TST Mock 4

EGMOTC 2023 - Rohan

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Problems

Problem 1. Consider a circle Γ, a point A on its exterior, and the points of tangency B and C from A to Γ. Let P be a point on the segment AB, distinct from A and B, and let Q be the point on AC such that PQ is tangent to Γ. Points R and S are on lines AB and AC, respectively, such that $PQ \parallel RS$ and RS is tangent to Γ as well. Prove that $[APQ] \cdot [ARS]$ does not depend on the placement of point P.

Problem 2. Let n be an integer with $n \geq 2$. Over all real polynomials $p(x)$ of degree n, what is the largest possible number of negative coefficients of $p(x)^2$?

Problem 3. In an $n \times n$ array, each of the numbers $1, \cdots n$ appear exactly n times. Show that there is a row or a column in the array with at least $\sqrt{n}$ distinct numbers.