Probabilistic Method Pset 1

EGMOTC 2023 - Rohan

November 5, 2023

Problems

Remark. ** marked problems are considered harder.*
*** marked problems are strictly optional for the ones feeling extremely curious*

Problem. Suppose you have the whole numbers number line, labelled $0, 1, 2, \ldots$ and a drunk person say, Aditi, starts at $x = 7$ and every minute with equal probability goes either left(-1) or right$(+1)$. What's the probability that:

1. She reaches Ananya who's standing at $x = 10$ first or Sunaina who's standing at $x = 0$ first?
2. What's the expected amount of time before she reaches atleast one of them?
3. (*) Can you answer the same questions for general values instead of $0, 7$ and 10? What if she goes right with probability p and left with probability $1 - p$?
4. (**) Can we ask similar questions in higher dimensions? What if they are playing the same game on the entire integer plane?