Theorem:

Let $\mathbf{x} := (\mathbf{y}, \mathbf{z})$ and $F(\mathbf{x}) := (G(\mathbf{y}, \mathbf{z}), H(\mathbf{y}, \mathbf{z}))$. Define a sequence

$\mathbf{x}_k := (\mathbf{y}_k, \mathbf{z}_k)$ and $\mathbf{s}_k := \mathbf{x}_{k+1} - \mathbf{x}_k = (\mathbf{y}_{k+1} - \mathbf{y}_k, \mathbf{z}_{k+1} - \mathbf{z}_k) := (\mathbf{t}_k, \mathbf{u}_k)$

where

$\mathbf{t}_k = \{\delta_k \mathbf{I} - G'(\mathbf{y}_k, \mathbf{z}_k)\} G(\mathbf{y}_k, \mathbf{z}_k)$

$\mathbf{u}_k = \{\delta_k \mathbf{I} - H'(\mathbf{y}_k, \mathbf{z}_k)\} H(\mathbf{y}_k, \mathbf{z}_k)$

where G' and H' exists and invertible $\forall (\mathbf{y}_k, \mathbf{z}_k)$

and $\delta_k = min(\delta_0 ||F_0|| / ||F_k||, \delta_{max})$

Then, $\lim\limits_{k \to \infty} (\mathbf{y}_k, \lim\limits_{l \to \infty} z_k^l) = \mathbf{x}^*$, where $F(\mathbf{x}^*) = 0$

Proof:

From convergence result in [1] for pseudo-transient continuation, on applying to $H(\mathbf{y}_k, \mathbf{z}_k^l) := H_{\mathbf{y}_k}(\mathbf{z}_k^l)$

$\lim\limits_{k \to \infty} (\mathbf{y}_k, \lim\limits_{l \to \infty} \mathbf{z}_k^l) := \lim\limits_{k \to \infty} (\mathbf{y}_k, \mathbf{z}_k^*)$, it follows that $\lim\limits_{k \to \infty} H_{\mathbf{y}_k}(\mathbf{z}_k^*) = 0$ and thus $\lim\limits_{k \to \infty} H(\mathbf{y}_k, \mathbf{z}_k^*) = 0$

Now, $\lim\limits_{k \to \infty} (G(\mathbf{y}_k, \mathbf{z}_k^*), H(\mathbf{y}_k, \mathbf{z}_k^*)) = \lim\limits_{k \to \infty} (G(\mathbf{y}_k, \mathbf{z}_k^*), 0)$

Let $G(\mathbf{y}_k, \mathbf{z}_k^*) := G_{\mathbf{z}_k^*}(\mathbf{y}_k)$

From convergence result in [1] applied to $G_{\mathbf{z}_k^*}(\mathbf{y}_k)$, we have $\lim\limits_{k \to \infty} G_{\mathbf{z}_k^*}(\mathbf{y}_k) := G_{\mathbf{z}^*}(\mathbf{y}^*) = 0$

Thus, $G(\mathbf{y}^*, \mathbf{z}^*) = 0$ and $H(\mathbf{y}^*, \mathbf{z}^*) = 0$ as $\lim\limits_{k \to \infty} H_{\mathbf{y}_k}(\mathbf{z}^*) = 0$

References

[1] Kelley, Carl Timothy, and David E. Keyes. "Convergence analysis of pseudo-transient continuation." SIAM Journal on Numerical Analysis 35.2 (1998): 508-523.