A Stochastic Non-Linear Model for Bird Flocking

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A Stochastic Nonlinear Model for Coordinated Bird Flocks

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Abstract

Certain small birds such as pigeons, starlings, and shorebirds fly in coordinated flocks that display strong synchronization in turning, initiation of flight, and landing. Experimental efforts to find leaders in such flocks have to date failed. We propose that synchronization of movement may be a byproduct of "rules" for movement followed by each bird in the flock. Accordingly, we have developed a computer-simulated bird flock employing stochastic differential equations which demonstrates realistic "flocking" behavior. The functions employed in our model include attraction to a roost, a nonlinear attraction to flockmates, preservation of flight velocity, and an n-dimensional Poisson stochastic process. By varying the values of the parameters of the model, we have seen organized flight develop from chaotic milling and an organized flock break up into chaotic flight.

The coordinated flight formations typical of small birds such as pigeons, shorebirds, and blackbirds have been simulated with a model that assumes no leadership within the flock, with rules for movement as a function of the movements of neighbors, and containing a nonlinear component that produces occasional chaotic behavior. Changing the numerical values of the parameters in the flock model produces behavior typical of different bird species.

The apparently coordinated movements of large flocks of certain small bird species such as starlings and small shorebirds flying in cluster flocks *(1)* have fascinated observers since the time of Pliny *(2)*. How do the flocks coordinate their movements and decide when to turn and wheel? Although a "group mind" model has been proposed *(3)* in which the individual nervous systems of the flock members were somehow connected, many investigators have assumed a leadership model, in which a leader directs the movements of the flock *(4)*. Efforts to identify such a leader have thus far been unsuccessful, even using methods that permitted an analysis of the position of individually identified birds in a free-flying pigeon flock *(5)*. An alternate view to a leadership model has been proposed *(6)* in which a "self-generated synchrony" provides the mechanism for coordinated movements. In this proposed mechanism, coordinated turning is the result of individual birds "voting" with their bodies on the flight path the flock should take. When a "critical mass" is reached, the flock as a whole then turns in the direction expressed by the initiators of the turn. Some support for this hypothesis is provided by the observation *(7)* that waves of turning in dunlin (*Calidris alpina*) flocks can be initiated by individuals turning toward the interior of the flock.

We suggest here that the apparently coordinated movements of cluster flocks represent an organization whose collective movement is shaped by relatively simple rules, which determine the movements of its members. We present a computer simulation of a bird flock incorporating nonlinear elements, which produces "behavior" strikingly similar to natural flocks. We further suggest that such a model may be applicable to fish schools and other organized moving groups of animals.

The core of the simulation was initially inspired by a game concept developed by John Conway *(8)* in the 1960s. Conway's game was called "Life," the play of which was that squares on a grid were filled or not filled in subsequent moves of play, depending on the presence or absence of filled neighboring squares in the present move. "Colonies" of cells would be established, grow, send off new colonies, decline, and die, depending on the original state of the game. A significant feature of the game was that there was no self-generated behavior of the cells; what they did in future moves was *entirely* a function of each cell's response to neighbors. Striking patterns would develop, sometimes hundreds of generations after the initial state, which gave the appearance of orderliness or direction, despite a lack of direction in the "rules" of the game.

It occurred to us that a coordinated flock might be built around the same concept, where a behavior of the flock as a whole, e.g., a turn, might arise as a byproduct of rules influencing the movements of individuals, the majority of which might be governed by reactions to neighbors. Because real flocks will maintain an orderly pattern of turning and wheeling for a time, then spontaneously break up, and conversely form a synchronized group after flying without coordination for a time, we felt that the introduction of a nonlinear component as well as a random term in the simulation might account for this behavior.

Our initial model contains the following features: (i) it exhibits the collective behavior of flocking seen in some species, without assuming that the "flock" has a leader; (ii) it also exhibits the somewhat erratic, unsystematic behavior seen in flocks, due to wind gusts, external disturbances, and random causes; (iii) the flock aims for some roosting area; (iv) the flock turns and wheels for an indeterminate time over the roost; (v) individuals can change their position within the flock; (vi) from time to time, the flock can split into two or more "subflocks"; (vii) the flight paths of individual birds in the flock can be approximately parallel and to some degree random; and (viii) the model postulates no other systematic external influence acting on a totality of the "flock" to steer it in a flock-like fashion.

The model is given through a stochastic differential driven by a Poisson process with associated random variables equation, using the notation of stochastic differential equations

$$\begin{cases} dv_i(t) = (\mathrm{F}_{home} + \mathrm{F}_{vel} + \mathrm{F}_{interact})\, dt + dP(t) \\ v_i(t) = du_i(t),\ i = 1,2,..n \end{cases} \quad (1)$$

where $u_i(t) = (x_i(t), y_i(t)$ is the location of bird number i at time t. The equation involves the functions for

homing: F_{home} expresses the tendency towards the roosting area

velocity regulation: F_{vel} controls the velocity of an individual

interaction: $\mathrm{F}_{interact}$ measures the influence of one individual on another

randomness: $\mathrm{P}(t)$ is a Poisson stochastic process.

The first three terms are deterministic in contrast to the fourth one, which is intended to simulate the effects of wind gusts and random local disturbances. This term is obtained from an n-dimensional Poisson process with associated independent and identically distributed (*i.i.d.*) random variables.

More precisely, our model assumes the following. For the roosting tendency we postulated for the i^{th} component

$$\mathrm{F}^{(i)}_{home} = -u_i(t) f_{home}(|u_i(t)|);\ i = 1,2,\ldots n; \quad (2)$$

where u_i is the vector from the homing zone to bird i and where f_{home} is a scalar valued function (Fig. 1), where r_0 is the minimum radius of a circular homing area. No influence is felt from the homing area if the distance r is greater than r_1.

The velocity control term (Fig. 2) is postulated to be

$$\mathrm{F}^{(i)}_{vel} = -v_i(t) f_{vel}(|v_i(t)|), \quad (3)$$

where v_0 is the preferred velocity.

The third term, $\mathrm{F}_{interact}$ (Fig. 3), was the most difficult to choose. If $d_{ij}(t)$ means the

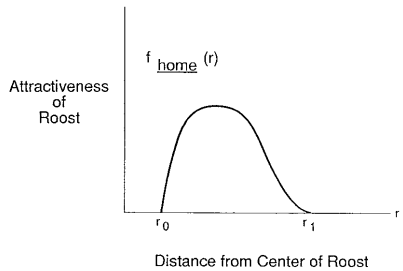

Fig. 1. Attractiveness of the roost.

vector difference from individual i to j: $d_{ij}(t) = u_j(t)-u_i(t)$ we postulate the i^{th} component to be

$$F^{(i)}_{interact}(t) = \sum_{j=1}^{n} f_{interact}(|d_{ij}(t)|)\, d_{ij}(t). \quad (4)$$

In Eq. 4 the summation is over all j. The interaction dies out if the inter-individual distance $d_{ij} > d_1$.

The random impact term is modeled by an n-dimensional, time-homogeneous Poisson process with stochastically independent components. The i^{th} component of $dP(t)$ is zero unless t happens to be an event of the i^{th} component of the Poisson process. In the latter case,

$$dP(t) = \text{random 3-vector with uniformly distributed components} \quad (5)$$

with a scalar parameter controlling the magnitude of the random vector.

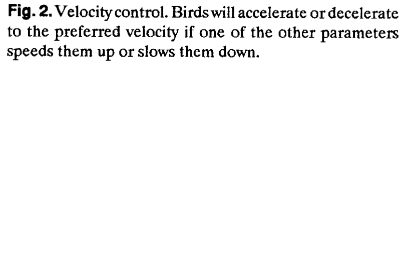

Fig. 2. Velocity control. Birds will accelerate or decelerate to the preferred velocity if one of the other parameters speeds them up or slows them down.

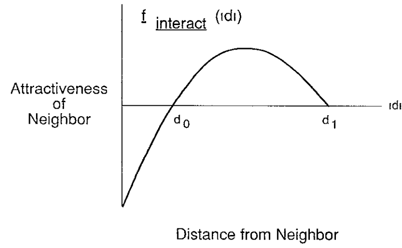

Fig. 3. Interaction function. Birds closer to each other than d_0 are repelled, and avoid collision. Between d_0 and d_1 they are attracted, and beyond d_1 they are not influenced by the proximity of a neighbor.

In terms of pattern theory, a striking feature of this model, which differs from other pattern models, is that the connector graph varies in time. It changes as the inter-individual distances change. One other case of this type could be mentioned, namely, for herd leader succession *(9)*.

We simulated the dynamics represented by Eq. 1 and displayed the solution graphically for small values of $n(<20)$ and for different values of the functions f_{home}, f_{vel}, $f_{interact}$, and $P(t)$. This was done by developing software written in the programming language APL, then translating into C, and run on a Silicon Graphics IRIS computer.

In practice, 15 parameters are available (Table 1) to vary the four terms. By changing the values of these parameters, different flock behaviors can be produced.

The appearance of the flock on the computer screen is striking (Fig. 4). By varying the values of the parameters of the terms, various group behaviors can be produced. "Birds" are initially lined up linearly, as on a telephone wire, with equidistant spacing between birds. With the set of values we call "c20.2," the birds initially fly toward the roost, mill around over it for about a minute in real time, then begin to form a cohesive flock which flies clockwise around the roost, "recruiting" stragglers, until all birds have joined the flock. In this flock, after an initial period of shifting positions, the position of each individual eventually stabilizes.

Table 1. Parameters available to modify "flock" behavior.

Parameter	Function
NB	Number of birds
FDISPLAYP	Scalar for force display
$HOMER_0$	Radius at which roost attraction forces begin
HOMERM	Radius of location of maximum attractive homing roost attracting force
HOMERE	Parameter for controlling the shape of the force
HOMERL	Analogous to a Gamma density exponent and Gamma density Lambda
FHOMERP	Scalar for roost attractiveness
$VELOCV_0$	Preferred bird velocity (V_0)
FVELOCP	Scalar for velocity
$INTDISTI_0$	Preferred interbird distance (d_0)
$INTDISTI_1$	Maximum distance at which a neighbor is attractive (d_1)
INTMAXR	Maximum force of repulsion for a neighbor closer than d_0
FINTP	Scalar for attractiveness of neighbor
HLAMBDA	Poisson frequency, Hlambda, for random term
$FRNDP_0$	Scalar for Poisson controlled impact force

In the value set called "Mill 1," the attractiveness of neighbors is increased. Immediately as the program is started, the birds fly toward each other, and form a grouping that resembles a swarm of gnats – individuals maintaining flight paths that contain them within the "ball" represented by the swarm itself, but not displaying any parallelism in their flight paths. This "swarm" then drifts over the roost and does not display the directional flight seen in c20.2.

In "2flock," the attractiveness of neighbors is reduced back to the level of c20.2, but the attractiveness of the roost is reduced. In this situation, the half-dozen birds most distant from the roost form an organized flock with roughly parallel flight paths which flies past the roost at a distance and continues, maintaining a straight direction of flight. The other birds fly toward the roost and form a cohesive group as in c20.2. The result is two coordinated flocks, one circling the roost, the other permanently leaving the roost area.

These patterns were discovered serendipitously, by simply changing values and seeing what happened. As more is learned about the properties of this system, it should eventually be possible to control the flight paths of selected individual birds qualitatively. This would enable the simulation of a predator attack *(10)* on a cluster-flying species and a test of the hypothesis that a turn can be initiated by an individual bird *(6,7)*, followed by a wave of turning in the flock.

Depending upon the choice of parameters, flocks tend to form even from fairly arbitrary initial locations around the roost or to break up from a well-formed flock. In the latter case, the behavior becomes more chaotic for a time, while in the former order emerges out of chaos. Our knowledge of the qualitative properties of this system are still limited. Our current computer experimentation suggests that the system exhibits chaotic regimes. Further study is required to resolve this question.

The model is simple and natural – an obvious extension of the classical n-body problem to a stochastic model with unconventional (non-Newtonian) forces. Such models involving stochastic differential equations are extensively used in applied probability theory. A pioneering and closely related model *(11)* also uses stochastic differential equations but with other functional forms and another type of randomness. It also presents some analytical results. Our reason for using this one was to serve as a tool in systematic mathematical experiments on the computer. The results of such experiments qualitatively resemble the flocking behavior of real birds.

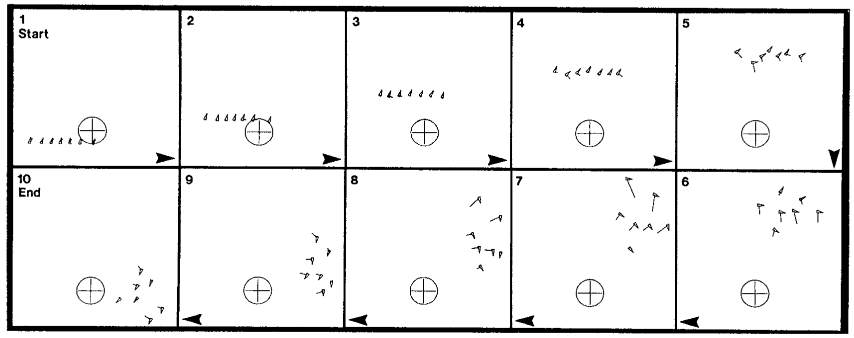

Fig. 4. A series of time-equidistant "frames" from a continuous flock movement. The circled cross is the "roost," triangles are "birds," with the vertex indicating the direction of "flight," and the line segment attached to each bird indicating the vector sum of the four forces – homing, velocity, interaction, and random impact – acting on the bird at the time depicted in the frame.

To our knowledge, the first operating computer model of a bird flock was reported in 1987 *(12)*. Although the mathematical expressions used to prepare this model were not presented, it appears generally to differ from ours in that our birds do not attempt to match velocity with neighbors but attempt to return to preset individual velocities, and our birds are attracted to a central roost. Further, it was not specifically reported in the earlier model that a stochastic differential element was included in its rules for movement, although such an element was necessary to produce flocking in ours. The fact that both models produced flocking under certain circumstances, using different parameters, provides fuel for further investigation.

We plan to extend the code to three dimensions, to give the user several options to choose the way he wishes to see the birds, and to remove the attractions of a roost. We hope to make the resulting code available to other researchers interested in this problem.

This model is mechanistic. Flocking here is the result of attractiveness of neighbors in flight, attractiveness of a roost, or other focal point, and the particular functional forms selected for the force terms in Eq. 1. Indeed, it took considerable trial-and-error experimentation with these functions before flock-like behavior was produced. Early attempts, for example, using Newtonian gravitation forces, failed to produce flock-like behavior. Also, the choice of $P(t)$ as a Poisson-based force succeeded where a Gaussian one did not. Unanswered is the question of why a neighbor should be attractive, and this is where possible selective forces such as predation deterrence *(13)* might enter.

Acknowledgment

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