Probabilistic Method Pset 1

EGMOTC 2023 - Rohan

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Problems

Remark. * marked problems are considered harder.

** marked problems are strictly optional for the ones feeling extremely curious about this particular setup.

Remark. Try to do the first two parts at least and submit whatever progress you get on the last two parts.

Problem. Suppose you have the whole numbers number line, labelled $0, 1, 2, \ldots$ and a drunk person say, Aditi, starts at x = 7 and every minute with equal probability goes either left(-1) or right(+1). What's the probability that:

- 1. She reaches Ananya who's standing at x=10 first or Sunaina who's standing at x=0 first?
- 2. What's the expected amount of time before she reaches at least one of them?
- 3. (*) Can you answer the same questions for general values instead of 0,7 and 10? What if she goes right with probability p and left with probability 1 p?
- 4. (**) Can we ask similar questions in higher dimensions? What if they are playing the same game on the entire integer plane?