Lab Assignment B

Multivariable Control

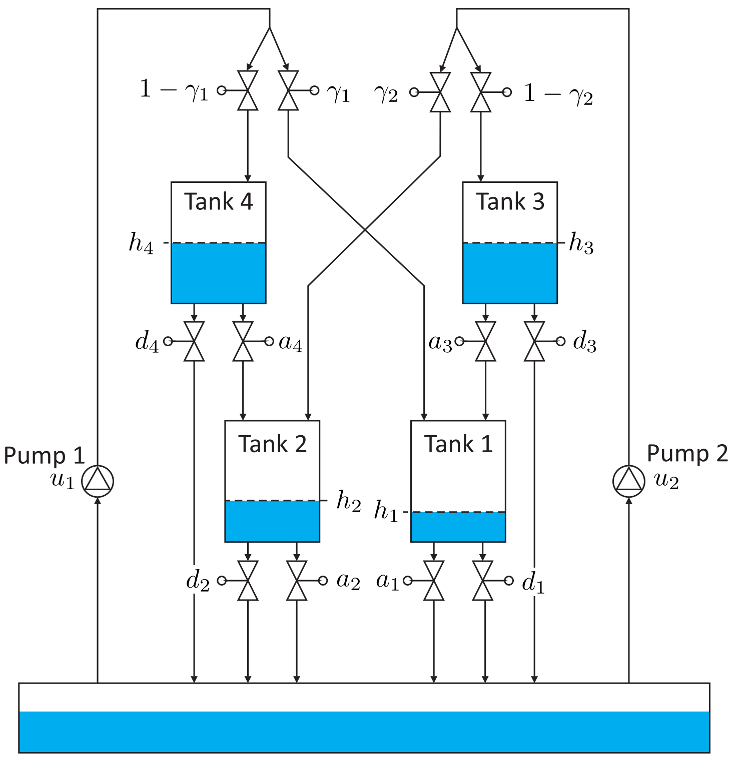
Examination: Submitted report in CANVAS.

1 Introduction

In this lab assignment we will explore multivariable control and robustness issue. Further, a comparison of the multivariable control scheme with a decentralized control scheme will be conducted. The lab assignment will be conducted in simulation on a quadruple tank process, as depicted in Fig. 1.

2 Process model

The process model for the quadruple tank process in Fig. 1 makes use of the mass balances and the simplified Bernoulli law, and can be stated as follows



Figur 1: Schematics drawing of the quadruple tank process

$$\dot{h}_1 = -\frac{a_1}{A_1}\sqrt{2gh_1} - \frac{d_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_3}\sqrt{2gh_3} + \frac{\gamma_1}{A_1}k_1u_1 \tag{1}$$

$$\dot{h}_2 = -\frac{a_2}{A_2}\sqrt{2gh_2} - \frac{d_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_4}\sqrt{2gh_4} + \frac{\gamma_2}{A_2}k_2u_2 \tag{2}$$

$$\dot{h}_3 = -\frac{a_3}{A_3}\sqrt{2gh_3} - \frac{d_3}{A_3}\sqrt{2gh_3} + \frac{1-\gamma_2}{A_3}k_2u_2 \tag{3}$$

$$\dot{h}_4 = -\frac{a_4}{A_4}\sqrt{2gh_4} - \frac{d_4}{A_4}\sqrt{2gh_4} + \frac{1-\gamma_1}{A_4}k_1u_1 \tag{4}$$

where A_i is the cross-section of tank i and a_i is the cross-section of the outlet hole. The pump models are simplified to the static gains k_1 and k_2, which map the respective input voltages u_1 and u_2 into flows q_1 and q_2. Clearly, this is not true in reality since there is a dynamic behavior present in the pump. This dynamics can be approximated with the following models:

$$\tau_1\dot{q}_1 = -q_1 + k_1u_1 \tag{5}$$

$$\tau_2\dot{q}_2 = -q_2 + k_2u_2 \tag{6}$$

For these pump models the time constants τ_1 and τ_2 are unknown, and usually assumed to be zero. The resulting flow that is generated by the pumps is then split into two flows for the respective tanks. The split ratios for the flows q_1 and q_2 are denoted γ_1 and γ_2, respectively. The acceleration of gravity is denoted g. In addition the measurement of the levels is done by a sensor which has a static gain constant k_c, and maps the level into a voltage.

A linear model for the system in transfer function form is given by

$$G(s) = \begin{bmatrix} \dfrac{\gamma_1 c_1}{T_1s+1} & \dfrac{(1-\gamma_2)c_1}{(T_1s+1)(T_3s+1)} \\ \dfrac{(1-\gamma_1)c_2}{(T_2s+1)(T_4s+1)} & \dfrac{\gamma_2 c_2}{T_2s+1} \end{bmatrix} \tag{7}$$

where

$$T_i = \frac{A_i}{a_i}\sqrt{\frac{2h_{i0}}{g}}$$

$$c_1 = \frac{T_1k_1k_c}{A_1}$$

$$c_2 = \frac{T_2k_2k_c}{A_2}$$

2.1 Model parameters

The following table summarizes the nominal model parameters

Parameter	Unit	Value
A_1, A_3	$[cm^2]$	28
A_2, A_4	$[cm^2]$	32
a_1, a_3	$[cm^2]$	0.071
a_2, a_4	$[cm^2]$	0.057
k_c	$[V/cm]$	0.5
g	$[cm/s^2]$	982

The disturbance inputs d_i are adjustable in the range $[0, a_i]$.

2.2 Operating conditions

There are several operating conditions that can be explored with this setup, but there are two primary operating conditions which have very different behavior. These operating conditions are the defined by variables and parameters.

Variable	Unit	Range	P_-	P_+
(h_{10}, h_{20})	$[cm]$	$[0, 20]$	$(12.3, 12.8)$	$(12.6, 13.0)$
(h_{30}, h_{40})	$[cm]$	$[0, 20]$	$(1.6, 1.4)$	$(4.8, 4.9)$
(u_{10}, u_{20})	$[V]$	$[0, 10]$	$(3.0, 3.0)$	$(3.15, 3.15)$

Parameter	Unit	P_-	P_+
k_1	$[cm^3/(Vs)]$	3.33	3.14
k_2	$[cm^3/(Vs)]$	3.35	3.29
γ_1	$[-]$	0.70	0.43
γ_2	$[-]$	0.60	0.34

2.3 Uncertainties

As we know the system is affected by uncertainty. We assume a parametric uncertainty for the parameters a_1, a_2, γ_1, and γ_2, which means the values for the parameters will be in an interval.

Parameter	Unit	Intervall
a_1	$[cm^2]$	$\pm 10\%$
a_2	$[cm^2]$	$\pm 10\%$
γ_1	$[-]$	$[0.56, 0.70]$
γ_2	$[-]$	$[0.48, 0.60]$

3 Problem description

Your task is to design and implement a multivariable control scheme which keeps the level h_1 in tank 1 and the level h_2 in tank 2 at a desired level using the two pumps u_1 and u_2, despite disturbances in the form of leakages represented by d_1, d_2, d_3, and d_4. The resulting design should also be compared with a decentralized control scheme, which treats tank 1 and tank 2 as two systems without couplings.

In addition, the model of the quadruple tank process is to some degree uncertain, which means that parameters in the model are not exactly known or that certain components are not modeled correctly. Modeling errors can be due to approximations, conscious simplifications, but also due to unknown properties of a system.

The controller design should therefore consider the uncertainties, but not necessarily use robust control design methods. This means that the controller design should fulfill a robustness criteria.

4 Required tasks

Task 1: Development environment

The lab assignment will be performed in a simulated environment, where the designed controller will be tested and evaluated. For this end a development environment is needed. You have received a simulink model that contains the complete model.

The development environment shall implement the model equations above with the following exogenous inputs u_1, u_2, d_3, and d_4.

Note: Check that when you add d_1 and d_2 the system became unobservable. Can you guess why from physical point of view?

It is preferred that the model is structured and that the model parameters are accessible through variables on the work space. Thereby, it would be possible to update them programmatically, when robustness is analyzed later. For this you need to replace the hard coded parameters in the mask (block dialog window) with the parameters that you use on your work space.

It is also important that the model can be initialized to a certain operating point, or even that this can be chosen by the user. This makes the use later on more flexible.

Create a script that makes use of the parameters and generates the linear model for the process.

Task 2:Calculate poles and zeros of the system(s)

The system will have a number of poles and zeros. Calculate the poles and zeros of both P_- and P_+. Also, Calculate the value of γ_1 and γ_2 that will yield a minimum phase system.

Task 3: Decentralized control for P_-

Design a decentralized controller for the levels h_1 and h_2, which controls the levels individually. You are not allowed to use h_3 and h_4 as measurable signals. You may use your preferred design scheme for the individual controllers, but we suggest to try a new design scheme, like for example the IMC design methodology as described in section 8.3 of *Control Theory*. The resulting controller should be a PI controller.

Implement the controller in the development environment and test the control scheme for the following scenarios:

- Immediate step changes in h_{R1} and h_{R2} with an amplitude of 2 cm

- Immediate steps in d_1 and d_2 of at most 25% of their range.

Task 4: Multivariable control for P_-

Design a multivariable LQG controller for the levels h_1 and h_2. You are not allowed to use h_3 and h_4 as measurable signals. Thus, the system shall be treated as a 2×2 system. The controller shall be tuned for two cases:

1. Both levels h_1 and h_2 respond equally fast. Tune the controller such that the system is as fast as possible, while not saturating.
2. Level h_1 is tracked fast, while the level in h_2 is allowed to deviate more. Thus, make use of u_1 and u_2 to enable a faster tracking and disturbance rejection of h_1.

The following operational and disturbance scenarios have to be tested and analyzed:

- Immediate step changes in h_{R1} and h_{R2} with an amplitude of 2 cm
- Immediate steps in d_1 and d_2 of at most 25% of their range.

Steps that need to be performed during the design.

1. Realize the linear model as a state space model.
2. Augment the state space model such that the resulting state feedback will contain integral action.
3. Implement the LQG (including the Kaalm,an filter)controller with the integral action.
4. Test the controller for the above scenarios.

Task 5: Performance analysis

The performance of the control schemes from task 3 and task 4 can be evaluated using the RMSE on the control error, as well as settling time and overshoots. For this end you have to log the control error for the different operational and disturbance scenarios.

Draw conclusions on the performance of the different control schemes.

Task 6: Robustness analysis

In accordance with section 6.3 an uncertainty model has to be derived, which takes the following form

$$G_0 = (I + \Delta_G)G$$

There an upper bound for Δ_G has to be found which captures all possible model errors. For this end a large number of G_0^i has to be defined where the parameters $\gamma_1^i,\ \gamma_2^i,\ a_1^i$, and a_2^i are randomly picked from the respective interval.

Now a Δ_G is designed which creates an upper bound for the four bode magnitude plots of all the $(G_0^i - G)G^{-1}$. The chosen transfer functions should have a low order.

Perform the robustness analysis for the multivariable control scheme by evaluating

$$||\Delta_G T||_\infty < 1$$

Draw conclusions closed loop system's robustness towards uncertainties.

5 Optional tasks

Task 7: Practical robustness analysis

For each of the randomly picked parameter sets in task 5, a simulation run can be made and the performance can be evaluated. Create a script which enables you to do the following in an automated way:

- Randomly pick a parameter value for each parameter and store them. The number of picks need to be adjustable.
- Run a simulation in Simulink with the closed loop system for the currently picked parameters
- Quantify the performance of the closed loop systems from task 3 and 4, and store it
- Determine the worst case performance and the best case performance
- Generate plots for the performance in relation to the parameter variations

Note: In case that the closed loop system looses stability, then this need to be detected and stored.

Do the results reflect the outcome from task 5?

Task 8: Decentralized control for P_+

Redo the design of the decentralized control system for the operating point P_+. Is it possible to attain similar performance for control scheme as it was achieved in task 4?

Task 9: Multivariable control for P_+

Redo the design of the multivariable control system for the operating point P_+, but choose only the case where both levels are tuned equally fast.

- Is it possible to attain similar performance for control scheme as it was achieved in task 4? - Reflect also on the cause!
- Is it possible to outperform the decentralized control scheme from task 5? - How much better are we doing?