Gowtham Kuntumalla

HW4 ECE498 GOWTHAM4

Part 1 :-

HMM Forward-Backward Algorithm

Codes submitted :- 1) HMM.py

Functions :-

Forward algorithm :- returns a $T * n\_states$ matrix

Backward algorithm :- returns a "

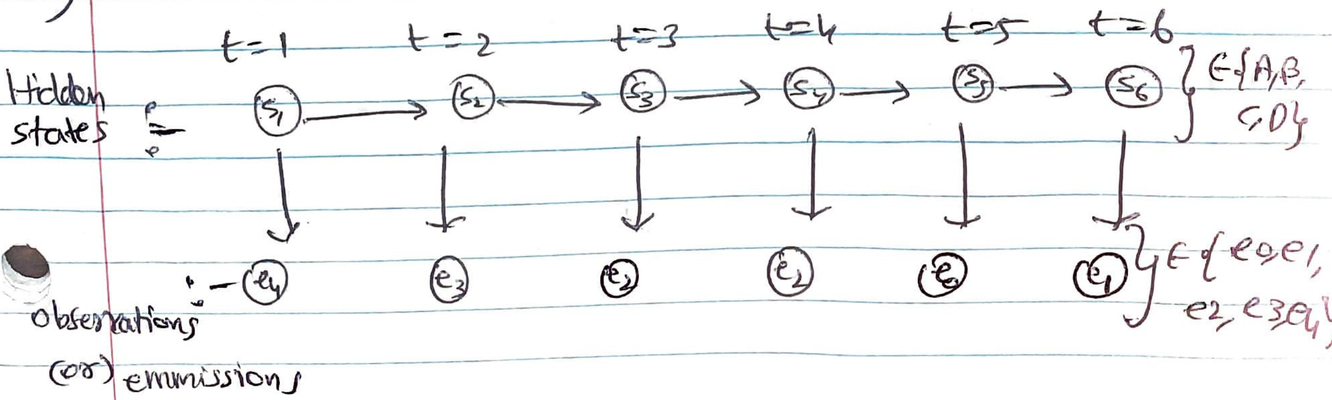
Forward-Backward algorithm :- returns a "

✓ verified with ICA4 solutions.

$$\gamma_2 = \begin{array}{cc} BS & CS \\ 0.244 & 0.756 \end{array}$$

Part 2 :- HMM model

1) A, B, π, observations are given



2) we use the Hmm-example.py to verify Hmm.py and use it for inference.

result :- $\boxed{[D, D, C, C, A, B]}$

$P(S_t \mid E_1, E_2, E, --E_t)$:- alpha

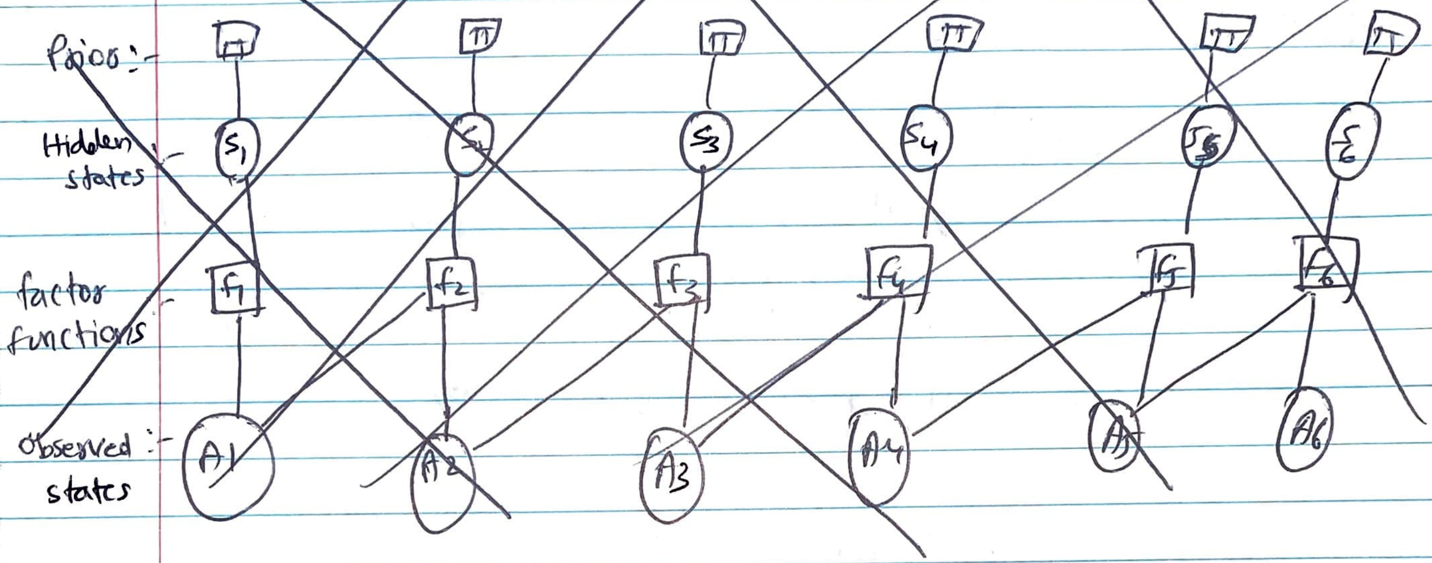
$P(E_{t+1}, ---, E_n \mid S_t)$:- Beta

$P(S_t \mid E_1, — E_n)$:- Gamma

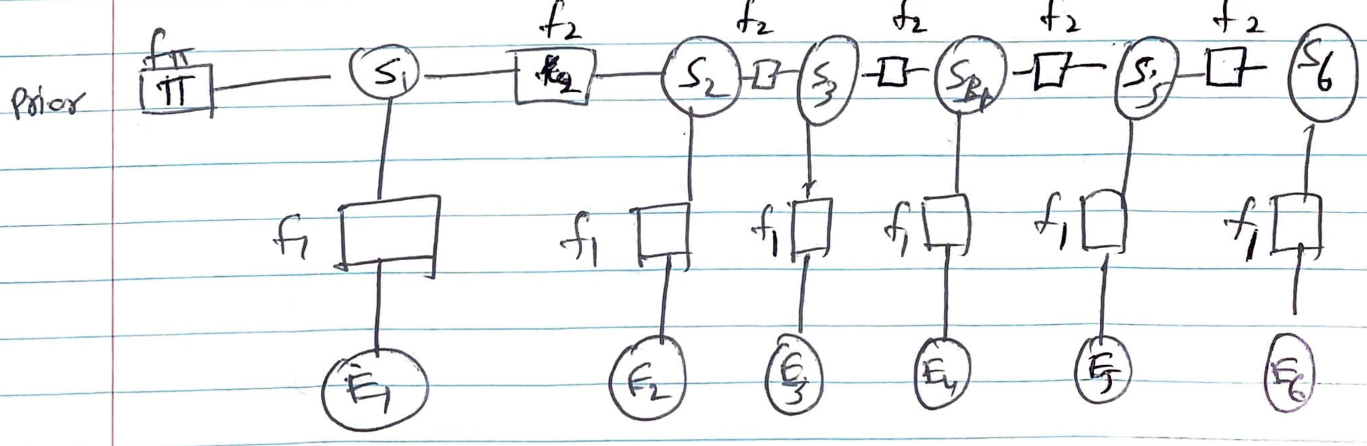
For most likely sequence only

	Alpha (α_t)	Beta (β_t)	Gamma (γ_t)	Most likely state
t=1	0.5	9.34 e-5	0.435	D
t=2	0.42394	0.000614	0.487	D
t=3	0.542	0.00495	0.568	C
t=4	0.474	0.0424	0.429	C
t=5	0.792	0.215	0.802	A
t=6	0.677	1	0.677	B

3) ~~HMM converted to Factor Graph~~



3) Drawing Factor Graph from H.M.M



f_π = Prior probabilities of states function

f_1 = Affinity b/w states & observation

f_2 = Affinity b/w consecutive states (transition probabilities)

Factor Graphs & Belief Propogation

Problem 1 :-

1. $$P(y) = \frac{1}{Z} \sum_{x \in \{0,1\}} m_{x \to f_A}(x) \; f_A \qquad -(1)$$

Z is the normalization factor $(y = 0, 1)$

2. $$m_{x \to f_A}(x) = \begin{array}{c|c} x & \\ 0 & \\ 1 & \end{array} \begin{bmatrix} 0.4 \times 0.3 \\ 0.6 \times 0.7 \end{bmatrix}$$

$$\Rightarrow m_{x \to f_A}(x) = \begin{array}{c} 0 \\ 1 \end{array} \begin{bmatrix} 0.12 \\ 0.42 \end{bmatrix}$$

Since $P(y)$ is given by eq in (1).

For $y = 0$:- $$f_A = \begin{array}{cc} y & x \\ 0 & 0 \\ 0 & 1 \end{array} \begin{bmatrix} 0.3 \\ 0.4 \end{bmatrix}$$

$$P(y=0) = \begin{array}{cc} y & x \\ 0 & 0 \\ 0 & 1 \end{array} \begin{bmatrix} 0.12 \\ 0.42 \end{bmatrix} \begin{bmatrix} 0.3 \\ 0.4 \end{bmatrix}$$

$$P(y=0) = 0.12 \times 0.3 + 0.42 \times 0.4$$

$$= 0.036 + 0.168$$

$$= 0.204$$

Similarly for $y=1$:- $f_A = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}$

$$P(y=1) = \begin{array}{cc|c} y & x & \\ 1 & 0 & \\ 1 & 1 & \end{array} \begin{bmatrix} 0.12 \\ 0.42 \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}$$

$$= 0.12 \times 0.1 + 0.42 \times 0.2$$

$$= 0.012 + 0.084$$

$$= 0.096$$

$$\Rightarrow \quad P(y) = \frac{1}{Z} \begin{bmatrix} P(y=0) \\ P(y=1) \end{bmatrix}$$

also normalization term, $Z = 0.204 + 0.096$

$$= 0.3$$

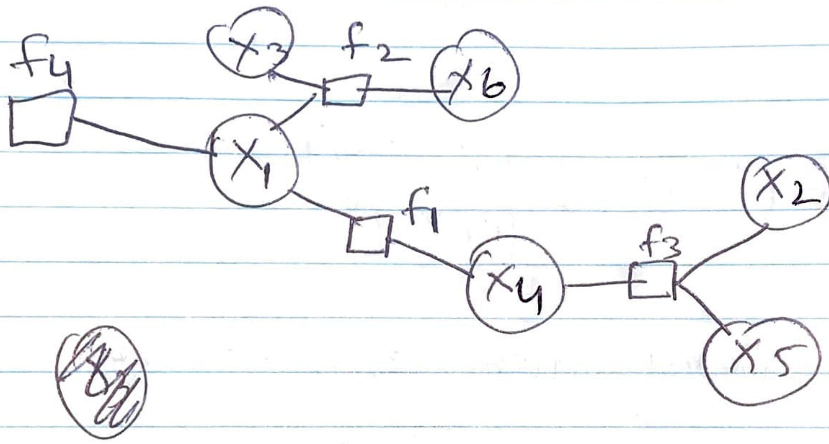
$$\Rightarrow \quad P(y) = \begin{array}{c} y \\ 0 \\ 1 \end{array} \begin{bmatrix} 0.204 \\ 0.096 \end{bmatrix} \frac{1}{0.3}$$

$$\Rightarrow \quad \boxed{P(y) = \begin{array}{c} y \\ 0 \\ 1 \end{array} \begin{bmatrix} 0.68 \\ 0.32 \end{bmatrix}}$$

Problem 2 :

$$f(x_1,x_2,x_3,x_4,x_5,x_6) = f_1(x_1,x_4)\, f_2(x_1,x_3,x_6)\, f_3(x_2,x_4,x_5)\, f_4(x_1)$$

1.



2. Marginal of x_1

$$f(x_1) = \sum_{x_2,x_3,x_4,x_5,x_6} f(x_1,x_2,x_3,x_4,x_5,x_6)$$

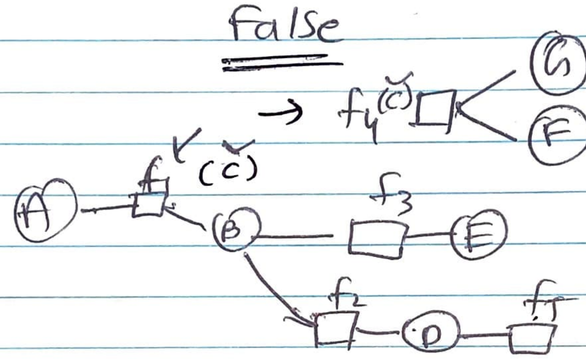
$$\Rightarrow f(x_1) = \sum_{x_2,x_3,x_4,x_5,x_6} f_1(x_1,x_4)\, f_2(x_1,x_3,x_6)\, f_3(x_2,x_4,x_5)\, f_4(x_1)$$

$$\Rightarrow \boxed{f(x_1) = f_4(x_1) \sum_{\substack{x_2,x_3,x_4\\x_5,x_6}} f_1(x_1,x_4)\, f_2(x_1,x_3,x_6)\, f_3(x_2,x_4,x_5)}$$

Problem 3 :- Conditional Independences

a) $F \perp\!\!\!\perp G \mid C$

False



G, F are still connected, Messages can flow

b) $A \perp\!\!\!\perp G \mid C$

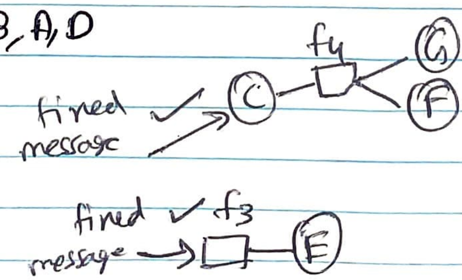
True, independent conditionally

Same factor graph as above

Graphs are disconnected. No message propogation b/w $A, G/c$

c) $E \perp\!\!\!\perp F \mid B, A, D$

True,



Graphs are disjoint

So, E, F are conditionally independent.