

NOTES OF ASTROPHYSICAL PROCESSES

version 1.0

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*Knowledge without understanding
Is but a sword stuck in its sheat.*

Arthur Leywin

PREFACE

Not long ago, it occurred to me how cool it is when someone unexpectedly releases a very detailed and all-comprehensive version of their notes, especially when dealing with a course that has a handful of really different topics that often interact together in unpredictable, yet fascinating, ways—as it's the case for the Astrophysical Processes class.

This notes will be mainly based on *my* own notes of the lectures by Professor Walter del Pozzo and Professor Marco Crisostomi during the academic year 2025-2026. Since, however, I take little to no pride in my messy notes, I'll be often using some of the many references you can find on the course catalogue page or in the bibliography of this humble collection.

You can report errors (whatever their nature might be) and suggestions for additions at g.pannocchia3@studenti.unipi.it (institutional) or by any means (conventional or not) you deem the best¹.

Without further ado, we'd better not lose much more time on a preface and get started with it.

There was Eru, the One, who in Arda is called Ilúvatar; and he made first the Ainur [...] But for a long while they sang only each alone, or but few together, while the rest hearkened; for each comprehended only that part of the mind of Ilúvatar from which he came, and in the understanding of their brethren they grew but slowly.

Yet ever as they listened they came to deeper understanding, and increased in unison and harmony.

*Ainulindalë, "The music of the Ainur",
Silmarillion, J. R. R. Tolkien*

¹ I'd like, however, not to see my house stormed by homing pigeons.

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Part I

RADIATIVE TRANSPORT

1

INTERACTION OF RADIATION WITH MATTER

1.1 INTRODUCTION

Most of our knowledge about the Universe is based on the electromagnetic radiation that reaches us from far far away. EM radiation is obviously not the only way we can probe the Universe we live in but, in respect to neutrinos, cosmic rays or even gravitational waves, it's not a long stretch to claim it is by far the most understood.

It is most important then that an astrophysicist worthy of his (or her) name has a good grasp of the theory of radiative transfer and of its applications.

Apart from a few more key differences, I'll follow the description of radiative transfer of [1], but I won't fail to emphasize whenever I'll be doing otherwise.

1.2 RELEVANT QUANTITIES FOR RADIATIVE TRANSFER

Although some books often start their description of radiative transfer from the definition of *monochromatic energy* and *monochromatic intensity*, I found that it is most misleading, since, in all but a few cases, what we experimentally measure are fundamentally *fluxes*.

We shall then consider the *monochromatic flux* F_ν ($\text{erg s}^{-1} \text{Hz}^{-1} \text{cm}^{-2}$) produced by some source passing through a small area dA located somewhere in space.

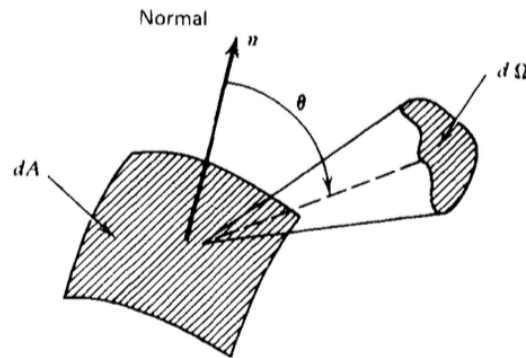


Figure 1: Schematic geometrical representation of the system.
Credits: G. Rybicki, A. Lightman [2].

If we call \hat{k} the propagation direction of the flux and \hat{n} the unit vector emerging from the surface dA , it's easy to get convinced that what is actually passing through the surface is somewhat proportional to $F_\nu(\hat{k} \cdot \hat{n})$.

From the monochromatic flux we can define the *bolometric flux*, which is just the monochromatic flux integrated over all frequencies (or wavelengths)

$$F = \int_0^{+\infty} F_\nu d\nu = \int_0^{+\infty} F_\lambda d\lambda \quad (1)$$

This also tells us how to convert a flux per unit frequency to a flux per unit wavelength

$$F_\nu d\nu = F_\lambda d\lambda$$

By now it should be clear that, despite being experimentally sensible to use the flux, we're losing much information sticking with it, namely directional information.

We consider then the amount of radiation $E_\nu d\nu$ passing through the same area in time dt and solid angle $d\Omega$. Hence we can write

$$dE_\nu d\nu = I_\nu(\mathbf{r}, t, \hat{k})(\hat{k} \cdot \hat{n}) dt d\Omega dA d\nu \quad (2)$$

where the quantity $I_\nu(\mathbf{r}, t, \hat{k})$ is called the *specific monochromatic intensity*. If $I_\nu(\mathbf{r}, t, \hat{k})$ is specified for all directions at every point in a certain region of spacetime, then we'd have a complete prescription of the radiation field we intend on studying.

Capitalizing on the blatant similarities with distribution functions, we can evaluate the moments of the monochromatic intensity.

Definition 1.2.1. *Monochromatic mean intensity J_ν*

$$J_\nu = \frac{1}{4\pi} \int_\Omega I_\nu d\Omega = \frac{c}{4\pi} U_\nu$$

with U_ν the total energy density of radiation. Note that J_ν is pretty much just an average of the monochromatic intensity over all solid angles.

Definition 1.2.2. *Monochromatic flux \vec{F}_ν*

$$\vec{F}_\nu = \frac{1}{4\pi} \int_\Omega I_\nu(\hat{k}) \hat{k} d\Omega = \frac{1}{4\pi} \vec{F}_\nu$$

I haven't explicitly proved the last equality, but it shouldn't be hard for you to convince yourself (or prove it yourself) that it is indeed true.

Definition 1.2.3. *Monochromatic radiation pressure p_ν* The monochromatic pressure is defined starting from the different directions correlations of the monochromatic intensity

$$K_\nu^{ij} = \frac{1}{4\pi} \int_\Omega I_\nu(\hat{k}) n^i n^j d\Omega$$

The pressure in particular is usually expressed as

$$P_\nu = \frac{1}{c} \int_\Omega I_\nu(\hat{k}) \cos^2 \theta d\Omega$$

where $\cos^2 \theta = (\hat{k} \cdot \hat{n})^2$.

1.3 BLACKBODY RADIATION

Even at an undergraduate level, we're all fairly familiar with *blackbody radiation*. The easiest way to deduce the expression for the energy density of photons in *thermal equilibrium* (STE) inside a cavity is by the means of statistical mechanics.

Remember the Bose-Einstein distribution

$$n = \frac{1}{\exp(h\nu/kT) - 1}$$

and the phase space density of states

$$\rho(\nu) d\nu = \frac{4\pi g \nu^3}{c^3} d\nu$$

from which deducing the expression from internal energy is straightforward. Remembering $g = 2$ is the quantum degeneracy of photons, a simple multiplication of the previous expressions yields

$$U_\nu d\nu = \frac{8\pi\nu^3}{c^3} \frac{1}{\exp(h\nu/kT) - 1} d\nu$$

Since blackbody radiation is isotropic (it actually depends only on the absolute temperature T), the definition of mean monochromatic intensity yields

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp(h\nu/kT) - 1} \quad (3)$$

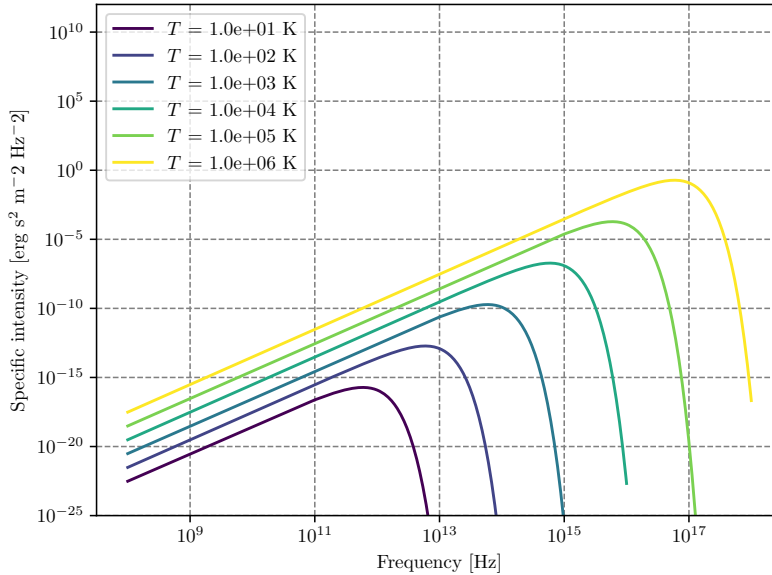


Figure 2: Blackbody frequency spectrum.

It's important to notice that, in principle, such a fundamental result holds only in *strict thermodynamic equilibrium* (STE), but we'll soon see how to generalize this formulation for less "restrictive" environments.

An incredible number of important results descends from (3), and it may be worthwhile to at least cite some of them, starting from Stefan-Boltzmann law.

We'll use the following result without proving it

$$\int_0^{+\infty} B_\nu(T) d\nu = \frac{2h}{c^2} \frac{\pi^2}{15} \left(\frac{kT}{h} \right)^4$$

Computing the bolometric flux and the bolometric energy density by integrating over all frequencies using what we've just written down, you find the following

$$U(T) = aT^4 \quad F(T) = \sigma_{SB}T^4$$

Clearly the two constants a and σ_{SB} cannot be independent, and are actually related by the integral we've previously calculated. Using for example

$$F(T) = \pi \int_0^{+\infty} B_\nu(T) d\nu$$

you can easily find out that the *Stefan-Boltzmann constant* is equal to

$$\sigma_{SB} = \frac{2\pi^5 k^4}{15c^2 h^3}$$

and the relation with a is simply $\sigma_{SB} = ac/4$.

The equation

$$F(T) = \frac{2\pi^5 k^4}{15c^2 h^3} T^4 \tag{4}$$

is what is usually known as the *Stefan-Boltzmann law*.

Let us now consider two different regimes for eq.3: $h\nu/kT \ll 1$ and $h\nu/kT \gg 1$. The first yields what is commonly known as the Rayleigh-Jeans Law which is, sadly, pretty much relevant only for radioastronomy.

Since

$$\exp\left(\frac{h\nu}{kT}\right) = 1 + \frac{h\nu}{kT} + o\left(\frac{h\nu}{kT}\right)^2$$

the blackbody radiation assumes the much simpler form of

$$B_\nu^{RJ} = \frac{2\nu^2}{c^2} kT \tag{5}$$

Another important results is achieved in the opposite regime, when the exponential term is rather larger than unity

$$B_\nu^W = \frac{2h\nu^3}{c^2} \exp\left(-\frac{h\nu}{kT}\right) \tag{6}$$

This expression is often known as Wien's Law.

1.4 RADIATIVE TRANSFER EQUATION

In the presence of matter, it is not immediately obvious what changes may occur in the specific intensity as we move along a ray path. The aim of this section will be to eviscerate the matter.

Let's consider the following geometric construction

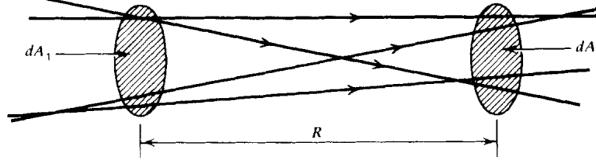


Figure 3: Geometrical construction for ray paths propagating in empty space.
Credits: G. Rybicki, A. Lightman.

It won't take a lot of effort to convince yourself that in empty space the monochromatic intensity I_ν is actually conserved. Simply writing down the definitions and imposing the conservation of energy

$$I_{\nu_2} dA_2 dt d\Omega_2 d\nu = I_{\nu_1} dA_1 dt d\Omega_1 d\nu$$

the conclusion follows from observing that $dA_2 d\Omega_2 = dA_1 d\Omega_1$.

If we consider an affine parameter of the form $\vec{x} = \vec{x}_0 + \hat{k}s$, we may as well write the previous results in a more familiar fashion

$$\frac{dI_\nu}{ds} = 0 \implies (\hat{k} \cdot \nabla) I_\nu = 0 \quad (7)$$

What changes if matter is present along the ray path? Clearly it will no longer be true that $(\hat{k} \cdot \nabla) I_\nu = 0$, but we're not that far off. All that we need is some little work on both terms.

How the right member of the equation should change is obvious: It needs to keep track of the "creation" and "destruction" of photons in the considered volume of spacetime.

The left member requires a little more care. Consider infinitesimal time and space displacements along the ray path, respectively dt and $d\vec{x}$

$$\Delta E_\nu d\nu = \left(I_\nu(\vec{x} + d\vec{x}, t + dt, \hat{k}) - I_\nu(\vec{x}, t, \hat{k}) \right) dt d\Omega dA d\nu$$

Taking a first order expansion in respect to the affine parameter s along the ray path yields

$$\left(\frac{1}{c} \partial_t I_\nu + \partial_s I_\nu \right) dt ds d\Omega dA d\nu = \text{photon addition} - \text{photon removal}$$

This equation is clearly a generalization of eq.7 for non-stationary radiative transport and in the presence of matter. It's about time we get to know what "lives" in the right side of the equation.

1.4.1 Monochromatic emission coefficient

For the moment, we'll define the *spontaneous* monochromatic emission coefficient j_ν as

$$dE_\nu d\nu = j_\nu dV dt d\Omega d\nu \quad (8)$$

which in general has a non-zero dependence on the emission direction. Sometimes the spontaneous emission coefficient is defined by the *emissivity* ϵ_ν (**please note** that rather often the two names are used almost interchangeably), which is the energy emitted spontaneously per unit frequency per unit time per unit mass

$$j_\nu = \frac{\epsilon_\nu \rho}{4\pi}$$

where ρ is the mass density of the emitting medium.

If we perform the decomposition $dV = dA ds$, the contribution of spontaneous emission to the specific intensity is

$$dI_\nu = j_\nu ds$$

1.4.2 Absorption coefficient

Similarly, we can consider the energy that is absorbed from the radiation when passing through a medium. There exists similar definition; I'll use the one we gave in class and that is incidentally the one used in [1] and [2] as well.

We define the *absorption coefficient* α_ν through the following relation

$$dI_\nu = -\alpha_\nu I_\nu ds \quad (9)$$

If we use a microscopic model, then the absorption coefficient can be understood as particles with numeric density n presenting an effective absorbing area, the *cross section*. The coefficient α_ν can thus be rewritten in terms of

$$\alpha_\nu = n\sigma_\nu = \rho\kappa_\nu$$

where κ_ν is called the mass absorption coefficient or the *mass-weighted opacity coefficient*.

I should probably point out that in eq.9, we consider “absorption” to include both “true absorption” and stimulated emission, because both are proportional to the intensity of the incoming beam. Depending on the entity of the contribution, the α_ν coefficient may be positive or even negative, giving raise to curious phenomena.

Making full use of what we've just defined, we can finally present the celebrated *equation of radiative transfer* (although in the notable absence of scattering)

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu \quad (10)$$

which is actually fairly easy to solve when one of the two coefficients vanishes.

Emission only

We set $\alpha_v = 0$ and the equation may be solved by direct integration

$$I_v(s) = I_v(s_0) + \int_{s_0}^s j_v(s') ds'$$

the result is not that interesting per se.

Absorption only

This time we set $j_v = 0$. The equation is easily solved this time as well

$$I_v(s) = I_v(s_0) \exp\left(-\int_{s_0}^s \alpha_v(s') ds'\right)$$

In this case, it's rather common to write down the equation in terms of a new variable, namely the *optical depth* τ_v

$$d\tau_v = \alpha_v ds \quad (11)$$

Given this definition we'll say that if

- $\tau_v \gg 1$: the medium is *optically thick or opaque*
- $\tau_v \ll 1$: the medium is *optically thin or transparent*

this has some crucial implications we'll be going through in a moment.

In the stationary limit, the equation of radiative transport may be written as

$$(\hat{k} \cdot \nabla) I_v(\hat{k}, \vec{x}) = j_v(\vec{x}) - \alpha_v(\vec{x}) I_v(\hat{k}, \vec{x})$$

In terms of the *source function* $S_v = j_v / \alpha_v$ it can now be written as

$$\frac{dI_v}{d\tau_v} = -I_v + S_v \quad (12)$$

which can be integrated to yield the formal solution

$$I_v(\hat{k}, \tau_v) = I_v(\tau_{v,0}) \exp(-\tau_v) + \int_{\tau_{v,0}}^{\tau_v} d\tau'_v S_v \exp(-(\tau_v - \tau'_v))$$

Assume for the moment that the matter through which radiation is passing has constant properties and has no background source. Then the source function S_v is constant and the formal equation becomes

$$I_v = I_v(\tau_{v,0}) e^{-\tau_v} + S_v (1 - e^{-\tau_v})$$

If the medium is optically thin, then the equation is reduced to

$$I_v = S_v \tau_v = j_v L \quad (13)$$

by taking the Taylor expansion of the exponential term and calling L some typical length of the medium.

If, on the other hand, the medium is optically thick, we can neglect the exponential $e^{-\tau_v}$ to obtain

$$I_v = S_v \quad (14)$$

1.5 KIRCHHOFF'S LAW AND LTE

The most notable implication of eq.14 is if we consider the specific intensity coming out of a small hole on a box kept in thermodynamic equilibrium. We know that what's going to come out of there is the blackbody radiation

$$I_\nu = B_\nu(T)$$

but what if we were to put an optically thick object just behind the hole?

If the object is in thermodynamic equilibrium with the surroundings (and it *will* be, given an appropriate amount of time), then the radiation coming out of the hole will still be blackbody radiation. But eq.14 tells us that the source function will tend to be equal to the specific intensity, hence

$$S_\nu = B_\nu(T) \quad (15)$$

which actually puts a constraint on the possible values of the emission coefficient in terms of the absorption coefficient. This is exactly what is expressed in Kirchhoff's law

$$j_\nu = \alpha_\nu B_\nu \quad (16)$$

Let us briefly consider what we have just derived. Often matter tends to emit and absorb at specific frequencies corresponding to what are commonly called *spectral lines*. We would expect then both j_ν and α_ν to have peaks (or depression) around these lines. But Kirchhoff's law forces their ratio to be equal to a smooth blackbody profile.

Thus we can expect to observe two very different scenarios if the medium is optically thin rather than optically thick. In the former, the radiation emerging from the medium is essentially determined by its emission coefficient; since j_ν is expected to present peaks, so will the radiation spectrum, which will appear in spectral lines, as shown in Fig.4 and Fig.5.

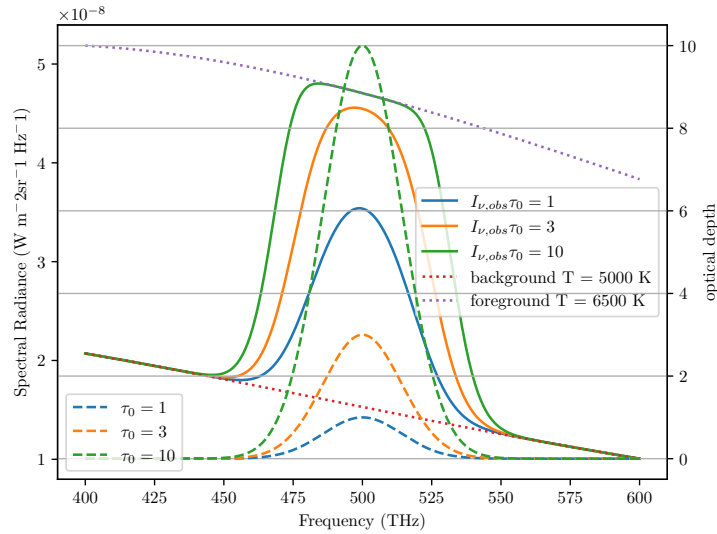


Figure 4: An example of emission features formation for different temperatures and different values of τ . Credits: Prof. Walter del Pozzo.

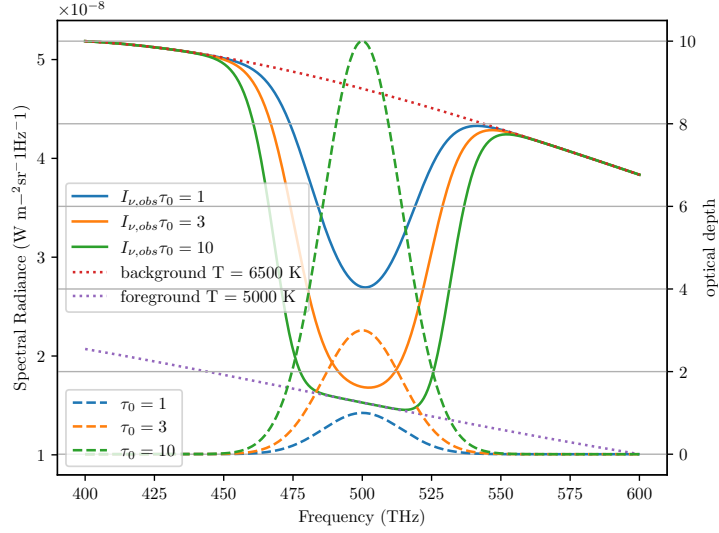


Figure 5: An example of absorption features formation for different temperatures and different values of τ . Credits: Prof. Walter del Pozzo.

On the other hand, the intensity coming out of an optically thick body is its source function, which must be equal to the blackbody function. Hence we expect the medium the emit in a continuum, pretty much like a blackbody.

All throughout this description, we've been assuming the medium to have constant properties, which has the perk of being a good approximation for many objects of interest, but still turns out to be a really poor one for many other objects. Stars, for example.

Ingenuously, we may expect stars to emit radiation like blackbodies, but they're not. Actually, stars present absorption lines—possibly many, depending on the class of star. What we cannot assume in stars is them having constant properties, starting from temperature.

In fact, we could take a guess and claim that stars are in *strict* thermodynamic equilibrium. It would be a very bad guess indeed.

1.5.1 Local Thermodynamic Equilibrium (LTE)

Let's be honest: In a realistic situation, we *rarely* have strict thermodynamic equilibrium. If a body is in thermodynamic equilibrium, we can assume a number of important physical principles to hold, like the Maxwellian distribution

$$dn_v = 4\pi n \left(\frac{m}{2\pi kT} \right)^{3/2} v^2 \exp \left(-\frac{mv^2}{2kT} \right) dv \quad (17)$$

where n is the total number of particles per unit volume and m is the mass of each particle.

Similarly, we can expect certain laws to hold, like Boltzmann's law for occupation numbers

$$\frac{n_E}{n_0} = \frac{g_E}{g_0} \exp \left(-\frac{E - E_0}{kT} \right) \quad (18)$$

and Saha's equation

$$\frac{N_{j+1}n_e}{N_j} = 2 \frac{Z_{j+1}(T)}{Z_j(T)} \left(\frac{2\pi mkT}{h^2} \right)^{3/2} \exp \left(-\frac{\chi_{j,j+1}}{kT} \right) \quad (19)$$

where n_e is the density of electrons and $\chi_{j,j+1}$ is the ionization potential. Saha's equation in particular is expected to be crucial in interpreting the effect that ionization has on the emission/absorption spectrum.

The proverbial "one-million-dollar-question" then is: When can we expect a system to be in thermodynamic equilibrium and when can we expect the previous principles to hold?

Even if the system initially does not obey the, say, Maxwellian distribution, it will eventually relax to it after undergoing some *collisions*.

Collisions are crucial in establishing thermodynamic equilibrium.

When collisions are frequent, the mean free path of particles will be small, and particles will interact more effectively. When this happens, we can expect the principles aforementioned to hold. Since we're physicists, vague sentences like "*the mean free path of particles will be small*" are destined to elicit a deep sense of unease and distress. How small does the free path have to be? One meter? Two micrometers? Below the Planck lengthscale?

When we've defined the absorption coefficient α_ν , the sharpest among my four readers total may have noticed that α_ν has the dimension of the inverse of a length. It is safe to assume that α_ν^{-1} may define some distance over which a significant fraction of the radiation would get absorbed by matter.

Such a "mean-distance" is defined in a homogeneous medium as

$$\langle \tau_\nu \rangle = \alpha_\nu l_\nu = 1$$

Thus, if l_ν is sufficiently small such that the temperature can be taken as a constant over such distance, we can safely say that the useful relations we have defined earlier still hold, although only locally.

In such a fortunate scenario, known as *Local Thermodynamic Equilibrium* (LTE), all the important laws requiring thermodynamic equilibrium are expected to hold, provided that we use the local temperature $T(\vec{x})$.

In the interiors of stars, for example, LTE will prove to be a very good approximation, that will get progressively worse as we inch towards the "surface" of the star.

2 | THE EINSTEIN COEFFICIENTS

2.1 INTRODUCTION

Coming soon.

Part II

FLUID DYNAMICS

Part III

GRAVITATION

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