Gabriel Colangelo

```
clear
close all
clc

% Earth Parameters
mu_earth        = 398600.4415;
R_earth         = 6378.1363;

% OTV Parameters - Circular Orbit
a_otv           = 1.5*R_earth;
e_otv           = 0;
i_otv           = 0;
theta_D         = 0;

% Space Station Parameters
a_ss            = 8*R_earth;
e_ss            = 0.2;
i_ss            = 30;
RAAN_ss         = 45;
AOP_ss          = 60;
theta_A         = 240;
```

Problem 2a)

Find: a, type, e, p, e, v_D, γ_D, θ^*_D, v_A, γ_A, θ^*_A, $|\Delta\bar{v}_D|$, α_D, $|\Delta\bar{v}_A|$, α_A

Assume: TOF = 8 hours, Type 1 transfer, mass of space station and OTV negligble. Transfer from OTV to Space Station

```
% Desired time of flight
TOF             = 8*3600;

% Orbit radius at space station arrival
r_ss            = (a_ss*(1 - e_ss^2))/(1 + e_ss*cosd(theta_A - AOP_ss));

% Direction cosine matrix from rotating frame to inertial frame
DCM_ss          =  I_DCM_R(i_ss,theta_A,RAAN_ss);

% Calculate position vectors in inertial frame
r_ss_I          = DCM_ss*[r_ss;0;0];
r_otv_I         = [a_otv;0;0];

% Calculate transfer angle (angle between position vectors)
TA              = acosd(dot(r_ss_I,r_otv_I)/(r_ss*a_otv));

% Solve Lamberts problem
probA           = LambertsProblem(TOF, a_otv, r_ss, TA, mu_earth);
fprintf('The semi major axis of the transfer orbit is %.2f km',probA.a)
```

```
The semi major axis of the transfer orbit is 32773.37 km
```

$$r_{ss} = \frac{a_{ss}(1 - e_{ss}^2)}{1 + e_{ss}\cos(\theta_{ss} - \omega_{ss})} = 61230 \ \ [km]$$

$$[IR] = \begin{pmatrix} C_\Omega C_\theta - S_\Omega C_i S_\theta & -C_\Omega S_\theta - S_\Omega C_i C_\theta & S_\Omega S_i \\ S_\Omega C_\theta + C_\Omega C_i S_\theta & -S_\Omega S_\theta + C_\theta C_i C_\Omega & -C_\Omega S_i \\ S_i S_\theta & S_i C_\theta & C_i \end{pmatrix}$$

$[IR]_{ss} = [IR]|_{i=30°, \theta=240°, \Omega=45°}$ - Orbit fixed frame to Inertial frame direction cosine matrix, at arrival location in space station orbit

$$[\bar{r}_{ss}]^I = [IR]_{ss}[\bar{r}_{ss}]^R = 10824 \ \ \hat{x} - 54120 \ \ \hat{y} - 26513 \ \ \hat{z} \ \ [km]$$

$$[\bar{r}_{OTV}]^I = 9567 \ \ \hat{x} \ \ [km]$$

$$\cos(TA) = \frac{\bar{r}_{OTV} \cdot \bar{r}_{ss}}{|\bar{r}_{OTV}||\bar{r}_{ss}|}$$

$$TA = 79.82, 280.18 \ \ [deg]$$

Type 1, TA < 180: $TA = \phi = 79.82 \ \ [deg]$

Law of cosines from space triangle: $c^2 = r_{ss}^2 + r_{OTV}^2 - 2 r_{ss} r_{OTV} \cos\phi$

Semi-perimeter: $s = (r_{ss} + r_{OTV} + c)/2$

Minimum energy semi major axis $a_{min} = s/2 = 32769 \ \ [km]$

$$\beta_{min} = 2\sin^{-1}\sqrt{\frac{s-c}{2a_{min}}} = 32.92 \ \ [deg]$$

$$TOF_{min} = \sqrt{\frac{a^3}{\mu}}[(\pi - \beta_{min}) - (\sin\pi - \sin\beta_{min})] = 8.12 \ \ [hour]$$

The minimum energy time of flight is greater than the desired time of flight, therefore the transfer type is an A. The transfer type is <u>1A.</u>

For a type 1A, the true α and β values are related to their principal values through:

$$\alpha = \alpha_0$$
$$\beta = \beta_0$$

Lamberts problem is solved numerically for a using a bisection method:

$$\beta_0 = 2\sin^{-1}\sqrt{\frac{s-c}{2a}}$$

$$\alpha_0 = 2\sin^{-1}\sqrt{\frac{s}{2a}}$$

$$TOF = \sqrt{\frac{a^3}{\mu}}[(\alpha - \beta) - (\sin\alpha - \sin\beta)] = 8 \ \ [hour]$$

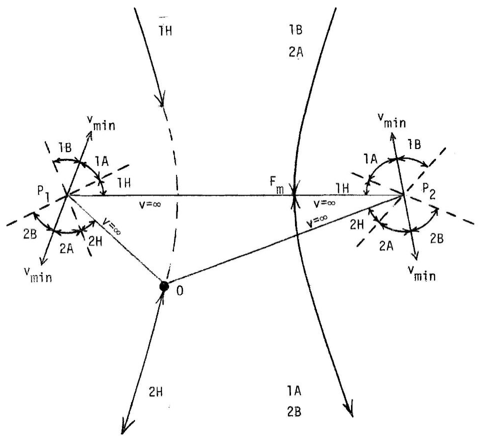
<u>**a = 32773.37 [km]**</u>

```
% Calculate semi latus rectum
p_type1     = max((4*probA.a*(probA.s - a_otv)*(probA.s - r_ss)/probA.c^2)*...
                [sin((probA.alpha + probA.beta)/2)^2, sin((probA.alpha - probA.beta)/2)^2]);
fprintf('The semi latus rectum of the transfer ellipse is %.3f km', p_type1)
```

```
The semi latus rectum of the transfer ellipse is 8054.629 km
```

$\alpha = 178.68\ \ [deg]$
$\beta = 32.91\ \ [deg]$
$p = a(1-e^2) = \dfrac{4a(s-r_{ss})(s-r_{OTV})}{c^2}\sin^2(\dfrac{\alpha \pm \beta}{2}) = 8054.63\ \text{ or }\ 7946.20\ \ [km]$

$\bar{OF} = 2ae$



For 1A transfer type, a smaller e is needed as observed in the solution space diagram above. A smaller e will cause an increase in semi-latus rectum. Therefore, the necessary semi-latus rectum value is the larger option.

$\underline{\mathbf{p = 8054.63\ \ [km]}}$

```
% Calculate eccentricity
e_type1     = sqrt(1 - p_type1/probA.a);
fprintf('The transfer eccentricity is %.4f ',e_type1)
```

```
The transfer eccentricity is 0.8685
```

$e = \sqrt{1 - \dfrac{p}{a}}$

$\mathbf{e = 0.8685}$

```
% Calculate energy of transfer ellipse
energy_type1   = -mu_earth/(2*probA.a);

% Velocity magnitude at departure
v_D            = sqrt(2*(energy_type1 + mu_earth/a_otv));
fprintf('The velocity at departure on the transfer ellipse is %.3f km/s',v_D)
```

```
The velocity at departure on the transfer ellipse is 8.436 km/s
```

$r_D = r_{OTV}$

$r_A = r_{ss}$

$v_D = \sqrt{2(\frac{\mu}{r_D} - \frac{\mu}{2a})}$

$v_D = 8.436 \ \ [km/s]$

```
% Calculate true anomaly at departure
true_anomaly_D  = acosd((p_type1/a_otv - 1)/e_type1);

% Calculate true anomaly at arrival
true_anomaly_A  = -acosd((p_type1/r_ss - 1)/e_type1);
```

$\theta^* = \cos^{-1}(\frac{1}{e_m}(\frac{p}{r} - 1))$

$\theta_D^* = \pm 100.49 \ \ [deg]$

$\theta_A^* = \pm 179.69 \ \ [deg]$

4 possible combinations of θ_A^* and θ_D^*. However, only one pair produces a transfer angle of 79.82 deg.

Therefore, the pair of true anomaly's must be:

$\boldsymbol{\theta_D^* = 100.49 \ \ [deg]}$

$\boldsymbol{\theta_A^* = -179.69 \ \ [deg] = 180.31 \ \ [deg]}$

```
% Minimum energy transfer specific angular momentum
h_type1         = sqrt(mu_earth*p_type1);

% Flight path angle at departure
gamma_D         = sign(true_anomaly_D)*acosd(h_type1/(a_otv*v_D));
fprintf('The flight path angle at departure is %.3f deg',gamma_D)
```

```
The flight path angle at departure is 45.407 deg
```

$h = \sqrt{\mu p}$

$h = r_D v_D \cos \gamma_D$

$\gamma_D = 45.41 \ \ [deg]$ Ascending based on θ_D^*, so $\gamma > 0$

```
% Velocity magnitude at arrival
v_A            = sqrt(2*(energy_type1 + mu_earth/r_ss));
fprintf('The velocity at arrival on the transfer ellipse is %.3f km/s',v_A)
```

```
The velocity at arrival on the transfer ellipse is 0.926 km/s
```

$v_A = \sqrt{2(\frac{\mu}{r_A} - \frac{\mu}{2a})}$

$v_A = 0.926 \ \ [km/s]$

```
% Flight path angle at arrival
gamma_A        = sign(true_anomaly_A)*acosd(h_type1/(r_ss*v_A));
```

```
fprintf('The flight path angle at arrival is %.3f deg',gamma_A)
```

```
The flight path angle at arrival is -2.025 deg
```

$h = \sqrt{\mu p}$

$h = r_A v_A \cos \gamma_A$

$\gamma_A = -2.025 \ \ [deg]$ Descending based on θ_A^*, so $\gamma < 0$

```
% F and G Equations
f           = 1 - (r_ss/p_type1)*(1 - cosd(TA));
g           = (r_ss*a_otv)*sind(TA)/h_type1;

% Departure velocity in inertial coordinates
v_D_I       = (r_ss_I - f*r_otv_I)/g;

% Initial circular orbit velocity in orbit frame
v_otv       = [0;sqrt(mu_earth/a_otv);0];

% OTV velocity in inertial frame
v_OTV_I     = I_DCM_R(0, 0, 0)*v_otv;

% Departure deltaV in intertial frame
dv_D_I      = v_D_I - v_OTV_I;
fprintf('The departure deltaV magnitude is %.4f km/s',norm(dv_D_I))
```

```
The departure deltaV magnitude is 13.4717 km/s
```

$\bar{v}_{c,otv} = \sqrt{\frac{\mu}{r_D}} \ \hat{\theta} = 6.455 \ \hat{\theta} \ \ [km/s]$ Prior to departure maneuver , $\hat{\theta}$ is aligned with $\hat{y}$

$\bar{r}_A = f\bar{r}_D + g\bar{v}_D$

$f = 1 - \frac{r_A}{p}(1 - \cos(TA))$, where $TA = \theta_A^* - \theta_D^*$. $f = -5.2580$

$g = \frac{r_A r_D}{\sqrt{\mu p}} \sin(TA) = 10175.7 \ \ [s]$

$\bar{v}_D = \frac{\bar{r}_A - f\bar{r}_D}{g} = 6.007 \ \hat{x} - 5.318 \ \hat{y} - 2.605 \ \hat{z} \ \ [km/s]$

$\Delta\bar{v}_D = \bar{v}_D - \bar{v}_{c,otv} = 6.007 \ \hat{x} - 11.773 \ \hat{y} - 2.605 \ \hat{z} \ \ [km/s]$

$\mathbf{|\Delta\bar{v}_D| = 13.47 \ \ [km/s]}$

```
% Departure deltaV rotated into orbit frame
dv_D          = I_DCM_R(0, 0, 0)'*dv_D_I;

% Extract alpha and beta
beta_D        = asind(dv_D(3)/norm(dv_D));
alpha_D       = atan2d(dv_D(1),dv_D(2));
dv_D_VNC      = norm(dv_D)*[cosd(beta_D)*cosd(alpha_D),cosd(beta_D)*sind(alpha_D), sind(beta_D)];
fprintf('The departure maneuver alpha is %.3f deg',alpha_D)
```

```
The departure maneuver alpha is 152.967 deg
```

$[IR]_D = [IR]|_{i=0°,\theta=0°,\Omega=0°}$ - Orbit frame to Inertial frame direction cosine matrix, at departure location in OTV orbit. At this point, $[IR]_D$ is the identity matrix

$[\Delta\bar{v}_D]_R = [IR]_D^T[\Delta\bar{v}_D]_I$

$[\Delta\bar{v}_D]_R = 6.007\ \hat{r} - 11.773\ \hat{\theta} - 2.605\ \hat{h}\ \ [km/s]$

γ_1 is zero (circular orbit) $\therefore\ \ \hat{\theta} = \hat{V}, \hat{r} = \hat{C}$, and $\hat{h} = \hat{N}$

$[\Delta\bar{v}_D]_{VNC} = \Delta v_D[\cos\beta_D\cos\alpha_D\ \ \hat{V} + \cos\beta_D\sin\alpha_D\ \ \hat{C} + \sin\beta_D\ \ \hat{N}] = -11.773\ \ \hat{V} + 6.007\ \ \hat{C} - 2.605\ \ \hat{N}$

$\beta_D = \sin^{-1}\dfrac{-2.605}{\Delta v_D} = -11.15\ \ [deg]$

$\tan\alpha_D = \dfrac{\Delta v_D\cos\beta_D\sin\alpha_D}{\Delta v_D\cos\beta_D\cos\alpha_D} = \dfrac{6.007}{-11.773}$ based off of the signs of numerator and denominator, α_D must be in quadrant 2.

$\alpha_D = 152.97\ \ [deg]$

```
% F & G equations
fdot        = (dot(r_otv_I,v_D_I)/(p_type1*a_otv))*(1 - cosd(TA)) - (1/a_otv)*(sqrt(mu_earth/p_type1))*sind(TA);
gdot        = 1 - (a_otv/p_type1)*(1 - cosd(TA));

% Velocity vector at arrival
v_A_I       = fdot*r_otv_I + gdot*v_D_I;

% Velocity in final orbit at arrival in orbit frame - at apoapsis so tangential
v_ss        = [0;sqrt(2*((-mu_earth/(2*a_ss)) + mu_earth/r_ss));0];

% Velocity in final orbit at arrival in inertial frame
v_ss_I      = DCM_ss*v_ss;

% Arrival deltaV in intertial frame
dv_A_I      = v_ss_I - v_A_I;
fprintf('The arrival deltaV magnitude is %.4f km/s',norm(dv_A_I))
```

```
The arrival deltaV magnitude is 3.1634 km/s
```

$\dot{f} = \dfrac{\bar{r}_D\cdot\bar{v}_D}{pr_D}[1 - \cos(TA))] - \dfrac{1}{r_D}\sqrt{\dfrac{\mu}{p}}\sin(TA) = -1.0974\times10^{-4}\ \ [1/s]$

$\dot{g} = 1 - \dfrac{r_D}{p}(1 - \cos(TA)) = 0.0222$

$\bar{v}_A = \dot{f}\bar{r}_D + \dot{g}\bar{v}_D = -0.9166\ \ \hat{x} - 0.1180\ \ \hat{y} - 0.0578\ \ \hat{z}\ \ [km/s]$

$\bar{v}_{ss} = \sqrt{2(\dfrac{\mu}{r_A} - \dfrac{\mu}{2a_{ss}})}\ \ \hat{\theta} = 2.282\ \ \hat{\theta}$ At apoapsis of final orbit, therefore velocity is tangential, $\gamma_2 = 0$

$[\bar{v}_{ss}]_I = [IR]_{ss}[\bar{v}_{ss}]_R = 2.0962\ \ \hat{x} + 0.6987\ \ \hat{y} - 0.5705\ \ \hat{z}\ \ [km/s]$

$\Delta\bar{v}_A = [\bar{v}_{ss}]_I - [\bar{v}_A]_I = 3.0128\ \ \hat{x} + 0.8167\ \ \hat{y} - 0.5127\ \ \hat{z}\ \ [km/s]$

$|\Delta\bar{v}_A| = 3.163\ \ [km/s]$

```
% Transfer Orbit unit vectors
h_T_hat       = cross(r_ss_I,v_A_I)/norm(cross(r_ss_I,v_A_I));
rA_T_hat      = r_ss_I/r_ss;
thetaA_T_hat  = cross(h_T_hat,rA_T_hat);

% Transfer orbit inclination and RAAN
```

```
i_T          = acosd(h_T_hat(3));
RAAN_T       = atan2d(h_T_hat(1),-h_T_hat(2));
thetaA_T     = atan2d(rA_T_hat(3),thetaA_T_hat(3));

% Arrival deltaV rotated into orbit frame
dv_A         = I_DCM_R(i_T,thetaA_T,RAAN_T)'*dv_A_I;

% Extract alpha and beta
beta_A       = asind(dv_A(3)/norm(dv_A));
alpha_A      = atan2d(dv_A(1),dv_A(2)) - gamma_A;
dv_A_VNC     = norm(dv_A)*[cosd(beta_A)*cosd(alpha_A),cosd(beta_A)*sind(alpha_A), sind(beta_A)];
fprintf('The arrival maneuver alpha is %.3f deg',alpha_A)
```

```
The arrival maneuver alpha is 181.411 deg
```

$\hat{h}_T = \frac{\bar{r}_{ss} \times \bar{v}_A}{|\bar{r}_{ss} \times \bar{v}_A|} = \sin \Omega_T \sin i_T \ \hat{x} - \cos \Omega_T \sin i_T \ \hat{y} + \cos i_T \ \hat{z} = 0\hat{x} + 0.4399 \ \hat{y} - 0.8980 \ \hat{z}$

$i_T = \pm \cos^{-1} -0.8980 = 153.89 \ \ [deg]$ - Choose positive value

$\Omega_T = \tan^{-1} \frac{\sin \Omega_T \sin i_T}{\cos \Omega_T \sin i_T} = \tan^{-1} \frac{0}{-0.4399} = 180 \ \ [deg]$ based off of the signs of numerator and denominator, must be in quadrant 2.

$\hat{r}_{A,T} = \frac{\bar{r}_{ss}}{r_{ss}} = (\cos \Omega_T \cos \theta_{A,T} - \sin \Omega_T \cos i_T \sin \theta_{A,T})\hat{x} + (\sin \Omega_T \cos \theta_{A,T} + \cos \Omega_T \cos i_T \sin \theta_{A,T})\hat{y} + (\sin i_T \sin \theta_{A,T})\hat{z} = 0.1768\hat{x} - 0.8839\hat{y} - 0.4330\hat{z}$

$\hat{\theta}_{A,T} = \hat{h}_T \times \hat{r}_{A,T} = (-\cos \Omega_T \sin \theta_{A,T} - \sin \Omega_T \cos i_T \cos \theta_{A,T})\hat{x} + (-\sin \Omega_T \sin \theta_T + \cos \Omega_T \cos i_T \cos \theta_{A,T})\hat{y} + (\sin i_T \cos \theta_{A,T})\hat{z} = -0.984\hat{x} - 0.159\hat{y} - 0.078\hat{z}$

$\theta_{A,T} = \tan^{-1} \frac{\sin i_T \sin \theta_{A,T}}{\sin i_T \cos \theta_{A,T}} = \frac{-0.433}{-0.0778} = -100.2 \ \ [deg]$ based off of the signs of numerator and denominator, must be in quadrant 3

$[IR]_A = [IR]|_{i=153.89°, \theta=-100.2°, \Omega=180°}$ - Orbit frame to Inertial frame direction cosine matrix, at arrival location in transfer orbit

$[\Delta \bar{v}_A]_R = [IR]_A^T [\Delta \bar{v}_A]_I$

$[\Delta \bar{v}_A]_R = \Delta v_A [\cos \beta_A \sin \phi_A \ \hat{r} + \cos \beta_A \cos \phi_A \ \hat{\theta} + \sin \beta_A \ \hat{h}] = 0.0327 \ \hat{r} - 3.0552 \ \hat{\theta} + 0.8197 \ \hat{h} \ \ [km/s]$

$\beta_A = \sin^{-1} \frac{0.8197}{\Delta v_A} = 15.02 \ \ [deg]$

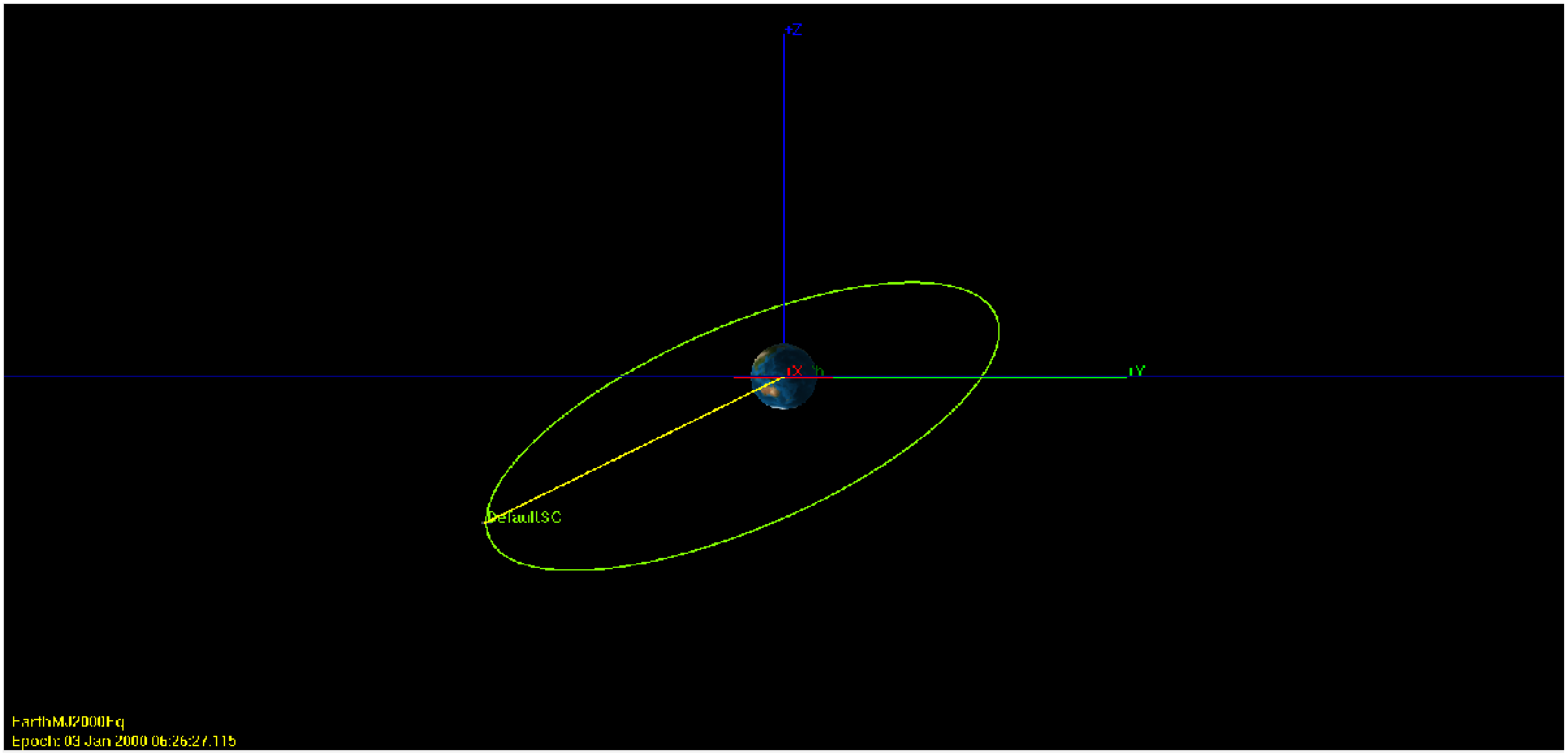
$\tan \phi_A = \frac{\Delta v_A \cos \beta_A \sin \phi_A}{\Delta v_A \cos \beta_A \cos \phi_A} = \frac{0.0327}{-3.0552}$ based off of the signs of numerator and denominator, must be in quadrant 2

$\phi_A = 179.3864 \ \ [deg] = \gamma_A + \alpha_A$

$\boldsymbol{\alpha_A = 181.41 \ \ [deg]}$

$[\Delta \bar{v}_A]_{VNC} = \Delta v_A [\cos \beta_A \cos \alpha_A \ \hat{V} + \cos \beta_A \sin \alpha_A \ \hat{C} + \sin \beta_A \ \hat{N}] = -3.0544 \ \hat{V} - 0.0753 \ \hat{C} + 0.8197 \ \hat{N} \ \ [km/s]$

Supplementary GMAT Plots - used for self check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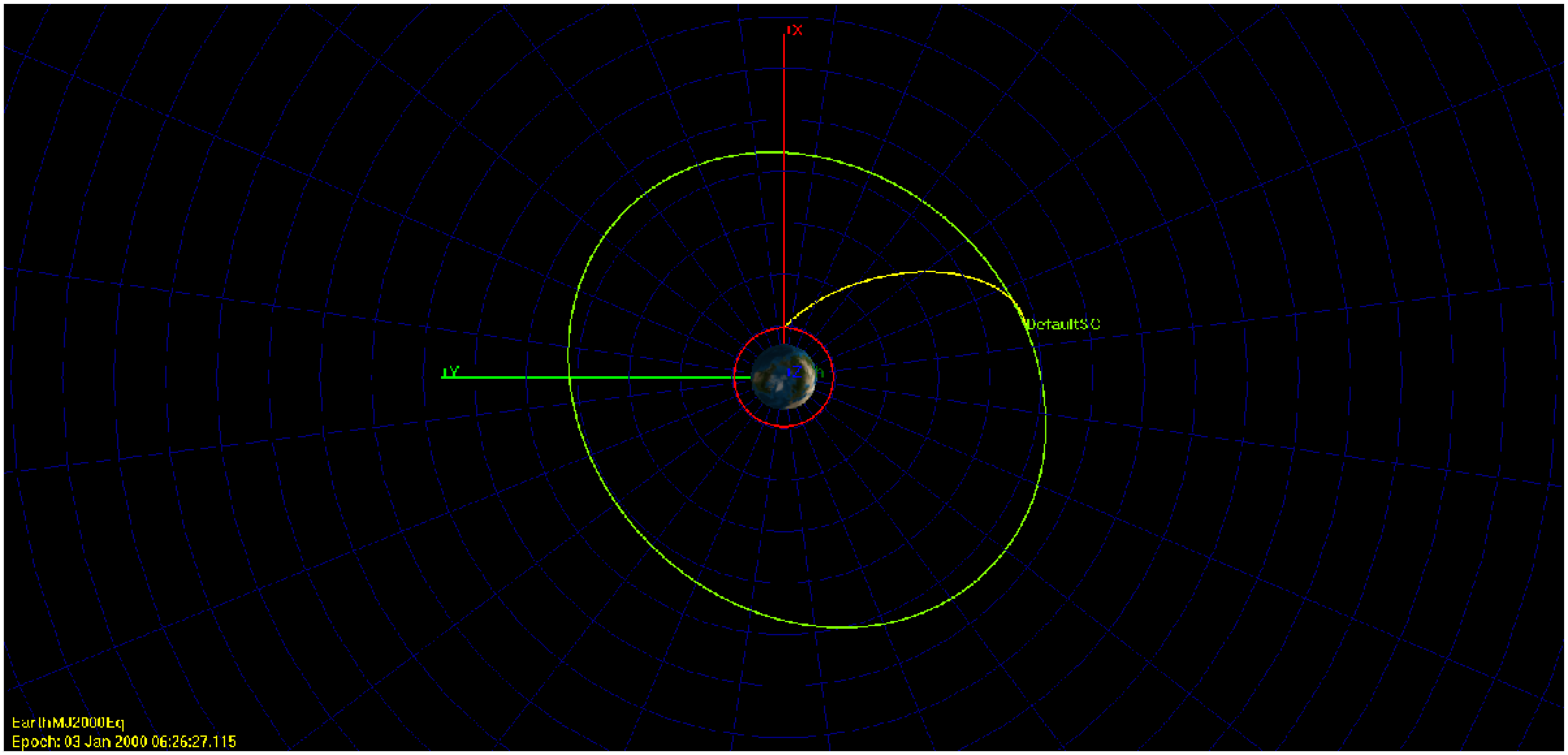
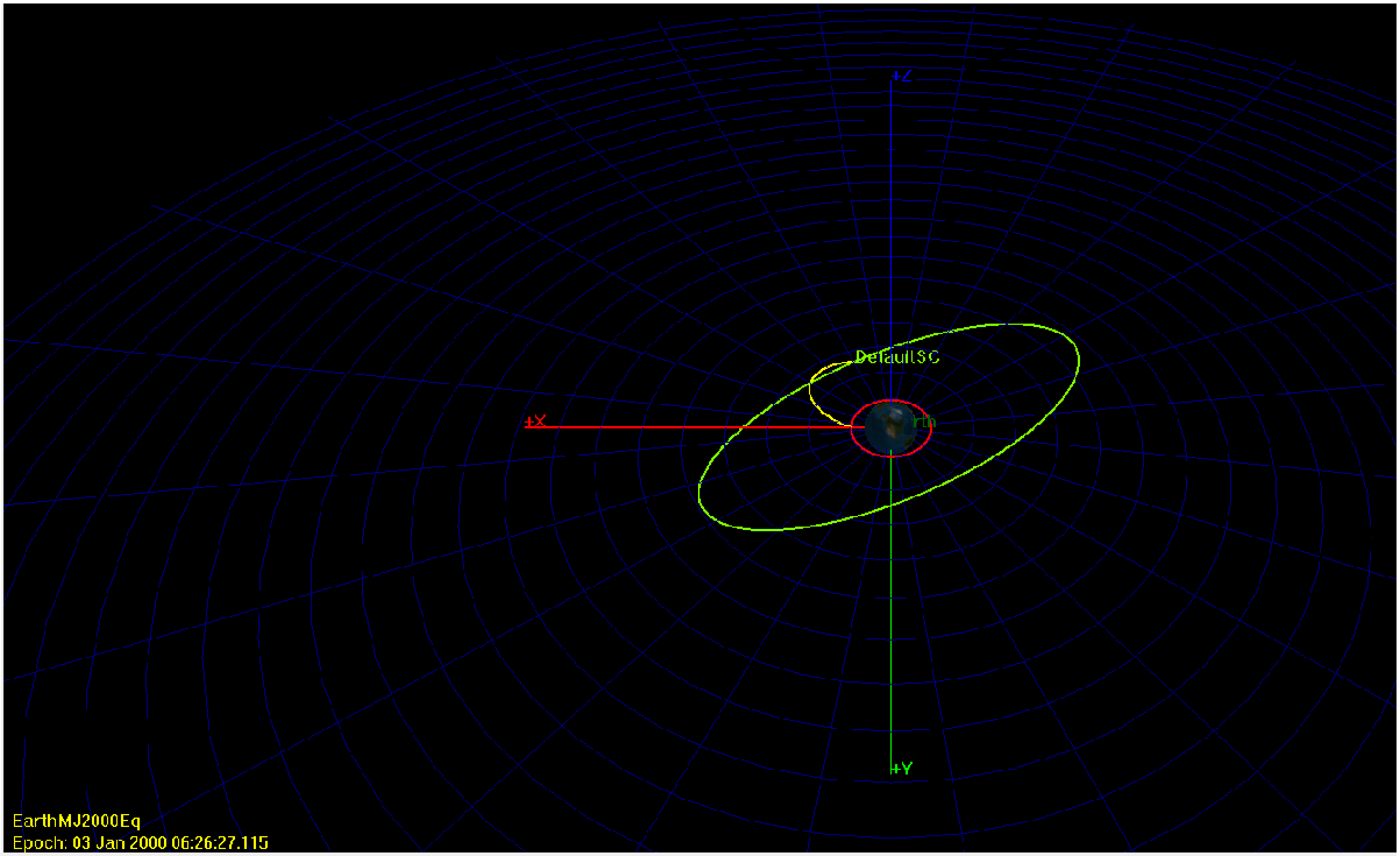

+X
+Y
DefaultSC
EarthMJ2000Eq
Epoch: 03 Jan 2000 06:26:27.115



Problem 2b)

Find: For the lowest $|\Delta\bar{v}|$, determine i, Ω, ω for the transfer orbit

Assume: Neglect mass of space station and OTV, maintain same TOF

```
% Calculate transfer angle (angle between position vectors)
TA_2            = wrapTo360(-acosd(dot(r_ss_I,r_otv_I)/(r_ss*a_otv)));

% Solve Lamberts problem
probB           = LambertsProblem(TOF, a_otv, r_ss, TA_2, mu_earth);

% Calculate semi latus rectum
p_type2         = min((4*probB.a*(probB.s - a_otv)*(probB.s - r_ss)/probB.c^2)*...
                  [sin((probB.alpha + probB.beta)/2)^2, sin((probB.alpha - probB.beta)/2)^2]);

% Calculate eccentricity
e_type2         = sqrt(1 - p_type2/probB.a);

% Calculate true anomaly at departure
true_anomaly_D2 = -acosd((p_type2/a_otv - 1)/e_type2);

% Calculate true anomaly at arrival
true_anomaly_A2 = acosd((p_type2/r_ss - 1)/e_type2);
```

```
% Calculate energy of transfer ellipse
energy_type2    = -mu_earth/(2*probB.a);

% Velocity magnitude at departure
v_D2            = sqrt(2*(energy_type2 + mu_earth/a_otv));

% Minimum energy transfer specific angular momentum
h_type2         = sqrt(mu_earth*p_type2);

% Flight path angle at departure
gamma_D2        = sign(true_anomaly_D2)*acosd(h_type2/(a_otv*v_D2));

% Velocity magnitude at arrival
v_A2            = sqrt(2*(energy_type2 + mu_earth/r_ss));

% Flight path angle at arrival
gamma_A2        = sign(true_anomaly_A2)*acosd(h_type2/(r_ss*v_A2));

% F and G Equations
f2              = 1 - (r_ss/p_type2)*(1 - cosd(TA_2));
g2              = (r_ss*a_otv)*sind(TA_2)/h_type2;

% Departure velocity in inertial coordinates
v_D2_I          = (r_ss_I - f2*r_otv_I)/g2;

% Departure deltaV in intertial frame
dv_D2_I         = v_D2_I - v_OTV_I;

% Departure deltaV rotated into orbit frame
dv_D2          = I_DCM_R(0, 0, 0)'*dv_D2_I;

% Extract alpha and beta
beta_D2       = asind(dv_D2(3)/norm(dv_D2));
alpha_D2      = atan2d(dv_D2(1),dv_D2(2));
dv_D2_VNC     = norm(dv_D2)*[cosd(beta_D2)*cosd(alpha_D2),cosd(beta_D2)*sind(alpha_D2), sind(beta_D2)];

fprintf('The departure deltaV magnitude is %.4f km/s',norm(dv_D2_I))
```

```
The departure deltaV magnitude is 6.7057 km/s
```

For a lower Δv while maintaining the same TOF, use a type 2 transfer (i.e. TA > 180).
From part A, $TA = 79.82, 280.18 \ \ [deg]$
Type 2, TA > 180: $TA = 280.18 \ \ [deg]$
$\phi = 360 - TA = 79.82 \ \ [deg]$

Law of cosines from space triangle: $c^2 = r_{ss}^2 + r_{OTV}^2 - 2r_{ss}r_{OTV}\cos\phi$
Semi-perimeter: $s = (r_{ss} + r_{OTV} + c)/2$
Minimum energy semi major axis $a_{min} = s/2 = 32769 \ \ [km]$

$$\beta_{min} = 2\sin^{-1}\sqrt{\frac{s-c}{2a_{min}}} = 32.92 \ \ [deg]$$

$$TOF_{min} = \sqrt{\frac{a^3}{\mu}}[(\pi - \beta_{min}) - (\sin\pi - \sin\beta_{min})] = 8.12 \ \ [hour]$$

The minimum energy time of flight is greater than the desired time of flight, therefore the transfer type is an A. The transfer type is 2A.

For a type 2A, the true α and β values are related to their principal values through:

$\alpha = \alpha_0$
$\beta = -\beta_0$

Lamberts problem is solved numerically for a using a bisection method:

$$\beta_0 = 2\sin^{-1}\sqrt{\frac{s-c}{2a}}$$

$$\alpha_0 = 2\sin^{-1}\sqrt{\frac{s}{2a}}$$

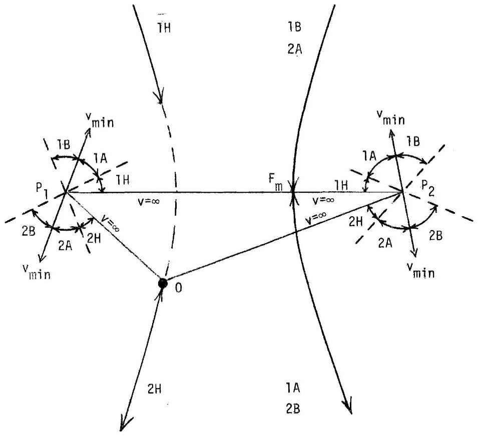
$$TOF = \sqrt{\frac{a^3}{\mu}}[(\alpha - \beta) - (\sin\alpha - \sin\beta)] = 8 \ \ [hour]$$

$\underline{a = 32794.27 \ \ [km]}$

$\alpha = 178.68 \ \ [deg]$
$\beta = -32.90 \ \ [deg]$

$$p = a(1-e^2) = \frac{4a(s - r_{ss})(s - r_{OTV})}{c^2}\sin^2(\frac{\alpha \pm \beta}{2}) = 7870.22 \ \text{ or } \ 8132.38 \ \ [km]$$

$\bar{OF} = 2ae$



For 2A transfer type, a larger e is needed as observed in the solution space diagram above. A larger e will cause an decrease in semi-latus rectum. Therefore, the necessary semi-latus rectum value is the smaller option.

$\underline{p = 7870.22 \;\; [km]}$

$e = \sqrt{1 - \frac{p}{a}} = 0.872$

$\theta^* = \cos^{-1}(\frac{1}{e_m}(\frac{p}{r} - 1))$

$\theta^*_D = \pm 101.74 \;\; [deg]$

$\theta^*_A = \pm 178.44 \;\; [deg]$

4 possible combinations of θ^*_A and θ^*_D. However, only one pair produces a transfer angle of 280.18 deg.

Therefore, the pair of true anomaly's must be:

$\theta^*_D = -101.74 \;\; [deg]$

$\theta^*_A = 178.44 \;\; [deg]$

$v_D = \sqrt{2(\frac{\mu}{r_D} - \frac{\mu}{2a})} = 8.436 \;\; [km/s]$

$h = \sqrt{\mu p}$

$h = r_D v_D \cos \gamma_D$

$\gamma_D = -46.057 \;\; [deg]$ Descending based on θ^*_D, so $\gamma < 0$

$v_A = \sqrt{2(\frac{\mu}{r_A} - \frac{\mu}{2a})} = 0.93 \;\; [km/s]$

$h = r_A v_A \cos \gamma_A$

$\gamma_A = 10.44 \;\; [deg]$ Ascending based on θ^*_A, so $\gamma > 0$

$\bar{v}_{c,otv} = \sqrt{\frac{\mu}{r_D}}\ \hat{\theta} = 6.455\ \ \hat{\theta}\ \ [km/s]$ Prior to departure maneuver , $\hat{\theta}$ is aligned with $\hat{y}$

$\bar{r}_A = f\bar{r}_D + g\bar{v}_D$

$f = 1 - \frac{r_A}{p}(1 - \cos(TA))$, where $TA = \theta_A^* - \theta_D^*$. $\quad f = -5.404$

$g = \frac{r_A r_D}{\sqrt{\mu p}}\sin(TA) = -10294.2\ \ [s]$

$\bar{v}_D = \frac{\bar{r}_A - f\bar{r}_D}{g} = -6.0744\ \ \hat{x} + 5.2573\ \ \hat{y} + 2.5756\ \ \hat{z}\ \ [km/s]$

$\Delta\bar{v}_D = \bar{v}_D - \bar{v}_{c,otv} = -6.0744\ \ \hat{x} - 1.1974\ \ \hat{y} + 2.5756\ \ \hat{z}\ \ [km/s]$

$|\Delta\bar{v}_D| = 6.7057$ [km/s]

$[IR]_D = [IR]|_{i=0^\circ,\theta=0^\circ,\Omega=0^\circ}$ - Orbit frame to Inertial frame direction cosine matrix, at departure location in OTV orbit. At this point, $[IR]_D$ is the identity matrix

$[\Delta\bar{v}_D]_R = [IR]_D^T[\Delta\bar{v}_D]_I$

$[\Delta\bar{v}_D]_R = -6.0744\ \ \hat{r} - 1.1974\ \ \hat{\theta} + 2.5756\ \ \hat{h}\ \ [km/s]$

γ_1 is zero (circular orbit) $\therefore\ \ \hat{\theta} = \hat{V}, \hat{r} = \hat{C}$, and $\hat{h} = \hat{N}$

$[\Delta\bar{v}_D]_{VNC} = \Delta v_D[\cos\beta_D\cos\alpha_D\ \ \hat{V} + \cos\beta_D\sin\alpha_D\ \ \hat{C} + \sin\beta_D\ \ \hat{N}] = -1.1974\ \ \hat{V} - 6.0744\ \ \hat{C} + 2.5756\ \ \hat{N}$

$\beta_D = \sin^{-1}\frac{2.5756}{\Delta v_D} = 22.5872\ \ [deg]$

$\tan\alpha_D = \frac{\Delta v_D\cos\beta_D\sin\alpha_D}{\Delta v_D\cos\beta_D\cos\alpha_D} = \frac{-6.0744}{-1.1974}$ based off of the signs of numerator and denominator, α_D must be in quadrant 3.

$\alpha_D = -101.1509$ [deg]

```
% F & G equations
fdot2       = (dot(r_otv_I,v_D2_I)/(p_type2*a_otv))*(1 - cosd(TA_2)) - (1/a_otv)*(sqrt(mu_earth/p_type2))*sind(TA_2);
gdot2       = 1 - (a_otv/p_type2)*(1 - cosd(TA_2));

% Velocity vector at arrival
v_A2_I      = fdot2*r_otv_I + gdot2*v_D2_I;

% Arrival deltaV in intertial frame
dv_A2_I     = v_ss_I - v_A2_I;
fprintf('The arrival deltaV magnitude is %.4f km/s',norm(dv_A2_I))
```

```
The arrival deltaV magnitude is 1.4754 km/s
```

$\dot{f} = \dfrac{\overline{r}_D \cdot \overline{v}_D}{p r_D}[1 - \cos(TA)] - \dfrac{1}{r_D}\sqrt{\dfrac{\mu}{p}}\sin(TA) = 9.676 \times 10^{-5}\ \ [1/s]$

$\dot{g} = 1 - \dfrac{r_D}{p}(1 - \cos(TA)) = 0.00072$

$\overline{v}_A = \dot{f}\,\overline{r}_D + \dot{g}\overline{v}_D = 0.9301\ \ \hat{x} - 0.0038\ \ \hat{y} - 0.00187\ \ \hat{z}\ \ [km/s]$

$\overline{v}_{ss} = \sqrt{2(\dfrac{\mu}{r_A} - \dfrac{\mu}{2a_{ss}})}\ \ \hat{\theta} = 2.282\ \ \hat{\theta}$ At apoapsis of final orbit, therefore velocity is tangential, $\gamma_2 = 0$

$[\overline{v}_{ss}]_I = [IR]_{ss}[\overline{v}_{ss}]_R = 2.0962\ \ \hat{x} + 0.6987\ \ \hat{y} - 0.5705\ \ \hat{z}\ \ [km/s]$

$\Delta\overline{v}_A = [\overline{v}_{ss}]_I - [\overline{v}_A]_I = 1.1661\ \ \hat{x} + 0.7026\ \ \hat{y} - 0.5686\ \ \hat{z}\ \ [km/s]$

$\mathbf{|\Delta\overline{v}_A| = 1.475\ \ [km/s]}$

```
% Transfer Orbit unit vectors
h2_T_hat      = cross(r_ss_I,v_A2_I)/norm(cross(r_ss_I,v_A2_I));
thetaA2_T_hat = cross(h2_T_hat,rA_T_hat);

% Transfer orbit inclination, RAAN, AOP
i_T2          = acosd(h2_T_hat(3));
RAAN_T2       = atan2d(h2_T_hat(1),-h2_T_hat(2));
thetaA_T2     = atan2d(rA_T_hat(3),thetaA2_T_hat(3));
AOP_T2        = wrapTo360(thetaA_T2) - true_anomaly_A2;

fprintf(['The transfer orbit inclination is %.3f deg \n'...
         'The transfer orbit argument of periapsis is %.3f deg \n'...
         'The transfer orbit RAAN is %.3f deg'],i_T2,AOP_T2,RAAN_T2)
```

```
The transfer orbit inclination is 26.100 deg
The transfer orbit argument of periapsis is 101.739 deg
The transfer orbit RAAN is 0.000 deg
```

$\hat{h}_T = \frac{\bar{r}_{ss} \times \bar{v}_A}{|\bar{r}_{ss} \times \bar{v}_A|} = \sin\Omega_T \sin i_T \;\; \hat{x} - \cos\Omega_T \sin i_T \;\; \hat{y} + \cos i_T \;\; \hat{z} = 0\hat{x} - 0.4399 \;\; \hat{y} + 0.8980 \;\; \hat{z}$

$i_T = \pm\cos^{-1} 0.8980 = 26.1 \;\; [deg]$ - Trasnfer Orbit Inclination, choose positive value

$\underline{i_T = 26.1 \;\; [deg]}$

$\Omega_T = \tan^{-1}\frac{\sin\Omega_T \sin i_T}{\cos\Omega_T \sin i_T} = \tan^{-1}\frac{0}{0.4399} = 0 \;\; [deg]$ based off of the signs of numerator and denominator, must be in quadrant 1.

$\underline{\Omega_T = 0 \;\; [deg]}$

$\hat{r}_{A,T} = \frac{\bar{r}_{ss}}{r_{ss}} = (\cos\Omega_T \cos\theta_{A,T} - \sin\Omega_T \cos i_T \sin\theta_{A,T})\hat{x} + (\sin\Omega_T \cos\theta_{A,T} + \cos\Omega_T \cos i_T \sin\theta_{A,T})\hat{y} + (\sin i_T \sin\theta_{A,T})\hat{z} = 0.1768\hat{x} - 0.8839\hat{y} - 0.4330\hat{z}$

$\hat{\theta}_{A,T} = \hat{h}_T \times \hat{r}_{A,T} = (-\cos\Omega_T \sin\theta_{A,T} - \sin\Omega_T \cos i_T \cos\theta_{A,T})\hat{x} + (-\sin\Omega_T \sin\theta_T + \cos\Omega_T \cos i_T \cos\theta_{A,T})\hat{y} + (\sin i_T \cos\theta_{A,T})\hat{z} = 0.984\hat{x} + 0.159\hat{y} + 0.078\hat{z}$

$\theta_{A,T} = \tan^{-1}\frac{\sin i_T \sin\theta_{A,T}}{\sin i_T \cos\theta_{A,T}} = \frac{-0.433}{0.0778} = -79.82 \;\; [deg]$ based off of the signs of numerator and denominator, must be in quadrant 4

$\omega_t = \theta_{A,T} - \theta_A^*$

$\underline{\omega_t = 101.74 \;\; [deg]}$

```
% Arrival deltaV rotated into orbit frame
dv_A2         = I_DCM_R(i_T2,thetaA_T2,RAAN_T2)'*dv_A2_I;

% Extract alpha and beta
beta_A2       = asind(dv_A2(3)/norm(dv_A2));
alpha_A2      = atan2d(dv_A2(1),dv_A2(2)) - gamma_A2;
dv_A2_VNC     = norm(dv_A2)*[cosd(beta_A2)*cosd(alpha_A2),cosd(beta_A2)*sind(alpha_A2), sind(beta_A2)];
```

$[IR]_A = [IR]|_{i=26.1°,\theta=-79.82°,\Omega=0°}$ - Orbit frame to Inertial frame direction cosine matrix, at arrival location in transfer orbit

$[\Delta\bar{v}_A]_R = [IR]_A^T[\Delta\bar{v}_A]_I$

$[\Delta\bar{v}_A]_R = \Delta v_A[\cos\beta_A \sin\phi_A \;\; \hat{r} + \cos\beta_A \cos\phi_A \;\; \hat{\theta} + \sin\beta_A \;\; \hat{h}] = -0.1686 \;\; \hat{r} + 1.2150 \;\; \hat{\theta} - 0.8197 \;\; \hat{h} \;\; [km/s]$

$\beta_A = \sin^{-1}\frac{-0.8197}{\Delta v_A} = -33.75 \;\; [deg]$

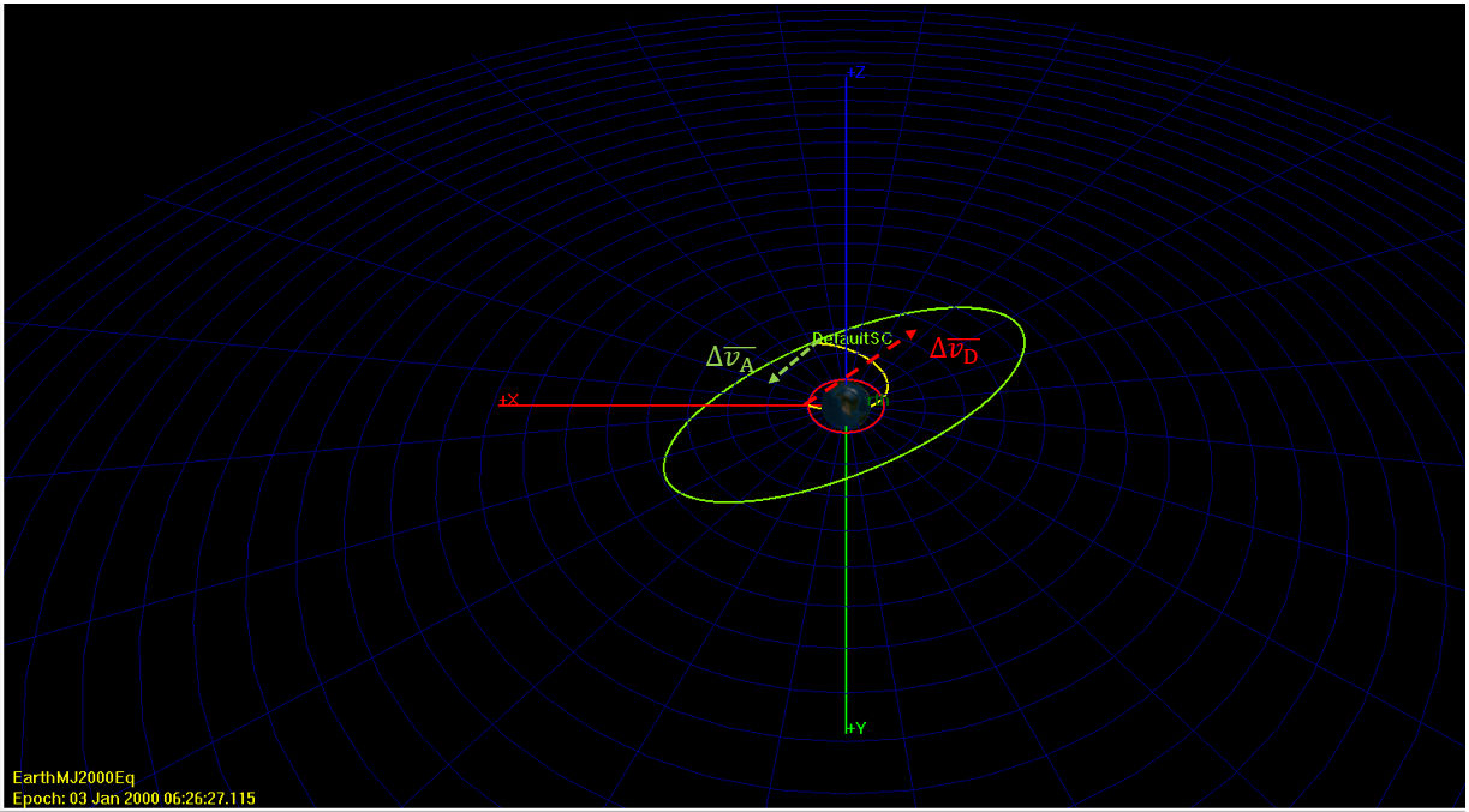
$\tan\phi_A = \frac{\Delta v_A \cos\beta_A \sin\phi_A}{\Delta v_A \cos\beta_A \cos\phi_A} = \frac{-0.1686}{1.2150}$ based off of the signs of numerator and denominator, must be in quadrant 4

$\phi_A = -7.9 \;\; [deg] = \gamma_A + \alpha_A$

$\boldsymbol{\alpha_A = -18.34 \;\; [deg]}$

$[\Delta\bar{v}_A]_{VNC} = \Delta v_A[\cos\beta_A \cos\alpha_A \;\; \hat{V} + \cos\beta_A \sin\alpha_A \;\; \hat{C} + \sin\beta_A \;\; \hat{N}] = 1.1643 \;\; \hat{V} - 0.3861 \;\; \hat{C} - 0.8197 \;\; \hat{N} \;\; [km/s]$

The transfer passes through periapsis. This is known because the transfer starts at a location on the transfer orbit that is descending (true anomaly < 0) towards periapis and ends at an arrival location that is ascending (true anomaly > 0). Observing the transfer arc in GMAT, it is clear that the transfer passes through periapsis, as it actually intersects with the Earth, which is problematic. Therefore, if maintaining the same TOF the only option is the option from part A, which had very large Δv requirements. The best course of action may be to increase the required time of flight, to lower the required propellant cost and avoid colliding with Earth.



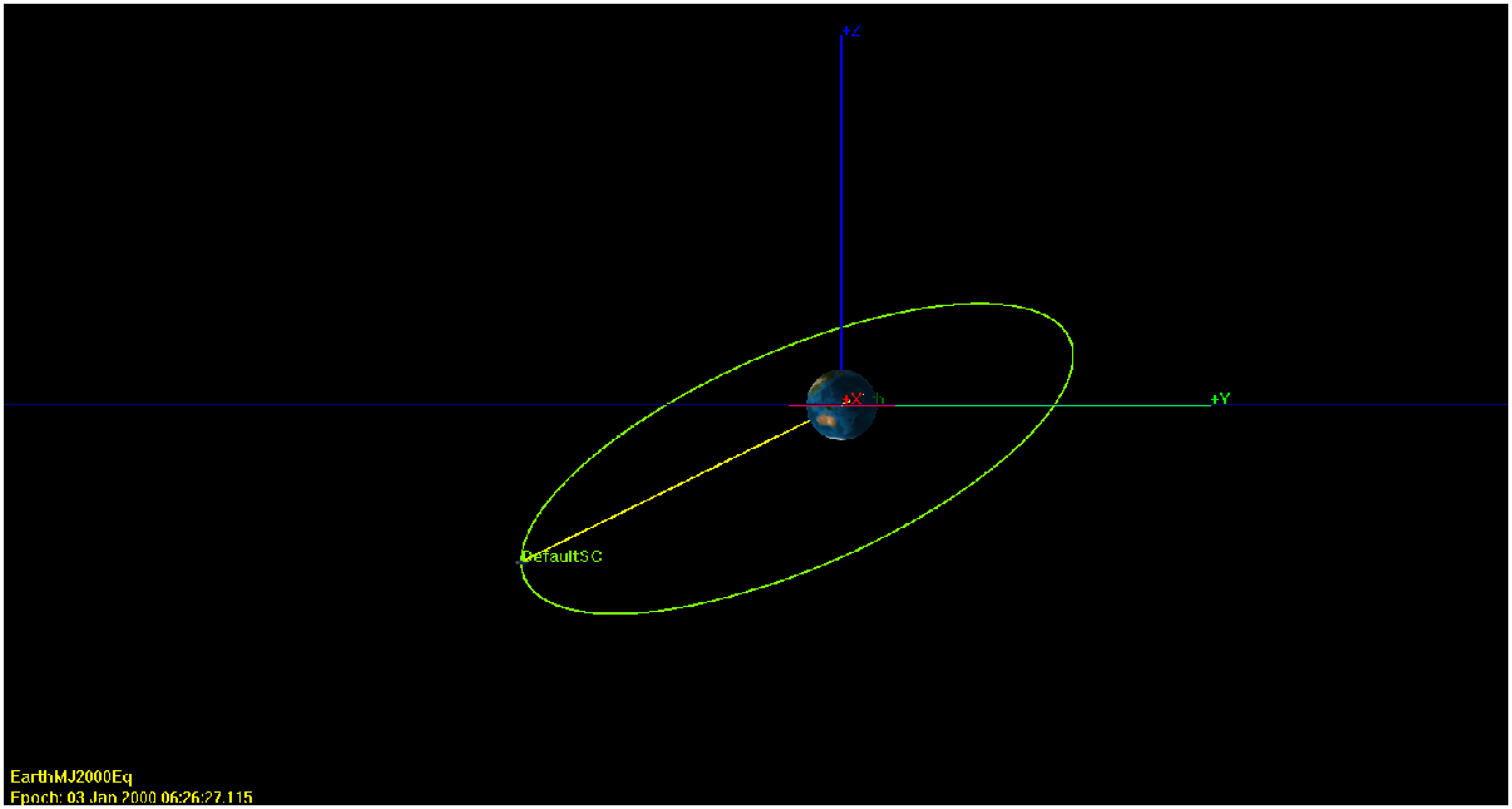
Red Line: Original Orbit

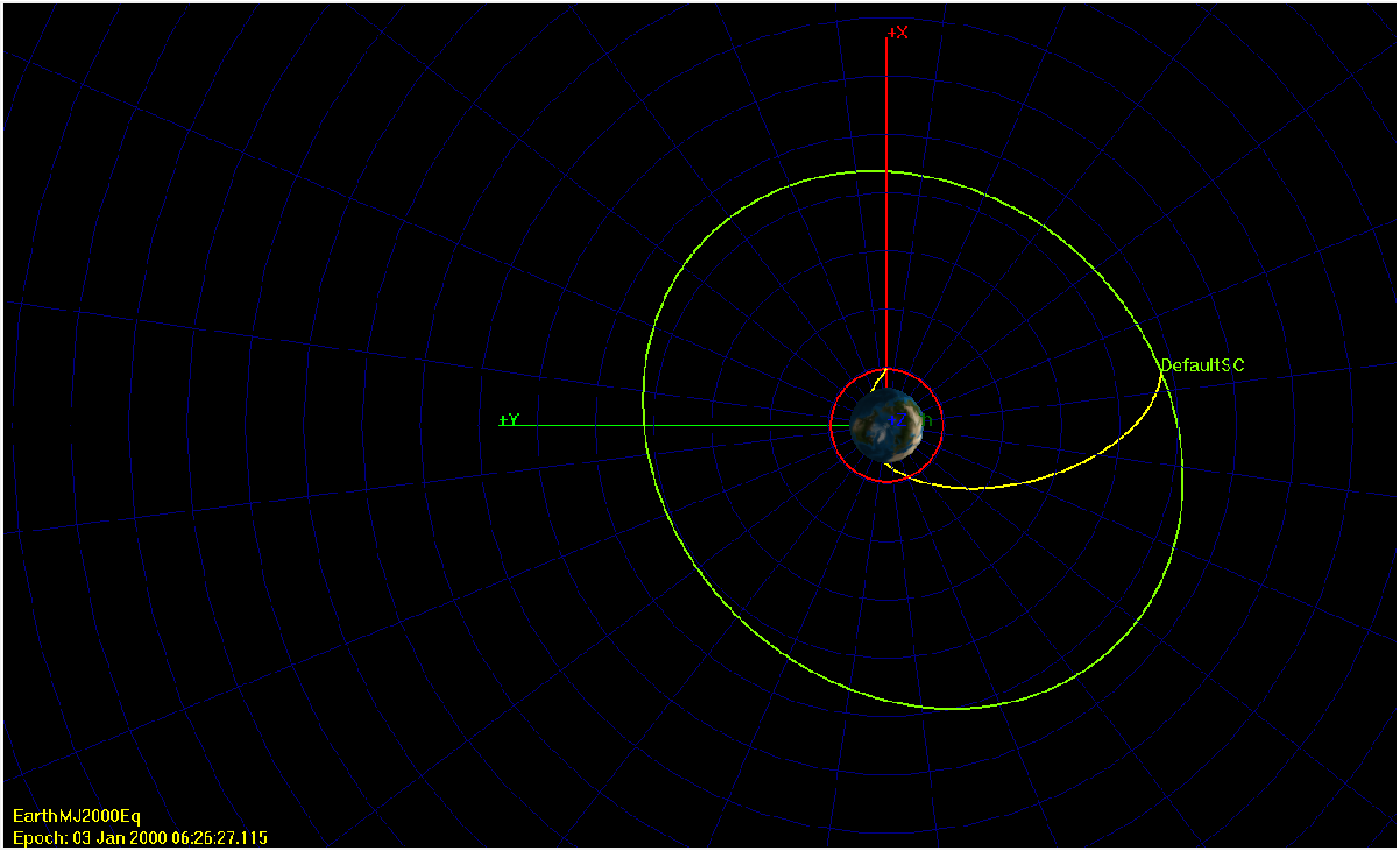
Dashed Red Line: 1st Maneuver

Yellow Line: Transfer Arc

Green Dashed Line: 2nd Maneuver

Green Line: Final Orbit

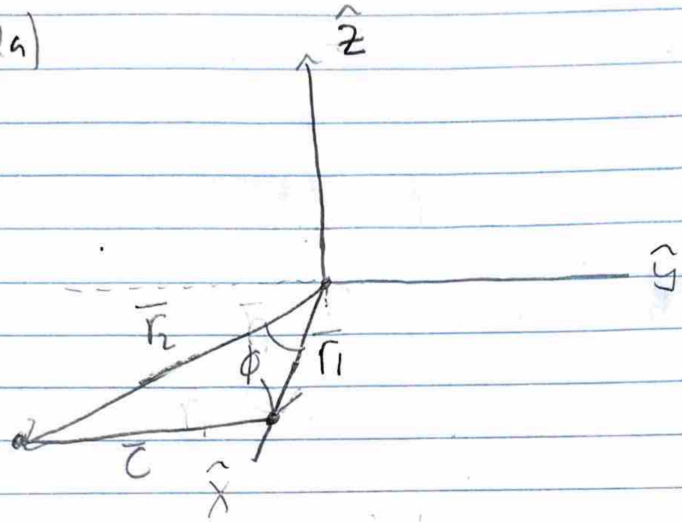

+Z
+Y
+X
DefaultSC
EarthMJ2000Eq
Epoch: 03 Jan 2000 06:26:27.115



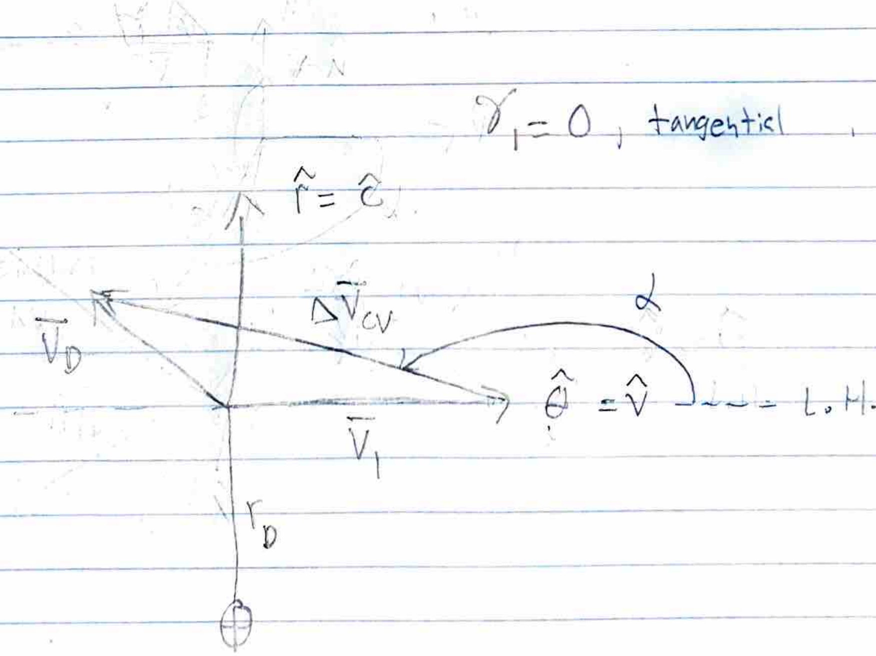
Functions

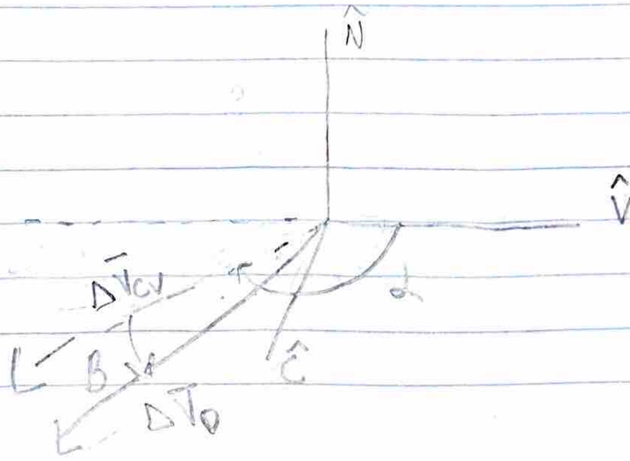
```
function DCM = I_DCM_R(i, theta, RAAN)
    C      = @(x) cosd(x);
    S      = @(x) sind(x);

   DCM     = [C(RAAN)*C(theta) - S(RAAN)*C(i)*S(theta), -C(RAAN)*S(theta) - S(RAAN)*C(i)*C(theta),...
                   S(RAAN)*S(i); S(RAAN)*C(theta) + C(RAAN)*C(i)*S(theta), -S(RAAN)*S(theta) + C(theta)*C(i)*C(RAAN),...
                   -C(RAAN)*S(i);S(i)*S(theta), S(i)*C(theta), C(i)];
end
```

#2a)

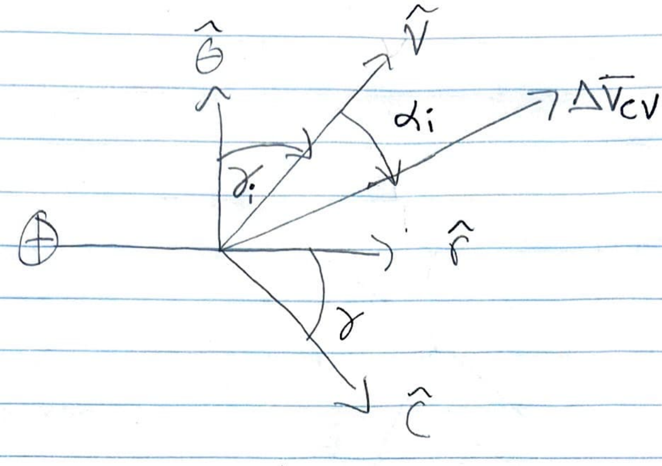


$\gamma_1 = 0$, tangential





#2)



$$\phi_i = \gamma_i + \alpha_i$$

VNC Maneuver defintions