Resolution Proof Example.

(a) Marcus was a man.

(b) Marcus was a Roman.

(c) All men are people.

(d) Caesar was a ruler.

(e) All Romans were either loyal to Caesar or hated him (or both).

(f) Everyone is loyal to someone.

(g) People only try to assassinate rulers they are not loyal to.

(h) Marcus tried to assassinate Caesar.

Convert to First order Logic.

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(b) Marcus was a Roman.

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(h) Marcus tried to assassinate Caesar.

(a) man(marcus)

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(h) Marcus tried to assassinate Caesar.

(a) man(marcus)
(b) roman(marcus)

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(h) Marcus tried to assassinate Caesar.

(a) man(marcus)
(b) roman(marcus)
(c) $\forall X.man(X) \rightarrow person(X)$

Convert to First order Logic

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(a) man(marcus)
(b) roman(marcus)
(c) $\forall X.man(X) \rightarrow person(X)$
(d) ruler(caesar)

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(h) Marcus tried to assassinate Caesar.

(a) man(marcus)

(b) roman(marcus)

(c) $\forall X.man(X) \rightarrow person(X)$

(d) ruler(caesar)

(e) $\forall X.roman(x) \rightarrow loyal(X,caesar) \lor hate(X,caesar)$

Convert to First order Logic

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(h) Marcus tried to assassinate Caesar.

(a) man(marcus)
(b) roman(marcus)
(c) $\forall X.man(X) \rightarrow person(X)$
(d) ruler(caesar)
(e) $\forall X.roman(x) \rightarrow loyal(X,caesar)$
     $\vee\ hate(X,caesar)$
(f) $\forall X\exists Y.\ loyal(X,Y)$

Convert to First order Logic

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(a) man(marcus)
(b) roman(marcus)
(c) $\forall X.\ man(X) \rightarrow person(X)$
(d) ruler(caesar)
(e) $\forall X.\ roman(x) \rightarrow loyal(X,caesar) \vee hate(X,caesar)$
(f) $\forall X \exists Y.\ loyal(X,Y)$
(g) $\forall X \forall Y.\ person(X) \wedge tryassasin(X,Y) \rightarrow \neg loyal(X,Y)$

Convert to First order Logic

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(h) Marcus tried to assassinate Caesar.

(a) man(marcus)
(b) roman(marcus)
(c) ∀X. man(X) → person(X)
(d) ruler(caesar)
(e) ∀X. roman(x) → loyal(X,caesar) ∨ hate(X,caesar)
(f) ∀X∃Y. loyal(X,Y)
(g) ∀X∀Y. person(X) ∧ ruler(Y) tryassasin(X,Y) → ¬loyal(X,Y)
(h) tryassasin(marcus,caesar)

Convert to Clausal Form

(a) man(marcus)

(b) roman(marcus)

(c) $\forall X.\ man(X) \rightarrow person(X)$

(d) ruler(caesar)

(e) $\forall X.\ roman(x) \rightarrow loyal(X,caesar) \lor hate(X,caesar)$

(f) $\forall X \exists Y.\ loyal(X,Y)$

(g) $\forall X \forall Y.\ person(X) \land ruler(Y) \land tryassasin(X,Y) \rightarrow \neg loyal(X,Y)$

(h) tryassasin(marcus,caesar)

Convert to Clausal Form

(a) man(marcus)

(b) roman(marcus)

(c) $\forall$X. man(X) $\rightarrow$ person(X)

(d) ruler(caesar)

(e) $\forall$X. roman(x) $\rightarrow$ loyal(X,caesar) $\vee$ hate(X,caesar)

(f) $\forall$X$\exists$Y. loyal(X,Y)

(g) $\forall$X$\forall$Y. person(X) $\wedge$ ruler(Y)$\wedge$ tryassasin(X,Y) $\rightarrow$ $\neg$loyal(X,Y)

(h) tryassasin(marcus,caesar)

Convert to Clausal Form

(a) man(marcus)

(b) roman(marcus)

(c) $\forall$X. man(X) $\rightarrow$ person(X)

($\neg$man(X), person(X))

(d) ruler(caesar)

(e) $\forall$X. roman(x) $\rightarrow$ loyal(X,caesar) $\vee$ hate(X,caesar)

(f) $\forall$X$\exists$Y. loyal(X,Y)

(g) $\forall$X$\forall$Y. person(X) $\wedge$ ruler(Y) $\wedge$ tryassasin(X,Y) $\rightarrow$ $\neg$loyal(X,Y)

(h) tryassasin(marcus,caesar)

Convert to Clausal Form

(a) man(marcus)

(b) roman(marcus)

(c) ∀X. man(X) → person(X)

(¬man(X), person(X))

(d) ruler(caesar)

(e) ∀X. roman(x) → loyal(X,caesar) ∨ hate(X,caesar)

(f) ∀X∃Y. loyal(X,Y)

(g) ∀X∀Y. person(X) ∧ ruler(Y) ∧ tryassasin(X,Y) → ¬loyal(X,Y)

(h) tryassasin(marcus,caesar)

Convert to Clausal Form

(a) man(marcus)

(b) roman(marcus)

(c) ∀X. man(X) → person(X)

(¬man(X), person(X))

(d) ruler(caesar)

(e) ∀X. roman(X) → loyal(X,caesar) ∨ hate(X,caesar)

(¬roman(X), loyal(X,caesar), hate(X,caesar))

(f) ∀X∃Y. loyal(X,Y)

(g) ∀X∀Y. person(X) ∧ ruler(Y)∧ tryassasin(X,Y) → ¬loyal(X,Y)

(h) tryassasin(marcus,caesar)

Convert to Clausal Form

(a) man(marcus)

(b) roman(marcus)

(c) $\forall$X. man(X) $\rightarrow$ person(X)

   ($\neg$man(X), person(X))

(d) ruler(caesar)

(e) $\forall$X. roman(X) $\rightarrow$ loyal(X,caesar) $\vee$ hate(X,caesar)

   ($\neg$roman(X), loyal(X,caesar), hate(X,caesar))

(f) $\forall$X$\exists$Y. loyal(X,Y)

   (loyal(X,f(X)))

(g) $\forall$X$\forall$Y. person(X) $\wedge$ ruler(Y) $\wedge$ tryassasin(X,Y) $\rightarrow$ $\neg$loyal(X,Y)

(h) tryassasin(marcus,caesar)

Convert to Clausal Form

(a) man(marcus)

(b) roman(marcus)

(c) $\forall$X. man(X) $\rightarrow$ person(X)

  ($\neg$man(X), person(X))

(d) ruler(caesar)

(e) $\forall$X. roman(X) $\rightarrow$ loyal(X,caesar) $\vee$ hate(X,caesar)

  ($\neg$roman(X), loyal(X,caesar), hate(X,caesar))

(f) $\forall$X$\exists$Y. loyal(X,Y)

  (loyal(X,f(X))

(g) $\forall$X$\forall$Y. person(X) $\wedge$ ruler(Y) $\wedge$ tryassasin(X,Y) $\rightarrow$ $\neg$loyal(X,Y)

  ($\neg$person(X), $\neg$ruler(Y), $\neg$tryassasin(X,Y), $\neg$loyal(X,Y))

(h) tryassasin(marcus,caesar)

Convert to Clausal Form

(a) man(marcus)

(b) roman(marcus)

(c) ∀X. man(X) → person(X)

    (¬man(X), person(X))

(d) ruler(caesar)

(e) ∀X. roman(X) → loyal(X,caesar) ∨ hate(X,caesar)

    (¬roman(X), loyal(X,caesar), hate(X,caesar))

(f) ∀X∃Y. loyal(X,Y)

    (loyal(X,f(X))

(g) ∀X∀Y. person(X) ∧ ruler(Y) ∧ tryassasin(X,Y) → ¬loyal(X,Y)

    (¬person(X), ¬ruler(Y), ¬tryassasin(X,Y), ¬loyal(X,Y))

(h) tryassasin(marcus,caesar)

Convert to Clausal Form

1. man(marcus)
2. roman(marcus)
3. ($\neg$man(X), person(X))
4. ruler(caesar)
5. ($\neg$roman(X), loyal(X,caesar), hate(X,caesar))
6. (loyal(X,f(X))
7. ($\neg$person(X), $\neg$ruler(Y), $\neg$tryassasin(X,Y), $\neg$loyal(X,Y))
8. tryassasin(marcus,caesar)

Query

Who hated Caesar?

Query

Who hated Caesar?

In First Order logic.

1. $\exists X.\ hate(X, caesar)$

Query

Who hated Caesar?

In First Order logic.

1. $\exists X. hate(X,caesar)$

Negate!

1. $\forall X. \neg hate(X,caesar)$

Query

Who hated Caesar?

In First Order logic.

1. ∃X. hate(X,caesar)

Negate!

1. ∀X. ¬hate(X,caesar)

Clausal Form, with answer literal.
9. (¬hate(X,caesar), ans(X))

Resolution Proof

1. man(marcus)
2. roman(marcus)
3. (¬man(X), person(X))
4. ruler(caesar)
5. (¬roman(X), loyal(X,caesar), hate(X,caesar))
6. (loyal(X,f(X))
7. (¬person(X), ¬ruler(Y), ¬tryassasin(X,Y), ¬loyal(X,Y))
8. tryassasin(marcus,caesar)
9. (¬hate(X,caesar), ans(X))
10. R[9,5c] (¬roman(X), loyal(X,caesar), ans(X))
Note: In general we have to be cautious about variable names. The X in clause 5 is NOT the same as the X in clause 9!

Resolution Proof

Note: In general we have to be cautious about variable names. The X in clause 5 is NOT the same as the X in clause 9!

1. (p(X), h(Y))
2. (¬p(X), q(Y))

3. R[1,2] (h(Y), q(Y))

This is incorrect, as now h and q seem to have the same variable. In fact the Y in 1 and the Y in 2 are different. The correct operation is to first rename the variables in one of the clauses so that the two clauses each have distinct variable names.

Resolution Proof

Note: In general we have to be cautious about variable names. The X in clause 5 is NOT the same as the X in clause 9!

1. ~~(p(X), h(Y))~~ (p(Z), h(W))
2. (¬p(X), q(Y))

3. R[1,2] {Z=X} (h(W), q(Y))

Note that 3 is now more general than (h(Y), q(Y)).

In our example if we applied this more correct rule we would still get the same answer...so I took a short cut.

Resolution Proof

1. man(marcus)
2. roman(marcus)
3. (¬man(X), person(X))
4. ruler(caesar)
5. (¬roman(X), loyal(X,caesar), hate(X,caesar))
6. (loyal(X,f(X))
7. (¬person(X), ¬ruler(Y), ¬tryassasin(X,Y), ¬loyal(X,Y))
8. tryassasin(marcus,caesar)
9. ¬hate(X,caesar)
10. R[9,5c] (¬roman(X), loyal(X,caesar), ans(X))

Resolution Proof

1. man(marcus)
2. roman(marcus)
3. (¬man(X), person(X))
4. ruler(caesar)
5. (¬roman(X), loyal(X,caesar), hate(X,caesar))
6. (loyal(X,f(X))
7. (¬person(X), ¬ruler(Y), ¬tryassasin(X,Y), ¬loyal(X,Y))
8. tryassasin(marcus,caesar)
9. ¬hate(X,caesar)
10. R[9,5c] (¬roman(X), loyal(X,caesar), ans(X))
11. R[10a,2] {X=marcus} (loyal(markus,caesar), ans(markus))

Resolution Proof

1. man(marcus)
2. roman(marcus)
3. (¬man(X), person(X))
4. ruler(caesar)
5. (¬roman(X), loyal(X,caesar), hate(X,caesar))
6. (loyal(X,f(X))
7. (¬person(X), ¬ruler(Y), ¬tryassasin(X,Y), ¬loyal(X,Y))
8. tryassasin(marcus,caesar)
9. ¬hate(X,caesar)
10. R[9,5c] (¬roman(X), loyal(X,caesar), ans(X))
11. R[10a,2] {X=marcus} (loyal(markus,caesar), ans(markus))
12. R[11,7c] {X=markus, Y=caesar} (¬person(markus), ¬ruler(caesar), ¬tryassasin(markus,caesar), ans(markus))

Resolution Proof

1. man(marcus)
2. roman(marcus)
3. (¬man(X), person(X))
4. ruler(caesar)
5. (¬roman(X), loyal(X,caesar), hate(X,caesar))
6. (loyal(X,f(X))
7. (¬person(X), ¬ruler(Y), ¬tryassasin(X,Y), ¬loyal(X,Y))
8. tryassasin(marcus,caesar)
9. ¬hate(X,caesar)
10. R[9,5c] (¬roman(X), loyal(X,caesar), ans(X))
11. R[10a,2] {X=marcus} (loyal(markus,caesar), ans(markus))
12. R[11,7c] {X=markus, Y=caesar} (¬person(markus), ¬ruler(caesar), ¬tryassasin(markus,caesar), ans(markus))
13. R[12a,3b] {X=markus} (¬man(markus), ¬ruler(caesar), ¬tryassasin(markus,caesar), ans(markus))

Resolution Proof

1. man(marcus)
2. roman(marcus)
3. (¬man(X), person(X))
4. ruler(caesar)
5. (¬roman(X), loyal(X,caesar), hate(X,caesar))
6. (loyal(X,f(X))
7. (¬person(X), ¬ruler(Y), ¬tryassasin(X,Y), ¬loyal(X,Y))
8. tryassasin(marcus,caesar)
9. ¬hate(X,caesar)
10. R[9,5c] (¬roman(X), loyal(X,caesar), ans(X))
11. R[10a,2] {X=marcus} (loyal(markus,caesar), ans(markus))
12. R[11,7c] {X=markus, Y=caesar} (¬person(markus), ¬ruler(caesar), ¬tryassasin(markus,caesar), ans(markus))
13. R[12a,3b] {X=markus} (¬man(markus), ¬ruler(caesar), ¬tryassasin(markus,caesar), ans(markus))
14. R[13a,1] (¬ruler(caesar), ¬tryassasin(markus,caesar), ans(markus))

Resolution Proof

1. man(marcus)
2. roman(marcus)
3. (¬man(X), person(X))
4. ruler(caesar)
5. (¬roman(X), loyal(X,caesar), hate(X,caesar))
6. (loyal(X,f(X))
7. (¬person(X), ¬ruler(Y), ¬tryassasin(X,Y), ¬loyal(X,Y))
8. tryassasin(marcus,caesar)
9. ¬hate(X,caesar)
10. R[9,5c] (¬roman(X), loyal(X,caesar), ans(X))
11. R[10a,2] {X=marcus} (loyal(markus,caesar), ans(markus))
12. R[11,7c] {X=markus, Y=caesar} (¬person(markus), ¬ruler(caesar), ¬tryassasin(markus,caesar), ans(markus))
13. R[12a,3b] {X=markus} (¬man(markus), ¬ruler(caesar), ¬tryassasin(markus,caesar), ans(markus))
14. R[13a,1] (¬ruler(caesar), ¬tryassasin(markus,caesar), ans(markus))
15. R[13a,2] (¬tryassasin(markus,caesar), ans(markus))

Resolution Proof

1. man(marcus)
2. roman(marcus)
3. (¬man(X), person(X))
4. ruler(caesar)
5. (¬roman(X), loyal(X,caesar), hate(X,caesar))
6. (loyal(X,f(X))
7. (¬person(X), ¬ruler(Y), ¬tryassasin(X,Y), ¬loyal(X,Y))
8. tryassasin(marcus,caesar)
9. ¬hate(X,caesar)
10. R[9,5c] (¬roman(X), loyal(X,caesar), ans(X))
11. R[10a,2] {X=marcus} (loyal(markus,caesar), ans(markus))
12. R[11,7c] {X=markus, Y=caesar} (¬person(markus), ¬ruler(caesar), ¬tryassasin(markus,caesar), ans(markus))
13. R[12a,3b] {X=markus} (¬man(markus), ¬ruler(caesar), ¬tryassasin(markus,caesar), ans(markus))
14. R[13a,1] (¬ruler(caesar), ¬tryassasin(markus,caesar), ans(markus))
15. R[13a,2] (¬tryassasin(markus,caesar), ans(markus))
16. R[15a,8] ans(markus)