

## B) DYNAMIC BALANCING:

- A system is said to be dynamically balanced if the force polygon as well as couple polygon both are closed.

Since the force polygon is closed:

Since the couple polygon is closed:

$$\sum F_x = 0 \quad \sum F_y = 0$$

$$\sum M_x = 0 \quad \sum M_y = 0$$

- A system which is dynamically balanced will also be statically balanced whereas reverse is not true.

$m_A, m_B, m_C, m_D$  -----

are unbalanced masses.

$e_A, e_B, e_C, e_D$  -----

are radial position of unbalanced masses.

$\theta_A, \theta_B, \theta_C, \theta_D$  -----

are angular pos'n. of unbalanced masses measured from +x-axis. i.e. (c.c.w)

$l_A, l_B, l_C, l_D$  ----- distance

from the plane of mass  $m_1$  to the planes  $m_A, m_B, m_C, m_D$ .

$m_1, m_2$  - Balancing masses.

$e_1, e_2$  - Radial position of masses  $m_1, m_2$ .

$$\sum F_x = 0 \quad m_1 e_1 w^2 \cos \theta_1 + m_A e_A w^2 \cos \theta_A + \dots + m_2 e_2 w^2 \cos \theta_2 = 0$$

$$\sum_{i=1}^n m_i e_i \cos \theta_i = 0$$

$$\sum F_y = 0 \quad m_1 e_1 w^2 \sin \theta_1 + m_A e_A w^2 \sin \theta_A + \dots + m_2 e_2 w^2 \sin \theta_2 = 0$$

$$\sum_{i=1}^n m_i e_i \sin \theta_i = 0$$

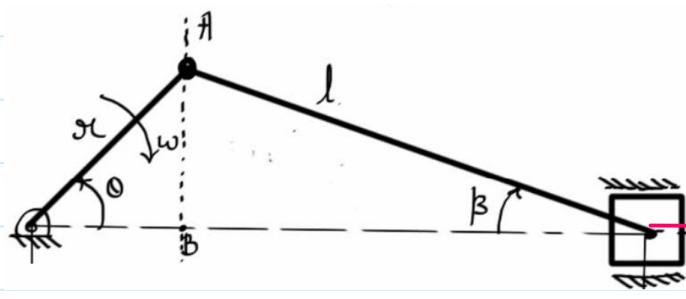
Moments about  $m_1$  (mass) Plane.

$$\sum M_x = 0 \quad m_A e_A w^2 \cos \theta_A \cdot l_A + m_B e_B l_B w^2 \cos \theta_B + \dots + m_2 e_2 l_2 w^2 \cos \theta_2 = 0$$

$$\sum_{i=1}^n m_i e_i l_i \cos \theta_i = 0$$

$$\sum M_y = 0 \quad m_A e_A \cdot l_A w^2 \sin \theta_A + \dots + m_2 e_2 \cdot l_2 w^2 \sin \theta_2 = 0$$

$$\sum_{i=1}^n m_i e_i l_i \sin \theta_i = 0$$



Calong the line of stroke  
 $F_{un} = m_p \cdot a_p$

Mass of crank and connecting rod is ignored.

$$\omega_{crank} = \text{constant}$$

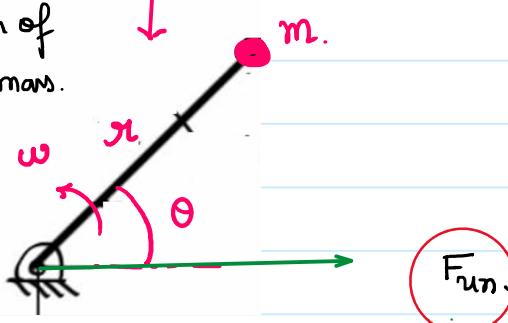
$$F_{un} = m \cdot r \omega^2 \left[ \cos \theta + \frac{\cos 2\theta}{n} \right] = m r \omega^2 \cos \theta + \frac{m r \omega^2 \cos(2\theta)}{n}$$

$$n > 1$$

$$F_{un} = m r \omega^2 \cos \theta$$

is converted into rotating unbalance by assuming mass @ crank pin.

$F_{un}$  is represented in the form of a rotating mass.



$F_{un}$  is acting along the line of stroke.

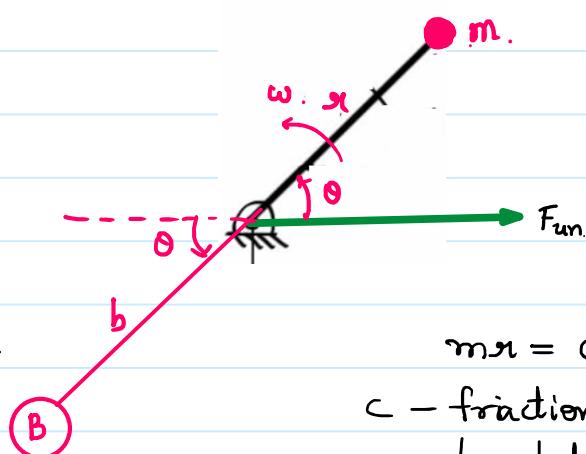
$$\leq F_x = 0$$

$$m r \omega^2 \cos \theta - B b \omega^2 \cos \theta$$

$$(-c) m r \omega^2 \cos \theta$$

$$\leq F_y$$

$$B b \omega^2 \sin \theta = c m r \omega^2 \sin \theta$$



$$m r = c \cdot B \cdot b$$

c - fraction of mass to be balanced.

$$\text{Resultant Unbalance force} = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$R_{un} = \sqrt{[(1-c)m\omega^2 \cdot \cos\theta]^2 + [cm\omega^2 \cdot \sin\theta]^2}$$

 FACULTY **WAHEED UL HAQ**

$$R_{un} = f(c, m, r, \omega, \theta)$$

↗ Variable  
 ↘ constant  
 ↓ variable

$$R_{un} = \text{Min. } \frac{dR_{un}}{dc} = 0 \Rightarrow c = 0.5$$

$$R_{min} = \frac{m\omega^2}{2}$$

### COMPLETE BALANCING:

- Initially the reciprocating mass is assumed at crank pin i.e., the problem is converted into rotating unbalance mass and thus location of balancing mass obtained.
- In a complete balancing the unbalanced force along the line of stroke become zero and the unbalanced force perpendicular to the line of stroke is  $Bbw^2 \sin(\theta)$ , having maximum magnitude  $Bbw^2$ .
- Therefore in complete balancing only the direction of maximum unbalanced force has been changed. Whereas its magnitude is unaffected that's why complete balancing is not preferred for reciprocating masses.

$$\begin{aligned}
 & F_p & F_s \\
 R_{un} &= m\omega^2 \cdot \cos\theta + \frac{m\omega^2 \cdot \cos(2\theta)}{n} \\
 &= m\omega^2 \cdot \cos(\omega t) + \frac{m \cdot r \cdot (2\omega)^2 \cdot \cos(2\omega t)}{4n}
 \end{aligned}$$

 $F_p$  - Primary Unbalance force.

 $F_s$  - Secondary Unbalance force.

$m$  - mass of piston.

$r$  - crank length

$\omega$  - angular velocity of crank.

Force	Crank length	Speed	Crank angle
Primary	$r$	$\omega$	$\theta$
Secondary	$\frac{r}{4n} = \frac{r}{4(\frac{l}{m})} = \frac{r^2}{4l}$	$2\omega$	$2\theta$

 FACULTY **WAHEED UL HAQ**

## DIFFERENCE BETWEEN ROTATING AND RECIPROCATING BALANCE:

- The rotating unbalance force is always constant in magnitude but changes its direction continuously, whereas reciprocating unbalance force magnitude changes continuously but it always acts along the line of stroke.
- In case of rotating unbalance the system can be balanced by either adding or removing the masses, whereas in case of reciprocating unbalance removal of masses is not possible.  

$$F_i = mrw^2 + mrw^2 \cos(2\theta)/n$$

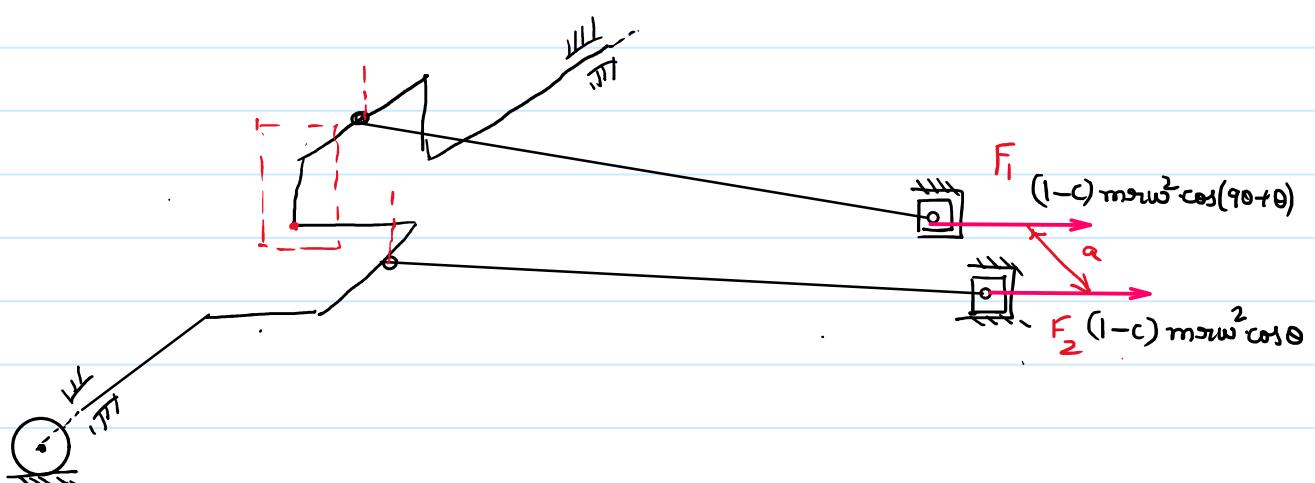
$$F_i = (F_i)_{\text{primary}} + (F_i)_{\text{secondary}}$$
- Effect of Unbalance cannot be eliminated for reciprocating that is why we opt for partial balancing.

## EFFECT OF THE PARTIAL BALANCING OF LOCOMOTIVE:

- The partial balancing results in unbalanced force along the line of stroke (FH) it results in variation in tractive force along the line of stroke and swaying couple about centre line.
- Unbalanced force perpendicular to line of stroke (FV) due to which there is variation in pressure on the rails which results in hammering action which is known as "Hammer Blow".

### SWAYING COUPLE:

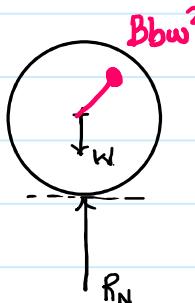
- The two unbalanced force acting along the line of stroke of the two cylinders constitutes a couple between the centre plane is known as swaying couple.
- It tends to sway the engine alternately in clockwise and counter clockwise direction.



$$\begin{aligned} \text{Traction force} &= (1-c)mrw^2 \cdot \cos\theta + (1-c)mrw^2 \cdot \cos(90+\theta) \\ &= (1-c)mrw^2 \cdot [\cos\theta - \sin\theta] = \pm \sqrt{2}(1-c)mrw^2 \end{aligned}$$

$$\begin{aligned} \text{Swaying Couple.} &= (1-c)mrw^2 \cdot \cos\theta \times a_2 - (1-c)mrw^2 \cdot \cos(90+\theta) \times a_2 \\ &= (1-c)mrw^2 \times a_2 \cdot [\sin\theta + \cos\theta] = \frac{(1-c)mrw^2}{\sqrt{2}} \times a_2 \end{aligned}$$

$$\theta = 90^\circ$$



$$R_N = W \pm Bbw^2 \sin\theta.$$

$$R_N = W \pm Bbw^2.$$

$$R_N = W + Bbw^2 \rightarrow \text{safe.}$$

$$R_N \geq 0$$

$$W - Bbw^2 \geq 0.$$

$$\omega \leq \sqrt{\frac{W}{Bb}}$$

### BALANCING OF LOCOMOTIVE:

- Most of the locomotive have two cylinder of same dimension which are placed at right angle to each other in order to have uniform turning moment diagram. There are 4 type of locomotive:
  - INSIDE CYLINDER LOCOMOTIVE: If the cylinders are placed inside the two wheels it is known as inside cylinder locomotive.
  - OUTSIDE CYLINDER LOCOMOTIVE: If the cylinders are placed outside the two wheels it is known as outside cylinder locomotive.

### BALANCING OF LOCOMOTIVE:

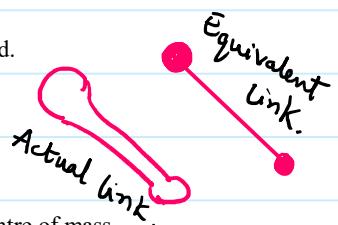
- SINGLE OR UNCOUPLED LOCOMOTIVE: In this the effort is transmitted to one pair of wheel only.
- COUPLED LOCOMOTIVE: In it the driving wheel are connected to the leading or trailing wheels with the help of an outside coupling rod.

#### NOTE:

- In the locomotive the value of n is very large therefore we can neglect the secondary inertia force.
- In locomotive to balance the primary inertia force the balancing mass is kept on the wheels and partial balancing was done.

### DYNAMICALLY EQUIVALENT LINK FOR CONNECTING ROD:

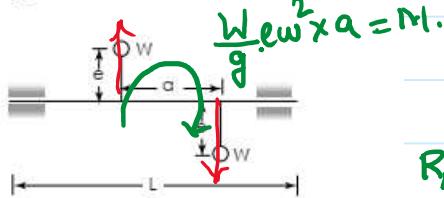
- The two member are equivalent for dynamic analysis purpose if the following three condition are satisfied.
  - The location of centre of mass must be same.
  - The total mass of the system must be same.
  - The mass moment of inertia of actual and equivalent systems must be same with respect to the centre of mass.



01. Static balancing is satisfactory for low speed rotors, but with increasing speeds, dynamic balancing becomes necessary. This is because, the

  - unbalanced couples are caused only at higher speeds
  - effect of unbalance is directly proportional to speed
  - effect of unbalance is directly proportional to square of speed
  - unbalanced forces are not significant at higher speeds

02. A statically-balanced system is shown in the given figure. Two equal weights  $W$ , each with an eccentricity  $e$ , are placed on opposite sides of the axis in the same axial plane. The axial distance between them is 'a'. The total dynamic reactions at the supports will be



- (a) zero      (b)  $\frac{W}{g} \omega^2 e \frac{a}{L}$   
                   (c)  $\frac{2W}{g} \omega^2 e \frac{a}{L}$       (d)  $\frac{W}{g} \omega^2 e \frac{L}{a}$

03. In a rotor having several masses rotating in different planes the bearing reactions are found to be equal and opposite. It indicates that the system is in  
(a) Dynamic balance    (b) Static balance  
(c) Couple balance    (d) Complete balance

$$\sum F_x = 0$$

$$R_{A,B} = \pm \frac{M}{L}$$

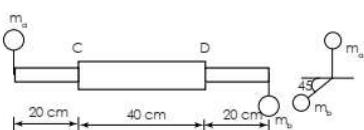
$$R_{AB} = \pm \frac{W}{q} \cdot \frac{ew^2 \times a}{L}$$

$$R_A + R_B = 0.$$

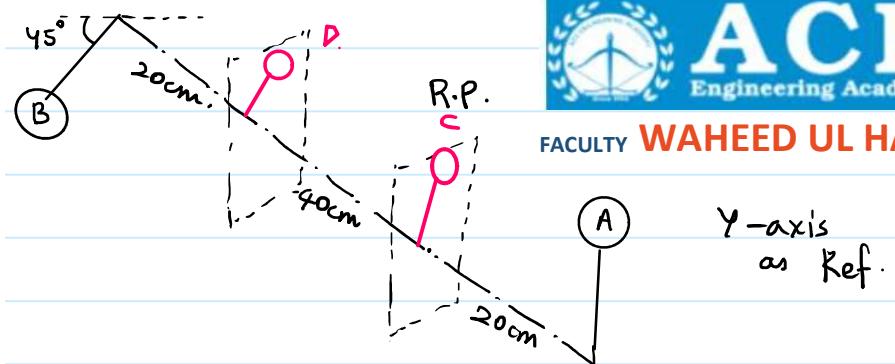
$$R_A = -R_B$$

Common data for Questions 04 & 05 :

A rotor shown in figure is balanced by attaching two masses  $m_A = 5 \text{ kg}$  and  $m_B = 6 \text{ kg}$  in the planes A and B at a radius of 20 cm at the locations shown. It is required to balance the rotor by removing masses in the planes C & D at a radius of 20 cm. Magnitude and angular location of the masses to be removed in Planes C and D.



04. Mass in plane C and its angular location with respect to  $m_A$   
 (a) 9.85 kg @ 167.5°    (b) 10.5 kg @ 23.8°  
 (c) 11 kg @ 215.7°    (d) 10.5 kg @ 203.8°
05. Mass in plane D and its angular location with respect to  $m_A$   
 (a) 10 kg @ 23.8°    (b) 10.91 kg @ 35.7°  
 (c) 11 kg @ 215.7°    (d) 11 kg @ 203.8°



$$e_A = e_B = e_C = e_D = 20 \text{ cm} = 0.2 \text{ cm}$$

$$\theta_A = 0^\circ \quad \theta_B = 135^\circ \quad \theta_C, \theta_D = ?$$

SNo.	Mass.	Radial Location $e_i$	Angular position $\theta_i$	$\sum m_i e_i l_i \sin \theta_i$	$\sum m_i e_i l_i \cos \theta_i$	Dist. b/w plane $L_i$	$\sum m_i e_i l_i \sin \theta_i$	$\sum m_i e_i l_i \cos \theta_i$
1	$m_A = 5 \text{ kg}$	0.2	0°	$5 \times 0.2 \times 0.2 \sin 0^\circ = 1$	$5 \times 0.2 \times 0.2 \cos 0^\circ = 0$	-0.2	-0.02	0
2	$m_C$	0.2	$\theta_C$	$0.2 m_C \sin \theta_C$	$0.2 m_C \cos \theta_C$	0	0	0
3	$m_D$	0.2	$\theta_D$	$0.2 m_D \sin \theta_D$	$0.2 m_D \cos \theta_D$	0.4	$0.08 m_D \cos \theta_D$	$0.08 m_D \sin \theta_D$
4.	$m_B = 6 \text{ kg}$	0.2	225°	$6 \times 0.2 \times 0.2 \sin 225^\circ = -0.6$	$6 \times 0.2 \times 0.2 \cos 225^\circ = -0.6$	0.6	$6 \times 0.2 \times 0.6 \cos 135^\circ = -0.6$	$6 \times 0.2 \times 0.6 \sin 135^\circ = 0.6$

$$\sum m_i e_i l_i \sin \theta_i = 0$$

$$-0.02 + 0.08 m_D \cos \theta_D + 6 \times 0.2 \times 0.6 \sin 135^\circ = 0$$

$$m_D = 10.981$$

$$\theta_D = 35.7^\circ$$

$$\sum m_i e_i l_i \cos \theta_i = 0$$

$$-0.02 + 0.08 m_D \cos \theta_D + 6 \times 0.2 \times 0.6 \cos 135^\circ = 0$$

$$\sum m_i e_i \cos \theta_i = 0$$

$$1 + 0.08 m_D \cos \theta_D + 6 \times 0.2 \times 0.6 \cos 135^\circ = 0$$

$$\sum m_i e_i \sin \theta_i = 0$$

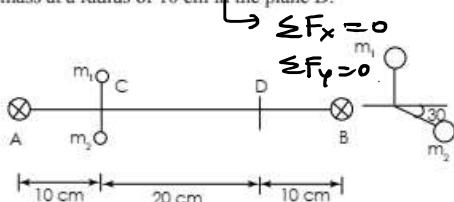
$$0 + 0.08 m_D \sin \theta_D + 6 \times 0.2 \times 0.6 \sin 135^\circ = 0$$

$$m_D = 10.981$$

$$\theta_D = 35.7^\circ$$

Common data for Questions 06 & 07 :

A rotor supported in the bearings A & B has two unbalanced masses in the plane C,  $m_1 = 10\text{kg}$  at an eccentricity of 10 cm and  $m_2 = 5\text{kg}$  at an eccentricity of 20 cm. It is statically balanced by attaching a single mass at a radius of 10 cm in the plane D.



06. The magnitude and angular position (with respect to the horizontal) of the mass attached in the plane D is

- (a) 10 kg @  $210^\circ$
- (b) 14.14 kg @  $210^\circ$
- (c) 14.14 kg @  $120^\circ$   $0.1m_p \cos\theta_p = -0.816$
- (d) 10 kg @  $180^\circ$   $0.1m_p \sin\theta_p = -0.5$

07. When the rotor rotates 600 rpm the dynamic reaction in the bearings is about

- (a) 40 kN
- (b) 4 kN
- (c) 3 kN
- (d) 2 kN

$$\tan\theta_p = \frac{-0.5}{-0.816}$$

$$\theta_p = 210^\circ$$

$$m_p = 10\text{kg}$$

08. A rotor of mass 2000 kg is balanced statically by attaching two masses one in a plane 20 cm to the left of the centroidal plane and the second in a plane 25 cm to the right. Both masses are attached at a radius of 10 cm and mass 1 is 52 kg and the mass 2 is 75 kg, mass 1 is in the horizontal direction and the second mass is in the vertical direction. Locate the eccentricity of the rotor and its angular location with reference to the first one

- (a) 0.45 cm @  $235.2^\circ$
- (b) 0.02 cm @  $71.5^\circ$
- (c) 0.4 cm @  $71.5^\circ$
- (d) 0.02 cm @  $251.6^\circ$

$$m_1 = 10\text{kg} \quad m_2 = 5\text{kg}$$

$$e_1 = 20\text{cm} \quad e_2 = 20\text{cm}$$

$$m_D = ?$$

$$e_D = 10\text{cm}$$

$$\theta_D = ?$$

$$\theta_1 = 90^\circ$$

$$\theta_2 = 330^\circ$$

$$\sum F_x = 0$$

$$m_1 e_1 w^2 \cos\theta_1 + m_2 e_2 w^2 \cos\theta_2$$

$$+ m_D e_D w^2 \cos\theta_D = 0 \rightarrow (A)$$

$$(10 \times 0.2 \times \cos 90^\circ) + (5 \times 0.2 \times \cos 330^\circ) + (m_D \times 0.1 \times \cos\theta_D) = 0$$

$$\sum F_y = 0$$

$$m_1 e_1 \sin\theta_1 = 0$$

$$m_1 e_1 \sin\theta_1 + m_2 e_2 \sin\theta_2 + m_D e_D \sin\theta_D = 0 \rightarrow (B)$$

$$(10 \times 0.2 \times \sin 90^\circ) + (5 \times 0.2 \times \sin 330^\circ) + m_D \times 0.1 \times \sin\theta_D = 0$$

$$M = m_e w^2 \times a$$

$$R_A, R_B = \pm \frac{m_e w \times a}{L} = \pm (10) \times 0.1 \times \left( \frac{2\pi \times 600}{60} \right)^2 \times 0.2$$

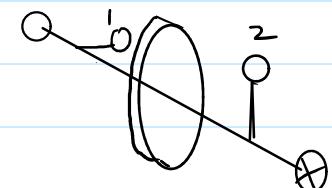
$$m_R = 2000\text{kg} \quad e_R = ? \quad \theta_R = ?$$

$$e_1 = e_2 = 10\text{cm}$$

$$m_1 = 52\text{kg}$$

$$m_2 = 75\text{kg}$$

$$\theta_1 = 0^\circ, \quad \theta_2 = 90^\circ$$



$$\sum m_i r_i \cos\theta_i = 0$$

$$52 \times 0.1 \times \cos 0^\circ + 75 \times 0.1 \times \cos 90^\circ + 2000 \times e_R \cos\theta_R = 0$$

$$\sum m_i r_i \sin\theta_i = 0$$

$$52 \times 0.1 \times \sin 0^\circ + 75 \times 0.1 \times \sin 90^\circ + 2000 \times e_R \sin\theta_R = 0$$

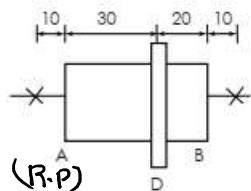
$$2000 e_R \cos\theta_R = -5.2$$

$$2000 e_R \sin\theta_R = -7.5$$

$$e_R = \frac{-5.2}{2000 \times \cos 235.26} \\ = 0.456\text{cm.}$$

$$\tan\theta_R = -\frac{7.5}{5.2}$$

$$\theta_R = 180 + 55.26 \\ = 235.26^\circ$$



For the rotor shown in figure has an unbalance of 2kg-m in the plane D along the x-axis. It is desired to balance this by removing masses in the planes A and B at a radius of 0.5 m. The dimensions shown in the figure are in cm. The masses to be removed in plane A, plane B and their angular positions with

x axis respectively are

- (a) 1.6 kg, 2.4 kg @  $0^\circ$  &  $0^\circ$
- (b) 0.48 kg, 0.72 kg @  $180^\circ$  &  $0^\circ$
- (c) 0.24 kg, 0.48 kg @  $0^\circ$  &  $180^\circ$
- (d) 0.72 kg, 0.24 kg @  $180^\circ$  &  $0^\circ$

SNo.	Mass.	Radial Location $e_i$	Angular position $\theta_i$	$\sum m_i e_i \cos \theta_i$	$\sum m_i e_i \sin \theta_i$	Dist. b/w plane $L_i$	$\sum m_i e_i \cos \theta_i$	$\sum m_i e_i \sin \theta_i$
1	2 kg-m		$0^\circ$	$2 \times 0.5 \cos 0^\circ$	$2 \times 0.5 \sin 0^\circ$	0.3	$2 \times 0.5 \times 0.3 = 0.6$	0
2	$-m_A$	0.5	$\theta_A$	$-0.5 m_A \cos \theta_A$	$-0.5 m_A \sin \theta_A$	0	0	0
3	$-m_B$	0.5	$\theta_B$	$-0.5 m_B \cos \theta_B$	$-0.5 m_B \sin \theta_B$	0.5	$-0.25 m_B \cos 180^\circ$	$-0.25 m_B \sin 180^\circ$

$$\sum m_i e_i \sin \theta_i = 0$$

$$0.025 m_B \sin \theta_B = 0 \quad \sin \theta_B = 0^\circ, 180^\circ$$

$$\sum m_i e_i \cos \theta_i = 0$$

$$0.6 - 0.025 m_B \cos \theta_B = 0 \quad m_B = \frac{-0.6}{-0.25} = 2.4 \text{ kg}$$

$$\sum m_i e_i \sin \theta_i = 0$$

$$0 - 0.5 m_B \sin \theta_B - 0.5 m_A \sin \theta_A = 0$$

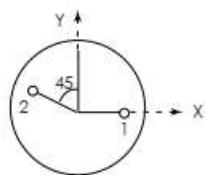
$\theta_A = 0^\circ$

$$\sum m_i e_i \cos \theta_i = 0$$

$$2 - 0.5 m_B \cos 0^\circ - 0.5 \times 2.4 \times \cos 0^\circ = 0$$

$$m_B = 2.4 \text{ kg}$$

10. A wheel is balanced by placing two trial masses as shown in figure. The mass 1 is of 20 gms at radius of 15 cm and the mass 2 is 25 grams at a radius of 20 cm. If it is to be balanced by a single balancing mass at a radius of 20 cm, the magnitude and angular position of the balancing mass is



- (a) 17.88 gm & 98.7°  
 (b) 16 gm & 208°  
 (c) 52 gm & 45°  
 (d) 16 gm & 45°

11. A slider crank mechanism has slider mass of 10 kg, stroke of 0.2 m and rotates with a uniform angular velocity of 10 rad/s. The primary inertia forces of the slider are partially balanced by a revolving mass of 6 kg at the crank, placed at a distance equal to crank radius. Neglect the mass of connecting rod and crank. When the crank angle (with respect to slider axis) is 30°, the unbalanced force (in Newton) normal to the slider axis is \_\_\_\_\_  
 (GATE-14)

12. Which one of the following statements in the context of balancing in engines is correct?

- (a) Magnitude of the primary unbalancing force is less than the secondary unbalancing force  
 (b) The primary unbalancing force attains its maximum value twice in one revolution of the crank, **Secondary force will attain 4 times in 1 rev.**  
 (c) The hammer blow in the locomotive engines occurs due to unbalanced force along the line of stroke of the piston  
 (d) The unbalanced force due to reciprocation masses varies in magnitude and direction

13. In balancing of single-cylinder engine, the rotating unbalance is

- (a) completely made zero and so also the reciprocating unbalance  
 (b) completely made zero and the reciprocating unbalance is partially reduced  
 (c) partially reduced and the reciprocating unbalance is completely made zero  
 (d) partially reduced and so also the reciprocating unbalance

$$m_1 = 20 \text{ gms} \quad e_1 = 15 \text{ cm}$$

$$m_2 = 25 \text{ gms} \quad e_2 = 20 \text{ cm}$$

$$\theta_1 = 0^\circ \quad \theta_2 = 135^\circ$$

$$m_b = ? \quad e_b = 20 \text{ cm} \quad \theta_b = ?$$

$$\sum F_x = 0$$

$$m_1 e_1 \cos \theta_1 + m_2 e_2 \cos \theta_2 + m_b e_b \cos \theta_b = 0 \\ = (20 \times 0.15 \times \cos 0^\circ) + (25 \times 0.2 \times \cos 135^\circ) + (m_b \times 0.2 \times \cos \theta_b) = 0$$

$$\sum F_y = 0$$

$$m_1 e_1 \sin \theta_1 + m_2 e_2 \sin \theta_2 + m_b e_b \sin \theta_b = 0 \\ = (20 \times 0.15 \times \sin 0^\circ) + (25 \times 0.2 \times \sin 135^\circ) + (m_b \times 0.2 \times \sin \theta_b) = 0$$

$$m = 10 \text{ kg}$$

$$\theta = 30^\circ$$

$$r = 0.2 \text{ m}$$

$$\omega = 10 \text{ rad/s}$$

$$B = 6 \text{ kg}$$

$$b = 0.2 \text{ m}$$

Unbalanced  $\perp$  to line of stroke

$$\text{stroke} = B b \omega^2 \sin \theta$$

$$= 6 \times 0.2 \times 10^2 \times \sin 30^\circ$$

= —

$$F_p > F_s$$

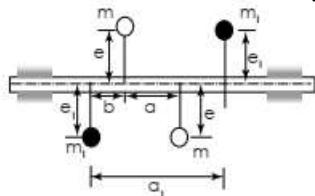
$$F_p = m r \omega^2 \cos \theta \xrightarrow[2\pi]{\text{Periodicity}}$$

$$F_s = \frac{m r \omega^2}{n} \cos 2\theta \quad n > 1$$

$$\xrightarrow{\text{Periodicity}} = \frac{2\pi}{2} = \pi$$

14. In a reciprocating machine based on regular slider crank mechanism, the mass of reciprocating parts is 10 kg and the crank radius is 15 cm. 60% of the primary unbalance is balanced using a counter mass. What will be the residual unbalance along the line of stroke for a crank position of  $60^\circ$  at a speed of 4 rad/sec.
- (a) 2.4 N      (b) 4.8 N  
 (c) 12 N      (d) 6.6 N

15. Two masses  $m$  are attached to opposite sides of a rigid rotating shaft in the vertical plane. Another pair of equal masses  $m_1$  is attached to the opposite sides of the shaft in the vertical plane as shown in figure. Consider  $m = 1 \text{ kg}$ ,  $e = 50 \text{ mm}$ ,  $e_i = 20 \text{ mm}$ ,  $b = 0.3 \text{ m}$ ,  $a = 2 \text{ m}$  and  $a_i = 2.5 \text{ m}$ . For the system to be dynamically balanced,  $m_1$  should be \_\_\_\_\_ kg.



(GATE-16)  $c_1 = c_2$

$$m_e w^2 \times a = m_1 e_i w^2 \times a_i \\ 1 \times 50^2 \times 2 = m_1 \times 20 \times 2.5 \\ m_1 = 2 \text{ kg}$$

statically &  
Dynamically  
balanced.

6. A connecting rod of a steam engine has a length of 100 cm and a mass of 100 kg. The CG is at a distance of 40cm from the big end with a radius of gyration of 30 cm about CG. If it is to be replaced with a two mass system with  $m_b$  at the big end and  $m_s$  at small end. Then  $m_b$ ,  $m_s$  and the moment of inertia of the equivalent link are respectively.

- (a) 60 kg, 40 kg, 24 kg.m<sup>2</sup>  
 (b) 60 kg, 40 kg, 9 kg.m<sup>2</sup>  
 (c) 40 kg, 60 kg, 9 kg.m<sup>2</sup>  
 (d) 60 kg, 40 kg, 90 kg.m<sup>2</sup>

$$l = 100 \text{ cm.}$$

$$l_b = 40 \text{ cm.}$$

$$l_s = 60 \text{ cm.}$$

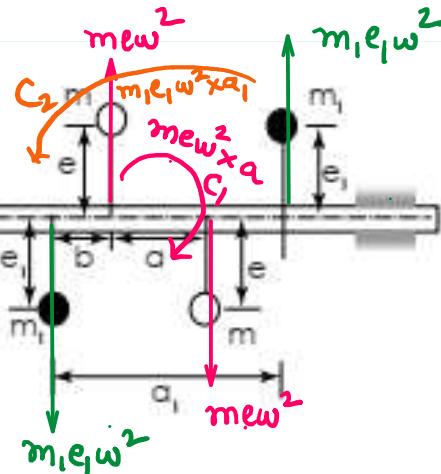
$$l_s + l_b = l.$$

$$m = 10 \text{ kg} \quad r = 15 \text{ cm} \\ = 0.15 \text{ m.}$$

$$c = 60\% = 0.6 \quad \theta = 60^\circ$$

$$F_x = (1-c)m r \omega^2 \cdot \cos \theta = (1 - 0.6) \times 10 \times 0.15 \times 4^2 \times \cos 60^\circ$$

$$F_x = 4.8 \text{ N.}$$



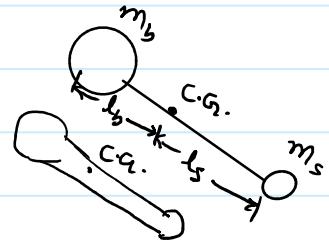
$$\sum F = 0$$

$$C_2 = C_1$$

$$m = 100 \text{ kg}$$

$$l_b = 40 \text{ cm}$$

$$k = 30 \text{ cm.}$$



$$m_s + m_b = m.$$

$$m_s + m_b = 100 \rightarrow (1)$$

$$m_s \cdot l_s = m_b \cdot l_b$$

$$m_s (60) = m_b (40)$$

$$m_b = 1.5 m_s$$

$$m_s + 1.5 m_s = 100 \Rightarrow m_s = \frac{100}{2.5} = 40 \text{ kg}$$

$$m_b = 60 \text{ kg.}$$

$$I = m_s \cdot l_s^2 + m_b \cdot l_b^2 = 40 \times 0.6^2 + 60 \times (0.4)^2 \\ = 24 \text{ kg-m}^2$$