

EGMO TST Day 2

Date: 30 December 2023

Instructions:

- i) You have 4 hours and 30 minutes for three problems.
- ii) Each problem is worth 10 points. Attempt all three.
- iii) Any claim you make must be accompanied by a proper justification.

Rubric P3

Problem 3.

Let ABC be an acute angled triangle and orthocentre H . Let $E = BH \cap AC$ and $F = CH \cap AB$. Let D, M, N be the midpoints of segments AH, BD, CD respectively, and $T = FM \cap EN$. Suppose D, E, T, F are concyclic. Prove that DT passes through the circumcentre of ABC .

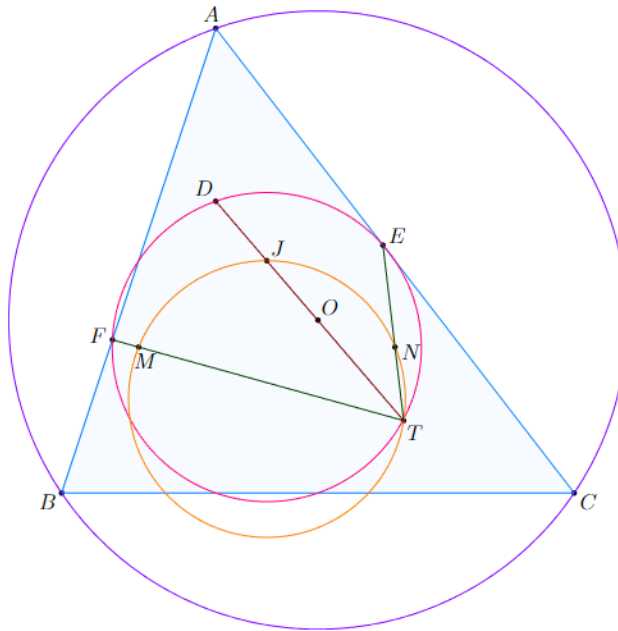
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Solution. Let O be the circumcenter of (ABC) and J be the midpoint of DO . Now it is sufficient to prove that TD and TJ coincide. We first prove that M, N, T, J are concyclic.

$$\angle MJN = \angle BOC = 2\angle BAC = \angle EDF = 180 - \angle FTE = \angle 180 - \angle NJM$$

This proves our claim!

Now note that $JM = \frac{OB}{2} = \frac{OC}{2} = JN$, so TJ is the angle bisector of $\angle NTM = \angle ETF$. But, D is the midpoint of arc EF in the nine-point circle, so TD is the angle bisector of $\angle ETF$ as well. Thus, TD and TJ must coincide! \square



Rubric

- (A) **+3:** Introducing J that is the midpoint of O and D (Or any equivalent definition of J). J should be introduced in fair and not just in rough or a diagram. There should be some try using point J
- (B) **+4:** Proving $MNTJ$ is concyclic. Note that
- **+1:** Proving $\angle BOC = 180 - \angle FTE$
 - any other angle chase progress will be given marks only when the chase has the involvement of J AND this claim is written in fair AND the angle chase can lead to prove this claim.
- (C) **+2:** TJ is the angle bisector $\angle ETF$
- (D) **+1:** TD is angle bisector of $\angle ETF$

Remarks

- Introducing nine-point circle but not any significant progress is 0.