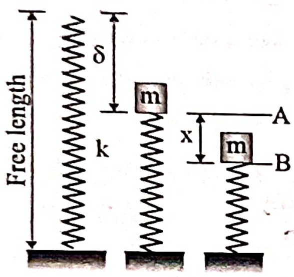
Chapter 5 Mechanical Vibrations

One Mark Questions

01. In the figure shown, the spring deflects by δ to position A (the equilibrium position) when a mass m is kept on it. During free vibration, the mass is at position B at some instant. The change in potential energy of the spring mass system from position A to position B is **(GATE–ME-01)**



(a) $\frac{1}{2}kx^2$ (b) $\frac{1}{2}kx^2 - mgx$

(c) $\frac{1}{2}k(x+\delta)^2$ (d) $\frac{1}{2}kx^2 + mgx$

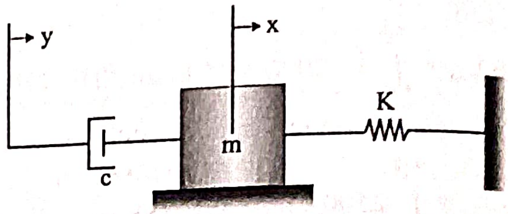
02. A vibrating machine is isolated from the floor using springs. If the ratio of excitation frequency of vibration of machine to the natural frequency of the isolation system is equal to 0.5, the transmissibility ratio of isolation is **(GATE–ME-04)**

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{4}{3}$ (d) 2

03. There are four samples P, Q, R and S with natural frequencies 64, 96, 128 and 256 Hz, respectively. They are mounted on test setups for conducting vibration experiments. If a loud pure note of frequency 144 Hz is produced by some instrument, which of the samples will show the most perceptible induced vibration? **(GATE–ME-05)**

(a) P (b) Q (c) R (d) S

04. The differential equation governing the vibrating system is **(GATE–ME-06)**



(a) $m\ddot{x} + c\dot{x} + k(x-y) = 0$

(b) $m(\ddot{x}-\ddot{y}) + c(\dot{x}-\dot{y}) + kx = 0$

(c) $m\ddot{x} + c(\dot{x}-\dot{y}) + kx = 0$

(d) $m(\ddot{x}-\ddot{y}) + c(\dot{x}-\dot{y}) + k(x-y) = 0$

05. For an under damped harmonic oscillator, resonance **(GATE–ME-07)**

(a) occurs when excitation frequency is greater than undamped natural frequency.

(b) occurs when excitation frequency is less than undamped natural frequency

(c) occurs when excitation frequency is equal to undamped natural frequency

(d) never occurs

06. The rotor shaft of a large electric motor supported between short bearings at both the ends shows a deflection of 1.8 mm in the middle of the rotor. Assuming the rotor to be perfectly balanced and supported at knife edges at both the ends, the likely critical speed (in rpm) of the shaft is **(GATE–ME-09)**

(a) 350 (b) 705 (c) 2810 (d) 4430

07. The natural frequency of a spring mass system on earth is ω_n. The natural frequency of this system on the moon ($g_{moon} = g_{earth}/6$) is **(GATE–ME-10)**

(a) ω_n (b) $0.408\omega_n$

(c) $0.204\omega_n$ (d) $0.167\omega_n$

08. If two nodes are observed at a frequency of 1800 rpm during whirling of a simply supported long slender rotating shaft, the first critical speed of the shaft in rpm is **(GATE–ME-13)**
(a) 200 (b) 450 (c) 600 (d) 900

09. Critical damping is the **(GATE–ME-14)**
(a) largest amount of damping for which no oscillation occurs in free vibration
(b) smallest amount of damping for which no oscillation occurs in free vibration
(c) largest amount of damping for which the motion is simple harmonic in free vibration
(d) smallest amount of damping for which the motion is simple harmonic in free vibration

10. In vibration isolation, which one of the following statements is NOT correct regarding Transmissibility (T)? **(GATE–ME-14)**
(a) T is nearly unity at small excitation frequencies
(b) T can be always reduced by using higher damping at any excitation frequency
(c) T is unity at the frequency ratio of $\sqrt{2}$
(d) T is infinity at resonance for undamped systems

11. Consider a single degree-of-freedom system with viscous damping excited by a harmonic force. At resonance, the phase angle (in degree) of the displacement with respect to the exciting force is **(GATE–ME-14)**
(a) 0 (b) 45 (c) 90 (d) 135

12. A point mass is executing simple harmonic motion with an amplitude of 10 mm and frequency of 4 Hz. The maximum acceleration (m/s^2) of the mass is ______ **(GATE–ME-14)**

13. In a spring-mass system the mass is m and the spring constant is k. The critical damping coefficient of the system is 0.1 kg/s. In another spring mass system, the mass is 2 m and the spring constant is 8 k. The critical damping coefficient (in kg/s) of this system is ______ **(GATE –15 –Set 2)**

14. Which of the following statements are TRUE for damped vibrations?
P. For a system having critical damping, the value of damping ratio is unity and system does not undergo a vibratory motion.
Q. Logarithmic decrement method is used to determine the amount of damping in a physical system.
R. In case of damping due to dry friction between moving surfaces resisting force of constant magnitude acts opposite to the relative motion.
S. For the case of viscous damping, drag force is directly proportional to the square of relative velocity. **(GATE –15 –Set 3)**
(a) P and Q only
(b) P and S only
(c) P, Q and R only
(d) Q and S only

15. A single degree of freedom spring mass system with viscous damping has a spring constant of 10 kN/m. The system is excited by a sinusoidal force of amplitude 100 N. If the damping factor (ratio) is 0.25, the amplitude of steady state oscillation at resonance is ______ mm **(GATE – 16 – SET – 1)**

16. The static deflection of a spring under gravity, when a mass of 1 kg is suspended from it, is 1 mm. Assume the acceleration due to gravity g = 10 m/s^2. The natural frequency of this spring-mass system (in rad/s) is ______ **(GATE – 16 – SET – 3)**

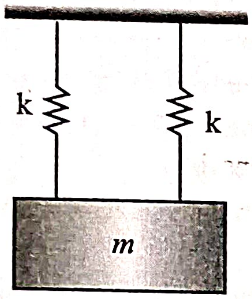
17. A single degree of freedom mass-spring – viscous damper system with mass m, spring constant k and viscous damping coefficient q is critically damped. The correct relation among m, k and q is **(GATE – 16 – SET – 2)**
(a) $q = \sqrt{2km}$ (b) $q = 2\sqrt{km}$
(c) $q = \sqrt{\frac{2k}{m}}$ (d) $q = 2\sqrt{\frac{k}{m}}$

18. The damping ratio for a viscously damped spring mass system, governed by the relationship $m\frac{d^2x}{dt^2}+c\frac{dx}{dt}+kx=F(t)$, is given by

(GATE – 17 – SET – 1)

(a) $\sqrt{\frac{c}{mk}}$ (b) $\frac{c}{2\sqrt{km}}$ (c) $\frac{c}{\sqrt{km}}$ (d) $\sqrt{\frac{c}{2mk}}$

19. A mass m is attached to two identical springs having spring constant k as shown in figure. The natural frequency ω of this single degree of freedom system is



(GATE – 17 – SET – 2)

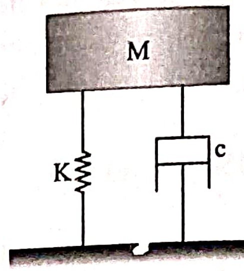
(a) $\sqrt{\frac{2k}{m}}$ (b) $\sqrt{\frac{k}{m}}$ (c) $\sqrt{\frac{k}{2m}}$ (d) $\sqrt{\frac{4k}{m}}$

20. The equation of motion for a spring-mass system excited by a harmonic force is $M\ddot{x}+Kx=F\cos(\omega t)$, Where M is the mass, K is the spring stiffness, F is the force amplitude and ω is the angular frequency of excitation. Resonance occurs when ω is equal to

(GATE – 18 – SET – 1)

(a) $\sqrt{\frac{M}{K}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{K}{M}}$ (c) $2\pi\sqrt{\frac{K}{M}}$ (d) $\sqrt{\frac{K}{M}}$

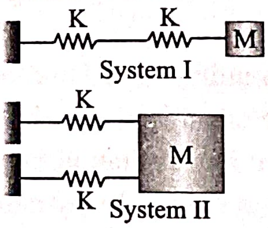
21. In a single degree of freedom underdamped spring-mass-damper system as shown in the figure, an additional damper is added in parallel such that the system still remains underdamped. Which one of the following statements is ALWAYS true ?



(GATE – 18 – SET – 2)

(a) Transmissibility will increase.
(b) Transmissibility will decrease.
(c) Time period of free oscillations will increase.
(d) Time period of free oscillations will decrease.

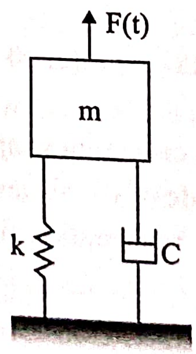
22. The natural frequencies corresponding to the spring-mass systems I and II are ω_I and ω_{II} respectively. The ratio $\frac{\omega_I}{\omega_{II}}$ is



((GATE – 19 – SET – 1)

(a) $\frac{1}{2}$ (b) 4 (c) 2 (d) $\frac{1}{4}$

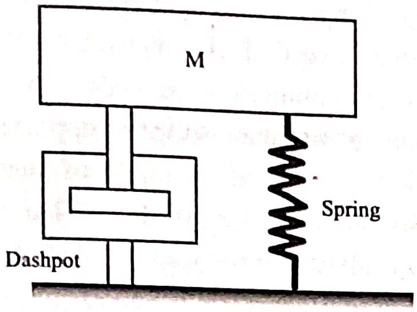
23. A single-degree-of-freedom oscillator is subjected to harmonic excitation $F(t)=F_0\cos(\omega t)$ as shown in the figure. **(GATE–20–SET–1)**



The non-zero value of ω, for which the amplitude of the force transmitted to the ground will be F_0, is

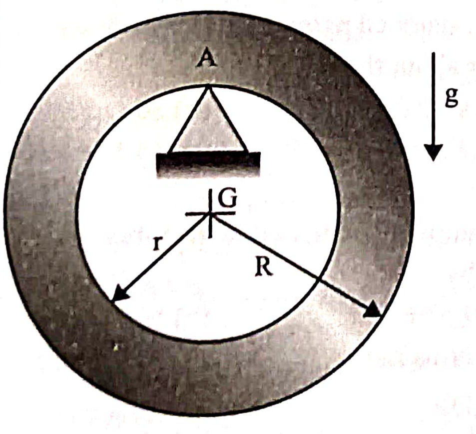
(a) $\sqrt{\frac{k}{m}}$ (b) $\sqrt{\frac{k}{2m}}$ (c) $\sqrt{\frac{2k}{m}}$ (d) $2\sqrt{\frac{k}{m}}$

24. Consider a single degree of freedom system comprising a mass M, supported on a spring and a dashpot as shown in the figure.



If the amplitude of the free vibration response reduces from 8 mm to 1.5 mm in 3 cycles, the damping ratio of the system is ______ (round off to three decimal places). **(GATE–21_SET–1)**

25. A rigid uniform annular disc is pivoted on a knife edge A in a uniform gravitational field as shown, such that it can execute small amplitude simple harmonic motion in the plane of the figure without slip at the pivot point. The inner radius r and outer radius R are such that $r^2 = R^2/2$, and the acceleration due to gravity is g. If the time period of small amplitude simple harmonic motion is given by $T = \beta\pi\sqrt{R/g}$, where π is the ratio of circumference to diameter of a circle, then β = ________ (round off to 2 decimal places).



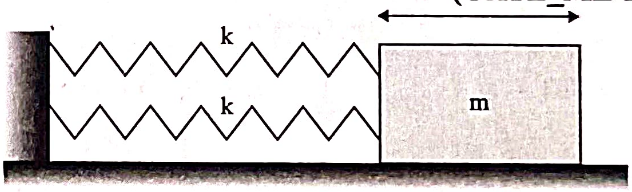
(GATE-22_SET-1)

26. For a dynamical system governed by the equation,

$$\ddot{x}(t)+2\xi\,\omega_n\dot{x}(t)+\omega_n^2 x(t)=0$$

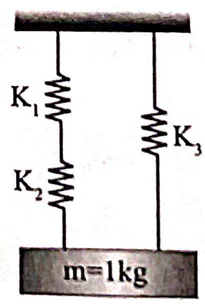
the damping ratio ζ is equal to $\frac{1}{2\pi}\log_e 2$. The displacement x of this system is measured during a hammer test. A displacement peak in the positive displacement direction is measured to be 4 mm. Neglecting higher powers (>1) of the damping ratio, the displacement at the next peak in the positive direction will be ______ mm (in integer). **(GATE-22_SET-2)**

27. The figure shows a block of mass m = 20 kg attached to a pair of identical linear springs, each having a spring constant k = 1000 N/m. The block oscillates on a frictionless horizontal surface. Assuming free vibrations, the time taken by the block to complete ten oscillations is ________ seconds. (Rounded off to two decimal places) Take π = 3.14. **(GATE_ME-23)**



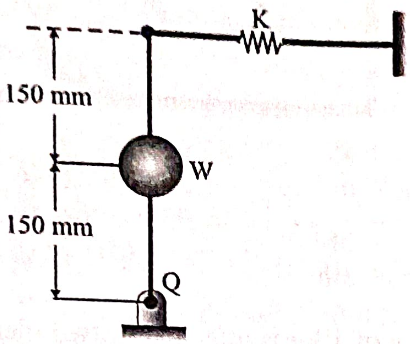
Two Marks Questions

01. A mass of 1 kg is suspended by means of 3 springs as shown in Figure. The spring constants K_1, K_2 and K_3 are respectively 1 kN/m, 3 kN/m and 2 kN/m. The natural frequency of the system is approximately) **(GATE–ME-96)**



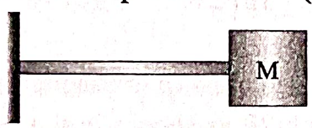
(a) 46.90 rad/sec
(b) 52.44 rad/sec
(c) 60.55 rad/sec
(d) 77.46 rad/sec

vertical in a stable the minimum value of spring constant K needed is (GATE–ME-04)



(a) 300 N/m (b) 400 N/m
(c) 500 N/m (d) 1000 N/m

09. A mass M, of 20 kg is attached to the free end of a steel cantilever beam of length 1000 mm having a cross-section of 25 × 25 mm. Assume the mass of the cantilever to be negligible and E_{steel} = 200 GPa. If the lateral vibration of this system is critically damped using a viscous damper, the damping constant of the damper is (GATE–ME-04)



(a) 1250 Ns/m (b) 625 Ns/m
(c) 312.50 Ns/m (d) 156.25 Ns/m

10. In a spring-mass system, the mass is 0.1 kg and the stiffness of the spring is 1 kN/m, By introducing a damper, the frequency of oscillation is found to be 90% of the original value. What is the damping coefficient of the damper? (GATE–ME-05)
(a) 1.2 N.s/m (b) 3.4 N.s/m
(c) 8.7 N.s/m (d) 12.0N.s/m

11. A machine of 250 kg mass is supported on springs of total stiffness 100 kN/m. Machine has an unbalanced rotating force of 350 N at speed of 3600 rpm. Assuming a damping factor of 0.15, the value of transmissibility ratio is (GATE–ME-06)
(a) 0.0531 (b) 0.9922 (c) 0.0162 (d) 0.0028

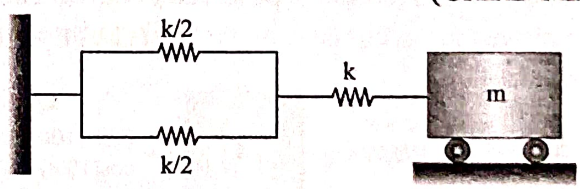
Statement for Linked Answer Q.Nos. 12 & 13
A vibratory system consists of a mass 12.5 kg, a spring of stiffness 1000 N/m, and a dashpot with damping coefficient of 15 Ns/m

12. The value of critical damping of the system is: (GATE–ME-06)
(a) 0.223 Ns/m (b) 17.88 Ns/m
(c) 71.4 Ns/m (d) 223.6 Ns/m

13. The value of logarithmic decrement is (GATE–ME-06)
(a) 1.35 (b) 0.42
(c) 0.68 (d) 0.66

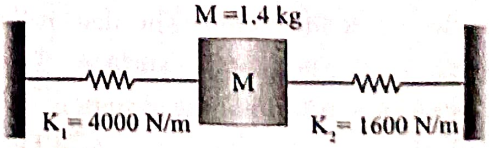
14. The equation of motion of a harmonic oscillator is given by $\frac{d^2x}{dt^2} + 2\xi\omega_n\frac{dx}{dt} + \omega_n^2 x = 0$ and the initial conditions at $t = 0$ are $x(0) = X$, $X = \frac{dx}{dt}(0) = 0$ The amplitude of x(t) after n complete cycles is (GATE–ME-07)
(a) $Xe^{-2n\pi\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)}$ (b) $Xe^{2n\pi\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)}$
(c) $Xe^{-2n\pi\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)}$ (d) X

15. The natural frequency of the system shown below is (GATE–ME-07)



(a) $\sqrt{\frac{k}{2m}}$ (b) $\sqrt{\frac{k}{m}}$ (c) $\sqrt{\frac{2k}{m}}$ (d) $\sqrt{\frac{3k}{m}}$

16. The natural frequency of the spring mass system shown in the figure is closest to (GATE–ME-08)



(a) 8 Hz (b) 10 Hz (c) 12 Hz (d) 14 Hz

17. A uniform rigid rod of mass m = 1 kg and length L = 1 m is hinged at its centre & laterally supported at one end by a spring of spring constant k = 300 N/m. The natural frequency ω_n in rad/s is

(GATE–ME-08)

(a) 10 (b) 20 (c) 30 (d) 40

18. An automotive engine weighting 240 kg is supported on four springs with linear characteristics. Each of the front two springs have a stiffness of 16 MN/m while the stiffness of each rear spring is 32 MN/m. The engine speed (in rpm), at which resonance is likely to occur, is **(GATE–ME-09)**

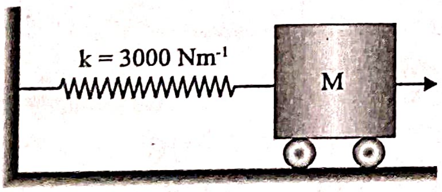
(a) 6040 (b) 3020
(c) 1424 (d) 955

19. A vehicle suspension system consists of a spring and a damper. The stiffness of the spring is 3.6 kN/m and the damping constant of the damper is 400 Ns/m. If the mass is 50 kg, then the damping factor (ξ) and damped natural frequency (f_n) respectively are

(GATE–ME-09)

(a) 0.47 and 1.19 Hz (b) 0.471 and 7.48 Hz
(c) 0.666 and 1.35 Hz (d) 0.666 and 8.50 Hz

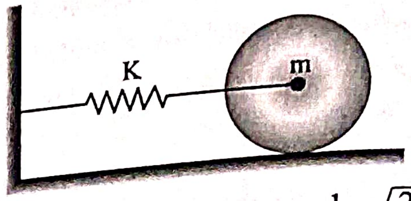
20. A mass m attached to a spring is subjected to a harmonic force as shown in figure. The amplitude of the forced motion is observed to be 50 mm. The value of m (in kg) is **(GATE–ME-10)**



(a) 0.1 (b) 1.0 (c) 0.3 (d) 0.5

21. A disc of mass m is attached to a spring of stiffness k as shown in the figure. The disc rolls without slipping on a horizontal surface. The natural frequency of vibration of the system is

(GATE–ME-11)

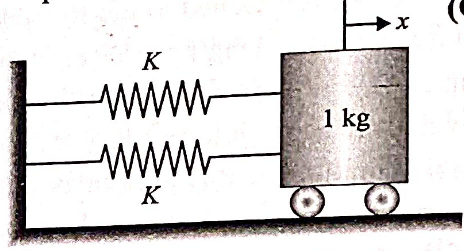


(a) $\frac{1}{2\pi}\sqrt{\frac{K}{m}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{2K}{m}}$

(c) $\frac{1}{2\pi}\sqrt{\frac{2K}{3m}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{3K}{2m}}$

22. A mass of 1 kg is attached to two identical springs each with stiffness k = 20 kN/m as shown in the figure. Under frictionless condition, the natural frequency of the system in Hz is close to

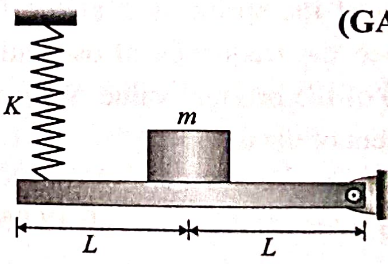
(GATE–ME-11)



(a) 32 (b) 23 (c) 16 (d) 1

23. A concentrated mass m is attached at the centre of a rod of length 2L as shown in the figure. The rod is kept in a horizontal equilibrium position by a spring of stiffness k. For very small amplitude of vibration, neglecting the weights of the rod and spring, the undamped natural frequency of the system is

(GATE–ME-12)



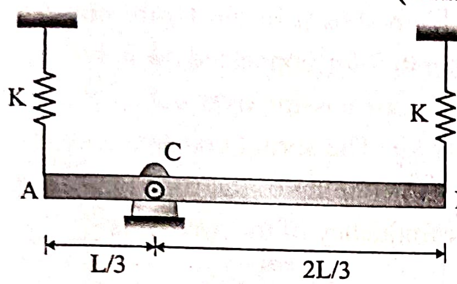
(a) $\sqrt{\frac{K}{m}}$ (b) $\sqrt{\frac{2K}{m}}$ (c) $\sqrt{\frac{K}{2m}}$ (d) $\sqrt{\frac{4K}{m}}$

24. A single degree of freedom system having mass 1 kg and stiffness 10 kN/m initially at rest is subjected to an impulse force of magnitude 5 kN for 10^{-4} seconds. The amplitude in mm of the resulting free vibration is **(GATE–ME-13)**

(a) 0.5 (b) 1.0 (c) 5.0 (d) 10.0

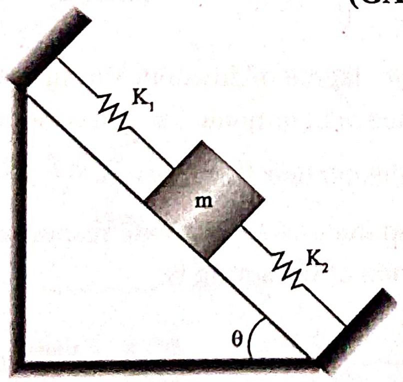
25. Consider a cantilever beam, having negligible mass and uniform flexural rigidity, with length 0.01 m. The frequency of vibration of the beam, with a 0.5 kg mass attached at the free tip, is 100 Hz. The flexural rigidity (in N.m²) of the beam is _______ **(GATE-ME-14)**

26. A rigid uniform rod AB of length L and mass m is hinged at C such that AC = L/3, CB = 2L/3. Ends A and B are supported by springs of spring constant k. The natural frequency of the system is given by **(GATE-ME-14)**



(a) $\sqrt{\frac{K}{2m}}$ (b) $\sqrt{\frac{K}{m}}$ (c) $\sqrt{\frac{2K}{m}}$ (d) $\sqrt{\frac{5K}{m}}$

27. What is the natural frequency of the spring mass system shown below? The contact between the block and the inclined plane is frictionless. The mass of the block is denoted by m and the spring constants are denoted by k_1 and k_2 as shown below. **(GATE-ME-14)**

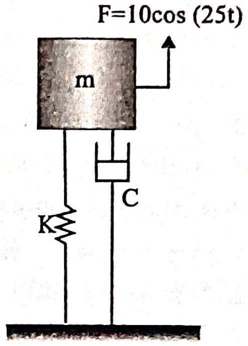


(a) $\sqrt{\frac{K_1+K_2}{2m}}$ (b) $\sqrt{\frac{K_1+K_2}{4m}}$

(c) $\sqrt{\frac{K_1-K_2}{m}}$ (d) $\sqrt{\frac{K_1+K_2}{m}}$

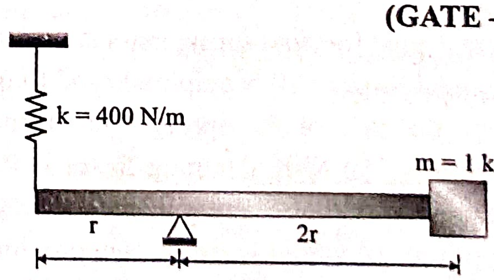
28. The damping ratio of a single degree of freedom spring-mass-damper system with mass of 1 kg, stiffness 100 N/m and viscous damping coefficient of 25 N.s/m is _______ **(GATE-ME-14)**

29. A mass-spring-dashpot system with mass m = 10 kg, spring constant K = 6250 N/m is excited by a harmonic excitation of 10cos(25t) N. At the steady state, the vibration amplitude of the mass is 40 mm. The damping coefficient (C, in N.s/m) of the dashpot is _______ **(GATE-ME-14)**



30. A single degree of freedom system has a mass of 2 kg, stiffness 8 N/m and viscous damping ratio 0.02. The dynamic magnification factor at an excitation frequency of 1.5 rad/s is _____ **(GATE-ME-14)**

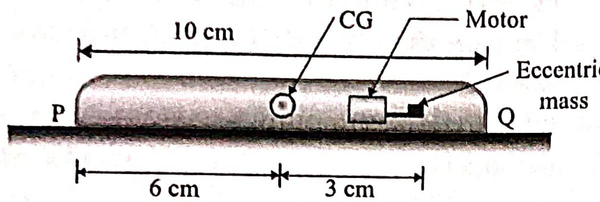
31. Considering massless rigid rod and small oscillations, the natural frequency (in rad/s) of vibration of the system shown in the figure is **(GATE –15 –Set 1)**



(a) $\sqrt{\frac{400}{1}}$ (b) $\sqrt{\frac{400}{2}}$

(c) $\sqrt{\frac{400}{3}}$ (d) $\sqrt{\frac{400}{4}}$

32. A mobile phone has a small motor with an eccentric mass used for vibrator mode. The location of the eccentric mass on motor with respect to center of gravity (CG) of the mobile and the rest of the dimensions of the mobile phone are shown. The mobile is kept on a flat horizontal surface.



Given in addition that the eccentric mass = 2 grams, eccentricity = 2.19 mm, mass of the mobile = 90 grams, g = 9.81 m/s² .Uniform speed of the motor in RPM for which the mobile will get just lifted off the ground at the end Q is approximately

(GATE –15 –Set 1)

(a) 3000 (b) 3500 (c) 4000 (d) 4500

33. A precision instrument package (m = 1 kg) needs to be mounted on a surface vibrating at 60 Hz. It is desired that only 5% of the base surface vibration amplitude be transmitted to the instrument. Assume that the isolation is designed with its natural frequency significantly lesser than 60 Hz, so that the effect of damping may be ignored. The stiffness (in N/m) of the required mounting pad is ________

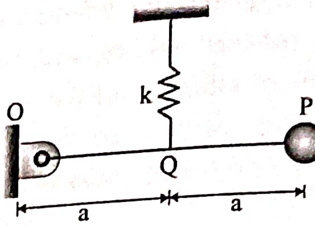
(GATE –15 –Set 1)

34. A single degree freedom spring mass is subjected to a sinusoidal force of 10 N amplitude and frequency ω along the axis of the spring. The stiffness of the spring is 150 N/m, damping factor is 0.2 and undamped natural frequency is 10 ω. At steady state, the amplitude of vibration (in m) is approximately

(GATE –15 –Set 2)

(a) 0.05 (b) 0.07 (c) 0.70 (d) 0.90

35. Figure shows a single degree of freedom system. The system consists of a mass less rigid bar OP hinged at O and a mass m at end P. The natural frequency of vibration of the system is

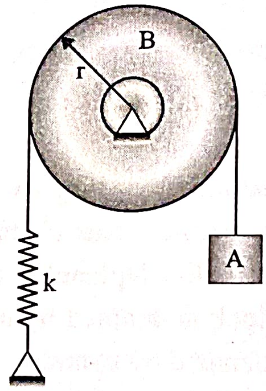


(GATE –15 –Set 3)

(a) $f_n = \frac{1}{2\pi}\sqrt{\frac{k}{4m}}$ (b) $f_n = \frac{1}{2\pi}\sqrt{\frac{k}{2m}}$

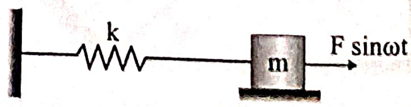
(c) $f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (d) $f_n = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

36. The system shown in the figure consists of block A of mass 5 kg connected to a spring through a massless rope passing over pulley B of radius r and mass 20 kg. The spring constant k is 1500 N/m. If there is no slipping of the rope over the pulley, the natural frequency of the system is ______ rad/s.



(GATE – 16 – SET – 2)

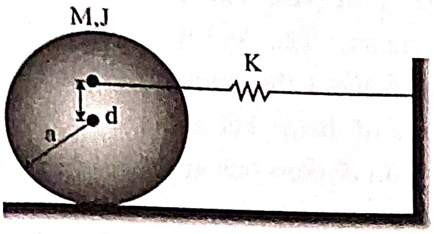
37. A single degree of freedom spring-mass system is subjected to a harmonic force of constant amplitude. For an excitation frequency of $\sqrt{\frac{3k}{m}}$ the ratio of the amplitude of steady state response to the static deflection of the spring is ________



(GATE – 16 – SET – 3)

38. A solid disc with radius a is connected to a spring at a point d above center of the disc. The other end of the spring is fixed to the vertical wall. The disc

is free to roll without slipping on the ground. The mass of the disc is M and the spring constant is K. The polar moment of inertia for the disc about its center is $J = \frac{Ma^2}{2}$

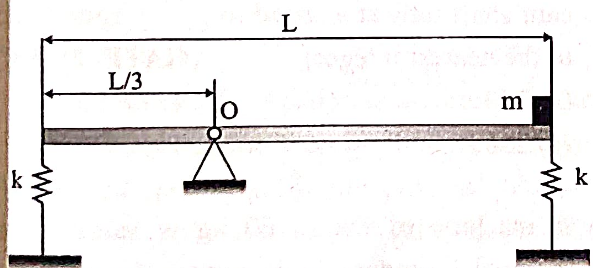


The natural frequency of this system in rad/s is given by **(GATE – 16 – SET – 1)**

(a) $\sqrt{\frac{2K(a+d)^2}{3Ma^2}}$ (b) $\sqrt{\frac{2K}{3M}}$

(c) $\sqrt{\frac{2K(a+d)^2}{Ma^2}}$ (d) $\sqrt{\frac{K(a+d)^2}{Ma^2}}$

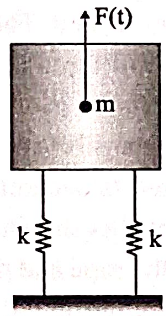
39. A thin uniform rigid bar of length L and mass M is hinged at point O, located at a distance of $\frac{L}{3}$ from one of its ends. The bar is further supported using springs, each of stiffness k, located at the two ends. A particle of mass $m = \frac{M}{4}$ is fixed at one end of the bar, as shown in the figure. For small rotations of the bar about O, the natural frequency of the system is **(GATE – 17 – SET – 1)**



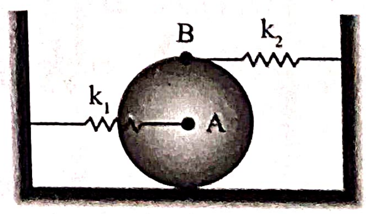
(a) $\sqrt{\frac{5k}{M}}$ (b) $\sqrt{\frac{5k}{2M}}$ (c) $\sqrt{\frac{3k}{2M}}$ (d) $\sqrt{\frac{3k}{M}}$

40. The radius of gyration of a compound pendulum about the point of suspension is 100 mm. The distance between the point of suspension and the centre of mass is 250 mm. Considering the acceleration due to gravity as 9.81 m/s², the natural frequency (in radian/s) of the compound pendulum is ________ **(GATE – 17 – SET – 2)**

41. A machine of mass m = 200 kg is supported on two mounts, each of stiffness k = 10 kN/m. The machine is subjected to an external force (in N) F(t) = 50cos5t. Assuming only vertical translatory motion, the magnitude of the dynamic force (in N) transmitted from each mount to the ground is ________ (correct of two decimal places). **(GATE – 18 – SET – 1)**



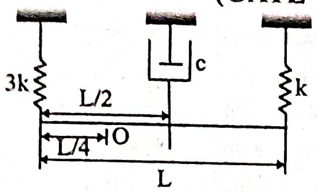
42. A uniform thin disk of mass 1 kg and radius 0.1 m is kept on a surface as shown in the figure. A spring of stiffness k_1 = 400 N/m is connected to the disk center A and another spring of stiffness k_2 = 100 N/m is connected at point B just above point A on the circumference of the disk. Initially, both the springs are unstretched. Assume pure rolling of the disk. For small disturbance from the equilibrium, the natural frequency of vibration of the system is ______ rad/s (round off to one decimal place). **(GATE – 19 – SET – 1)**



43. A slender uniform rigid bar of mass m is hinged at O and supported by two springs, with stiffnesses 3k and k, and a damper with damping coefficient c, as

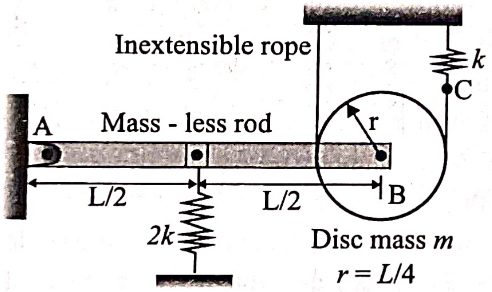
shown in the figure. For the system to be critically damped, the ratio $c/\sqrt{km}$ should be

(GATE – 19 – SET – 2)



(a) $2\sqrt{7}$ (b) $4\sqrt{7}$ (c) 4 (d) 2

44. A rigid mass-less rod of length L is connected to a disc (pulley) of mass m and radius r = L/4 through a friction-less revolute joint. The other end of that rod is attached to a wall through a friction-less hinge. A spring of stiffness 2k is attached to the rod at its mid-span. An inextensible rope passes over half the disc periphery and is securely tied to a spring of stiffness k at point C as shown in the figure. There is no slip between the rope and the pulley. The system is in static equilibrium in the configuration shown in the figure and the rope is always taut.

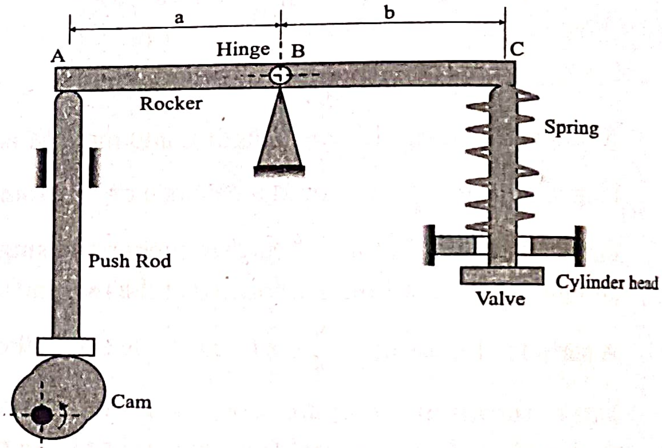


Neglecting the influence of gravity, the natural frequency of the system for small amplitude vibration is (GATE–20–SET–1)

(a) $\sqrt{\frac{3}{2}}\sqrt{\frac{k}{m}}$ (b) $\sqrt{3}\sqrt{\frac{k}{m}}$ (c) $\frac{3}{\sqrt{2}}\sqrt{\frac{k}{m}}$ (d) $\sqrt{\frac{k}{m}}$

45. A hollow spherical ball of radius 20 cm floats in still water, with half of its volume submerged. Taking the density of water as 1000 kg/m³, and the acceleration due to gravity as 10 m/s², the natural frequency of small oscillations of the ball, normal to the water surface is _______ radian/s (round off to 2 decimal places). (GATE–20–SET–2)

46. A tappet valve mechanism in an IC engine comprises a rocker arm ABC that is hinged at B as shown in the figure. The rocker is assumed rigid and it oscillates about the hinge B. The mass moment of inertia of the rocker about B is 10^{-4} kgm². The rocker arm dimensions are a = 3.5 cm and b = 2.5 cm. A pushrod pushes the rocker at location A, when moved vertically by a cam that rotates at N rpm. The pushrod is assumed massless and has a stiffness of 15 N/mm. At the other end C, the rocker pushes a valve against a spring of stiffness 10 N/mm. The valve is assumed massless and rigid.



Resonance in the rocker system occurs when the cam shaft runs at a speed of _____ rpm (round off to the nearest integer). (GATE–21_SET–1)

(a) 790 (b) 4739
(c) 2369 (d) 496

47. A machine of mass 100 kg is subjected to an external harmonic force with a frequency of 40 rad/s. The designer decides to mount the machine on an isolator to reduce the force transmitted to the foundation. The isolator can be considered as a combination of stiffness (K) and damper (damping factor, ξ) in parallel. The designer has the following four isolators:

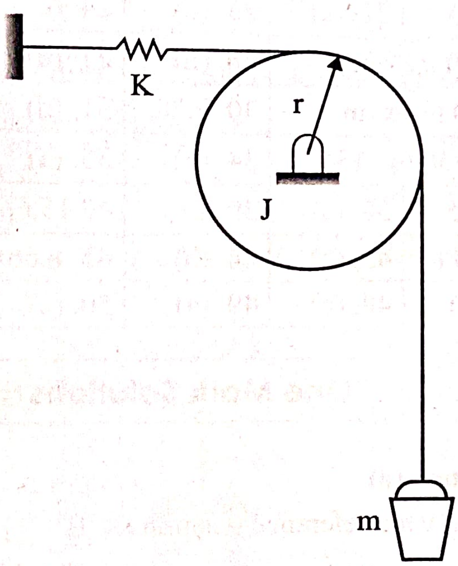
1. $K = 640$ kN/m, $\xi = 0.70$
2. $K = 640$ kN/m, $\xi = 0.07$
3. $K = 22.5$ kN/m, $\xi = 0.70$
4. $K = 22.5$ kN/m, $\xi = 0.07$

Arrange the isolators in the ascending order of the force transmitted to the foundation.

(GATE–21_SET–2)

(a) 1-3-4-2
(b) 1-3-2-4
(c) 4-3-1-2
(d) 3-1-2-4

48. Consider the system shown in the figure. A rope goes over a pulley. A mass, m, is hanging from the rope. A spring of stiffness, k, is attached at one end of the rope. Assume rope is inextensible, massless and there is no slip between pulley and rope.



The pulley radius is r and its mass moment of inertia is J. Assume that the mass is vibrating harmonically about its static equilibrium position. The natural frequency of the system is **(GATE–21_SET–2)**

(a) $\sqrt{\frac{k r^2}{J}}$
(b) $\sqrt{\frac{k r^2}{J - mr^2}}$
(c) $\sqrt{\frac{k r^2}{J + mr^2}}$
(d) $\sqrt{\frac{k}{m}}$

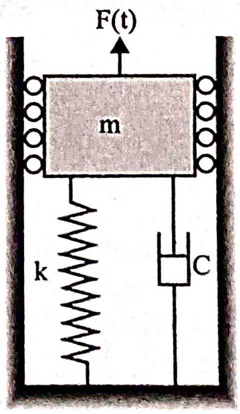
49. Consider a forced single degree-of-freedom system governed by

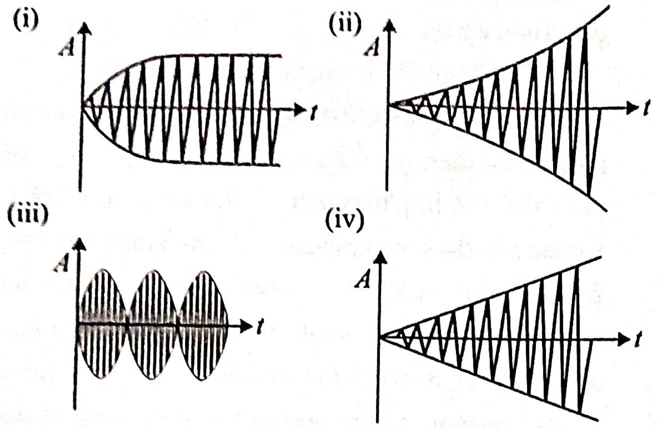
$$\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2 x(t) = \omega_n^2\cos(\omega t),$$

where ζ and ω_n are the damping ratio and undamped natural frequency of the system, respectively, while ω is the forcing frequency. The amplitude of the forced steady state response of this system is given by $[(1 - r^2)^2 + (2\zeta r)^2]^{-1/2}$, where $r = \omega/\omega_n$. The peak amplitude of this response occurs at a frequency $\omega = \omega_p$. If ω_d denotes the damped natural frequency of this system, which one of the following options is true? **(GATE–22_SET-1)**

(a) $\omega_p < \omega_d < \omega_n$
(b) $\omega_p = \omega_d < \omega_n$
(c) $\omega_d < \omega_n = \omega_p$
(d) $\omega_d < \omega_n < \omega_p$

50. A spring mass damper system (mass m, stiffness k, and damping coefficient c) excited by a force $F(t) = B \sin \omega t$, where B, ω and t are the amplitude, frequency and time, respectively, is shown in the figure. Four different responses of the system (marked as (i) to (iv)) are shown just to the right of the system figure. In the figures of the responses, A is the amplitude of response shown in red color and the dashed lines indicate its envelope. The responses represent only the qualitative trend and those are not drawn to any specific scale.





Four different parameter and forcing conditions are mentioned below.

(P) $C > 0$ and $\omega = \sqrt{k/m}$

(Q) $C < 0$ and $\omega \neq 0$

(R) $C = 0$ and $\omega = \sqrt{k/m}$

(S) $C = 0$ and $\omega \cong \sqrt{k/m}$

Which one of the following options gives correct match (indicated by arrow →) of the parameter and forcing conditions to the responses?

(GATE-22_SET-2)

(a) (P) → (i), (Q) → (iii), (R) → (iv), (S) → (ii)
(b) (P) → (ii), (Q) → (iii), (R) → (iv), (S) → (i)
(c) (P) → (i), (Q) → (iv), (R) → (ii), (S) → (iii)
(d) (P) → (iii), (Q) → (iv), (R) → (ii), (S) → (i)

KEY & Detailed Solutions

ONE MARK QUESTIONS				
01. (a)	02. (c)	03. (c)	04. (c)	05. (c)
06. (b)	07. (a)	08. (a)	09. (b)	10. (b)
11. (c)	12. 6.32 m/sec^2		13. 0.38 to 0.42	
14. (c)	15. 20	16. 100	17. (b)	18. (b)
19. (a)	20. (d)	21. (c)	22. (a)	23. (c)
24. 0.088	25. 2.65	26. 2	27. 6.28	
TWO MARKS QUESTIONS				
01. (b)	02. (a)	03. (c)	04. (d)	05. (b)
06. (a)	07. (c)	08. (c)	09. (a)	10. (c)
11. (c)	12. (d)	13. (b)	14. (a)	15. (a)
16. (b)	17. (c)	18. (a)	19. (a)	20.(a&d)
21. (c)	22. (a)	23. (d)	24. (c)	
25. 0.0658 N.m^2		26. (d)	27. (d)	28. 1.25
29. 10 N.sec/m		30. 2.28	31. (d)	32. (b)
33. 6750 to 7150		34. (b)	35. (a)	36. 10
37. 0.5	38. (a)	39 (b)	40.15.66	41. 33.33
42. 23.09	43. (b)	44. (b)	45. 8.66	46. (b)
47. (c)	48. (c)	49. (a)	50. (c)	

One Mark Solutions

01. Ans : (a)

Sol: Take the reference position as 'B'

$$(PE)_A = \frac{1}{2}k\delta^2 + mgx$$

$$(PE)_B = \frac{1}{2}k(\delta + x)^2$$

Change in potential energy:

$$= \frac{1}{2}k(\delta + x)^2 - \frac{1}{2} \times k \times \delta^2 - mgx$$

$$= \frac{1}{2} \times k \times x^2 + k\delta x - mgx$$

$$= \frac{1}{2} \times k \times x^2 + mgx - mgx \quad [\because k\delta = mg]$$

$$= \frac{1}{2} \times kx^2$$