# Probabilistic Method Lecture 2

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## Erdös-Ko-Rado

**Intersecting Set-families** : A family $\mathcal{F}$ of sets is called intersecting if, $A, B \in \mathcal{F} \implies A \cap B \neq \Phi$. Now, suppose $n \geq 2k$ and let $\mathcal{F}$ be an intersecting family of $k$-element subsets of an $n$ set, say $[0, 1, 2, \cdots n-1]$. Then, Erdös-Ko-Rado Theorem states that $|\mathcal{F}| \leq \binom{n-1}{k-1}$. [1]

## Maximal Antichains

**Antichains:** A family $\mathcal{F}$ of subsets of $[n]$ is called an anti-chain if no set of $\mathcal{P}$ is contained in another.

**Theorem 1.** *Let $\mathcal{F}$ be an antichain then,*

$$\sum_{A \in \mathcal{F}} \frac{1}{\binom{n}{|A|}} \leq 1$$

The above theorem is also called the Lubell–Yamamoto–Meshalkin inequality or more frequently, the LYM inequality.

**Corollary 1.** ***(Sperner's Theorem)*** *Let $\mathcal{F}$ be an antichain, then*

$$|\mathcal{F}| \leq \binom{n}{\lfloor n/2 \rfloor}$$

Try to prove these three results.

[1] Taken from the probabilistic lens 1 in Alon-Spencer.

# Law of Large Numbers

Take this result as something to remember intuitively and not necessarily too formally but more as an intuitive statement:

**Law of Large Numbers.** Let $X_1, \cdots X_n$ be indepedent identically distributed random values with expected value 0. Then, if you define $\overline{X_n} = \frac{X_1 + \cdots X_n}{n}$ then for any $\epsilon > 0$, we have

$$\lim_{n \mapsto \infty} \mathbb{P}[|\overline{X_n}| < \epsilon] = 1$$