

# TST Mock 1

EGMOTC 2023 - Rohan

December 7, 2023

## Problems

**Problem 1.** A point  $P$  lies in  $\triangle ABC$ . The lines  $BP, CP$  meet  $AC, AB$  at  $Q, R$  respectively. Given that  $AR = RB = CP, CQ = PQ$ , find  $\angle BRC$ .

**Problem 2.** The two cats Fitz and Will play the following game. On a blackboard is written the expression

$$x^{100} + \square x^{99} + \square x^{98} + \square x^{97} + \cdots + \square x^2 + \square x + 1.$$

Both cats take alternate turns replacing one  $\square$  with a 0 or 1, with Fitz going first, until (after 99 turns) all the blanks have been filled. If the resulting polynomial obtained has a real root, then Will wins, otherwise Fitz wins. Determine, with proof, which player has a winning strategy.

**Problem 3.** We call a natural number  $n$  honourable, if when a single corner cell is removed from an  $n \times n$  grid, there are an odd number of ways of tiling the remaining cells using L-trominoes. Prove that a number is honourable if and only if it is a power of 2