CSCI-GA.1170-001/002 Fundamental Algorithms	November 19, 2015

Solutions to Problem 1 of Homework 8 (6 points)

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Using dynamic programming, find the optimum printing of the text *"Not all those who wander are lost"*, i.e. $\ell_1 = 3, \ell_2 = 3, \ell_3 = 5, \ell_4 = 3, \ell_5 = 6, \ell_6 = 3, \ell_7 = 4$, with line length $L = 14$ and penalty function $P(x) = x^3$. Will the optimal printing you get be consistent with the strategy "print the word on as long as it fits, and otherwise start a new line"? Once again, you have to actually find the alignment, as opposed to only finding its penalty.

Solution:

Following the algorithm given in the notes, we maintain two arrays m and s such that $m(i)$ denotes the total badness of the text $l_1, \dots, l_i$. Initialize $m(0) = 0$. As long as $l_1, \dots, l_i$ fit in one line, set $m(i)$ to be the badness of that line. Loop on i going up to n. For a given i let k range over $i, i-1, \dots$ as long as $l_k, \dots, l_i$ fits in one line. For each of those k values calculate $m(k-1)$ plus the badness of the line $l_k, \dots, l_i$. Pick the k that gives the smallest sum and set $m(i)$ equal to that sum. We can also $s(i) = k$ which means that in the optimal splitting of text $l_1, \dots, l_i$ the last line starts with l_k

m(1), s(1)

l_1 fits in the line. So $m(1) = (14-3)^3 = 1131$ and $s(1) = 1$

m(2), s(2)

l_1, l_2 fit in the line. So $m(2) = (14 - (3+3+1))^3 = 343$ and $s(2) = 1$

m(3), s(3)

l_1, l_2, l_3 fit in the line. So $m(3) = (14 - (3+3+5+2))^3 = 1$ and $s(3) = 1$

m(4), s(4)

$$
\begin{aligned}
k = 4 &\implies P = (14-3)^3 = 1131 \\
k = 3 &\implies P = (14 - (3+5+1))^3 = 125 \\
k = 2 &\implies P = (14 - (3+5+3+2))^3 = 1 \\
m(4) &= Min\{m(3) + 1131, m(2) + 125, m(1) + 1\} \\
m(4) &= m(2) + 125 \\
m(4) &= 468 \\
\implies s(4) &= 3
\end{aligned}
$$

m(5), s(5)

$$\begin{aligned}
k = 5 &\implies P = (14 - (6))^3 = 512 \\
k = 4 &\implies P - (14 - (3 + 6 + 1))^3 = 64 \\
m(5) &= Min m(4) + 512, m(3) + 64 \\
m(5) &= m(3) + 64 \\
m(5) &= 65 \\
\implies s(5) &= 4
\end{aligned}$$

m(6), s(6)

$$\begin{aligned}
k = 6 &\implies P = (14 - 3)^3 = 1131 \\
k = 5 &\implies P = (14 - (6 + 3 + 1))^3 = 64 \\
k = 4 &\implies P = (14 - (3 + 6 + 3 + 2))^3 = 0 \\
m(6) &= Min\{m(5) + 1131, m(4) + 64, m(3) + 0\} \\
m(6) &= m(3) + 0 \\
m(6) &= 1 \\
\implies s(6) &= 4
\end{aligned}$$

m(7), s(7)

$$\begin{aligned}
k = 7 &\implies P = (14 - 4)^3 = 1000 \\
k = 6 &\implies P = (14 - (3 + 4 + 1))^3 = 216 \\
m(7) &= Min\{m(6) + 1000, m(5) + 216\} \\
m(7) &= m(5) + 216 \\
m(7) &= 281 \\
\implies s(7) &= 6
\end{aligned}$$

$s(7) = 6 \implies$ in the optimal splitting of the given text, the last line starts with l_6 which is *"are"*. Therefore this line is *"are lost"*

$s(5) = 4 \implies$ in the optimal splitting the line ending with *"wander"* starts with *"who"*. Therefore this line is *"who wander"*

$s(3) = 1 \implies$ in the optimal splitting the line ending with *"those"* starts with *"Not"*. Therefore this line is *"Not all those"*

Therefore in the optimal splitting, the first line contains the text *"Not all those"*. Second line contains the text *"who wander"*. Third line contains the text *"are lost"*. The penalty turns out to be 281. If we used the greedy strategy, the first line contains the text *"Not all those"*. Second line contains the text *"who wander are"*. Third line contains the text *"lost"*. The penalty in this case is 1132

Therefore the optimal printing derived using dynamic programming is not consistent with the greedy strategy □