Size and Guesstimation: Computational

EGMOTC 2023 - Rohan

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Problems

Problem 1. (Newton Iteration)

- Find $\sqrt{2023}$ upto 20 decimal places (without using the $\sqrt{\cdot}$ operation). You are free to write code for this or use a calculator.[1]
- Find the first 10 digits of π. [2]

Problem 2. (Big-O) Order the function by their sizes as $n \mapsto \infty$?

- $f(x) = 2023 \log(n)^{2023}$, $g(x) = \log(\log(F_n^{F_n^2} + 3^{3^n}))^3$, $h(x) = 1.001^n$
- $f(n) = 3f\left(\lfloor n/2 \rfloor\right) + 2023n$ with $f(1) = 1$, $g(n) = 1.01g(n-1) - g(n-2)$ with $g(0) = 0$ and $g(1) = 1$
- $f(n) = 2023^{2023^{2023n}}$, $g(n) = 2^{2^{2^{2n}}}$, and $h(n) = 1.01^{1.01^{1.01^{1.01^{1.01n}}}}$

Problem 3. (Some contest problems)

- Compute $\left\lceil \displaystyle\sum_{k=2023}^{\infty} \frac{2024! - 2023!}{k!} \right\rceil$
- For any natural number n, expressed in base 10 , let $s(n)$ denote the sum of all its digits. Find all natural numbers m and n such that $m < n$ and

$$(s(n))^2 = m \text{ and } (s(m))^2 = n$$

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[1] 4 iterations of Newton's algorithm are definitely sufficient, 3 iterations might be enough too. Can you argue that 4 iterations are sufficient?

[2] $sin\pi = 0$ and you can use Newton Iteration)

[3] F_n is the nth Fibonacci term