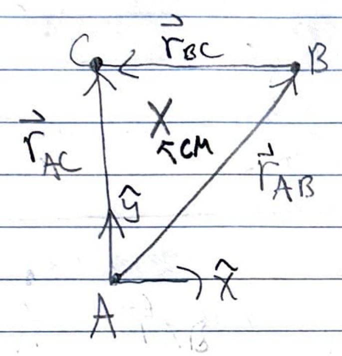
Exam 1

Problem 1:

a) The spherically symmetric assumption is signifgant because it allows the body's to be treated like point masses.



$$\vec{r}_{cm} = \frac{\mu_c \vec{r}_{AC} + \mu_B \vec{r}_{AB}}{\mu_A + \mu_B + \mu_c} = \frac{(1\times10^8)(4\times10^8\hat{y}) + (5\times10^8)(4\sqrt{3}\times10^8\hat{x} + 4\times10^8\hat{y})}{(1\times10^8) + (5\times10^8) + (2\times10^8)}$$

$$\boxed{\vec{r}_{cm} = 4.33\times10^8\,\hat{x} + 3\times10^8\,\hat{y} \quad [km]}$$

b)

C wrt A:

$$\boxed{\ddot{\vec{r}}_{AC} = -\frac{(\mu_A + \mu_C)}{r_{AC}^3}\vec{r}_{AC} + \mu_B\left(\frac{\vec{r}_{CB}}{r_{CB}^3} - \frac{\vec{r}_{AB}}{r_{AB}^3}\right)}$$

C wrt B:

$$\boxed{\ddot{\vec{r}}_{BC} = -\frac{(\mu_B + \mu_C)}{r_{BC}^3}\vec{r}_{BC} + \mu_A\left(\frac{\vec{r}_{CA}}{r_{CA}^3} - \frac{\vec{r}_{BA}}{r_{BA}^3}\right)}$$

Both expressions are correct, they just describe the motion relative to a different base point.

Choose C relative to A.

$$\boxed{\ddot{\vec{r}}_{AC} = -\frac{(\mu_A + \mu_C)}{r_{AC}^3}\vec{r}_{AC} + \mu_B\left(\frac{\vec{r}_{CB}}{r_{CB}^3} - \frac{\vec{r}_{AB}}{r_{AB}^3}\right)}$$

Dependent Variables: $\boxed{\begin{matrix}\dot{\vec{r}}_A, \vec{r}_A \\ \dot{\vec{r}}_B, \vec{r}_B \\ \dot{\vec{r}}_C, \vec{r}_C\end{matrix}}$ 18 dependent variables

Independent Variable: $\boxed{\text{time}}$

c)

Dominant: $-\frac{(\mu_A + \mu_C)\vec{r}_{AC}}{r_{AC}^3} = \frac{-(2\times10^8 + 1\times10^8)(4\times10^8\,\hat{y})}{(4\times10^8)^3}$

$$\boxed{\text{Dominant acceleration} = -1.875\times10^{-9}\,\hat{y}\ [\text{km/s}^2]}$$

Direct Acceleration: $\mu_B \left(\frac{\vec{r}_{CB}}{r_{CB}^3} \right) = (5 \times 10^8) \left(\frac{4\sqrt{3} \times 10^8 \, \hat{x}}{(4\sqrt{3} \times 10^8)^3} \right)$

$$\boxed{\text{Direct Acceleration: } 1.0417 \times 10^{-9} \, \hat{x} \ (\text{Km}/\text{s}^2)}$$

Indirect Acceleration: $\mu_B \left(\frac{\vec{r}_{AB}}{r_{AB}^3} \right) = (5 \times 10^8) \left(\frac{4\sqrt{3} \times 10^8 \, \hat{x} + 4 \times 10^8 \, \hat{y}}{(8 \times 10^8)^3} \right)$

$$\boxed{\text{Indirect Acceleration: } 6.766 \times 10^{-10} \, \hat{x} + 3.906 \times 10^{-10} \, \hat{y} \ [\text{Km}/\text{s}^2]}$$

Net perturbing: Direct - Indirect

$$\boxed{\text{Net perturbing: } 3.651 \times 10^{-10} \, \hat{x} - 3.906 \times 10^{-10} \, \hat{y} \ [\text{Km}/\text{s}^2]}$$

The dominant term has the largest magnitude, followed by the direct acceleration, then the indirect acceleration. The dominant acceleration is 2 orders of magnitude larger than the net perturbing acceleration. At this instant the perturbing term is neither increasing nor decreasing the distance between A & C due to the magnitude of the perturbing term.

ii & iii)

Problem 2:

a) $p = a(1-e^2)$

Semi-latus rectum: $200(1-0.25^2)$

$$\boxed{p = 187.5 \text{ [Km]}}$$

$\mathcal{E} = \frac{-\mu}{2a}$ (Specific energy)

$$\mathcal{E} = \frac{-(GM_B + GM_R)}{2a}$$

$$\boxed{\mathcal{E} = -1.75 \times 10^{-4} \ \frac{Km^2}{s^2}}$$

$$\boxed{\text{Distance between Foci} = 2ae = 100 \text{ Km}}$$

Apoapsis distance: $r_a = a(1+e)$

$r_a = 200(1+.25)$ $\boxed{r_a = 250 \text{ Km}}$

$h = \sqrt{\mu p}$ (specific angular momentum)

$$h = \sqrt{(GM_B + GM_R)p}$$

$$\boxed{h = 3.623 \ Km^2/s}$$

Period : $\tau = 2\pi\sqrt{\frac{a^3}{\mu}}$ $= 67170\,[s]$

$\boxed{\tau = 18.66 \text{ hours}}$

Average angular velocity : $n = \sqrt{\frac{\mu}{a^3}}$

$n = \frac{2\pi}{P}$

$\boxed{\dot{\theta}_{avg} = 9.3541 \times 10^{-5} \text{ rad/s} = 5.359 \times 10^{-3} \text{ deg/s}}$

Angular Velocity at Periapsis : $\dot{\theta} = \frac{h}{r_p^2} = \frac{h}{(a(1-e))^2}$

$\boxed{\dot{\theta} = \frac{36623}{(2606(1-.27))^2} = 1.610 \times 10^{-4} \text{ rad/s} = 9.226 \times 10^{-3} \text{ deg/s}}$

Velocity at Periapsis : $\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r}$

$V_p = \sqrt{2\left(\mathcal{E} + \frac{\mu}{r_p}\right)} = \sqrt{2\left(\mathcal{E} + \frac{\mu}{a(1-e)}\right)}$

$\boxed{V_p = 2.415 \times 10^{-2} \text{ km/s}}$

b) $\varepsilon = \frac{V^2}{2} - \frac{\mu}{r}$

$$V = \sqrt{2(\varepsilon + \frac{\mu}{r})} = \sqrt{\frac{\mu}{r}} = V_c$$

$$2(\varepsilon + \frac{\mu}{r}) = \frac{\mu}{r}$$

$$\frac{\mu}{r} = -2\varepsilon$$

$$\boxed{r = \frac{-\mu}{2\varepsilon} = \frac{-.07}{-1.75\times10^{-4}} = 200 \text{ km}}$$

$$\boxed{V_c = \sqrt{\frac{.07}{200}} = 0.0187 \text{ km/s}}$$

$\underline{V = V_c \text{ at } r = a}$ (semi-major axis)

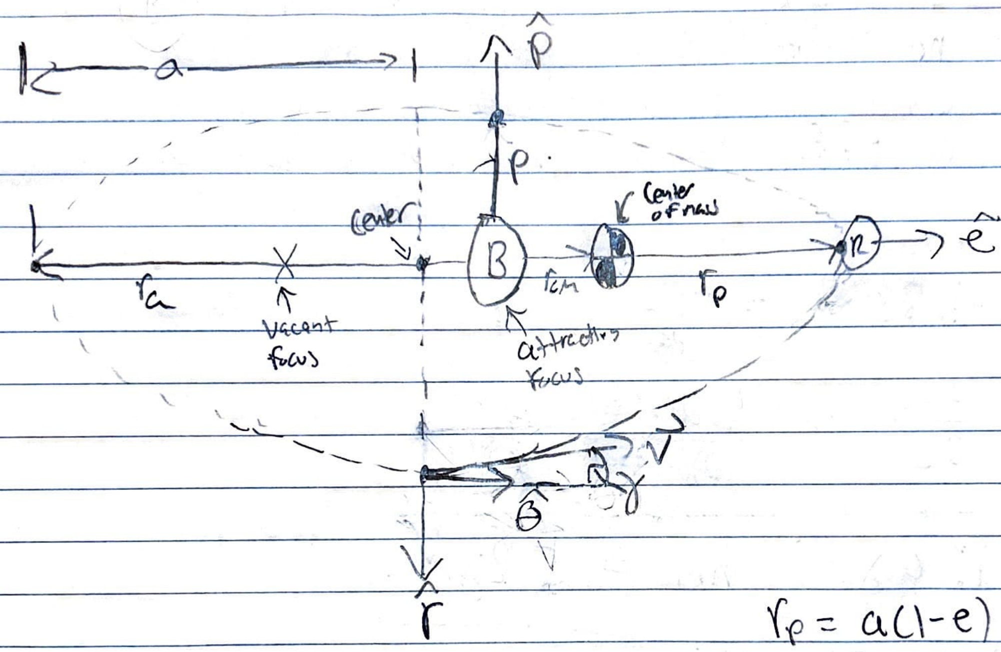
$$r = \frac{p}{1+e\cos(\theta^*)} = a \quad \therefore$$

$$\theta^* = \cos^{-1}\left(\frac{\frac{p}{r}-1}{e}\right) \quad \text{- true anomaly}$$

$$\theta^* = \cos^{-1}\left(\frac{\frac{187.5}{200}-1}{.25}\right) = 104.48^\circ$$

$$\boxed{\theta^* = 104.48^\circ}$$

c)



$r_p = a(1-e)$

$$r_{cm} = \frac{m_R\, r_p}{m_R + m_B} = \frac{(.03)(150)}{.03 + .04}$$

$$\boxed{\vec{r}_{cm} = 64.28\ \hat{e}\ [km]}$$

Problem 3:

a) System linear momentum being zero says that the center of mass is stationary & there are no external forces acting on the system. If external forces were non-zero, V_{cm} would also be non-zero, therefore $\vec{p}$ would be non-zero.

b) $\mathcal{E} = -1.2351 = \frac{-\mu}{2a}$

$$a = \frac{-\mu}{2\mathcal{E}} = \frac{-4.2\times10^4}{(2)(-1.2351)}$$

$$\boxed{a = 1.70\times10^4 \text{ Km} = 5 R_{\sigma}}$$

$$\boxed{P = \frac{h^2}{\mu} = \frac{(2.5\times10^4)^2}{4.2\times10^4} = 1.488\times10^4 \text{ [Km]} = 4.376 R_{\sigma}}$$

$$a = \frac{P}{1-e^2} \Rightarrow a(1-e^2) = P \Rightarrow$$

$$e = \sqrt{1-\frac{P}{a}}$$

$$\boxed{e = 0.353}$$

$$r_p = a(1-e)$$

$$\boxed{r_p = 1.10\times10^4 \text{ Km} = 3.23 R_{\sigma}}$$

$r_a = a(1+e)$

$$\boxed{r_a = 2.3 \times 10^4 \text{ Km} = 6.76\ R_{♂}}$$

Period: $\tau = 2\pi\sqrt{\frac{a^3}{\mu}} = 67956$ secs $= \boxed{18.88 \text{ hours}}$

C) The orbit is now a $\boxed{\text{parabolic orbit}}$

$\therefore$ $\boxed{e=1}$, $\boxed{a=\infty}$

$|h|$ & μ are unchanged

$$\boxed{p = \frac{h^2}{\mu} = 1.488 \times 10^4\ [\text{Km}] = 4.376\ R_{♂}}$$

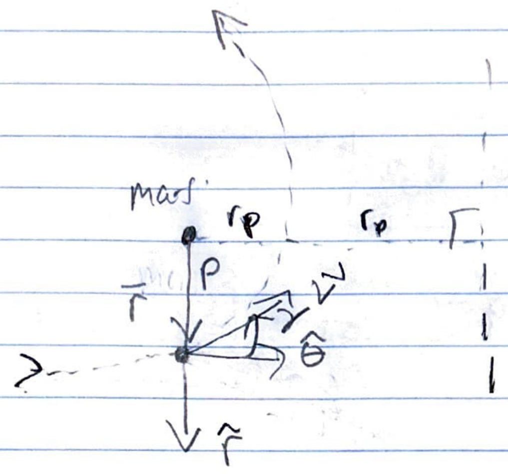
$$\boxed{r_p = \frac{p}{2} = 7.44 \times 10^3\ [\text{Km}] = 2.188\ R_{♂}}$$

$\boxed{r_a = \infty}$, $\boxed{\text{period} = \infty}$

At $\theta^* = 90°$ $\boxed{r = p = 1.488 \times 10^4\ [\text{Km}]}$

$\frac{V^2}{2} - \frac{\mu}{r} = 0 \Rightarrow V = \sqrt{2}\sqrt{\frac{\mu}{p}} = \sqrt{2}\left(\sqrt{\frac{4.2\times10^4}{1.488\times10^4}}\right)$

$$\boxed{V = 2.375\ \text{Km/s}}$$



Mars
r_p
r_p
P
$\bar{r}$
$\vec{v}$
$\hat{\theta}$
$\hat{r}$