Exam 3

1a) Find: r, v, γ at encounter

Assume: Coplanar orbit with same angular momentum direction, encounter at end of minor axis of Themis ($\gamma > 0$). (Assume relative 2BP about Jupiter, Neglect mass of s/c & themis)

$r_p = a(1-e)$

$$a_{s/c} = \frac{r_p}{1-e} = \frac{7.5\ R_{Jup}}{1-0.5} = 15\ [R_{Jup}] = 1.05\times10^6\ \text{Km}$$

$$P_{s/c} = a_{s/c}(1-e^2) = 15(1-0.5^2) = 11.25\ [R_{Jup}] = 7.875\times10^5\ \text{Km}$$

$$P_{themis} = a_{themis}(1-e^2) = 15(1-.25^2) = 14.0625\ [R_{Jup}] = 9.84375\times10^5\ \text{Km}$$

At end of minor axis, $\underline{E = 90^\circ}$

$$r = a(1-e\cancel{\cos E}^{\,0}) \quad \therefore \quad r_{themis} = a_{themis} = r_{s/c}$$

$$\boxed{\begin{aligned} r_{themis} &= 15\ [R_{Jup}] = 1.05\times10^6\ [\text{Km}] \\ r_{s/c} &= 1.05\times10^6\ [\text{Km}] \end{aligned}}$$

← r at encounter

$$\mathcal{E} = -\frac{\mu}{2a} = \frac{V^2}{2} - \frac{\mu}{r}$$

$$V = \sqrt{2\left(-\frac{\mu}{2a} + \frac{\mu}{r}\right)}$$

$$V_{themis} = \sqrt{2\left(\frac{-1.26\times10^8}{(2)(15)(7\times10^4)} + \frac{1.26\times10^8}{(15)(7\times10^4)}\right)} = 10.954 \ [Km/s]$$

$$V_{s/c} = \sqrt{2\left(\frac{-1.26\times10^8}{(30)(7\times10^4)} + \frac{1.26\times10^8}{(15)(7\times10^4)}\right)} = 10.954 \ [Km/s]$$

$$\boxed{\begin{aligned} V_{themis} &= 10.954 \ [Km/s] \\ V_{s/c} &= 10.954 \ [Km/s] \end{aligned}}$$

← Velocity at encounter, wrt Jupiter

$$h = \sqrt{\mu p} = rV\cos(\gamma)$$

$$\gamma = \pm \cos^{-1}\left(\frac{h}{rV}\right)$$

$$h_{the} = \sqrt{(1.26\times10^8)(9.84375\times10^5)} = 1.11\times10^7 \ Km^2/s$$

$$h_{sc} = \sqrt{(1.26\times10^8)(7.875\times10^5)} = 9.61\times10^6 \ Km^2/s$$

$$\gamma_{the} = \pm\cos^{-1}\left(\frac{1.11\times10^7}{(10.954)(1.05\times10^6)}\right) = \cos^{-1}(.9683)$$

$$\gamma_{the} = 14.47^\circ \quad (\gamma > 0 \ , \text{given})$$

$$\gamma_{sc} = \pm \cos^{-1}\left(\frac{9.61 \times 10^6}{(10.959)(1.05 \times 10^6)}\right) = \pm \cos^{-1}(.866)$$

$$\gamma_{sc} = \pm 30$$

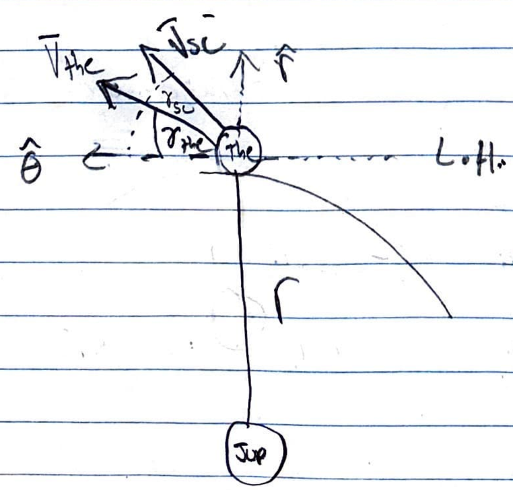
Spacecraft is outbound, with same direction of rotation as themis $\therefore$ $\gamma > 0$

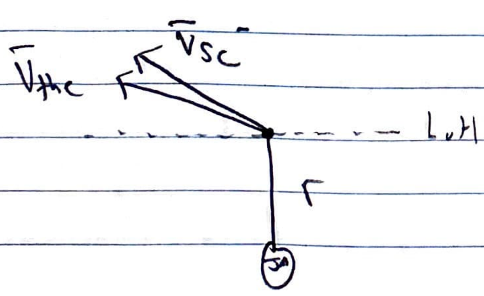
$$\boxed{\begin{aligned} \gamma_{themis} &= 14.47^\circ \\ \gamma_{sc} &= 30^\circ \end{aligned}}$$

← Flight path angle at encounter

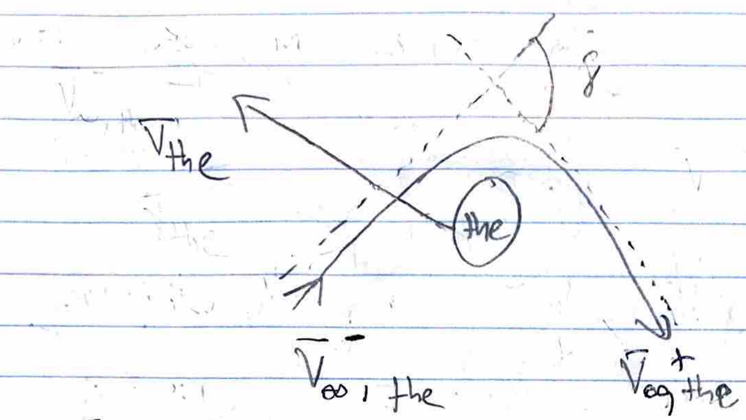
b) In order to decrease energy, the spacecraft should pass $\boxed{\text{ahead}}$ of themis.

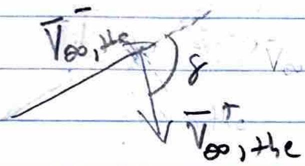
Jupiter Centric View

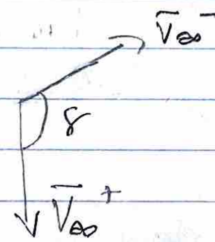




Themis Centric View:

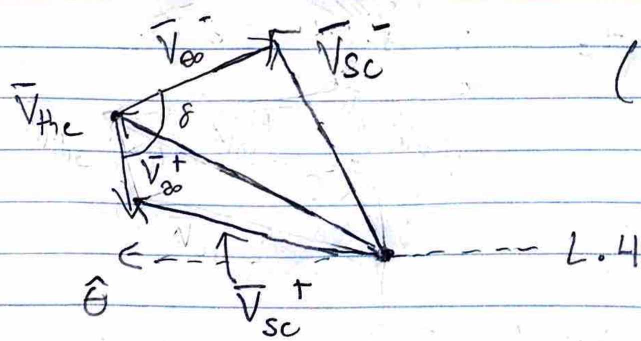






$$\bar{V}_{sc}^- = \bar{V}_{the} + \bar{V}_\infty^-$$

$$\bar{V}_{sc}^+ = \bar{V}^{the} + \bar{V}_\infty^+$$



(Exxagerated for Clarity)

c) Find: v^+, γ^+ relative to Jupiter

Assume: Patched conic approximation for themis assist

$$\bar{V}_\infty^- = \bar{V}_{sc}^- - \bar{V}_{the}$$

$$= V_{sc}^- [\sin(\gamma_{sc})\hat{r} + \cos(\gamma_{sc})\hat{\theta}] - V_{the}[\sin(\gamma_{the})\hat{r} + \cos(\gamma_{the})\hat{\theta}]$$

$$= \begin{bmatrix} 5.477\ \hat{r} \\ 9.486\ \hat{\theta} \end{bmatrix} - \begin{bmatrix} 2.737\ \hat{r} \\ 10.606\ \hat{\theta} \end{bmatrix} = \begin{bmatrix} 2.74\ \hat{r} \\ -1.12\ \hat{\theta} \end{bmatrix}$$

$$|\bar{V}_\infty|^- = 2.96\ [km/s] = |\bar{V}_\infty^+|$$ (Gravity assist only changes direction of V_∞, not magnitude)

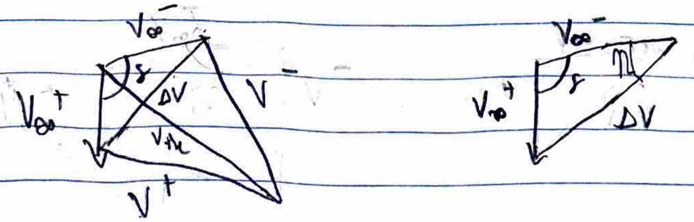
$$\mathcal{E} = \frac{V_\infty^2}{2} = 4.3808\ [km^2/s^2]$$

$$r_p = R_{the} + 1000 = 4000\ [km]$$

$$\mathcal{E}_H = \frac{\mu}{2|a|} \Rightarrow |a| = \frac{\mu_{the}}{2\mathcal{E}_H} = 1.1413 \times 10^4\ [km]$$

$$e_H = \frac{r_p}{|a|} + 1 = 1.35$$

$$\sin(\delta/2) = 1/e_H \Rightarrow \delta = 2\sin^{-1}(1/e_H) = 95.59^\circ$$

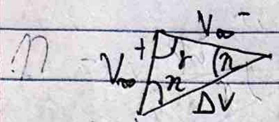


(Isoceles)

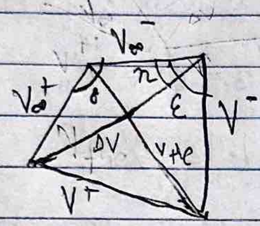
Isosceles triangle $\therefore$ $\Delta V = 2V_\infty \sin\left(\frac{\delta}{2}\right)$

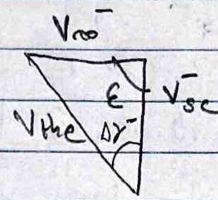
$\Delta V = (2)(2.96)\sin\left(\frac{95.59}{2}\right)$

$\Delta V = 4.38$ [Km/s]



$\eta = \frac{180 - \delta}{2} = 42.2^\circ$

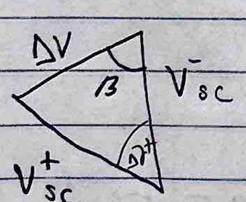




$\Delta\gamma^- = \gamma_{sc}^- - \gamma_{the} = 30 - 14.47 = 15.53^\circ$

$\frac{\sin(\Delta\gamma^-)}{V_\infty^-} = \frac{\sin(\varepsilon)}{V_{the}} \Rightarrow \sin(\varepsilon) = \frac{V_{the}\sin(\Delta\gamma^-)}{V_\infty^-}$

$\sin(\varepsilon) = \frac{10.954\sin(15.53)}{2.96} \Rightarrow \varepsilon = 82.22^\circ$



$V^{+2} = \Delta V^2 + V^{-2} - 2\Delta V V^- \cos(B)$

$B = \varepsilon - \eta = 40.02^\circ$

$V^{+2} = (4.38)^2 + (10.954)^2 - 2(4.38)(10.954)\cos(40.02)$

$\boxed{V_{sc}^+ = 8.104 \text{ [Km/s]}}$

$$\frac{\sin(\Delta\gamma^+)}{\Delta V} = \frac{\sin(B)}{V^+}$$

$$\Delta\gamma^+ = \sin^{-1}\left(\frac{\Delta V \sin(B)}{V^+}\right)$$

$$\Delta\gamma^+ = 20.34^\circ = \gamma_{sc}^- - \gamma_{sc}^+$$

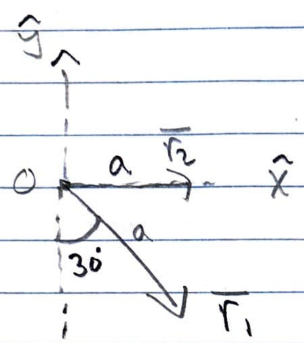
From sketch: $\gamma_{sc}^+ = \gamma_{sc}^- - \Delta\gamma^+ = 30^\circ - 20.34$

$$\boxed{\gamma^+ = 9.66^\circ}$$

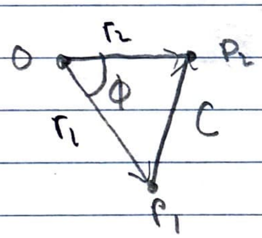
Exam 3

#2a) **Find** : chord

Assume: $a = R_\oplus$



$\bar{r}_2 = a\hat{x}$
$\bar{r}_1 = a(\sin(30)\hat{x} - \cos(30)\hat{y})$



(Space triangle)

$$\cos(\phi) = \frac{r_1 \cdot r_2}{|r_1||r_2|} = \frac{a^2 \sin(30)}{|a|a^2} = \sin(30) \Rightarrow \phi = 60$$

$$C^2 = r_1^2 + r_2^2 - 2r_1r_2\cos\phi = a^2 + a^2 - 2a^2\sin(30)$$
$$C^2 = 2a^2(1 - \sin(30))$$
$$C = \sqrt{(2)(64)(1-\sin(30))}$$

$$\boxed{C = 8 R_\oplus = 51200 \text{ [km]}}$$

b) **Find** : TA & both p

Assume : 2A for transfer type

2A - TA > 180 & F Not between chord & arc

TA > 180 $\therefore$ TA = 360 - ϕ = 360 - 60

$\boxed{TA = 300^\circ}$

$s = \frac{1}{2}(r_1 + r_2 + c) = 12$ R⊕
$\quad 8 + 8 + 8$

$\alpha_0 = 2\sin^{-1}\left(\sqrt{s/2a}\right)$ $\qquad B_0 = 2\sin^{-1}\left(\sqrt{\frac{s-c}{2a}}\right)$

$\alpha_0 = 2\sin^{-1}\left(\sqrt{\frac{12}{16}}\right) = 120^\circ$

$B_0 = 2\sin^{-1}\left(\sqrt{\frac{4}{16}}\right) = 60^\circ$

For 2A: $\alpha = \alpha_0 = 120^\circ$
$\quad B = -B_0 = -60^\circ = 300^\circ$

$$P = \frac{4a(s-r_1)(s-r_2)}{c^2}\sin^2\left(\frac{\alpha \pm B}{2}\right)$$

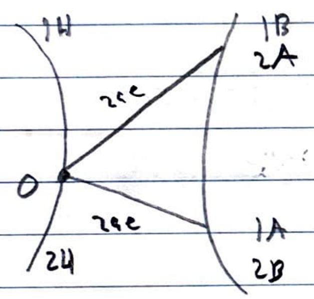
$$= \frac{(32)(12-8)(12-8)}{64}\sin^2\left(\frac{120 \pm 300}{2}\right)$$

$$= 8\sin^2\left(\frac{120 \pm 300}{2}\right)$$

$P_1 = 2\ R_\oplus$
$P_2 = 8\ R_\oplus$

$$\boxed{\begin{aligned} P_1 &= 12800\ [km] \\ P_2 &= 51200\ [km] \end{aligned}}$$

Check Solution space



For a 2A transfer type, a larger e is needed as shown is the sketch of the solution space, $p = a(1-e^2)$
$\therefore$ increasing e, decreases p. Choose the $\boxed{\text{smaller } p}$

$$\boxed{P = 2\ R_\oplus = 12800\ [km]}$$

c) $p = a(1-e^2)$

$$e = \sqrt{1-\frac{p}{a}} = \sqrt{1-\frac{2}{8}}$$

$$\boxed{e = 0.866}$$

$$r = \frac{p}{1+e\cos(\theta^*)} \Rightarrow \theta^* = \cos^{-1}\left(\frac{1}{e}\left(\frac{p}{r}-1\right)\right)$$

$\theta_1^* = \pm \cos^{-1}\left(\frac{1}{0.866}\left(\frac{2}{8} - 1\right)\right)$ $\quad r_1 = a = 8\, R_\oplus$

$\theta_1^* = \pm 150^\circ = 210, 150$

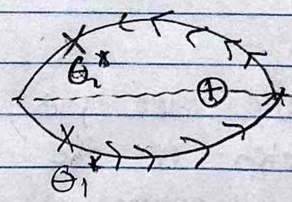
$\theta_2^* = \pm \cos^{-1}\left(\frac{1}{.866}\left(\frac{2}{8} \cdot 1\right)\right)$ $\quad r_2 = a = 8 = R_\oplus$

$\theta_2^* = \pm 150^\circ = 210, 150$

$TA = \theta_2^* - \theta_1^* = 300^\circ$

$$\boxed{\begin{aligned}\theta_2^* &= 150^\circ \\ \theta_1^* &= 210^\circ\ (-150^\circ)\end{aligned}}$$

Only pair of θ^* that yieds apprpricte TA



$h = \sqrt{\mu p} = \sqrt{(4\times10^5)(12800)} = 71554.17\ \text{Km}^2/\text{s}$

$\varepsilon = \frac{-\mu}{2a} = \frac{V_{1,2}^2}{2} - \frac{\mu}{r} \Rightarrow V = \sqrt{2\left(\frac{-\mu}{2a} + \frac{\mu}{r}\right)}$

$r_1 = r_2 = a \Rightarrow V_1 = V_2 = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{4\times10^5}{(8)(6400)}} = 2.795\ [\text{Km/s}]$

$h = rV\cos(\gamma) \Rightarrow \gamma = \pm\cos^{-1}\left(\frac{h}{rV}\right)$

$$\boxed{\begin{aligned}\gamma_1 &= -60^\circ \quad (\text{descending},\ \theta_1^* < 0) \\ \gamma_2 &= 60^\circ \quad (\text{ascending},\ \theta_2^* > 0)\end{aligned}}$$

Angle between $\bar{r}_1$ and x-axis is 60° (aligned with $\bar{r}_2$)

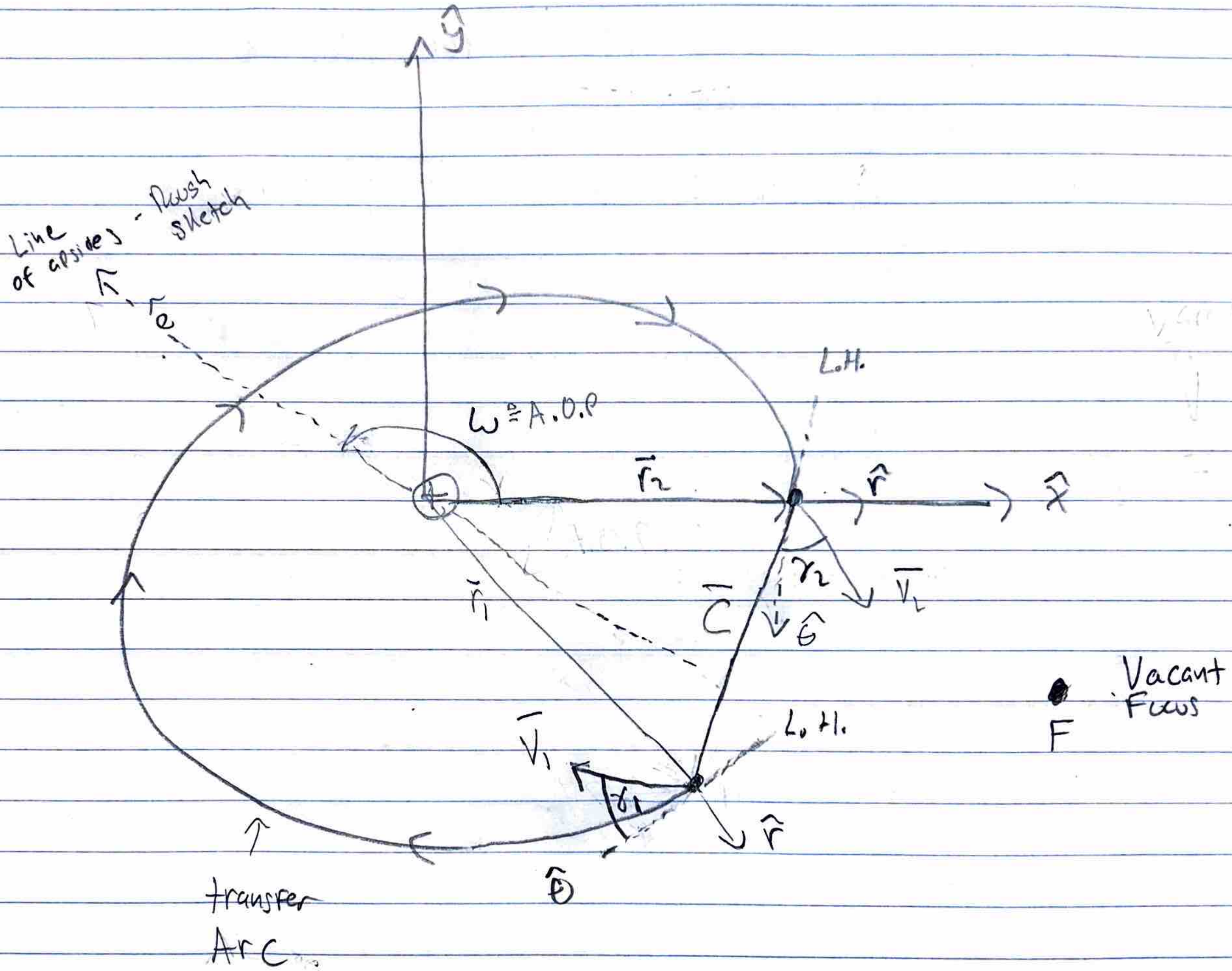
$\therefore \theta_2 = 0$, $\boxed{\omega = -150^\circ}$ from x-axis

or 210°

$\therefore \theta_1 = 60$

$210 + (-150)$

#2d)



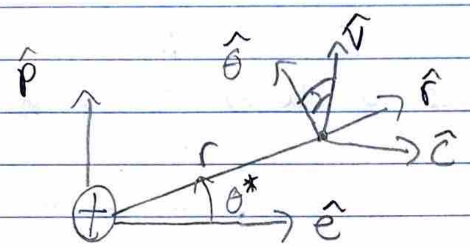
y
Line of apsides - Rough sketch
$\hat{r}_e$
$\omega \triangleq$ A.O.P
$\bar{r}_2$
L.H.
$\hat{r}$
$\hat{x}$
$\bar{r}_1$
$\bar{C}$
γ_2
$\hat{\theta}$
$\bar{V}_2$
Vacant Focus
F
L.H.
$\bar{V}_1$
γ_1
$\hat{r}$
$\hat{\theta}$
transfer Arc

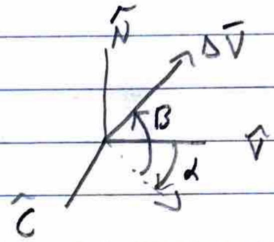
Exam 3

Find: $\Delta\bar{V}$ in $\hat{r}, \hat{\theta}, \hat{h}$ & $\hat{x}, \hat{y}, \hat{z}$

$i = 90°$, $\Omega = 90°$, $\omega = 30°$, $\theta^* = 60°$, $\gamma = 30°$

$\Delta\bar{V} = 1\,\hat{V} + 0.5\,\hat{C} + 1\,\hat{N}$





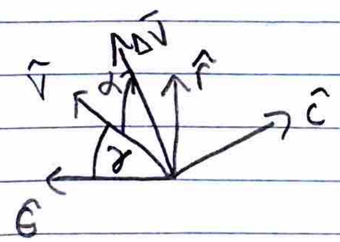
$\Delta\bar{V} = \Delta V\cos(\beta)\cos(\alpha)\,\hat{V} + \Delta V\cos(\beta)\sin(\alpha)\,\hat{C} + \Delta V\sin(\beta)\,\hat{N}$

$|\Delta V| = \sqrt{1^2 + 0.5^2 + 1^2} = 1.5$

$\Delta V\sin(\beta) = 1 \Rightarrow \beta = \sin^{-1}(1/1.5) = 41.81°$

$\tan(\alpha) = \dfrac{\cancel{\Delta V\cos(\beta)}\sin(\alpha)}{\cancel{\Delta V\cos(\beta)}\cos(\alpha)} = \dfrac{0.5}{1}$

$\alpha = 26.565°$ (α in Quad 1, both $\hat{C}$ & $\hat{V} > 0$)



$\phi = \gamma + \alpha = 56.565°$

$\hat{N} = \hat{h}$

$\Delta\bar{V}_\theta = \Delta V[\cos(\beta)\cos(\phi)]$

$\Delta\bar{V}_r = \Delta V[\cos(\beta)\sin(\phi)]$

$\Delta \bar{V}_{\theta} = 1.5 \cos(56.565) \cos(41.81)$

$\Delta V_{\theta} = 0.616 \quad [km/s]$

$\Delta \bar{V}_r = 1.5 \sin(56.565) \cos(41.81) = 0.933 \quad [km/s]$

$$\boxed{[\Delta \bar{V}]_{\hat{r}\hat{\theta}\hat{h}} = [0.933\,\hat{r} + 0.616\,\hat{\theta} + 1\,\hat{h}]\ [km/s]}$$

	$\hat{r}$	$\hat{\theta}$	h
$\hat{x}$	$c\Omega c\theta - s\Omega c i s\theta$	$-c\Omega s\theta - s\Omega c i c\theta$	$s\Omega s i$
$\hat{y}$	$s\Omega c\theta + c\Omega c i s\theta$	$-s\Omega s\theta + c\Omega c i c\theta$	$-c\Omega s i$
$\hat{z}$	$s i s\theta$	$s i c\theta$	$c i$

$i = \Omega = 90^\circ \quad \therefore \quad c\Omega = c i = 0$

$\theta = \omega + \theta^* = 60 + 30 = 90 \quad \therefore \quad c\theta = 0$

$$\begin{array}{c|ccc} & \hat{r} & \hat{\theta} & \hat{h} \\ \hline \hat{x} & 0 & 0 & 1 \\ \hat{y} & 0 & -1 & 0 \\ \hat{z} & 1 & 0 & 0 \end{array} = C^{\hat{x}\hat{y}\hat{z}}_{\hat{r}\hat{\theta}\hat{h}}$$

$$[\Delta \bar{V}]_{\hat{x}\hat{y}\hat{z}} = C^{\hat{x}\hat{y}\hat{z}}_{\hat{r}\hat{\theta}\hat{h}} \, [\Delta V]_{\hat{r}\hat{\theta}\hat{h}}$$

$$= \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} .933 \\ .616 \\ 1 \end{pmatrix}$$

$$\boxed{[\Delta \bar{V}]_{\hat{x}\hat{y}\hat{z}} = [1\,\hat{x} - 0.616\,\hat{y} + 0.933\,\hat{z}] \quad [km/s]}$$