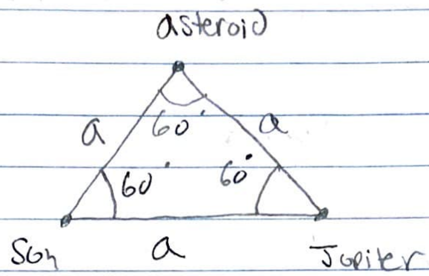
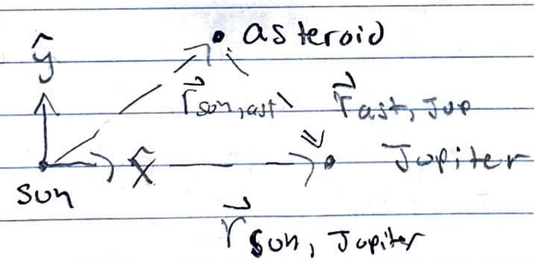
HW 3

Problem 1: Assume: Distance between Sun & Jupiter is equal to semi-major axis, $\mu = 10\ \frac{Km^3}{s^2}$



$a = 778279959$ Km

i) **Find:** $\ddot{r}_{sun,ast}$ (asteroid relative to sun)



$$\vec{r}_{sun,Jupiter} = a\,\hat{x}$$

$$\vec{r}_{sun,ast} = a\cos(60)\,\hat{x} + a\sin(60)\,\hat{y}$$

$$\ddot{\vec{r}}_{sun,ast} + \frac{G(m_{sun} + m_{ast})}{r_{sun,ast}^3}\vec{r}_{sun,ast} = G\,m_{Jupiter}\left(\frac{\vec{r}_{ast,Jup}}{r_{ast,Jup}^3} - \frac{\vec{r}_{sun,Jup}}{r_{sun,Jup}^3}\right)$$

$$\boxed{\ddot{\vec{r}}_{\odot,ast} = -\frac{G(m_\odot + m_{ast})}{r_{\odot,ast}^3}\vec{r}_{\odot,ast} + G\,m_{Jup}\left(\frac{\vec{r}_{ast,Jup}}{r_{ast,Jup}^3} - \frac{\vec{r}_{\odot,Jup}}{r_{\odot,Jup}^3}\right)}$$

Dominant term: $\boxed{\vec{a}_{dominant} = \frac{-G(m_{\odot} + m_{ast})}{r_{\odot,ast}^3} \vec{r}_{\odot,ast}}$

$$a_{dominant} = \frac{-(\mu_{\odot} + \mu_{ast})}{r_{\odot,ast}^3} \vec{r}_{\odot,ast} \qquad (\mu = Gm)$$

$$\vec{a}_{dominant} = (1.095 \times 10^{-7})(-\hat{x}) + (1.897 \times 10^{-7})(-\hat{y}) \ [km/s^2]$$

$$|\vec{a}_{dominant}| = 2.191 \times 10^{-7} \ [km/s^2]$$

Direct Pertubing term: $\boxed{\vec{a}_{direct} = Gm_{Jup} \left(\frac{\vec{r}_{ast,Jup}}{r_{ast,Jup}^3} \right)}$

$$\vec{r}_{sun,Jup} = \vec{r}_{sun,ast} + \vec{r}_{ast,Jup} \quad \therefore$$

$$\vec{r}_{ast,Jup} = \vec{r}_{sun,Jup} - \vec{r}_{sun,ast}$$

$$\vec{a}_{direct} = 1.046 \times 10^{-10} (\hat{x}) + 1.812 \times 10^{-10} (-\hat{y}) \ [km/s^2]$$

$$|\vec{a}_{direct}| = 2.092 \times 10^{-10} [km/s^2]$$

Indirect Perturbing term: $\boxed{\vec{a}_{indirect} = G\, m_{Jup} \left(\dfrac{\vec{r}_{Sun,Jup}}{r_{Sun,Jup}^{3}} \right)}$

$$\boxed{\begin{aligned} \vec{a}_{indirect} &= 2.092 \times 10^{-10} \, (\hat{x}) \ Km/s^2 \\ |\vec{a}_{indirect}| &= 2.092 \times 10^{-10} \ Km/s^2 \end{aligned}}$$

Net Perturbing acceleration: $\vec{a}_{direct} - \vec{a}_{indirect}$

$$\boxed{a_{perturbing} = 1.046 \times 10^{-10} (-\hat{x}) + 1.812 \times 10^{-10} (-\hat{y}) \ [Km/s^2]}$$

$|a_{perturbing}| = 2.092 \times 10^{-10} \ Km/s^2$

$\ddot{r}_{\odot,ast} = a_{net} = a_{dominant} + a_{perturbing}$

$$\boxed{\ddot{r}_{\odot,ast} = 1.096 \times 10^{-7} (-\hat{x}) + 1.899 \times 10^{-7} (-\hat{y}) \ [Km/s^2]}$$

$|\ddot{r}_{\odot,ast}| = 2.193 \times 10^{-7} \ Km/s^2$

ii) Find: $\ddot{r}_{Jupiter,asteroid}$

$$\boxed{\ddot{\vec{r}}_{Jup,ast} = \frac{-G(M_{Jup} + M_{ast})}{r_{Jup,ast}^{3}} \vec{r}_{Jup,ast} + G M_{\odot} \left(\frac{\vec{r}_{ast,\odot}}{r_{ast,\odot}^{3}} - \frac{\vec{r}_{Jup,\odot}}{r_{Jup,\odot}^{3}} \right)}$$

$\vec{r}_{ast,sun} = -\vec{r}_{sun,ast}$ $\odot$ - sun

$\vec{r}_{Jup,sun} = -\vec{r}_{sun,Jup}$

$\vec{r}_{Jup,ast} = -\vec{r}_{ast,Jup}$

Dominant term: $\boxed{\frac{-(\mu_{Jup} + \mu_{ast})\vec{r}_{Jup,ast}}{r_{Jup,ast}^{3}}} = a_{dominant}$

$$\boxed{\vec{a}_{dominant} = 1.046 \times 10^{-10}\,\hat{x} + 1.812 \times 10^{-10}\,(-\hat{y}) \; [km/s^2]}$$

$$|\vec{a}_{dominant}| = 2.092 \times 10^{-10} \; km/s^2$$

Direct Perturbing term: $G M_{\odot} \left(\frac{\vec{r}_{ast,\odot}}{r_{ast,\odot}^{3}} \right)$

$$\boxed{\vec{a}_{direct} = 1.095 \times 10^{-7}(-\hat{x}) + 1.897 \times 10^{-7}(-\hat{y}) \; [km/s^2]}$$

$$|\vec{a}_{direct}| = 2.191 \times 10^{-7} \; [km/s^2]$$

Indirect term: $\boxed{Gm_{\odot} \dfrac{\vec{r}_{Jup,\odot}}{r_{Jup,\odot}^3}}$

$$\boxed{\vec{a}_{indirect} = 2.191 \times 10^{-7} (-\hat{x}) \; [km/s^2]}$$

$$|\vec{a}_{indirect}| = 2.191 \times 10^{-7} \; [km/s^2]$$

Net perturbing Acceleration: $\vec{a}_{direct} - \vec{a}_{indirect}$

$$\boxed{\vec{a}_{perturbing} = 1.095 \times 10^{-7} (\hat{x}) + 1.897 \times 10^{-7} (-\hat{y}) \; [km/s^2]}$$

$$|a_{perturbing}| = 2.191 \times 10^{-7} \; [km/s^2]$$

Total net acceleration: $\ddot{\vec{r}}_{Jup,ast} = \vec{a}_{dominant} + \vec{a}_{perturbing}$

$$\boxed{\ddot{\vec{r}}_{Jup,ast} = 1.0965 \times 10^{-7} (\hat{x}) + 1.899 \times 10^{-7} (-\hat{y}) \; [km/s^2]}$$

$$|\ddot{\vec{r}}_{Jup,ast}| = 2.193 \times 10^{-7} \; km/s^2$$

iii) For the asteroid relative to the sun formulation, the dominant acceleration is the largest. For the asteroid relative to Jupiter formulation, the direct effect on the asteroid due to the sun is as equally large as the indirect effect on Jupiter due to the sun. For this formulation, the dominant effect is ~1000 times smaller than in the asteroid relative to the sun formulation.

The net perturbing acceleration in the asteroid relative to sun case is ~1000 smaller than in the asteroid relative to Jupiter case. For both formulations the sun has the largest impact.

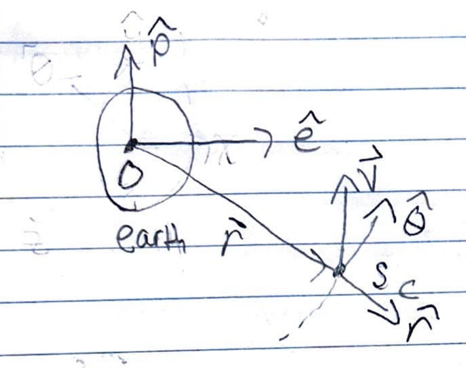
The total net acceleration on the asteroid has the same magnitude in both cases, but opposite $\hat{x}$ directions. This makes sense because the only thing changing is the reference point of view.

Both formulations are correct & provide the same net acceleration as the bodies are equally distanced. If they weren't, the various formulations could be used to gain more insight.

From these results, it is reasonable to model the asteroid motion as a two-body problem between the sun & the asteroid. This is because the gravitational effect of the sun is ~1000 times stronger than Jupiters.

Problem 3:

a)



$I = \{O, \hat{e}, \hat{p}, \hat{z}\}$

$C = \{SC, \hat{r}, \hat{\theta}, \hat{z}\}$

b) Find: C_3, specific angular momentum, Total Kinetic energy, C_4, specific energy, and areal velocity

Assume: Altitude (h) = 8560 Km, $V_r = -2.11$ Km/s, $V_\theta = 4.89$ Km/s, Perfect measurement, $m_{s/c} = 300$ Kg. Position along equator.

$$\bar{r} = r_{earth} + h\,\hat{r} = 14938.14\,\hat{r}\ [Km]$$

$$^{I}\vec{v} = V_r\,\hat{r} + V_\theta\,\hat{\theta} = \dot{\bar{r}} = {}^{I}\frac{d\bar{r}}{dt}$$

$$\bar{C}_3 = \frac{m_{earth}\, m_{sc}}{m_{earth} + m_{sc}} \left(\vec{r} \times {}^{I}\dot{\bar{r}}\right)$$

$$\boxed{\bar{C}_3 = 0\,\hat{r} + 0\,\hat{\theta} + 21914245.95\,\hat{z}\ \left[\frac{Kg\text{-}Km^2}{s}\right]}$$

$$\boxed{\bar{h} = \vec{r} \times \dot{\vec{r}} = 0\,\hat{r} + 0\,\hat{\theta} + 73047.49\,\hat{z}\ \left[\frac{Km^2}{s}\right]}$$

$$T = \frac{1}{2} \frac{m_e m_{sc}}{m_e + m_{sc}} (\dot{\vec{r}} \cdot \dot{\vec{r}})$$

$$\boxed{T = 4254.63 \ \frac{Kg\,Km^2}{s^2}}$$

$$C_4 = T - U$$

$$U = \frac{G m_e m_{sc}}{r} = 8005 \ \frac{Kg\,Km^2}{s^2}$$

$$\boxed{C_4 = -3750.39 \ \frac{Kg\,Km^2}{s^2}}$$

$$\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r} \qquad \mu = G(m_1 + m_2), \quad v = |\vec{v}|, \quad r = |\vec{r}|$$

$$\boxed{\mathcal{E} = -12.50 \ \frac{Km^2}{s^2}}$$

$$\dot{A} = \frac{h}{2} = \frac{|\vec{h}|}{2}$$

$$\boxed{\dot{A} = 36523.74 \ Km^2/s}$$

c) $C_4 \left(\frac{m_e + m_{sc}}{m_e\, m_{sc}}\right) = \mathcal{E}$

$$\boxed{\frac{m_e + m_{sc}}{m_e\, m_{sc}} = 0.00333\bar{3} \ [1/kg]}$$

d) <u>Find</u>: p, e, a, $\mathbb{P}$, γ, θ^*

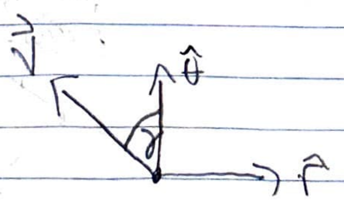
$$\boxed{p = \frac{h^2}{\mu} = 13386.68 \ km}$$

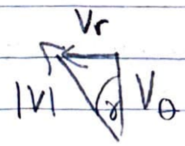
$\bar{e} = \frac{\dot{r} \times h}{\mu} - \hat{r}$ $\qquad \hat{r} = \frac{\bar{r}}{r}$, $e = |\bar{e}|$

$$\boxed{e = 0.4}$$

$$\boxed{a = \frac{p}{1-e^2} = 15942.3 \ [km]}$$

$$\boxed{\mathbb{P} = 2\pi\sqrt{\frac{a^3}{\mu}} = 20032.7 \ [s] = 5.56 \ [hours]}$$





$|V| \cos(\gamma) = V_\theta$

$$\gamma = \cos^{-1}\left(\frac{V_\theta}{|V|}\right)$$

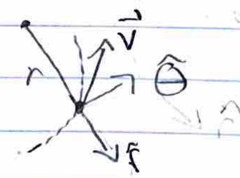
$$\boxed{\gamma = 23.34 \text{ [deg]}}$$

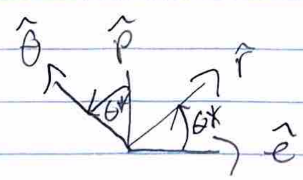
$$r = \frac{p}{1 + e\cos(\theta - \omega)} \qquad \theta - \omega = \theta^*$$

$$\therefore \quad 1 + e\cos(\theta^*) = \frac{p}{r}$$

$$\theta^* = \cos^{-1}\left(\frac{\frac{p}{r} - 1}{e}\right)$$

$$\boxed{\theta^* = 105.03 \text{ [deg]}}$$





$$\bar{r} = r\,\hat{r}$$

$$\hat{r} = \cos(\theta^*)\,\hat{e} + \sin(\theta^*)\,\hat{p}$$

$$\hat{\theta} = -\sin(\theta^*)\,\hat{e} + \cos(\theta^*)\,\hat{p}$$

$$\bar{r} = r\cos(\theta^*)\,\hat{e} + r\sin(\theta^*)\,\hat{p}$$

$$\boxed{\bar{r} = -3874.9\,\hat{e} + 14426.8\,\hat{p} \text{ [Km]}}$$

$$\bar{V} = V_r \hat{r} + V_\theta \hat{\theta}$$

$$= V_r \cos(\theta^*)\hat{e} - V_\theta \sin(\theta^*)\hat{e} + V_r \sin(\theta^*)\hat{p} + V_\theta \cos(\theta^*)\hat{p}$$

$$\boxed{\bar{V} = -4.175\,\hat{e} - 3.306\,\hat{p}\ [\text{Km/s}]}$$

e) $V_c = \sqrt{\frac{\mu}{r}}$

$V_c = 5.166$ Km/s

$|\vec{V}| = 5.326$ Km/s $= v$

$\sqrt{2}\, V_c = 7.305$ Km/s $\therefore$

The relative velocity is less than $\sqrt{2}\, V_c$ $\therefore$ this is an elliptical orbit

Verified by OCEL

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Problem 1- i) - Acceleration of asteroid relative to Sun

```
% Semi-major axis of Jupiters orbit
a                = 778279959;                % [km]

% Gravitational Parameters: mu = G*m [km^3/s^2]
mu_sun           = 132712440017.99;
mu_asteroid      = 10;
mu_jupiter       = 126712767.8578;

% Define 2-D position vectors [x-y] plane
% Assume equilateral triangle of 60 deg
r_sun_jupiter    = [a;0];                    % [km]
r_sun_asteroid   = [a*cosd(60); a*sind(60)]; % [km]

% Dominant acceleration of asteroid wrt sun [km/s^2]
a_sa_dominant       = -(mu_sun + mu_asteroid)*r_sun_asteroid...
                  /(norm(r_sun_asteroid)^3)
```

```
a_sa_dominant = 2×1
     -1.0954938498391e-07
    -1.89745100730055e-07
```

```
% Magnitude of dominant acceleration of asteroid wrt sun
a_sa_dominant_mag   = norm(a_sa_dominant)
```

```
a_sa_dominant_mag =
      2.1909876996782e-07
```

```
% Position vector from asteroid to Jupiter [km]
r_asteroid_jupiter = r_sun_jupiter - r_sun_asteroid;

% Direct perturbing acceleration due to Jupiter on asteroid wrt sun [km/s^2]
a_sa_direct           = mu_jupiter*r_asteroid_jupiter/(norm(r_asteroid_jupiter)^3)
```

```
a_sa_direct = 2×1
       1.045968696341e-10
    -1.81167092518919e-10
```

```
% Magnitude of direct perturbing acceleration on asteroid wrt sun
a_sa_direct_mag         = norm(a_sa_direct)
```

```
a_sa_direct_mag =
       2.091937392682e-10
```

```
% Indirect perturbing acceleration due to Jupiter on asteroid wrt sun [km/s^2]
a_sa_indirect           = mu_jupiter*r_sun_jupiter/(norm(r_sun_jupiter)^3)
```

```
a_sa_indirect = 2×1
       2.091937392682e-10
                        0
```

```
% Net perturbing acceleration on asteroid wrt sun
a_sa_pertubing           = a_sa_direct - a_sa_indirect
```

```
a_sa_pertubing = 2×1
    -1.045968696341e-10
  -1.81167092518919e-10
```

```
% Magnitude of perturbing acceleration on asteroid wrt sun
a_sa_pertubing_mag        = norm(a_sa_pertubing)
```

```
a_sa_pertubing_mag =
     2.091937392682e-10
```

```
% Net acceleration of asteroid wrt sun
a_sa_net                  = a_sa_dominant + a_sa_pertubing
```

```
a_sa_net = 2×1
  -1.09653981853544e-07
  -1.89926267822574e-07
```

```
% Magnitude of net acceleration of asteroid wrt sun
a_sa_net_mag               = norm(a_sa_net)
```

```
a_sa_net_mag =
   2.19307963707088e-07
```

Problem 1- ii) - Acceleration of asteroid relative to Jupiter

```
% Relevant position vectors
r_asteroid_sun          = -r_sun_asteroid;
r_jupiter_sun           = -r_sun_jupiter;
r_jupiter_asteroid      = -r_asteroid_jupiter;

% Dominant acceleration of asteroid wrt Jupiter [km/s^2]
a_ja_dominant           = -(mu_jupiter + mu_asteroid)*r_jupiter_asteroid...
                           /(norm(r_jupiter_asteroid)^3)
```

```
a_ja_dominant = 2×1
   1.04596877888743e-10
  -1.81167106816381e-10
```

```
% Magnitude of dominant acceleration of asteroid wrt Jupiter
a_ja_dominant_mag       = norm(a_ja_dominant)
```

```
a_ja_dominant_mag =
   2.09193755777486e-10
```

```
% Direct perturbing acceleration due to sun on asteroid wrt Jupiter [km/s^2]
a_ja_direct             = mu_sun*r_asteroid_sun/(norm(r_asteroid_sun)^3)
```

```
a_ja_direct = 2×1
  -1.09549384975655e-07
  -1.89745100715758e-07
```

```
% Magnitude of direct perturbing acceleration on asteroid wrt Jupiter
a_ja_direct_mag        = norm(a_ja_direct)
```

```
a_ja_direct_mag =
```

```
2.19098769951311e-07
```

```
% Indirect perturbing acceleration due to sun on asteroid wrt Jupiter [km/s^2]
a_ja_indirect           = mu_sun*r_jupiter_sun/(norm(r_jupiter_sun)^3)
```

```
a_ja_indirect = 2×1
    -2.19098769951311e-07
                        0
```

```
% Net perturbing acceleration on asteroid wrt Jupiter
a_ja_pertubing           = a_ja_direct - a_ja_indirect
```

```
a_ja_pertubing = 2×1
     1.09549384975655e-07
    -1.89745100715758e-07
```

```
% Magnitude of perturbing acceleration on asteroid wrt Jupiter
a_ja_pertubing_mag         = norm(a_ja_pertubing)
```

```
a_ja_pertubing_mag =
     2.19098769951311e-07
```

```
% Net acceleration of asteroid wrt Jupiter
a_ja_net                   = a_ja_dominant + a_ja_pertubing
```

```
a_ja_net = 2×1
     1.09653981853544e-07
    -1.89926267822574e-07
```

```
% Magnitude of net acceleration of asteroid wrt Jupiter
a_ja_net_mag                = norm(a_ja_net)
```

```
a_ja_net_mag =
     2.19307963707088e-07
```

Problem 3b)

```
r_earth     = 6378.1363;        % Radius of Earth [km]
altitude    = 8560;             % [km]

% Position vector from Earth CG to spacecraft (polar coordinates)
r_earth_sc  = [r_earth + altitude;0;0];

% Inertial velocity of spacecraft relative to earth (polar coordinates)
v_earth_sc  = [-2.11;4.89;0];   % [km/s]
m_sc        = 300;              % Spacecraft mass [kg]

% Gravitational constant km^3/kg-s^2
G           = 6.6743e-11*(1/1000)^3;

m_earth     = 398600.4415/G;    % Earth mass

% Total system angular momentum
C3          = (m_earth*m_sc)*cross(r_earth_sc,v_earth_sc)...
              /(m_earth + m_sc) % [kg-km^2/s]
```

```
C3 = 3×1
                         0
                         0
             21914245.9521
```

```
% Specific angular momentum
h_vec       = cross(r_earth_sc,v_earth_sc) % [km^2/s]
```

```
h_vec = 3×1
                         0
                         0
              73047.486507
```

```
% Total system kinetic energy
T             = 1/2*(m_earth*m_sc)*dot(v_earth_sc,v_earth_sc)...
               /(m_earth + m_sc) % [kg-km^2/s^2]
```

```
T =
                   4254.63
```

```
% Gravitional potential
U              = G*m_earth*m_sc/norm(r_earth_sc);

% Total energy
C4             = T - U           % [kg-km^2/s^2]
```

```
C4 =
         -3750.39352157545
```

```
% Magnitude of angular momentum
h              = norm(h_vec);

% Gravitional parmeter
mu             = G*(m_earth + m_sc);

% Specific energy
E              = norm(v_earth_sc)^2/2 - mu/norm(r_earth_sc)
```

```
E =
         -12.5013117385848
```

```
% Areal velocity
Adot             = h/2 % [km^2/s]
```

```
Adot =
             36523.7432535
```

Problem 3d)

```
% Semi-latus rectum
p                = h^2/mu   % [km]
```

```
p =
          13386.6768057515
```

```
% Eccentricity vector
e_vec               = cross(v_earth_sc,h_vec)/mu -...
```

```
                     r_earth_sc/norm(r_earth_sc);

% Eccentricity
e                  = norm(e_vec)
```

```
e =
       0.400383443095008
```

```
% Semi-major axis
sma                 = p/(1 - e^2) % [km]
```

```
sma =
        15942.3446849075
```

```
% Orbital Period
period               = 2*pi*sqrt(sma^3/mu)/3600 % [hours]
```

```
period =
        5.56463613853122
```

```
% Flight path angle
gamma                = acosd(v_earth_sc(2)/norm(v_earth_sc))
```

```
gamma =
        23.3398520112336
```

```
% True anomaly
theta_star           = acosd((p/norm(r_earth_sc) - 1)/e) % [deg]
```

```
theta_star =
         105.03439032896
```

```
% Rotation matrix from Inertial p & e frame to polar frame
DCM_C_I              = [cosd(theta_star), sind(theta_star), 0;...
                       -sind(theta_star), cosd(theta_star), 0;...
                       0, 0, 1];

% Position vector in semi-latus rectum and eccentricity unit vectors
r_earth_sc_ep         = DCM_C_I'*r_earth_sc
```

```
r_earth_sc_ep = 3×1
       -3874.93419372084
        14426.8084173774
                       0
```

```
% Velocity vector in semi-latus rectum and eccentricity unit vectors
v_earth_sc_ep          = DCM_C_I'*v_earth_sc
```

```
v_earth_sc_ep = 3×1
       -4.17528537426884
       -3.30623532789436
                       0
```

Problem 3e)

```
% Circular velocity
Vc                      = sqrt(mu/norm(r_earth_sc_ep))
```

```
Vc =
         5.16559887511456
```

```
% Magnitude of relative velocity
v_mag                    = norm(v_earth_sc_ep)
```

```
v_mag =
         5.32580510345619
```