CSCI-GA.1180:
Mathematical Techniques for Computer Science Applications
New York University, Fall 2016

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Programming Assignment 1

Assigned Monday, 26 September 2016;

due 11:59pm, Wednesday, 5 October 2016

Problem P1.1. A famous optimization problem, traditionally called *linear programming* (LP), involves optimization (maximization or minimization) of a linear function subject to linear constraints. Linear programming is important in finance, operations research, machine learning, and data science.

This (short) programming assignment has two purposes: (1) to familiarize students with Matlab and (2) to apply some of the knowledge already acquired about vectors and matrices.

Be sure to use `format shortG` *to make the numbers more readable. (Usually, scientific notation would be preferred.)*

This problem is a baby version of the "diet problem", one of the first real-world problems ever solved using linear programming, and also a problem that is understandable by everyone. Note that this diet problem is not meant to be realistic!

Assume that your diet every day is chosen from the following six items: milk, coffee, chocolate chip cookies, muesli (cereal), broccoli, and pizza.

You have recently been informed by an authoritative source that you need to think about nutrition, in particular, your daily consumption of 5 items:

- nutrient 1: vitamin X;

- nutrient 2: vitamin Y;

- nutrient 3: fiber;

- nutrient 4: calories;

- nutrient 5: fat.

The following tables display the same information in slightly different forms, namely the values (made up) of vitamin X, vitamin Y, fiber, calories, and fat per indicated quantity of the six foods.

	X	Y	fiber	calories	fat
Milk (pint)	55	12	7	103	2
Coffee (cup)	0	0	0	0	0
Cookies (each)	2	14	12	300	8
Muesli (bowl)	15	32	210	60	1
Broccoli (ounce)	25	40	122	54	1
Pizza (slice)	34	45	7	721	4

	Milk	Coffee	Cookie	Muesli	Broccoli	Pizza
X	55	0	2	15	25	34
Y	12	0	14	32	40	45
fiber	7	0	12	210	122	7
calories	103	0	300	60	54	721
fat	2	0	8	1	1	4

As of today, your daily diet consists of precisely 3 pints of milk, 2 cups of coffee, 30 cookies, 1 bowl of muesli, 6 ounces of broccoli, and 5 slices of pizza.

1. Define a vector x, i.e., write down the dimension of x and the meaning of each component of x, such that x represents your daily diet.

2. Define a vector p in a similar fashion that represents the nutritional properties of one slice of pizza, and explain the meaning of each component of p.

3. Define two *column* vectors y and z, such that the inner product $y^T z$ gives the amount of fiber in your daily diet. Explain how you obtained y and z, and give the value of their inner product.

4. Is the amount of fat in your daily diet less than 290? Explain.

5. Define a matrix A and a vector x such that the i-th component of the matrix-vector product Ax gives the amount of the i-th nutrient, numbered as above, in your daily diet. Explain how you obtained A and x, and give Ax.

6. Suppose that you consume 10 cups of coffee every day rather than 2. What would be the effect on the amounts of the nutrients in your daily diet? Explain your answer mathematically in terms of properties of the matrix A in the matrix-vector product Ax from part 5 (above).

7. Suppose that we wish to choose a daily diet that satisfies the following conditions:

   - At least 500 units of vitamin X;
   - At least 500 units of vitamin Y;
   - At least 1500 units of fiber;
   - No more than $5,000$ calories;
   - No more than 100 units of fat.

   Does your present daily diet satisfy these constraints? Explain.

8. Find a daily diet that does satisfy these constraints, and explain how you found it.