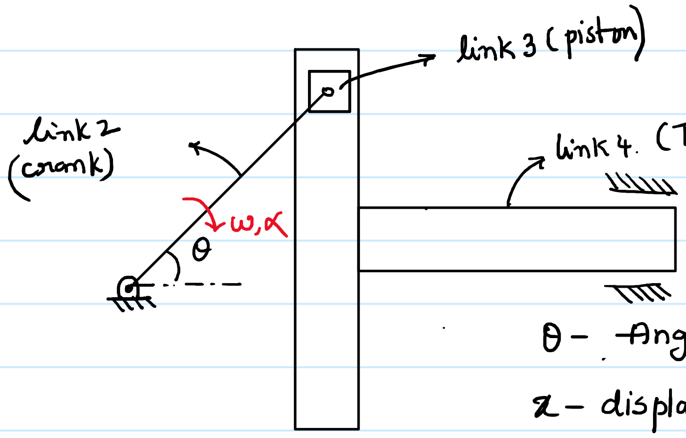
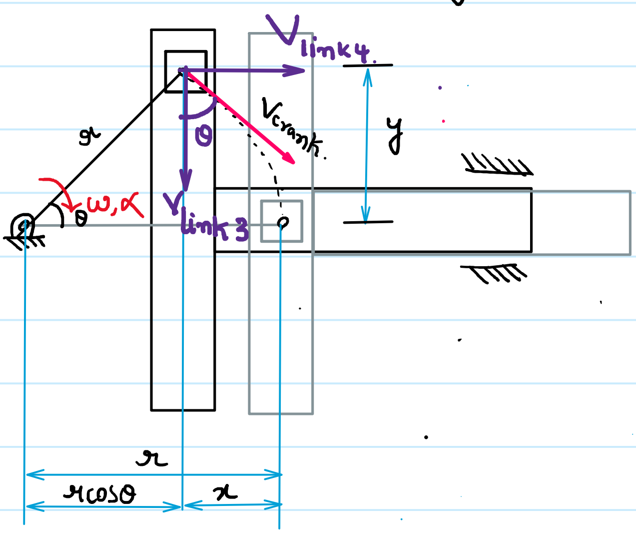
Scotch Yoke Mechanism.



θ – Angular displacement of link 2.

x – displacement of link 4.

y – displacement of link.



$$x = r - r\cos\theta$$

$$x = r[1-\cos\theta]$$

$$y = r\sin\theta$$

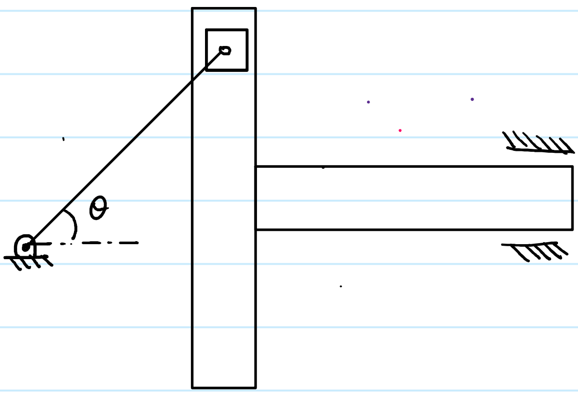
Velocity of link 3

$$V_{link\,3} = \frac{dy}{dt} = r\cos\theta\cdot\frac{d\theta}{dt}$$

$$V_{link\,3} = r\cdot\omega\cdot\cos\theta.$$

$$V_{link\,4} = r\cdot\left[0-(-\sin\theta)\right]\frac{d\theta}{dt}$$

$$= r\omega\sin\theta$$



Acceleration of link 3/slider

$$a_{link3.} = \frac{d}{dt}(r\omega\cdot\cos\theta)$$

$$a_{link\,3} = r\cdot\omega\cdot(-\sin\theta)\frac{d\theta}{dt} + r\cdot\cos\theta\cdot\frac{d\omega}{dt}$$

$$a_{link\,3} = r\alpha\cos\theta - r\omega^2\sin\theta.$$

Acceleration of link 4.

$$a_{link\,4} = \frac{d}{dt}(r\omega\sin\theta) = r\omega\cos\theta\cdot\frac{d\theta}{dt} + r\sin\theta\cdot\frac{d\omega}{dt} = r\omega^2\cos\theta + r\alpha\sin\theta.$$

12. In a four bar mechanism at a particular configuration the $\boxed{\text{angular velocity of the coupler is zero}}$. The input link is half the length of the output link. The input link rotates at an angular velocity of 2 rad/sec. The pins at all the joints have a radius of 10cm. The rubbing velocity at the pins starting from the joint 1 (connecting fixed link to input link) to joint 4 (connecting output link to fixed link) are respectively in cm/sec.

(a) 20, 0, 0, 40 (b) 20, 20, 10, 0
(c) 20, 20, 10, 10 (d) 20, 0, 0, 10

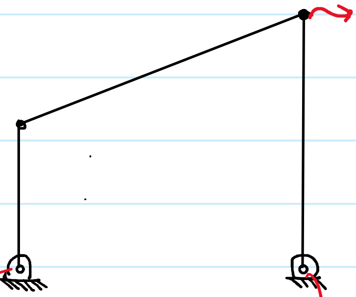
→ i/p and o/p link are ∥le.

$l_{i/p} = \frac{1}{2} \cdot l_{o/p}$

$\omega_{i/p} = 2\,rad/s$.

$r_{pin} = 10\,cm$.

rubbing velocity @ all joints.



$V_R = r_{pin} \cdot (\omega_2 \pm \omega_3)$, $\omega_3 = 0$

$= 10 \times 2$
$= 20\,cm/s$

$V_R = r_{pin} (\omega_1 \pm \omega_2)$, $\omega_1 = 0$

$= 10 \times 2 = 20\,cm/s$.

$V_R = r_{pin} \cdot (\omega_3 \pm \omega_4)$, $\omega_3 = 0$

$= 10 \times 1 = 10\,cm/s$.

$l_2 \cdot \omega_2 = l_4 \cdot \omega_4$

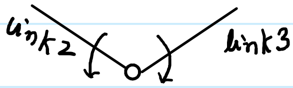
$\frac{1}{2} \cdot l_{o/p} \times 2 = l_{o/p} \times \omega_4$

$\omega_4 = 1\,rad/s$.

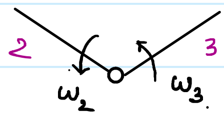
$V_R = r_{pin} \cdot (\omega_4 \pm \omega_1)$, $\omega_1 = 0$

$= 10 \times 1 = 10\,cm/s$.

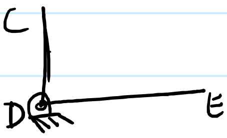
Rubbing velocity



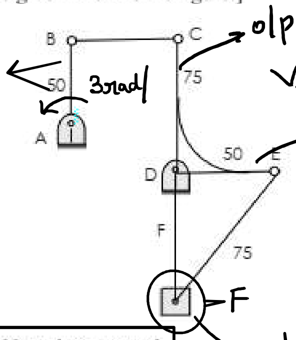
$V_{Rubbing} = r_{pin} \times (\omega_2 - (-\omega_3))$



$V_{Rubbing} = r_{pin} \cdot (\omega_2 - \omega_3)$

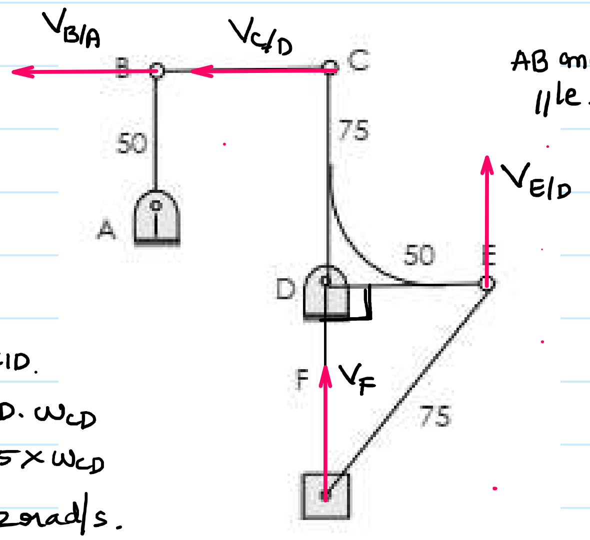


13. In a four bar mechanism shown in figure if the link AB has an instantaneous velocity of 3 rad/sec CCW. The velocity of the slider F is. [All the dimensions are given in cm is the figure.]



$V_F = ?$

$\boxed{\text{(a) 100 cm/sec upward}}$
(b) 100 cm/sec downwards
(c) 150 cm/sec upward
(d) 125 cm/sec upwards



AB and CD are ∥le. ⇒ $\boxed{\omega_{BC} = 0}$

$\boxed{\omega_{EF} = 0}$

$V_{B/A} = V_{C/D}$.

$AB \cdot \omega_{AB} = CD \cdot \omega_{CD}$

$50 \times 3 = 75 \times \omega_{CD}$

$\omega_{CD} = 2\,rad/s$.

$\omega_{DE} = \omega_{CD} = 2\,rad/s$.

$V_{E/D} = V_F$

$ED \cdot \omega_{CE} = V_F$

$50 \times 2 = V_F$

$V_F = 100\,cm/s$.

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14. In a slider crank mechanism the length of the crank is 1m and that of the connecting rod is 2 m. When the velocity of the slider is 2 m/sec. The angular velocity of the connecting rod is zero. The angular velocity of the crank, assuming to be uniform is

(a) 2 rad/sec (b) 1 rad/sec
(c) 0.5 rad/sec (d) 4 rad/sec

$r = 1m.$ $V_{piston} = 2m/s.$

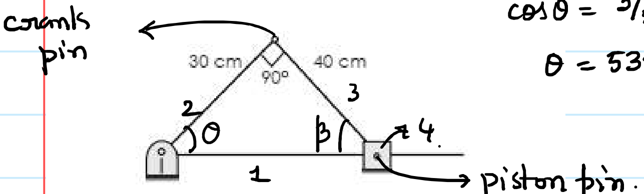
$\theta = 90^\circ$

$V_{crank} = V_{piston}.$

$r\omega = 2$

$\omega = 2/1 = 2 rad/s.$

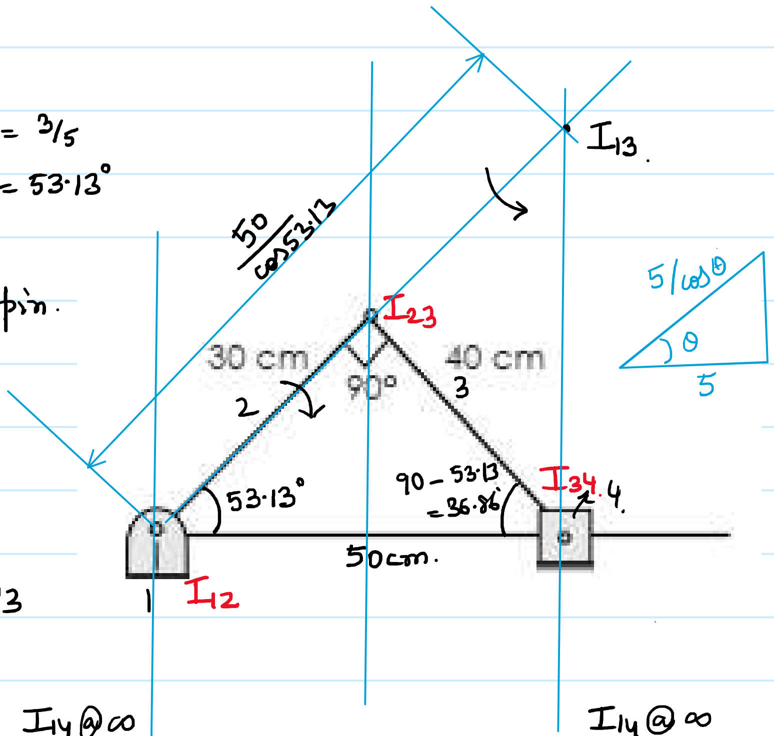
15. In a slider crank mechanism shown in figure at the instant the crank is perpendicular to the connecting rod. If the angular velocity of the crank is 10 rad/sec CCW. The rubbing velocity at the crank pin if its radius is 2.5 cm.



$\cos\theta = 3/5$

$\theta = 53.13^\circ$

(a) 0 (b) 25 cm/sec
(c) 50 cm/sec (d) 39 cm/sec



$r = 30cm$ $\omega_2 = 10 rad/s.$ c.c.w.

$V_{I_{23}} = I_{12}I_{23} \cdot \omega_2 = I_{13} \cdot I_{23} \cdot \omega_3$

$\Rightarrow 30 \times 10 = \left(\frac{50}{\cos 53.13} - 30\right) \times \omega_3$

$\omega_3 = \frac{300 \times 0.8 + 30 \times 0.8}{50} = 5.625 rad/s.$

@ Crank Pin

$V_{Rubbing} = r_{pin} \cdot (\omega_2 - (-\omega_3))$

$= 2.5(10 + 5.625) = 39 cm/s.$

@ piston pin.

$V_{Rubbing} = r_{pin}(\omega_3 \pm \cancel{\omega_4}^{\,0})$

$= 2.5 \times 5.625 = 14.06 cm/s.$

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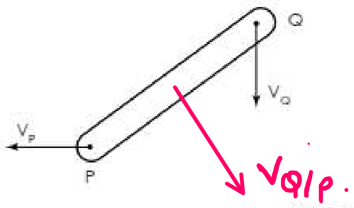
19. A rigid link PQ is undergoing plane motion as shown in the figure (V_P and V_Q are non-zero). V_{QP} is the relative velocity of points Q with respect to point P.



(GATE – 16)

Which one of the following is TRUE?

(a) V_{QP} has components along and perpendicular PQ

(b) V_{QP} has only one component directed from P to Q

(c) V_{QP} has only one component directed from Q to P

(d) V_{QP} has only one component perpendicular to PQ

$\vec{V}_Q = \vec{V}_P + \vec{V}_{Q/P}$

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$\vec{V}_{Q/P} = \vec{V}_Q - \vec{V}_P$

$\hat{V}_{Q/P} = \hat{V}_Q - \hat{V}_P$

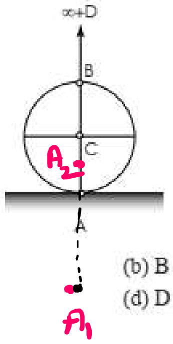
$= -\hat{j} - (-\hat{i})$

$= \hat{i} - \hat{j}$

Velocity analysis:

Instantaneous center method

20. The instantaneous centre of motion of a rigid-thin-disc-wheel rolling on plane rigid surface shown in the figure is located at the point.



(a) A (b) B

(c) C (d) D

Higher Pair.

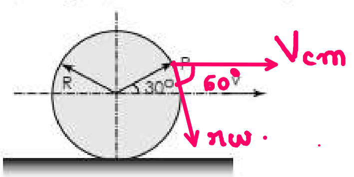
Pure Rolling – (A) – I-centre.

(A_1) – I-centre.

Rolling + slipping / Foward slipping. (Translation > Rotation)

A_2. — Rolling + slipping | Backward slipping (Rotation > Translation)

21. A circular disk of radius 'R' rolls without slipping at a velocity v. The magnitude of the velocity at point 'P' (see figure) is (GATE-08)



(a) $\sqrt{3}\,v$ (b) $\frac{\sqrt{3}\,v}{2}$

(c) $\frac{v}{2}$ (d) $\frac{2v}{\sqrt{3}}$

$\vec{V}_P = V_{cm}(\hat{i}) + r\omega\cdot[\hat{i}\cos 60 - \hat{j}\sin 60]$

$\vec{V}_P = V_{cm}(\hat{i}) + V_{cm}\cdot\times\frac{1}{2}\cdot\hat{i} - V_{cm}\cdot\frac{\sqrt{3}}{2}\hat{j}$

$\vec{V}_P = V_{cm}\cdot\left[\frac{3}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right]$

$V_P = \sqrt{3}\,V_{cm}.$

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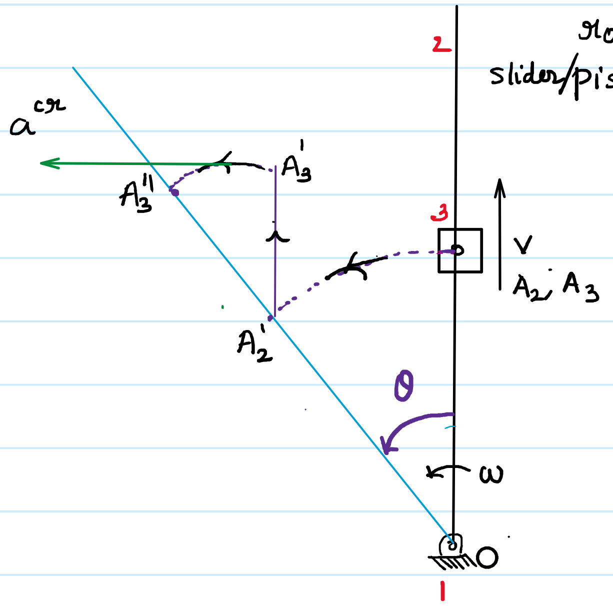


A_2, A_3 are points on link 2 and link 3 instantly co-incident.

Displacement of link 3 / Slider $= \overset{\frown}{A_2A_2'} + \overline{A_2'A_3'} + \overset{\frown}{A_3'A_3''}$

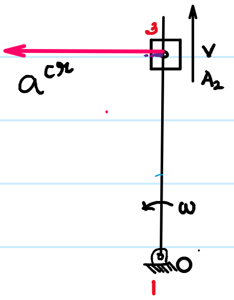
$\overset{\frown}{A_2A_2'} \rightarrow$ Angular displacement due to velocity ω.

$\overline{A_2'A_3'} \rightarrow$ linear displacement due to velocity V.

$\overset{\frown}{A_3'A_3''} \rightarrow$ Addition displacement of slider due to coriolis acceleration

$$\vec{a} = \vec{a}^{\,r}_{rod} + \vec{a}^{\,t}_{rod} + \vec{a}^{\,cr} + \vec{a}^{\,slider}$$

$$\vec{a}^{\,cr} = 2\cdot\dot{r}\dot{\theta}(\hat{e}_t) = 2\cdot(\vec{\omega}\times\vec{V}).$$



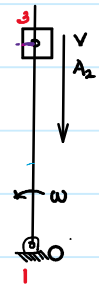
$$a^{cr}\,\hat{a}^{cr} = 2\cdot V_{slider}\cdot \omega_{Rod}\cdot(\hat{\omega}_{Rod}\times\hat{V}_{slider})$$

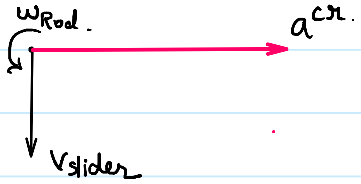
$$\hat{\omega}_{Rod} = +\hat{k}$$

$$\hat{V}_{slider} = +\hat{j}$$

$$\hat{a}^{cr} = \hat{\omega}_{Rod}\times\hat{V}_{slider} = \hat{k}\times\hat{j} = -\hat{i}$$

Rotate V_{slider} in the direction of ω by $90°$.





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$$\hat{a}^{cr} = \hat{\omega} \times \hat{V}$$
$$= \hat{k} \times -\hat{j} = \hat{i}$$

Newton's laws
D'Alembert Principle
FBD
Second order D.E.

Coriolis acceleration exists in
1.Crank Slotted Lever Mechanism 2.Whitworth Mechanism 3.Oscillating Cylinder Mechanism.

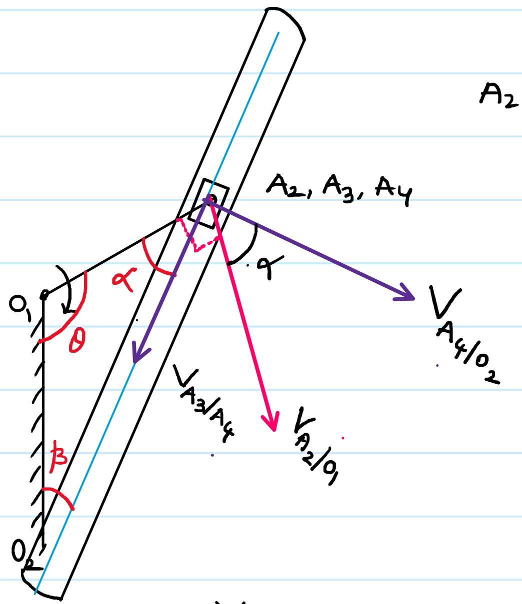
Analysis of Crank Slotted Lever Mechanism

$$\alpha = 180 - (\theta + \beta)$$

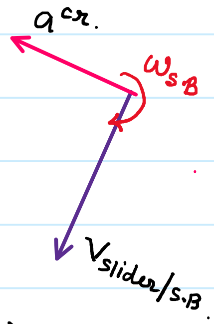
V_{A_2/O_1} – V_{crank}.

V_{A_3/A_4} – $V_{slider/slotted\ bar}$.

V_{A_4/O_2} – $V_{slotted\ bar/O_2}$



A_2, A_3, A_4 are points on link 2 (crank), link 3 (slider) and link 4 (slotted bar)



$$V_{A_3/A_4} = V_{A_2/O_1} \cdot \sin\alpha.$$

$$V_{A_4/O_2} = O_2A_4 \cdot \omega_{S.B}.$$

$$V_{A_4/O_2} = V_{A_2/O_1} \cdot \cos\alpha = O_2A_4 \cdot \omega_{S.B}.$$

Coriolis acceleration

$$\vec{a}^{cr} = 2 \cdot (\vec{\omega}_{S.B} \times \vec{V}_{slider/S.B})$$

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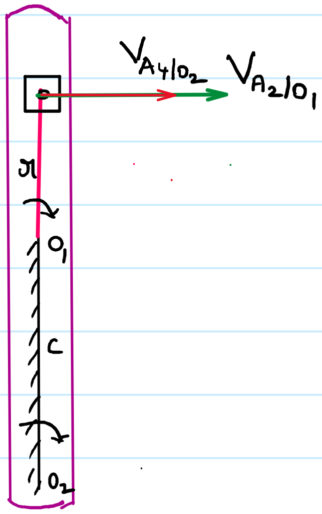
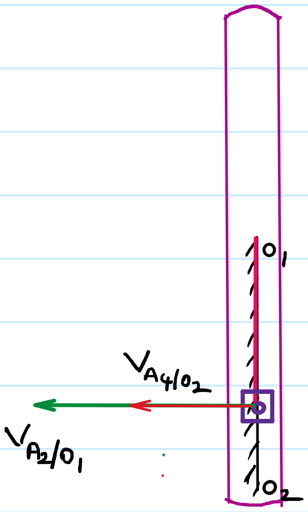
Case I When the Slotted bar is at the Mean position





$V_{A_3/A_4} = 0$
because link 3 is @ IDC / ODC.

$\alpha = 0, 180$

$V_{A_3/A_4} = 0$

O_1O_2 – fixed link.
C – distance b/w fixed centers.
O_1A_1 – crank.
r – lenght of crank.

$$V_{A_2/O_1} = V_{A_4/O_2}$$
$$O_1A_2 \cdot \omega_{O_1A_2} = O_2 \cdot A_4 \cdot \omega_{O_2 \cdot A_4}$$
$$(r) \cdot \omega = (C+r) \cdot (\omega_{s.B})_{max.1} \longrightarrow A$$

$$V_{A_2/O_1} = V_{A_4/O_2}$$
$$r \cdot \omega = (C-r) \cdot (\omega_{s.B})_{max\ 1} \rightarrow (B)$$

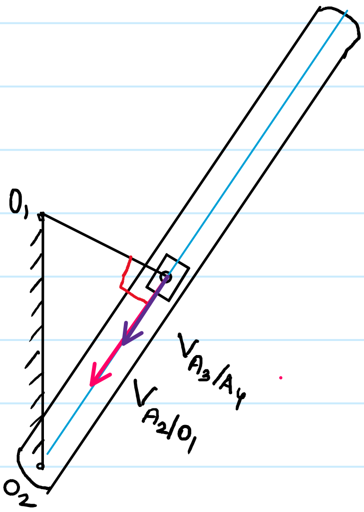
$$\frac{A}{B} = 1 = \frac{(C+r) \cdot \omega_{max\,1}}{(C-r) \cdot \omega_{max\,2}}$$

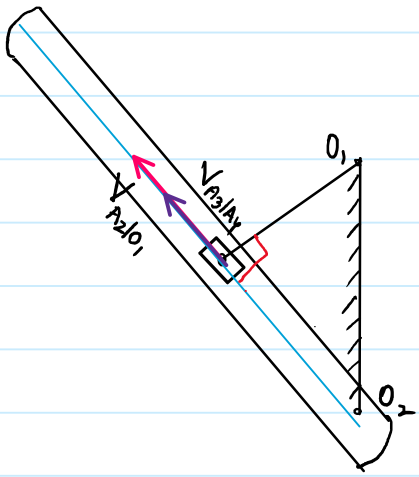
$$\frac{(\omega_{max-2})_{\text{Return}}}{(\omega_{max-1})_{\text{cutting}}} = \frac{C+r}{C-r} > 1$$

Coriolis acceleration

$$\vec{a}^{cr} = 2 \cdot (\vec{\omega}_{s.B} \times \underbrace{\vec{V}_{slider/s.B}}_{0})$$

$$\vec{a}^{cr} = 0$$



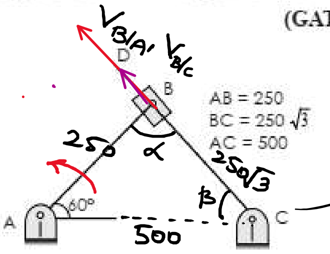


$\alpha = 90^\circ$ $\quad V_{A_4/O_2} = V_{A_2/O_1} \cdot \cos\alpha = 0 \Rightarrow \omega_{S.B} = 0$

$\vec{a}^{cr} = 0$

$$V_{A_3/A_4} = V_{A_2/O_1} \cdot \sin\alpha.$$
$$= r\omega.$$

16. For the configuration shown, the angular velocity of link AB is 10 rad/s counterclockwise. The magnitude of the relative sliding velocity (in ms^{-1}) of slider B with respect to rigid link CD is (GATE – 10)



(a) 0 (b) 0.86 (c) 1.25 ~~(d) 2.50~~

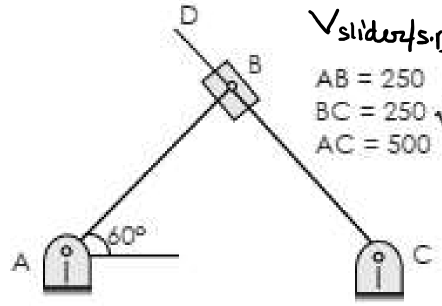
Toggle position.
slotted is @ extreme position.

$\omega_{SB} = 0$

$V_{S.B/O_2} = V_{crank} \cdot \cos\alpha$.

$V_{slider/S.B.} = V_{crank} \cdot \sin\alpha$.

$V_{slider/S.B} = V_{crank}$.



$V_{slider/SB} = r\omega$

$= 0.25 \times 10$

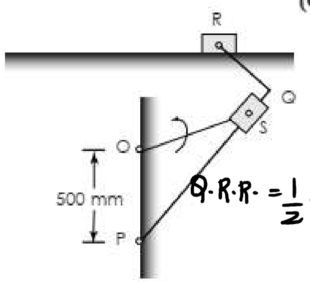
$= 2.5 m/s$.

Sine Rule.

$$\frac{250}{\sin 60} = \frac{500}{\sin\alpha} \qquad \alpha = 90^\circ \qquad \beta = 30^\circ$$

Statement for Common data Q. 17 & Q.18

A quick return mechanism is shown below. The crank OS is driven at 2 rev/s in counterclockwise direction. (GATE-07)



$$Q.R.R. = \frac{1}{2} = \frac{\theta_{Return}}{\theta_{cutting}}$$

17. If the quick return ratio is 1:2 then the length of the crank in mm is

(a) 250 (b) $250\sqrt{3}$
(c) 500 (d) $500\sqrt{3}$

18. The angular speed PQ in rev / s when the block R attains maximum speed during forward stroke (stroke with slower speed) is

slotted bar.

(ω_{max}) cutting

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$
(c) 2 (d) 3

$\omega_{crank} = 2 rev/s$ C.C.W.

lenght of fixed link = 500.

Q.R.R. = 1:2 lenght of crank = ?

Whitworth. Mechanism.

$$\frac{\text{lenght of fixed link}/L_{min}}{\text{lenght of crank}} = \frac{1}{Q.R.R.}$$

lenght of crank = 1000 mm.

Crank. slotted Lever

$$\frac{\text{lenght of crank}/L_{min}}{\text{lenght of fixed link}} = \frac{1}{Q.R.R.}$$

$$\text{lenght of crank} = \frac{500}{2} = 250mm.$$

$r \cdot \omega = (c+r) \cdot \omega_{max}$,

$250 \times 2 = (500+250)(\omega_{max})_{cutting}$

$(\omega_{max})_{cutting} = \frac{2}{3} rev/s$.

$r\omega = (c-r) \cdot (\omega_{max-2})_{Return}$.

$$(\omega_{max})_{Return} = \frac{250 \times 2}{(500-250)}$$

$= 2 rev/s$.

Q.48 At the instant when OP is vertical and AP is horizontal, the link OD is rotating counter clockwise at a constant rate ω = 7 rad/s. Pin P on link OD slides in the slot BC of link ABC which is hinged at A, and causes a clockwise rotation of the link ABC. The magnitude of angular velocity of link ABC for this instant is ________ rad/s (*rounded off to 2 decimal places*).

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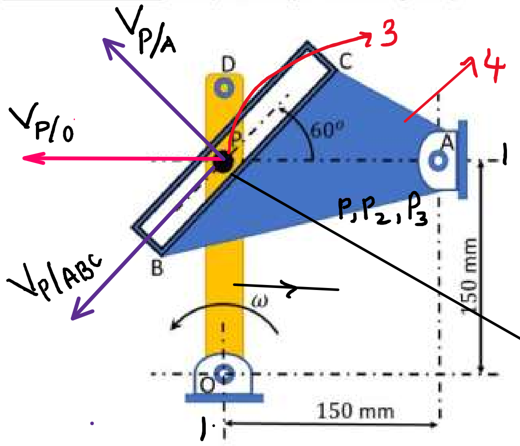


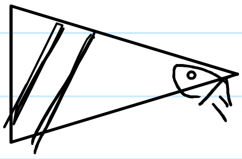
Pin is moving along slot.

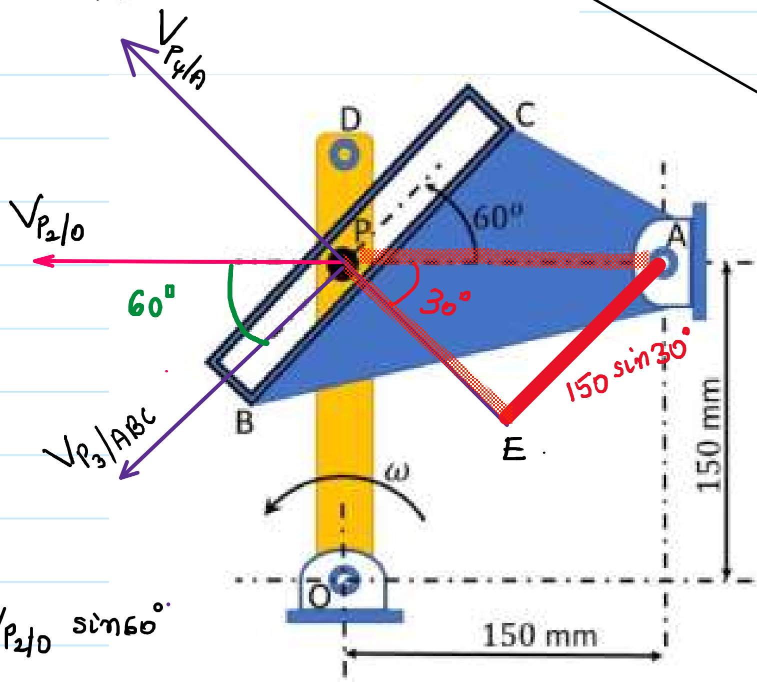
disp. of pin wrt. slot of link ABC.

ω_{OD} = 7 rad/s C.C.W.

OP = 150 mm

AP = 150 mm.





P_2, P_3, P_4 is the point on link 2, link 3, link 4.

$V_{P_2/O}$ – velocity of link 2

$V_{P_3/ABC}$ – velocity of link 3 wrt link 4

$V_{P_4/A}$ – velocity of link 4.

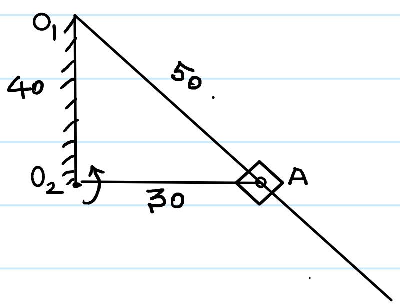
$V_{P_4/A} = V_{P_2/O} \sin 60°$

$AE \cdot \omega_{ABC} = OP \cdot \omega_{OP} \cdot \sin 60°$

$150 \sin 30° \times \omega_{ABC} = 150 \times 7 \times \sin 60°$

$\omega_{ABC} = \frac{7 \sin 60}{\sin 30°} = 12.124$ rad/s.

C.R.P.Q. - volume - 2



O_2A = 4 rad/s.

V_{slider} = ? $\omega_{S.B.}$ = ?

$\frac{W_{cutting}}{W_{Return}}$ = ?   Coriolis acc'n. a^{cr} = ?

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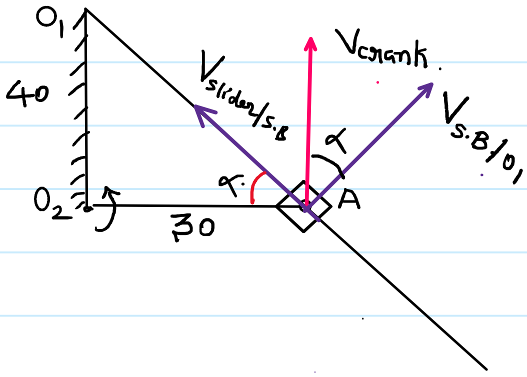
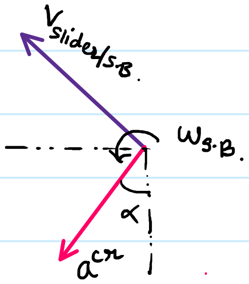




$\tan\alpha = \frac{40}{30}$

$\alpha = 53.13^\circ$

$V_{slider/s.B} = V_{crank} \cdot \sin\alpha$.

$= 30 \times 4 \sin 53.13 = 120 \times 0.8 = 96\,cm/s$.

V_{slider} – $V_{s.B/O_1} = V_{crank} \cdot \cos\alpha$

$= 30 \times 4 \times \cos 53.13 = 120 \times 0.6 = 72\,cm/s$.

$V_{s.B.}$ – $V_{s.B/O_1} = 72 = O_1A \cdot \times \cdot \omega_{s.B.} \Rightarrow \omega_{s.B.} = 72/50 = 1.44\ rad/s$.

Ratio of cutting velocity to Return Velocity.

$$\frac{(\omega_{max})_{cutting}}{(\omega_{max})_{Return}} = \frac{c-r}{c+r} = \frac{40-30}{40+30} = \frac{1}{7}.$$

Coriolis acceleration

$$a^{cr} = 2 \cdot \times V_{slider/s.B.} \times \omega_{s.B.}$$

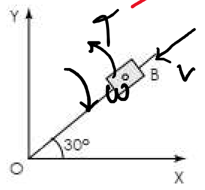
$$= 2 \times 96 \times 1.44$$

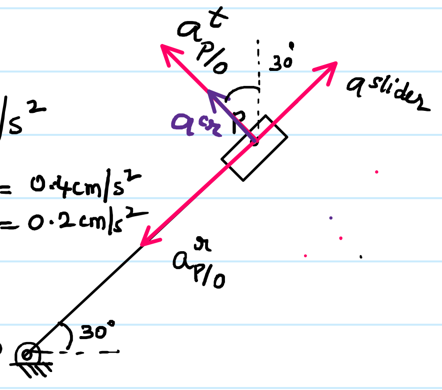
$$= 276.48\,(-\hat{i}\sin 53.13 - \hat{j}\cos 53.13)$$

$$= -276.48 \times 0.8\,\hat{i} - 276.48 \times 0.6\,\hat{j} \quad cm/s.$$

$$= -221.184\,\hat{i} - 168.44\,\hat{j} \quad cm/s.$$

In the mechanism shown at the instant OB = 40 cm and the block B slides inward with a velocity of 0.2 m/sec and decelerates at the rate of 0.5 m/~~sec². The link~~ rotates at 1 rad/sec in CW direction and decelerates at the rate of 0.5 rad/sec². Determine





$OB = 40\,cm.$

$V_{slider} = 0.2\,m/s.$

$a_{slider} = -0.5\,m/s^2$

$\omega_{OB} = 1\,rad/s.$

$\alpha_{OB} = 0.5\,rad/s^2$

$a^r = OP\cdot\omega^2 = 0.4\times 1^2 = 0.4\,cm/s^2$

$a^t = OP\cdot\alpha = 0.4\times 0.5 = 0.2\,cm/s^2$

$a^{slider} = 0.5\,m/s^2$

$a^{cr} = 2\cdot V\cdot\omega = 2\times 0.2\times 1 = 0.4\,m/s^2$

31. The resultant acceleration of the block centre B with respect to the centre of rotation of the link 'O'.
   (a) 0.608 m/sec²   (b) 1.64 m/sec²
   (c) 0.88 m/sec²   (d) 0.94 m/sec²

$$\vec{a}_{Resultant} = \vec{a}^r + \vec{a}^t + \vec{a}^{cr} + \vec{a}^{slider}$$

$$\vec{a}_{Resultant} = 0.4(-\hat{i}\cos 30^\circ - \hat{j}\sin 30) + 0.2(-\hat{i}\sin 30^\circ + \hat{j}\cos 30^\circ) + 0.4(-\hat{i}\sin 30^\circ + \hat{j}\cos 30) + 0.5(\hat{i}\cos 30 + \hat{j}\sin 30)$$

32. The direction of the above acceleration with respect to OX
   (a) 63.7°   (b) 110.53°
   (c) 93.7°   (d) 176.3°

$$a_{Resultant} = (-0.4\cos 30 - 0.2\sin 30 - 0.4\sin 30^\circ + 0.5\cos 30)\,\hat{i} + \hat{j}(-0.4\sin 30^\circ + 0.2\cos 30^\circ + 0.4\cos 30^\circ + 0.5\sin 30)$$

$$a_{Resultant} = 0.608\,m/s^2$$

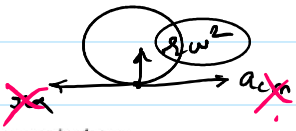
Direction $\alpha = Tan^{-1}\left(\frac{j\text{-coff}}{i\text{-coff}}\right) = 110^\circ$

$\alpha = 180 - Tan^{-1}\left(\frac{j\text{-coff}}{i\text{-coff}}\right) \rightarrow -ve$

Pure Rolling

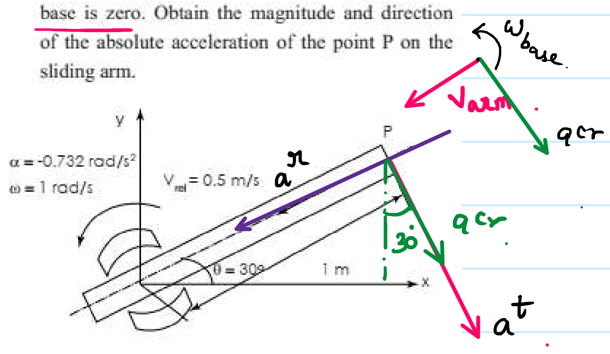
33. A solid disc of radius r rolls without slipping on a horizontal floor with angular velocity ω and angular acceleration α. The magnitude of the acceleration of the point of contact on the disc is (GATE-12)
   (a) zero   (b) rα
   (c) $\sqrt{(r\alpha)^2 + (r\omega)^2}$ (d) $r\omega^2$

$a_{cm} = r\alpha.$



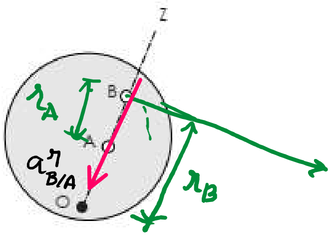
38. Fig. shows a 2 degree of freedom manipulator consisting of a rotary base and a sliding arm which slides radially with respect to the base. The instantaneous position, angular velocity, angular acceleration and relative velocity of arm with respect to the base are as shown in fig. The radial relative acceleration of the arm with respect to the base is zero. Obtain the magnitude and direction of the absolute acceleration of the point P on the sliding arm.

$a^{slider} = 0$



Common Data for Question 34 & 35 :

The circular disc shown in its plan view in the figure rotates in a plane parallel to the horizontal plane about the point O at a uniform angular velocity ω. Two other points A and B are located on the line OZ at distances r_A and r_B from O respectively. **(GATE-03)**



V_B

$\perp^{lar}$ to line joining AB.

$V_{B/A} = V_{B/O} - V_{A/O}$

$= (OB \cdot \omega - OA\,\omega)$

$= (OB - OA) \cdot \omega$

$= (r_B - r_A) \cdot \omega$

34. The velocity of point B with respect to point A is a vector of magnitude

(a) 0 ✗

(b) $\omega(r_B - r_A)$ and direction opposite to the direction of motion of point B ✗

(c) $\omega(r_B - r_A)$ and direction same as the direction of motion of point B ✓

(d) $\omega(r_B - r_A)$ and direction being from O to Z

35. The acceleration of point B with respect to point A is a vector of magnitude

(a) 0

(b) $\omega(r_B^2 - r_A^2)$ and direction same as the direction of motion of point B

(c) $\omega^2(r_B - r_A)$ and direction opposite to the direction of motion of point B ✗

(d) $\omega^2(r_B - r_A)$ and direction being from Z to 'O' ✓

$a^r_{B/A} = a^r_{B/O} - a^r_{A/O}$

$= (OB \cdot \omega^2 - OA \cdot \omega^2)$

$= (r_B - r_A)\,\omega^2$

39. In a reciprocating engine mechanism, the crank and connecting rod of same length r meters. At a given instant, when the crank makes an angle of 45° with TDC and the crank rotates with a uniform velocity of ω rad/s, the angular acceleration of the connecting rod will be

(a) $2\omega^2 r$ (b) $\omega^2 r$

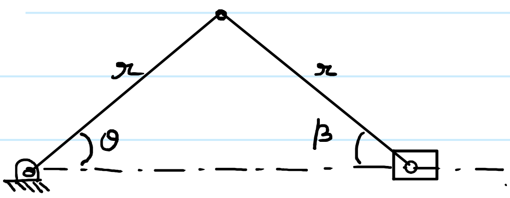
(c) ω^2/r (d) zero

$\theta = 45^\circ$

$\omega = \text{constant}$

$\alpha_{\text{conn.Rod}} = ?$

$r = l$



$r \sin\theta = l \cdot \sin\beta$

$\sin\theta = \sin\beta$

$\cos\theta \cdot \frac{d\theta}{dt} = \cos\beta \cdot d\beta/dt$

$\cos\theta \cdot \omega_{\text{crank}} = \cos\beta \cdot \omega_{\text{conn.Rod}}$ →

$\cos 45^\circ \times \omega = \cos 45^\circ\ \omega_{\text{conn.Rod}}$

@ $\theta = 45^\circ$ $\boxed{\omega_{\text{crank}} = \omega_{\text{conn.Rod}}}$

$$(-\sin\theta)\cdot\frac{d\theta}{dt}\cdot\omega_{crank} = -\sin\beta\cdot\frac{d\beta}{dt}\cdot\omega_{C.R.} + \cos\beta\cdot\frac{d^2\beta}{dt^2}$$

$$-\sin\theta\cdot\omega^2_{crank} = -\sin\beta\cdot\omega^2_{C.R} + \cos\beta\cdot\alpha_{C.R}$$

$$-\sin 45^\circ\cdot\cancel{\omega^2_{crank}} = -\sin 45^\circ\times\cancel{\omega^2_{C.R}} + \cos 45^\circ\,\alpha_{C.R}.$$

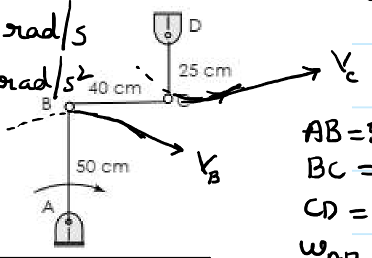
$$\alpha_{C.R.} = 0$$

36. A four bar mechanism along with the dimensions is shown in figure, The links AB and CD are vertical while the link BC is horizontal in the given configuration. The input link has an instantaneous angular velocity of 0.2 rad/sec and acceleration of 0.1 rad/sec². The angular velocity and acceleration of the output link DC are

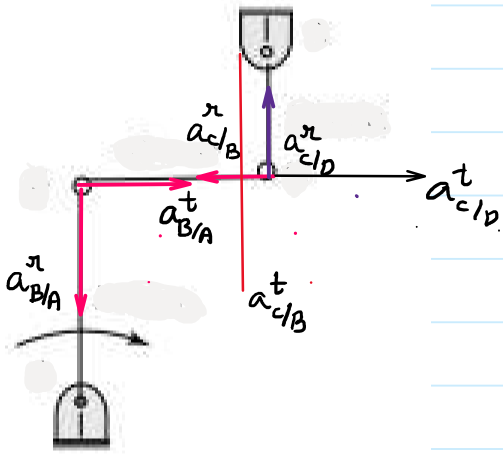
$\alpha_{DC} = ?$

$\omega_{AB} = 0.2\ rad/s$

$\alpha_{AB} = 0.1\ rad/s^2$



$AB = 50cm$
$BC = 40cm$
$CD = 25cm$
$\omega_{AB} = 0.2\ rad/s$
$\omega_{CD} = 0.4\ rad/s.$
$\alpha_{AB} = 0.1\ rad/s^2$



$\boxed{\text{(a) 0.4 rad/sec and 0.2 rad/sec}^2}$
(b) 0.2 rad/sec and 0.2 rad/sec²
(c) 0.1 rad/sec and 0.4 rad/sec²
(d) 0.4 rad/sec and 0.1 rad/sec²

$$\vec{a}_c = \vec{a}_B + \vec{a}_{c/B}$$

$$\vec{a}^r_{c/D} + \vec{a}^t_{c/D} = \vec{a}^r_{B/A} + \vec{a}^t_{B/A} + \vec{a}^r_{c/B} + \vec{a}^t_{c/B}$$

Velocity ~~Analysis~~.

AB is $\parallel^{le}$ to CD

$\boxed{\omega_{BC} = 0}$

$$V_{B/A} = V_{c/D}$$

$$AB\cdot\omega_{AB} = CD\cdot\omega_{CD}.$$

$$50\times0.2 = 25\times\omega_{CD}$$

$$\omega_{CD} = 0.4\ rad/s.$$

$$CD\cdot\omega^2_{CD}(+\hat{j}) + CD\cdot\alpha_{CD}(\hat{i}) = AB\cdot\omega^2_{AB}(-\hat{j}) + AB\,\alpha_{AB}(\hat{i}) + CB\cdot\cancel{\omega^2_{CB}}(-\hat{i}) + CB\,\alpha_{CB}(\hat{j})$$

$$25(0.4)^2(\hat{j}) + 25\times\alpha_{CD}(\hat{i}) = 50\times0.2^2(-\hat{j}) + 50\times0.1(\hat{i}) + 0 + 40\,\alpha_{CB}(\hat{j})$$

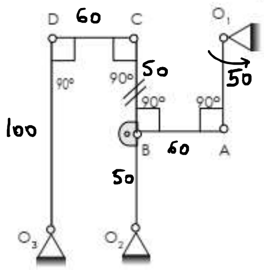
$$4\hat{j} + 25\,\alpha_{CD}(\hat{i}) = -2\hat{j} + 5\hat{i} + 40\,\alpha_{CB}(\hat{j})$$

$\hat{j}$-coff $\quad 4 = -2 + 40\,\alpha_{CB} \Rightarrow \alpha_{CB} = 6/40 = 0.15\ rad/s^2$

$\hat{i}$-coff $\quad 25\,\alpha_{CD} = 5 \Rightarrow \alpha_{CD} = 0.2\ rad/s^2$

37. In the linkage shown in figure, $O_1A = 50$ mm, AB $= 60$ mm, $O_2B = 50$ mm, BC $= 50$ mm, CD $= 60$ mm, $O_3D = 100$ mm. In the position shown in the figure, if O_1A has momentarily an angular velocity of 2 rad/s without any angular acceleration, then the velocity and acceleration of D will be:

(GATE-04)



(a) 100 mm/s, 100 mm/s² (b) 200 mm/s, 100 mm/s²
(c) 200 mm/s, 300 mm/s² (d) 200 mm/s, 400 mm/s²

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$\omega_{O_1A} = 2$ rad/s.

$\alpha_{O_1A} = 0$ rad/s²

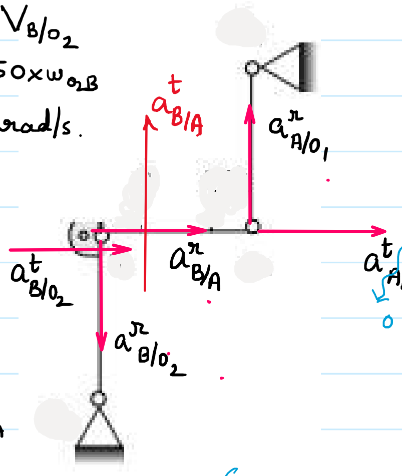
$V_D,\ a_D = ?$

O_1A and O_2B are $\parallel^{le}$ $\Rightarrow$ $\boxed{\omega_{AB} = 0}$

$V_{A/O_1} = V_{B/O_2}$

$50 \times 2 = 50 \times \omega_{O_2B}$

$\omega_{O_2B} = 2$ rad/s.



$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

$\vec{a}^{\,r}_{B/O_2} + \vec{a}^{\,t}_{B/O_2} = \vec{a}^{\,r}_{A/O_1} + \underbrace{\vec{a}^{\,t}_{A/O_1}}_{0} + \underbrace{\vec{a}^{\,r}_{B/A}}_{0} + \vec{a}^{\,t}_{B/A}$

$O_2B\cdot\omega^2_{O_2B}(-\hat{j}) + O_2B\cdot\alpha_{O_2B}\cdot(\hat{i}) = O_1A\cdot\omega^2_{O_1A}(\hat{j}) + O_1A\cdot\underbrace{\alpha_{O_1A}}_{0}(\hat{i}) + AB\cdot\underbrace{\omega^2_{AB}}_{0}(-\hat{i}) + AB\cdot\alpha_{AB}(\hat{j})$

$50 \times 2^2(-\hat{j}) + 50\,\alpha_{O_2B}(\hat{i}) = 50 \times 2^2(\hat{j}) + 0 + 0 + 60\,\alpha_{AB}$

j-coff $\quad -200 = 200 + 60\,\alpha_{AB} \Rightarrow \alpha_{AB} = -400/60$ rad/s²

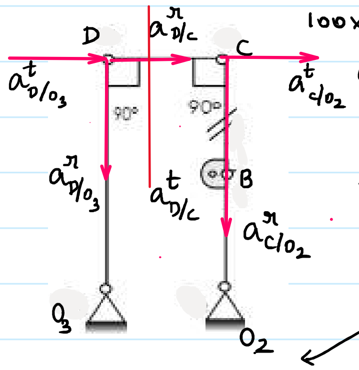
i-coff $\quad 50\,\alpha_{O_2B} = 0 \Rightarrow \boxed{\alpha_{O_2B} = 0}$

$\omega_{O_2C} = \omega_{O_2D} = 2$ rad/s. $\quad V_{C/O_2} = V_{D/O_3} \Rightarrow \omega_{CD} = 0,\ \alpha_{O_2C} = 0$

O_2C is $\parallel^{le}$ to O_3D $\quad O_2C \times \omega_{O_2C} = O_3D \times \omega_{O_3D}$

$100 \times 2 = 100 \times \omega_{O_3D}$

$\omega_{O_3D} = 2$ rad/s.



$\vec{a}_D = \vec{a}_C + \vec{a}_{D/C}$

$\vec{a}^{\,r}_{D/O_3} + \vec{a}^{\,t}_{D/O_3} = \vec{a}^{\,r}_{C/O_2} + \vec{a}^{\,t}_{C/O_2} + \vec{a}^{\,r}_{D/C} + \vec{a}^{\,t}_{D/C}$

$O_3D\cdot\omega^2_{O_3D}\cdot(-\hat{j}) + O_3D\cdot\alpha_{O_3D}(\hat{i}) = O_2C\cdot\omega^2_{O_2C}(-\hat{j}) + O_2C\,\underbrace{\alpha_{O_2C}}_{0}(\hat{i}) + DC\cdot\underbrace{\omega^2_{CD}}_{0}(-\hat{i}) + DC\,\alpha_{CD}(\hat{j})$

$100 \times 2^2(-\hat{j}) + 100\,\alpha_{O_3D}(\hat{i}) = 100 \times 2^2(-\hat{j}) + 0 + 0 + 60\,\alpha_{CD}(\hat{j})$

j-coff $\quad -400 = -400 + 60\,\alpha_{CD} \Rightarrow \alpha_{CD} = 0$

i-coff $\quad 100\,\alpha_{O_3D} = 0 \Rightarrow \alpha_{O_3D} = 0$.

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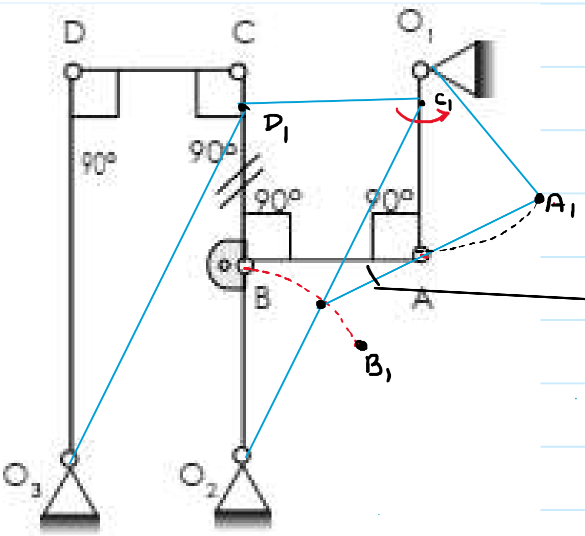
CD and C_1D_1 are $\parallel^{le}$.

$\omega_{CD} = 0$

$\alpha_{CD} = 0$



$\omega_{AB} = 0$

$\alpha_{AB} = -400/60$

Link AB having angular retardation.

Link CD is in Translation throughout the transmission of motion.

$a_D = ?$

$V_D = ?$

$V_{D/O_3} = O_3D \cdot \omega_{O_3D} = 100 \times 2 = 200 mm/s$.

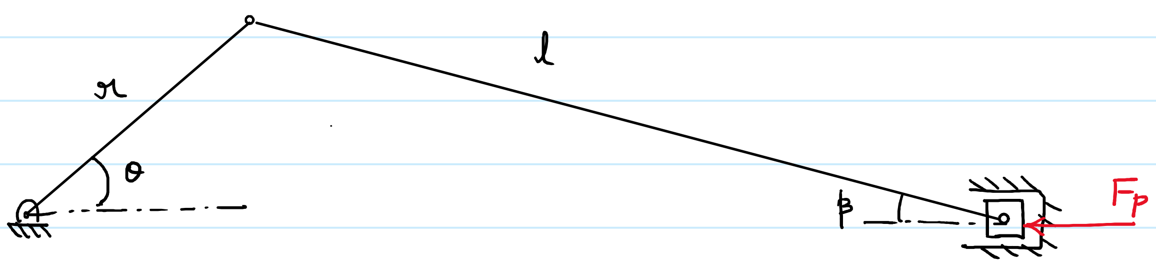
$a^{r}_{D/O_3} = O_3D \cdot \omega^2_{O_3D} = 100 \times 2^2 = 400 mm/s$.

$\alpha_{O_3D} = 0$.

Dynamic Analysis of Crank Slider Mechanism

Assumptions

1. Mass of Crank and Connecting Rod is neglected.
2. Crank is rotating with constant angular velocity.



F_p – Piston Effort.

Piston Effort:

$$F_p = F_G \pm F_I \pm F_W \pm f$$

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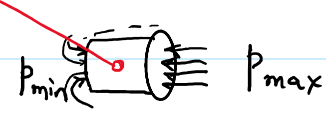
F_G - Gas force.

$$F_G = p_{max} \cdot \frac{\pi}{4} D^2 - p_{min} \cdot \frac{\pi}{4} (D^2 - d^2)$$



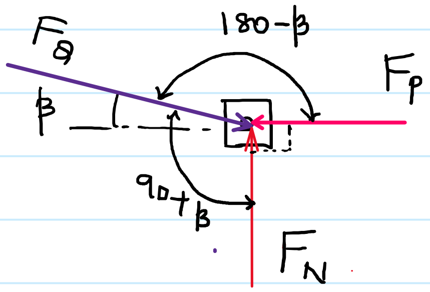
D - Dia of piston, d - dia of connecting

F_I - Inertia force $\quad F_I = m_{piston} \cdot a_{piston}$

$$a_p = r\omega^2 \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

F_W - Weight of piston - (Considered only for vertical cylinder engine)

f - frictional force



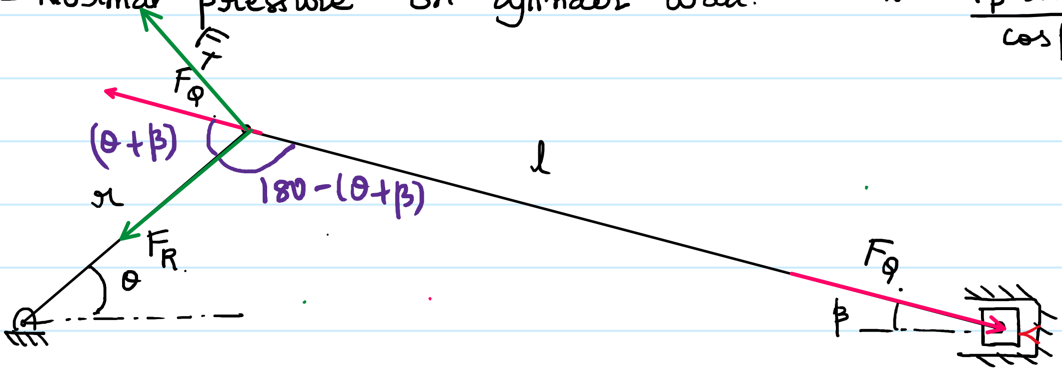
$$\frac{F_p}{\sin(90+\beta)} = \frac{F_Q}{\sin 90^\circ} = \frac{F_N}{\sin(180-\beta)}$$

$$F_Q = \frac{F_p}{\cos\beta}$$

F_Q - Thrust on connecting Rod.

F_N - Normal pressure on cylinder wall.

$$F_N = \frac{F_p \cdot \sin\beta}{\cos\beta} = F_p \tan\beta$$



F_T - Tangential force / Crank Effort.

F_R - Radial load on crank Bearings.

$$F_T = F_Q \cdot \sin(\theta+\beta)$$

$$F_T = \frac{F_p}{\cos\beta} \sin(\theta+\beta)$$

$$F_R = F_Q \cdot \cos(\theta+\beta) = \frac{F_p}{\cos\beta} \cos(\theta+\beta)$$

$$\text{Torque on crank shaft} = F_T \times r = \frac{F_p}{\cos\beta} \sin(\theta+\beta) \times r.$$

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Dynamic Analysis

Statement for Common data Q.40 & Q. 41

In a single acting steam engine when the piston is at the middle of the expansion stroke the net gas force on the Piston is 2 kN. The crank length is 20 cm and the connecting rod length is 80 cm

40. The thrust in the connecting rod is
    (a) 2 kN (b) 2.065 kN
    (c) 1.935 kN (d) 1.5 kN

41. The turning moment is
    (a) 0.4 kN-m (b) 4 kN-m
    (c) 0.3 kN-m (d) None

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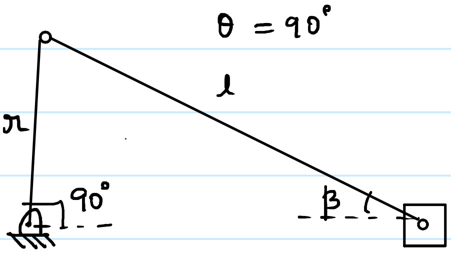
Piston effort

$F_p = 2\ KN$

$r = 20cm. \quad l = 80cm.$

$\theta = 90°$



$r \sin\theta = l \sin\beta$

$20 \sin 90° = 80 \sin\beta$

$\beta = \sin^{-1}\left(\frac{20}{80}\right)$

$= 14.47°$

Thrust in connecting Rod.

$F_Q = \frac{F_p}{\cos\beta} = \frac{2}{\cos 14.47°} = 2.06\ KN.$

Turning Moment.

$T = \frac{F_p}{\cos\beta} \cdot \sin(\theta+\beta) \times r$

$T = \frac{2}{\cos 14.47} \sin(90 + 14.47) \times 0.2$

$T = 0.4\ KN\text{-}m.$

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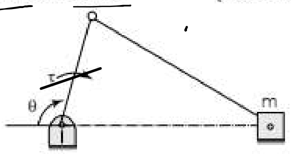
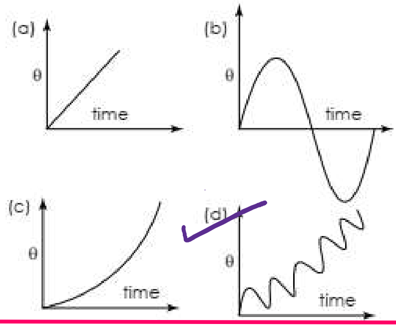
45. Consider a slider crank mechanism with nonzero masses and inertia. A constant torque τ is applied on the crank as shown in the figure. Which of the following plots best resembles variation of crank angle, θ versus time (GATE –15 –Set 1)





I - inertia of crank and connecting Rod.

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$$\tau = I\alpha \pm F_p \times r$$

$$F_I = mr\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

$$\tau = I\alpha \pm F_I \times r$$

$f(\theta)$

$$\alpha = \frac{d^2\theta}{dt^2}$$

$$\alpha = \frac{\tau \pm F_I \times r}{I}$$

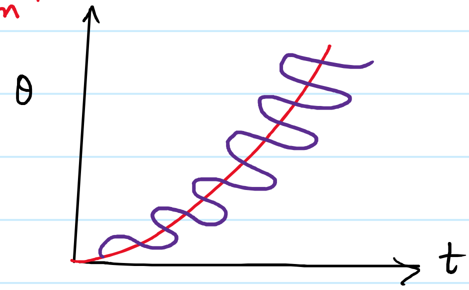
τ = constant.

I = constant.

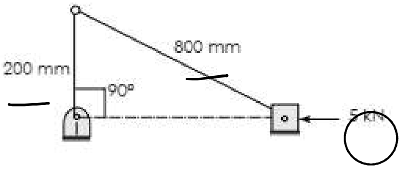
$$\theta = \frac{\tau}{I}t^2 \pm \iint \frac{F_I \times r}{I}$$

Parabolic Variation.

cyclic / harmonic.



46. A slider crank mechanism with crank radius 200 mm and connecting rod length 800 mm is shown. The crank is rotating at 600 rpm in the counterclockwise direction. In the configuration shown, the crank makes an angle of 90° with the sliding direction of the slider, and a force of 5 kN is acting on the slider. Neglecting the inertia forces, the turning moment on the crank (in kN-m) is _______ (GATE –16)



$F_p = 5$ kN. , $\theta = 90°$

$$T = \frac{F_p}{\cos\beta}\sin(\theta+\beta) \times r$$

$$T = \frac{F_p \cdot \sin(90+\beta)}{\cos\beta} \times r = F_p \times r$$

$$T = 5 \times 0.2 = 1\ \text{kN-m}$$

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Vibrations

↓

Pythagorus.

↳ sound waves have repeated pattern.

Repeated pattern → oscillations.

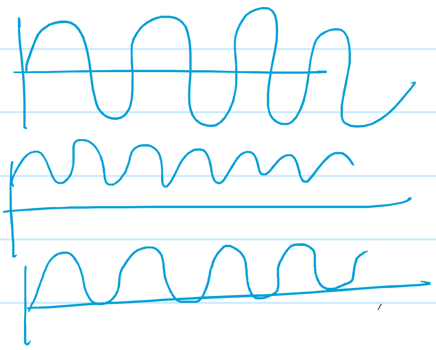
Oscillations produced under the influence of Mechanical energy are called Mechanical Vibrations.

Mechanical Energy = (P.E) + (K.E)

Vibrations → caused due to presence of unbalanced element.

↓

Dynamic Loads / Unsteady loads / Transient loading.



↓

Variable stresses are setup in the m/c component.

↓

Design criteria → Endurance strength
Fatigue strength.

Theories of failure.

$\underbrace{\text{static}\quad \text{Load}}$. = const. $\quad \frac{d(\text{Load})}{dt} = 0$

↳ Yield strength
ultimate strength.