Midterm 2 Review Problems

1. Use the method of variation of parameters to solve

$$2y'' - 3y' + y = \frac{6e^{\frac{t}{2}}}{1 + e^t}.$$

2. $y'' + 4y' + 4y = 8t^2 + 10\cos(4t) + e^{-2t}$, $y(0) = -1$, $y'(0) = 2$. Solve the initial value problem by the method of undetermined coefficients.

3. Consider the initial value problem $y''(t) + 2y'(t) + y(t) = 8e^{3t}$, $y(0) = -3$, $y'(0) = 2$.

- Transform the above initial value problem into an algebraic equation for $Y(s) = \mathcal{L}\{y(t)\}$.
- Find $Y(s)$; i.e., solve the equation in the s-domain.
- Find the solution of the initial value problem by using $y(t) = \mathcal{L}^{-1}\{Y(s)\}$.