Gabriel Colangelo

```
clear
close all
clc
```

Problem 3b

```
% Calender days of ineterest
t_calendar = datetime([ "2006-01-19 12:00:00",...
                        "2007-02-28 12:00:00",...
                        "2011-03-18 12:00:00",...
                        "2015-07-14 12:00:00",...
                        "2019-01-01 12:00:00",...
                        "2021-04-17 12:00:00",...
                        "2024-10-15 12:00:00"]);

% Julian dates
jd          = juliandate(t_calendar)
```

```
jd = 1×7
     2453755     2454160     2455639     2457218     2458485     2459322     2460599
```

Problem 3c)

Find: $|\Delta\bar{v}|_{total}$ and TOF for Hohmann transfer from Earth to Pluto, assuming coplanar circular orbit, while neglecting local gravity fields.

```
% Orbit radius from Sun to Earth - assumed to be semi major axis of orbit
r_sun_earth = 149597898;     % [km]

% Orbit radius from Sun to Pluto - assumed to be semi major axis of orbit
r_sun_pluto = 5907150229;    % [km]

% Sun gravitional parameter
mu_sun      = 132712440017.99;

% Orbital speed prior to maneuver - circular velocity
v1_minus    = sqrt(mu_sun/r_sun_earth);

% Periapsis of transfer ellipse
rp_T        = r_sun_earth;

% Apoapsis of transfer ellipse
ra_T        = r_sun_pluto;

% Semi major axis of transfer ellipse
a_T         = (rp_T + ra_T)/2;

% Eccentricity of transfer ellipse
e_T         = 1 - rp_T/a_T;
```

```
% Orbital speed post maneuver
v1_plus     = sqrt(2*(mu_sun/rp_T - mu_sun/(2*a_T)));

% Calculate first deltaV
deltaV_1    = v1_plus - v1_minus;
fprintf('The first deltaV magnitude is %.3f km/s',deltaV_1);
```

```
The first deltaV magnitude is 11.814 km/s
```

$a_T = (r_{a_T} + r_{p_T})/2$

$e_T = 1 - \frac{r_{p_T}}{a_T}$

$v_1^- = \sqrt{\frac{\mu}{r_{p_T}}}$ - Circular Orbit

$v_1^+ = \sqrt{2(\frac{\mu}{r_{p_T}} - \frac{\mu}{2a_T})}$

$|\Delta \bar{v}_1| = v_1^+ - v_1^-$

$|\Delta \bar{v}_1| = 11.8 \ [km/s]$

```
% Orbital speed prior to 2nd maneuver
v2_minus   = sqrt(2*(mu_sun/ra_T - mu_sun/(2*a_T)));

% Orbit speed after 2nd maneuver - circular velocity
v2_plus    = sqrt(mu_sun/r_sun_pluto);

% Calculate second deltaV
deltaV_2   = v2_plus - v2_minus;
fprintf('The second deltaV magnitude is %.3f km/s',deltaV_2);
```

```
The second deltaV magnitude is 3.686 km/s
```

$v_2^+ = \sqrt{\frac{\mu}{r_{a_T}}}$ - Circular Orbit

$v_2^- = \sqrt{2(\frac{\mu}{r_{a_T}} - \frac{\mu}{2a_T})}$

$|\Delta \bar{v}_2| = v_2^+ - v_2^-$

$|\Delta \bar{v}_2| = 3.7 \ [km/s]$

```
% Total deltaV
deltaV      = deltaV_1 + deltaV_2;
fprintf('The Hohmann transfer total deltaV is %.3f km/s',deltaV)
```

```
The Hohmann transfer total deltaV is 15.500 km/s
```

$|\Delta \bar{v}|_{total} = |\Delta \bar{v}_2| + |\Delta \bar{v}_1|$

$|\Delta \bar{v}|_{total} = 15.5 \ [km/s]$

The total $|\Delta v|$ for the departing maneuever is more than 3 times larger than the $|\Delta v|$ for the arrival manuever.

```
% Period of transfer orbit
period_T     = 2*pi*sqrt(a_T^3/mu_sun);

% Time of flight for hohmann transfer - Julian years
TOF          = (period_T/2);
fprintf('The time of flight is %.3f Julian years',TOF/(86400 * 365.25))
```

```
The time of flight is 45.541 Julian years
```

$$Period = 2\pi\sqrt{\frac{a^3}{\mu}}$$

$$T.O.F = Period/2$$

$$T.O.F. = 45.54\ [\text{Julian Years}]$$

The time of flight using a Hohmann transfer was more than 45 Julian years. The actual time of flight from Earth to Pluto used in the New Horizons mission was 9.48 Julian years. This is 20% of the travel time required by the Hohmann transfer, and likely why a Hohmann transfer wasn't used in the New Horizons mission as the Hohmann transfer would take far too long. Therefore, it is not likely that we could generally use a Hohmann transfer path to get to Pluto, unless we had no time constraints. Interplanetary missions are planned to be no longer than 14 years, this is likely due to the limited lifespan of a typical propulsion/power system. Any longer than this time frame, and the propulsion system may fail to perform the required maneuvers.

Problem 3d)

Find: The phase angle at Earth departure and the synodic period

```
% Mean motion for Earth
n_earth    = sqrt(mu_sun/r_sun_earth^3);

% Mean motion  for Pluto
n_pluto    = sqrt(mu_sun/r_sun_pluto^3);

% Synodic period
ts         = 2*pi/(n_earth - n_pluto);
fprintf('The synodic period is %.3f Julian Years',ts/(86400 * 365.25));
```

```
The synodic period is 1.004 Julian Years
```

$$n = \sqrt{\frac{\mu}{a^3}}$$

$$t_s = \frac{2\pi}{n_{earth} - n_{pluto}}$$

$$t_s = 1.004\ [Julian\ \ Years] = 366.73\ [Julian\ \ Days]$$

```
% Calculate phase angle of Earth
phase        = (pi - (n_pluto*TOF))*180/pi;
fprintf('The phase angle at Earth departure is %.2f deg',phase)
```

```
The phase angle at Earth departure is 113.93 deg
```

$$(n_{pluto})(T.O.F) = 180 - \phi$$

$$\phi = 113.9 \ [deg]$$

Problem 3e)

Find: $|\Delta\bar{v}|_{total}$ and TOF for Hohmann transfer from Earth (circular orbit) to Pluto perihelion (elliptic orbit), while neglecting local gravity fields

```
% Pluto eccentricity about sun
e_pluto     = 0.24885238;

% Pluto semi major axis about sun
a_pluto     = 5907150229;

% Pluto perihelion about sun
rp_pluto    = a_pluto*(1 - e_pluto);

% Define transfer orbit radii - r1 is transfer ellipse periapsis, r2 is apoapsis
r1          = r_sun_earth;
r2          = rp_pluto;

% Transfer ellipse semi major axis for elliptic pluto orbit
a_Te        = (r1 + r2)/2;

% Eccentricity of transfer ellipse for ellipctic pluto orbit
e_Te        = 1 - r1/a_Te;

% Orbital speed post maneuver - at perigee of transfer ellipse
vp          = sqrt(2*(mu_sun/r1 - mu_sun/(2*a_Te)));

% Calculate first deltaV - Earth velocity unchanged from above
dv_1        = vp - v1_minus;
fprintf('The first deltaV magnitude is %.3f km/s',dv_1);
```

```
The first deltaV magnitude is 11.645 km/s
```

$$r_2 = a_{pluto}(1 - e_{pluto})$$

$$a_T = (r_1 + r_2)/2$$

$$e_T = 1 - \frac{r_1}{a_T}$$

$$v_1^- = \sqrt{\frac{\mu}{r_1}} \text{ - Circular Orbit}$$

$$v_p = \sqrt{2(\frac{\mu}{r_1} - \frac{\mu}{2a_T})}$$

$$|\Delta\bar{v}_1| = v_p - v_1^-$$

$$|\Delta\bar{v}_1| = 11.6 \ [km/s]$$

```
% Orbital speed prior to 2nd maneuver - apogee of transfer ellipse
va          = sqrt(2*(mu_sun/r2 - mu_sun/(2*a_Te)));

% Orbit speed after 2nd maneuver - pluto elliptic orbit
v2          = sqrt(2*(mu_sun/r2 - mu_sun/(2*a_pluto)));

% Calculate second deltaV
dv_2        = v2 - va;
fprintf('The second deltaV magnitude is %.3f km/s',dv_2);
```

```
The second deltaV magnitude is 4.715 km/s
```

$$v_2 = \sqrt{2(\frac{\mu}{r_2} - \frac{\mu}{2a_{pluto}})}$$

$$v_a = \sqrt{2(\frac{\mu}{r_2} - \frac{\mu}{2a_T})}$$

$$|\Delta\bar{v}_2| = v_2 - v_a$$

$$|\Delta\bar{v}_2| = 4.7 \ [km/s]$$

```
% Total deltaV
dv           = dv_1 + dv_2;
fprintf('The Hohmann transfer total deltaV is %.3f km/s',dv)
```

```
The Hohmann transfer total deltaV is 16.360 km/s
```

$$|\Delta\bar{v}|_{total} = |\Delta\bar{v}_2| + |\Delta\bar{v}_1|$$

$$|\Delta\bar{v}|_{total} = 16.36 \ [km/s]$$

The total $|\Delta v|$ for the higher fidelity Pluto model is 0.85 km/s higher than in the model which assumed Pluto's orbit is circular. This is a 5.54% increase, and shows that the assumption that Pluto's orbit is circular to be a poor one, as the increase in total $|\Delta v|$ is non-negligble. The asssumption does not save $|\Delta v|$.

```
% Period of transfer orbit
period_Te    = 2*pi*sqrt(a_Te^3/mu_sun);

% Time of flight for hohmann transfer - Julian years
TOF_Te       = (period_Te/2);
fprintf('The time of flight is %.3f Julian years',TOF_Te/(86400 * 365.25))
```

```
The time of flight is 30.012 Julian years
```

$$Period = 2\pi\sqrt{\frac{a^3}{\mu}}$$

$$T.O.F = Period/2$$

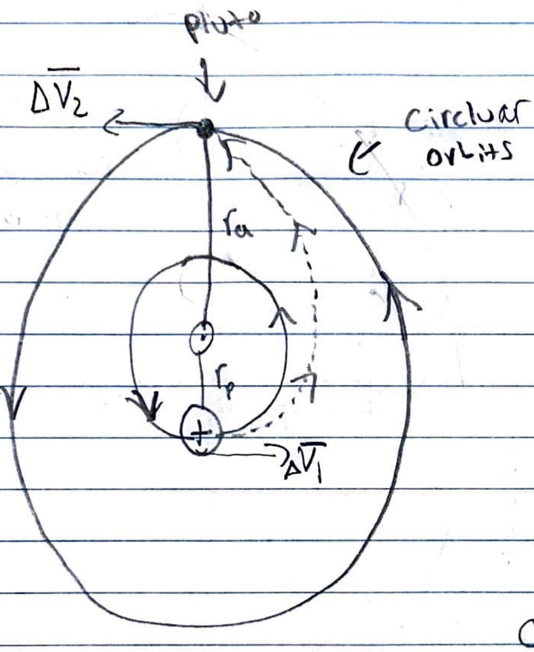
$$T.O.F. = 30.01 \ [\text{Julian Years}]$$

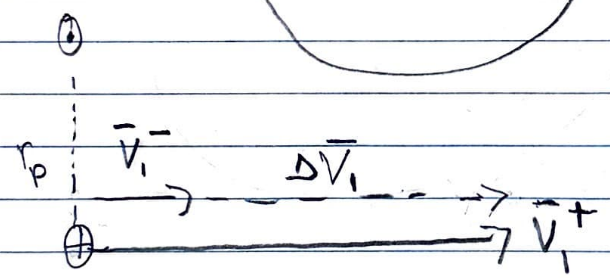
The higher fidelity Pluto model saves 15 years on the time of flight, a substantial improvement. This is because the resulting semi major axis of the transfer ellipse is much smaller, and

therefore decreases the period of the transfer orbit.

HW7

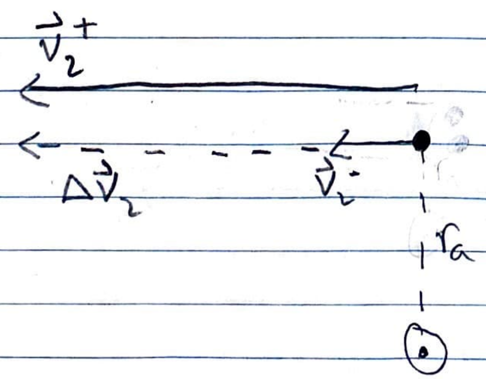
3c)





Circle, so tangential

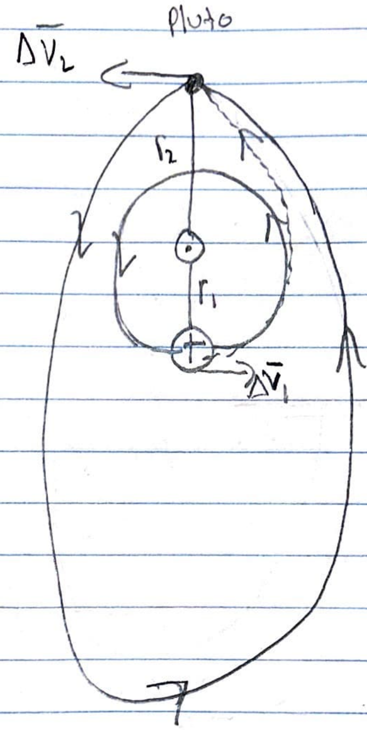
$\underline{\alpha = \beta = 0}$ $\gamma = 0$



Circle, so tangential

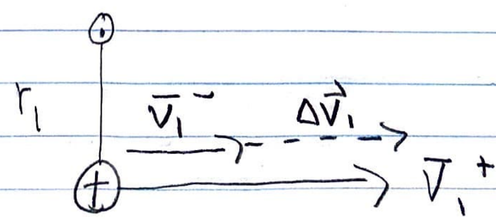
$\underline{\alpha = \beta = 0}$ $\gamma = 0$

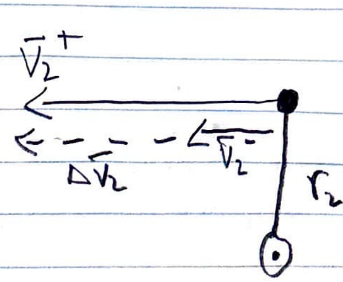
3d)



Exaggerated
Circle to Ellipse
Hohmann transfer

$\alpha = \beta = 0 = \gamma$
(circular orbit,
tangential)

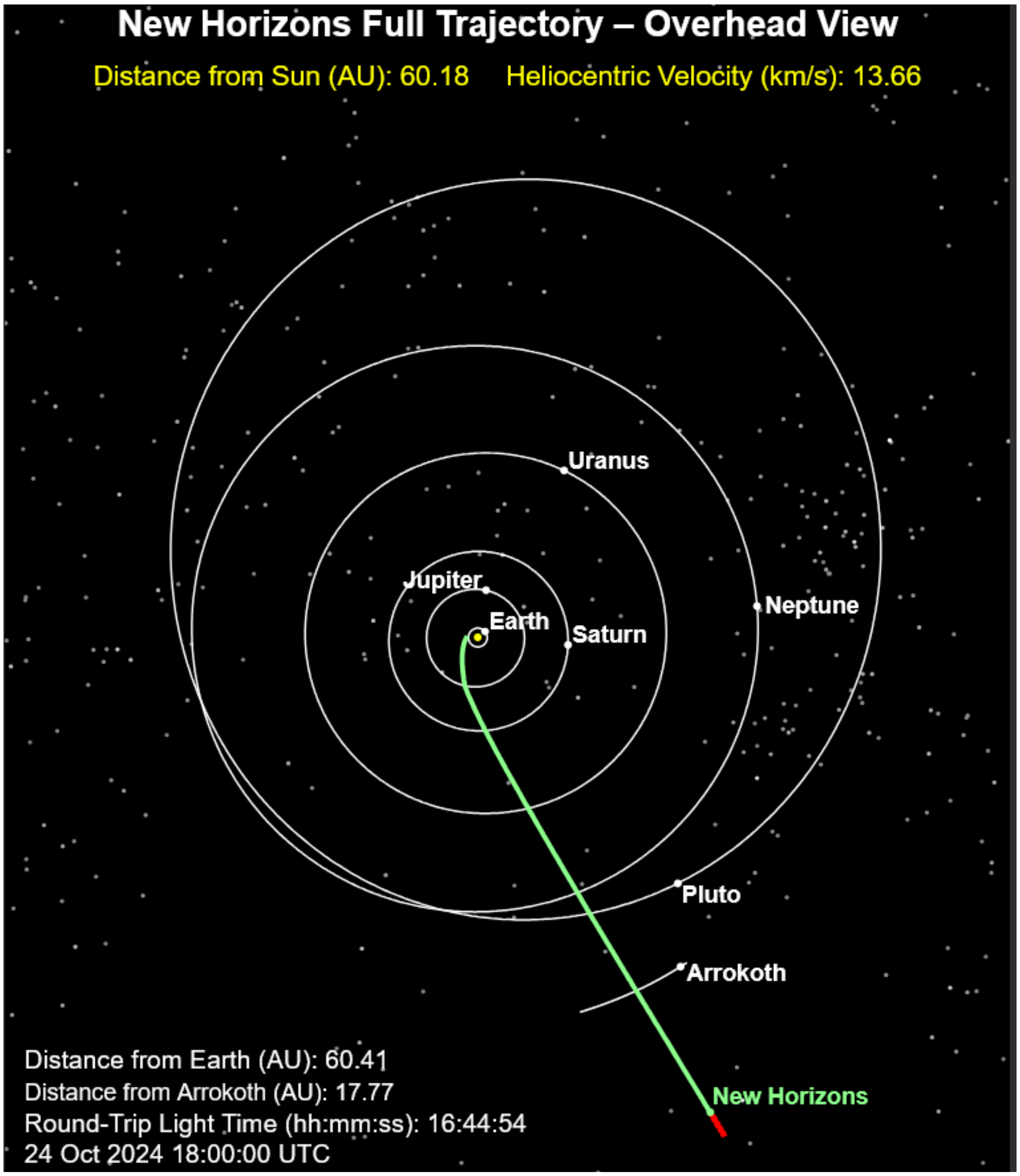




At periapsis of pluto orbit, tangential
$\alpha = \beta = 0 = \gamma$

Problem 3a)

In the figure below, it is shown that on October 24, 2024 the New Horizons spacecraft is 60.18 AU away from the sun, and moving with a velocity of 13.66 km/s with respect to the sun. Assuming this velocity to be constant, it can be calculated that the New Horizons spacecraft traveled 10622016 km or 0.071 AU since October 15, 2024. Therefore, **on October 15, 2024 the New Horizons spacecraft was 60.11 AU from the Sun**.



References: JHU APL Current Position – New Horizons Beyond Pluto

https://pluto.jhuapl.edu/Mission/Where-is-New-Horizons.php

Problem 3b)

Calendar Date	Julian Days
2006-01-19 12:00	2453755
2007-02-28 12:00	2454160
2011-03-18 12:00	2455639
2015-07-14 12:00	2457218
2019-01-01 12:00	2458485
2021-04-17 12:00	2459322
2024-10-15 12:00	2460599

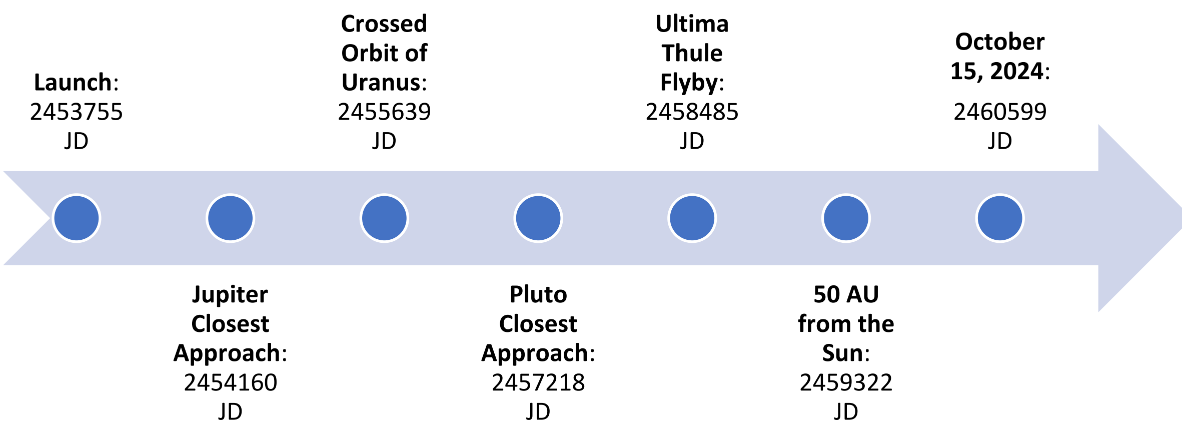


Figure 1: New Horizons Timeline in Julian days

Between the New Horizons launch and its closest approach of Jupiter, 405 Julian days (1.1088 Julian Years) passed. The next major event was the crossing of the Uranus' orbit, which occurred 1479 Julian days (4.0493 Julian years) later. New Horizons closest approach of Pluto occurred 1579 Julian days (4.3231 Julian years) after it crossed Uranus' orbit. The flyby of Ultima Thule took place 1267 Julian days later (3.4689 Julian years). 837 Julian days (2.2916 Julian years) after the Ultima Thule flyby, New Horizons reach a distance of 50 AU from the sun. Since this accomplishment, 1277 Julian days have passed (3.4962 Julian years) until the latest date of interest 10/15/24. In total, the New Horizons mission has been going on for 6844 Julian days (18.7379 Julian years).