Impulse function (5.7) & Convolution (5.8)

Solve the following initial value problems.

1. $y'' - y = -20\delta(t - 3)$, $y(0) = 4$, $y'(0) = 4$

2. $y'' + y = \delta(t - \pi/2)\dfrac{1 + \cos t}{1 + t^2}$, $y(0) = 0$, $y'(0) = 0$

3. Find the inverse Laplace transform using the convolution theorem: $\dfrac{s}{(s+1)(s^2+4)}$

4. Find the Laplace transform of $f(t) = \frac{1}{240}\int_0^t (t-\tau)^6 \sin(4\tau)d\tau$.