

Solutions to Problem 1 of Homework 2 (14 points)

Name: **** INSERT YOUR NAME HERE ****

Due: Tuesday, September 22

Consider the recurrence $T(n) = 8T(n/4) + n$ with initial condition $T(1) = 1$.

- (a) (2 points) Solve it asymptotically using the “master theorem”.

Solution: ***** INSERT YOUR SOLUTION HERE *****

□

- (b) (4 points) Solve it by the “guess-then-verify method”. Namely, guess a function $g(n)$ — presumably solving part (a) will give you a good guess — and argue by induction that for all values of n we have $T(n) \leq g(n)$. What is the “smallest” $g(n)$ for which your inductive proof works?

Solution: ***** INSERT YOUR SOLUTION HERE *****

□

- (c) (4 points) Solve it by the “recursive tree method”. Namely, draw the full recursive tree for this recurrence, and sum up all the value to get the final time estimate. Again, try to be as precise as you can (i.e., asymptotic answer is OK, but would be nice if you preserve a “leading constant” as well).

Solution: ***** INSERT YOUR SOLUTION HERE *****

□

- (d) (4 points) Solve it *precisely* using the “domain-range substitution” technique. Namely, make several changes of variables until you get a basic recurrence of the form $S(k) = S(k-1) + f(k)$ for some f , and then compute the answer from there. Make sure you carefully maintain the correct initial condition.

Solution: ***** INSERT YOUR SOLUTION HERE *****

□

- (e) This part will not be graded. However, briefly describe your personal comparison of the above 4 methods. Which one was the fastest? The easiest? The most precise?

Solution: ***** INSERT YOUR SOLUTION HERE *****

□

Solutions to Problem 2 of Homework 2 (10 points)

Name: **** INSERT YOUR NAME HERE ****

Due: Tuesday, September 22

Consider the following recursive procedure.

 $\text{BLA}(n)$:**If** $n = 1$ **Then Return** 1**Else Return** $\text{BLA}(n/3) + \text{BLA}(n/3)$

- (a) (3 points) What function of n does BLA compute (assume it is always called on n which is a power of 3)?

Solution: ***** INSERT YOUR SOLUTION HERE *****

- (b) (3 points) What is the running time $T(n)$ of BLA?

Solution: ***** INSERT YOUR SOLUTION HERE *****

- (c) (4 points) How do the answers to (a) and (b) change if we replace the last line by “**Else Return** $2 \cdot \text{BLA}(n/3)$ ”?

Solution: ***** INSERT YOUR SOLUTION HERE *****

Solutions to Problem 3 of Homework 2 (16 points)

Name: **** INSERT YOUR NAME HERE ****

Due: Tuesday, September 22

Let $A[1 \dots n]$ be an array of pairwise different numbers. We call pair of indices $1 \leq i < j \leq n$ an *inversion* of A if $A[i] > A[j]$. The goal of this problem is to develop a divide-and-conquer based algorithm running in time $\Theta(n \log n)$ for computing the number of inversions in A .

- (a) (8 points) Suppose you are given a pair of *sorted* integer arrays A and B of length $n/2$ each. Let C an n -element array consisting of the concatenation of A followed by B . Give an algorithm (in pseudocode) for counting the number of inversions in C and analyze its runtime. Make sure you also argue (in English) why your algorithm is correct.

Solution: ***** INSERT YOUR SOLUTION HERE *****

□

- (b) (8 points) Give an algorithm (in pseudocode) for counting the number of inversions in an n element array A that runs in time $\Theta(n \log n)$. Make sure you formally prove that your algorithm runs in time $\Theta(n \log n)$ (e.g., write the recurrence and solve it.)
(**Hint:** Combine Merge Sort with part (a).)

Solution: ***** INSERT YOUR SOLUTION HERE *****

□

Solutions to Problem 4 of Homework 2 (9 points)

Name: **** INSERT YOUR NAME HERE ****

Due: Tuesday, September 22

Solve the following recurrences using any method you like. If you use “master theorem”, use the version from the book and justify why it applies. Assume $T(1) = 2$, and be sure you explain every important step.

(a) $T(n) = T(9n/10) + n$.

Solution: ***** INSERT YOUR SOLUTION HERE *****

□

(b) $T(n) = 2T(n/2) + n \log n$.

Solution: ***** INSERT YOUR SOLUTION HERE *****

□

(c) $T(n) = T(\sqrt{n}) + 1$. (**Hint:** Substitute ... until you are done!)

Solution: ***** INSERT YOUR SOLUTION HERE *****

□