Shifted Systems

1. Consider $\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} x_1 - 2 \\ x_2 - 3 \end{bmatrix}$.

a. Find all equilibria.

b. Find general solutions.

c. Solve under the initial condition $\vec{\mathbf{x}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

d. Sketch the phase portrait.

e. Determine whether each equilibrium is stable, asymptotically stable or unstable.

2. Consider $\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 1 & -2 \end{bmatrix} \vec{\mathbf{x}} + \begin{bmatrix} -2 \\ 2 \end{bmatrix}$.

a. Find all equilibria.

b. Convert the system to the form $\frac{d\vec{\mathbf{x}}}{dt} = A(\vec{\mathbf{x}} - \vec{\mathbf{a}})$.

c. Find general solutions.

d. Sketch the phase portrait.

e. Determine whether each equilibrium is stable, asymptotically stable or unstable.

3. We have a system of two interconnected tanks initially with Tank A holding 20 L of pure water and Tank B holding 30 L of pure water. A brine with salt concentration 3 g/L flows into Tank at a rate of 1.5 L/min, and the solution in Tank A is discharged from the system at a rate of 0.8 L/min. A brine with salt concentration 7 g/L flows into Tank B at a rate of 0.5 L/min, and the solution in Tank B is discharged from the system at a rate of 1.2 L/min. The two tanks are interconnected by two pipes: through one of the pipes the solution flows from Tank A to Tank B at a rate of 1 L/min, and through the other pipe the solution flows from Tank B to Tank A at a rate of 0.3 L/min. Denote by $Q_1(t)$ the amount of salt in Tank A, and by $Q_2(t)$ the amount of salt in Tank B.

a. Write down differential equations and initial conditions for $Q_1(t)$ and $Q_2(t)$.

b. Find the general solutions of the system of differential equations.