Closed- versus open-loop active vibration control in the presence of finite precision arithmetic

Martina Maggio, Alberto Leva, Luigi Piroddi

***Abstract*— A closed-loop and an open-loop active vibration control technique are critically compared, accounting for the effects of finite precision arithmetic. The main result is that the relevance of such effects is pointed out and characterised, as rigorously as possible at the present state of the research, in a suitably idealised context, and with specifically devised indicators. Also, the differences between the compared methods, and in particular the different relevance of finite precision-related facts, are put to evidence and synthetically investigated. The ideas proposed herein can be extended to other types of vibration control methods, the final goal of such an overall research being a general framework to study and compare those methods, in suitably defined nominal conditions, and accounting for realistic implementation facts.**

I. Introduction

Active control of vibrations is a well-established research area, as witnessed by a huge literature, examples of which are [1], [2], [3], [4]. The numerous approaches attempted are historically classified in two broad categories, namely "closed-loop" and "open-loop". To give a quick picture, in the closed-loop context the main idea is to set up a feedback system, the loop gain of which is as large as possible at the frequency/ies of the vibration(s) to eliminate, and as small as possible at any other frequency. This is typically achieved by means of anti-notch filters, possibly adaptive if the vibration spectrum is unknown and/or time-varying. Closed-loop techniques are quite simple from a computational standpoint, but may pose significant stability problems, especially if the number of filters is high and/or the behaviour of the controlled system is not easily modelled (particularly at high frequencies).

Open-loop techniques of the adaptive type considered herein, on the other hand, try to eliminate the unwanted vibration(s) by generating an anti-vibration signal, with the same amplitude of the undesired one, and opposite phase. This can be accomplished in several ways, the Filtered-X Least Mean Squares (FXLMS) method [2] being probably the most widely employed. Open-loop methods tend to be more complex to implement than closed-loop ones, but are far less likely to produce stability problems. Quite intuitively, several papers were published that test both types of methods in various contexts. It is not possible here to do justice to so vast a literature, some samples of which (chosen among the most relevant for the particular scope of this work) are e.g. [5], [6], [7], [8]. It is however worth noticing that, apart to some extent from the examples above, it is rare to find works that, irrespective of their particular scope and purpose, include in the treatise the effects of finite precision arithmetic.

M. Maggio is a former student at the Dipartimento di Elettronica e Informazione, A. Leva (corresponding author) and L. Piroddi are with the Dipartimento di Elettronica e Informazione, Politecnico di Milano, Italy {leva,piroddi}@elet.polimi.it;.

This manuscript proposes such a comparison, in the simplest available context. In detail, a single-frequency, non-adaptive notch-based closed-loop vibration canceller is compared to its FXLMS-based counterpart, in totally nominal conditions. Even in so simple a comparison setup, some interesting considerations emerge.

II. The comparison problem statement

The purpose of this work is to compare two vibration control frameworks from the point of view of the effects of finite precision. To achieve meaningful (and methodologically sound) results, it is apparently necessary to adopt an extremely simplified comparison setup, though preserving the relevant facts. In extreme synthesis, an answer is sought here for the following two questions.

- In the adopted comparison setup, do closed- and open-loop schemes differ, owing (essentially) to finite precision arithmetic, in terms of the achievable vibration attenuation?
- In the same conditions, do closed- and open-loop schemes differ in terms of the proximity of their behaviour to that of their infinite-precision (ideal) counterparts (having agreed some indicators of such proximity)?

To proceed, we first need to define the comparison setup, then to briefly discuss the possible effects of finite precision *in a way abstracted from the comparisons treated herein*, and finally to devise the required indicators.

III. The comparison setup

Owing probably to the numerous and heterogeneous domains and cultures interacting in the vibration control arena, the used terminology can sometimes be ambiguous. We therefore spend some words to agree the notation used herein (apologising in advance to readers with expertise in the field).

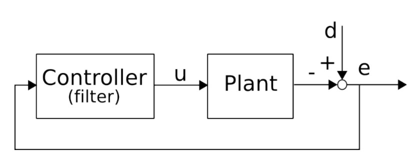

Fig. 1. Closed-loop vibration control scheme.

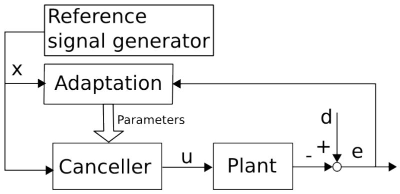

Fig. 2. Open-loop (adaptive) vibration control scheme.

The most general closed- and open-loop (adaptive) vibration control schemes are shown in figures 1 and 2, respectively. In the control-theoretical jargon, both would be termed "closed-loop", since the former actually encompasses a control loop in the strict sense of the term, whereas the latter includes an *adaptation* loop. Assuming however a stationary problem, and the convergence of the adaptation algorithm, the latter scheme tends to be open-loop, whence the name adopted in the vibration control field. Obviously, stability of the plant dynamics is a necessary requirement for open-loop control.

In the context of this research, the "plant" is described by a transfer function $P(z)$ in the $\mathcal{Z}$-transform domain. That transfer function accounts for the dynamics between the "control signal" $u(k)$ and the "residual vibration" (or "error") $e(k)$, both treated as signals in the discrete time k. Finally, $d(k)$ is "the disturbance", i.e., a discrete-time signal modelling the ultimate (external) cause of the unwanted vibration. In such a context, the vibration canceller takes the form of a linear filter in the closed-loop context, and of a signal generator in the open-loop one. The adaptation mechanism, when present, has the purpose of determining the filter or generator parameters, based on the necessary measurements.

A. Closed-loop scheme

In this work, a fixed-parameters, single-frequency notch-based canceller is taken as the representative of the closed-loop approach. Such a canceller is here obtained by using an extremely standardised technique, namely by discretising the continuous-time anti-notch filter

$$R_c(s) = \frac{1}{1 + w\frac{\xi}{\omega_d}s + \frac{s^2}{\omega_d^2}} \tag{1}$$

with the pole-zero matching method, and the sampling time denoted by T_s. Doing so yields the discrete-time regulator

$$R(z) = \frac{\mu_R(z+1)}{z^2 - 2\rho_R\cos(\theta_R)z + \rho_R^2} \tag{2}$$

where

$$\begin{aligned} \rho_R = e^{-\xi\omega_d T_s}, \quad \theta_R = \omega_d T_s\sqrt{1-\xi^2} \\ \mu_R = \frac{1}{2}\left(1 - 2\rho_R\cos(\theta_R) + \rho_R^2\right) \end{aligned} \tag{3}$$

Having fixed the disturbance frequency ω_d, that is assumed in the following as known, and the sampling time T_s (or, more significantly, the product of the two), the only canceller parameter is the continuous-time damping factor ξ, or its discrete-time counterpart provided by the pole radius ρ_R. That parameter determines at the same time the attenuation level, and the distance of the canceller from the stability limit—a relevant fact, as will emerge in the following. As a final remark, direct-form realisation was used here. Of course other types of realisation may affect the results—for instance, the delta form is being considered for further research. However the system (2) is structurally so simple that the effects of changing the realisation type can be expected of modest entity.

B. Open-loop scheme

The open-loop framework, conversely, is represented in this work by the FXLMS-based adaptive notch filter [2] shown in figure 3.

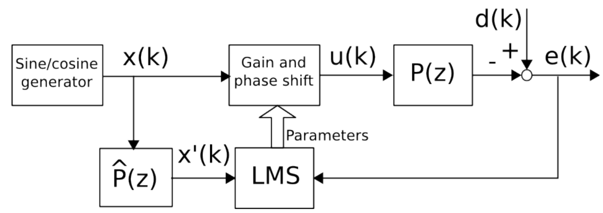

Fig. 3. Open-loop vibration control (via FXLMS): basic scheme.

In the scheme of figure 3, the "gain and phase shift" block modifies the amplitude and phase of the generated reference sine/cosine wave(s) $x_{s,c}(k)$, that must cover the frequency/ies of the disturbance d to be cancelled. Typically, at each frequency ω a sine and cosine wave $x_s(k)$ and $x_c(k)$ are generated, and the "gain and phase shift" block therefore computes the component $u(k)$ of its output at the frequency ω as

$$u(k) = w_s x_s(k) + w_c x_c(k). \tag{4}$$

The "weights" w_s and w_c are computed by the LMS block with an updating law of the type

$$w_{s,c}(i+1) = w_{s,c}(i) + \mu x'_{s,c}(i)e(i), \tag{5}$$

where $x'_{s,c}$ are the sine and cosine reference signals filtered by some estimate $\widehat{P}(z)$ of the dynamics $P(z)$ (reasonably valid at the frequencies of interest), μ is a parameter and i the index of the adaptation steps (that may not equal the control step, related to the temporal index k).

To generate the required sine and cosine signal, the oscillator described by

$$\begin{array}{rcl} q(k) & = & 2\cos(\omega_d T_s)q(k-1) - q(k-2) \\ x_s(k) & = & \sin(\omega_d T_s)q(k-1) \\ x_c(k) & = & q(k-2) - \cos(\omega_d T_s)q(k-1) \end{array} \tag{6}$$

is implemented, where y_s and y_c are respectively the sine and cosine outputs in the discrete time k, ω_d the frequency (of the vibration to be cancelled), and T_s the sampling time. The two past values of the state variable q are initialised as $q(-1) = A$ and $q(-2) = 0$, A being the required oscillator's output amplitude (the unity in the examples of this work, for simplicity and uniformity). Also in this case, the technique used to set up the canceller is absolutely standard, see e.g. [2], [1], [9] for discussions on the matter.

C. Final considerations

For simplicity, but without loss of conceptual generality, the following comparisons are in totally nominal conditions, and abstracted as much as possible from the peculiarities of any particular control problem, i.e.,

- with $\widehat{P}(z) = P(z)$, and with $P(z) = 1$, to enhance the significance of the comparison by abstracting from any particular dynamics (notice that in more complex cases only the values of the frequency response of P at the controlled frequencies come into play, so as far as stability is preserved and only steady-state attenuation is considered, generality is not violated here),
- with a disturbance composed of a single sinusoid of *known* frequency ω_d,
- and in the absence of adaptation, i.e., with the closed-loop regulator (2) tuned to ω_d, and the sine/cosine generator of the open-loop FXLMS regulator emitting the frequency ω_d.

The free parameters are therefore ξ (or, equivalently, ρ_R) in the closed-loop case, and μ in the open-loop one. Finally, for simplicity, the FXLMS sine/cosine generation and weights update steps are taken coincident.

An important remark is now in order. In both schemes, it is expected that y vanishes after an initial transient. It makes therefore sense to compare the achieved "final" attenuation, i.e., the limit of the ratio between the amplitude of y and that of d for $k \to \infty$.

It must be noted, however, that the schemes are very different in nature, and so different is the reason for the initial transient, even in the absence of adaptation, that is the case addressed herein. In the closed-loop (notch) case, that transient comes from the motion of a linear dynamic system, time-invariant, asymptotically stable, subject to sinusoidal input, and with an initial state in general different from the corresponding periodic generator (hence we assume zero initial state, for simplicity and uniformity). In the open-loop (FXLMS) case, conversely, the system is inherently nonlinear, and the transient comes from the dynamics of the weights as dictated by (5).

It would therefore make little sense to compare the initial transients of the two schemes. Far more significant is to compare each of the two schemes with its "ideal" counterpart, i.e., with the scheme as it would be in the presence of infinite-precision arithmetic.

IV. Effects of finite precision

Some effects of finite precision arithmetic, such as quantisation of the involved signals and filter coefficients and roundoff errors in the numerical computations, are well known [2]. An example is the "detuning" of notch filters, that owing to the imprecise evaluation of ρ_R and θ_R in (3), get tuned to a slightly different frequency than ω_d, to the obvious detriment of attenuation.

In this research, we focus however also on a seldom studied aspect, namely the inherent nonlinearity of the sum and product operators when implemented in finite precision arithmetic. That matter was previously discussed in [10], to which the interested reader is referred. We report here only the most relevant facts for the purpose of this work. Assuming that finite precision numbers are represented in an IEEE754-like format, a (floating point) finite precision arithmetic $\mathcal{A}$ is characterised by the numbers m and e of mantissa and exponent bits, that define the set $\mathbb{F}_{m,e}$ of finite precision numbers, and by the functions $\overline{f}_{\oplus}(\cdot,\cdot) : \mathbb{F}_{m,e} \times \mathbb{F}_{m,e} \mapsto \mathbb{F}_{m,e}$ and $\overline{f}_{\otimes}(\cdot,\cdot) : \mathbb{F}_{m,e} \times \mathbb{F}_{m,e} \mapsto \mathbb{F}_{m,e}$ that realise the finite precision sum and product in $\mathbb{F}_{m,e}$.

We denote the arithmetic under question by $\mathcal{A}(m, e, \overline{f}_{\oplus}, \overline{f}_{\otimes})$, and so define an '$\mathcal{A}$-FPLTI' dynamic system as the (nonlinear) dynamic system obtained from the LTI one

$$\begin{cases} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{cases} \tag{7}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$, and (A, B, C, D) are real matrices with the convenient dimensions, by substituting the $(+, \times)$ arithmetics over $\mathbb{R}$ with the arithmetic $\mathcal{A}(m, e, \overline{f}_{\oplus}, \overline{f}_{\otimes})$, i.e.

$$\begin{cases} \overline{x}(k+1) &= \overline{A} \otimes \overline{x}(k) \oplus \overline{B} \otimes \overline{u}(k) \\ \overline{y}(k) &= \overline{C} \otimes \overline{x}(k) \oplus \overline{D} \otimes \overline{u}(k) \end{cases} \tag{8}$$

where $\overline{x} \in \mathbb{F}^n_{m,e}$, $\overline{u} \in \mathbb{F}^m_{m,e}$, $\overline{y} \in \mathbb{F}^p_{m,e}$, and $(\overline{A}, \overline{B}, \overline{C}, \overline{D})$ are matrices of $\mathbb{F}_{m,e}$ numbers. We call (7) the 'LTI system', and (8) its '$\mathcal{A}$-FPLTI counterpart'.

In the floating point arithmetic theory and technology, it is assumed that the $\oplus$ and $\otimes$ operators adhere to the "floating point computation axiom" [11], i.e., that the relative error introduced by those operators is bounded in magnitude by the so-called "machine epsilon", defined as the largest positive number ϵ_m such that $1 \oplus \epsilon_m/2 = 1$. Plenty of literature is available that analyses the properties (notably, the stability) of algorithms under the hypothesis that the axiom holds true. The scope of this research is lateral with respect to that mainstream literature, and can be synthesised by saying that we are addressing the following two cases.

Case 1. The operators $\oplus$ and $\otimes$ of the used arithmetic $\mathcal{A}(m, e, \overline{f}_{\oplus}, \overline{f}_{\otimes})$ fulfil the axiom, yet the relevance of the introduced errors (which are only *bounded* by ϵ_m) may jeopardise the "correct operation" of a regulator designed as LTI, and realised as $\mathcal{A}$-FPLTI.

Case 2. It is not possible, or not convenient, or merely not easy enough, to guarantee that $\oplus$ and $\otimes$ fulfil the axiom: this is for example the case when floating point computations are emulated by hardware architectures designed for a specific purpose. In such situations, not so infrequent in high-demanding vibration control applications, employing the minimum sufficient precision can save time and/or silicon area, and the quest for that "minimum sufficient precision" is an important part of the overall controller design and implementation. The interested reader can find examples of such problems in [12], [13], while [14], [15] contain the description of a high-performance, FPGA-based vibration canceller, the design and realisation of which put most of the problems addressed here to evidence.

We would like to spend some further words to stress the importance of case 2. Suppose that an *ad hoc* canceller has to be designed, including the necessary arithmetics, e.g., because an ASIC (Application-Specific Integrated Circuit) implementation is necessary. In principle, in such a situation, also an arithmetic not fulfilling the axiom may "do the job" for the particular application considered; what is required is simply that the dynamics of the "difference" between the so obtained $\mathcal{A}$-FPLTI system and its LTI linear model does not diverge, and maintains the behaviour of the former "close enough" to the latter. Being capable of deciding whether or not the designed arithmetic results in an acceptable $\mathcal{A}$-FPLTI system is apparently very relevant for the design process, and from a merely engineering, implementation-oriented standpoint, it is not the same problem as guaranteeing the fulfilment of the floating point computation axiom.

To address the problems sketched above, it is not enough to describe "precision" with truncations, roundoffs and so forth. Those descriptions are perfectly adequate to qualify the behaviour of *operators*, and to ground the analysis of *algorithms*, but when it comes to assess the properties of *dynamic systems*, much more is required. To study the matter rigorously, one should ideally re-formulate as much as possible of the LTI systems theory in the $\mathcal{A}$-FPLTI context. This is apparently a formidable task, and far beyond the present state of the research. In this work, we therefore just face the questions of section II, with the definition just given, by devising some indicators that measure, in a data-based (and therefore experiment-related) but objective way, the proximity of an $\mathcal{A}$-FPLTI system $\overline{\mathcal{S}}$ to the LTI system $\mathcal{S}$ it is meant to realise. After doing so, we apply the proposed indicators to the comparison of closed- and open-loop vibration cancellers (using the representatives).

V. Comparative indicators

The rationale of the proposed indicators is to evaluate the proximity of an $\mathcal{A}$-FPLTI system $\overline{\mathcal{S}}$ to its LTI $\mathcal{S}$ counterpart. Given the linear nature of the latter, and therefore (with a slight abuse of notation) the "almost" or "ideally linear" character of the former, the indicators are based on the outputs of the LTI and the $\mathcal{A}$-FPLTI system. For systems that approach or touch the stability boundary, suitable indicators are

- the sum η of squared differences between the responses of $\overline{\mathcal{S}}$ and $\mathcal{S}$, from the initial time up to a number N of samples sufficient to represent the transient response under question (for vibration cancellers, from the time when the canceller is activated up to the time when a pre-specified vibration attenuation is reached);
- the total number Φ of samples for which the responses of $\overline{\mathcal{S}}$ and $\mathcal{S}$ have different sign (recall that in this contexts we are invariantly talking about highly oscillatory dynamics and therefore responses);
- the maximum number J of *subsequent* samples for which the responses of $\overline{\mathcal{S}}$ and $\mathcal{S}$ have different sign.

It is worth noticing that, to compute the indicators, in principle one needs infinite precision computations. This is not possible, of course, and therefore a software library was created with an *ad hoc* number representation, capable of emulating any possible operator algorithm with any (finite but arbitrarily fine) precision. The library allows to compute at precisions that are far higher than those typically employed, such as the IEEE standards, and from an operational point of view can be taken as "infinite" (actually 63 bits were used). Notice that doing so is not a theoretical flaw, since to evaluate a precision versus a better one it is only necessary that the latter allows to represent numbers that do not have a representation in the former, irrespective of whether the "better" precision is actually infinite or not.

VI. Comparisons and remarks

This section reports an example of the comparison test introduced above. In the reported example, a (floating point) disturbance signal with an angular frequency of 1/20 (corresponding e.g. to a disturbance frequency of 500 Hz and a sampling frequency 0f 10 kHz) was fed to the closed-loop (notch-based) and the open-loop (FXLMS-based) schemes illustrated above, with $P(z) = 1$.

The pole radius ρ_R of the closed-loop canceller was set to 0.999, while its angular frequency θ_R was tuned to 1/20; the weights' adaptation gain μ of the FXLMS was set to 2^{-16}, while also the sine/cosine oscillator was tuned to an angular frequency of 1/20.

The numbers above were selected so as to perform the comparison in nominal condition, as previously said, and also to have more or less the same final vibration attenuation and the same duration for the cancellation transient in the closed- and open-loop schemes, in order to enhance the significance of the following comparisons. Also, with the selected numbers there is no *apparent* degradation of the $\mathcal{A}$-FPLTI responses with respect to their LTI counterparts. In fact, we are not interested here in studying what happens when the $\mathcal{A}$-FPLTI system shows e.g. unstable behaviours; the goal of this comparison is that *even when the effects of finite precision definitely elude human sight*, the same effects are already of very different entity in the closed- and open-loop cases.

Figures 4, 5, and 6 report respectively the behaviour of the η, J, and Φ indicators versus the precision level, i.e., the number of mantissa bits, in the closed-loop case. Figures 7, 8, and 9 report the same indicators, also in this case versus the precision level, for the open-loop case.

The depicted results are definitely self-explanatory. Nonetheless, some remarks are worth reporting.

In the first place, the absolute value of all the indicators is invariantly much better in the open-loop case, and frequently it is "better" by orders of magnitude, as shown herein. Incidentally, that is the reason why the presented figures report absolute values, and not (for example) values normalised with respect to that obtained with the best available precision.

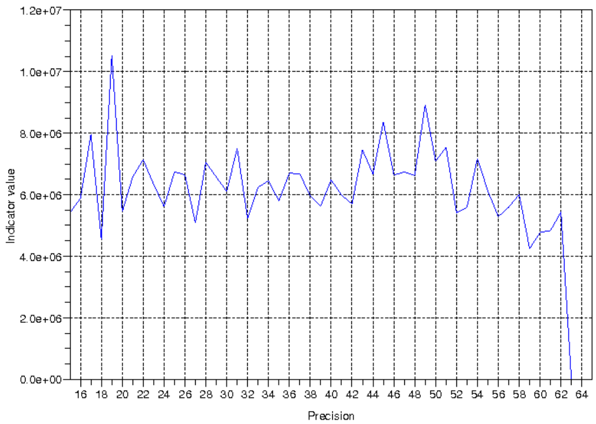

Fig. 4. Closed-loop (notch) vibration control test: the η indicator versus precision (number of mantissa bits).

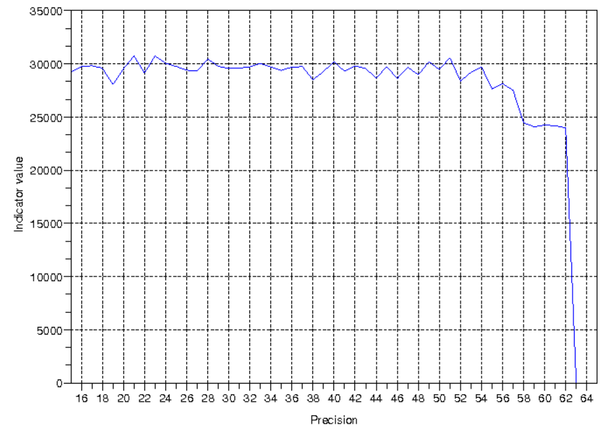

Fig. 6. Closed-loop (notch) vibration control test: the Φ indicator versus precision (number of mantissa bits).

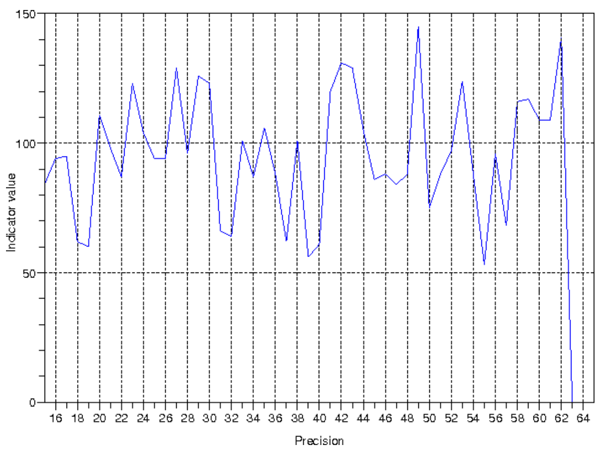

Fig. 5. Closed-loop (notch) vibration control test: the J indicator versus precision (number of mantissa bits).

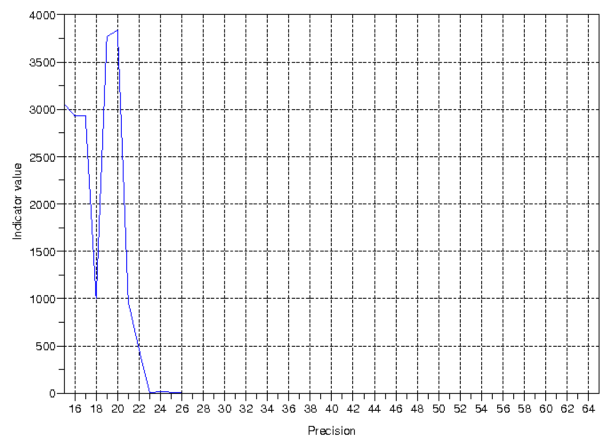

Fig. 7. Open-loop (FXLMS) vibration control test: the η indicator versus precision (number of mantissa bits).

Secondly, even disregarding for a while the absolute values of the indicators, there are apparent differences between the closed- and the open-loop cases even with respect to the degradation of those indicators when the precision level is decreased.

For the closed-loop scheme, it is quite easy to figure out some "threshold" precision that causes an abrupt degradation of the scheme's performance (see figures 4 through 6). Beyond that threshold, the measure of the performance degradation can be slightly different depending on which indicator is considered (consider e.g. figure 4 versus figure 6).

Also for the open-loop scheme some "threshold" precision can be defined, but that precision turns out to be invariantly (and significantly) coarser then the same threshold precision for the closed-loop scheme (see for the open-loop case figures 7 through 9). In addition, for precisions just slightly lower than the threshold, more graduality can (as a tendency) be observed for the performance degradation than in the closed-loop case. Also for the open-loop scheme, the description of the performance degradation for precisions lower than the identified threshold depends on the particular indicator considered.

The above results appear to vote unanimously for the open-loop (FXLMS-based) approach versus the closed-loop (notch-based) one. Of course, one could object that the presented results were obtained in a highly idealised context, and therefore are not guaranteed to carry over to "real-life" cases.

In the humble opinion of the authors, however, the ideality of the comparison setup adopted herein *enhances* the strength of the results. Avoiding a tedious and inessential description of a huge design and experimental work, the experience summarised in [14], [15] has demonstrated that there exists at least one category of *industrial applications* where the open-loop approach is invariantly superior, namely those applications in which the dynamics of $P(z)$ is (probably) high-order, not lowpass-like in the control relevant-band, and above all so dependent on the particular plant conditions that "you just tighten or loosen a bolt, and everything changes". In such cases, employing a closed-loop scheme results in very hard stability problems, to say nothing about robustness; using an open-loop scheme is conversely far less critical. Seeing the same situation arise in a completely idealised

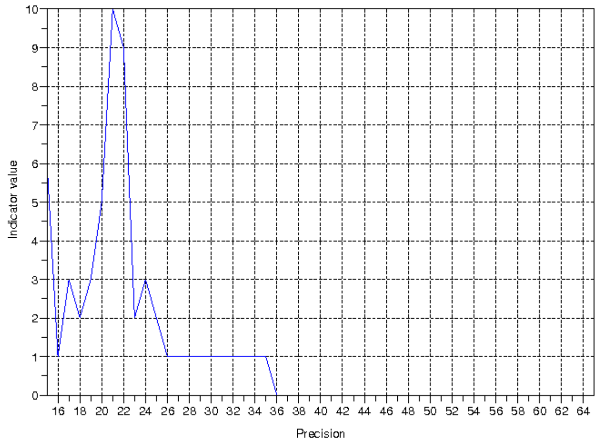

Fig. 8. Open-loop (FXLMS) vibration control test: the J indicator versus precision (number of mantissa bits).

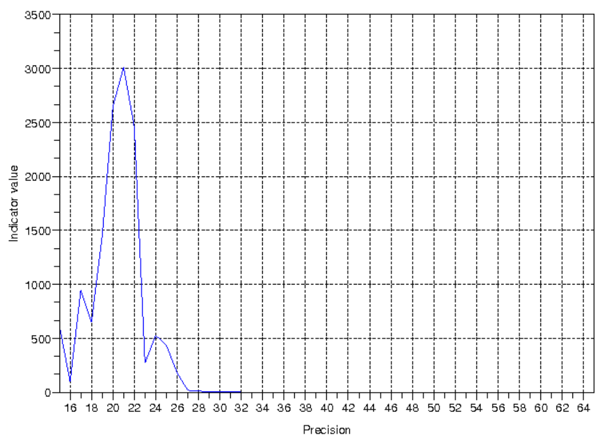

Fig. 9. Open-loop (FXLMS) vibration control test: the Φ indicator versus precision (number of mantissa bits).

setup significantly confirms the idea that the superiority of the open-loop scheme is somehow structural, in a sense still to be fully qualified, but for sure not only due to the high uncertainties on the process dynamics so frequently encountered in practical applications.

As a final point, a couple of additional facts (the illustration of which would exceed space limitations) are worth noticing. First, the comparison results above are strengthened by the proximity of the canceller to the stability limit (which backs up the conjecture formulated in [10]). Second, in the open-loop context, performance immediately degrades if the oscillator precision is affected, while if the precision of the weights' adaptation is reduced, the performance detriment is of much lesser entity. A detailed presentation and comments on the facts above are deferred to future works.

VII. Conclusions

A closed- and an open-loop vibration control technique, each one taken as a representative of the two approaches based on the results of a huge literature and many applications, were critically compared. The qualifying points of the reported comparison are that a suitably idealised (or "nominal") comparison setup was devised, and that the comparison does explicitly account for the effects of finite precision arithmetic.

The main result of the reported tests, based on performance indicators specifically devised for the evaluation of the proximity of vibration control systems to their ideal (infinite-precision) counterpart, is that the relevance of finite precision effects is very (and structurally) different in the closed- and open-loop cases.

The ideas, comparison setup, and considerations proposed herein can be extended, in quite a straightforward way based on the devised indicators or similar ones, to other types of vibration control methods. The final goal of the research is a general framework to study and compare such methods, accounting for realistic implementation conditions, and allowing to derive effective design clues, useful in particular for applications where the computational machinery required for vibration control has to be implemented, owing to high performance request, with some specialised technology.

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