1

If I bid x, the expected payoff

is $x \times x + 0 = x^2$

$\therefore$ I should bid 1.

12

$$\frac{1}{5}(2+3+4+5+6) = \underline{\underline{4}}$$

③

$999 — \left(\frac{1}{2}\right)^{10}$

$1 — 1$

$$Pr(\text{The penny is fair}) = \frac{\frac{999}{1000}\left(\frac{1}{2}\right)^{10}}{\frac{999}{1000}\left(\frac{1}{2}\right)^{10} + \frac{1}{1000} \times 1}$$

$$= 49\%$$

$Pr(\text{the penny is two-headed}) = 1 - 49\% = 51\%$

14

$$\frac{3! + 2}{4!} = \frac{1}{3}$$

5

10% infected $\langle$ PPN : ${}_3C_2 \times 0.95^2 \times 0.05$

90% not infected $\langle$ PPN : ${}_3C_2 \times 0.01^2 \times 0.99$

$\therefore$ p (infected | PPN)

$$= \frac{0.1 \times {}_3C_2 \times 0.95^2 \times 0.05}{0.1 \times {}_3C_2 \times 0.95^2 \times 0.05 + 0.9 \times {}_3C_2 \times 0.01^2 \times 0.99}$$

$= \underline{\underline{98\%}}$

(6)

$X \sim B(1600, \frac{3}{4})$

$\sim N(1200, 300)$

$P(X < q - 200) < 0.1$

$P(Z < \frac{q - 1400}{\sqrt{300}}) < 0.1$

$\therefore \frac{q - 1400}{\sqrt{300}} < -1.282$

$\therefore q \leq \underline{\underline{1377}}$

7

(a) There is no φ_1 and φ_2 satisfying

$$\begin{pmatrix} 100 \\ 70 \\ 180 \end{pmatrix} = \begin{pmatrix} 124 & 71 \\ 83 & 61 \\ 92 & 160 \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$$

$\therefore$ There is arbitrage opportunity

(b) Long $7.94 \times B$ and Short $4.57 \times A$, $1 \times C$.

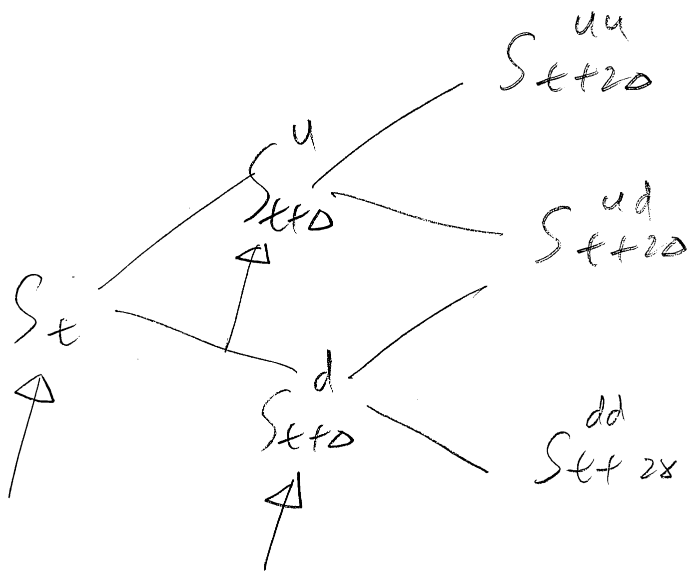
(c) $$\left\{ \begin{pmatrix} A \\ B \\ C \end{pmatrix} \middle| \begin{pmatrix} 124 & 71 \\ 83 & 61 \\ 92 & 160 \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}, \varphi_1, \varphi_2 > 0, \varphi_1 + \varphi_2 = \frac{1}{1+r} \right\}$$

(d) $70(1 + r\Delta t)$

18 (a)

$$\begin{pmatrix} 1 \\ S_t \\ C_t \end{pmatrix} = \begin{pmatrix} 1+r\delta & 1+r\delta \\ S^u_{t+\delta} & S^d_{t+\delta} \\ C^u_{t+\delta} & C^d_{t+\delta} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

(b) 3 three-equation systems including above one.



⑨

(a) $u = e^{\sigma\sqrt{\Delta}}$

$1.15 = e^{\sigma\sqrt{\frac{1}{12}}}$

$\therefore \sigma = 0.48$

(b)

$t=0$	$t=1$	$t=2$	$t=3$	$t=4$
				174.9
			152.1	
		132.3		132.3
	115		115	
100		100		100
	87		87	
		75.6		75.6
			65.8	
				57.2

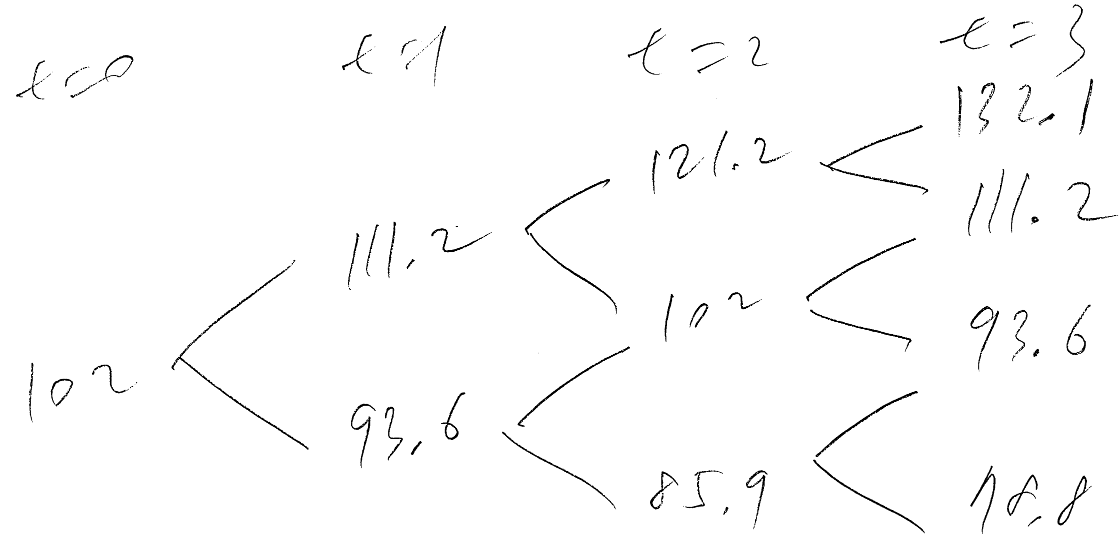
(c) Risk-neutral (upside) $p = \dfrac{1+\frac{0.05}{12} - d}{u-d} = 0.48$

$$\therefore \frac{(174.9-100)}{1+\frac{0.05}{12}\times 4} \times p^4 + \frac{(132.3-100)}{1+\frac{0.05}{12}\times 4} \times \left(4p^3(1-p)\right)$$

$$= \underline{\underline{11.22}}$$

$\boxed{10}$ -1

(a) $u = e^{\sigma\sqrt{\Delta t}} = e^{0.3\sqrt{\frac{1}{12}}} = 1.09$



Replicating portfolio

At t=0: Borrow 14.88 and long 0.159 stocks.

At t=1: Price down → do nothing

Price up → borrow 16.02 and buy 0.144 stocks

(long 0.303 stocks, balance 30.9)

At t=2: Price down → do nothing

Price up → long 0.579 stocks, balance 64.38

(b) 1.34

$\boxed{10}$ — 2

(c) $\underline{\underline{(a) \times 100}}$

(d) Sell call and do (a) using 1.34

Profit : 3.66

$\boxed{11-2)}$ -1

(a) $5\delta = \frac{200}{365} \Rightarrow \delta = \frac{40}{365}$

(b) $u = e^{\sigma\sqrt{\delta}} = e^{0.12\sqrt{\frac{40}{365}}} = 1.04$

$d = \frac{1}{u} = 0.961$

(c) $p = \frac{1 + r\delta - d}{u - d} = 0.577$

(d)

t=0	t=1	t=2	t=3	t=4	t=5
					121.7
				117	
			112.5		112.5
		108.2		108.2	
	104		104		104
100		100		100	
	96.1		96.1		96.1
		92.4		92.4	
			88.8		88.8
				85.3	
					82

11-2) −2

(e) Calculate backward from $t=5$.

$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$
					21.7
				17.69	
			13.86		12.5
		10.48		8.85	
	7.69		6.04		4
5.51		4.01		2.29	
	2.61		1.31		0
		0.75		0	
			0		0
				0	
					0

12

$$\begin{pmatrix} 100 \\ 70 \\ 180 \\ x \end{pmatrix} = \begin{pmatrix} 120 & 70 & 80 \\ 80 & 60 & 50 \\ 90 & 150 & 190 \\ 30 & 20 & 30 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$$

From first three rows,

$$\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} = \begin{pmatrix} 120 & 70 & 80 \\ 80 & 60 & 50 \\ 90 & 150 & 190 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 100 \\ 70 \\ 180 \end{pmatrix} = \frac{1}{247} \begin{pmatrix} 65 \\ 94 \\ 129 \end{pmatrix}$$

$$\therefore x = 30\psi_1 + 20\psi_2 + 30\psi_3 = \underline{\underline{31}}$$

13

(a) 0

(b) 1

(14)

$$E(L_n) = \frac{n}{{}_{2n}C_2}\left(1 + E(L_{n-1})\right) + \left(1 - \frac{n}{{}_{2n}C_2}\right)E(L_{n-1})$$

$$= E(L_{n-1}) + \frac{n}{{}_{2n}C_2} = E(L_{n-1}) + \frac{1}{2n-1}$$

$$E(L_1) = 1.$$

$$\therefore E(L_{100}) = \sum_{n=1}^{100} \frac{1}{2n-1} \approx 3.28$$

15

a) It's a fair game.

$p(\text{survival}) = \frac{1}{2}$

b) I should go first.

$$p(\text{survival}) = \frac{\frac{1}{6}}{1 - \frac{25}{36}} = \frac{6}{11}.$$

c) I should spin.

$p(\text{survival} \mid \text{spin}) = \frac{1}{5}$.

$$p(\text{survival} \mid \text{not spin}) = \frac{\frac{4}{{}_6C_2}}{\frac{4}{6}} = \frac{2}{5}$$

d) I should spin

$p(\text{survival} \mid \text{spin}) = \frac{1}{5}$

$$p(\text{survival} \mid \text{not spin}) = \frac{\frac{1}{6}}{\frac{4}{6}} = \frac{1}{4}$$