A Combinatorial Algorithm for Large-Scale Power System Islanding

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***Abstract*—Intentional Controlled Islanding (ICI) is an online measure employed to prevent cascaded system outage after a disturbance in the power system. By switching off select lines, the system operator can create smaller, easier to control islands. An algorithm for ICI should be fast to implement in real time, as well as capable of integrating islanding requirements such as coherency of the generators in an island and minimum disruption of the power balance caused by the switching of lines. In this work, we approximate the solution to a common ICI formulation by employing a combinatorial approximation scheme of the normalized cut. This approach is easily implementable and numerically robust, exhibits high computational efficiency and allows for a natural integration of islanding requirements (such as generator minimum load and inflexible lines) into the problem solution. Experimental results on systems with up to 3000 buses verify the effectiveness of our approach.**

***Index Terms*—Controlled Islanding, Power Systems Resilience, Graph Partitioning, Topology Optimization, Minimum Cut, Maximum Flow, Combinatorial Optimization**

I. Introduction

Mitigating the impacts of an extended outage is among the main characteristics of a resilient grid. Intentional Controlled Islanding (ICI) is a well studied technique employed to prevent widespread blackout after a large scale grid disturbance. The idea is that, by disconnecting branches of the power system, the operator can create small islands that are stable or easily controllable. That way, a cascaded outage can be avoided. Since time is of the essence after a power disturbance, any algorithm employed for intentional islanding needs to be robust and executable in real time, while also resulting in an effective islanding scheme.

A common objective for the islanding is to split the power grid into islands that only contain coherent generators. Coherent generators are generators whose phase angle difference does not change much after a disturbance, i.e. generators that swing together. The goal is to eliminate interarea oscillations, which are a common cause of blackouts. More specifically, interarea oscillations occur when two incoherent generators (or groups of generators) swing against each other after a disturbance, at frequencies of 1Hz or less, leading to large power variations in the tie-line [1]. If the system also suffers from insufficient oscillation damping, this power variation can lead to an extended blackout. Hence, by disconnecting these two groups of generators from each other, the operator may prevent a cascaded outage.

The idea that generator coherency with respect to the slowest modes (which are the ones responsible for interarea oscillations) can be used for determining an islanding scheme appeared in some of the seminal works in the field [2], [3]. Generator coherency with respect to the slowest modes has been associated with weak coupling between the state variables of the generators belonging to incoherent sets [4], [5]. This gives rise to a generator islanding scheme based on minimizing the coupling between generators in different islands. In [6], bipartitioning of the generators into two coherent groups is formulated as a normalized cut problem [7] and approximated through solving a generalized eigenvector problem and a clustering problem based on the second eigenvector. Similar approaches are used in [8]–[10].

Following the generator grouping, the specific set of lines to switch off in order to create islands that contain the corresponding generator groups needs to be defined. To that end, the set of lines is commonly chosen to minimize the total power imbalance (i.e absolute value of algebraic sum of power flows of the switched lines) or the power flow disruption (i.e. sum of absolute values of the power flows of the switched lines). This can be achieved through variations of constrained spectral clustering [6], [8], through mixed integer programing [11], or through graph cuts [2]. Often, grid simplification and aggregation steps are required to ensure computational efficiency [12].

While the aforementioned schemes are the most dominant in literature, there are also many approaches that utilize different techniques and include further islanding considerations. Among them, a submodular optimization problem is formulated in [13], an efficient multilevel graph partitioning algorithm is used in [14], and multiple mixed integer programming models have been proposed [15]–[18].

In this work, a normalized cut problem that combines generator coherency and minimum power flow disruption is formulated. The formulation can allow for the integration of further islanding constraints, such as inflexible lines, minimum generator limits, or forcing components of the system to belong to the same island. An adaptation of the approximation algorithm proposed in [19], [20] is employed. The resulting scheme runs at the complexity of a minimum cut, which is generally faster and more numerically robust than eigenvector computations. The efficiency of the approach allows it to run fast even for large scale systems, obviating the need

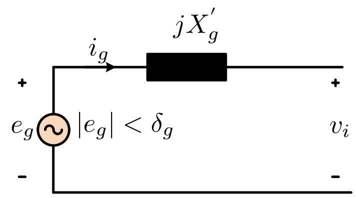

Fig. 1: Classical transient generator model. The internal generator node, of voltage e_g, is connected to the terminal (power system) node $i \in N$, of voltage v_i, through a transient reactance $X_g^{'}$. The transient internal voltage e_g is calculated based on the steady state operation using the transient reactance. Following that, the magnitude $|e_g|$ is assumed constant and the angle δ_g is initialized based on the steady state value $\hat{\delta}_g$ and then follows the dynamics of the differential equations (5).

for grid simplification. The performance in terms of the two objectives of the islanding problem (coherency and power flow disruption) is better or comparable to the spectral clustering approach performance.

The rest of this paper is organized as follows: section II presents the background to setup the problem, section III presents the problem formulation and the algorithm employed, section IV shows experimental results, and section V draws conclusions and defines directions of future work.

II. Controlled Islanding Objectives

In this section we review the necessary background for the problem formulation, i.e. the logic behind the two main objectives considered in this work (generator coherency and minimal power flow disruption).

A. Generator Coherency

We briefly describe a typical model for small signal system analysis to motivate the discussion on coherent generator sets. The analysis examines two instances of the system: the pre-disturbance system and the post-disturbance system. The pre-disturbance system is assumed in steady state. Its state is calculated through the power flow equations and is used for initializing the transient phenomena. After the disturbance, the transient dynamics are captured through the second order differential equations of the generators (swing equations), as well as the power flow equations.

The generators are represented using a classical model, as shown in Fig 1. Each generator g in the set of generators G is modeled through an internal node with complex transient voltage e_g, that is connected to the power system bus (terminal generator node) through a transient reactance $X_g^{'}$. The magnitudes $|e_g|$ of internal voltages e_g are constant throughout the transient phenomenon, based on the assumption of constant flux linkage in the machine. The phase angles δ_g of the internal voltages, on the contrary, are the state variables of our system. The coupling between the phase angle responses from different generators will eventually be the criterion used for grouping generators in coherent sets.

A number of approaches can be used for load modeling [21], but a typical one for stability studies is to represent the load as a constant impedance. For a given node, if the pre-disturbance load active and reactive power P_D, Q_D and the load bus voltage V_D are known, the impedance is calculated by:

$$Y_D = \frac{P_D - jQ_D}{|V_D|^2} \tag{1}$$

and is assumed constant in the post-disturbance system.

We can now form a generalized $(|G|+|N|) \times (|G|+|N|)$ admittance matrix $\boldsymbol{Y}$, that considers the buses (set N) as well as the internal generator nodes (therefore takes into account the load impedances, the generator transient reactances, and the rest of the system). The following equality holds:

$$\begin{bmatrix} \boldsymbol{i}_G \\ \boldsymbol{i}_N \end{bmatrix} = \boldsymbol{Y} \begin{bmatrix} \boldsymbol{e}_G \\ \boldsymbol{v}_N \end{bmatrix} = \begin{bmatrix} \boldsymbol{Y}_{GG} & \boldsymbol{Y}_{GN} \\ \boldsymbol{Y}_{NG} & \boldsymbol{Y}_{NN} \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_G \\ \boldsymbol{v}_N \end{bmatrix} \tag{2}$$

where $\boldsymbol{e}_G$, $\boldsymbol{i}_G$ are the $|G|$-dimensional complex vectors of voltage and current injections in the internal generator nodes and $\boldsymbol{v}_N$, $\boldsymbol{i}_N$ the $|N|$-dimensional vectors of voltage and current injections in the buses of the power system. Note that, due to the fact that loads are modeled through constant impedances (included in the matrix $\boldsymbol{Y}$) and generators are modeled through additional nodes connected to the buses through constant reactances, the current injections for all the buses of the power system are zero $\boldsymbol{i}_N = \boldsymbol{0}$. Following that, by using Kron reduction [22], we eliminate the variables $\boldsymbol{v}_N$ and obtain the $|G| \times |G|$ effective admittance matrix $\boldsymbol{Y}^{'} = \boldsymbol{Y}_{GG} - \boldsymbol{Y}_{GN}\boldsymbol{Y}_{NN}^{+}\boldsymbol{Y}_{NG}$ (where $\boldsymbol{Y}_{NN}^{+}$ the Moore-Penrose inverse), which satisfies $\boldsymbol{i}_G = \boldsymbol{Y}^{'}\boldsymbol{e}_G$. The imaginary part of this matrix is usually dominating the real part in terms of order of magnitude, so a common assumption is to neglect it and consider: $\boldsymbol{Y}^{'} \approx -j\boldsymbol{B}^{'}$, where the effective transient susceptance matrix $\boldsymbol{B}^{'}$ is assumed real symmetric with its off diagonal entries non negative.

Restricting ourselves to the reduced network that only contains the internal generator nodes, notice that for $g, g' \in G$, the nodes g and g' are connected through a branch of reactance $1/\boldsymbol{B}^{'}_{gg'}$, therefore the active power transfer from g to g' is:

$$P_{gg'} = |e_g||e_{g'}|\boldsymbol{B}^{'}_{gg'} \sin(\delta_g - \delta_{g'}) \tag{3}$$

and the total active power that a generator g sends to the grid is equal to

$$P_g^e = \sum_{g' \in G, g' \neq g} P_{gg'} \tag{4}$$

If for a generator g this power is not equal to the mechanical input of the generator, P_g^m, the imbalance will cause a change in the internal voltage phase angle according to the swing equation for that generator (with inertia constant H_g, angular frequency ω_0, damping neglected):

$$\frac{2H_g}{\omega_0}\ddot{\delta}_g = P_g^m - P_g^e \tag{5}$$

Substituting (3) and (4) into (5), we get a set of $|G|$ second order equations for the $|G|$ dimensional vector of internal generator angles $\boldsymbol{\delta}$ (state variables). By linearizing the system around the pre-disturbance operating point $\hat{\boldsymbol{\delta}}$ we obtain:

$$\boldsymbol{M}\ddot{\boldsymbol{\delta}} = \boldsymbol{K}\boldsymbol{\delta} \tag{6}$$

where the g, g' entry of the $|G| \times |G|$ matrix $\boldsymbol{K}$ for $g \neq g'$ equals:

$$\begin{aligned} \boldsymbol{K}_{gg'} &= -\frac{\partial P_{gg'}}{\partial \delta_{g'}}\bigg|_{\substack{\delta_g=\hat{\delta}_g \\ \delta_{g'}=\hat{\delta}_{g'}}} \\ &= |e_g||e_{g'}|\boldsymbol{B}^{'}_{gg'} \cos\left(\hat{\delta}_g - \hat{\delta}_{g'}\right) \end{aligned} \tag{7}$$

and for $g = g'$ equals $\boldsymbol{K}_{gg} = -\sum_{g'' \in G, g'' \neq g} \boldsymbol{K}_{gg''}$. The $|G| \times |G|$ matrix $\boldsymbol{M}$ is diagonal with $\boldsymbol{M}_{gg} = \frac{2H_g}{\omega_0}$ for all $g \in G$. Note for the linearization that the internal voltage magnitudes $|e_g|$ are constants (equal to their pre-disturbance value) and so is the mechanical input of each generator P_g^m (equal to the electrical output of the generator before the disturbance, where the system was at steady state).

Two generators are characterized as "coherent" if their internal voltage angle difference (which is a function of time) does not change much after a disturbance. Therefore, coherent generators swing together and can be aggregated in transient system simulations. While there are many formal definitions of coherency, one that has particularly nice structural properties characterizes two generators as coherent with respect to a subset of the modes of the system of differential equations (6), if none of these modes are observable from the voltage angle difference. In [5], coherency with respect to the slowest modes is related to small values of a scalar quantity ζ that depends on the off-diagonal entries of the matrix $\boldsymbol{K}$. More specifically, for the case of partitioning the generator set G into two sets of coherent generators V_G and $\bar{V}_G = G \setminus V_G$, we have:

$$\zeta(V_G) = \frac{\sum_{(gg') \in \delta(V_G)} \boldsymbol{K}_{gg'}}{\sum_{g \in V_G} \boldsymbol{M}_{gg}} + \frac{\sum_{(gg') \in \delta(V_G)} \boldsymbol{K}_{gg'}}{\sum_{g \in \bar{V}_G} \boldsymbol{M}_{gg}} \tag{8}$$

where $\delta(\cdot)$ is the undirected cutset of a set and its complement, i.e. $\delta(V_G)$ contains all pairs (g, g') with one generator in V_G and one in $\bar{V}_G$.

It has been verified that splitting the post-disturbance grid based on groups of coherent generators leads to stable islands and prevents fault propagation [23], [24]. Furthermore, an objective similar to (8) has been recognized as a normalized graph cut [7] for the bipartition of the generator set in [6]. Since solving the minimum normalized graph cut problem is NP-hard, a spectral clustering approximation algorithm based on a generalized eigenvalue problem was obtained in [6]. Partitioning of the grid into more than two islands can be accomplished, if necessary, by repeating the same procedure for the resulting islands.

B. Minimal power flow disruption

A common objective when identifying a set of lines to switch off in order to isolate coherent generator groups is that of minimal power imbalance. If the set of buses N is partitioned into the sets S and $\bar{S} = N \setminus S$, the power imbalance is calculated by:

$$\sum_{(i,j) \in \delta(S)} |P_{ij}| \tag{9}$$

where P_{ij} the active power on the transmission lines between buses i and j (algebraic sum for the case of multiple lines or flow directions). The idea behind this penalty is that we seek to remove lines in a way that causes minimum change from the pre-disturbance power flows within the resulting islands. The advantages and disadvantages of using this objective have been extensively examined in literature [6].

III. A Combinatorial Algorithm for Optimal Islanding

A. Problem Formulation

We formulate an optimal islanding problem that incorporates both generator coherency and power imbalance with a trade-off, i.e. the problem of interest is:

$$\underset{S \subseteq N}{\text{minimize}} \quad \frac{\sum_{(ij) \in \delta(S)} \boldsymbol{W}_{ij}}{\sum_{i \in S} \boldsymbol{Q}_{ii}} + \frac{\sum_{(ij) \in \delta(S)} \boldsymbol{W}_{ij}}{\sum_{i \in \bar{S}} \boldsymbol{Q}_{ii}} \tag{10}$$

In the equation above, the weights are defined for nodes $i, j \in N$, $i \neq j$:

$$\boldsymbol{W}_{ij} = \sum_{(g,g') \in (G(i) \times G(j))} \boldsymbol{K}_{gg'} \; + \; \lambda \, |P_{ij}| \, \mathbb{I}_E\{(ij)\} \tag{11}$$

where $G(i)$ denotes the set of generators connected to node i, λ a trade-off coefficient (which is the only tuning parameter of the optimization problem), and $\mathbb{I}_E$ is the indicator function of the set of undirected branches E. Note that the weight W_{ij} can be nonzero only if both buses i and j have a connected generator or if there is a branch in the power system connecting i and j. The balancing weights for a node $i \in N$ are given below (zero if $G(i) = \emptyset$).

$$\boldsymbol{Q}_{ii} = \sum_{g \in G(i)} \boldsymbol{M}_{gg} \tag{12}$$

The output of the optimization problem (10) is an optimal partitioning of the nodes S. All the lines in the cutset $\delta(S)$ will be switched off to create (at least) two islands. If further partitioning of the grid is required, the optimization can be formulated for each of the remaining islands. Note that lines that are inflexible (cannot be remotely switched off) can be assigned a large weight $\boldsymbol{W}_{ij}$, which will ensure that they will not belong to the cutset.

Another critical concern for stable islands is that a generator operates in a more stable fashion if its generation exceeds a minimum, which means that one or more load nodes should belong to the same island as this generator. We can easily force the generator to belong to the same island as a load node by assigning large weights to the edges in one or more paths between them. These paths and load nodes can be efficiently found by graph search algorithms (such as Breadth First Search) around the generators.

The alternative, two step approach to formulate the problem, common in literature, would be to pick the partition of the generators in the first step and in the second step the optimal set of lines to switch off to minimize power flow disruption in a way that respects the generator grouping. The techniques described in what follows can be used in this two-step setting as well. However, the same result can be simulated using the single step optimization problem (10) by picking a small trade-off coefficient for the power flow disruption: The optimal generator grouping will be selected based only on generator coherency (dominant terms), and then (since there can be multiple ways to select lines to switch off and achieve the same generator grouping), the solution among them that minimizes power imbalance will be chosen.

B. Theoretical Justification for the Algorithm

The problem in (10) can be recognized as a normalized cut problem on a graph with $|N|$ nodes and at most $|G| + |E|$ edges. Since the problem is NP-hard in general, we use an algorithm from [20] to solve a relaxation of the problem. This algorithm has been used, among others, in neuroscience [25] and nuclear material identification [26]. In this section, we adapt some results from [20] to our problem and illustrate the main idea behind the algorithm. First, define the problem:

$$\underset{S\subseteq N, b\in\mathbb{R}_+}{\text{minimize}} \quad \frac{(1+b)^2\sum_{(ij)\in\delta(S)} \boldsymbol{W}_{ij}}{\sum_{i\in S}\boldsymbol{Q}_{ii} + b^2\sum_{i\in\bar{S}}\boldsymbol{Q}_{ii}} \tag{13a}$$

$$\text{subject to} \quad G(S)\neq\emptyset, G(\bar{S})\neq\emptyset \tag{13b}$$

$$b = \frac{\sum_{i\in S}\boldsymbol{Q}_{ii}}{\sum_{i\in\bar{S}}\boldsymbol{Q}_{ii}} \tag{13c}$$

where $G(S), G(\bar{S})$ the set of generators connected to nodes of S and to the complement of S respectively. The problem (13) is equivalent to (10). To see that, first note that in (10) neither of $G(S)$, $G(\bar{S})$ can be empty for a finite objective value. The equivalence of the objectives can be seen by substituting $b = \frac{\sum_{i\in S}\boldsymbol{Q}_{ii}}{\sum_{i\in\bar{S}}\boldsymbol{Q}_{ii}}$ into the objective of (13). The algorithm developed in [20] solves (13) with constraint (13c) relaxed. For the different values of the parameter β, define the following problem:

$$P(\beta) = \underset{S\subseteq N: G(S), G(\bar{S})\neq\emptyset}{\text{minimize}} \sum_{(ij)\in\delta(S)} \boldsymbol{W}_{ij} \; + \; \beta \sum_{i\in S}\boldsymbol{Q}_{ii} \tag{14}$$

Claim 1: Any optimal solution (partition S, $\bar{S}$) of problem (13) with constraint (13c) relaxed is an optimal solution to problem $P(\beta)$, for some value of the parameter β. The proof of the claim is provided in the Appendix.

Based on the previous claim, instead of solving the relaxation of (13), we will solve problem (14) for all values of the parameter β. To that end, first note that (14) only allows partitions in which both sets S and $\bar{S}$ must contain at least one generator. For two generators $g, g' \in G$, with $g \neq g'$, let $P^{gg'}(\beta)$ denote the optimization problem $P(\beta)$ with the additional constraint that $n(g)\in S$, $n(g')\in\bar{S}$, where $n(g)\in N$ denotes the node to which generator g is connected. Then, the problem $P(\beta)$ can be solved as:

$$P(\beta) = \underset{g,g'\in G, g\neq g'}{\text{minimize}} \quad P^{gg'}(\beta) \tag{15}$$

Therefore, by solving at most $O(|G|^2)$ problems of the type $P^{gg'}(\beta)$, we can solve $P(\beta)$. However, in order to reduce the real time computational burden, we will instead heuristically pick two generators g, g' and force them to belong to different sets. The generators g and g' can be picked based on a heuristic, such as weakest coupling $\boldsymbol{K}_{gg'}$, or empirical knowledge of the particular power system. Even though this approach yields a worse objective, it is reasonable for cases in which we may want to force separation between two generators.

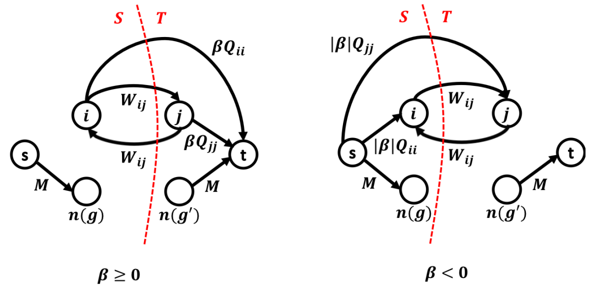

Fig. 2: Directed graphs to solve $P^{gg'}(\beta)$ for the different values of β, following the construction in [20]. The graph has all the nodes in N, plus two dummy (source and sink) nodes s,t. The partition is S, $T = \bar{S}$. The weights $\boldsymbol{W}_{ij}, \boldsymbol{Q}_{ii}$ refer to the objective of (14). M is a large number. For every pair i, j with non zero weight $\boldsymbol{W}_{ij}$, two directed edges are added. For every generator node i (i.e. $G(i)\neq\emptyset$), one directed edge of weight $\beta\boldsymbol{Q}_{ii}$ is added.

Finally, for a given pair g, g', the problem $P^{gg'}(\beta)$ can be solved efficiently using the graphs of Fig 2. To see that, note first that the capacity of an $s-t$ cut in the graph for $\beta\geq 0$ is exactly the objective from (14). The big-M capacity arcs from s to $n(g)$ and from $n(g')$ to t ensure that $n(g)\in S$ and $n(g')\in T$. Therefore, the minimum-cut problem on the graph solves $P^{gg'}(\beta)$. For $\beta < 0$, the capacity of the cut is $\sum_{(ij)\in\delta(S)}\boldsymbol{W}_{ij} + (-\beta)\sum_{i\in T}\boldsymbol{Q}_{ii} = \sum_{(ij)\in\delta(S)}\boldsymbol{W}_{ij} + \beta\sum_{i\in T}\boldsymbol{Q}_{ii} - \beta\sum_{i\in N}\boldsymbol{Q}_{ii}$, which again solves $P^{gg'}(\beta)$ since the last term in the summation is a constant. In both problems, the parameter $|\beta|$ appears on strictly increasing functions of capacities only from the source/only to the sink. This ensures we can solve for all values of β efficiently at the complexity of a maximum flow problem (parametric cut) [27]. By the same theory, we know there will be at most $|N|$ different partitions generated from the parametric cut solution, for all the parameters β. Therefore, we can efficiently calculate the objective of interest for all of them and pick the best.

C. The algorithm

Based on the analysis and theoretical justification presented in the previous sections, the proposed algorithm for optimal islanding following an extended grid disturbance is presented a step-wise fashion below.

Step 1: Identify the surviving power system, the operational generators, buses and edges. Based on the last known pre-disturbance measurements and the analysis presented in subsection II-A, calculate the matrices $\boldsymbol{K}_{gg'}$, $\boldsymbol{M}_{gg}$ for the surviving generators. Note that if these calculations are periodically performed online for the power system in steady state, the matrices for the surviving system can be updated more efficiently (but we will not focus on that aspect in this work). Calculate the weights $\boldsymbol{W}_{ij}$ and $\boldsymbol{Q}_{ii}$ based on equations (11) and (12) and any other requirements that we want to impose on the system (such as inflexible lines or minimum load requirements, as described in subsection III-A).

Step 2: Formulate the graphs from Fig 2 and solve the parametric minimum cut problem on both of them. The output of the algorithm for each graph will be at most $|N|$ different partitions of N, each one corresponding to a value of β [27]. Calculate the objective value of (10) for each one of them and pick the partition with the best objective. On a technical note, the calculation of the objective for each partition can be done in $O(|N|)$ by using the optimal parametric cut objective that yielded this partition. Note that if an implementation of the parametric cut is not available, one can simply pick some values of β instead and solve the problem only for them, as a heuristic.

Step 3: Repeat the process from Step 2 for each of the partition sets to further split the grid into smaller islands.

IV. Simulation Results

We simulated the algorithm on the IEEE-9, IEEE-39, IEEE-300 and Polish test systems. Based on [21], we assumed that the transient reactance of the generators is $X_g' = \max\{0.1, 92.8(P_g^{\max})^{-1.3}\}$, where X_g' is expressed in p.u. with respect to the system basis $S_{\text{base}} = 100\text{MVA}$ and $P_g^{\max}$ is the nominal power of generator g is MW. We also assumed that $H_g = 0.04 P_g^{\max}$, where H_g is in p.u. with respect to the system basis. Matpower [28] was used to calculate the pre-disturbance ac power flow.

The algorithm from [6], which is conceptually close to our approach and uses a well established partitioning technique based on spectral clustering, was also simulated. We implemented the two algorithms in Matlab. For our implementation, since the Matlab graph environment does not support parametric maximum flow, we simply solved problem (14) for 20 values of β evenly spaced between -1 to 1 and the solution with the best objective was chosen. The trade-off was set to $\lambda = 1$, however the optimal solution in the instances solved was often not sensitive to changes in the value of λ, which is an indication that both objectives are solved to optimality. We focus on bipartitions in the results. Multiple applications of the algorithm can break the system into smaller islands if necessary.

Alg.	Node Partition $(\lvert S\rvert, \lvert\bar{S}\rvert)$	Generator Partition $(\lvert V_G\rvert, \lvert\bar{V}_G\rvert)$	Power Disruption MW	ζ	Time [s]
	IEEE-9				
ICI[1]	(2,7)	(1,2)	71.7	68.44	0.055
ICI[2]	(1,8)	(1,2)	163.0	67.82	0.081
	IEEE-39				
ICI[1]	(3,36)	(1,9)	85.4	57.97	0.068
ICI[2]	(8,31)	(5,5)	4611.8	31.94	0.462
	IEEE-300				
ICI[1]	(4,296)	(1,68)	140.1	2.33	0.067
ICI[2]	(83,217)	(1,68)	33434.5	2.33	2.030
	3375-bus Polish system				
ICI[1]	(52,3322)	(1,440)	554.5	582.13	0.960
ICI[2]	(478,2896)	(1,440)	81495.5	9918.4	277

TABLE I: Optimal bipartition based on the algorithm of this paper (ICI[1]) and the algorithm from [6] (ICI[2]) for IEEE test cases and the Polish system. The metrics compared are the quantity ζ from (8), the power flow disruption, and the algorithm execution time.

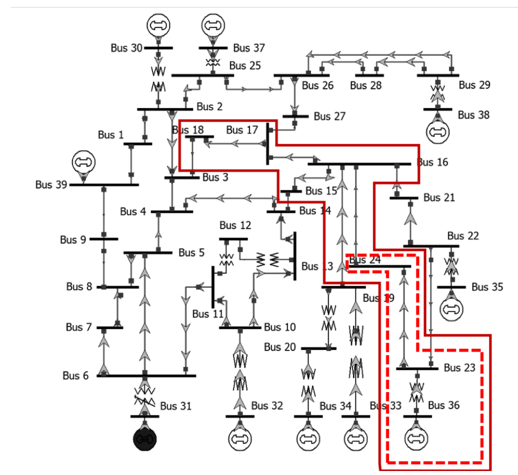

Fig. 3: The optimal bipartition for the IEEE-39 bus system is shown with red dashed line. The purple solid line indicates the partitioning when the generator in bus 36 is required to be connected to load in node 16. This is imposed by using large weights for the path leading to the bus, in this case for branches $(36, 23)$, $(23, 24)$, $(24, 16)$.

Table I shows the main computational results. Note that, both the power disruption values and the normalized generator cut ζ values are comparable or lower for our approach. The time of execution is also significantly better using the proposed approach. Compared to the spectral clustering approximation, our approach tends to lead to smaller islands, but it turns out that these solutions still have better objectives. If the problem changes, by introducing further islanding constraints, larger islands can be obtained. An example of introducing islanding constraints is shown in Figure 3.

V. Conclusions and Future Work

In this work, we examined an efficient approach for controlled islanding based on a combinatorial approximation of the normalized cut. The algorithm allows integration of further requirements through the use of large arc weights, a practice that will not influence the computational efficiency due to the strongly polynomial nature of the algorithms for minimum cut. Experimental results showed that the computations are fast even for large systems, hence no grid simplification is required. Future research directions include an efficient implementation of the parametric minimum cut procedure, an algorithmic description and implementation of the generator-load mapping for every island, possible generalizations to multicut partitioning, and additional simulations to verify the effectiveness of the approach.

Appendix A
Proof of Claim 1

The proof is based on ideas in [19]. Let S^*, b^* be an optimal solution to (13) with constraint (13c) relaxed and let the value of the objective be z^*, i.e.:

$$z^* = \frac{(1+b^*)^2 \sum_{(ij)\in\delta(S^*)} \boldsymbol{W}_{ij}}{\sum_{i\in S^*} \boldsymbol{Q}_{ii} + (b^*)^2 \sum_{i\in\bar{S}^*} \boldsymbol{Q}_{ii}} \tag{16}$$

We will show that S^* is an optimal solution to the optimization problem $P(\hat{\beta})$, as defined in (14), with $\hat{\beta} = z^* \frac{b^*-1}{b^*+1}$. By the optimality of S^*, b^*, we have that for any $S \subseteq N : G(S), G(\bar{S}) \neq \emptyset$:

$$\frac{(1+b^*)^2 \sum_{(ij)\in\delta(S)} \boldsymbol{W}_{ij}}{\sum_{i\in S} \boldsymbol{Q}_{ii} + (b^*)^2 \sum_{i\in\bar{S}} \boldsymbol{Q}_{ii}} \geq z^* \tag{17}$$

which can be equivalently written, after a few algebraic manipulations and substituting $\sum_{i\in\bar{S}} \boldsymbol{Q}_{ii} = \sum_{i\in N} \boldsymbol{Q}_{ii} - \sum_{i\in S} \boldsymbol{Q}_{ii}$, as follows:

$$\sum_{(ij)\in\delta(S)} \boldsymbol{W}_{ij} + z^* \frac{b^*-1}{b^*+1} \sum_{i\in S} \boldsymbol{Q}_{ii} \geq z^* b^* \sum_{i\in N} \boldsymbol{Q}_{ii}, \quad \forall\ S \subseteq N : G(S), G(\bar{S}) \neq \emptyset \tag{18}$$

Note that the left hand side is the objective of $P(\hat{\beta})$, the right hand side is a constant, and the inequality holds for all feasible S in $P(\hat{\beta})$. Now, for the particular choice $S = S^*$, we can see from (16) that (18) holds with equality. Therefore, S^* is an optimal solution for $P(\hat{\beta})$ and the proof is complete.

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