CSCI-GA.1180-001	November 4, 2016

Solutions to Problem 1 of Homework 4

Name: GOWTHAM GOLI (N17656180) *Due: Friday, November 4*

(a) **Solution:**

Let x^* be the solution to the linear least squares problem of minimizing $\|b - Ax\|_2$ then we know that, $b_R = Ax^*$ and $r_N = b_N$, $r_R = b_R - Ax^* = 0$

$$\begin{aligned} r &= r_R + r_N \\ \implies 0 &= 0 + r_N \\ \implies r_N &= 0 \\ \implies b_N &= 0 \end{aligned}$$

$\therefore$ b can be expressed only in terms of the range space component of A as the null space component of A^T is 0 i.e. $b = b_R = Ax^*$ $\square$

(b) **Solution:**

Let x^* be the solution to the linear least squares problem of minimizing $\|b - Ax\|_2$ then we know that, $b_R = Ax^* \implies r_R = b_R - Ax^* = 0$ and $r_N = b_N$.

$\because x^* = 0 \implies b_R = Ax^* = 0$ i.e. $r_R = b_R = 0$ and $r_N = b_N$

$\therefore$ b can be expressed only in terms of the null space component of A^T as the range space component of A is 0 i.e. $b = b_N$.

$\square$

(c) **Solution:**

Let x_1^*, x_2^* be the solutions to the linear least squares problems of minimizing $\|b_1 - Ax_1\|_2$ and $\|b_2 - Ax_2\|_2$ then we know that $b_{R_1} = Ax_1^*$ and $b_{R_2} = Ax_2^*$. But is given that $x_1^* = x_2^* \implies b_{R_1} = b_{R_2}$.

Also it is given that $b_1 \neq b_2 \implies b_{R_1} + b_{N_1} \neq b_{R_2} + b_{N_2} \implies b_{N_1} \neq b_{N_2}$

$\therefore b_1, b_2$ have equal range space components and unequal null space components $\square$

CSCI-GA.1180-001 November 4, 2016

Solutions to Problem 2 of Homework 4

Name: GOWTHAM GOLI (N17656180) *Due: Friday, November 4*

(a) **Solution:**

$$
\begin{aligned}
H(v) &= I - \frac{2vv^T}{\|v\|_2^2} \\
\implies H^T(v) &= I - \frac{2(vv^T)^T}{\|v\|_2^2} = I - \frac{2vv^T}{\|v\|_2^2} = H(v) \qquad (1) \\
\implies H^T(v)H(v) &= (I - \frac{2vv^T}{\|v\|_2^2})(I - \frac{2vv^T}{\|v\|_2^2}) \\
&= I - \frac{4vv^T}{\|v\|_2^2} + \frac{4v(v^Tv)v^T}{\|v\|_2^4} \\
&= I - \frac{4vv^T}{\|v\|_2^2} + \frac{4vv^T}{\|v\|_2^2} \\
&= I \qquad (2)
\end{aligned}
$$

From (1) and (2), we can conclude that $H^T(v)H(v) = H(v)H^T(v) = I$ i.e., the associated Householder matrix is orthogonal □

(b) **Solution:**

$$
\begin{aligned}
H(\alpha v) &= I - \frac{2\alpha v(\alpha v)^T}{\|\alpha v\|_2^2} \\
&= I - \frac{2\alpha^2 vv^T}{\alpha^2\|v\|_2^2} \\
&= I - \frac{2vv^T}{\|v\|_2^2} \\
&= H(v)
\end{aligned}
$$

□

(c) **Solution:** Given that

$$
\begin{aligned}
Ha &= b \\
\implies H^THa &= H^Tb \\
\implies a &= Hb \quad \text{Using (1) and (2)} \qquad (3)
\end{aligned}
$$

Now consider $\|a\|_2^2 = a^T a$

$$\begin{aligned}\|a\|_2^2 = a^T a &= (Hb)^T (Hb)\\ &= b^T (H^T H) b\\ &= b^T b\\ &= \|b\|_2^2\end{aligned}$$

$\therefore$ If $Ha = b$ it inherently implies that $\|a\|_2^2 = \|b\|_2^2$. (This isn't any additional information)
Now consider $Ha = b \implies (I - 2uu^T)a = b \implies a - b = 2uu^T a$

From (3), $Hb = a \implies (I - 2uu^T)b = a \implies a - b = -2uu^T b$

From the above two equations we get

$uu^T(a+b) = 0 \implies (u^T u)u^T(a+b) = u^T 0 \implies u^T(a+b) = 0 \implies (a+b)^T u = 0$ $\square$

(d) **Solution:** Let $u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \implies u_1^2 + u_2^2 = 1$ From part (c)

$$\begin{aligned}&(a+b)^T u = 0\\ \implies &\begin{pmatrix} 8 & -4 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = 0\\ \implies &u_2 = 2u_1\\ \implies &5u_1^2 = 1\\ \implies &u_1 = \pm 1/\sqrt{5}\\ \implies &u_2 = \pm 2/\sqrt{5}\\ \implies &u = \begin{pmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{pmatrix}, \begin{pmatrix} -1/\sqrt{5} \\ -2/\sqrt{5} \end{pmatrix}\end{aligned}$$

$\square$

(e) **Solution:**

```
a = [9/2;-1];
b = [7/2;-3];
u1 = [1/sqrt(5); 2/sqrt(5)];
u2 = [-1/sqrt(5); -2/sqrt(5)];
H1 = [1 0; 0 1] - 2*(u1*u1');
H2 = [1 0; 0 1] - 2*(u2*u2');
H1a = H1*a
H2a = H2*a
```

$$Ha = \begin{pmatrix} 3.5 \\ -3 \end{pmatrix} = b$$

$\square$

CSCI-GA.1180-001 November 4, 2016

Solutions to Problem 3 of Homework 4

Name: GOWTHAM GOLI (N17656180) *Due: Friday, November 4*

(1) **Solution:**

Any m-vector b can be expressed as a linear combinations of the columns of U.

Let $b = u_1\lambda_1 + \cdots + u_m\lambda_m$ i.e. $b = U\lambda$, where $\lambda = \begin{pmatrix} \lambda_1 \\ \vdots \\ \lambda_m \end{pmatrix}$

$$
\begin{aligned}
x &= A^\dagger b \\
&= VS^\dagger U^T U\lambda \\
&= VS^\dagger \lambda \text{ as } U^T U = I \\
&= V \begin{pmatrix} S_n^{-1} & 0 \end{pmatrix} \lambda \\
&= V \begin{pmatrix} 1/\sigma_1 & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & 1/\sigma_2 & \dots & 0 & 0 & \dots & 0 \\ & & \ddots & & 0 & \dots & 0 \\ 0 & 0 & \dots & 1/\sigma_n & 0 & \dots & 0 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \vdots \\ \lambda_m \end{pmatrix} \\
&= V \begin{pmatrix} \lambda_1/\sigma_1 \\ \lambda_2/\sigma_2 \\ \vdots \\ \lambda_n/\sigma_n \end{pmatrix} \\
\implies x &= v_1\lambda_1/\sigma_1 + v_2\lambda_2/\sigma_2 + \cdots + v_n\lambda_n/\sigma_n
\end{aligned}
$$

□

(2) (a) **Solution:**

```
format long e;
A = [1 1; 1 1+10^-6; 1 1+10^-6];
b1 = [1;2.22474;-0.22474];
b2 = [-2;1;1];

[U,S,V] = svd(A)
alpha = U\b1
beta = U\b2

b1_verify = U(:,1)*alpha(1,1) + U(:,2)*alpha(2,1) + U(:,3)*alpha(3,1)
b2_verify = U(:,1)*beta(1,1) + U(:,2)*beta(2,1) + U(:,3)*beta(3,1)
```

Let $U\alpha = b_1$ and $U\beta = b_2$. Therefore, we have

$$\begin{aligned}
b_1 &= u_1\alpha_1 + u_2\alpha_2 + u_3\alpha_3 \\
b_2 &= u_1\beta_1 + u_2\beta_2 + u_3\beta_3 \\
\text{where } \alpha &= \begin{pmatrix} -1.732050807568829e+00 \\ 4.082482900763943e-07 \\ -1.732043918380825e+00 \end{pmatrix} \\
\beta &= \begin{pmatrix} -5.773502695703939e-07 \\ -2.449489742783110e+00 \\ 1.922962686383511e-16 \end{pmatrix}
\end{aligned}$$

□

(b) **Solution:**

```
Sinv = [1/S(1,1) 0 0;
        0 1/S(2,2) 0];
Apseud = V*Sinv*U'
```

From the above matlab code we get

$$A^\dagger = \begin{pmatrix} 1.000001000052131e+06 & -5.000000000260655e+05 & -5.000000000260654e+05 \\ -1.000000000052131e+06 & 5.000000000260654e+05 & 5.000000000260653e+05 \end{pmatrix}$$

□

(c) **Solution:**

```
x1 = Apseud*b1
alphasigma = [alpha(1,1)/S(1,1) ; alpha(2,1)/S(2,2)];
x1_verify = V*alphasigma
norm_x1 = norm(x1)
norm_b1 = norm(b1);
ratio_x1_b1 = norm_x1/norm_b1;
```

From the above matlab code we get

$$\begin{aligned}
x_1 &= \begin{pmatrix} 1.000000000001855e+00 \\ -4.795693371184925e-12 \end{pmatrix} \\
||x_1|| &= 1.000000000001855e+00 \\
||x_1||/||b_1|| &= 4.082491023640258e-01 \\
x_1 &= v_1\alpha_1/\sigma_1 + v_2\alpha_2/\sigma_2 \\
\text{where } \alpha_1/\sigma_1 &= -7.071065454842872e-01, \alpha_2/\sigma_2 = 7.071070162546135e-01
\end{aligned}$$

□

(d) **Solution:**

```
x2 = Apseud*b2
betasigma = [beta(1,1)/S(1,1) ; beta(2,1)/S(2,2)];
x2_verify = V*betasigma
norm_x2 = norm(x2)
norm_b2 = norm(b2);
ratio_x2_b2 = norm_x2/norm_b2;
```

From the above matlab code we get

$$x_2 = \begin{pmatrix} -3.000002000156393e+06 \\ 3.000000000156393e+06 \end{pmatrix}$$
$$||x_2|| = 4.242642101554256e+06$$
$$||x_2||/||b_2|| = 1.732051385009536e+06$$
$$x_2 = v_1\beta_1/\sigma_1 + v_2\beta_2/\sigma_2$$
$$\text{where } \beta_1/\sigma_1 = -2.357021819835502e-07, \beta_2/\sigma_2 = -4.242642101554258e+06$$

□

(e) **Solution:**

```
norm_x1_x2 = norm(x1-x2);
norm_b1_b2 = norm(b1-b2);
ratio = norm_x1_x2/norm_b1_b2
```

$$\frac{\|x_1 - x_2\|}{\|b_1 - b_2\|} = 1.224746701675926e+06$$

So we can conclude that though they are very near by problems $\because \|b_1\| - \|b_2\| = 4.87 \times 10^{-6}$, the solutions to these problems i.e., x_1 and x_2 need not be close to each other

□

CSCI-GA.1180-001	November 4, 2016

Solutions to Problem 4 of Homework 4

Name: GOWTHAM GOLI (N17656180) *Due: Friday, November 4*

(a) **Solution:**

Given that $Z = \begin{pmatrix} -B^{-1}S \\ I_{n-m} \end{pmatrix}$. From this we can conclude that there are $n-m$ columns of Z and for the matrix product AZ to be feasible, Z must have n rows. Therefore, the dimension of Z is $n \times n-m \implies -B^{-1}S$ is of the form $m \times n-m \implies B^{-1}$ is of the form $m \times m \implies B$ is of the form $m \times m$ and S is of the form $m \times n-m$.

$$\begin{aligned}
AZ &= \begin{pmatrix} B & S \end{pmatrix} \begin{pmatrix} -B^{-1}S \\ I_{n-m} \end{pmatrix} \\
&= \underbrace{\underbrace{B}_{m\times m} \cdot \underbrace{(-B^{-1}S)}_{m\times n-m}}_{m\times n-m} + \underbrace{\underbrace{S}_{m\times n-m} \cdot \underbrace{I_{n-m}}_{n-m\times n-m}}_{m\times n-m} \\
&= \underbrace{\underbrace{-I}_{m\times m} \cdot \underbrace{S}_{m\times n-m}}_{m\times n-m} + \underbrace{\underbrace{S}_{m\times n-m} \cdot \underbrace{I_{n-m}}_{n-m\times n-m}}_{m\times n-m} \\
&= \underbrace{-S + S}_{m\times n-m} \\
&= 0
\end{aligned}$$

Let $B^{-1}S = \begin{pmatrix} B^{-1}s_1 & B^{-1}s_2 & \dots & B^{-1}s_{n-m} \end{pmatrix} \implies Z = \begin{pmatrix} -B^{-1}s_1 & -B^{-1}s_2 & \dots & -B^{-1}s_{n-m} \\ 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ & & \ddots & \\ 0 & 0 & \dots & 1 \end{pmatrix}$

The columns of Z will be linearly independent if we can show that $Zy = 0$ only if $y = 0$

$$
\begin{aligned}
&Zy = 0\\
\implies &\begin{pmatrix} -B^{-1}s_1 & -B^{-1}s_2 & \dots & -B^{-1}s_{n-m} \\ 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ & & \ddots & \\ 0 & 0 & \dots & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_{n-m} \end{pmatrix} = 0\\
\implies &\begin{pmatrix} -y_1B^{-1}s_1 - y_2B^{-1}s_2 + \dots - y_{n-m}B^{-1}s_{n-m} \\ y_1 \\ y_2 \\ \vdots \\ y_{n-m} \end{pmatrix} = 0\\
\implies &\begin{pmatrix} -B^{-1}Sy \\ y \end{pmatrix} = 0\\
\implies &y = 0
\end{aligned}
$$

$\therefore$ The columns of Z are linearly independent $\square$

(b) **Solution:**

- The dimension of Z is $n \times n-m \implies v$ must be a $n-m$ vector $\implies$
  Let $v = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_{n-m} \end{pmatrix}$ and $B^{-1}S = \begin{pmatrix} B^{-1}s_1 & \dots & B^{-1}s_{n-m} \end{pmatrix}$
  Now consider $B^{-1}s_j$, some j^{th} column of the matrix $B^{-1}S$. Let λ_j be a m vector such that $B\lambda_j = s_j \implies \lambda_j = B^{-1}s_j$. But we know the LU factorization of B. Therefore we can solve the the triangular system of equations $LU\lambda_j = s_j$ to get λ_j (so the explicit matrix B^{-1} is not needed $\implies -B^{-1}S = \begin{pmatrix} -\lambda_1 & \dots & -\lambda_{n-m} \end{pmatrix}$

$$
\begin{aligned}
\implies Zv &= \begin{pmatrix} -\lambda_1 & \dots & -\lambda_{n-m} \\ & I_{n-m} & \end{pmatrix} v\\
&= \begin{pmatrix} -\lambda_1 v_1 - \dots - \lambda_{n-m}v_{n-m} \\ I_{n-m}v \end{pmatrix}\\
&= \begin{pmatrix} -\lambda_1 v_1 - \dots - \lambda_{n-m}v_{n-m} \\ v \end{pmatrix}
\end{aligned}
$$

- The dimension of Z^T is $n-m \times n \implies q$ must be a n vector $\implies$
  Let $q = \begin{pmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{pmatrix}$, $q_{1,m} = \begin{pmatrix} q_1 \\ \vdots \\ q_m \end{pmatrix}$, $q_{m+1,n} = \begin{pmatrix} q_{m+1} \\ \vdots \\ q_n \end{pmatrix}$

Similar to the above part, let $-B^{-1}S = \begin{pmatrix} -\lambda_1 & \dots & -\lambda_{n-m} \end{pmatrix}$ where λ_j is the solution the the triangular system of equations $LU\lambda_j = s_j$

$$
\begin{aligned}
\implies Z^T q &= \begin{pmatrix} -B^{-1}S \\ I_{n-m} \end{pmatrix}^T q \\
&= \begin{pmatrix} \underbrace{(-B^{-1}S)^T}_{n-m \times m} & \underbrace{I_{n-m}}_{n-m \times n-m} \end{pmatrix} \begin{pmatrix} q_1 \\ \vdots \\ q_m \\ q_{m+1} \\ \vdots \\ q_n \end{pmatrix} \\
&= (-B^{-1}S)^T \begin{pmatrix} q_1 \\ \vdots \\ q_m \end{pmatrix} + I_{n-m} \begin{pmatrix} q_{m+1} \\ \vdots \\ q_n \end{pmatrix} \\
&= \begin{pmatrix} -\lambda_1 & \dots & -\lambda_{n-m} \end{pmatrix}^T q_{1,m} + q_{m+1,n} \\
&= \begin{pmatrix} -\lambda_1^T \\ \vdots \\ -\lambda_{n-m}^T \end{pmatrix} q_{1,m} + q_{m+1,n} \\
&= \begin{pmatrix} -\lambda_1^T q_{1,m} \\ \vdots \\ -\lambda_{n-m}^T q_{1,m} \end{pmatrix} + q_{m+1,n}
\end{aligned}
$$

$\square$

Solutions to Problem 5 of Homework 4

Name: GOWTHAM GOLI (N17656180) *Due: Friday, November 4*

The given model doesn't fit the given description because as c increases i.e., as the person is more blonder, $\exp(-c)$ decreases and thus h decreases. Therefore for the model to fit the given description, the definition of c must be modified so that c ranges from 0 to 1 with smaller values meaning blonder hair i.e. $c = 0$ is the blondest person.

$$h \approx x_1\sqrt{t-4} + x_2(\frac{100}{w}) + x_3 \exp(-c)$$

(a) **Solution:**

As shown in the notes, we can formulate this into a linear least squares problem of minimizing $||Ax - h||_2$ where the model error of the person j is

$$r_j = h_j - (x_1\sqrt{t_j - 4} + x_2(\frac{100}{w_j}) + x_3 \exp(-c_j))$$

Therefore this can be expressed in a linear algebraic form where the residual r is

$$r = \begin{pmatrix} r_1 \\ \vdots \\ r_m \end{pmatrix} = h - Ax$$

where Row j of A is $\begin{pmatrix} \sqrt{t_j - 4} & \frac{100}{w_j} & \exp(-c_j) \end{pmatrix}$, $h = \begin{pmatrix} h_1 \\ \vdots \\ h_m \end{pmatrix}$ □

(b) **Solution:**

```
format short e
A=[ sqrt(5.2-4) 100/240    exp(-0.13);
    sqrt(6.0-4) 100/162.3  exp(-0.83);
    sqrt(5.9-4) 100/130.8  exp(-1.0)  ;
    sqrt(5.6-4) 100/150.1  exp(-0.24);
    sqrt(6.2-4) 100/95.9   exp(-0.31);
    sqrt(5.7-4) 100/141.2  exp(-0.47) ]
cond2A = cond(A)
```

$$A = \begin{pmatrix} 1.0954e+00 & 4.1667e-01 & 8.7810e-01 \\ 1.4142e+00 & 6.1614e-01 & 4.3605e-01 \\ 1.3784e+00 & 7.6453e-01 & 3.6788e-01 \\ 1.2649e+00 & 6.6622e-01 & 7.8663e-01 \\ 1.4832e+00 & 1.0428e+00 & 7.3345e-01 \\ 1.3038e+00 & 7.0822e-01 & 6.2500e-01 \end{pmatrix}$$

$$cond(A) = 1.4426e+01$$

$\square$

(c) (i) **Solution:** From part(a), we can see that $b = \begin{pmatrix} 9 \\ 8 \\ 7 \\ 10 \\ 12 \\ 9 \end{pmatrix} \implies ||b||_2 = 2.2782e + 01$ $\square$

(ii) **Solution:**

```
b = [9;8;7;10;12;9];
norm_b = norm(b);

[U,S,V] = svd(A);

Sinv = [1/S(1,1) 0 0 0 0 0;
        0 1/S(2,2) 0 0 0 0;
        0 0 1/S(3,3) 0 0 0];

Apseud = V*Sinv*U';
x = Apseud*b
```

$$x = \begin{pmatrix} 8.8085e - 01 \\ 5.1051e + 00 \\ 6.9423e + 00 \end{pmatrix}$$

$\square$

(iii) **Solution:**

```
Ax = A*x
r = Ax-b
norm_r = norm(r)
norm_r_b = norm(r)/norm(b)
```

$$Ax = \begin{pmatrix} 9.1880e+00 \\ 7.4184e+00 \\ 7.6711e+00 \\ 9.9763e+00 \\ 1.1722e+01 \\ 9.1030e+00 \end{pmatrix}$$

$$r = \begin{pmatrix} 1.8805e-01 \\ -5.8163e-01 \\ 6.7109e-01 \\ -2.3659e-02 \\ -2.7831e-01 \\ 1.0296e-01 \end{pmatrix}$$

$$\|r\|_2 = 9.5532e-01$$

$$\|b\|_2 = 2.2782e+01$$

$$\|r\|_2/\|b\|_2 = 4.1934e-02$$

□

(iv) **Solution:** Yes, the model is effective at predicting the happiness because $\|r\|_2/\|b\|_2$ is and $\|r\|_2$ is two orders less than the magnitude of $\|b\|_2$ □

(v) **Solution:** From (iii), the components of Ax gives the predicted happiness for the given 6 people. We can see that the 5th person is predicted to be the happiest with a happiness of 11.722.

This is expected because we can see that $t_5 = 6.2, w_5 = 95.9, c_5 = 0.31$ i.e., he is the tallest, the thinnest and the second most blondest person of all the 6 people. Therefore the model correctly predicts that he is the happiest person. □

(vi) **Solution:** From (iii), the components of Ax gives the predicted happiness for the given 6 people. We can see that the 2nd person is predicted to be the least happiest with a happiness of 7.4184.

This is kind of expected behavior too because we can see that $t_2 = 6.0, w_2 = 162.3, c_2 = 0.83$ i.e., he is the second heaviest, second least blondest.

□

(vii) **Solution:** The components of r gives the model error of each person. We can see that the minimum error occurs for person 4 with an absolute error of 0.023659. This is because $t_4 = 5.6, w_4 = 150.1, c_4 = 0.24$ i.e, he has an average height, average weight, average blondness, so he roughly falls around the median of all the attributes combined for the 6 persons. □

(viii) **Solution:** The components of r gives the model error of each person. We can see that the maximum error occurs for person 3 with an absolute error of 0.67109. This could be because $t_3 = 5.9, w_3 = 130.8, c_3 = 1.0$ i.e, he has a good height, good weight but he's the least blonde person i.e., his attributes are scattered or non uniform. □

(d) (i) **Solution:**

```
format short e;
A= [sqrt(5.2-4) 100/240    exp(-0.13);
    sqrt(6.0-4) 100/162.3  exp(-0.83);
    sqrt(5.9-4) 100/130.8  exp(-1.0);
    sqrt(5.6-4) 100/150.1  exp(-0.24);
    sqrt(6.2-4) 100/95.9   exp(-0.31);
    sqrt(5.7-4) 100/141.2  exp(-0.47)];
b = [4.5;6.5;10;5.5;11;6];
[U,S,V] = svd(A);
Sinv = [1/S(1,1) 0 0 0 0 0;
        0 1/S(2,2) 0 0 0 0;
        0 0 1/S(3,3) 0 0 0];
Apseud = V*Sinv*U';
x = Apseud*b
Ax = A*x
r = Ax-b
norm_r = norm(r)
norm_b = norm(b)
norm_r_b = norm(r)/norm(b)
```

$$b = \begin{pmatrix} 4.5 \\ 6.5 \\ 10 \\ 5.5 \\ 11 \\ 6 \end{pmatrix}$$

$$x = \begin{pmatrix} 1.4064e+00 \\ 1.0149e+01 \\ -2.7482e+00 \end{pmatrix}$$

$$r = \begin{pmatrix} -1.1440e+00 \\ 5.4350e-01 \\ -1.3137e+00 \\ 8.7831e-01 \\ -3.4727e-01 \\ 1.3034e+00 \end{pmatrix}$$

$$\|r\|_2 = 2.4332e+00$$
$$\|b\|_2 = 1.8702e+01$$
$$\|r\|_2/\|b\|_2 = 1.3011e-01$$

□

(ii) **Solution:** Because $\|r\|_2/\|b\|_2$ is greater than that of part(c), the model is not as good as the model in part(c) but it still does an okay job at predicting happiness as $\|r\|_2$ is an order of magnitude less than $\|b\|_2$ □

(iii) **Solution:** If we compare x to the optimal coefficients in part(c), x_1 has increased, x_2 has increased, x_3 has become negative

* $\delta x_1/x_1 = .59664$. Therefore the impact of tallness will be increased by 0.6 times
* $\delta x_2/x_2 = .9880$. Therefore the impact of weight will be increased by 0.98 times
* $\delta x_3/x_3 = -1.395$. Because x_3 is negative, people who with blonde hair will be less happier

The predicted happiness are as follows

$$Ax = \begin{pmatrix} 3.3560e+00 \\ 7.0435e+00 \\ 8.6863e+00 \\ 6.3783e+00 \\ 1.0653e+01 \\ 7.3034e+00 \end{pmatrix}$$

For example, if we consider person 1, $t_1 = 5.2, w_1 = 240, c_1 = 0.13$. In part (c), since, he had the most blondest hair he wasn't the least happiest person although he has least height and highest weight. But now he's the least happiest because the effect of blonde hair on happiness has been reversed □