

Pathological PSet

EGMOTC 2023 - Rohan

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Problems

Problem 1. (Memories!) Let k be a positive integer. Lexi has a dictionary \mathbb{D} consisting of some k -letter strings containing only the letters A and B . Lexi would like to write either the letter A or the letter B in each cell of a $k \times k$ grid so that each column contains a string from \mathbb{D} when read from top-to-bottom and each row contains a string from \mathbb{D} when read from left-to-right. What is the smallest integer m such that if \mathbb{D} contains at least m different strings, then Lexi can fill her grid in this manner, no matter what strings are in \mathbb{D} ?

Problem 2. Each cell of a 100×100 grid is colored with one of 101 colors. A cell is diverse if, among the 199 cells in its row or column, every color appears at least once. Determine the maximum possible number of diverse cells.

Problem 3. Find all positive integers $k < 202$ for which there exist a positive integers n such that

$$\left\{ \frac{n}{202} \right\} + \left\{ \frac{2n}{202} \right\} + \cdots + \left\{ \frac{kn}{202} \right\} = \frac{k}{2}$$