HW10

#1a) Find: $\bar{V}_1$, $\bar{r}_1$ in Earth Inertial Coordinates at departure (Ascending Node)

Assume: Circular orbit about Earth, Departure at Ascending node
$a = 4R_\oplus$, $\Omega = 60^\circ$, $e = 0$, $\omega = 45^\circ$, $i = 28.5^\circ$, $M = 90^\circ$

Circular Orbit $\therefore$ $r_1 = a = 4R_\oplus$

$\bar{r} = 25512.5452\ [Km]\ \hat{r}$

$$[IR] = \begin{pmatrix} C_\Omega C_\theta - S_\Omega C_i S_\theta & -C_\Omega S_\theta - S_\Omega C_i C_\theta & S_\Omega S_i \\ S_\Omega S_\theta + C_\Omega C_i S_\theta & -S_\Omega S_\theta + C_\Omega C_i C_\theta & -C_\Omega S_i \\ S_i S_\theta & S_i C_\theta & C_i \end{pmatrix}$$

At ascending node: $\theta = 0^\circ$ $\therefore$ at departure

$$[IR] = \begin{pmatrix} 0.5 & -0.761 & 0.413 \\ 0.866 & 0.439 & -0.238 \\ 0 & 0.477 & 0.8788 \end{pmatrix}$$

$$[\bar{r}_1]_I = [IR][\bar{r}_1]_R$$

$$= [IR]\begin{bmatrix} r_1 \\ 0 \\ 0 \end{bmatrix} = 0.5\, r_1 \hat{x} + 0.866\, r_1 \hat{y}$$

$$\boxed{r_1 = 12756.27\,\hat{x} + 22094.51\,\hat{y}\ [Km]}$$

$\mathcal{E} = \frac{-\mu_\oplus}{2a} = \frac{V_1^2}{2} - \frac{\mu_\oplus}{r_1}$, circular orbit $\therefore r = a$

$V_1 = \sqrt{\frac{\mu_\oplus}{r_1}}$, circular orbit so $\bar{V}$ is tangential to path, $\gamma_1 = 0°$

$\bar{V}_1 = V_1 \sin(\gamma_1)\,\hat{r} + V_1\cos(\gamma_1)\,\hat{\theta} = \sqrt{\frac{\mu}{r_1}}\,\hat{\theta} = 3.952\ [Km/s]\ \hat{\theta}$

$\bar{V}_1 = 3.952\ \hat{\theta}\ [Km/s]$

$[\bar{V}_1]_I = [IR]\,[\bar{V}_1]_R = [IR]\begin{bmatrix} 0 \\ V_1 \\ 0 \end{bmatrix} = -.761\,V_1\,\hat{x} + .439\,V_1\,\hat{y} + .477\,V_1\,\hat{z}$

$$\boxed{\bar{V}_1 = -3.0083\,\hat{x} + 1.7368\,\hat{y} + 1.8860\,\hat{z}\quad [Km/s]}$$

#1b) Find: $\bar{r}_2$, $\bar{V}_2$ in Earth Inertial Frame at arrival

Assume: $a = 6R_\oplus$, $e = 0$, $\Omega = 60°$, $\omega = 45$, $i = 45°$, $M = 0$, $\theta = 120°$

$$[IR] = \begin{pmatrix} -.7803 & -.1268 & 0.6123 \\ -.1268 & -0.9267 & -0.3535 \\ 0.6123 & -0.3535 & 0.7071 \end{pmatrix}$$

$r_2 = 6R_\oplus = a = 38268.82\ [Km]$ (Circular orbit)

$\bar{r}_2 = 38268.82\ \hat{r}\ [Km]$

$[\bar{r}_2]_I = [IR]\,[\bar{r}_2]_R = -.7803\,r_2\,\hat{x} - .1268\,r_2\,\hat{y} + 0.6123\,r_2\,\hat{z}$

$$\boxed{\bar{r}_2 = -29862.31\,\hat{x} - 4853.50\,\hat{y} + 23434.77\,\hat{z}\ [Km]}$$

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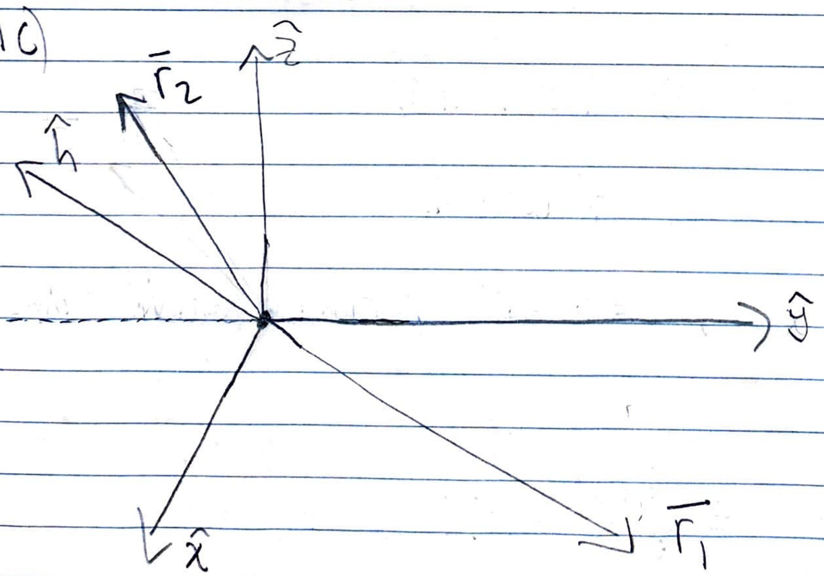
$V_2 = \sqrt{\frac{\mu_\oplus}{r_2}} = 3.227 \text{ Km/s}$

Circolar orbit, $\gamma_2 = 0 \therefore \quad \bar{V}_2 = 3.227\, \hat{\theta}$ [Km/s]

$[\bar{V}_2]_I = [IR][\bar{V}_2]_R = -.1268 V_2 \hat{x} - 0.9267 V_2 \hat{y} - 0.3535 V_2 \hat{z}$

$$\boxed{\bar{V}_2 = -0.4093\,\hat{x} - 2.9910\,\hat{y} - 1.1410\,\hat{z} \text{ [Km/s]}}$$

#1c)



#1(c-ii)

$\hat{h} = \dfrac{\bar{r}_1 \times \bar{r}_2}{|\bar{r}_1 \times \bar{r}_2|}$ ($\bar{r}_1$ & $\bar{r}_2$ are in same transfer arc plane, $\therefore$ $\hat{h}$ is $\perp$ to plane of transfer)

$$\bar{r}_1 \times \bar{r}_2 = \begin{array}{ccc} \hat{x} & \hat{y} & \hat{z} \\ \end{array} \begin{pmatrix} 12756.27 & 22094.51 & 0 \\ -29862.31 & -4853.50 & 28434.77 \end{pmatrix}$$

$= 517779474.64\,\hat{x} - 298940303.81\,\hat{y} + 597880607.63\,\hat{z}$

$|\bar{r}_1 \times \bar{r}_2| = 845530863.99$

$\hat{h} = 0.612\,\hat{x} - 0.3535\,\hat{y} + 0.7071\,\hat{z}$

$\hat{h} = S\Omega\, si\,\hat{x} - C\Omega\, si\,\hat{y} + \cos i\,\hat{z}$

$\cos(i) = 0.7071$, $i = \pm 45°$, Choose Positive Value

$\boxed{i = 45°}$

$\tan(\Omega) = \dfrac{0.612}{0.3535} = \dfrac{S\Omega\, si}{C\Omega\, si} \Rightarrow \Omega = 60, 240$

Both 0.612 & 0.3535 are positive $\therefore$ Ω in Quadrant 1

$\boxed{\Omega = 60°}$

$\hat{\theta}_1 = \hat{h} \times \hat{r}_1$ $\hat{\theta}_2 = \hat{h} \times \hat{r}_2$

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$$\hat{r}_1 = \frac{r_1}{|r_1|} = 0.5\,\hat{x} + 0.866\,\hat{y}$$

$$\hat{r}_2 = \frac{r_2}{|r_2|} = -0.7803\hat{x} - .1268\hat{y} + 0.6124\hat{z}$$

$$\hat{\theta}_1 = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0.612 & -0.3535 & 0.7071 \\ 0.5 & 0.866 & 0 \end{vmatrix} = -0.612\,\hat{x} + 0.3535\,\hat{y} + 0.7071\,\hat{z}$$

$$\hat{\theta}_2 = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0.612 & -0.3535 & 0.7071 \\ -0.7803 & -.1268 & 0.6124 \end{vmatrix} = -0.1268\hat{x} - 0.92677\hat{y} - 0.3535\hat{z}$$

$$\hat{z}_1 = s_i s_{\theta_1}\hat{r}_1 + s_i c_{\theta_1}\hat{\theta}_1 + c_i\hat{h}$$

$$\hat{z}_1 = 0.0\,\hat{r}_1 + 0.7071\,\hat{\theta}_1 + 0.7071\,\hat{h}$$

$$\tan\theta_1 = \frac{\sin(i)\sin(\theta_1)}{\sin(i)\cos(\theta_1)} = \frac{0}{.7071} \Rightarrow \theta_1 = 0, 180$$

0.7071 is positive $\therefore$ θ_1 is in Quad 1

$\underline{\theta_1 = 0}$

$$\tan\theta_2 = \frac{\sin(i)\sin(\theta_2)}{\sin(i)\cos(\theta_2)} = \frac{0.6124}{-0.3535} \Rightarrow \theta_2 = 120, 300$$

0.6124 is positive & -0.3535 is negative $\therefore$ θ_2 is in Quad 2.

$\underline{\theta_2 = 120}$

$TA = \theta_2 - \theta_1$ (Transfer angle)

$$\boxed{TA = 120^\circ}$$

1c-iii) $p = 5R_\oplus = 31890.68$ [Km]

$h = \sqrt{\mu p}$

$$\boxed{h = 112745.91 \; [Km^2/s]}$$

$$\bar{r}_2 = \underbrace{\left[1 - \frac{r_2}{p}(1-\cos(\Delta\theta))\right]}_{f}\bar{r}_1 + \underbrace{\frac{r_2 r_1}{\sqrt{\mu p}}\sin(\Delta\theta)}_{g}\bar{V}_1$$

$$\bar{V}_1 = \frac{\bar{r}_2 - f\bar{r}_1}{g}$$

$$f = \left[1 - \frac{6\cancel{R_\oplus}}{5\cancel{R_\oplus}}(1-\cos(120))\right] = -0.8$$

$g = 7499.44$ [s]

$$\boxed{\bar{V}_1 = -2.621\hat{x} + 1.7097\hat{y} + 3.1248\hat{z} \quad [Km/s]}$$

$$\mathcal{E} = \frac{-\mu}{2a} = \frac{V_1^2}{2} - \frac{\mu}{r_1}$$

$V_1 = 4.422$ Km/s

$\mathcal{E} = -5.844$ Km^2/s^2

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$a = \frac{-\mu}{2\varepsilon}$

$\boxed{a = 34100.93 \ [Km]}$

$P = a(1-e^2)$

$e = \sqrt{1 - \frac{p}{a}}$

$\boxed{e = 0.2546}$

$r_1 = \frac{p}{1 - e\cos(\theta_1^*)} \Rightarrow \theta_1^* = \pm\cos^{-1}\left(\frac{1}{e}\left(\frac{p}{r_1} - 1\right)\right)$

$\theta_1^* = \pm 10.89^\circ \qquad \bar{r}_1 \cdot \bar{V}_1 = 4339.59 > 0 \ \therefore \ \theta_1^* > 0$

$\therefore$ ascending

$\boxed{\theta_1^* = 10.89^\circ}$

$h = r_1 V_1 \cos(\gamma_1) \Rightarrow \gamma_1 = \cos^{-1}\left(\frac{h}{r_1 V_1}\right)$, ascending so $\gamma_1 > 0$

$\boxed{\gamma_1 = 2.204^\circ}$

$\bar{V}_2 = \dot{f}\,\bar{r}_1 + \dot{g}\,\bar{V}_1$

$\dot{f} = \frac{\bar{r}_1 \cdot \bar{V}_1}{p r_1}[1 - \cos(\Delta\theta)] - \frac{1}{r_1}\sqrt{\frac{\mu}{p}}\sin(\Delta\theta) = -1.12 \times 10^{-4} \ [1/s]$

$\dot{g} = 1 - \frac{r_1}{p}(1 - \cos(\Delta\theta)) = -0.2$

$\boxed{\bar{V}_2 = -0.9046\,\hat{x} - 2.8167\,\hat{y} - 0.6250\,\hat{z} \ [Km/s]}$

$\theta_2^* = \cos^{-1}\left(\frac{1}{e}\left(\frac{p}{r_2} - 1\right)\right) = \pm 130.9^\circ$

$\bar{r}_2 \cdot \bar{V}_2 = 26037 > 0 \quad \therefore \quad \theta_2^* > 0$, ascending

$$\boxed{\theta_2^* = 130.9^\circ}$$

$h = r_2 V_2 \cos(\gamma_2) \qquad \gamma_2 > 0$, ascending

$\gamma_2 = \cos^{-1}\left(\frac{h}{r_2 V_2}\right)$

$$\boxed{\gamma_2 = 13.0^\circ}$$

$\tan\left(\frac{\theta^*}{2}\right) = \sqrt{\frac{1+e}{1-e}} \tan\left(\frac{E}{2}\right) \Rightarrow$

$E = 2\tan^{-1}\left(\frac{\tan(\theta^*/2)}{\sqrt{\frac{1+e}{1-e}}}\right)$

$E_1 = 0.1467$ [rad]
$E_2 = 2.0715$ [rad]

$M_1 = E_1 - e\sin(E_1) = 0.1095$
$M_2 = E_2 - e\sin(E_2) = 1.8482$

$M_1 = n(t_1 - t_p) \Rightarrow t_1 = \frac{M_1}{n} + t_p$
$M_2 = n(t_2 - t_p) \Rightarrow t_2 = \frac{M_2}{n} + t_p$

$t_2 - t_1 = \frac{1}{n}(M_2 - M_1) = $ T.O.F.

$$n = \sqrt{\frac{\mu}{a^3}} = 1.0026 \times 10^{-4} \ [1/s]$$

$$TOF = \frac{1}{1.0026 \times 10^{-4}} (1.848 - 0.1095) = 17342.26 \ [s]$$

$$\boxed{TOF = 4.82 \ [hours]}$$

Problem 1a & 1b)

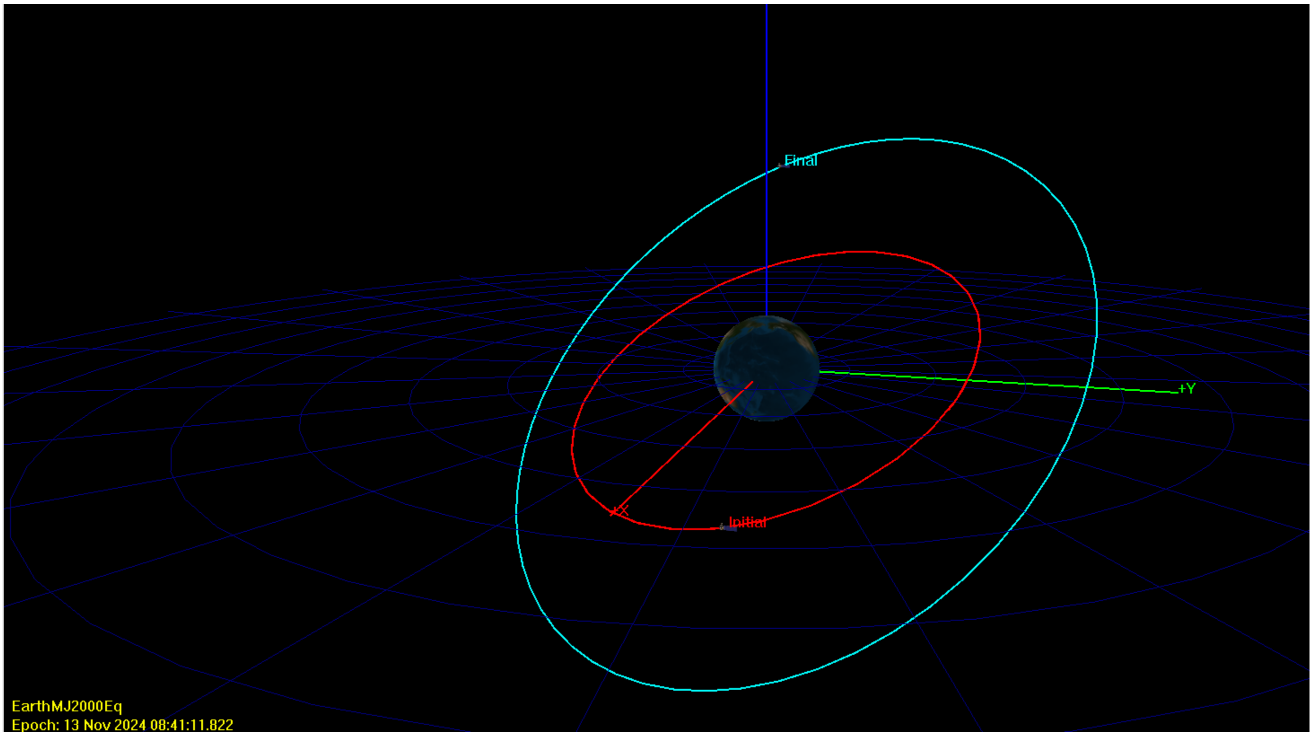

Figure 1: Problem 1a and 1b

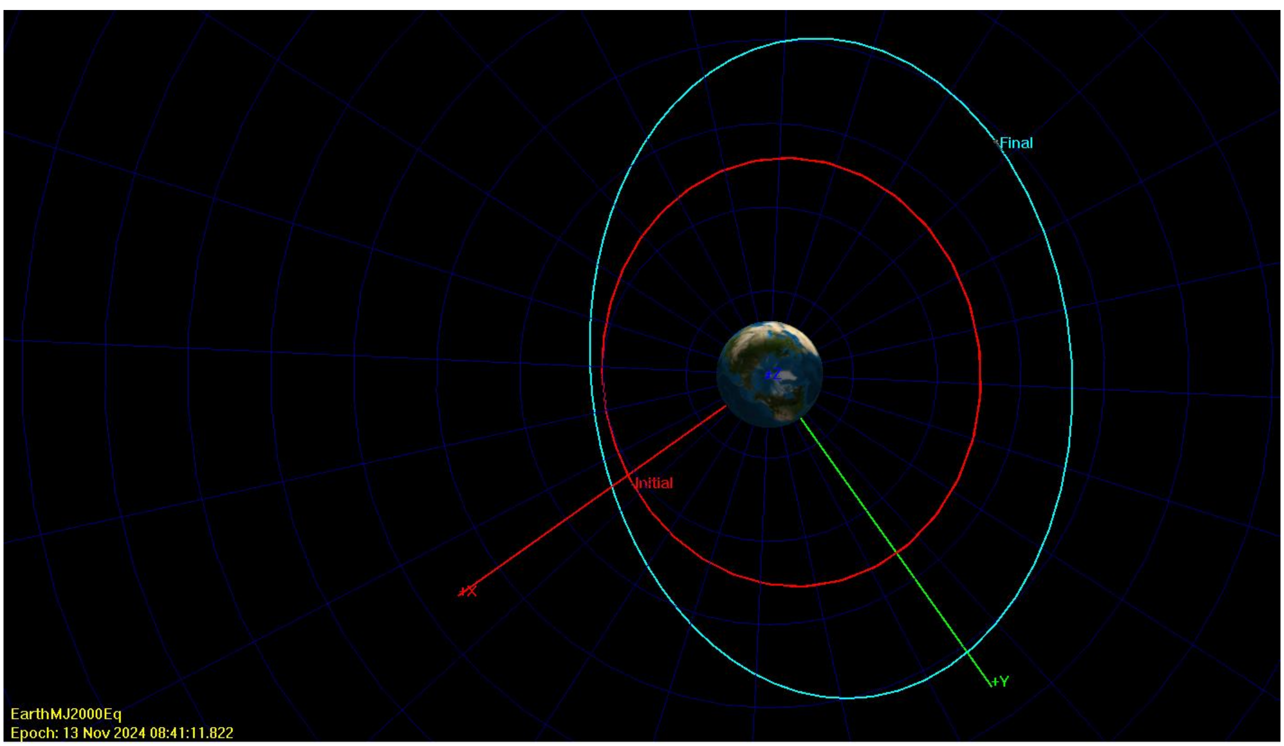

Figure 2: Problem 1a and

	Initial	Final
X [km]	12756.2726	-29862.30988
Y [km]	22094.51226	-4853.49961
Z [km]	0	23434.76917
Vx [km/s]	-3.008299911	-0.409313734
Vy [km/s]	1.736842764	-2.991034821
Vz [km/s]	1.886057356	-1.141041319

The values obtained from GMAT are listed in the table above. These results match the handwritten results, confirming their accuracy. In the figures on the previous pages, the red line is the orbit from problem 1a, and the blue line is the orbit from problem 1b.

Problem 1d)

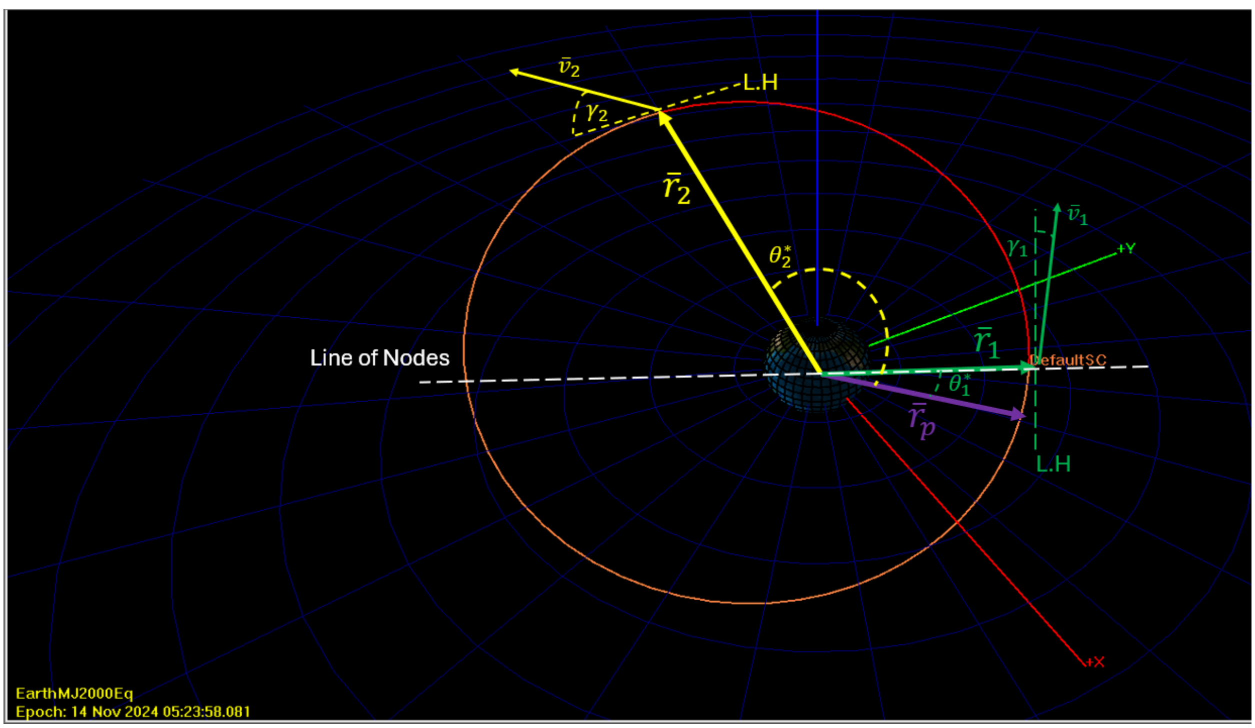

Figure 3: Problem 1d

The transfer ellipse is plotted in the figure above. The transfer arc is denoted by the red line, while the orange line comprises the rest of the transfer ellipse. Based off the orbit, it can be determined that the transfer type is a **1A** because the transfer angle is < 180 (transfer angle is 120) and the vacant focus is outside the chord and arc.