# Gabriel Colangelo

```
clear
close all
clc
```

## Problem 3a)

Find: $|\Delta\bar{v}|_{total}$ for two plane change approaches

(i) a single maneuver at the current altitude to simply shift the orbit to an inclination of 90

ii) A bi-elliptic strategy that includes three maneuvers: a maneuver to raise apoapsis to 19,000 km , a plane change maneuver at apoapsis, a maneuver to insert back into the 100 km altitude polar orbit

```
% Moon Parameters
mu_moon      = 4902.8005821478;
R_moon       = 1738.2;

% Circular orbit altitude [km]
alt          = 100;

% Circular orbit radius
rc           = R_moon + alt;

% New inclination
i            = 90;
```

## Problem 3a) - Option 1

```
% Orbital speed prior to manuever - circlular orbit
vc           = sqrt(mu_moon/rc);

% Delta v for orbital inclination change
dv_1         = 2*vc*sind(i/2);
fprintf('The deltaV magnitude for the first option is %.3f km/s',dv_1)
```

```
The deltaV magnitude for the first option is 2.310 km/s
```

$|\bar{v}|^- = \sqrt{\dfrac{\mu}{r}}$ circular orbit

$|\bar{v}|^+ = |\bar{v}|^-$ for pure inclination change

$\Delta v = 2|\bar{v}|^- \sin\dfrac{\Delta i}{2}$ Isoceles triangle

$|\Delta\bar{v}|_{total} = 2.31\ \ [km/s]$

## Problem 3a) - Option 2

```
% Apoapsis of transfer ellipse
ra           = 19000;
```

```
% Transfer ellipse semi major axis
a           = (rc + ra)/2;

% Velcoity post 1st maneuver at periapsis
v1_plus_ii  = sqrt(2*(mu_moon/rc - mu_moon/(2*a)));

% 1st maneuver deltaV - subtract vectors, along same direction
deltaV1     = v1_plus_ii  - vc;
fprintf('The deltaV magnitude for the first maneuver is %.3f km/s',deltaV1)
```

```
The deltaV magnitude for the first maneuver is 0.572 km/s
```

$a = (r_p + r_a)/2$

$|\bar{v}_1^-| = \sqrt{\frac{\mu}{r_p}}$

$|\bar{v}_1^+| = \sqrt{2(\frac{\mu}{r_p} - \frac{\mu}{2a})}$

$\Delta\bar{v}_1 = \bar{v}_1^+ - \bar{v}_1^-$

$|\Delta\bar{v}_1| = 0.57 \ \ [km/s]$

$\alpha = 0$**, <u>no change in direction during manuever, applied tangentially at periapsis.</u>**

```
% Orbital speed prior to plane change at apoapsis
v2_minus_ii = sqrt(2*(mu_moon/ra - mu_moon/(2*a)));

% Delta v for orbital inclination change at apoapsis
deltaV2     = 2*v2_minus_ii*sind(i/2);
fprintf('The deltaV magnitude for the second maneuver is %.3f km/s',deltaV2);
```

```
The deltaV magnitude for the second maneuver is 0.302 km/s
```

$|\bar{v}|^- = \sqrt{2(\frac{\mu}{r_a} - \frac{\mu}{2a})}$

$|\bar{v}|^+ = |\bar{v}|^-$ for pure inclination change

$\Delta v = 2|\bar{v}|^- \sin\frac{\Delta i}{2}$ Isoceles triangle

$|\Delta\bar{v}|_{total} = 0.3 \ \ [km/s]$

```
% Velocity prior to third maneuever at periapsis
v3_minus_ii = sqrt(2*(mu_moon/rc - mu_moon/(2*a)));

% 3rd maneuver deltaV - subtract vectors, along same direction
deltaV3     = vc - v3_minus_ii;
fprintf('The deltaV magnitude for the third maneuver is %.3f km/s', abs(deltaV3))
```

```
The deltaV magnitude for the third maneuver is 0.572 km/s
```

$$|\bar{v}_3^+| = \sqrt{\frac{\mu}{r_p}}$$

$$|\bar{v}_3^-| = \sqrt{2(\frac{\mu}{r_p} - \frac{\mu}{2a})}$$

$$\Delta\bar{v}_3 = \bar{v}_3^+ - \bar{v}_3^-$$

$$|\Delta\bar{v}_3| = 0.57 \ [km/s]$$

$\alpha = 180$, **applied in direction opposite of $\bar{v}_3^-$.**

```
% Total deltaV
dv_2             = deltaV1 + deltaV2 + abs(deltaV3);
fprintf('The deltaV magnitude for the second option is %.3f km/s',dv_2)
```

```
The deltaV magnitude for the second option is 1.446 km/s
```

$$|\Delta\bar{v}|_{total} = |\Delta\bar{v}_3| + |\Delta\bar{v}_2| + |\Delta\bar{v}_1|$$

$$|\Delta\bar{v}|_{total} = 1.45 \ [km/s]$$

**The bi-elliptic option saves 0.86 km/s of $|\Delta\bar{v}|$. This option is 37% cheaper than the single maneuver option, a substantial saving.**

```
% Calculate bi-ellipitic time of flight
TOF               = 2*pi*sqrt(a^3/mu_moon);
fprintf('The time of flight for the bi-elliptic transfer is %.2f hours',TOF/3600)
```

```
The time of flight for the bi-elliptic transfer is 26.51 hours
```

$$\tau_1 = 2\pi\sqrt{\frac{a^3}{\mu}}$$

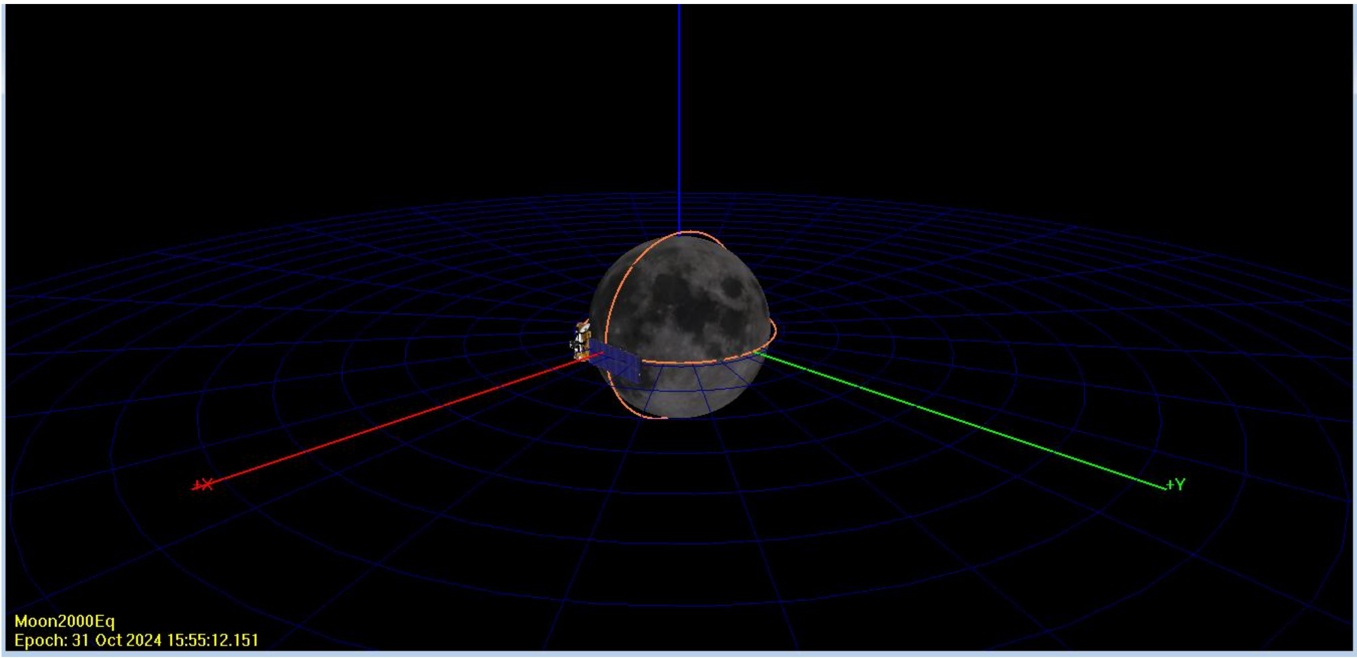
$$\tau_2 = 2\pi\sqrt{\frac{a^3}{\mu}}$$

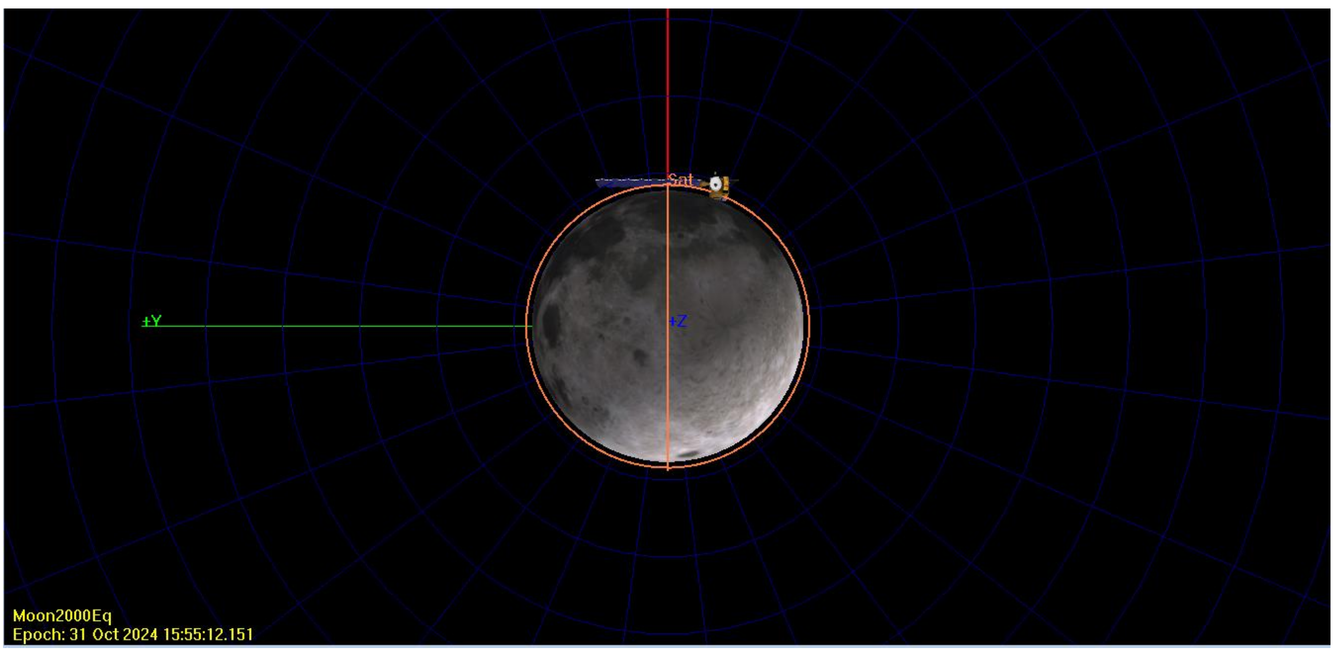
$$TOF = \tau_1/2 + \tau_2/2 = 2\pi\sqrt{\frac{a^3}{\mu}}$$

$$TOF = 26.5 \ [hours] = 1.1 \ [days]$$

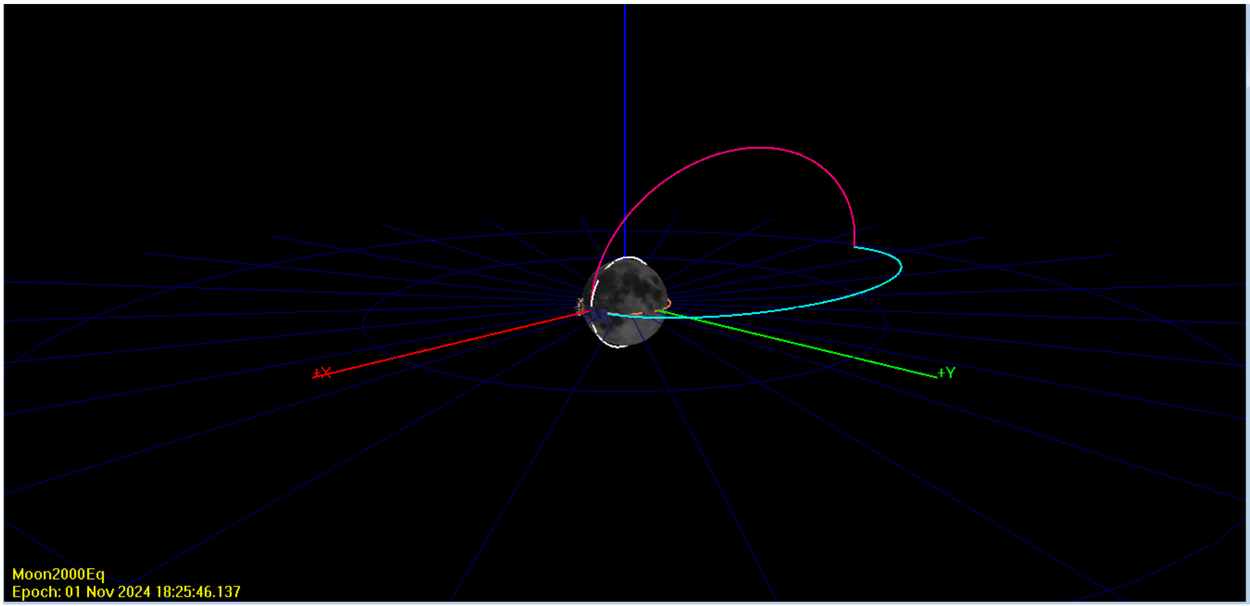
**The time of flight for the bi-elliptic option is 26.5 days, a considerable amount of time given the instanteneous nature of the first option. However, if the plane change is not time critical, the $|\Delta\bar{v}|$ savings may be worth the increased time of flight.**

**Problem 3b) Single Maneuver for plane change**



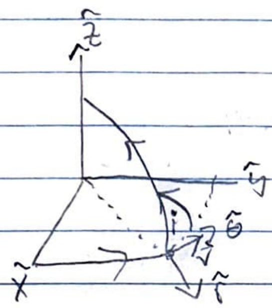
**Problem 3b) Bi-elliptic transfer for plane change**



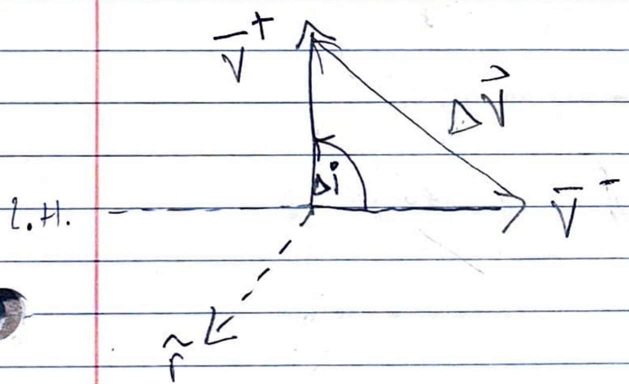

Orange line is original orbit. Blue line is 1st half of bi-elliptic transfer. Pink line is second half of bi-elliptic transfer, post plane change. The white line is the final orbit.

# HW8

3a)





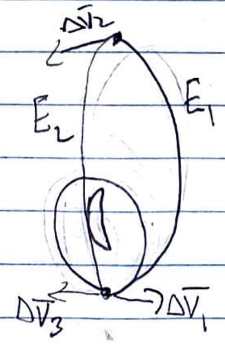




**Option 1**

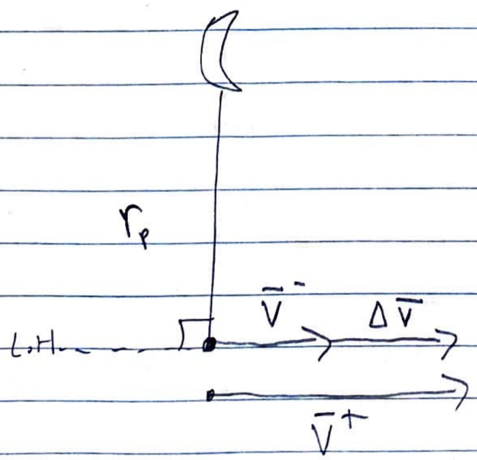
$\Delta V = 2v^- \sin(\Delta i)$

(Isosceles triangle - right triangle also)





**Option 2:**

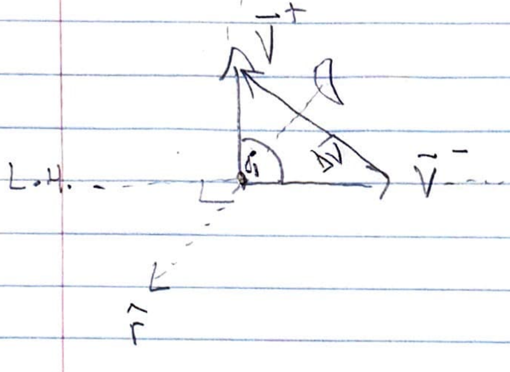




Circular orbit ↓ Periapsis of $E_1$ ↓

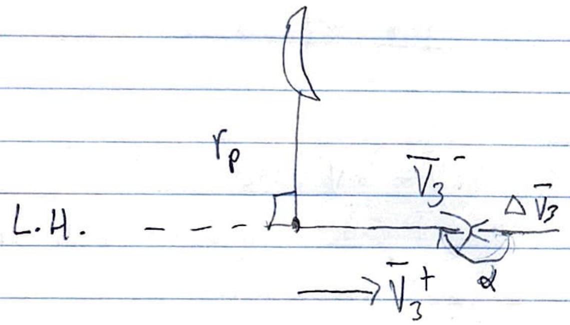
$\gamma_1^- = \gamma_1^+ = 0$

$\boxed{\alpha = 0}$, applied tangentially at periapsis of E1




$$|\bar{V}^-| = |\bar{V}^+|$$

$$\Delta V = 2|\bar{V}^-|\sin(\Delta i)$$




Periapsis of E2     Circular orbit

$$\gamma_3^- = \gamma_3^+ = 0$$

$$\boxed{\alpha = 180^\circ}$$