Midterm 1 Review Problems

1. A tank initially contains 100 gallons of brine with 10 lb of salt dissolved in it. Brine containing 0.4 lb of salt per gallon flows from an outside source into the tank at a rate of 5 gal/min. The mixture is discharged out of the tank at the same rate of 5 gal/min.. Assume that solutions are well-stirred at all times.

a. Write the initial value problem for Q(t).

b. Find the amount Q(t) of salt in the tank at time t $\geq$ 0.

c. Find the limit of Q(t) as $t \to \infty$.

2. Consider the equation $\frac{dy}{dt} = -0.5y^2(y^2+1)(y^2-4)(y+1)e^{\sin(y)}$.

a. Find all equilibrium solutions.

b. Sketch the phase portrait.

c. Determine whether each equilibrium is stable, asymptotically stable, or unstable.

3. $\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 3 & 10 \\ -5 & -7 \end{bmatrix} \begin{bmatrix} x_1 - 1 \\ x_2 + 2 \end{bmatrix}$

a. Find all equilibria;

b. Solve the initial value problem with initial condition $x_1(0) = 2$, $x_2(0) = 1$;

c. Sketch the phase portrait, identify the type of each equilibrium, and determine the stability of each equilibrium.