Gabriel Colangelo

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clear
close all
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Problem 2a) Hohmann Transfer

Find: Total $|\Delta\bar{v}|$, time of flight, phase angle at departure, and time until geomerty repeats.

Assume: Circular, coplaner orbits, OTV and space station masses are negligible

```
% Earth Parameters
mu_earth        = 398600.4415;
R_earth         = 6378.1363;

% OTV original circular orbit radius
r1              = 2*R_earth;

% Space station circular orbit radius
r2              = 6*R_earth;

% OTV original circular orbit speed
v1              = sqrt(mu_earth/r1);

% Semi-major axis of hohmann transfer ellipse
a               = (r1 + r2)/2;

% Transfer ellipse energy
energy          = -mu_earth/(2*a);

% Velocity at periapsis of transfer ellipse
vp              = sqrt(2*(energy + mu_earth/r1));

% First maneuver deltaV: vectors along same direction. Treat as scalars
dv1             = vp - v1;

% Velocity at apoapsis of transfer ellipse
va              = sqrt(2*(energy + mu_earth/r2));

% OTV orbit speed in final space station orbit
v2              = sqrt(mu_earth/r2);

% 2nd maneuver deltaV: vectors along same direction. Treat as scalars
dv2             = v2 - va;

% Total deltaV
dv_hohmann      = dv1 + dv2;
fprintf('The total deltaV for the Hohmann transfer is %.3f km/s',dv_hohmann)
```

```
The total deltaV for the Hohmann transfer is 2.202 km/s
```

$a_T = (r_a + r_p)/2$

$v_1^- = \sqrt{\frac{\mu}{r_p}}$ OTV Initial Circular Orbit Velocity Magnitude

$v_1^+ = \sqrt{2(\frac{\mu}{r_p} - \frac{\mu}{2a_T})}$ Velocity magnitude at periapsis of transfer ellipse

$\Delta\bar{v}_1 = \bar{v}_1^+ - \bar{v}_1^-$ Vectors along same direction, treat as scalars, $\alpha = 0$

$|\Delta\bar{v}_1| = 1.256\ [km/s]$

$v_2^+ = \sqrt{\frac{\mu}{r_a}}$ Final Circular Orbit Velocity Magnitude

$v_2^- = \sqrt{2(\frac{\mu}{r_a} - \frac{\mu}{2a_T})}$ Velocity magnitude at apoapsis of transfer ellipse

$|\Delta\bar{v}_2| = \bar{v}_2^+ - \bar{v}_2^-$ Vectors along same direction, treat as scalars, $\alpha = 0$

$|\Delta\bar{v}_2| = 0.945\ [km/s]$

$|\Delta\bar{v}|_{total} = |\Delta\bar{v}_2| + |\Delta\bar{v}_1|$

$|\Delta\bar{v}|_{total} = 2.202\ [km/s]$

```
% Hohmann transfer ellipse period
tau_hohmann     = 2*pi*sqrt(a^3/mu_earth);

% Time of flight
TOF_hohmann     = tau_hohmann/2;
fprintf('The time of flight is %.2f hours',TOF_hohmann/3600)
```

```
The time of flight is 5.63 hours
```

$Period = 2\pi\sqrt{\frac{a_T^3}{\mu}}$

$T.O.F. = period/2$

$T.O.F. = 5.63\ [hours]$

```
% Mean motion for space station about earth
n_ss           = sqrt(mu_earth/r2^3);

% Calculate phase angle of space station at depature
phase_ss       = (pi - n_ss*TOF_hohmann)*180/pi;
fprintf('The phase angle at departure is %.2f deg',phase_ss)
```

```
The phase angle at departure is 82.02 deg
```

$n_{ss} = \sqrt{\frac{\mu_{earth}}{r_a^3}}$ mean motion for circular orbit

$(n_{ss})(T.O.F) = 180 - \phi$

$\phi = 82.02\ [deg]$ Phase angle at departure

```
% Mean motion for OTV in initial orbit
n_OTV          = sqrt(mu_earth/r1^3);

% Synodic period
ts             = 2*pi/(n_OTV - n_ss);
fprintf('The synodic period is %.3f Hours',ts/3600);
```

```
The synodic period is 4.932 Hours
```

$n_{OTV} = \sqrt{\frac{\mu}{r_p^3}}$ mean motion for circular orbit

$t_s = \frac{2\pi}{n_{OTV} - n_{ss}}$ synodic period

$t_s = 4.93 \ \left[hours\right] = 17755 \ \left[sec\right]$

The geometry repeats in 4.93 hours

Problem 2b) Minimum energy transfer, transfer angle of 240 deg

$\underline{Find}$: $a, e, r_p, TOF, r_D, v_D, \gamma_D, \theta^*_D, r_A, v_A, \gamma_A, \theta^*_A$ for minimum energy transfer

```
% TA = 240 = 360 - phi
phi      = 120;

% Chord, law of cosines
c        = sqrt(r1^2 + r2^2 - 2*r1*r2*cosd(phi));

% Minimum semi major axis
a_m      = (r1 + r2 + c)/4;
fprintf('The minimum energy semi major axis is %.2f km',a_m)
```

```
The minimum energy semi major axis is 24254.62 km
```

$\phi = 360 - TA = 120 \ \left[deg\right]$

Law of cosines from space triangle: $c^2 = r_1^2 + r_2^2 - 2r_1r_2\cos\phi$

Minimum energy semi-perimeter: $4a_m = r_1 + r_2 + c$

$a_m = 24254.62 \ \left[km\right]$

```
% Law of sines
x        = asind(r1*sind(phi)/c);

% Law of cosines
OF       = sqrt(r2^2 + (2*a_m - r2)^2 -(2*r2*(2*a_m-r2)*cosd(x)));
```

```
% Solve for eccentricity
e_m     = OF/(2*a_m);
fprintf('The minimum energy ellipse eccentricity is %.3f',e_m)
```

```
The minimum energy ellipse eccentricity is 0.586
```

Law of sines from space triangle: $\dfrac{c}{\sin \phi} = \dfrac{r_1}{\sin x}$

Law of cosines from space triangle: $OF^2 = r_2^2 + (2a_m - r_2)^2 - 2r_2(2a_m - r_2)\cos x$

$OF = 2a_m e_m$

$e_m = 0.586$

```
% Periapsis distance for minimum energy ellipse
rp_m     = a_m*(1 - e_m);
fprintf('The periapsis distance for the minimum energy ellipse is %.2f km', rp_m);
```

```
The periapsis distance for the minimum energy ellipse is 10037.24 km
```

$r_p = a_m(1 - e_m)$

$r_p = 10037.24 \ [km]$

```
% Semi latus rectum
p          = a_m*(1 - e_m^2);

% Calculate true anomaly at departure
theta_D    = -acosd((p/r1 - 1)/e_m);

% Calculate true anomaly at arrival
theta_A    = acosd((p/r2 - 1)/e_m);
```

$r_D = r_1 = 12756.27 \ [km]$

$r_A = r_2 = 38268.82 \ [km]$

$p = a_m(1 - e_m^2)$

$\theta^* = \cos^{-1}\left(\frac{1}{e_m}\left(\frac{p}{r} - 1\right)\right)$

$\theta_D^* = \pm 64.96 \ [deg]$

$\theta_A^* = \pm 175.04 \ [deg]$

4 possible combinations of θ_1^* and θ_2^*. However, only one pair produces a transfer angle of 240 deg.

Therefore, the pair of true anomaly's must be:

$\theta_D^* = -64.96 \ [deg]$

$\theta_A^* = 175.04 \ [deg]$

```
% Calculate eccentric anomaly at arrival and departure
```

```
E_A      = 2*atan(tand(theta_A/2)/sqrt((1+e_m)/(1-e_m)));
E_D      = 2*atan(tand(theta_D/2)/sqrt((1+e_m)/(1-e_m)));

% Calculate Mean Anomaly at arrival and
M_A      = E_A - e_m*sin(E_A);
M_D      = E_D - e_m*sin(E_D);

% Mean motion of transfer arc
n        = sqrt(mu_earth/a_m^3);

% Time of flight
TOF_min = (M_A - M_D)/n;
fprintf('The time of flight for the minimum energy transfer is %.2f hours',TOF_min/3600);
```

```
The time of flight for the minimum energy transfer is 5.25 hours
```

$$\tan\left(\frac{\theta^*}{2}\right) = \sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E}{2}\right)$$

$$E = 2\tan^{-1}\left[\tan\left(\frac{\theta^*}{2}\right)/\sqrt{\frac{1+e}{1-e}}\right]$$

$$M = E - e_m \sin E$$

$$M_A = M_D + \sqrt{\frac{\mu}{a_m^3}}(t_A - t_D)$$

$$(t_A - t_D) = T.O.F = \frac{M_A - M_D}{\sqrt{\frac{\mu}{a_m^3}}}$$

$$T.O.F. = 5.25\ [hours]$$

```
% Calculate energy of transfer ellipse
energy_m    = -mu_earth/(2*a_m);

% Velocity magnitude at departure
v_D         = sqrt(2*(energy_m + mu_earth/r1));
fprintf('The velocity at departure on the minimum energy transfer ellipse is %.3f km/s',v_D)
```

```
The velocity at departure on the minimum energy transfer ellipse is 6.787 km/s
```

$$v_D = \sqrt{2\left(\frac{\mu}{r_D} - \frac{\mu}{2a_m}\right)}$$

$$v_D = 6.787\ [km/s]$$

```
% Velocity magnitude at arrival
v_A         = sqrt(2*(energy_m + mu_earth/r2));
fprintf('The velocity at arrival on the minimum energy transfer ellipse is %.3f km/s',v_A)
```

```
The velocity at arrival on the minimum energy transfer ellipse is 2.097 km/s
```

$v_A = \sqrt{2(\frac{\mu}{r_A} - \frac{\mu}{2a_m})}$

$v_A = 2.097 \ [km/s]$

```
% Minimum energy transfer specific angular momentum
h_m         = sqrt(mu_earth*p);

% Flight path angle at departure
gamma_D     = sign(theta_D)*acosd(h_m/(r1*v_D));
fprintf('The flight path angle at departure is %.3f deg',gamma_D)
```

```
The flight path angle at departure is -23.051 deg
```

$h = \sqrt{\mu p}$

$h = r_D v_D \cos \gamma_D$

$\gamma_D = -23.05 \ [deg]$ Descending based on θ_D^*, so $\gamma < 0$

```
% Flight path angle at arrival
gamma_A     = sign(theta_A)*acosd(h_m/(r2*v_A));
fprintf('The flight path angle at arrival is %.3f deg',gamma_A)
```

```
The flight path angle at arrival is 6.949 deg
```

$h = \sqrt{\mu p}$

$h = r_A v_A \cos \gamma_A$

$\gamma_A = 6.949 \ [deg]$ ascending based on θ_A^*, so $\gamma > 0$

```
% Calculate phase angle of space station at depature
phase_min   = (240*pi/180 - n_ss*TOF_min)*180/pi;
fprintf('The phase angle at departure for the minimum energy transfer is %.2f deg',phase_min)
```

```
The phase angle at departure for the minimum energy transfer is 148.71 deg
```

$(n_{ss})(T.O.F) = 240 - \phi$

$\phi = 148.71 \ [deg]$ Phase angle at departure

Problem 2c) Determine maneuvers

$\underline{Find}$: $|\Delta\bar{v}_D|, \alpha_D, |\Delta\bar{v}_A|, \alpha_A,$ express also in VNB coordinates

```
% Departure velocity vector - orbit frame
v_D_vec     = [v_D*sind(gamma_D);v_D*cosd(gamma_D)];

% Departure deltaV
deltaV_D    = norm(v_D_vec - [0;v1]);
fprintf('The departure deltaV magnitude is %.4f km/s',deltaV_D)
```

```
The departure deltaV magnitude is 2.7369 km/s
```

$\bar{v}_D = v_D \sin\gamma_D \ \hat{r} + v_D \cos\gamma_D \ \hat{\theta}$

$\bar{v}_1 = v_1 \ \hat{\theta}$

$\Delta\bar{v}_D = \bar{v}_D - \bar{v}_1 = -2.657 \ \hat{r} + 0.655 \ \hat{\theta} \ [km/s]$

$|\Delta\bar{v}_D| = 2.7369 \ [km/s]$

```
alpha_D      = sign(gamma_D - 0)*asind(v_D*sind(abs(gamma_D))/deltaV_D);
fprintf('The angle alpha for the departure maneuver is %.3f deg',alpha_D)
```

```
The angle alpha for the departure maneuver is -76.154 deg
```

Law of sines: $\dfrac{\Delta v_D}{\sin|\gamma_D|} = \dfrac{v_D}{\sin\zeta}$

$\zeta = 76.1538, 103.8462 \ [deg]$

$v_D^2 > \Delta v_D^2 + v_1^2$, therefore $\zeta > 90$, $\zeta = 103.8462$

$\zeta_D = 180 - |\alpha_D|$

$\alpha_D = -76.1538 \ [deg]$ Negative due to decrease in \gamma, $\Delta\gamma = \gamma_D - \gamma_1 < 0$

```
% Beta for manuever
beta_D       = 0;

% Departure deltaV in VNC frame
dv_D_VNC     = deltaV_D*[cosd(beta_D)*cosd(alpha_D), cosd(beta_D)*sind(alpha_D), sind(beta_D)];

% Departure deltaV in VNB frame
dv_D_VNB     = dv_D_VNC.*[1,-1,1];
```

Planar maneuver, $\beta_D \ = \ 0$

$[\Delta\bar{v}_D]_{VNC} = \Delta v_D[\cos\beta_D \cos\alpha_D \ \hat{V} + \cos\beta_D \sin\alpha_D \ \hat{C} + \sin\beta_D \ \hat{N}]$

$[\Delta\bar{v}_D]_{VNC} = 0.655 \ \hat{V} - 2.6574 \ \hat{C} \ [km/s]$

$\hat{C} = \hat{V} \times \hat{N}$

$\hat{B} = \hat{N} \times \hat{V}$

Therefore $\hat{C} = -\hat{B}$

$[\Delta\bar{v}_D]_{VNB} = 0.655 \ \hat{V} + 2.6574 \ \hat{B} \ [km/s]$

```
% Arrival velocity vector - orbit frame
v_A_vec      = [v_A*sind(gamma_A);v_A*cosd(gamma_A)];

% Arrival deltaV
deltaV_A     = norm([0;v2] - v_A_vec);
fprintf('The arrival deltaV magnitude is %.4f km/s',deltaV_A)
```

```
The arrival deltaV magnitude is 1.1735 km/s
```

$\bar{v}_A = v_A \sin\gamma_A \ \hat{r} + v_A \cos\gamma_A \ \hat{\theta}$

$\bar{v}_2 = v_2 \ \hat{\theta}$

$\Delta\bar{v}_A = \bar{v}_2 - \bar{v}_A = -0.2537 \ \hat{r} + 1.1457 \ \hat{\theta} \ [km/s]$

$|\Delta\bar{v}_A| = 1.1735 \ [km/s]$

```
% Arrival alpha
alpha_A     = sign(0 - gamma_A)*asind(v2*sind(abs(gamma_A))/deltaV_A);
fprintf('The angle alpha for the arrival maneuver is %.3f deg',alpha_A)
```

```
The angle alpha for the arrival maneuver is -19.435 deg
```

Law of sines: $\dfrac{\Delta v_A}{\sin|\gamma_A|} = \dfrac{v_2}{\sin\zeta}$

$\zeta = 19.4353, 160.5647 \ [deg]$

$v_2^2 > \Delta v_A^2 + v_A^2$, therefore $\zeta > 90$, $\zeta = 160.5647$

$\zeta_A = 180 - |\alpha_A|$

$\alpha_A = -19.435 \ [deg]$ Negative due to decrease in γ, $\Delta\gamma = \gamma_2 - \gamma_A < 0$

```
% Beta for manuever
beta_A      = 0;

% Arrrival deltaV in VNC frame
dv_A_VNC    = deltaV_A*[cosd(beta_A)*cosd(alpha_A), cosd(beta_A)*sind(alpha_A), sind(beta_A)];

% Arrival deltaV in VNB frame
dv_A_VNB    = dv_A_VNC.*[1,-1,1];
```

Planar maneuver, $\beta_A = 0$

$[\Delta\bar{v}_A]_{VNC} = \Delta v_A[\cos\beta_A \cos\alpha_A \ \hat{V} + \cos\beta_A \sin\alpha_A \ \hat{C} + \sin\beta_A \ \hat{N}]$

$[\Delta\bar{v}_A]_{VNC} = 1.1066 \ \hat{V} - 0.3905 \ \hat{C} \ [km/s]$

$\hat{C} = \hat{V} \times \hat{N}$

$\hat{B} = \hat{N} \times \hat{V}$

Therefore $\hat{C} = -\hat{B}$

$[\Delta\bar{v}_A]_{VNB} = 1.1066 \ \hat{V} + 0.3905 \ \hat{B} \ [km/s]$

```
% Total deltv
deltaV_tot      = deltaV_A + deltaV_D;
fprintf('The total deltaV for the minimum energy transfer is %.3f km/s',deltaV_tot)
```

```
The total deltaV for the minimum energy transfer is 3.910 km/s
```

$$|\Delta\bar{v}_{total}| = |\Delta\bar{v}_A| + |\Delta\bar{v}_D|$$
$$|\Delta\bar{v}_{total}| = 3.91 \ [km/s]$$

The time of flight for the minimum energy transfer is 5.25 hours, which is slightly shorter than the hohmann transfer time of flight of 5.63 hours. However, the deltaV required for the minimum energy transfer is 3.91 km/s, a 77% percent increase from the deltaV required for the Hohmann transfer (2.2 km/s). Therefore, the tradeoff of increased deltaV for a slightly shorter TOF is not ideal, and the Hohmann transfer is the better option.

Problem 2d)

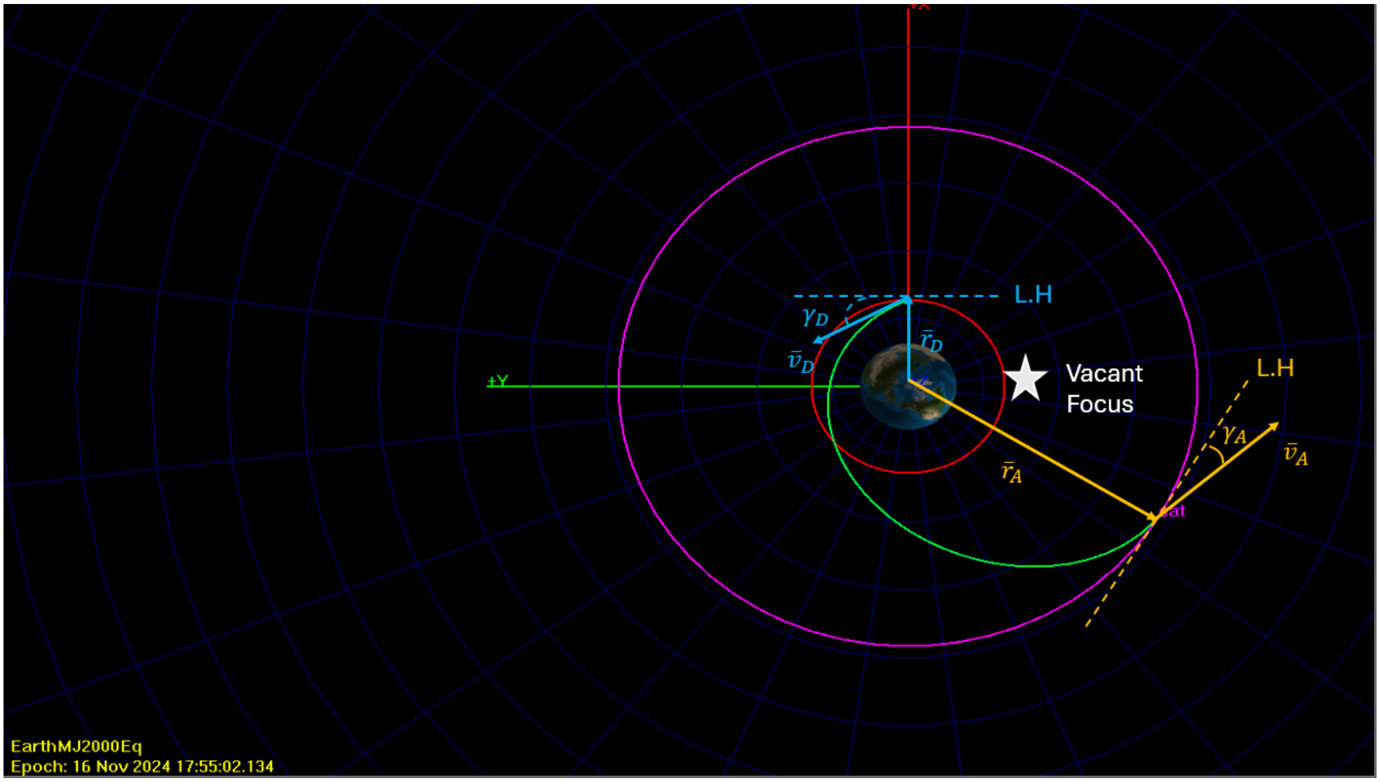


Figure 1: Problem 2d

The figure above shows initial orbit (red line), the transfer arc (green line), and the final orbit (pink line). On the following page, there are two sets of tables. One contains all the transfer ellipse quantities at the departure and arrival points. In this table, the FPA from GMAT was converted to our conventions FPA by subtracting the GMAT FPA from 90 degrees. The TOF was calculated by using the elapsed time at departure and subtracting that from the elapsed time at arrival, then this value was converted into hours. The other table contains the deltaV calculation for both arrival and departure. This was done by taking the difference in the velocity vectors at the time step prior and after the maneuver, then the deltaV was the norm of this difference. The quantities in both tables align very closely with what was calculated in problems 2b and 2c.

	Departure	Arrival
SMA [km]	24254.62134	24254.62134
ECC	0.586172027	0.586172027
RadPer [km]	10037.24078	10037.24078
R MAG [km]	12756.00686	38268.8178
V MAG [km/s]	6.786907697	2.097046705
TA [deg]	295.04039	175.037585
FPA [deg]	-23.05015476	6.948943123
Elpased Time [s]	14338.36715	33230.3129
TOF [hours]	0	5.247762709

Figure 2: Transfer Ellipse Properties

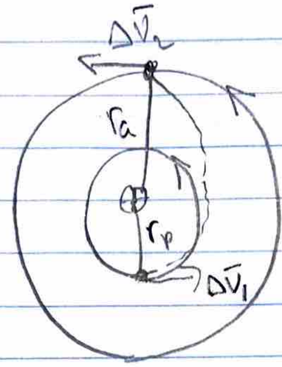
	Before Departure	After Departure
Vx [km/s]	3.04615E-10	-2.657629565
Vy [km/s]	5.589938012	6.244927637
Vz [km]	-1.51569E-17	9.3828E-18
DeltaV [km/s]	2.737152958	

	Before Arrival	After Arrival
Vx [km/s]	1.675899912	2.794982615
Vy [km/s]	-1.2605413	-1.613652537
Vz [km]	4.36057E-19	-1.88071E-18
DeltaV [km/s]	1.173470768	

Figure 3: DeltaV Calculations

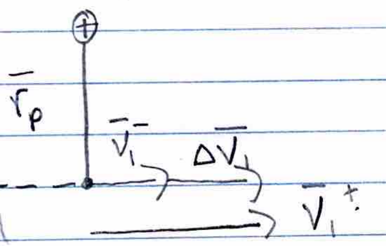
HW10

#2a)

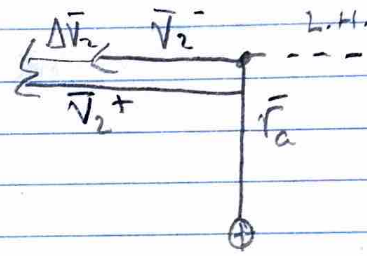


circular orbit ↓ periapsis ↓

$$\gamma_1^- = \gamma_1^+ = 0$$



$\underline{\alpha = 0}$, tangential

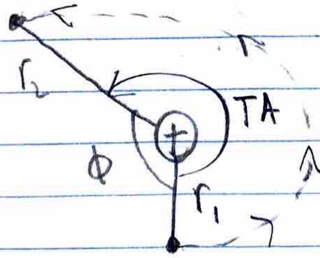


$\underline{\alpha = 0}$, tangent

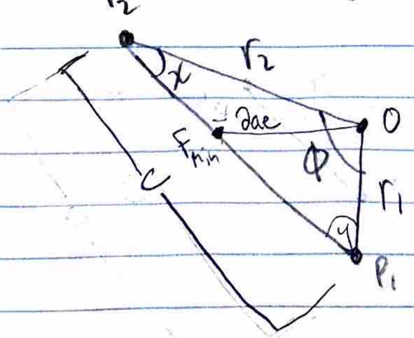
$$\gamma_2^- = \gamma_2^+ = 0$$

↑ apoapsis ↑ circular orbit

#2b)



TA = 240 , $\phi = 360 - 240 = 120°$

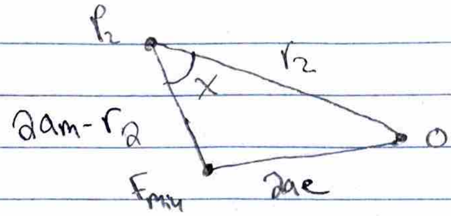


$$c^2 = r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\phi)$$

$$\frac{c}{\sin(\phi)} = \frac{r_1}{\sin(x)}$$

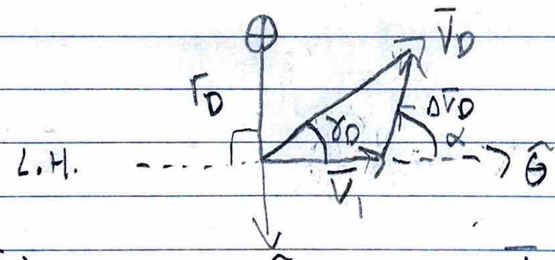
$$y = 180 - x - \phi$$

HW10



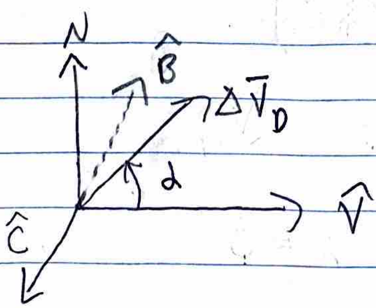
$$(2ae)^2 = r_2^2 + (2a_m - r_2)^2 - 2r_2(2a_m - r_2)\cos(x)$$

#2C)



$\gamma_1 = 0$ (circular orbit)

$$\frac{|\Delta \bar{V}_D|}{\sin(\gamma_D)} = \frac{|\bar{V}_D|}{\sin(180-\alpha)} \qquad \Delta\bar{V}_D = \bar{V}_D - \bar{V}_1$$



$\underline{\beta = 0}$, planar maneuver

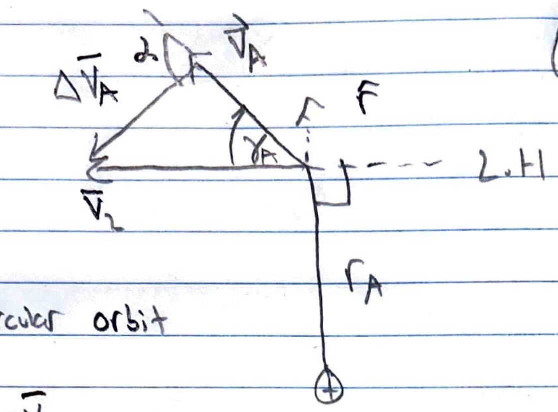
$$\Delta\bar{V}_D = \Delta V[\cancelto{1}{\cos(\beta)}\cos(\alpha)\,\hat{V} + \cancelto{1}{\cos(\beta)}\sin(\alpha)\,\hat{C} + \cancelto{0}{\sin(\beta)}\,\hat{N}]$$

$$\Delta\bar{V}_D = \Delta V[\cos(\alpha)\,\hat{V} + \sin(\alpha)\,\hat{C}]$$

$$\hat{C} = -\hat{B}$$

HW10

#2c)

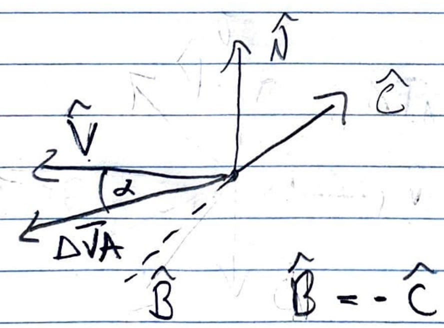


(Exagerrated)

$\gamma_2 = 0$, circular orbit

$\Delta\bar{V}_A = \bar{V}_2 - \bar{V}_A$

$$\frac{|\Delta\bar{V}_A|}{\sin(\gamma_A)} = \frac{|\bar{V}_2|}{\sin(180-\alpha)}$$



Maneuver in VNC Frame