2/20

CSCI 7000 - 11

Lambda Encodings

* true, false, ite
* and, or, not
* Thm: $a \wedge b \Rightarrow a$
* pair: $(\lambda f. \lambda s. \lambda b.\ b\ f\ s)$
* Fixpoint.

$$Y = \lambda f. (\lambda x. f (x\, x)) (\lambda x. f (x\, x))$$

$$Z = \lambda f. (\lambda x. f. (\lambda y\ x\, x\, y)) (\lambda x. f (\lambda y. x\, x\, y))$$

* factorial

$$true = \lambda x. \lambda y.\ x$$

$$false = \lambda x. \lambda y.\ y$$

$$ite = \lambda b. \lambda x. \lambda y.\ b\ x\ y$$

$$\boxed{\begin{aligned} &ite(true, e_1, e_2) = e_1 \\ &ite(false, e_1, e_2) = e_2 \end{aligned}}$$

CBV Semantics

$$\underbrace{ite}\ true\ 0\ inf$$

$$(\lambda b\, x. y.\ b\ x\ y)\ true\ 0\ inf$$

a

true 0 inf

$(\lambda x. \lambda y. x)$ 0 inf

$\underbrace{(\lambda y. 0)\ \text{inf}}$

0

ite true $(\lambda y. e_1)$ $(\lambda z. e_2)$

$(\lambda y. e_1)$ x

e_1

Boolean combinators
(operators)

and = $\lambda b. \lambda c.\ \underbrace{b\ c\ b}$ → $\sim$(b c fls)

or = $\lambda b. \lambda c.\ \underbrace{b\ b\ c}$

or b true c

not = $\lambda b.\ b$ fls true

$a \Rightarrow b \iff (\neg a) \vee b$

Thm $\forall a. \forall b (a \wedge b \Rightarrow a)$

$\lambda a. \lambda b. \underbrace{\text{or} \; (\text{not} \; (\text{and} \; a \; b)) \; b}$

true

$\rightarrow \quad (\lambda x. x\, x)\; (\lambda x. x\, x) \quad \Omega$

$\Downarrow$

Ω

$\underbrace{(x, y)}$

tuple:

$\lambda x. \lambda y. \lambda z. \lambda b.\, x. \; (\lambda b. \; y \; z)$

$P = \lambda f. \lambda s. \lambda b. \; b \; f \; s$

$\text{fst} \; P = \lambda f. \lambda s. ((P \; f \; s) \; \text{true})$

$(\text{Snd} \; P) = \lambda f. \lambda s. \; (P \; f \; s) \; \text{false}$

Recursion:

$\Omega = (\lambda x. x\, x) \quad (\lambda x. x\, x)$

$$(\lambda x. f(x\ x))\ (\lambda x. f(x\ x))$$

$$f\ (\underbrace{(\lambda x. f(x\ x))\ (\lambda x. f(x\ x))}_{(Y\ f)})$$

$$(Y\ f) = f\ (Y\ f)$$

$$= f(f\ (Y\ f))$$

$$= f\ (f\ (f\ (Y\ f)))$$

$$\boxed{(Y\ f) = f\ (Y\ f)}$$

$$\boxed{Y = \lambda f. (\lambda x. f(x\ x))\ (\lambda x. f(x\ x))}$$

Y – Combinator!

$$Y\ f = f\ (\underbrace{Y\ f}_{x})$$

$$\boxed{x = f(x)}$$

$f(x) = x^2$

$x = 1$ or $x = 0$ $\quad f(x) = x$

$fact = \lambda f. \lambda n.$ if $n = 0$ then 1

else $n * f(n-1)$

factorial

factorial 1

$(Y\ fact)\ 1$

$\rightarrow fact\ (Y\ fact)\ 1$

$\rightarrow$ if $1 = 0$ then 1

else $1 * ((Y\ fact)\ 0)$

$\rightarrow 1 * ((Y\ fact)\ 0)$

$\rightarrow 1 * (fact\ (Y\ fact)\ 0)$

$\rightarrow 1 * ($ if $0 = 0$ then 1 else $0 * (Y\ fact)(-1)$

$\to \quad 1 * 1$

$\to \quad 1$

Exercise: . implement fibonacci

$(fib\ n) = \lambda f.\ \lambda n.\ \ldots\ldots$

$(fib\ 4) = (Y\ fib)\ 4$

Under CBV:

$fact \quad (Y\ fact)\ 1$

$(fact\ (Y\ fact))$

$(fact\ (fact\ (Y\ fact)))$

$Z = \lambda f.\ (\lambda x.\ f\ (x\ \cancel{x}))\ (\lambda x.\ f\ (x\ x))$

$\lambda z.\ (x\ x\ z) \qquad (\lambda z.\ x\ x\ z)$

$f\ (Y\ f)$

$f\ (\underbrace{\lambda z.\ Y\ f\ z})$

let rec f = $\underline{\ldots f}$

$\cong \quad f = \mathrm{fix}(\lambda g. \ \ldots g)$

$f = \underset{Y}{\mathrm{fix}}(\lambda g \ \underline{\ldots g})$

Z