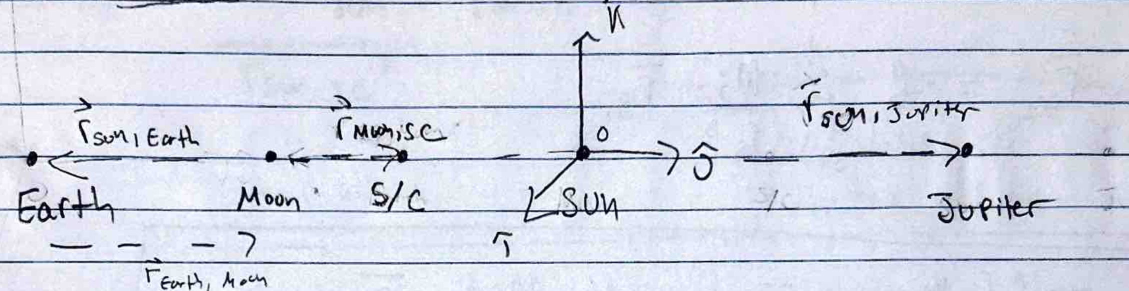
$\vec{r}_{A/C} = \vec{r}_{A/B} + \vec{r}_{B/C}$

HW2

1a) Find: System Center of Mass

Given: Earth - Moon - s/c - Sun - Jupiter aligned collinearly. $M_{s/c} = 130$ Kg. $r_{moon,s/c} = 77,500$ Km

Assume: Point O of Reference frame is aligned at sun



$\vec{r}_{sun,Jupiter} = 778279959\ \hat{j}$ [Km]

$\vec{r}_{sun,Earth} = -149597898\ \hat{j}$ [Km]

$\vec{r}_{sun,Moon} = \vec{r}_{sun,Earth} + \vec{r}_{Earth,Moon} = (-149597898 + 384400)\ \hat{j}$ [Km]

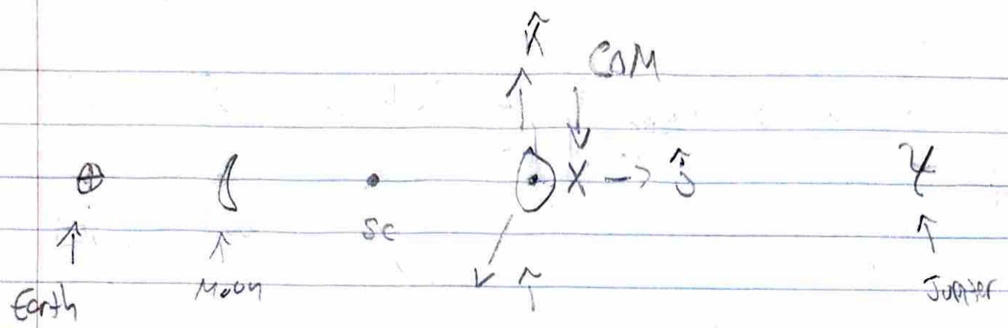
$\vec{r}_{sun,SC} = \vec{r}_{sun,Moon} + \vec{r}_{moon,SC} = -149213498 + 77500$

$$G = 6.6743 \times 10^{-11}\ \frac{m^3}{Kg\,s^2}\left(\frac{1\ Km}{1000\ m}\right)^3 = 6.6743 \times 10^{-20}\ \frac{Km^3}{Kg\,s^2}$$

$$R = \frac{\Sigma r_i m_i}{\Sigma m_i} = \left[(r_{sun,Jupiter})(M_{Jupiter}) + (r_{sun,earth})(M_{earth}) + (r_{sun,moon})(M_{moon}) + (r_{sun,sc})(M_{sc})\right] / M_{total}$$

$$M_{total} = M_{sc} + M_{earth} + M_{Jupiter} + M_{moon} + M_{sun}$$

$$\boxed{R = 74,129.96\ \hat{j}\ [Km]}$$ ← center of Mass



b) $$\ddot{\vec{r}}_{s/c} = -G \sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{m_j}{r_{ji}^3} \vec{r}_{ji}$$

$$\boxed{\ddot{\vec{r}}_{s/c} = -G\left[\frac{m_{earth}}{r_{earth,sc}^3}\vec{r}_{earth,sc} + \frac{m_{sun}}{r_{sun,sc}^3}\vec{r}_{sun,sc} + \frac{m_{moon}}{r_{moon,sc}^3}\vec{r}_{moon,sc} + \frac{m_{Jupiter}}{r_{Jupiter,sc}^3}\vec{r}_{Jupiter,sc} \right]}$$

$$\vec{r}_{earth,sc} = \vec{r}_{sun,sc} - \vec{r}_{sun,earth}$$

$$\vec{r}_{Jupiter,sc} = \vec{r}_{sun,sc} - \vec{r}_{sun,Jupiter}$$

($\vec{r}_{sun,com}$ terms cancel out in subtraction)

acceleration of spacecraft due to Earth

$$a_{earth,sc} = \frac{-G M_{earth} \vec{r}_{earth,sc}}{r_{earth,sc}^3} = \boxed{-1.8683 \times 10^{-6}\ \hat{j}\ [\mathrm{Km/s^2}]}$$

$$a_{moon,sc} = \frac{-G M_{moon} \vec{r}_{moon,sc}}{r_{moon,sc}^3} = \boxed{-8.1628 \times 10^{-7}\ \hat{j}\ [\mathrm{Km/s^2}]}$$

acceleration of s/c due to moon

acceleration of s/c due to Jupiter

$$a_{\text{Jupiter},sc} = \frac{-G\, m_{\text{jupiter}}\, \bar{r}_{\text{jupiter},sc}}{r_{\text{jupiter},sc}^{3}} = \boxed{1.4732 \times 10^{-10}\ \hat{j}\ [\mathrm{km/s^2}]}$$

$$a_{\text{sun},sc} = \frac{-G\, m_{\text{sun}}\, \bar{r}_{\text{sun},sc}}{r_{\text{sun},sc}^{3}} = \boxed{5.9669 \times 10^{-6}\ \hat{j}\ [\mathrm{km/s^2}]}$$

Acceleration of s/c due to the sun

The sun produces the largest acceleration on the space craft, while Jupiter produces the smallest. In order of acceleration magnitude, sun is the largest followed by the Earth, then moon, then Jupiter. The net acceleration is 3.2825×10^{-6} km/s along the positive $\hat{j}$ direction.

I used consistent number of sig figs in my computations.

c) The order of influence is not what I expected, as I expected the Earth to have the largest contribution as the acceleration is inversly proportional to the square of relative distance. Instead the sun is the most dominant due to the extreme mass difference. Outside of this the results align with expectation.

2a) <u>Find:</u> Relative acceleration of s/c wrt the moon

<u>Given:</u> Same orientation as problem 1

$$\bar{r}_{moon,sc} = \bar{r}_{sc} - \bar{r}_{moon}$$

$$^I\ddot{\bar{r}}_{moon,sc} = \ddot{\bar{r}}_{sc} - \ddot{\bar{r}}_{moon}$$

$$\ddot{\bar{r}}_{qi} + \frac{G(m_i + m_q)}{r_{qi}^3}\bar{r}_{qi} = G\sum_{\substack{j=1\\ j\neq i,q}}^{n}\left(\frac{\bar{r}_{ij}}{r_{ij}^3} - \frac{\bar{r}_{qj}}{r_{qj}^3}\right)m_j$$

Let $q = \text{moon}$, $i = s/c$

$$\boxed{\begin{aligned} ^I\ddot{\bar{r}}_{moon,sc} = {} & G\,m_{sun}\left(\frac{\bar{r}_{sc,sun}}{r_{sc,sun}^3} - \frac{\bar{r}_{moon,sun}}{r_{moon,sun}^3}\right) + G\,m_{Jupiter}\left(\frac{\bar{r}_{sc,Jupiter}}{r_{sc,Jupiter}^3} - \frac{\bar{r}_{moon,Jupiter}}{r_{moon,Jupiter}^3}\right) + \\ & - \frac{G(m_{sc} + m_{moon})\,\bar{r}_{moon,sc}}{r_{moon,sc}^3} + G\,m_{earth}\left(\frac{\bar{r}_{sc,earth}}{r_{sc,earth}^3} - \frac{\bar{r}_{moon,earth}}{r_{moon,earth}^3}\right)\end{aligned}}$$

$\bar{r}_{sc,Jupiter} = \bar{r}_{Jupiter} - \bar{r}_{sc} = 927415957 \ \hat{j}$ [Km]

$\bar{r}_{moon,Jupiter} = \bar{r}_{Jupiter} - \bar{r}_{moon} = 927493457 \ \hat{j}$ [Km]

Dominant Accelerations: $\dfrac{G(M_{sc} + M_{moon})\,\bar{r}_{moon,sc}}{r_{moon,sc}^3}$

$$= \boxed{8.1628 \times 10^{-7} \ Km/s} \ \hat{j}$$

Direct Acceleration: $G\,M_{sun}\left(\dfrac{\bar{r}_{sc,sun}}{r_{sc,sun}^3}\right) + G\,M_{Jupiter}\left(\dfrac{\bar{r}_{sc,Jupiter}}{r_{sc,Jupiter}^3}\right)$

$$+ G\,M_{earth}\left(\frac{\bar{r}_{sc,earth}}{r_{sc,earth}^3}\right) = \boxed{4.0987 \times 10^{-6} \ Km/s \ \hat{j}}$$

Indirect Acceleration: $G\,M_{sun}\left(\dfrac{\bar{r}_{moon,sun}}{r_{moon,sun}^3}\right) + G\,M_{Jupiter}\left(\dfrac{\bar{r}_{moon,Jupiter}}{r_{moon,Jupiter}^3}\right)$

$$+ G\,M_{earth}\left(\frac{\bar{r}_{moon,earth}}{r_{moon,earth}^3}\right) = \boxed{3.2633 \times 10^{-6} \ Km/s \ \hat{j}}$$

Total Pertubation: Direct - Indirect

$$= \boxed{8.3548 \times 10^{-7} \ Km/s \ \hat{j}}$$

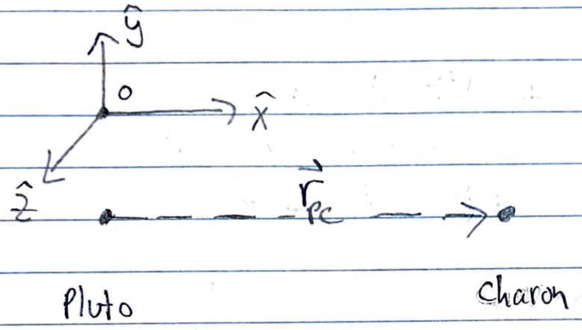
2b) The direct effect of the sun on the s/c has the same magnitude as the acceleration of the s/c due to the sun from problem 1 (5.9669×10^{-6} km/s). The same is true for the direct effect on the s/c from Jupiter (1.4732×10^{-10} km/s). Similarly, the direct effect of the earth on s/c is also the same magnitude as problem 1 (1.8683×10^{-6} km/s). This is expected as the direct effect is independent of the relative body.

2c) The largest acceleration on s/c was produced by the sun, while the smallest on s/c was by Jupiter. The spacecraft location would affect the impact the influence of bodies because the acceleration depends on the inverse square of the relative distance between bodies. The direction of the net pertubing force makes sense as it's towards the system center of mass.

2d) It is not reasonable to model the s/c motion with only the moon as it neglects the biggest two total pertubation acceleration contributors (Sun & Earth). Jupiter doesn't have significant influence (It's total pertubation is 2.4×10^{-14} km/s). Solar gravity is more significant (total pertubation is 6.19×10^{-9} km/s) than Jupiter's but is less significant than Earths total pertubation acceleration (8.2928×10^{-7} km/s). Because of this, I would remove Jupiter from the model. If 2 bodies were needed to be removed, it would be the sun & Jupiter. The total pertubation from sun & Jupiter is less than the dominant acceleration (At this instant).

3a) <u>Find:</u> Center of mass of Pluto - Chevron system

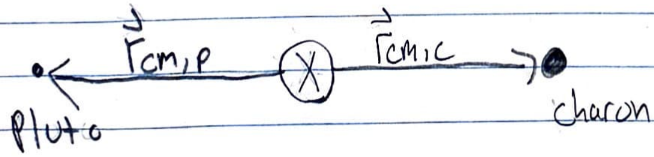
<u>Assume:</u> 2 Body Problem, no external Forces, Charon at semi-major axis



(Assume pluto is aligned at point O)

$$Cm = \frac{\sum m_i \vec{R}_i}{\sum m_i} = \frac{Gm_{Charon} \vec{r}_{pc}}{Gm_{charon} + Gm_{pluto}} = \frac{(119.480)(19596\,\hat{x})}{(981.601 + 119.480)}$$

$$\boxed{Cm = 2126.4 \text{ Km } \hat{x}}$$

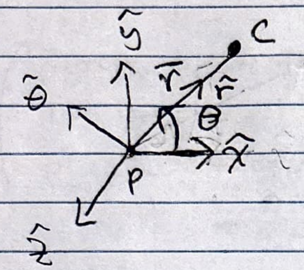


$$\vec{r}_{cm,p} = \vec{r}_p - \vec{r}_{cm} = -2126.4 \text{ Km } \hat{x}$$

$$\vec{r}_{cm,c} = \vec{r}_{pc} + \vec{r}_{cm,p} = 17469.6 \text{ Km } \hat{x}$$

The center of mass is outside the radios of pluto (581 Km).

3b)



$I = \{\hat{x}, \hat{y}, \hat{z}\}$

$C = \{\hat{r}, \hat{\theta}, \hat{z}\}$

$r = |\vec{r}_c - \vec{r}_p| = |\vec{r}_{pc}|$

$\boxed{\bar{r} = r\hat{r}}$

$^I V = \frac{^C d}{dt}(\bar{r}) + {}^{I}\bar{\omega}^{C} \times \bar{r}$

$^I V = \dot{r}\hat{r} + \dot{\theta}\hat{z} \times r\hat{r}$

$\boxed{^I V = \dot{r}\hat{r} + \dot{\theta} r \hat{\theta}}$

$\dot{r} = |\dot{\vec{r}}_c - \dot{\vec{r}}_p| = |\dot{\vec{r}}_{pc}|$

$|h| = |\bar{r} \times \dot{\bar{r}}| = r^2 \dot{\theta}$

$\bar{r} = 19596\,\hat{x}$

$\dot{\bar{r}} = |\dot{\bar{r}}_c - \dot{\bar{r}}_p| = .211319\,\hat{y} + .025717\,\hat{y} = .237\,\hat{y}$ Km/s

$\bar{r} \times \dot{\bar{r}} = 4644.96\ \mathrm{Km^2/s}\ \hat{z}$

$\dot{\theta} = \frac{|r \times \dot{r}|}{r^2} = \frac{4644.96}{19596^2} = \boxed{1.209 \times 10^{-5}\ \mathrm{rad/s}}$

angular velocity of charon wrt pluto

3c) $\bar{p} = \Sigma m_i \dot{\bar{r}}_i$

$m_{charon} \dot{\bar{r}}_c + m_{pluto} \dot{\bar{r}}_p = \bar{p}$

$$\boxed{\bar{p} = 6.834 \times 10^{16} \ \frac{Kg\,Km}{s} \ \hat{y}}$$

$$V_{cm} = \frac{\bar{p}}{m_{total}} = \frac{\bar{p}}{m_{pluto} + m_{charon}}$$

$$\boxed{V_{cm} = 4.142 \times 10^{-6} \ Km/s \ \hat{y}}$$

this result makes sense, center of mass is relatively constant

3d) $C_3 = \sum_i^N m_i (\bar{r}_i \times \dot{\bar{r}})$ $\qquad \hat{x} \times \hat{y} = \hat{z}$

$$C_3 = (m_{pluto})(\vec{r}_{cm,p} \times \dot{\vec{r}}_p) + (m_{charon})(\vec{r}_{cm,c} \times \dot{\vec{r}}_c)$$

$$\boxed{C_3 = 7.4128 \times 10^{24} \ \frac{Kg\,Km^2}{s} \ \hat{z}}$$

3e) $C_4 = T - U$

$$T = \frac{1}{2} m_{Pluto}\, \dot{\vec{r}}_p \cdot \dot{\vec{r}}_p + \frac{1}{2} m_{Charon}\, \dot{\vec{r}}_c \cdot \dot{\vec{r}}_c = 7.4128 \times 10^{24}$$

$$U = \frac{1}{2} G \frac{m_{pluto}\, m_{charon}}{r_{PC}} = 4.4836 \times 10^{19} \frac{Kg\, Km^2}{s^2}$$

$$\boxed{C_4 = -2.3968 \times 10^{15} \frac{Kg\, Km^2}{s^2}}$$

Contents


```
clear
close all
clc
```

Problem 1a

```
disp('-----Start Problem 1 ------------------')
% Earth -> Moon -> s/c -> sun -> Jupiter (Collinear)

% Assume 1-D position vectors along y axis. Positive y is right
r_sun_jupiter   = 778279959;                    % [km]
r_sun_earth     = -149597898;                   % [km]
r_earth_moon    = 384400;                       % [km]
r_sun_moon      = r_sun_earth + r_earth_moon;
r_moon_sc       = 77500;                        % [km]
r_sun_sc        = r_sun_moon + r_moon_sc;

% Gravitational constant
G               = 6.6743e-11*(1/1000)^3;        % [km^3/kg-s^2]

% Masses
m_sc            = 130;                          % [kg]
m_earth         = 398600.4415/G;                % [kg]
m_jupiter       = 126712767.8578/G;
m_moon          = 4902.8005821478/G;
m_sun           = 132712440017.99/G;

% Center of Mass Calculation [km]
disp('Center of mass location')
COM             = (r_sun_jupiter*m_jupiter + r_sun_earth*m_earth + ...
                   r_sun_moon*m_moon + r_sun_sc*m_sc)/ + (...
                   m_jupiter + m_earth + m_moon + m_sc + m_sun)
```

```
-----Start Problem 1 ------------------
Center of mass location

COM =

          741929.958019775
```

Problem 1b

```
% Relative position vectors
r_earth_sc       = r_sun_sc - r_sun_earth;
r_jupiter_sc     = r_sun_sc - r_sun_jupiter;

% Accelerations on spacecraft due to bodies
```

```
disp('Acceleration on s/c due to Earth')
a_earth_sc       = -G*m_earth*r_earth_sc/(norm(r_earth_sc)^3)
disp('Acceleration on s/c due to moon')
a_moon_sc        = -G*m_moon*r_moon_sc/(norm(r_moon_sc)^3)
disp('Acceleration on s/c due to Jupiter')
a_jupiter_sc     = -G*m_jupiter*r_jupiter_sc/(norm(r_jupiter_sc)^3)
disp('Acceleration on s/c due to Sun')
a_sun_sc         = -G*m_sun*r_sun_sc/(norm(r_sun_sc)^3)
disp('Net Acceleration on s/c')
a_net            = a_earth_sc + a_sun_sc + a_moon_sc + a_jupiter_sc

disp('-----End Problem 1 ------------------')
```

```
Acceleration on s/c due to Earth

a_earth_sc =

     -1.86827951052256e-06

Acceleration on s/c due to moon

a_moon_sc =

     -8.16283135425232e-07

Acceleration on s/c due to Jupiter

a_jupiter_sc =

      1.47323235925438e-10

Acceleration on s/c due to Sun

a_sun_sc =

      5.96687121560359e-06

Net Acceleration on s/c

a_net =

      3.28245589289172e-06

-----End Problem 1 ------------------
```

Problem 2a

```
disp('-----Start Problem 2 ------------------')

% More position vectors
r_sc_sun         = -r_sun_sc;
r_moon_sun       = -r_sun_moon;
r_sc_earth       = -r_earth_sc;
r_sc_jupiter     = r_sun_jupiter - r_sun_sc;
r_moon_jupiter   = r_sun_jupiter - r_sun_moon;
r_moon_earth     = -r_earth_moon;

disp('Dominant acceleration: ')
a_dominant       = G*(m_sc + m_moon)*r_moon_sc/(norm(r_moon_sc)^3)

disp('Total direct acceleration: ')
a_direct         = G*m_sun*(r_sc_sun/norm(r_sc_sun)^3) + ...
                   G*m_jupiter*(r_sc_jupiter/norm(r_sc_jupiter)^3) + ...
                   G*m_earth*(r_sc_earth/norm(r_sc_earth)^3)
```

```
disp('Total indirect acceleration: ')
a_indirect       = G*m_sun*(r_moon_sun/norm(r_moon_sun)^3) + ...
                   G*m_jupiter*(r_moon_jupiter/norm(r_moon_jupiter)^3) + ...
                   G*m_earth*(r_moon_earth/norm(r_moon_earth)^3)

disp('Total pertubation acceleration: ')
a_total_pert     = a_direct - a_indirect

disp('Pertubation acceleration due to Earth: ')
a_pert_earth     = G*m_earth*(r_sc_earth/norm(r_sc_earth)^3) - G*m_earth*(r_moon_earth/norm(r_moon_earth)^3)
disp('Pertubation acceleration due to Jupiter: ')
a_pert_jupiter   = G*m_jupiter*(r_sc_jupiter/norm(r_sc_jupiter)^3) -  G*m_jupiter*(r_moon_jupiter/norm(r_moon_jupiter)^3)
disp('Pertubation acceleration due to Sun: ')
a_pert_sun       = G*m_sun*(r_sc_sun/norm(r_sc_sun)^3) - G*m_sun*(r_moon_sun/norm(r_moon_sun)^3)
disp('-----End Problem 2 ------------------')
```

```
-----Start Problem 2 ------------------
Dominant acceleration:

a_dominant =

      8.16283135425232e-07

Total direct acceleration:

a_direct =

      4.09873902831695e-06

Total indirect acceleration:

a_indirect =

      3.26326245217686e-06

Total pertubation acceleration:

a_total_pert =

      8.35476576140096e-07

Pertubation acceleration due to Earth:

a_pert_earth =

      8.29279894596545e-07

Pertubation acceleration due to Jupiter:

a_pert_jupiter =

      2.46192006662886e-14

Pertubation acceleration due to Sun:

a_pert_sun =

      6.19665692435073e-09

-----End Problem 2 ------------------
```

Problem 3a

```
disp('-----Start Problem 3 ------------------')
```

```
Gm_charon       = 119.480; % [km^3/s^2]
Gm_pluto        = 981.601; % [km^3/s^2]
r_pc            = 19596;   % Positive xhat direction [km]

disp('Pluto, Charon Center of Mass: ')
COM_pc          = Gm_charon*r_pc/(Gm_charon + Gm_pluto)

disp('Position vector from Center of Mass to Pluto: ')
r_cm_pluto      = -COM_pc % Negative xhat direction
disp('Position vector from Center of Mass to Charon: ')
r_cm_charon     = r_pc + r_cm_pluto
```

```
-----Start Problem 3 ------------------
Pluto, Charon Center of Mass:

COM_pc =

          2126.39222727483

Position vector from Center of Mass to Pluto:

r_cm_pluto =

         -2126.39222727483

Position vector from Center of Mass to Charon:

r_cm_charon =

          17469.6077727252
```

Problem 3b

```
% Inertial Velocities along y axis
rdot_charon     = .211319;
rdot_pluto      = -.025717;

% Inertial velocity in y axis
rdot_pc         = rdot_charon - rdot_pluto; % [km/s]

% Specific Angular Momentum
h               = cross([r_pc;0;0],[0;rdot_pc;0]);

% Angular Velocity
disp('Angular Velocity of Charon relative to Pluto: ')
thetadot        = norm(h)/norm(r_pc)^2
```

```
Angular Velocity of Charon relative to Pluto:

thetadot =

      1.20961420698102e-05
```

Problem 3c

```
disp('Linear momentum of the system: ')
p               = Gm_pluto*rdot_pluto/G + Gm_charon*rdot_charon/G

disp('Velocity of center of mass: ')
v_cm            = p/(Gm_pluto/G + Gm_charon/G)
```

```
Linear momentum of the system:

p =

      6.83397959336591e+16

Velocity of center of mass:

v_cm =

      4.14247725644182e-06
```

Problem 3d

```
disp('C3 is: ')
C3                = Gm_pluto*r_cm_pluto*rdot_pluto/G + Gm_charon*r_cm_charon*rdot_charon/G
```

```
C3 is:

C3 =

      7.41287973673515e+24
```

Problem 3e

```
T                = 1/2*(Gm_pluto/G)*rdot_pluto^2 + 1/2*(Gm_charon/G)*rdot_charon^2;
U                = 1/2*G*(Gm_pluto/G)*(Gm_charon/G)/r_pc

disp('C4 is: ')
C4               = T - U
```

```
U =

      4.48360201818764e+19

C4 is:

C4 =

     -2.39681934576845e+15
```

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