NOTES OF
ASTROPHYSICAL PROCESSES

Giacomo Pannocchia

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Knowledge without understanding
Is but a sword stuck in its sheat.

Arthur Leywin

PREFACE

Not long ago, it occurred to me how cool it is when someone unexpectedly release a very detailed and all-comprehensive version of their notes, especially when dealing with a course that has a handful of really different topics that often interact together in unpredictable, yet fascinating, ways–as it's the case for the Astrophysical Processes class.

This notes will be mainly based on *my* own notes of the lectures by Professor Walter del Pozzo and Professor Marco Crisostomi during the academic year 2025-2026. Since, however, I take little to no pride in my messy notes, I'll be often using some of the many references you can find on the course catalogue page or in the bibliography of this humble collection.

You can report errors (whatever their nature might be) and suggestions for additions at g.pannocchia3@studenti.unipi.it (institutional) or by any means (conventional or not) you deem the best[1].

Without further ado, we'd better not lose much more time on a preface and get started with it.

There was Eru, the One, who in Arda is called Ilúvatar; and he made first the Ainur [...] *But for a long while they sang only each alone, or but few together, while the rest hearkened; for each comprehended only that part of the mind of Ilúvatar from which he came, and in the understanding of their brethren they grew but slowly.*

Yet ever as they listened they came to deeper understanding, and increased in unison and harmony.

Ainulindalë, "The music of the Ainur",
Silmarillion, J. R. R. Tolkien

1 I'd like, however, not to see my house stormed by homing pidgeons.

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Part I

RADIATIVE TRANSPORT

1 INTERACTION OF RADIATION WITH MATTER

1.1 INTRODUCTION

Most of our knowledge about the Universe is based on the electromagnetic radiation that reaches us from far far away. EM radiation is obviusly not the only way we can probe the Universe we live in but, in respect to neutrinos, cosmic rays or even gravitational waves, it's not a long stretch to claim it is by far the most understood.

It is most important then that an astrophysicist worthy of his (or her) name has a good grasp of the theory of radiative transfer and of its applications.

Apart from a few more key differences, I'll follow the description of radiative transfer of [1], but I won't fail to emphasize whenever I'll be doing otherwise.

1.2 RELEVANT QUANTITIES FOR RADIATIVE TRANSFER

Although some books often start their description of radiative transfer from the definition of *monochromatic energy* and *monochromatic intensity*, I found that it is most misleading, since, in all but a few cases, what we experimentally measure are fundamentally *fluxes*.

We shall then consider the *monochromatic flux* F_ν ($\mathrm{erg\,s^{-1}\,Hz^{-1}\,cm^{-2}}$) produced by some source passing through a small area $\mathrm{d}A$ located somewhere in space.

If we call $\hat{k}$ the propagation direction of the flux and $\hat{n}$ the unit vector emerging from the surface $\mathrm{d}A$, it's easy to get convinced that what is actually passing through the surface is somewhat proportional to $F_\nu(\hat{k} \cdot \hat{n})$.

From the monochromatic flux we can define the *bolometric flux*, which is just the monochromatic flux integrated over all frequencies (or wavelengths)

$$F = \int_0^{+\infty} F_\nu \,\mathrm{d}\nu = \int_0^{+\infty} F_\lambda \,\mathrm{d}\lambda \tag{1}$$

This also tells us how to convert a flux per unit frequency to a flux per unit wavelength

$$F_\nu \,\mathrm{d}\nu = F_\lambda \,\mathrm{d}\lambda$$

By now it should be clear that, despite being experimentally sensible to use the flux, we're losing much information sticking with it, namely directional information.

We consider then the amount of radiation $E_\nu \,\mathrm{d}\nu$ passing through the same area in time $\mathrm{d}t$ and solid angle $\mathrm{d}\Omega$. Hence we can write

$$\mathrm{d}E_\nu \,\mathrm{d}\nu = I_\nu(\mathbf{r}, t, \hat{k})(\hat{k} \cdot \hat{n}) \,\mathrm{d}t \,\mathrm{d}\Omega \,\mathrm{d}A \,\mathrm{d}\nu \tag{2}$$

where the quantity $I_\nu(\mathbf{r}, t, \hat{k})$ is called the *specific monochromatic intensity*. If $I_\nu(\mathbf{r}, t, \hat{k})$ is specified for all directions at every point in a certain region of spacetime, then we'd have a complete prescription of the radiation field we intend on studying.

Capitalizing on the blatant similarities with distribution functions, we can evaluate the moments of the monochromatic intensity.

Definition 1.2.1. *Monochromatic mean intensity J_ν*

$$J_\nu = \frac{1}{4\pi} \int_\Omega I_\nu \, d\Omega = \frac{c}{4\pi} U_\nu$$

with U_ν the total energy density of radiation. Note that J_ν is pretty much just an average of the monochromatic intensity over all solid angles.

Definition 1.2.2. *Monochromatic flux $\vec{F}_\nu$*

$$\vec{H}_\nu = \frac{1}{4\pi} \int_\Omega I_\nu(\hat{k}) \hat{k} \, d\Omega = \frac{1}{4\pi} \vec{F}_\nu$$

I haven't explicitly proved the last equality, but it shouldn't be hard for you to convince yourself (or prove it yourself) that it is indeed true.

Definition 1.2.3. *Monochromatic radiation pressure p_ν The monochromatic pressure is defined starting from the different directions correlations of the monochromatic intensity*

$$K_\nu^{ij} = \frac{1}{4\pi} \int_\Omega I_\nu(\hat{k}) n^i n^j \, d\Omega$$

The pressure in particular is usually expressed as

$$P_\nu = \frac{1}{c} \int_\Omega I_\nu(\hat{k}) \cos^2\theta \, d\Omega$$

where $\cos^2\theta = (\hat{k} \cdot \hat{n})^2$.

1.3 BLACKBODY RADIATION

Even at an undergraduate level, we're all fairly familiar with *blackbody radiation*. The easiest way to deduce the expression for the energy density of photons in *thermal equilibrium* (STE) inside a cavity is by the means of statistical mechanics.

Remember the Bose-Einstein distribution

$$n = \frac{1}{\exp(h\nu/kT) - 1}$$

and the phase space density of states

$$\rho(\nu) \, \mathrm{d}\nu = \frac{4\pi g \nu^3}{c^3} \, \mathrm{d}\nu$$

from which deducing the expression from internal energy is straightforward. Remembering $g = 2$ is the quantum degeneracy of photons, a simple moltiplication of the previous expressions yields

$$U_\nu \, \mathrm{d}\nu = \frac{8\pi\nu^3}{c^3} \frac{1}{\exp(h\nu/kT) - 1} \, \mathrm{d}\nu$$

Since blackbody radiation is isotropic (it actually depends only on the absolute temperature T), the definition of mean monochromatic intensity yields

$$\boxed{B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/kT) - 1}} \tag{3}$$

It's important to notice that, in principle, such a fundamental result holds only in *strict thermodynamical equilibrium* (STE), but we'll soon see how to generalize this formulation for less "restrictive" environments.

An incredible number of important results descends from (3), and it may be worthwhile to at least cite some of them, starting from Stefan-Boltzmann law.

We'll use the following result without proving it

$$\int_0^{+\infty} B_\nu(T)\,\mathrm{d}\nu = \frac{2h}{c^2}\frac{\pi^2}{15}\left(\frac{kT}{h}\right)^4$$

Computing the bolometric flux and the bolometric energy density by integrating over all frequencies using what we've just written down, you find the following

$$U(T) = aT^4 \quad F(T) = \sigma_{SB}T^4$$

Clearly the two constants a and σ_{SB} cannot be independent, and are actually related by the integral we've previously calculated. Using for example

$$F(T) = \pi\int_0^{+\infty} B_\nu(T)\,\mathrm{d}\nu$$

you can easily find out that the *Stefan-Boltzmann constant* is equal to

$$\sigma_{SB} = \frac{2\pi^5k^4}{15c^2h^3}$$

and the relation with a is simply $\sigma_{SB} = ac/4$.

The equation

$$F(T) = \frac{2\pi^5k^4}{15c^2h^3}T^4 \tag{4}$$

is what is usually known as the *Stefan-Boltzmann law*.

1.4 RADIATIVE TRANSFER EQUATION

Part II

FLUID DYNAMICS

Part III

GRAVITATION

BIBLIOGRAPHY

[1] A. R. Choudhuri. *Astrophysics for Physicists*. Astrophysics-Testbooks. Cambridge University Press, 2010.