

Constructions PSet

EGMOTC 2023 - Rohan

December 5, 2023

Problems

Problem 1. Determine whether for every real number t such that $0 < t < \frac{1}{2}$ there exists an infinite set S of positive integers such that

$$|x - my| > ty$$

for every pair of different elements x and y of S and every positive integer m (i.e. $m > 0$).

Problem 2. Prove that for every $n \in \mathbb{N}$, there exists a set S of n positive integers such that for any two distinct $a, b \in S$, $a - b$ divides a and b but none of the other elements of S .

Problem 3. For which integers $n > 1$ does there exist a rectangle that can be subdivided into n pairwise noncongruent rectangles similar to the original rectangle?

Problem 4. Euclid has a tool called cyclos which allows him to do the following:

- Given three non-collinear marked points, draw the circle passing through them.
- Given two marked points, draw the circle with them as endpoints of a diameter.
- Mark any intersection points of two drawn circles or mark a new point on a drawn circle.

Show that given two marked points, Euclid can draw a circle centered at one of them and passing through the other, using only the cyclos.