

Problem 1

let a, <az <az <az le hositive integers

Then

$$\frac{n-1}{\sum_{i=1}^{n-1} \operatorname{lem}(a_{i}, a_{i+1})} < 1$$

$$\sum_{i=1}^{n-1} \frac{gcd(a_i, a_{i+1})}{a_i a_{i+1}} < 1$$

$$i=1$$

$$0 \le a_i$$

$$\frac{1}{\operatorname{lcm}(a_{i},a_{i+1})} = \frac{\operatorname{gcd}(a_{i},a_{i+1})}{q_{i} \cdot a_{i+1}}$$

$$ged(a_{i}, a_{i+1}) \leq a_{i+1} - a_{i}$$

$$=) \frac{1}{lcm(a_{i}, o_{i+1})} \leq \frac{a_{i+1} - a_{i}}{a_{i} a_{i+1}} = \frac{1}{a_{i}} - \frac{1}{a_{i+1}}$$

$$= \sum_{i=1}^{n} \frac{1}{a_{i} a_{i+1}} = \sum_{i=1}^{n} \frac{1}{a_{i+1}} = \sum_{i=1}$$

Problem 2

Let a+b+c=1. Brove that: $\frac{1}{\epsilon R^{+}}$

$$\int a + \frac{(b-c)^2}{4} + \int b + \int c \leqslant \sqrt{3}$$

$$\sqrt{a} + \int b + \int c \leqslant \sqrt{3}$$

Can't directly apply AM-GM type ineq.

We ignore the $(b-c)^2$ torm for now.

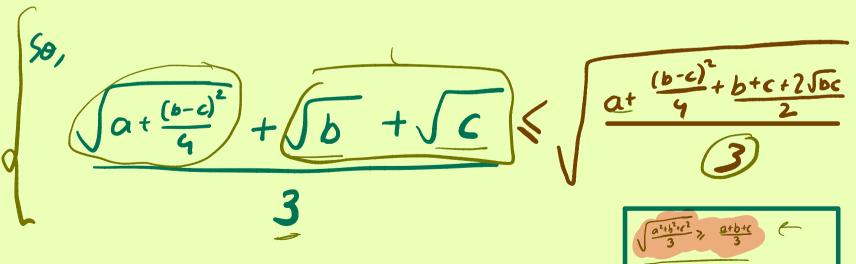
then

 $\int a + \int b + \int c < \int 3$ but close to equality

when $b \sim c$.

So we make them close!

$$\int a + \frac{(b-c)^2}{4} + \int b + \int c = \int a + \frac{(b-c)^2}{4} + \left(\frac{\sqrt{b}+\sqrt{c}}{2}\right) + \left(\frac{\sqrt{b}+\sqrt{c}}{2}\right)$$



Just need at
$$\frac{b+c}{2} + \sqrt{b}c + \frac{(b-c)^2}{9} \le 1$$

i.e
$$a + \frac{b+c}{2} + \sqrt{bc} + \frac{(b-c)^2}{4} \le \frac{d+b+c}{4}$$

$$(\Rightarrow) \int bc + (b-c)^2 \leq \frac{b+c}{2}$$

$$(=) \frac{b+c}{4} = \frac{b+c}{2}$$

$$(=) \frac{(b-c)^2}{2} \leq \frac{(5b-5c)^2}{2} = \frac{b+c+2\sqrt{bc}}{2} \leq \frac{2b+2c}{2} \leq 2$$

$$(=) \frac{(5b+5c)^2}{2} \leq \frac{2}{2} \leq$$

Thank You

- -> Try be problem set (1 will post robutions)
 by December 15)
- See you at TC