

# Evaluation of Solid-State Accelerometer for Positioning of Vehicle

S. Nikbakht<sup>\*,\*</sup>

nikbakht@shariaty.net

Masood Mazlom<sup>\*</sup>

m8a2@yahoo.com

A. Khayatian<sup>\*\*</sup>

khayatia@shirazu.ac.ir

<sup>\*\*</sup>Department of Electrical Engineering, Shiraz University, Shiraz, IRAN.

<sup>\*</sup>Department of Computer Engineering, Shariaty University, Tehran, IRAN.

**Abstract-** An evaluation of a low-cost, small sized solid state accelerometer in the absence of absolute positioning information is performed in this paper. Acceleration signal outputted by the sensor is doubly integrated with time that yields the travel distance. Due to bias and Thermal noise of the accelerometer which is accumulate in time, Kalman filter is need to be design to increase the accuracy of position estimation system.

A modeling for bias of solid state accelerometer ADXL250 QCG without aiding sensor evaluated in this paper. During the preceding time instants, the position information becomes increasingly untrustworthy and the validity of the position updates decays. So, we used a smoother algorithm based Kalman filter for better estimation accuracy in position estimation. Result show smoothing algorithm increase the accuracy of position estimate for short time duration.

**Keywords:** INS, IMU, Bias, solid state accelerometer, Kalman filter (KF), Smoothing.

## 1-introduction

Inertial navigation system has been used in aerospace vehicles, military applications, such as ships, submarines, missiles, and to much lesser extent, in land vehicles applications. Only few years ago the application of inertial sensing was limited to high performance-high cost aerospace and military applications. However, several contributions in non-military applications have recently been published making use of low cost inertial system.

A full inertial navigation system (INS) consists of at least three accelerometers and three orthogonal gyroscopes providing acceleration in three dimensions and rotation rates about three axes. Theoretically, single and double integration of the gyro and accelerometer outputs will provide velocity and position information. In practice when working with standard IMU units, the non-linearity and noise present in the sensors make the trajectory prediction valid for short periods of time. So Kalman filter need to utilize for solving this problem ([1], [2]).

The most relevant reference found that develop modeling the bias of solid accelerometer and evaluating it for position estimation. An evaluation of an INS consisting of a solid-state gyroscope and a solid state accelerometer for a mobile robot can be found in [3]. Burshan and Durrant Whyte investigated the use of a solid-state gyroscope and accelerometer for mobile robotics applications. They paid particular attention to the gyroscope error model, and came up with an exponential curve to fit the changes in bias as the gyroscope warms up. So, they do this test for solid state accelerometer. They then implemented a Kalman filter for estimating position and shown that the error of estimation decrease at least by factor of 5.

K. Mostov in reference [4] described systematic and random errors in an accelerometer. Methods are proposed to reduce the effects of random errors which include proper modeling of the accelerometer and using a feedback system. Reference [5] describes how to find the bias and scale factor errors by using inertial data and absolute position data. G. I. and at al. in reference [6] describe a low-cost GPS/INS car navigation system. GPS is used to compensate for errors such as drift rate and offset bias error for two gyroscopes and three accelerometers. In ([7], [8]), Mostov discuss an accelerometer-based low-cost gyro-free inertial device for automotive application. So, S. Park and Chin-Tan Woo used this system for long term duration with GPS sensor [9]. Hugh H.S. Liu and Grantham K.H. Pang evaluate a low-cost small-sized solid-state accelerometer [10]. They show that the accelerometer could be viable solution as a short-duration distance-measuring device for a mobile robot or platform. Stergios I. Roumeliotis in reference [11] described a technique to accurately estimate the state of a robot helicopter using a combination of gyroscopes, accelerometers, inclinometers and GPS. They used two Kalman filter, one for the gyroscope data and the other for the accelerometer data. The theory, implementation and usage of the ADXL250 accelerometer given in reference [12]. The primary motivation for the work reported in this paper has been the need to develop a system

that capable of providing low-cost, high precision, short time-duration position information. In particular, the interest has been in obtaining location information for short periods when the vehicle is not in contact with any aiding sensor for example beacon or landmark information. To make best use of low-cost inertial sensing system, it is important that a detailed understanding of the mechanisms causing drift error are understood and a model for these derived. The approach taken in this paper is to incorporate in the system a *priori* Information about the error characteristics of the inertial sensors and to use this directly in an Extended Kalman Filter (EKF) to estimate position before supplementing the INS with absolute sensing mechanisms.

In section 2 the experimental setup described. The bias modeling of solid state accelerometer described in section 3. Applications of Kalman filter for reduction of bias effect for position estimation evaluated in section 4. In conclusion, the usefulness of low cost solid state accelerometer for short time duration discussed

## 2- Experimental Setups and Bias modeling of solid-state accelerometer

The accelerometer that has been evaluated is the ADXL250 QCG produced by Analog Devices, Inc. It is a low-cost low-power two-axis micro machined accelerometer with a measurement range of  $\pm 50g$ . It can measure both dynamic acceleration and static acceleration. The outputs of accelerometer are analog and digital signals with duty cycles proportional to the acceleration in each of two axes. The sampling rate of the accelerometer depends on the dynamic of navigation system or robot. With high-speed, the accelerometer should be set to a higher bandwidth, which requires a faster sampling rate. The bandwidth of our evaluated accelerometers is from  $0.001Hz$  to  $200Hz$  be used. However, wider bandwidth would cause higher white noise in signal.

But in this research we used an accelerometer box that was illustrated in Figure 1. Accelerometer box contain two solid state accelerometer ADXL250 QGC that is a dual axis and we used the analog output of accelerometer. Building Error models for inertial sensors in motivated by an attempt to reduce the effect of unbounded position errors. Depending how successful these models are, inertial sensors may possibly be used in an unaided mode or for longer duration on their own.



Figure 1: Cube solid state accelerometer box

Drift on the outputs of inertial sensors is the most important contributor to navigation system errors, and is mainly dependent upon the device technology.

To develop an error model for the accelerometer, its output was recorded over long periods of time when subjected to zero input. The result of this experiment over a period of 6 hours is shown in Figure 1. In the following, let  $\varepsilon(t)$  be the bias error associated with measuring the true value of a quality using inertial sensors. A nonlinear parametric model of the following form was fitted to the data from the accelerometer:

$$\varepsilon_{model}(t) = c_1(1 - e^{-\frac{t}{T}}) + c_2 \quad (1)$$

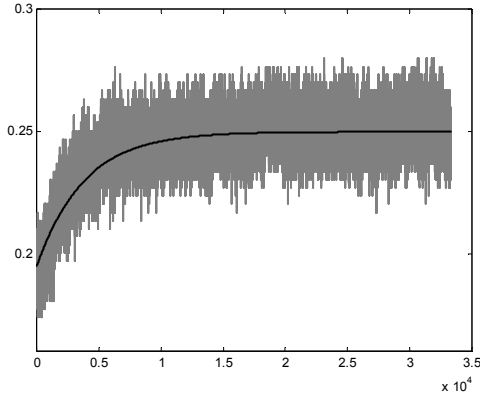
Where  $\varepsilon_{model}(t)$  is the fitted model to the accelerometer output when zero input was applied, with parameters  $c_1, c_2, T$  to be tuned.

In general, a model fitted to experimental data is regarded as being adequate if the *residuals* from the fitted model constitute a white, zero-mean process. Hence, one can start with any reasonable models based on inspecting the original data and test its residuals for whiteness. If the test fails, the model can be further developed until the residuals pass the whiteness test. This implies that the test for the validity of any model is basically reduced to a test for whiteness. The sufficiency of the above model in Equation 1 we attain the solid state accelerometer data in stationary for 6 hour and 1 hour. Figure 2 and Figure 3 show the result in time domain and frequency domain.

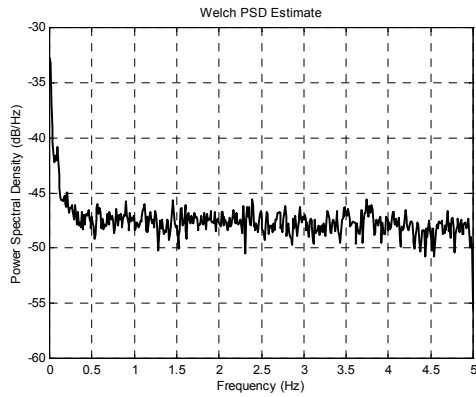
The parameterized model of Equation 1 for the deterministic bias error can be represented by the following differential equation:

$$\dot{\varepsilon} = \frac{(c_1 + c_2)}{T} - \frac{1}{T}\varepsilon \quad (2)$$

With initial conditions  $\varepsilon(0) = c_2$  and  $\dot{\varepsilon}(0) = \frac{c_1}{T}$ .



**Figure 2:** The output of accelerometer for one hour in time domain with scale  $(1/g) \text{ m/s}^2$



**Figure 3:** The output of accelerometer for one hour in frequency domain

After discretions:

$$\varepsilon(k+1) = \frac{T}{T+T_s} \varepsilon(k) + \frac{T_s}{T+T_s} (c_1 + c_2) \quad (3)$$

With  $\varepsilon(0) = c_2$ . Due to its recursive nature, this difference equation is independent of start-up time but relies on a good estimate of the initial bias. Also, the residual bias of model that was fitted on the raw data of accelerometer in no motion case is white stationary signal (equation 3). So, total bias model can be derived:

$$\varepsilon(k+1) = \frac{T}{T+T_s} \varepsilon(k) + \frac{T_s}{T+T_s} (c_1 + c_2) + \omega_\varepsilon(k) \quad (4)$$

$\omega_\varepsilon(t)$  is white Gaussian noise with variance  $\sigma_\varepsilon^2$ .

### 3- Reduction of Random Bias with Kalman filter

Kalman filter is method that used for reduction random noise and data fusion of multisensor in inertial navigation system. In this method stochastic characteristic of measurement model in recursive form used for estimate states of navigation. Estimation of this state achieve with

weighting function such that variance of error minimized.

#### 3-1 Design Kalman filter Based on Total State Kalman filter

A Kalman filter model for accelerometer is well described in [2]. However, we describe a one-dimensional Kalman filter here. We use Position-Velocity-Accelerometer (PVA) model, a stationary process such as Gauss-Markov process, for reduction the effect of bias in the solid state acceleration signal for position estimation. Figure 8 show the accelerometer bias that modeled as a random walk process. This bias is modeled as integrated white noise with power spectral density (PSD) being  $\mathcal{W}$ . PSD of input noise obtain from the variance of derivation of raw signal accelerometer in no motion state from some experiments to provide better results. So, if  $a(t)$  be acceleration in time  $t$ , we have:

$$\dot{a}(t) = \omega_n(t) \quad (5)$$

$\omega_n(t)$  is white noise with variance  $\sigma_n^2$  that equal with power spectral density (W). So, state equation for this method drive to following form:

$$\dot{r}(t) = u(t)$$

$$\dot{u}(t) = a(t)$$

$$\dot{a}(t) = \omega_n(t)$$

$$\dot{\varepsilon}(t) = \frac{(c_1 + c_2)}{T} - \frac{1}{T} \varepsilon(t) + \omega_\varepsilon(t) \quad (6)$$

The resulting state equations and state transition matrix  $F$  and process noise covariance matrix after discretization is derive as follows:

$$X(k+1) = F X(k) + u + w(k) \quad (7)$$

$$Z(k) = H X(k) + r$$

$$F = \begin{bmatrix} 1 & T_s & \frac{1}{2}T_s^2 & 0 \\ 0 & 1 & T_s & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{T}{T+T_s} \end{bmatrix}$$

$$u = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{T_s(c_1 + c_2)}{T + T_s} \end{bmatrix}, H = [0, 0, 1, 1]$$

$$Q = \begin{bmatrix} \frac{T_s^5}{20} \sigma_n^2 & \frac{T_s^4}{8} \sigma_n^2 & \frac{T_s^3}{6} \sigma_n^2 & 0 \\ \frac{T_s^4}{8} \sigma_n^2 & \frac{T_s^3}{3} \sigma_n^2 & \frac{T_s^2}{2} \sigma_n^2 & 0 \\ \frac{T_s^3}{6} \sigma_n^2 & \frac{T_s^2}{2} \sigma_n^2 & T_s \sigma_n^2 & 0 \\ 0 & 0 & 0 & T_s \sigma_\epsilon^2 \end{bmatrix}$$

We use raw data of accelerometer in motion state as measurement data for Kalman filter to estimate bias of sensor. Figure 9 shows the Kalman filter algorithm that used for TKF Kalman filter. Figure 10 illustrate acceleration estimated with Kalman filter and Figure 11 shows that the deterministic bias estimated with Kalman filter from raw data of solid state accelerometer in motion state. Result in Figure 11 show that Kalman filter has a better estimation for position from integration method (that attain from acceleration data with assumption decrease from  $c_2$ ) and decrease the effect of exponential and random bias in position estimation [16].

#### 4- Smoothing Based Kalman Filter

In previous section we built a kalman filter that estimates the position. During the preceding time instants, as we can see from the position plot in Figure 11, after 10 second, the position information becomes increasingly untrustworthy and the validity of the position updates decays. So we apply a smoothing algorithm every time we receive a good estimate of the position. The discrete Kalman filtering algorithm was presented in previous section. We will now consider the recursive *smoothing* problem. The algorithm to be presented here is due to Rauch, Tung, and Striebel [13-14] and its derivation is given in Meditch [15]. In the interest of brevity, the algorithm will be subsequently referred to as the RTS algorithm. Consider a fixed-length interval containing  $N+1$  measurements. These will be indexed in ascending order  $z_0, z_1, \dots, z_N$ . The assumptions relative to the process and measurement models are the same as for the filter problem. The computational procedure for the RTS algorithm consists of a forward recursive sweep followed by a backward sweep. We enter the algorithm as usual at  $k=0$  with the initial conditions  $\hat{x}_0^-$  and  $P_0^-$ .

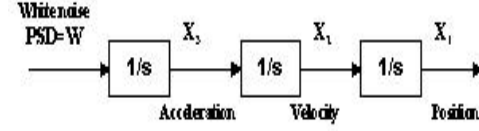


Figure 8: Process model of the accelerometer data for the Kalman filter

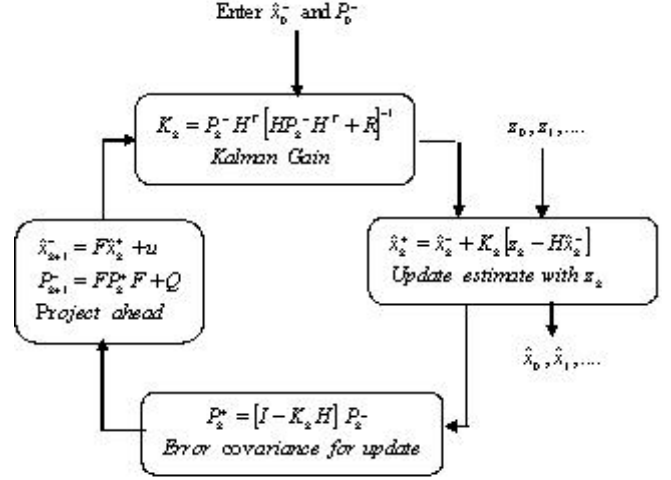


Figure 9: Kalman filter algorithm

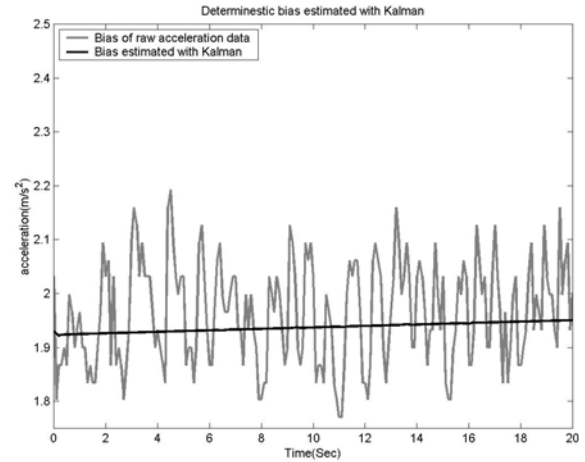
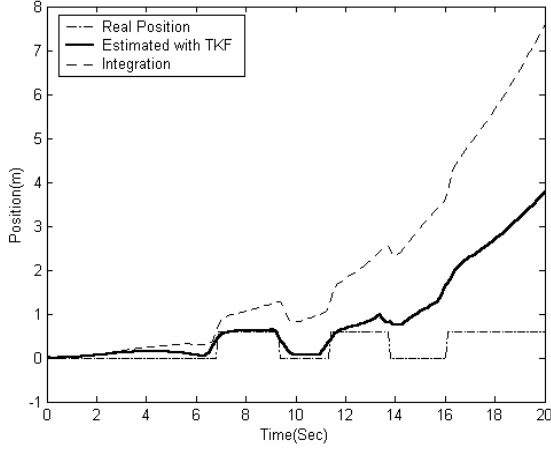
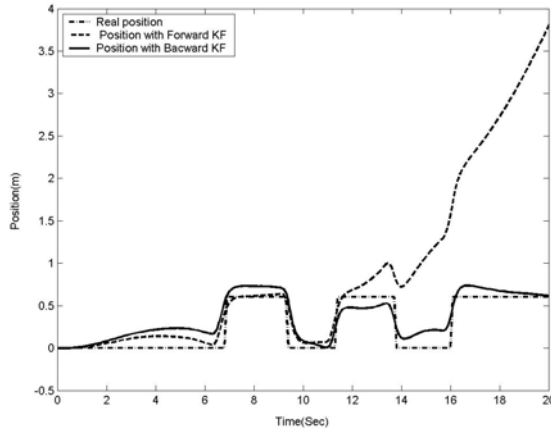


Figure 10: Total bias estimated with TKF Estimated with TKF (Solid line) and estimated and stationary data of acceleration with integration (dashed line)

We then sweep forward using the conventional filter algorithm. With each step of the forward sweep, we must save the computed a priori and a posteriori estimates ( $\hat{x}_k^-$  and  $\hat{x}_k^+$ ) and their associated  $P$  matrices ( $\hat{P}_k^-$  and  $\hat{P}_k^+$ ). These are needed for the backward sweep. After completing the forward sweep, we begin the backward sweep with "initial" conditions  $\hat{x}_N$ .



**Figure 11:** Real position (dash-dot line), position with integration (dashed line) and estimated with TKF (solid line)



**Figure 12:** Real position (dash-dot line), position with Forward KF (dashed line) and with Backward (solid line)

and  $\hat{P}_N$  obtained as the final computation in the forward sweep. With each step of the backward sweep, the old *filter* estimate is updated to yield an improved smoothed estimate, which is based on all the measurement data. The recursive equations for the backward sweep are:

$$\hat{x}_b(k) = \hat{x}_f(k) + A(k)^*$$

$$[\hat{x}_b(k+1) - \hat{x}_f^-(k+1)] \quad (8)$$

where the smoothing gain  $A(k)$  is given by:

$$A(k) = \hat{P}_f(k) F^T P_f^{-1}(k+1) \quad (9)$$

and  $k = N-1, N-2, \dots, 0$ .

The error covariance matrix for the smoothed estimates is given by the recursive equation:

$$\hat{P}_b(k) = \hat{P}_f(k) + A(k)^*$$

$$[\hat{P}_b(k+1) - \hat{P}_b^-(k+1)] A^T(k) \quad (10)$$

Position estimated with forward and backward filter is illustrated in Figure 12. Result

demonstrates the improvement in estimation with smoothing algorithm for short term duration [16].

## 5- Conclusion

The experimental results have provided a useful evaluation of a low-cost solid state accelerometer. The performance of the accelerometer is shown to be acceptable as a short-duration distance-measuring device for mobile platform or robot [17]. So, we applied a smoothing algorithm in the absence of absolute positioning information that constructed the best estimate of the position over a time period using all the measurements in that time interval. Further research would focus on the better model for stochastic bias of solid state accelerometer in order to reduce the effect of bias and thermal noise and fuse the solid state accelerometer data with other absolute sensors that measure position.

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