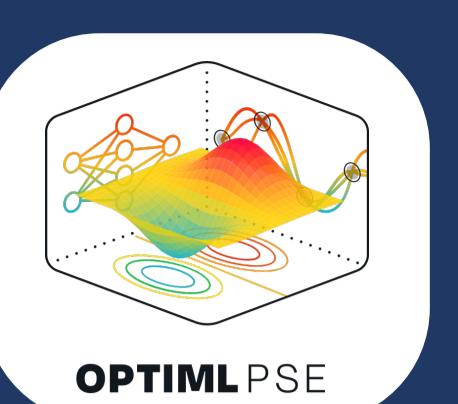


Deep Gaussian Process-based Multi-fidelity Bayesian Optimization for Simulated Chemical Reactors

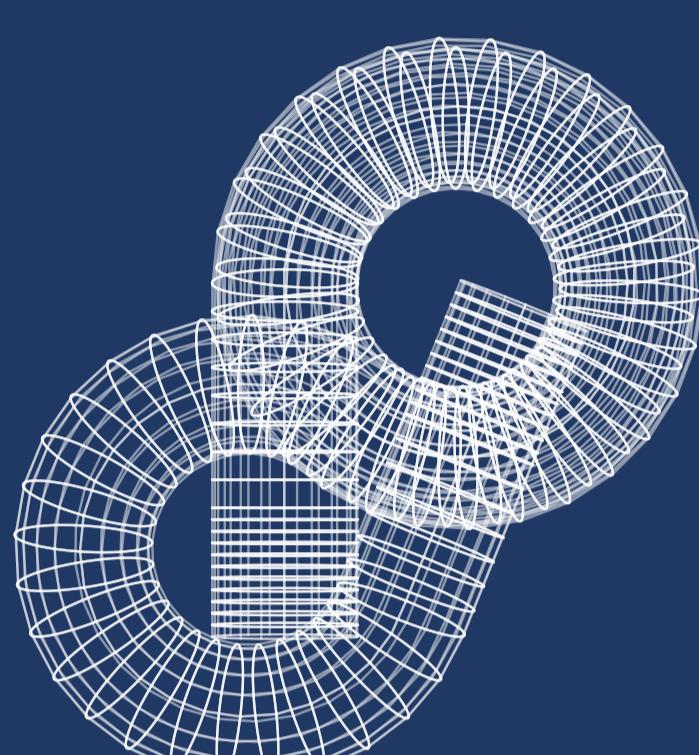
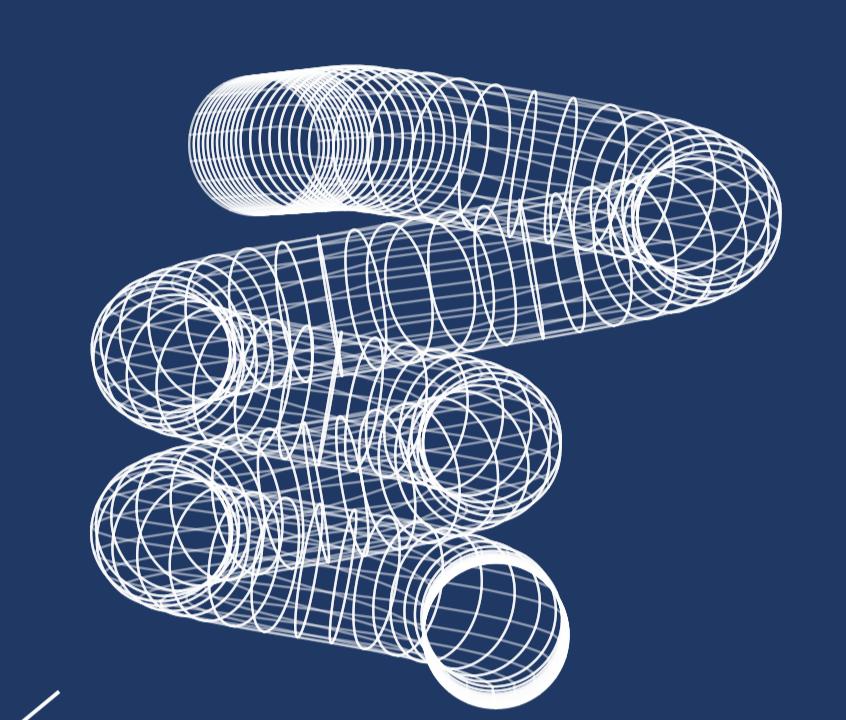
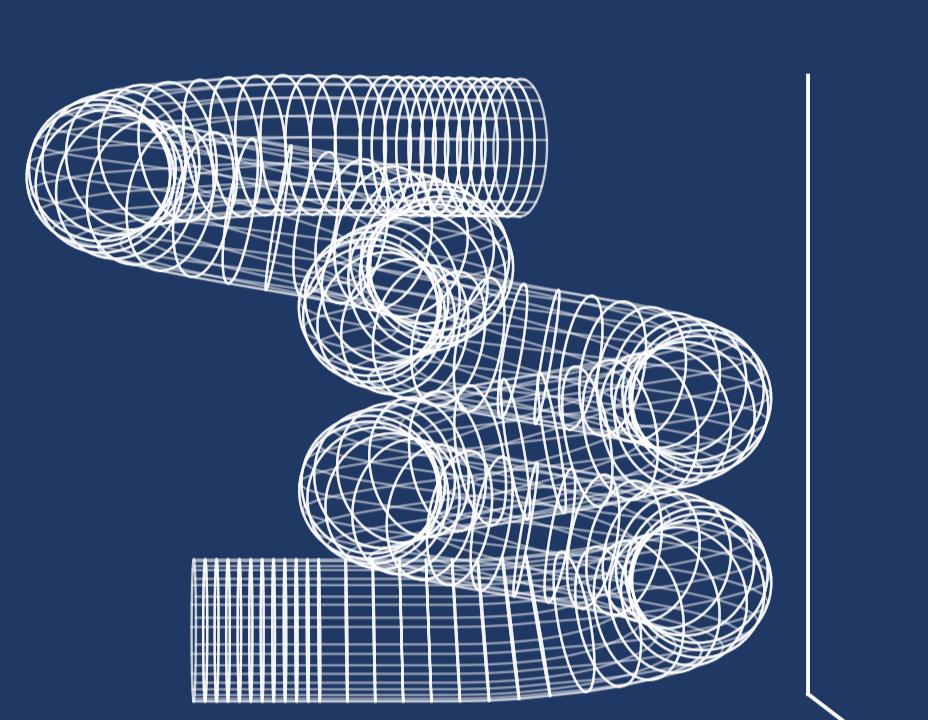
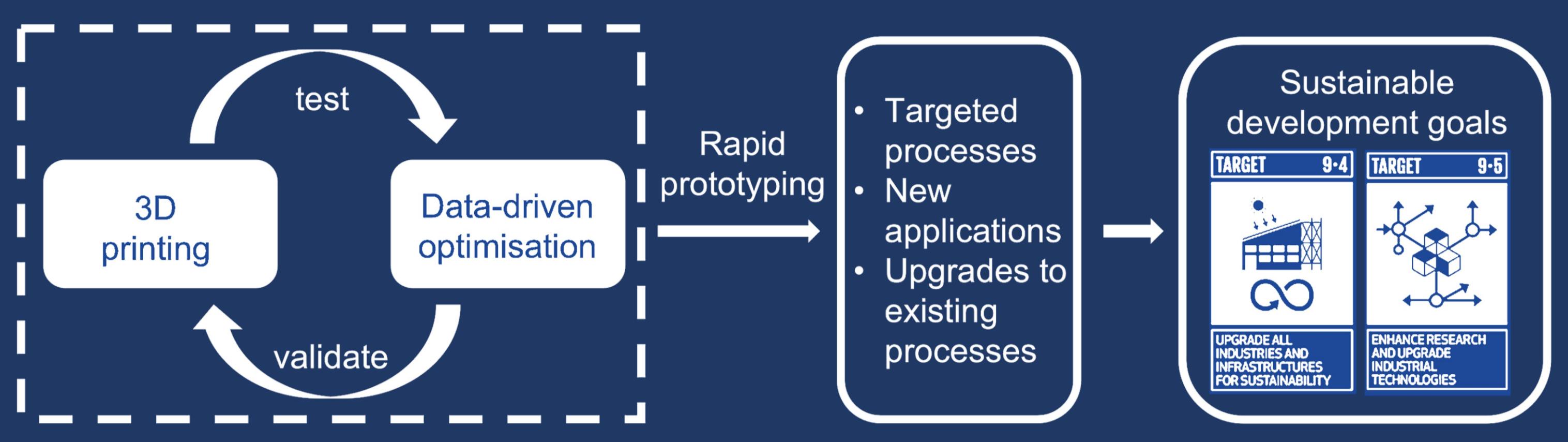
Tom Savage, Nausheen Basha, Omar Matar, Antonio del Rio Chanona

Imperial College London, United Kingdom

Imperial College London



- The development of new manufacturing techniques such as 3D printing have enabled the creation of previously infeasible chemical reactor designs.
- Now able to manufacture and optimize reactors with highly parameterised geometries.
 - Vital to ensure enhanced mixing characteristics;
 - Satisfy feasible manufacturability.

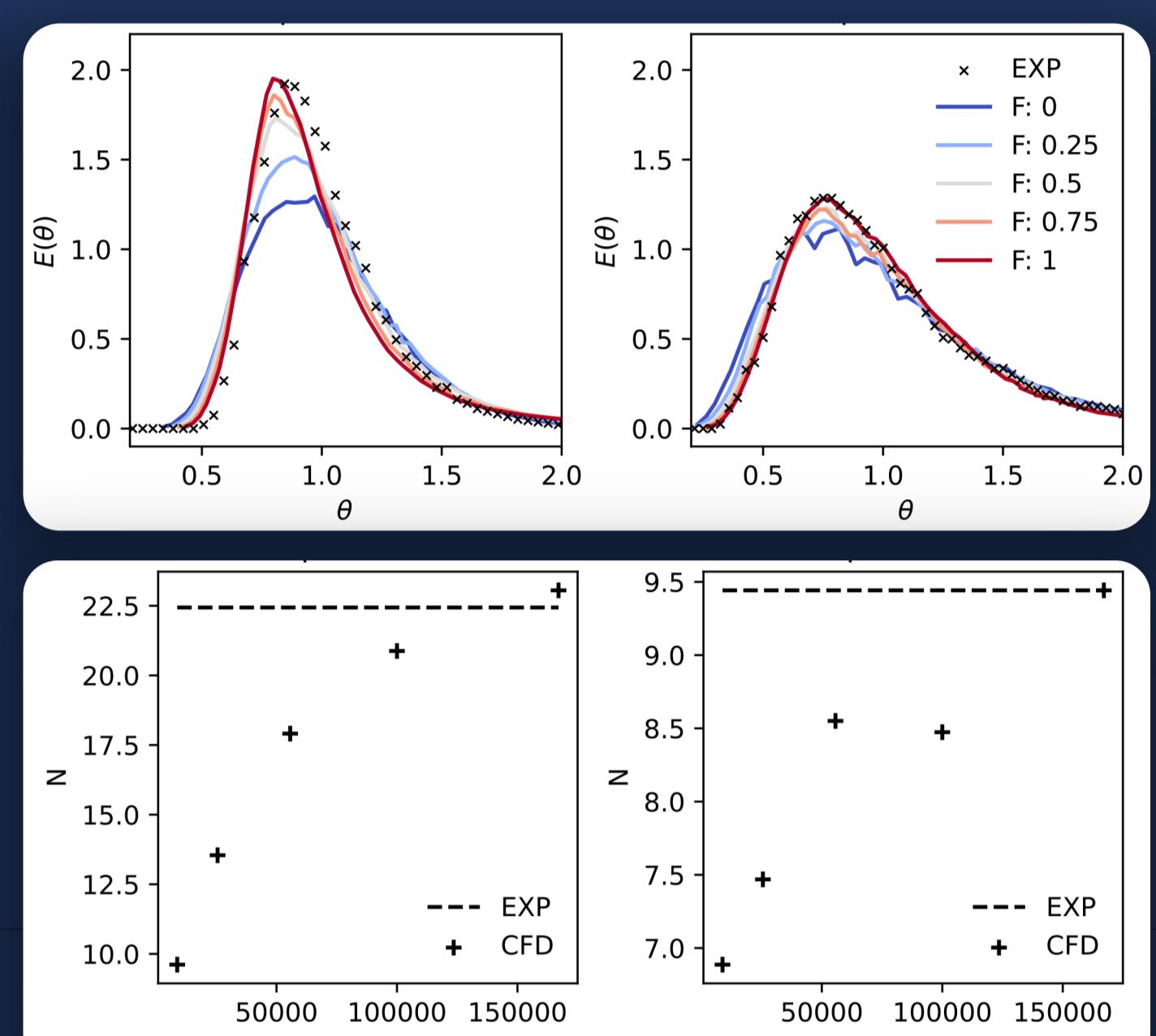


Objective: **maximise plug-flow characteristic**
Issues: Highly nonlinear, derivative-free, **expensive**

- Parameterize a pulsed-flow coiled tube reactor

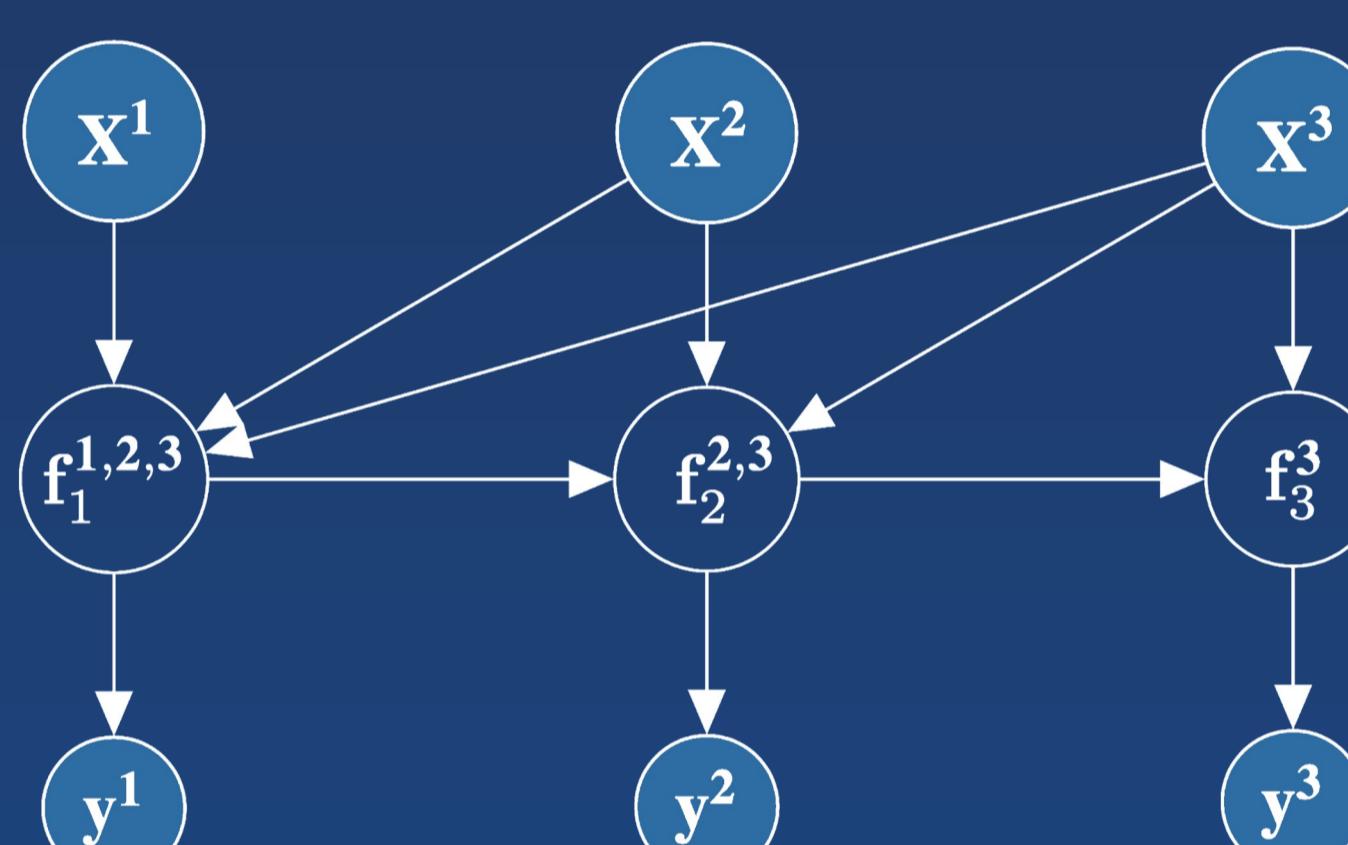
- Coil radius
- Pitch
- Inversion Location
- Frequency
- Amplitude
- Reynolds Number

- Define discrete simulation fidelities
- Experimentally validate different fidelity simulations



Multi-fidelity Deep Gaussian Processes

- $f_t(x) = \rho_t f_{t-1}(x) + \delta_t(x)$ [AR1]
 - Fails to capture nonlinear relationships between fidelities.
- $f_t(x) = \rho_t(f_{t-1}(x), x) + \delta_t(x)$ [NARGP]
 - Inaccurate uncertainty estimation.
- $f_t(x) = g_f(f_{t-1}^*(x), x)$ [MF-DGP]
 - End-to-end trained, higher fidelity data influences the prediction of lower fidelity functions.

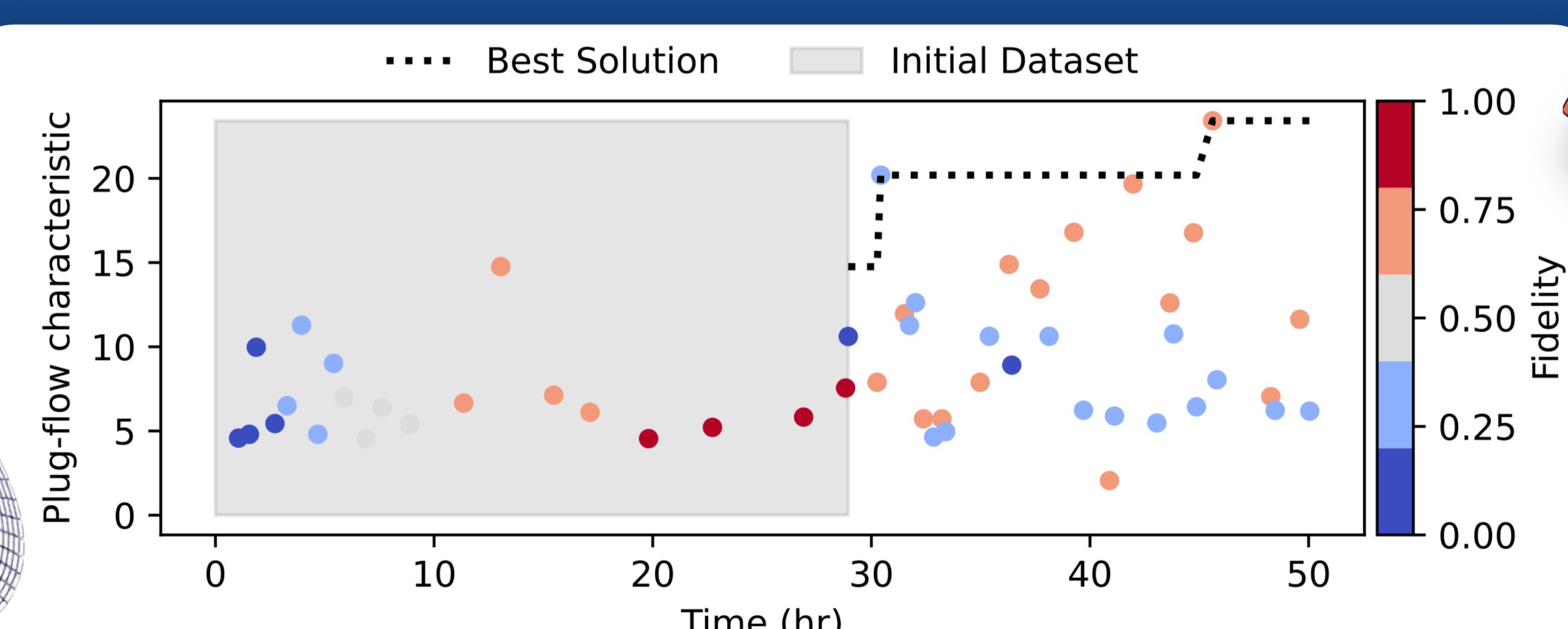


Algorithm 1 Deep GP-based Multi-fidelity Bayesian Optimization

```

Require:  $f_1(x) \dots f_T(x)$ ,  $\mathcal{X}$ ,  $n$ 
for  $t$  in  $1, \dots, T$  do
    Generate  $n$  samples,  $\mathbf{x}_t$ , and evaluate  $f_t(\mathbf{x})$  resulting in  $\mathbf{y}_t$ .
     $\tau_t \leftarrow$  average simulation time
end for
while Budget not exhausted do
    Train DGP using  $\mathbf{x}_1, \dots, \mathbf{x}_T$  and  $\mathbf{y}_1, \dots, \mathbf{y}_T$ 
    Solve UCB for highest-fidelity:  $x^* \leftarrow \arg \max_x \{\mu_T(x) + \beta^{1/2} \sigma_T(x) | x \in \mathcal{X}\}$ 
    Choose fidelity based on variance of DGP and simulation cost:  $t^* \leftarrow \operatorname{argmax}_t \{\gamma_t \beta^{1/2} \sigma_t(x^*)\}$ 
    where  $\gamma_t = \max(\tau)/\tau_t$ 
    Evaluate  $f_{t^*}(x^*)$ , add  $x^*$  to  $\mathbf{x}_{t^*}$  and  $f_{t^*}(x^*)$  to  $\mathbf{y}_{t^*}$ 
end while

```



Conclusions

- Additive manufacturing → highly parameterised chemical reactors.
- Optimization of coiled-tube reactor geometry → expensive, multi-fidelity black-box problem.
- Multi-fidelity Bayesian optimization using Deep Gaussian processes → enables solution.
- Framework → extended to other problems involving highly-parameterized CFD simulations.

References

- Deep Gaussian Processes for Multi-fidelity Modeling: arXiv:1903.07320
- Multi-fidelity Gaussian Process Bandit Optimisation: arXiv:1603.06288
- Oscillatory fluid motion unlocks plug flow operation in helical tube reactors at lower Reynolds numbers ($Re \leq 10$): DOI:10.1016/j.cej.2018.10.054

- Tom Savage would like to thank the Imperial College President's Scholarship Fund
- PREMIERE (EP/T000414/1)
- Dr Jonathan McDonough, Newcastle University
- Ilya Sandoval for discussion