$$F_I + F_B = 0$$

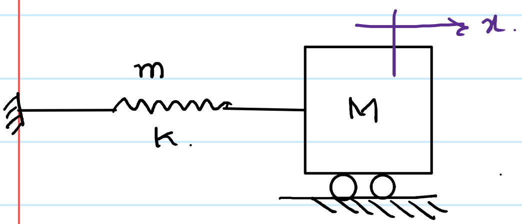
$$m_s \cdot \ddot{x}_s + F_B = 0$$

$$\rho_w \cdot \tfrac{2}{3}\pi R_o^3 \cdot \ddot{x} + \rho \cdot g \cdot \pi R_o^2 \cdot x = 0$$

$$\ddot{x} + \frac{3g}{2R_o} \cdot x = 0$$

$$\omega_n = \sqrt{\frac{3g}{2R_o}}$$

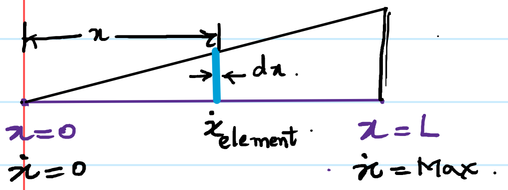
Considering mass of Spring.



M – mass of Block.
m – mass of spring.

Energy Approach.

$$(KE)_{block} + (KE)_{spring} + (SE) = constant$$



$$KE_{block} = \frac{1}{2} M \dot{x}^2$$

$$SE = \tfrac{1}{2} k x^2$$

$\dot{x}$ — velocity of block.

Velocity varies linearly along lenght of spring

$$(KE)_{spring} = \frac{1}{2} \cdot m_{element} \cdot \dot{x}_{element}^2$$

$$m_{element} = \frac{m}{L} \cdot dx$$

$$\frac{\dot{x}_{element}}{x} = \frac{\dot{x}}{L} \Rightarrow \dot{x}_{element} = \frac{\dot{x}}{L} \cdot x$$

$$(KE)_{spring} = \int \tfrac{1}{2} \cdot \left(\frac{m}{L}\right) \cdot \left(\frac{\dot{x}}{L} \cdot x\right)^2 \cdot dx = \frac{1}{2L^3} m \dot{x}^2 \int x^2 dx$$

$$= \frac{1}{2L^3} m \dot{x}^2 \cdot \left[\frac{x^3}{3}\right]_0^L = \frac{1}{6} \cdot m \cdot \dot{x}^2$$

$$= \tfrac{1}{6} m \dot{x}^2$$

$$(KE)_{Block} + (KE)_{spring} + (SE) = const.$$

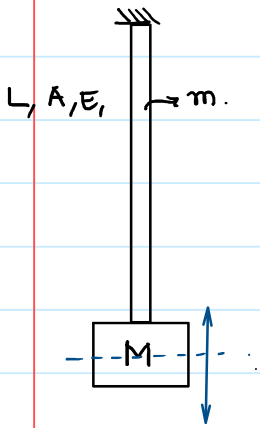
$$\frac{d}{dt}\left(\frac{1}{2} M \dot{x}^2 + \tfrac{1}{6} m \dot{x}^2 + \tfrac{1}{2} k x^2 = c.\right) \Rightarrow \cancel{\tfrac{1}{2}} \cdot M \cancel{2} \cancel{\dot{x}} \ddot{x} + \cancel{\tfrac{1}{6}}_3 m \cdot \cancel{2} \cancel{\dot{x}} \ddot{x} + \cancel{\tfrac{1}{2}} k \cancel{2} x \cdot \cancel{\dot{x}} = 0$$

$$M \ddot{x} + \frac{m}{3} \ddot{x} + k x = 0$$

$$\left(M + \frac{m}{3}\right)\ddot{x} + Kx = 0$$

Prismatic Rod.

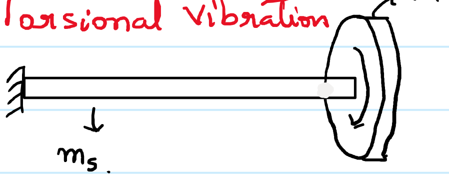
Longitudinal vibrations.



$$\left(M + \frac{m}{3}\right)\ddot{x} + K_{Axial} \cdot x = 0$$

$$\left(M + \frac{m}{3}\right)\ddot{x} + \left(\frac{AE}{L}\right)x = 0$$

Torsional Vibration



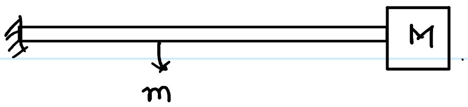
$$\left(I_{Rotor} + \frac{I_{shaft}}{3}\right)\ddot{\theta} + K_{Torsional} \cdot \theta = 0$$

M – Mass of Rotor.
m_s – Mass of shaft.

$$\left(I_{Rotor} + \frac{I_{shaft}}{3}\right)\ddot{\theta} + \frac{GJ}{L}\theta = 0$$

Transverse Vibration

$$(M + ml)\ddot{x} + K_{beam}x = 0$$



$$\omega_n = \sqrt{\frac{g}{\delta_{static}}}$$

$$\delta_{Point\,load} = \frac{MgL^3}{3EI}$$

m – mass of beam per unit lenght.
M – mass of block.

$$\delta_{UDL} = \frac{mgL^4}{8EI}$$

$$\delta_{Total} = \delta_{P.L.} + \frac{\delta_{UDL}}{1.27}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_{static}}} = \frac{0.495}{\sqrt{\delta_{P.L.} + \frac{\delta_{UDL}}{1.27}}}\ Hz.$$

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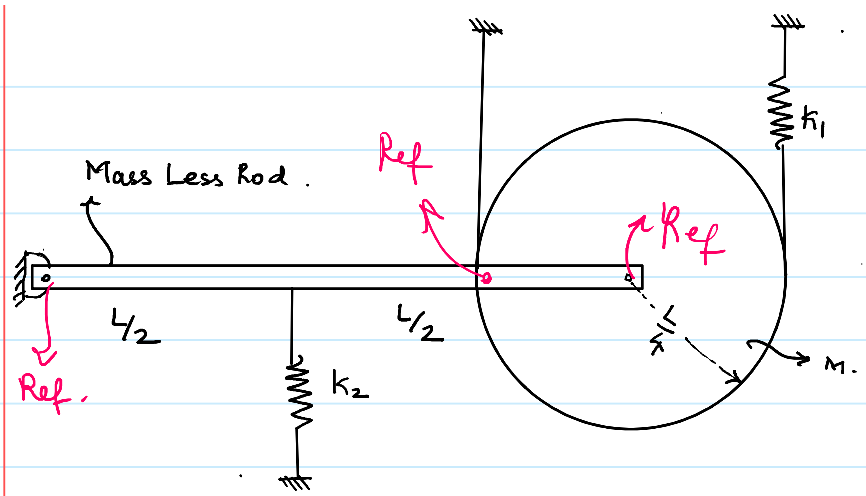
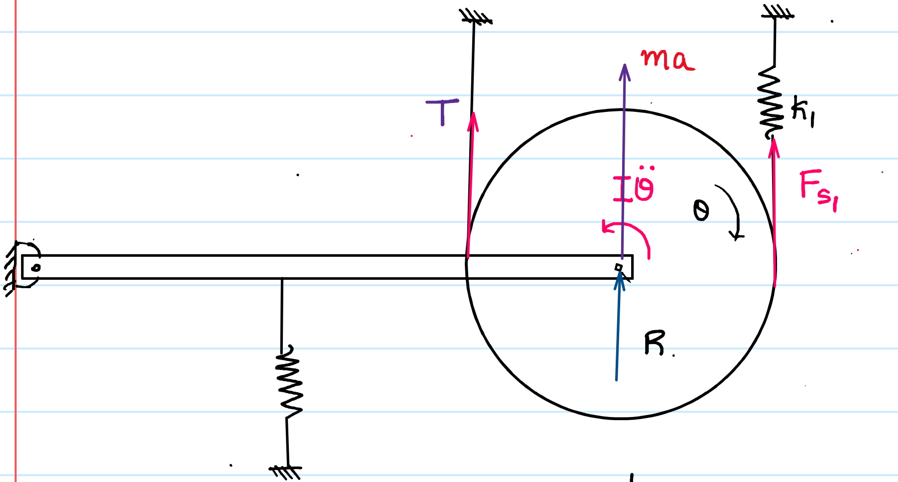
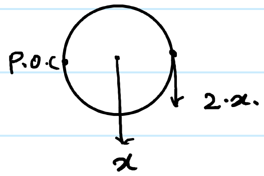
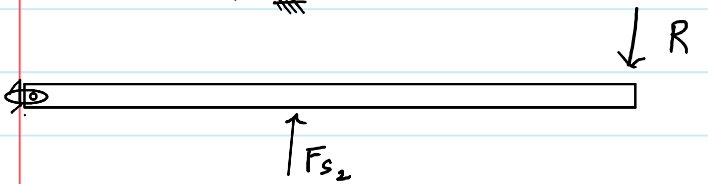




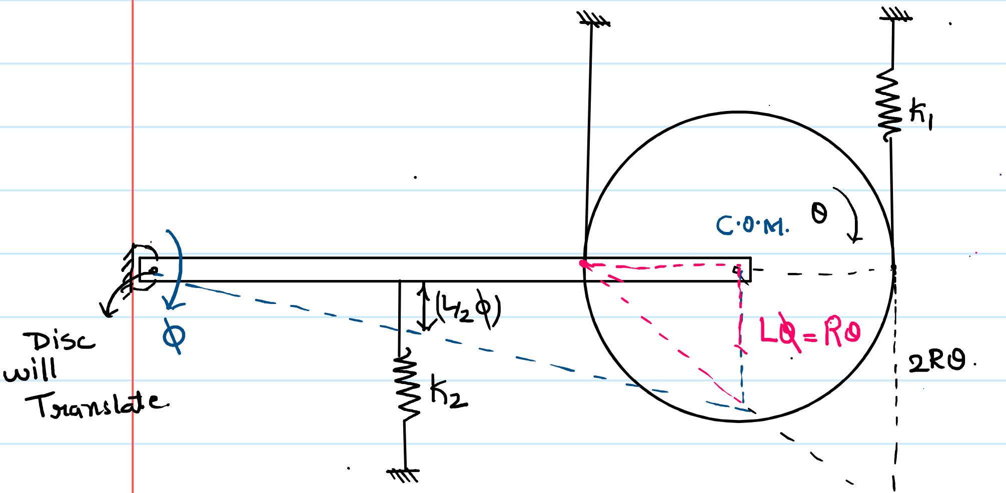
$\Sigma M_{c.o.m} = 0$

$\Sigma F_y = 0$





$\Sigma M = 0$



$R = L/4$

$L\phi = R\cdot\theta$

$\phi = \theta/4$

$\theta = 4\phi \Rightarrow \dot{\theta} = 4\dot{\phi}$

$I = \frac{MR^2}{2} = \frac{M(L/4)^2}{2} = \frac{ML^2}{32}$

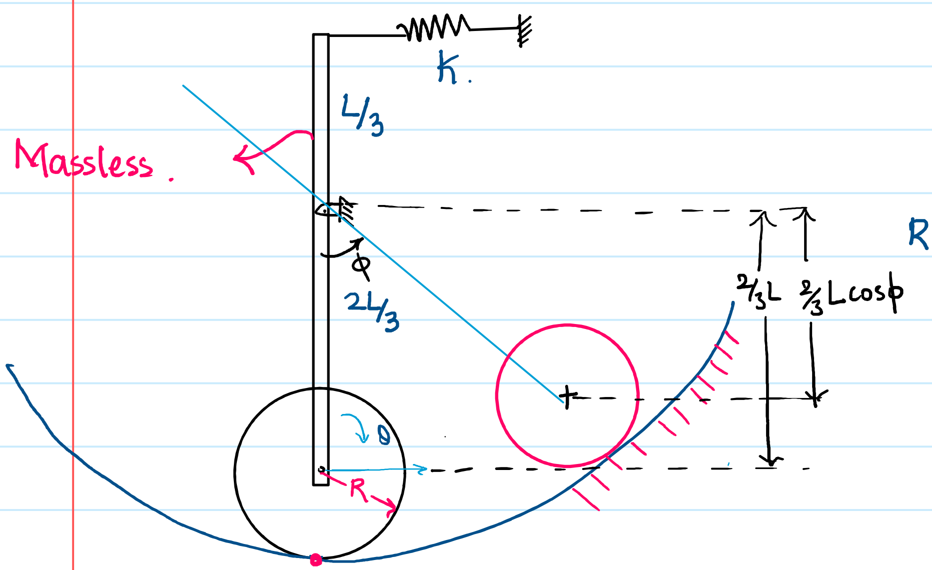
$$(KE)_{Rot} + (KE)_{Trans} + (SE)_1 + (SE)_2 = \text{constant}$$

$$\frac{1}{2}\cdot I\dot{\theta}^2 + \frac{1}{2}\cdot M\cdot(L\dot{\phi})^2 + \frac{1}{2}\cdot k_1(2R\theta)^2 + \frac{1}{2}k_2(L/2\,\phi)^2 = C$$

$$\frac{d}{dt}\left[\frac{1}{2}\cdot\left(\frac{ML^2}{32}\right)(4\dot{\phi})^2 + \frac{1}{2}\cdot ML^2\dot{\phi}^2 + \frac{1}{2}\cdot k_1\left(L/4\cdot 4\phi\right)^2\cdot 4 + 1/2\cdot k_2\cdot(L/2\,\phi)^2 = C\right]$$

$$\frac{1}{\cancel{2}}\cdot\frac{ML^2}{32}\times 16\cdot 2\cdot\cancel{\dot{\phi}}\,\ddot{\phi} + \frac{1}{\cancel{2}}ML^2\cdot\cancel{2}\cancel{\dot{\phi}}\ddot{\phi} + \frac{1}{\cancel{2}}\cdot k_1L^2\cdot\cancel{2}\phi\cdot\cancel{\dot{\phi}}\times 4 + \frac{1}{\cancel{2}}\cdot k_2\cdot\frac{L^2}{4}\cdot\cancel{2}\phi\cancel{\dot{\phi}} = 0$$

$$\left(\frac{ML^2}{2} + ML^2\right)\ddot{\phi} + \left(4k_1 + k_2/4\right)L^2\phi = 0$$



$$R\theta = \tfrac{2}{3} L\,\phi$$

$$R\dot{\theta} = \tfrac{2}{3} L\,\dot{\phi}$$

$$R\ddot{\theta} = \tfrac{2}{3} L\,\ddot{\phi}$$

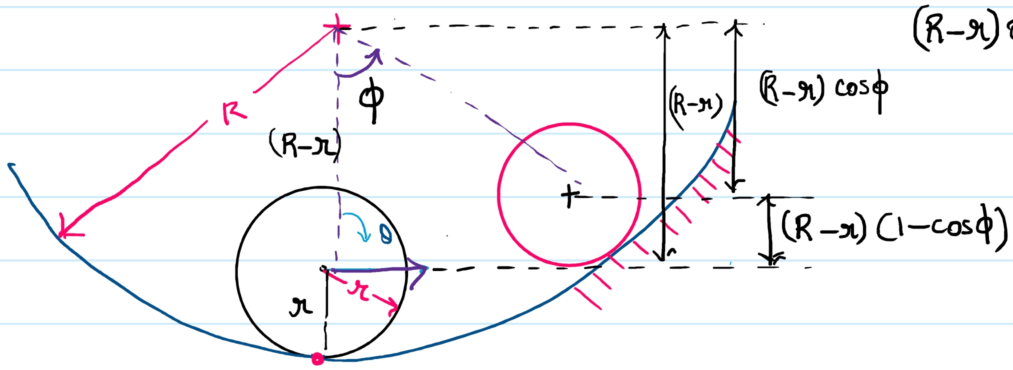
$$(KE)_{Rot} + (KE)_{Trans} + (SE) + mg\,\tfrac{2}{3}L[1-\cos\phi] = \text{const.}$$

$$\frac{1}{2}\left(\frac{MR^2}{2}\right)\dot{\theta}^2 + \frac{1}{2}\cdot M\cdot\left(\frac{2}{3}L\dot{\phi}\right)^2 + \frac{1}{2}\cdot k\cdot\left(\frac{L}{3}\phi\right)^2 + mg\,\tfrac{2}{3}L[1-\cos\phi] = C.$$

(2)

Pure Rolling

$$(R-r)\phi = r\theta.$$

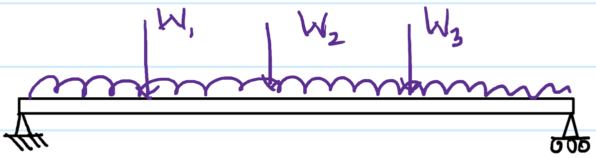


$$(PE) + (KE)_{Rot} + (KE)_{Trans} = C$$

$$mg(R-r)[1-\cos\phi] + \frac{1}{2}\left(\frac{Mr^2}{2}\right)\dot{\theta}^2 + \frac{1}{2}M\cdot(R-r)^2\cdot\dot{\phi}^2 = C$$

If $\omega_1, \omega_2, \omega_3, \ldots$ are natural frequencies of a Multi DOF system, then the lowest natural frequency is given by

$$\frac{1}{\omega^2} = \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} + \frac{1}{\omega_3^2} + \cdots\cdots$$



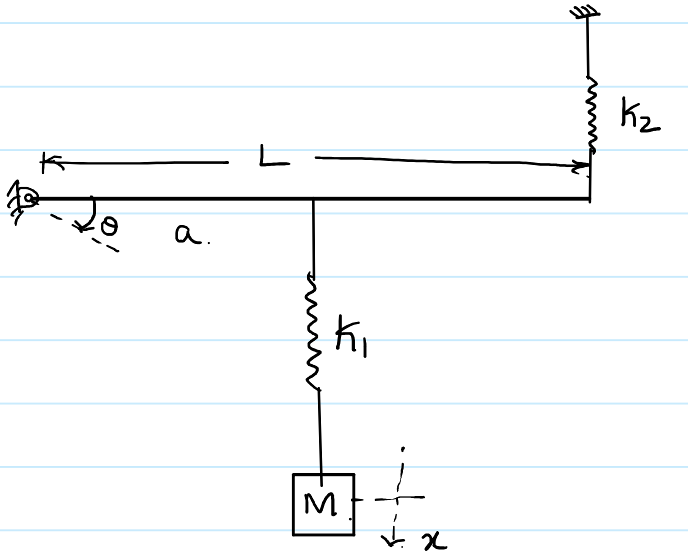
δ_1 – Deflection due to load W_1

δ_2 – " " " " W_2

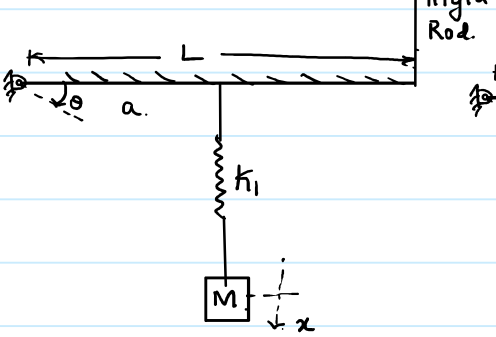
⋮

lowest natural frequency

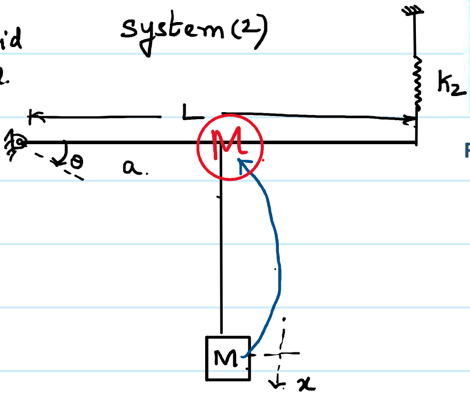
$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_{Total}}} \Rightarrow f_n = \frac{0.495}{\sqrt{\delta_1 + \delta_2 \cdots + \frac{\delta_{UDL}}{1.27}}}$$



System (1)



system (2)



$$M\ddot{x} + k_1 x = 0$$

$$\ddot{x} + \frac{k_1}{M}\cdot x = 0$$

$$\omega_1 = \sqrt{\frac{k_1}{M}}$$

$$\sum M_{Hinge} = 0$$

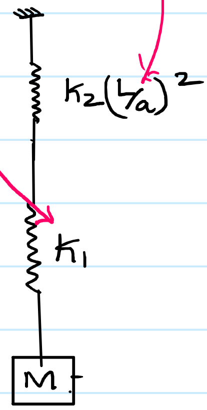
$$Ma^2\ddot{\theta} + k_2 L^2 \theta = 0$$

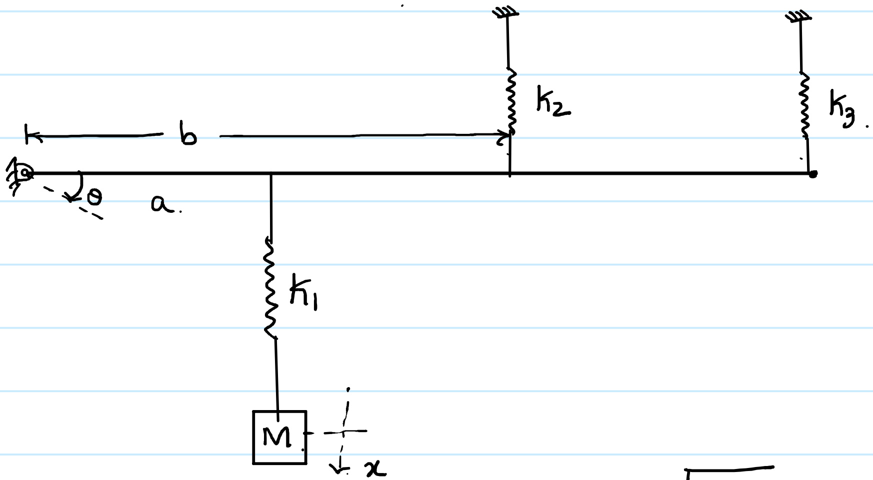
$$\ddot{\theta} + \frac{k_2 L^2}{Ma^2}\theta = 0$$

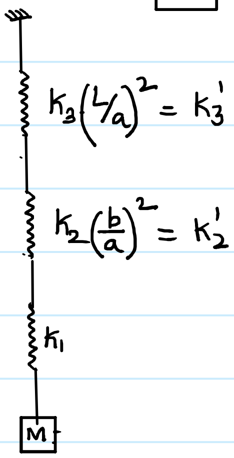
$$\omega_2 = \sqrt{\frac{k_2}{M}\left(\frac{L}{a}\right)^2}$$

$$\frac{1}{\omega^2} = \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2}$$

$$\omega = \sqrt{\frac{\omega_1^2 \cdot \omega_2^2}{\omega_1^2 + \omega_2^2}} = \sqrt{\frac{\left(\frac{k_1}{M}\right)\cdot\left(\frac{k_2}{M}\right)\left(\frac{L}{a}\right)^2}{\left(\frac{k_1}{M}\right) + \left(\frac{k_2 L^2}{Ma^2}\right)}} = \sqrt{\frac{k_1 \cdot k_2\left(\frac{L}{a}\right)^2}{M\left\{k_1 + k_2\left(\frac{L}{a}\right)^2\right\}}}$$

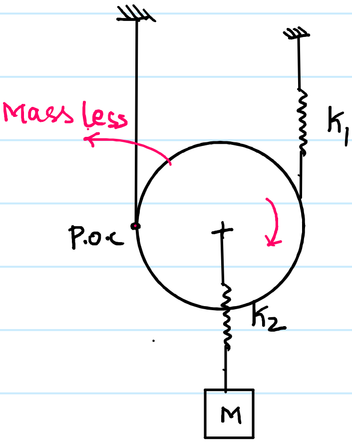






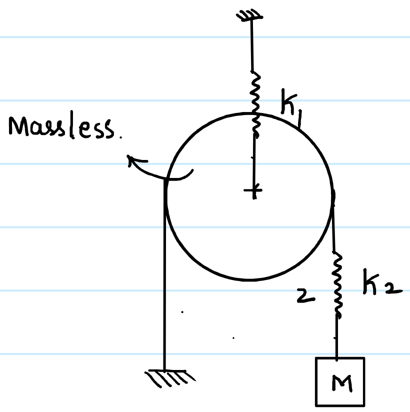
$$\omega_n = \sqrt{\frac{k_e}{M}}$$

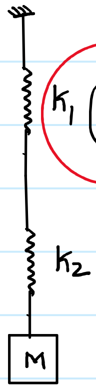
$$k_e = \frac{k_1 \cdot k_2' \cdot k_3'}{k_1 k_2' + k_2' k_3' + k_3' k_1}$$



$$k_1\left(\frac{2R}{R}\right)^2 = 4k_1$$

$$\omega_n = \sqrt{\frac{4k_1k_2}{(4k_1+k_2)M}}$$

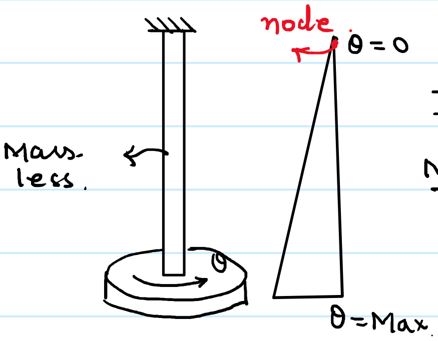




$$k_1\left(\frac{R}{2R}\right)^2 = \frac{k_1}{4}$$

$$\rightarrow k_1 \times \frac{(\perp^{lar}\text{ distance of spring from P.o.c})^2}{(\perp^{lar}\text{ distance of mass from P.o.c})^2}$$

Torsional Vibrations



S.D.O.F

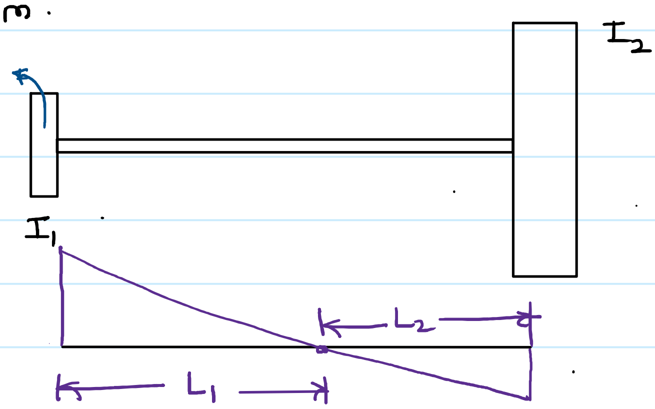
$$I\ddot{\theta} + k_T\theta = 0$$

$$\frac{MR^2}{2}\ddot{\theta} + \frac{GJ}{L}\theta = 0$$

$$\ddot{\theta} + \frac{GJ}{IL}\theta = 0 \qquad \omega_n = \sqrt{\frac{GJ}{IL}}$$

node – is a point having no displacement.

2. rotor system.



$$\omega_1 = \omega_2$$

$$\sqrt{\frac{GJ}{I_1L_1}} = \sqrt{\frac{GJ}{I_2 \cdot L_2}}$$

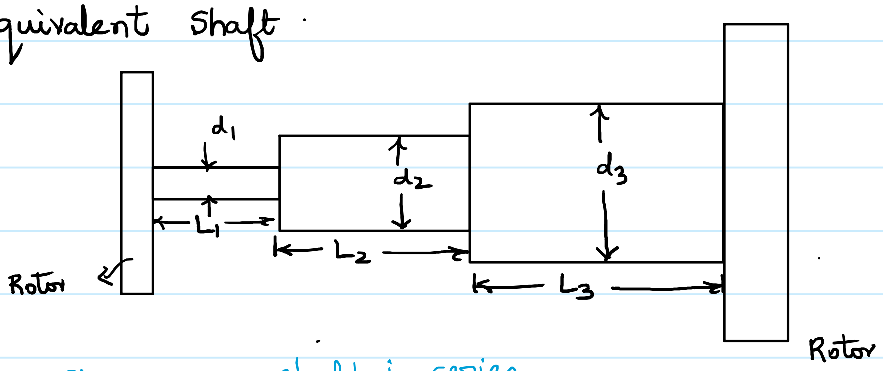
L_1, L_2 – distance of rotors 1, 2 from the node.

$$I \propto \frac{1}{\text{distance of node from rotor}}$$

$$\boxed{I_1L_1 = I_2L_2}$$

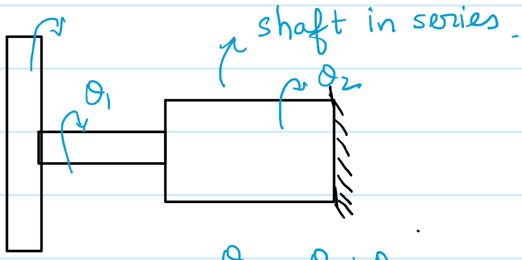
Number of nodes = No. of rotors – 1.

Equivalent Shaft.



$$T_1 = T_2 = T_3 = T_e$$

$$\theta = \frac{TL}{GJ}$$



$$\theta_e = \theta_1 + \theta_2$$

$$\theta_e = \theta_1 + \theta_2 + \theta_3 + \dots$$

$$\frac{T_e \cdot L_e}{G \cdot J_e} = \frac{T_1 L_1}{G \cdot J_1} + \frac{T_2 \cdot L_2}{G \cdot J_2} + \dots$$

$$J_e = \frac{\pi}{32} d_e^4$$

$d_e = d_1$

$d_e = d_2$

$$L_e = J_e \left[\frac{L_1}{J_1} + \frac{L_2}{J_2} + \dots \right]$$

$$L_e = d_e^4 \left[\frac{L_1}{d_1^4} + \frac{L_2}{d_2^4} + \dots \right]$$

$$L_e = L_1 \left(\frac{d_e}{d_1} \right)^4 + L_2 \left(\frac{d_e}{d_2} \right)^4 + L_3 \left(\frac{d_e}{d_3} \right)^4 + \dots$$

Gyroscope — Gyre → German word → circular motion

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Translation.

Linear Moment $\quad \vec{P} = m\vec{v}$

$$P.\hat{P} = m.v\hat{v}$$

$$P = mv \quad , \quad \hat{P} = \hat{v}$$

Newton 2nd law. $\quad \frac{d\vec{P}}{dt} = \vec{F}$

$$\vec{F} = m\vec{a}.$$

$$\hat{F} = \hat{a}$$

Angular Motion → cross-product.

Angular Moment $\quad L = mvr.$

$$\vec{L} = m.(\vec{r} \times \vec{v}) \quad \Big\} \text{ point mass.}$$

$$\vec{L} = I.\vec{\omega} \rightsquigarrow \text{complex body.}$$

$$L.\hat{L} = I.\omega.\hat{\omega} \qquad \hat{L} = \hat{\omega}, \quad L = I\omega.$$

Newton 2nd. $\quad \text{Torque} = \frac{d\vec{L}}{dt}$

$I = $ const.
ω – variable
$\hat{\omega}$ – variable.

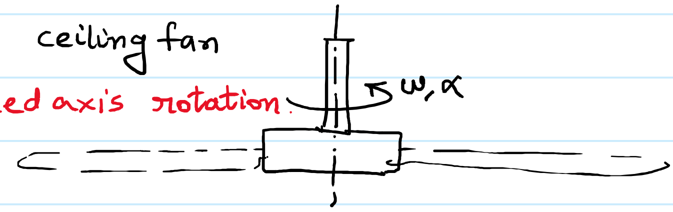
$$\vec{\tau} = \frac{d}{dt}(I.\omega.\hat{\omega})$$

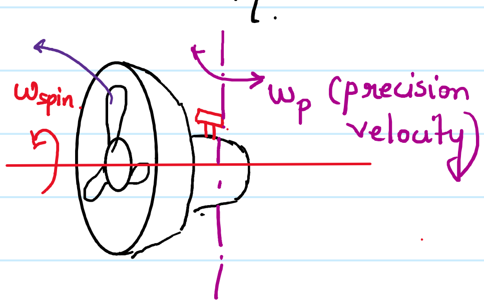
$$\vec{\tau} = I.\frac{d\omega}{dt}.\hat{\omega} + \underbrace{I.\omega.\frac{d\hat{\omega}}{dt}}$$

$$= I\alpha.\hat{\alpha} + I.\omega.\omega_p. \qquad \text{Gyroscopic Torque.} \qquad (\hat{\omega} = \hat{\alpha})$$

$I\alpha\hat{\alpha}$ – Torque exerted due to rate of change in angular velocity.
→ Fixed axis rotation.

$I\omega.\omega_p$ → Torque exerted due to rate of change in direction of angular velocity.
→ axis of rotation is moving with some velocity about another axis.





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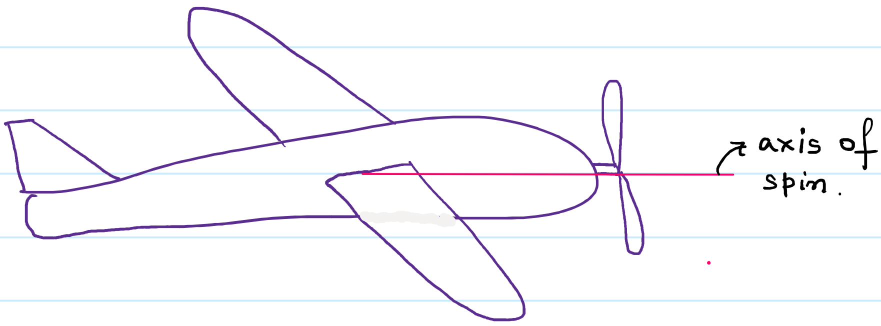
Effect of Gyroscope on
1. Aeroplane
2. Naval Ship
3. 4 wheeler
4. 2 wheeler

Effect of Gyroscope on the Aeroplane

1. Location of observer - Rear/Front
2. Direction of spin - CW/CCW
3. Direction of Precision (Steering) - Left/Right
   (Pitching) - Upwards/Downwards



Active Gyroscopic Torque $\vec{C} = I.(\vec{\omega_p} \times \vec{\omega_s})$

Reactive Gyroscopic Torque $\vec{C} = I.(\vec{\omega_s} \times \vec{\omega_p})$

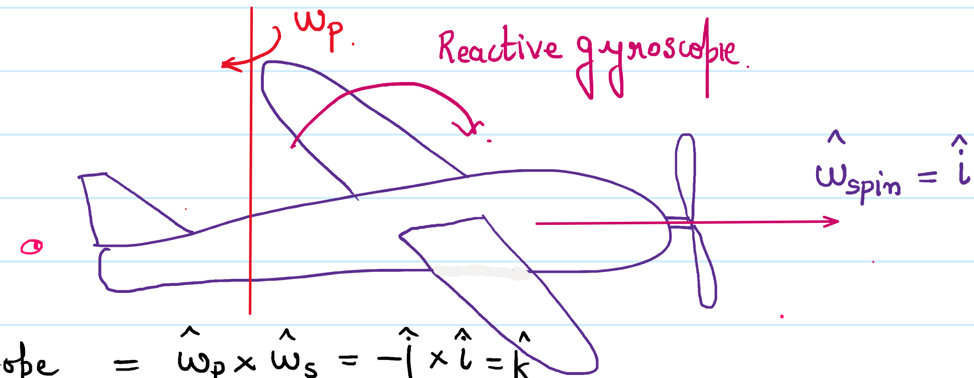
$\vec{C}, \vec{\omega_s}, \vec{\omega_p}$ are mutually $\perp^{lar}$ to each other.

case (i)

1. Location of observer - Rear
2. Direction of spin - CW
3. Direction of Precision (Steering) - Right

$\hat{\omega}_{spin} = \hat{i}$

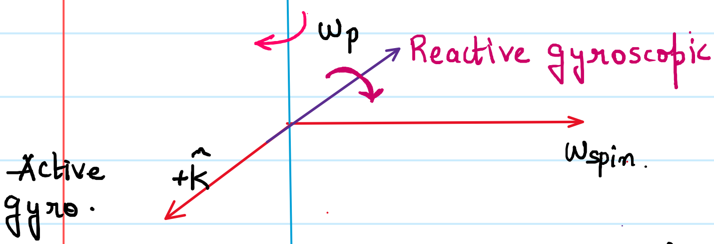
$\hat{\omega}_p = -\hat{j}$



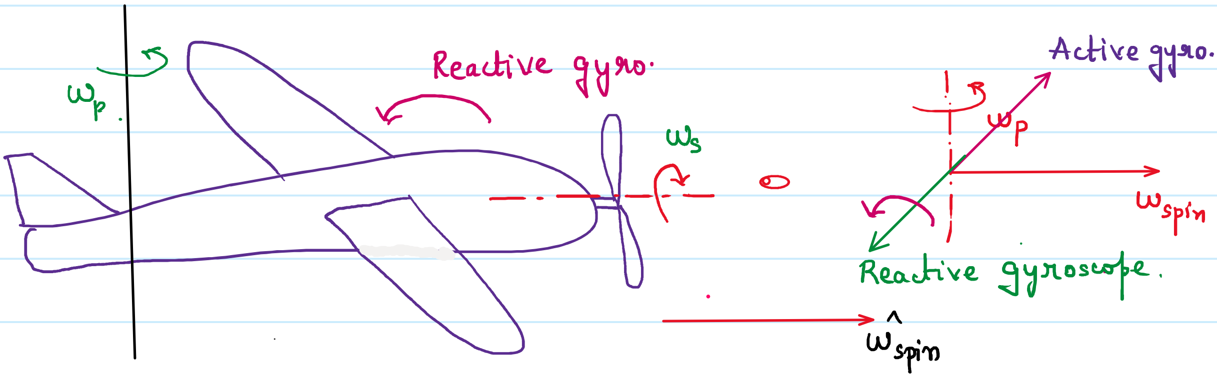
Active gyroscope $= \hat{\omega}_p \times \hat{\omega}_s = -\hat{j} \times \hat{i} = \hat{k}$

Reactive gyroscope $= \hat{\omega}_s \times \hat{\omega}_p = i \times -j = -\hat{k}$. (C.W)

Due to reactive gyroscope nose is dipped and tail is raised.



Rotate ω_{spin} in the direction of ω_p by 90°



1. Location of observer - Front
2. Direction of spin - CCW
3. Direction of Precision (Steering) - Left

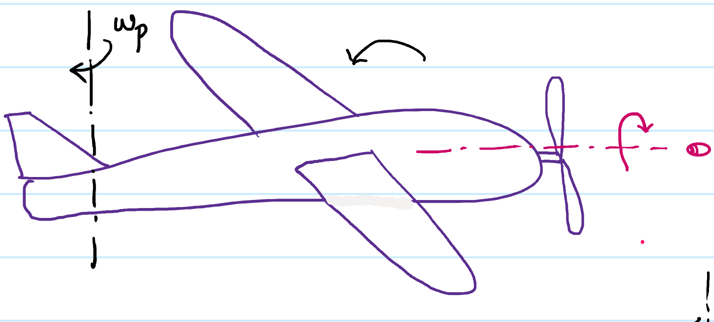
$\hat{\omega}_s = +\hat{i}$

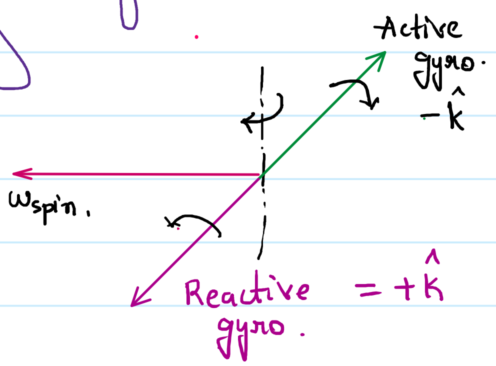
$\hat{\omega}_p = +\hat{j}$

Active gyro – $\hat{\omega}_p \times \hat{\omega}_s = \hat{j} \times \hat{i} = -\hat{k}$

Reactive gyro. – $\hat{\omega}_s \times \hat{\omega}_p = i \times j = +k$.

Effect of Reactive gyro. is to raise the nose and dip the tail.





1. Location of observer - Front
2. Direction of spin - CW
3. Direction of Precision (Steering) - Right

Active gyro. $= \hat{\omega}_p \times \hat{\omega}_s = -\hat{j} \times -\hat{i} = -\hat{k}$ $\quad \hat{\omega}_s = -\hat{i}$

Reactive gyro. $= \hat{\omega}_s \times \hat{\omega}_p = -\hat{i} \times -\hat{j} = +\hat{k}$ $\quad \hat{\omega}_p = -\hat{j}$

Due to Reactive gyro. nose is raised and tail is dipped.

Effect of Gyroscope on Naval Ships

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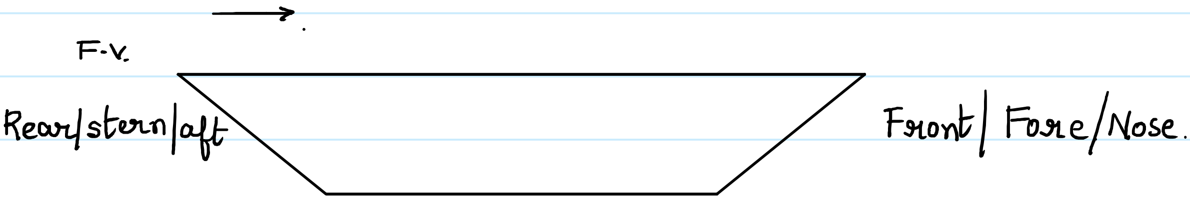
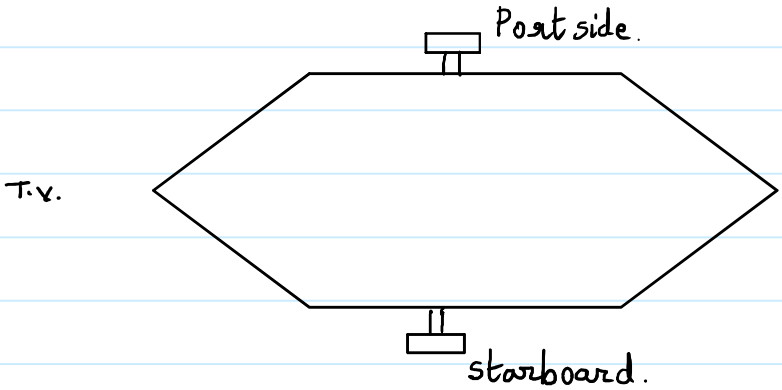
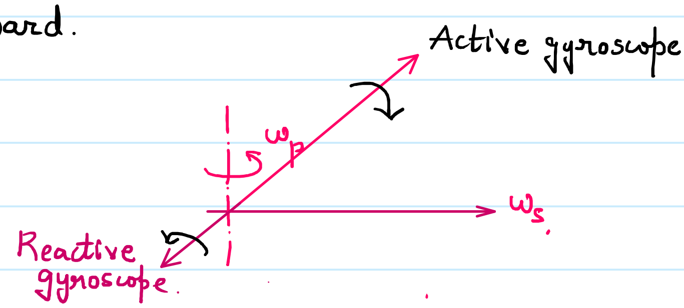
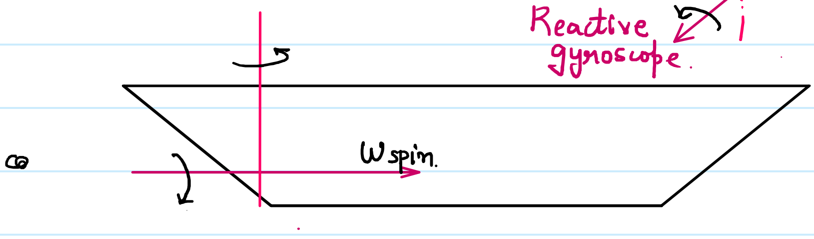




1. Location of observer - Rear
2. Direction of spin - CW
3. Direction of Precision (Steering) - Left



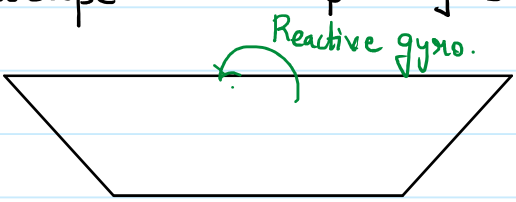


$\hat{\omega}_s = \hat{i}$

$\hat{\omega}_p = +\hat{j}$

Active gyroscope $= \hat{\omega}_p \times \hat{\omega}_s = \hat{j} \times \hat{i} = -\hat{k}$

Reactive gyroscope $= \hat{\omega}_s \times \hat{\omega}_p = \hat{i} \times \hat{j} = +\hat{k}$ (c.c.w)



Due to the effect of reactive gyro the stern is dipped and fore is raised.

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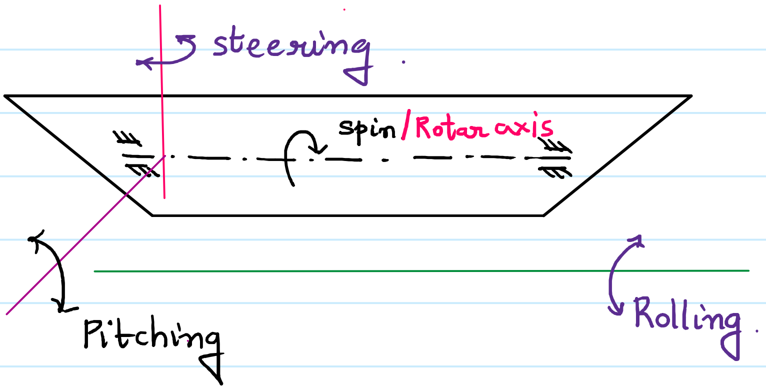
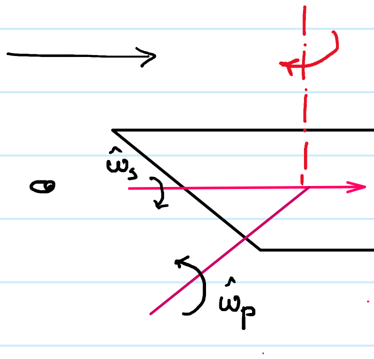


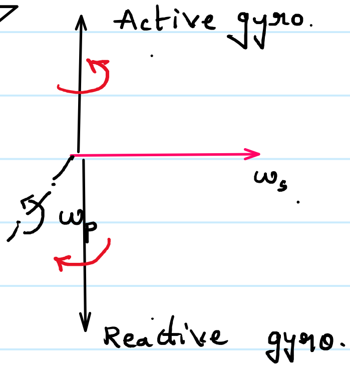
Pitching

1. Location of observer - Rear
2. Direction of spin - CW
3. Direction of Precision (Pitching) - Upwards



$\hat{\omega}_s = \hat{i}$

$\hat{\omega}_p = +\hat{k}$

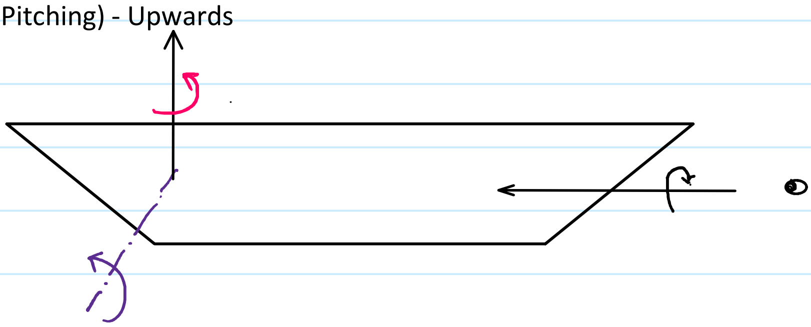


Active gyro. $= \hat{\omega}_p \times \hat{\omega}_s = \hat{k} \times \hat{i} = \hat{j}$

Reactive gyro. $= \hat{\omega}_s \times \hat{\omega}_p = \hat{i} \times \hat{k} = -\hat{j}$

The ship will turn towards right side / star board.

1. Location of observer - Front
2. Direction of spin - CW
3. Direction of Precision (Pitching) - Upwards



$\hat{\omega}_s = -\hat{i}$

$\hat{\omega}_p = +\hat{k}$

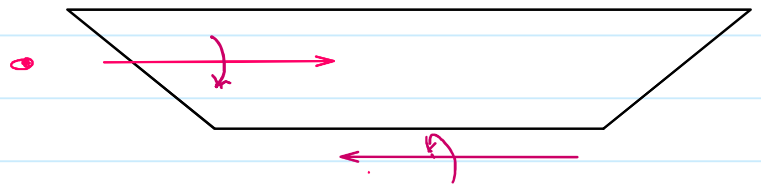
Active gyro. $= \hat{\omega}_p \times \hat{\omega}_s = \hat{k} \times -\hat{i} = -\hat{j}$

Reactive gyro $= \hat{\omega}_s \times \hat{\omega}_p = -\hat{i} \times \hat{k} = +\hat{j}$

Ship will turn to left side / Portside.

Rolling Motion

1. Location of observer - Rear
2. Direction of spin - CW
3. Direction of Precision (Rolling) - CCW



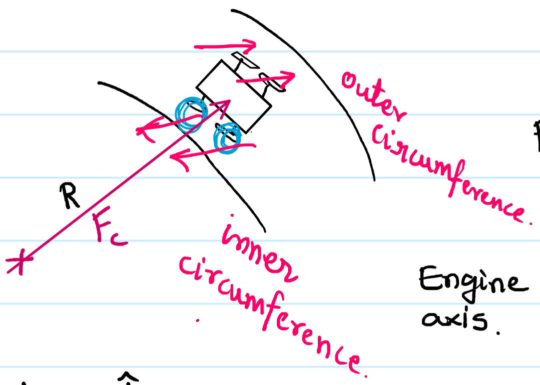
$\hat{\omega}_s = \hat{i}$

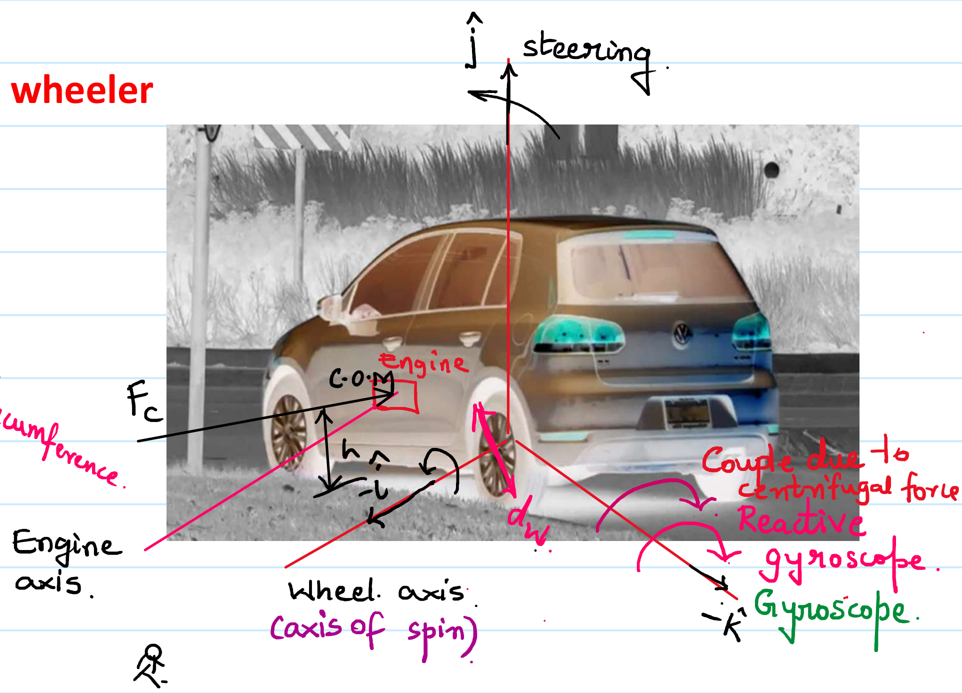
$\hat{\omega}_p = -\hat{i}$

$$\text{Active gyro} = \hat{\omega}_p \times \hat{\omega}_s = -\hat{i} \times \hat{i} = 0$$

$$\text{Reactive gyro} = \hat{\omega}_s \times \hat{\omega}_p = \hat{i} \times -\hat{i} = 0$$

Effect of Gyroscope on 4 wheeler





$\hat{\omega}_s = -\hat{i}$

$\hat{\omega}_p = +\hat{j}$

$$\text{Active gyro} = \hat{\omega}_p \times \hat{\omega}_s = \hat{j} \times -\hat{i} = +\hat{k}$$

$$\text{Reactive gyro} = \hat{\omega}_s \times \hat{\omega}_p = -\hat{i} \times \hat{j} = -\hat{k}$$

Due to the effect of Reactive gyro. the wheel on inner circumf-erence will be lifted from ground, wheels on outer circumference will be pressed against the ground.

Vehicle will tend to experience overturning due to Reactive gyro & Couple due to centrifugal force.

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$$\omega_p = \frac{V}{R}$$

V – Velocity of vehicle
R – Turning Radius.

Gear Ratio $= \frac{\omega_E}{\omega_W} = \quad \omega_E = G\cdot\omega_W$.

Gyroscope on Engine $= I_E\cdot\omega_E\cdot\omega_p$

$$= I_E\cdot G\,\omega_W\cdot\left(\frac{V}{R}\right)$$

Gyroscope on wheel $= 4\cdot I_W\cdot\omega_W\cdot\omega_p$

$$= \left(4\cdot I_W\cdot\frac{V}{R_W}\cdot\frac{V}{R}\right)$$

Angular velocity of wheel.

$$\omega_W = \frac{V}{R_W}.$$

$$C_E \pm C_W = I_E\cdot G\cdot\frac{V}{R_W}\cdot\frac{V}{R} \pm 4 I_W\cdot\frac{V^2}{R\cdot R_W}$$

R_W – Radius of wheel.

Net Gyroscopic Torque.

$$C_1 = \underbrace{(G\cdot I_E \pm 4 I_W)}\cdot\frac{V^2}{R\cdot R_W}$$

$$C_1 = I_{eq}\cdot\alpha_{eq}$$

I_{eq} – Equivalent Inertia
α_{eq} – Gyroscopic Acceleration.

Couple due to centrifugal force.

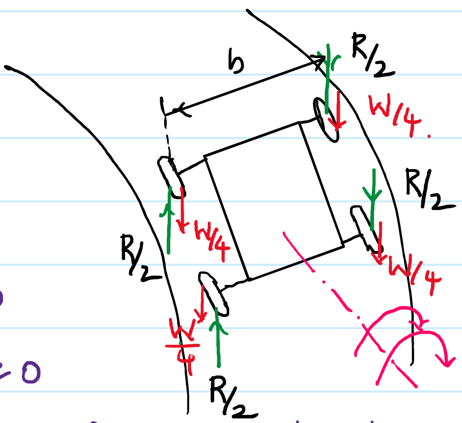
$$C_2 = F_c\times h = \frac{mV^2}{R}\times h.$$

Net Couple $= C_1 + C_2 = \left[(G\cdot I_E \pm 4 I_W)\cdot\frac{V^2}{R\cdot R_W} + \frac{mV^2}{R}\times h\right]$

Net couple on each axle.

$$= \frac{C_{Net}}{2} = \frac{R}{2}\times b.$$

$$\frac{R}{2} = \frac{C_{Net}}{2b}.$$



Reaction on outer wheel $= \frac{W}{4} + R/2$

Reaction on inner wheel $= W/4 - \frac{R}{2} \geq 0$

$$= \frac{W}{4} - \frac{C_{Net}}{2b} \geq 0$$

In order to maintain stability the value of V must be controlled.

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Condition of stability.

$$\frac{W}{4} - \frac{C_{Net}}{2b} \geq 0$$

$$\frac{W}{4} - \frac{1}{2b}\left[\frac{(G \cdot I_E \pm 4 I_W)}{R \cdot R_W} + \frac{m \cdot h}{R}\right] \cdot V^2 \geq 0$$

$$V^2 \leq \frac{W/4}{\frac{1}{2b}\left[(G I_E \pm 4 I_W) + \frac{mh}{R}\right]}$$

Effect of Gyroscope on 2 wheeler

$$\hat{\omega}_s = \hat{i}$$

$$\hat{\omega}_p = -\hat{j}$$

Active gyro. $= \hat{\omega}_p \times \hat{\omega}_s = -\hat{j} \times \hat{i} = \hat{k}$

Reactive gyro $= \hat{\omega}_s \times \hat{\omega}_p = \hat{i} \times -\hat{j} = -\hat{k}$

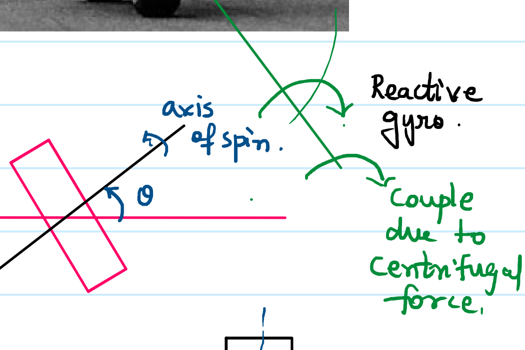
axis of spin.

Gyroscope on 2 wheel.

$$C = C_W + C_E$$

$$C = 2 \cdot I_W \cdot \omega_W \cdot \omega_P + I_E \cdot \omega_E \cdot \omega_P$$

$$= 2 \cdot I_W \cdot \frac{V}{R_W} \cos\theta \cdot \frac{V}{R} + I_E \cdot G \cdot \frac{V}{R_W} \cos\theta \cdot \frac{V}{R}$$

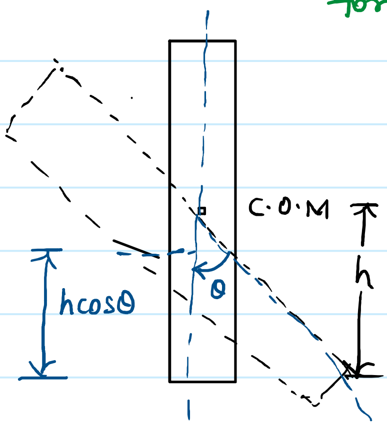


$$C_1 = (2 I_W + G I_E) \cdot \frac{V^2}{R \cdot R_W} \cos\theta \quad = I_{eq} \cdot \alpha_{eq}$$

Couple due to centrifugal force.

$$C_2 = F_c \times h\cos\theta$$

$$= \frac{mV^2}{R} \times h\cos\theta$$



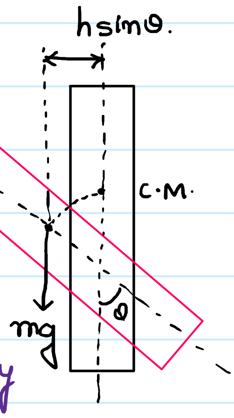
Net couple on vehicle

$$= C_1 + C_2$$

$$= \left[\left(2 \cdot I_w + G I_E\right) \cdot \frac{V^2}{R \cdot R_w} + \frac{mV^2}{R} \cdot h\right] \cos\theta \longrightarrow A$$

Balancing couple $= mg \cdot h \sin\theta \longrightarrow$ (B)

$A = B$



Centrifugal force and Reactive gyroscope together will cause disturbance/ unstability

MSQ

01. When a four-wheeler moving forward at a speed just above critical speed for stability (Assume the engine rotates on a parallel axis to the wheels in the same direction) takes a turn to the right the wheel(s) that tends to leave the ground is
(a) outer front wheel
(b) outer rear wheel
(c) both the inner wheels → Due reactive gyro. + couple due centrifugal force wheel are lifted.
(d) none of the four wheels

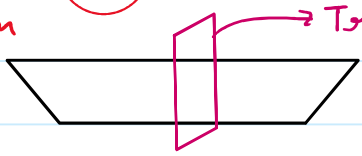
02. Pitching motion of ship, carrying a rotor that rotates in clockwise sense as seen from stern produces couple
Rolling motion {
(a) ~~in transverse vertical plane in clockwise sense as seen from stern~~
(b) ~~in transverse vertical plane in counterclockwise sense as seen from stern~~
}
(c) in longitudinal vertical plane → Pitching motion
(d) in horizontal plane → Gyroscope.

07. Which of the following statement(s) is/are CORRECT?
(a) Spin, Precession and Gyroscope axes are mutually perpendicular to each other. ✓
(b) Gyroscope couple becomes zero, if spin and precession axes become collinear. ✓ ← Rolling motion
(c) ~~Gyroscope effect and centrifugal force effects are opposing each other in case of automobiles.~~
(d) Rolling of ship causes no gyroscope effect. ✓

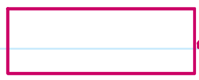
$\hat{\omega}_s = \pm\hat{i}$

$\hat{\omega}_p = \pm\hat{k}$

Gyroscope $= \pm\hat{j}$ ↑ plane normal to Y-axis.



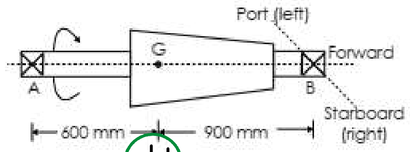
→ Tranverse vertical plane



→ longitudinal vertical plane

Horizontal plane.

03. The turbine rotor in a ship's power plant has a mass of 1000 kg with center of mass at G and a radius of gyration of 200 mm. The rotor shaft is mounted in bearings A and B with its axis in the horizontal fore-and-aft direction and turns ccw at a speed of 5000 rpm when viewed from the stern. Determine the vertical components of the bearing reactions at A and B if the ship is making a turn to port (left) of 400 m radius at a speed of 25 knots (1 knot = 0.514 m/s). Does the bow of the ship tend to rise or fall because of gyroscopic action.



$M = 1000\,kg$

$K = 200\,mm$.

$N = 5000\,rpm$. (spin) c.c.w. from stern.

turning left.

$R = 400\,m$ $V = 25$ Knots.

$V = 25 \times 0.514$ m/s.

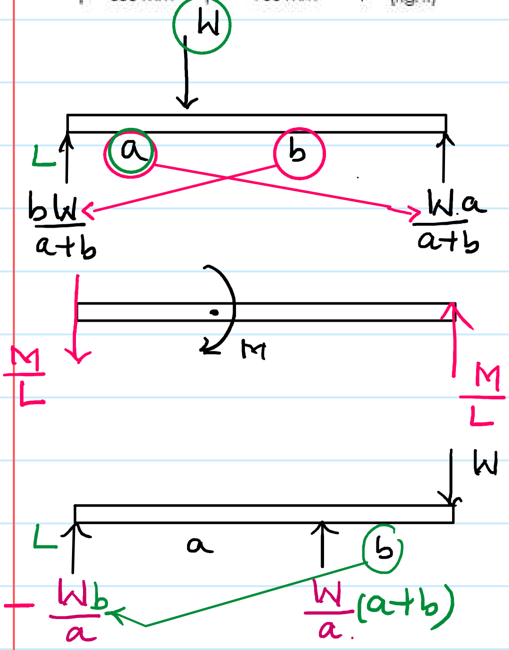
$V = 12.85$ m/s.

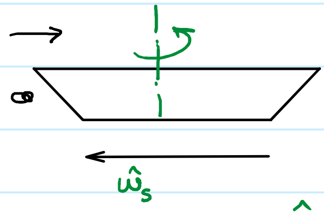
$C = I \cdot \omega_s \omega_p$.

$= mk^2 \cdot \left(\frac{2\pi N}{60}\right) \left(\frac{V}{R}\right)$

$= 1000 \times 0.2^2 \times \left(\frac{2\pi \times 5000}{60}\right) \times \left(\frac{12.85}{400}\right)$

$C = 672.824$ N-m.





$\hat{\omega}_s = -\hat{i}$

$\hat{\omega}_p = +\hat{j}$

Active gyro $= \hat{\omega}_p \times \hat{\omega}_s$

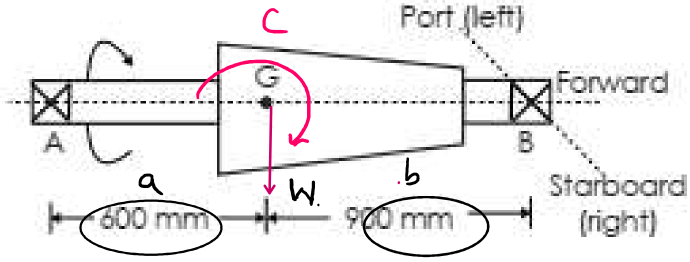
$= \hat{j} \times -\hat{i} = +\hat{k}$ c.c.w.

Reactive gyro. $= \hat{\omega}_s \times \hat{\omega}_p = -\hat{i} \times \hat{j} = -\hat{k}$ c.w.

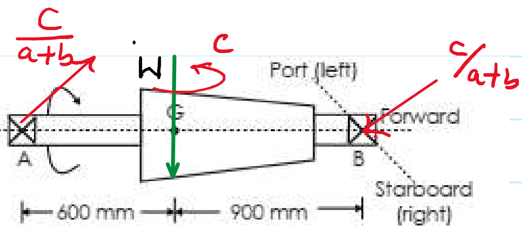
steering. Left.

$R_A = \frac{Wb}{a+b} - \frac{C}{a+b}$

$R_B = \frac{Wa}{a+b} + \frac{C}{a+b}$



Pitching Motion.



$\omega_s = \pm\hat{i}$ $C = \pm\hat{j}$

$\omega_p = \pm\hat{k}$

$R_A = \sqrt{\left(\frac{Wb}{a+b}\right)^2 + \left(\frac{C}{a+b}\right)^2}$

$R_B = \sqrt{\left(\frac{Wa}{a+b}\right)^2 + \left(\frac{C}{a+b}\right)^2}$

04. The 210 kg rotor of a turbojet aircraft engine has a radius of gyration of 220 mm and rotates ccw at 18,000 rpm as viewed from the front. If the aircraft is travelling at 1200 km/h and starts to execute an inside vertical loop of 3800 m radius, compute the gyroscopic moment 'M' transmitted to the airframe. What correction to the controls does the pilot have to make in order to remain in vertical plane?

$M = 210\,kg$

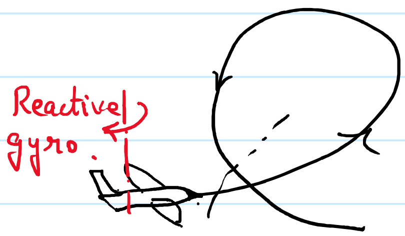
$k = 220\,mm$

$N = 18000\,rpm$. c.c.w from front.

$V = 1200\,kmph$

$R = 3800\,m$.

vertical loop



$M = ?$

$$M = I.\omega.\omega_p$$

$$= mk^2 \cdot \left(\frac{2\pi N}{60}\right)\left(\frac{V}{R}\right)$$

$$= 210 \times 0.22^2 \times \left(\frac{2\pi \times 18000}{60}\right)\left(\frac{1200 \times 1000}{3600 \times 3800}\right)$$

$$= \underline{1680.58}\ N\text{-}m.$$

$\longrightarrow \omega_{spin}$

$\hat{\omega}_{spin} = \hat{i} \qquad \hat{\omega}_p = +\hat{k}$

Active gyro $= \hat{\omega}_p \times \hat{\omega}_s = \hat{k} \times \hat{i} = \hat{j}$

Reactive gyro $= \hat{\omega}_s \times \hat{\omega}_p = \hat{i} \times \hat{k} = -\hat{j}$

Reactive gyro will cause aeroplane to turn right side, so the correction in control is to turn the aeroplane towards left, and vertical loop can be completed.

05. A car is moving on a curved horizontal road of radius 100 m with a speed of 20 m/s. The rotating masses of the engine have an angular speed of 100 rad/s in clockwise direction when viewed from the front of the car. The combined moment of inertia of the rotating masses is 10 kg-m². The magnitude of the gyroscopic moment (in N-m) is_____.

(GATE – 16)

$R = 100\,m$

$V = 20\,m/s$

$\omega_s = 100\,rad/s$

$I = 10\,kg\text{-}m^2$

$$C = I.\omega_s.\omega_p$$

$$= 10 \times 100 \times \frac{20}{100} = \underline{200\,N\text{-}m}.$$