Lattice-Field Medium (LFM): Core Equations and Theoretical Foundations

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Note: This version supersedes all prior releases (v1.x and v2.x) and adds No-Derivatives restrictions and defensive-publication language for intellectual property protection. All LFM Phase-1 documents are synchronized under this unified v3.0 release.

Abstract

This document defines the governing equations of the Lattice-Field Medium (LFM) and their continuum, discrete, and variational forms. It establishes the connection between the lattice update law and the variable-mass Klein–Gordon equation, outlines how Lorentz invariance emerges naturally in the continuum limit, and shows how quantization and gravitational analogues arise through the curvature field $\chi(x,t)$.

1 Introduction and Scope

The Lattice-Field Medium (LFM) treats spacetime as a discrete lattice of interacting energy cells. Each cell holds an energy amplitude $E(x,t)$ and curvature parameter $\chi(x,t)$. The purpose of this document is to define the mathematical foundation of LFM, connecting the discrete rule to its continuum form and providing validation targets used in Tier 1–3 testing.

2 Canonical Field Equation

The canonical continuum form of the LFM equation is:

$$\partial^2E/\partial t^2 = c^2\, \nabla^2E - \chi^2(x,t)\, E, \qquad \text{with} \quad c^2 = \alpha/\beta.$$

Here $E(x,t)$ is the local field energy, $\chi(x,t)$ is the curvature (effective mass), and c is the lattice propagation speed.

3 Discrete Lattice Update Law

We use a second-order, leapfrog scheme consistent with the canonical field equation

$$\partial^2E/\partial t^2 = c^2 \nabla^2E - \chi(x,t)^2\, E, \quad \text{with} \quad c^2 = \alpha/\beta.$$

where ∇_Δ^2 is the finite-difference Laplacian, $\gamma \geq 0$ is optional numerical

damping ($\gamma = 0$ for conservative runs), and $\chi(x,t)$ may be a scalar or a spatial field.

$$E^{t+1} = (2 - \gamma)\, E^t - (1 - \gamma)\, E^{t-1}$$

$$+ (\Delta t)^2\, [\, c^2\, \nabla\_\Delta^2\, E^t - \chi(x,t)^2\, E^t\,],$$

1D Laplacian (order-2):

$$\nabla\_\Delta^2 E\_i = (E\_\{i+1\} - 2E\_i + E\_\{i-1\}) / (\Delta x)^2$$

1D Laplacian (order-4):

$$\nabla\_\Delta^2 E\_i = [-E\_\{i+2\} + 16E\_\{i+1\} - 30E\_i + 16E\_\{i-1\} - E\_\{i-2\}] / (12 (\Delta x)^2)$$

Multi-D:

- 2D supports order-2 and order-4.
- 3D currently supports order-2 only (order-4/6 reserved for future tiers).

Boundary options (per test): periodic (canonical), reflective, or absorbing.

No stochastic (η) or exogenous coupling ($\Delta\varphi$) terms are part of the canonical law.

4 Derived Relations and (Continuum vs Lattice)

Continuum dispersion (χ constant):

$$\omega^2 = c^2 k^2 + \chi^2$$

Lattice dispersion (order-2 1D; used in Tier-1 validation):

$$\omega^2 = (4 c^2 / \Delta x^2) \sin^2(k \Delta x / 2) + \chi^2$$

Energy monitoring (numerical):

We track relative energy drift $|\Delta E| / |E_0|$ and target $\leq 10^{-6} \ldots 10^{-4}$ depending on grid and BCs.

Exact conservation holds in the continuum; simulations measure small drift.

Quantized exchange (interpretive):

$\Delta E = n\ \hbar\_\text{eff}$ with $\hbar\_\text{eff} = \Delta E\_\text{min}\ \Delta t$ arising from discrete time; this is interpretive, not an input law.

Cosmological feedback:

Terms such as $E_{t+1} = E_t + \alpha\nabla^2 E - nH E$ belong to higher-tier χ-feedback studies and are not part of the canonical kernel.

5 Analogues (Non-canonical, exploratory)

Electromagnetic and inertial behaviours can be constructed as analogues of the canonical kernel, but they are not part of it.

The following discrete Maxwell-like updates are included for context only and belong in Appendix A (Analogues).

Discrete EM Coupling (Eq. 5-1, 5-2):

$E_{I,t+1} = E_{I,t} + \alpha(\varphi_{i+1,t} - \varphi_{i-1,t}) - \beta B_{I,t}$

$B_{I,t+1} = B_{I,t} + \beta(\varphi_{i+1,t} - \varphi_{i-1,t}) + \alpha E_{I,t}$

6 Lorentz Continuum Limit

Starting from the discrete update rule and applying Taylor expansion in time, the LFM equation reduces to:

$$\partial^2 E/\partial t^2 = c^2 \nabla^2 E, \qquad \text{with} \quad c^2 = \alpha/\beta.$$

This form is invariant under Lorentz transformations, demonstrating that relativity emerges naturally from local lattice dynamics.

Formally, this corresponds to the joint limit $\Delta x, \Delta t \to 0$ (with $c = \Delta x/\Delta t$ fixed), where $\Sigma E_i \Delta x \to \int E(x)\, dx$ over $(-\infty,+\infty)$.

7 Quantization from Discreteness

Quantization arises from the finite time-step Δt. The minimal exchange of energy per step defines $\hbar_{eff} = \Delta E_{min} \Delta t$. The energy–frequency relation becomes $E = \hbar_{eff}\, \omega$, and the momentum–wavelength relation $p = \hbar_{eff}\, k$, reproducing the de Broglie relation.

8 Dynamic χ Feedback and Cosmological Scaling

The curvature field χ evolves according to the feedback law:

$$d\chi/dt = \kappa(\rho_{ref} - \rho_E) - \gamma\, \chi\, \rho_E.$$

This rule produces self-limiting cosmic expansion and links local energy density to curvature dynamics.

Edge-creation condition:
if $|\partial E/\partial r| > E_{th} \to$ new cell at boundary.

This mechanism replaces the classical singular Big Bang with a deterministic expansion cascade.

9 Variational Gravity for χ

Promoting χ to a dynamic field yields coupled Euler–Lagrange equations:

$$\sigma\_\chi(\partial_t^2\chi - v\_\chi^2\nabla^2\chi) + V'(\chi) = g\_\chi E^2 + \kappa\_EM(|\mathfrak{E}|^2 + c^2|\mathfrak{B}|^2).$$

In the weak-field limit, $\nabla^2\Phi = 4\pi G\_eff\rho\_eff$ reproduces Newtonian gravity and redshift/lensing analogues.

10 Numerical Stability and Validation

CFL stability (d spatial dimensions):

$c\,\Delta t / \Delta x \le 1 / \sqrt{d}$ (d = 1, 2, 3)

Energy diagnostics:

Measure $|\Delta E| / |E_0|$ each run; typical tolerances $\le 10^{-6} - 10^{-4}$ depending on Δx, Δt, stencil order, and boundary conditions.

Stencil availability:

1D / 2D → order-2 and order-4; 3D → order-2 only (order-4 / 6 reserved for future tiers).

Test alignment:

Tier-1 uses the lattice dispersion relation above;

Tier-2 uses static χ(x) gradients;

Tier-3 evaluates energy drift under conservative settings.

11 Relation to Known PDE Classes

PDE Class	Canonical Form	Relation to LFM	Reference
Klein–Gordon	$E\_tt - c^2\nabla^2E + m^2E = 0$	LFM with constant χ	—
Variable-mass KG	$E\_tt - c^2\nabla^2E + \chi(x,t)^2E = 0$	Identical continuum form	Ebert & Nascimento (2017)
Helmholtz	$\nabla^2u + k\_eff^2(x)u = 0$	Time-harmonic analogue	Yagdjian (2012)

Quantum-walk lattices	Discrete Dirac/KG	Emergent Lorentz symmetry	Bisio et al. (2015)

12 Summary and Outlook

The Lattice-Field Medium provides a deterministic, Lorentz-symmetric framework where quantization, inertia, gravity, and cosmic expansion emerge from one discrete rule. All formulations preserve conservation, isotropy, and CPT symmetry. Tier 1–3 validations confirm numerical stability and physical coherence, forming the foundation for higher-tier exploration.

The canonical PDE remains fixed across all tiers; all higher-tier phenomena emerge from this equation without modification.

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