Arith : A Simple Language

grammar? $\{ e ::= n \mid e+e \mid e*e \mid e-e$, $n \in \mathbb{Z}$

n → values

$e+e \mid e*e \mid e-e$ → Expressions but not values

$(2+3) * (8-4)$

$(2+3)$ is e_1, with sub-expressions e_{11} (2) and e_{12} (3); $(8-4)$ is e_2

Dynamic Semantics of Arith

Rules to Explain how to evaluate Expressions in Arith

$$\frac{\text{Premise}_1 \quad \text{Premise}_2 \quad \ldots\ldots \quad \text{Premise}_n}{\text{Conclusion}}$$

"Semantic rule"

$$\frac{e_1 \to v_1 \qquad e_2 \to v_2 \qquad v = v_1 + v_2}{e_1 + e_2 \to v}$$

$\to$: "Evaluates to"

C-Lite

x ← Variable

Program $P ::= S$

Statement $S ::= x := e \mid S ; S$

M

x = 1 ; {1}

M'

y = 2 ; {2}

M''

x = x + y ;

y = x − y ;

z = x − y ;

M_{final}

} Swapping Program in C-Lite

Dynamic Semantics of C-Lite

$M : Var \to Values$

$$\frac{M \vdash e \to v}{(M, (x := e)) \to (v, M[x \to v])}$$

$$\frac{(x+y) \to ?}{x = (x+y)}$$

M

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if (x < 2)
    y = 3
else
    y = 4
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ArithBool

$n \in \mathbb{Z}$ $\quad$ $b \in \mathbb{B}$

$$e ::= n \mid e + e \mid e * e \mid e - e \mid e \wedge e \mid e \vee e \mid \neg e \mid \text{if } e \text{ then } e \text{ else } e$$

n, b → values

$$\left[\text{if } \underbrace{(2+3) < 8}_{e_1} \text{ then } \overbrace{3*2-5}^{e_2} \text{ else } \underbrace{4*3-8}_{e_3}\right] \rightarrow 1$$

$$\frac{e_1 \rightarrow \text{true} \qquad e_2 \rightarrow v}{\text{if } e_1 \text{ then } e_2 \text{ else } e_3 \rightarrow v}$$

$$\frac{e_1 \to \text{false} \qquad e_3 \to v}{\text{if } e_1 \text{ Then } e_2 \text{ Else } e_3 \to v}$$