivalent systems must be same with respect to the centre of mass.



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01. Static balancing is satisfactory for low speed rotors, but with increasing speeds, dynamic balancing becomes necessary. This is because, the

(a) unbalanced couples are caused only at higher speeds

(b) effect of unbalance is directly proportional to speed

(c) effect of unbalance is directly proportional to square of speed

(d) unbalanced forces are not significant at higher speeds

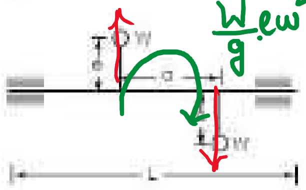
unbalance force $= mr\omega^2$.

unbalance couple $= mr\omega^2 \times a$.

$$\boxed{\text{unbalance} \propto \omega^2}$$

02. A statically-balanced system is shown in the given figure. Two equal weights W, each with an eccentricity e, are placed on opposite sides of the axis in the same axial plane. The axial distance between them is 'a'. The total dynamic reactions at the supports will be

$\frac{W}{g} e\omega^2 \times a = M.$



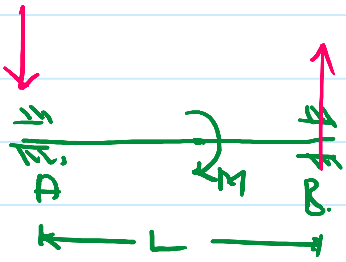
(a) zero

(b) $\frac{W}{g}\omega^2 e \frac{a}{L}$

(c) $\frac{2W}{g}\omega^2 e \frac{a}{L}$

(d) $\frac{W}{g}\omega^2 e \frac{L}{a}$

$\Sigma F_x = 0.$



$R_{A,B} = \pm \frac{M}{L}.$

$R_{A,B} = \pm \frac{\frac{W}{g} \cdot e\omega^2 \times a}{L}$

$\Sigma M \neq 0.$

$R_A + R_B = 0.$

03. In a rotor having several masses rotating in different planes the bearing reactions are found to be equal and opposite. It indicates that the system is in

(a) Dynamic balance (b) Static balance

(c) Couple balance (d) Complete balance

$$\boxed{R_A = -R_B}$$

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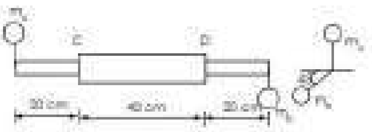
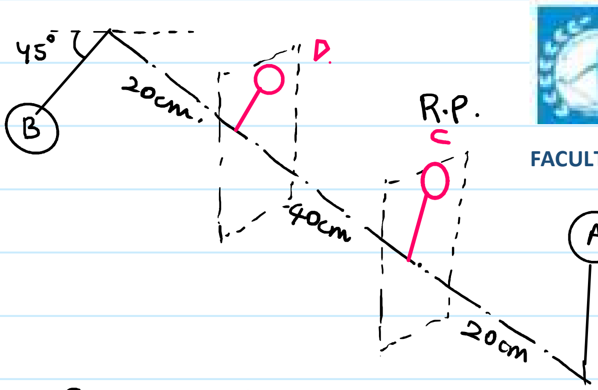
Common data for Questions 04 & 05:

A rotor shown in figure is balanced by attaching two masses $m_A = 5$ kg and $m_B = 6$ kg in the planes A and B at a radius of 20 cm at the locations shown. It is required to balance the rotor by removing masses in the planes C & D at a radius of 20 cm. Magnitude and angular location of the masses to be removed in Planes C and D.



04. Mass in plane C and its angular location with respect to m_A
(a) 9.85 kg @ 167.5° (b) 10.5 kg @ 23.8°
(c) 11 kg @ 215.7° (d) 10.5 kg @ 283.8°

05. Mass in plane D and its angular location with respect to m_A
(a) 10 kg @ 23.8° (b) 10.98 kg @ 35.7°
(c) 11 kg @ 215.7° (d) 11 kg @ 203.8°



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Y-axis as Ref.

$e_A = e_B = e_C = e_D = 20$ cm $= 0.2$ cm.

$\theta_A = 0°$ $\theta_B = 135°$ $\theta_C, \theta_D = ?$

SNo.	Mass.	Radial Location e_i	Angular position θ_i	$\Sigma m_i e_i \cos\theta_i$	$\Sigma m_i e_i \sin\theta_i$	Dist. b/w plane L_i	$\Sigma m_i e_i l_i \cos\theta_i$	$\Sigma m_i e_i l_i \sin\theta_i$
1	$m_A = 5$kg	0.2	0°	$5\times0.2\times\cos0° = 1$	$5\times0.2\times\sin0°$	−0.2	−0.02	0
2	m_C	0.2	θ_C	$0.2m_C\cos\theta_C$	$0.2m_C\sin\theta_C$	0	0	0
3	m_D	0.2	θ_D	$0.2m_D\cos\theta_D$	$0.2m_D\sin\theta_D$	0.4	$0.08\,m_D\cos\theta_D$	$0.08m_D\sin\theta_D$
4.	$m_B = 6$kg	0.2	225°	$6\times0.2\times\cos135°$	$6\times0.2\times\sin135°$	0.6	$6\times0.2\times0.6\times\cos135°$	$6\times0.2\times0.6\sin135°$

$$\Sigma m_i e_i l_i \sin\theta_i = 0$$

$$0.08m_D\sin\theta_D + 6\times0.2\times0.6\times\sin135° = 0.$$

$m_D = 10.981$
$\theta_D = 35.7°.$

$$\Sigma m_i e_i l_i \cos\theta_i = 0$$

$$-0.02 + 0.08m_D\cos\theta_D + 6\times0.2\times0.6\times\cos135° = 0.$$

$$\Sigma m_i e_i \cos\theta_i = 0$$

$$1 + 0.2m_C\cos\theta_C + 0.2m_D\cdot\cos\theta_D + 6\times0.2\times\cos135° = 0$$

$$\Sigma m_i e_i \sin\theta_i = 0$$

$$0 + 0.2m_C\sin\theta_C + 0.2m_D\sin\theta_D + 6\times0.2\times\sin135° = 0$$

$m_C = 9.85$kg
$\theta_C = 167.5°$

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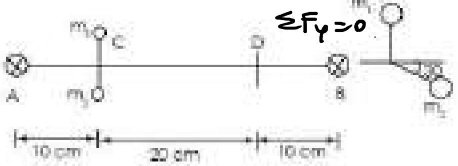
Common data for Questions 06 & 07 :

A rotor supported in the bearings A & B has two unbalanced masses in the plane C, m_1 = 10kg at an eccentricity of 10 cm and m_2 = 5 kg at an eccentricity of 20 cm. It is statically balanced by attaching a single mass at a radius of 10 cm in the plane D.

$\Sigma F_x = 0$

$\Sigma F_y = 0$



$m_1 = 10\,kg$ $\quad m_2 = 5\,kg$

$e_1 = 20\,cm$ $\quad e_2 = 20\,cm.$

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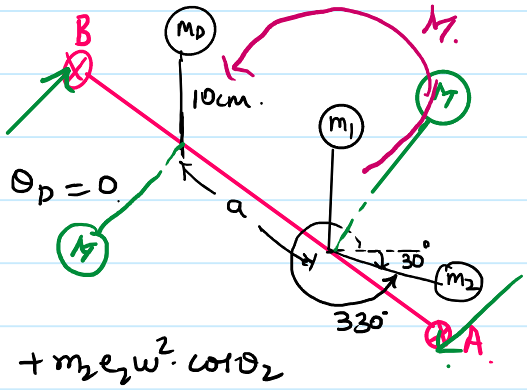
$m_D = ?$

$e_D = 10\,cm.$

$\theta_D = ?$

$\theta_1 = 90°$ $\quad \theta_D = 0$

$\theta_2 = 330°$



06. The magnitude and angular position (with respect to the horizontal) of the mass attached in the plane D is

(a) 10 kg @ 210°

(b) 14.14 kg @ 210°

(c) 14.14 kg @ 120°

(d) 10 kg @ 180°

$0.1 m_D \cos\theta_D = -0.866$

$0.1 m_D \sin\theta_D = -0.5$

$\Sigma F_x = 0$

$m_1 e_1 \omega^2 \cdot \cos\theta_1 + m_2 e_2 \omega^2 \cdot \cos\theta_2 + m_D e_D \omega^2 \cdot \cos\theta_D = 0. \longrightarrow (A)$

$(10 \times 0.2 \times \cos 90°) + (5 \times 0.2 \times \cos 330°) + (m_D \times 0.1 \times \cos\theta_D) = 0$

$\Sigma F_y = 0$

$m_i e_i \sin\theta_i = 0.$

$m_1 e_1 \sin\theta_1 + m_2 e_2 \cdot \sin\theta_2 + m_D e_D \sin\theta_D = 0. \longrightarrow (B)$

$(10 \times 0.2 \times \sin 90°) + (5 \times 0.2 \times \sin 330°) + m_D \times 0.1 \times \sin\theta_D = 0.$

07. When the rotor rotates 600 rpm the dynamic reaction in the bearings is about

(a) 40 kN (b) 4 kN

(c) 3 kN (d) 2 kN

$\tan\theta_D = \dfrac{-0.5}{-0.866}$

$\theta_D = 210°$

$m_D = 10\,kg$

$M = m e \omega^2 \times a$

$R_A, R_B = \pm \dfrac{m e \omega \times a}{L} = \dfrac{\pm (10) \times 0.1 \times \left(\dfrac{2\pi \times 600}{60}\right)^2 \times 0.2}{0.4}$

08. A rotor of mass 2000 kg is balanced statically by attaching two masses one in a plane 20 cm to the left of the centroidal plane and the second in a plane 25 cm to the right. Both masses are attached at a radius of 10 cm and mass 1 is 52 kg and the mass 2 is 75 kg, mass 1 is in the horizontal direction and the second mass is in the vertical direction. Locate the eccentricity of the rotor and its angular location with reference to the first one

$\boxed{\text{(a) 0.45 cm @ 235.2°}}$ (b) 0.02 cm @ 71.5°

(c) 0.4 cm @ 71.5° (d) 0.02 cm @ 251.6°

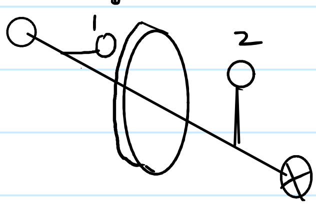
$m_R = 2000\,kg$ $\quad e_R = ?$ $\quad \theta_R = ?$

$e_1 = e_2 = 10\,cm.$

$m_1 = 52\,kg$

$m_2 = 75\,kg$

$\theta_1 = 0°, \quad \theta_2 = 90°$



$\Sigma m_i r_i \cos\theta_i = 0$

$52 \times 0.1 \times \cos 0° + 75 \times 0.1 \times \cos 90° + 2000 \times e_R \cdot \cos\theta_R = 0.$

$\Sigma m_i r_i \sin\theta_i = 0$

$52 \times 0.1 \times \sin 0° + 75 \times 0.1 \times \sin 90° + 2000 \times e_R \cdot \sin\theta_R = 0$

$2000 e_R \cdot \cos\theta_R = -5.2$

$2000\, e_R \cdot \sin\theta_R = -7.5$

$\tan\theta_R = \dfrac{-7.5}{5.2}$

$\theta_R = 180 + 55.26$

$= 235.26°.$

$e_R = -5.2 / 2000 \times \cos 235.26$

$= 0.456\,cm.$

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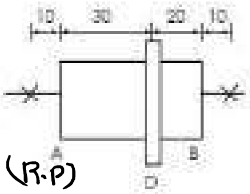
09.



For the rotor shown in figure has an unbalance of 2kg-m in the plane D along the x-axis. It is desired to balance this by removing masses in the planes A and B at a radius of 0.5 m. The dimensions shown in the figure are in cm. The masses to be removed in plane A, plane B and their angular positions with x axis respectively are

(a) 1.6 kg, 2.4 kg @ 0° & 0°
(b) 0.48 kg, 0.72 kg @ 180° & 0°
(c) 0.24 kg, 0.48 kg @ 0° & 180°
(d) 0.72 kg, 0.24 kg @ 180° & 0°

SNo.	Mass.	Radial Location e_i	Angular position θ_i	$\Sigma m_i e_i \cos\theta_i$	$\Sigma m_i e_i \sin\theta_i$	Dist. b/w plane L_i	$\Sigma m_i e_i l_i \cos\theta_i$	$\Sigma m_i e_i l_i \sin\theta_i$
1	2 kg-m.		0°	$2\times\cos 0^\circ$	$2\times\sin 0^\circ$	0.3	$2\times\cos 0^\circ\times 0.3 = 0.6$	0
2	$-m_A$	0.5	θ_A	$-0.5 m_A \cos\theta_A$	$-0.5\times m_A \sin\theta_A$	0	0	0
3	$-m_B$	0.5	θ_B	$-0.5 m_B \cos\theta_B$	$-0.5 m_B \sin\theta_B$	0.5	$-0.25 m_B \cos\theta_B$	$-0.25 m_B \sin\theta_B$

$\Sigma m_i e_i l_i \sin\theta_i = 0$

$$0.025 m_B \cdot \sin\theta_B = 0 \qquad \sin\theta_B = 0^\circ, 180^\circ$$

$\Sigma m_i e_i l_i \cos\theta_i = 0$

$$0.6 - 0.025\, m_B \cdot \cos\theta_B = 0 \qquad m_B = \frac{-0.6}{-0.25} = 2.4\,kg$$

$\Sigma m_i e_i \sin\theta_i = 0$

$$0 - 0.5 m_B \cdot \underbrace{\sin\theta_B}_{0} - 0.5 m_A \sin\theta_A = 0.$$

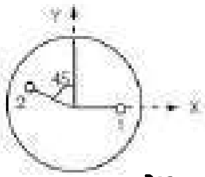
$$\theta_A = 0^\circ.$$

$\Sigma m_i e_i \cos\theta_i = 0$

$$2 - 0.5 m_B \cdot \cos 0^\circ - 0.5\times 2.4\times \cos 0^\circ = 0$$

$$m_B = 2.4\,kg.$$

10. A wheel is balanced by placing two trial masses as shown in figure. The mass 1 is of 20 gms at radius of 15 cm and the mass 2 is 25 grams at a radius of 20 cm. If it is to be balanced by a single balancing mass at a radius of 20 cm, the magnitude and angular position of the balancing mass is



$\boxed{\text{(a) 17.88 gm \& 98.7°}}$
(b) 16 gm & 208°
(c) 52 gm & 45°
(d) 16 gm & 45°

$m_1 = 20\,gms \quad e_1 = 15\,cm$

$m_2 = 25\,gms \quad e_2 = 20\,cm$

$\theta_1 = 0^\circ \quad \theta_2 = 135^\circ$

$m_b = ? \quad e_b = 20\,cm. \quad \theta_b = ?$

$\Sigma F_x = 0$

$m_1 e_1 \cos\theta_1 + m_2 e_2 \cdot \cos\theta_2 + m_b e_b \cdot \cos\theta_b = 0.$

$= (20 \times 0.15 \times \cos 0^\circ) + (25 \times 0.2 \times \cos 135^\circ) + (m_b \times 0.2 \times \cos\theta_b) = 0$

$\Sigma F_y = 0$

$m_1 e_1 \sin\theta_1 + m_2 e_2 \sin\theta_2 + m_b e_b \cdot \sin\theta_b = 0$

$= (20 \times 0.15 \times \sin 0^\circ) + (25 \times 0.2 \times \sin 135^\circ) + (m_b \times 0.2 \times \sin\theta_b) = 0$

$m_b \cdot \cos\theta_b = 0.535$

$m_b \sin\theta_b = 3.53$

$\theta_b = \tan^{-1}\left(\frac{-3.53}{0.535}\right)$

$\theta_b = -81.3^\circ$

$= 180 - 81.3^\circ$

$= 92^\circ.$

11. A slider crank mechanism has slider mass of 10 kg, stroke of 0.2 m and rotates with a uniform angular velocity of 10 rad/s. The primary inertia forces of the slider are partially balanced by a revolving mass of 6 kg at the crank, placed at a distance equal to crank radius. Neglect the mass of connecting rod and crank. When the crank angle (with respect to slider axis) is 30°, the unbalanced force (in Newton) normal to the slider axis is_______

(GATE–14)

$m = 10\,kg \qquad \theta = 30^\circ$

$r = 0.2\,m.$

$\omega = 10\,rad/s.$

$B = 6\,kg$

$b = 0.2\,m.$

Unbalanced $\perp$lar to line of stroke $= Bb\omega^2 \sin\theta$

$= 6 \times 0.2 \times 10^2 \times \sin 30^\circ$

$=$ ____

12. Which one of the following statements in the context of balancing in engines is correct?

(a) Magnitude of the primary unbalancing force is less than the secondary unbalancing force —— $F_p > F_s$

(b) The primary unbalancing force attains its maximum value twice in one revolution of the crank, Secondary force will attain 4 times in 1 rev.

(c) The hammer blow in the locomotive engines occurs due to unbalanced force ~~along~~ the line of stroke of the piston $\rightarrow \perp$lar.

(d) The unbalanced force due to reciprocation masses varies in magnitude and ~~direction~~

$F_p = mr\omega^2 \cos\theta \rightarrow$ Periodicity 2π.

$F_s = \frac{mr\omega^2}{n} \cos 2\theta. \quad n > 1.$

$\rightarrow$ Periodicity $= \frac{2\pi}{2} = \pi$

13. In balancing of single-cylinder engine, the rotating unbalance is

(a) completely made zero and ~~so also the~~ reciprocating unbalance

(b) $\boxed{\text{completely made zero}}$ and the reciprocating unbalance is partially reduced

(c) partially reduced and the reciprocating unbalance is completely made zero

(d) partially reduced and so also the reciprocating unbalance

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14. In a reciprocating machine based on regular slider crank mechanism, the mass of reciprocating parts is 10 kg and the crank radius is 15 cm. 60% of the primary unbalance is balanced using a counter mass. What will be the residual unbalance along the line of stroke for a crank position of 60° at a speed of 4 rad/sec.

(a) 2.4 N (b) 4.8 N

(c) 12 N (d) 6.6 N

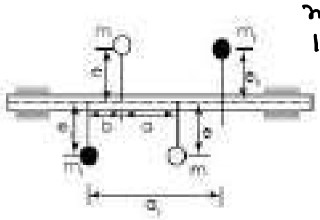
$m = 10\,kg$. $r = 15\,cm = 0.15\,m$.

$c = 60\% = 0.6$ $\theta = 60°$

$F_x = (1-c)\,m r \omega^2 \cdot \cos\theta = (1-0.6) \times 10 \times 0.15 \times 4^2 \times \cos 60°$

$F_x = 4.8\,N$.

15. Two masses m are attached to opposite sides of a rigid rotating shaft in the vertical plane. Another pair of equal masses m1 is attached to the opposite sides of the shaft in the vertical plane as shown in figure. Consider m = 1 kg, e = 50 mm, e_1 = 20 mm, b = 0.3 m, a = 2 m and a_1 = 2.5 m. For the system to be dynamically balanced, m_1 should be ______ kg. (GATE – 16)



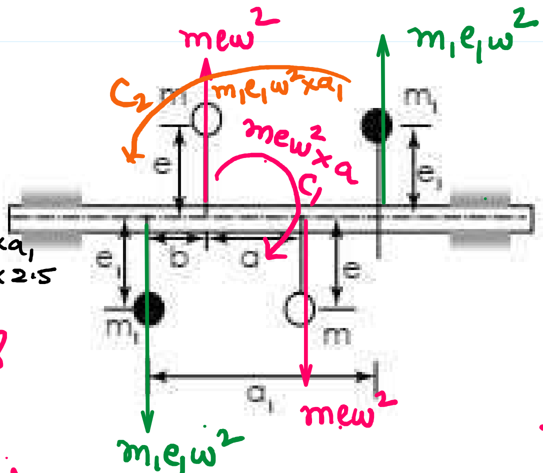
$C_1 = C_2$

$m e \omega^2 \times a = m_1 e_1 \omega^2 \times a_1$

$1 \times 50 \times 2 = m_1 \times 20 \times 2.5$

$m_1 = 2\,kg$

statically & Dynamically balanced.



$\boxed{\Sigma F = 0}$

$\boxed{C_2 = C_1}$

16. A connecting rod of a steam engine has a length of 100 cm and a mass of 100 kg. The CG is at a distance of 40cm from the big end with a radius of gyration of 30 cm about CG. If it is to be replaced with a two mass system with m_b at the big end and m_s at small end. Then m_b, m_s and the moment of inertia of the equivalent link are respectively.

$\boxed{\text{(a) 60 kg, 40 kg, 24 kg m}^2}$

(b) 60 kg, 40 kg, 9 kg.m²

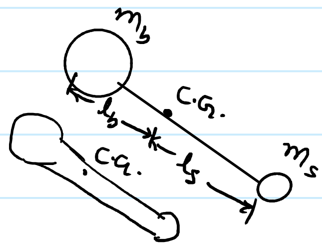
(c) 40 kg, 60 kg, 9 kg.m²

(d) 60 kg, 40 kg, 90 kg.m²

$m = 100\,kg$.

$l_b = 40\,cm$

$K = 30\,cm$.



$l = 100\,cm$.

$l_b = 40\,cm$.

$l_s = 60\,cm$.

$l_s + l_b = l$.

$m_s + m_b = m$.

$m_s + m_b = 100 \longrightarrow (1)$.

$m_s \cdot l_s = m_b \cdot l_b$.

$m_s(60) = m_b \cdot (40)$

$m_b = 1.5\,m_s$.

$m_s + 1.5 m_s = 100 \Rightarrow m_s = \frac{100}{2.5} = 40\,kg$

$m_b = 60\,kg$.

$I = m_s \cdot l_s^2 + m_b \cdot l_b^2 = 40 \times 0.6^2 + 60 \times (0.4)^2$

$= 24\,kg\text{-}m^2$

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