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```
clear
close all
clc
```

Problem 2a)

<u>Find</u>: $\Delta\bar{v}$ in rotating orbit unit vector, inertial orbit unit vectors, and VNC unit vectors

```
% Earth Parameters
mu_earth     = 398600.4415;
R_earth      = 6378.1363;

% Maneuver
deltaV       = 1;            % [km/s]
alpha        = 45;           % [deg]
beta         = 60;           % [deg]

% Original Orbit
a            = 8*R_earth;    % [km]
e            = 0.7;
i            = 30;           % [deg]
RAAN         = 60;           % [deg]
AOP          = 90;           % [deg]
ta_man       = 150;          % True anomaly at maneuver [deg]

% deltaV in VNC frame
deltaV_VNC   = deltaV*[cosd(beta)*cosd(alpha), cosd(beta)*sind(alpha), sind(beta)]';
```

$$[\Delta\bar{v}]^{VNC} = \Delta v[\cos\beta\cos\alpha\ \hat{V} + \cos\beta\sin\alpha\ \hat{C} + \sin\beta\ \hat{N}]$$
$$[\Delta\bar{v}]^{VNC} = 0.3536\ \hat{V} + 0.3536\ \hat{C} + 0.866\ \hat{N}\ [km/s]$$

```
% Calculate specific energy of original orbit
energy       = -mu_earth/(2*a);

% Semi-latus rectum of original orbit
p            = a*(1 - e^2);

% Calculate specific angular momentum of original orbit
h            = sqrt(mu_earth*p);

% Get orbital radius at maneuver point
r            = p/(1 + e*cosd(ta_man));

% Get original velocity at maneuver point
v            = sqrt(2*(energy + mu_earth/r));

% Get original FPA at maneuver point - sign based on true anomaly
gamma        = sign(ta_man)*acosd(h/(r*v));

% Determine phi for rotating orbit unit vectors
phi          = gamma + alpha;
```

$p = a(1 - e^2)$

$r = \dfrac{p}{1 - e\cos\theta^*}$

$h = \sqrt{\mu p} = rv\cos\gamma$

$\epsilon = \dfrac{-\mu}{2a} = \dfrac{v^2}{2} - \dfrac{\mu}{r}$

$v = \sqrt{2(\epsilon + \frac{\mu}{r})}$

$\gamma = \cos^{-1}\dfrac{h}{rv}$

$\phi = \gamma + \alpha$

```
% deltaV in rotating orbit unit vectors
deltaV_R     = deltaV*[cosd(beta)*sind(phi), cosd(beta)*cosd(phi), sind(beta)]';
```

$[\Delta\overline{v}]^R = \Delta v[\cos\beta\sin\phi\ \ \hat{r} + \cos\beta\cos\phi\ \ \hat{\theta} + \sin\beta\ \ \hat{h}]$

$[\Delta\overline{v}]^R = 0.4991\ \ \hat{r} + 0.0294\ \ \hat{\theta} + 0.866\ \ \hat{h}\ \ [km/s]$

```
% DCM rotating orbit frame to perifocal frame
P_DCM_R      = [ cosd(ta_man), -sind(ta_man), 0;...
                 sind(ta_man), cosd(ta_man), 0;...
                 0, 0, 1];

% Rotate position and velocity vectors from rotating orbit frame to perifocal inertial frame
deltaV_P     = P_DCM_R*deltaV_R;
```

$$[PR] = \begin{pmatrix} \cos\theta^* & -\sin\theta^* & 0 \\ \sin\theta^* & \cos\theta^* & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$[\Delta\overline{v}]^P = [PR][\Delta\overline{v}]^R$

$[\Delta\overline{v}]^P = -0.447\ \ \hat{e} + 0.2241\ \ \hat{p}\ \ + 0.866\ \ \hat{h}\ \ [km/s]$

Problem 2b)

$\underline{Find} : \overline{r}, \overline{v}, r, v, \gamma, \theta^*, M, E, \Delta\omega, a_N, e_N, i_N, \Omega_N, \omega$post manuever

```
% Position vector in original rotating orbit frame
r_R          = [r;0;0];

% DCM from rotating orbit frame to Earth Equatorial J2000 frame
C            = @(x) cosd(x);
S            = @(x) sind(x);

% Define arguement of latitude
theta        = AOP + ta_man;

I_DCM_R      = [C(RAAN)*C(theta) - S(RAAN)*C(i)*S(theta), -C(RAAN)*S(theta)...
                - S(RAAN)*C(i)*C(theta), S(RAAN)*S(i); S(RAAN)*C(theta) + ...
                C(RAAN)*C(i)*S(theta), -S(RAAN)*S(theta) + ...
                C(theta)*C(i)*C(RAAN),-C(RAAN)*S(i);S(i)*S(theta), S(i)*C(theta), C(i)];

% Rotate position and velocity vectors from rotating orbit frame to perifocal inertial frame
r_I          = I_DCM_R*r_R;
fprintf('The orbit radius is %.3f Earth Radii',norm(r_I)/R_earth)
```

```
The orbit radius is 10.361 Earth Radii
```

$$[IR] = \begin{pmatrix} C_\Omega C_\theta - S_\Omega C_i S_\theta & -C_\Omega S_\theta - S_\Omega C_i C_\theta & S_\Omega S_i \\ S_\Omega C_\theta + C_\Omega C_i S_\theta & -S_\Omega S_\theta + C_\theta C_i C_\Omega & -C_\Omega S_i \\ S_i S_\theta & S_i C_\theta & C_i \end{pmatrix}$$

$$[\overline{r}]^I = [IR][\overline{r}]^R$$

$$\overline{r} = 4.14\ \hat{x} - 8.37\ \hat{y} - 4.48\ \hat{z}\ [R_{earth}] = 26401.9\ \hat{x} - 53396.9\ \hat{y} - 28615.3\ \hat{z}\ [km]$$

$$r = |\overline{r}| = 66084.2\ [km] = 10.36\ [R_{earth}]$$

The position vector before and after the manuever is the same

```
% Define original velocity vector in orbit frame
v_R          = [v*sind(gamma), v*cosd(gamma),0]';

% Rotate original velocity vector into Earth J2000 frame
v_I          = I_DCM_R*v_R;

% Rotate deltaV vector from orbit frame to Earth J2000 frame
deltaV_I     = I_DCM_R*deltaV_R;

% Velocity vector post maneuver in Earth J2000 frame
v_new_I      = deltaV_I + v_I;
fprintf('The orbital speed post maneuver is %.3f km/s',norm(v_new_I))
```

```
The orbital speed post maneuver is 2.590 km/s
```

$$[\overline{v}^-]^R = v^- \sin(\gamma^-)\ \hat{r} + v^- \cos(\gamma^-)\ \hat{\theta}$$

$$[\overline{v}^-]^I = [IR][\overline{v}^-]^R$$

$$[\Delta\overline{v}]^I = [IR][\Delta\overline{v}]^R$$

$$[\overline{v}^+]^I = [\Delta\overline{v}]^I + [\overline{v}^-]^I$$

$$\overline{v}^+ = 2.39\ \hat{x} - 0.89\ \hat{y} - 0.45\ \hat{z}\ [km/s]$$

$$v^+ = |\overline{v}^+| = 2.59\ [km/s]$$

```
% Calculate new specific angular momentum
h_new_I      = cross(r_I,v_new_I);

% New specific angular momentum magnitude
h_new        = norm(h_new_I);

% Calculate new flight path angle - sign from dot product of r and v
gamma_new    = sign(dot(r_I,v_new_I))*acosd(h_new/(r*norm(v_new_I)));
fprintf('The new flight path angle is %.3f deg',gamma_new)
```

```
The new flight path angle is 46.180 deg
```

$$\overline{h}^+ = \overline{r} \times \overline{v}^+ = -1301.5\ \hat{x} - 56479.2\ \hat{y} + 104190.8\ \hat{z}\ [km^2/s]$$

$$\gamma^+ = \cos^{-1}\frac{|\overline{h}^+|}{rv^+}$$

$\gamma^+ = 46.18\ [deg]$ **Positive sign from** $\overline{r} \cdot \overline{v}$

```
% Calculate new true anomaly
ta_new      = atan2d((r*norm(v_new_I)^2/mu_earth)*cosd(gamma_new)*sind(gamma_new),...
               ((r*norm(v_new_I)^2/mu_earth)*cosd(gamma_new)^2 - 1));
fprintf('The new true anomaly at the maneuver point is %.3f deg', ta_new)
```

```
The new true anomaly at the maneuver point is 130.025 deg
```

$$\tan\theta^{*+} = \frac{\frac{rv^{+2}}{\mu}\cos\gamma^{+}\sin\gamma^{+}}{\frac{rv^{+2}}{\mu}\cos^2\gamma^{+} - 1}$$

$$\theta^{*+} = 130.0 \ [deg]$$

```
% Calculate new eccentricity
e_new       = sqrt((r*norm(v_new_I)^2/mu_earth - 1)^2*cosd(gamma_new)^2 + sind(gamma_new)^2);
fprintf('The new eccentricity is %.3f',e_new);
```

```
The new eccentricity is 0.726
```

$$e_N^2 = (\frac{rv^{+2}}{\mu} - 1)^2\cos\gamma^{+2} + \sin\gamma^{+2}$$

$$e_N = 0.726$$

```
% Calculate new eccentric anomaly
E_new       = 2*atan2d(tand(ta_new/2),sqrt((1+e_new)/(1-e_new)));
fprintf('The new eccentric anomaly is %.3f deg',E_new);
```

```
The new eccentric anomaly is 81.091 deg
```

$$\tan(\frac{\theta^*}{2}) = \sqrt{\frac{1+e}{1-e}}\tan(\frac{E}{2})$$

$$E^{+} = 2\tan^{-1}[\tan(\frac{\theta^{*+}}{2})/\sqrt{\frac{1+e_N}{1-e_N}}]$$

$$E^{+} = 81.09 \ [deg]$$

```
% Calculate new mean anomaly
M_new       = E_new*pi/180 - e_new*sind(E_new);
fprintf('The new mean anomaly is %.3f deg', M_new*180/pi)
```

```
The new mean anomaly is 40.013 deg
```

$$M^{+} = E^{+} - e_N\sin E^{+}$$

$$M^{+} = 40.01 \ [deg]$$

```
% Calculate new specific energy
energy_new  = norm(v_new_I)^2/2 - mu_earth/r;

% Calculate new semi major axis
a_new       = -mu_earth/(2*energy_new);
fprintf('The new semi major axis is %.3f Earth Radii',a_new/R_earth)
```

```
The new semi major axis is 11.673 Earth Radii
```

$$\epsilon_N = \frac{v^{+2}}{2} - \frac{\mu}{r}$$

$$a_N = \frac{-\mu}{2\epsilon_N}$$

$$a_N = 11.67 \ [R_{earth}] = 74451.2 \ [km]$$

```
% Calculate new angular momentum unit vector
h_hat       = h_new_I/h_new;

% Calculate new inclination angle, choose positive sign
i_new        = acosd(h_hat(3));
fprintf('The inclination angle is %.3f deg',i_new)
```

```
The inclination angle is 28.467 deg
```

$$\hat{h}^+ = \frac{\bar{h}^+}{|\bar{h}^+|} = \sin\Omega_N \sin i_N \ \hat{x} - \cos\Omega_N \sin i_N \ \hat{y} + \cos i_N \ \hat{z} = -0.011\hat{x} - 0.4765 \ \hat{y} + .8791 \ \hat{z}$$

$$i_N = \cos^{-1} 0.8791$$

$$i_N = 28.47 \ [deg]$$

Note: The positive inclination angle was chosen (inverse cosine yields a +/- result)

```
% Calculate new RAAN
RAAN_new     = atan2d(h_hat(1),-h_hat(2));
fprintf('The RAAN is %.3f deg', RAAN_new)
```

```
The RAAN is -1.320 deg
```

$$\tan\Omega_N = \frac{\sin\Omega_N \sin i_N}{\cos\Omega_N \sin i_N} = \frac{-0.011}{0.4765}$$

$$\Omega_N = -1.32 \ [deg]$$

Note: The sign check was taken care of by the atan2() function. This value of the RAAN satisifies both $\sin\Omega \sin i$ = -0.011 & $-\cos\Omega \sin i$ = -0.4765.

```
% Calculate rhat and thetahat unit vectors
r_hat     = r_I/norm(r_I);
theta_hat= cross(h_hat,r_hat);

% Calculate theta  = true anomaly + AOP
theta_new= wrapTo2Pi(atan2(r_hat(3),theta_hat(3)))*180/pi;

% Calculate argument of periapsis
AOP_new   = theta_new - ta_new;
fprintf('The new argument of periapsis is %.3f deg', AOP_new)
```

```
The new argument of periapsis is 164.685 deg
```

$\hat{r} = \frac{\bar{r}}{r} = (\cos\Omega_N \cos\theta_N - \sin\Omega_N \cos i_N \sin\theta_N)\hat{x} + (\sin\Omega_N \cos\theta_N + \cos\Omega_N \cos i_N \sin\theta_N)\hat{y} + (\sin i_N \sin\theta_N)\hat{z}$

$\hat{r} = .399\ \hat{x} - 0.808\ \hat{y} - 0.433\ \hat{z}$

$\hat{\theta} = \hat{h} \times \hat{r} = (-\cos\Omega_N \sin\theta_N - \sin\Omega_N \cos i_N \cos\theta_N)\hat{x} + (-\sin\Omega_N \sin\theta_N + \cos\Omega_N \cos i_N \cos\theta_N)\hat{y} + (\sin i_N \cos\theta_N)\hat{z}$

$\hat{\theta} = 0.9167\ \hat{x} + 0.3464\ \hat{y} + 0.1992\ \hat{z}$

$\tan\theta_N = \frac{\sin i_N \sin\theta_N}{\sin i_N \cos\theta_N} = \frac{-0.433}{0.1992}$

$\omega^+ = \theta_N - \theta^{*+}$

$\omega^+ = 164.68\ [deg]$

The quadrant check of θ was done using the atan2() function. This value of the θ satisifies both $\sin\theta \sin i$ = -0.433 & $\cos\theta \sin i$ = 0.1992.

```
% Calculate change in argument of periapsis
deltaAOP     = AOP_new - AOP;
fprintf('The change is the argument of periapsis is %.2f deg',deltaAOP)
```

```
The change is the argument of periapsis is 74.69 deg
```

$\Delta\omega = \omega^+ - \omega$

$\Delta\omega = 74.7\ [deg]$

Problem 2a)

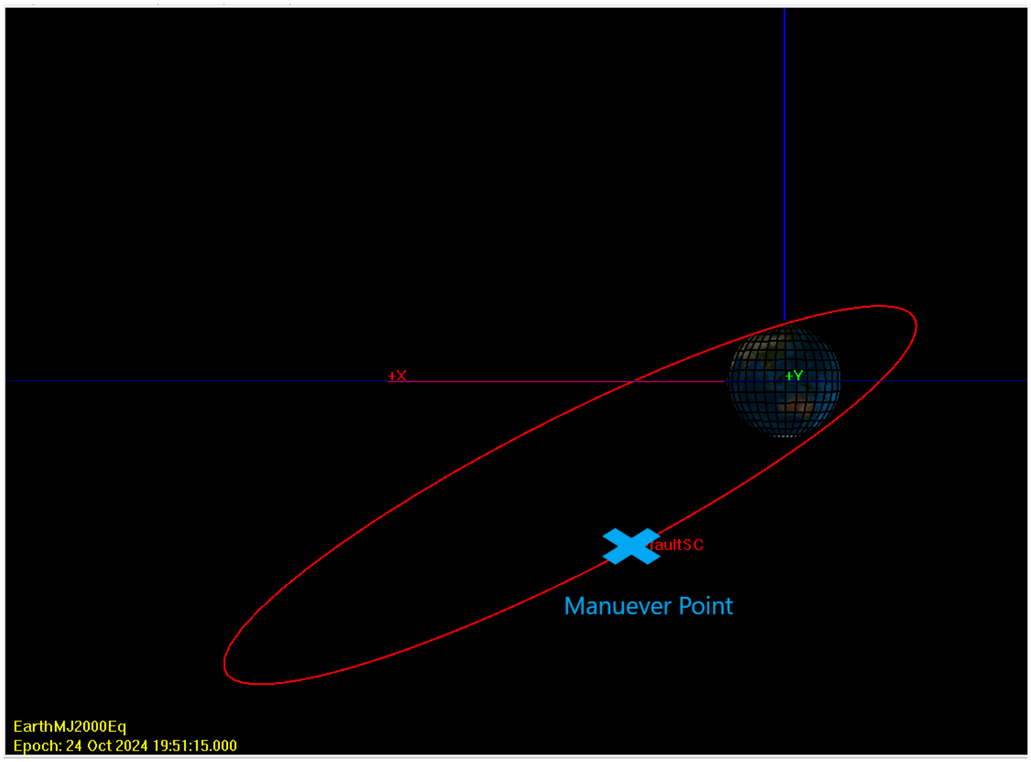

Figure 1: Problem 2a Orbit - Y Axis View

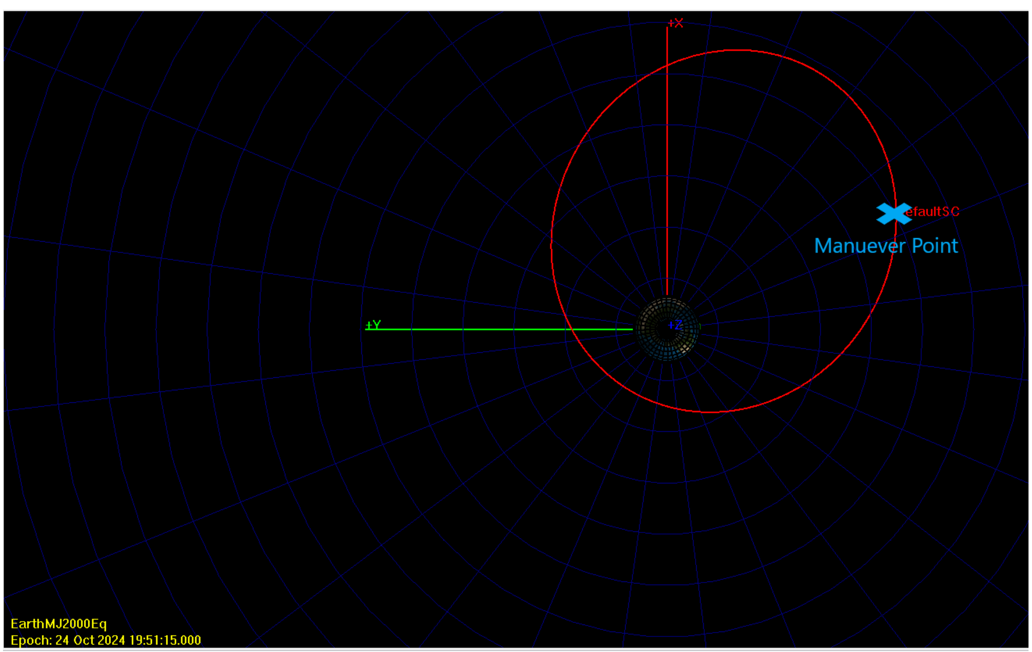

Figure 2: Problem 2a Orbit - Z Axis View

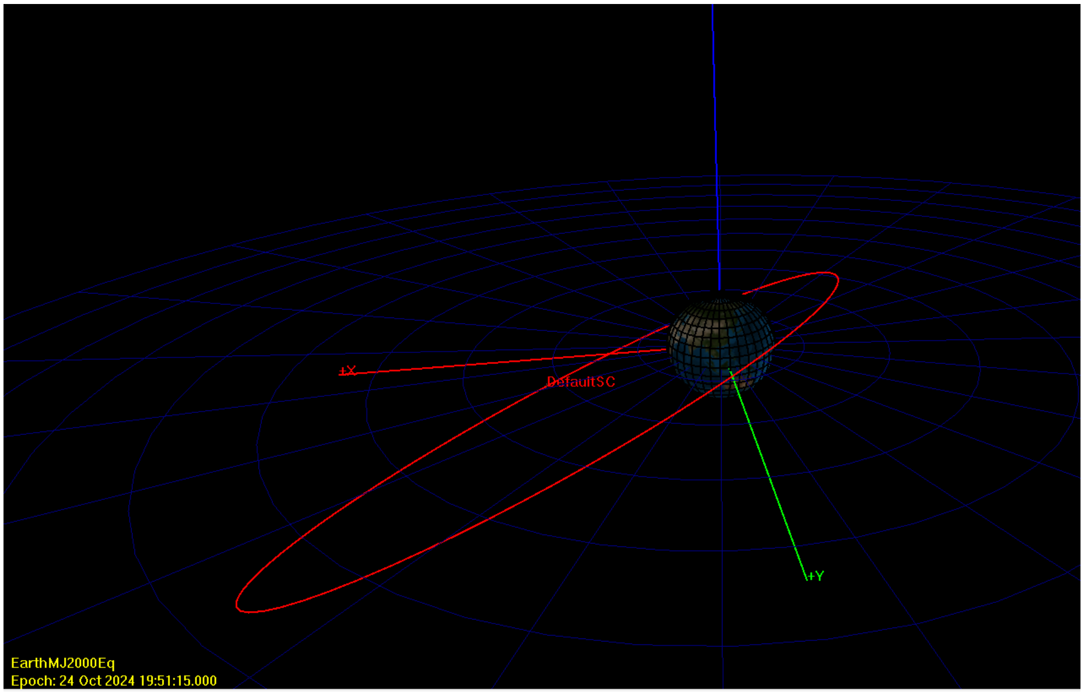

Figure 3: Problem 2a Orbit: Angled View

Problem 2c)

In the figures below, there are three colored lines. The pink line is the initial orbit, pre-maneuver. The white line, is the start of the initial orbit to the maneuver point in the initial orbit. The yellow line is the final orbit, post-maneuver.

The equatorial plane is denoted by the blue grid line spanning from the Earths equator (in the X-Y plane) in each image.

The J2000 unit vectors are given by red +X, green +Y, and blue +Z lines originating at the Earth in each image.

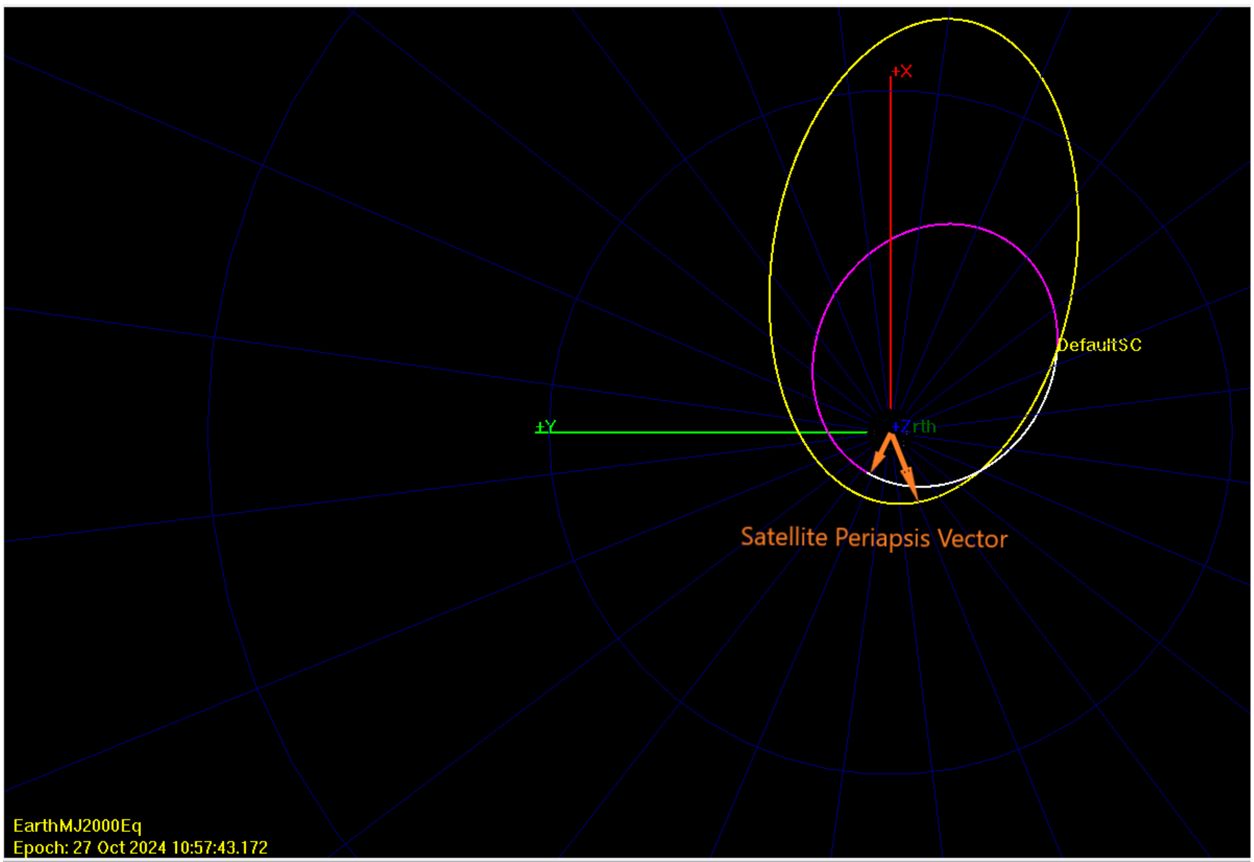

Figure 4: Problem 2c Orbits - Includes Periapsis Vectors, Z Axis View

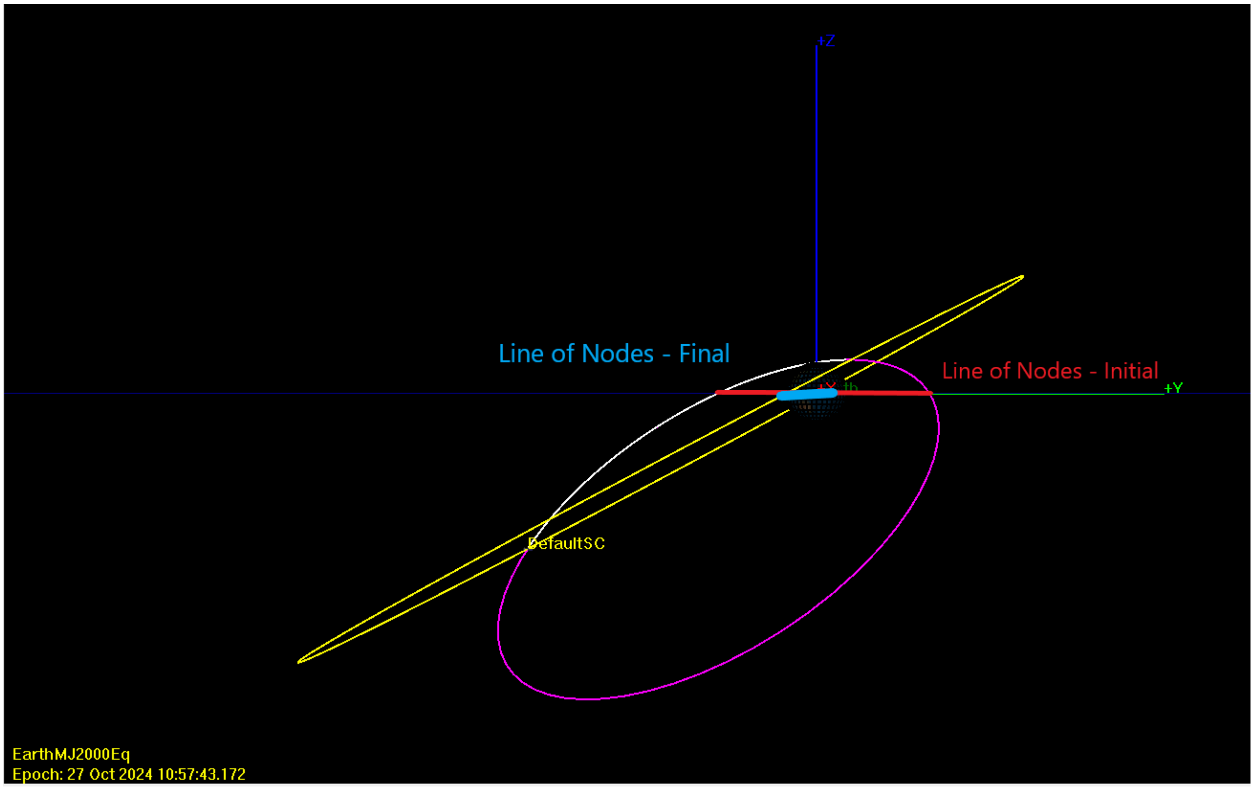

Figure 5: Problem 2c Orbits - Includes Line of Nodes, X Axis View

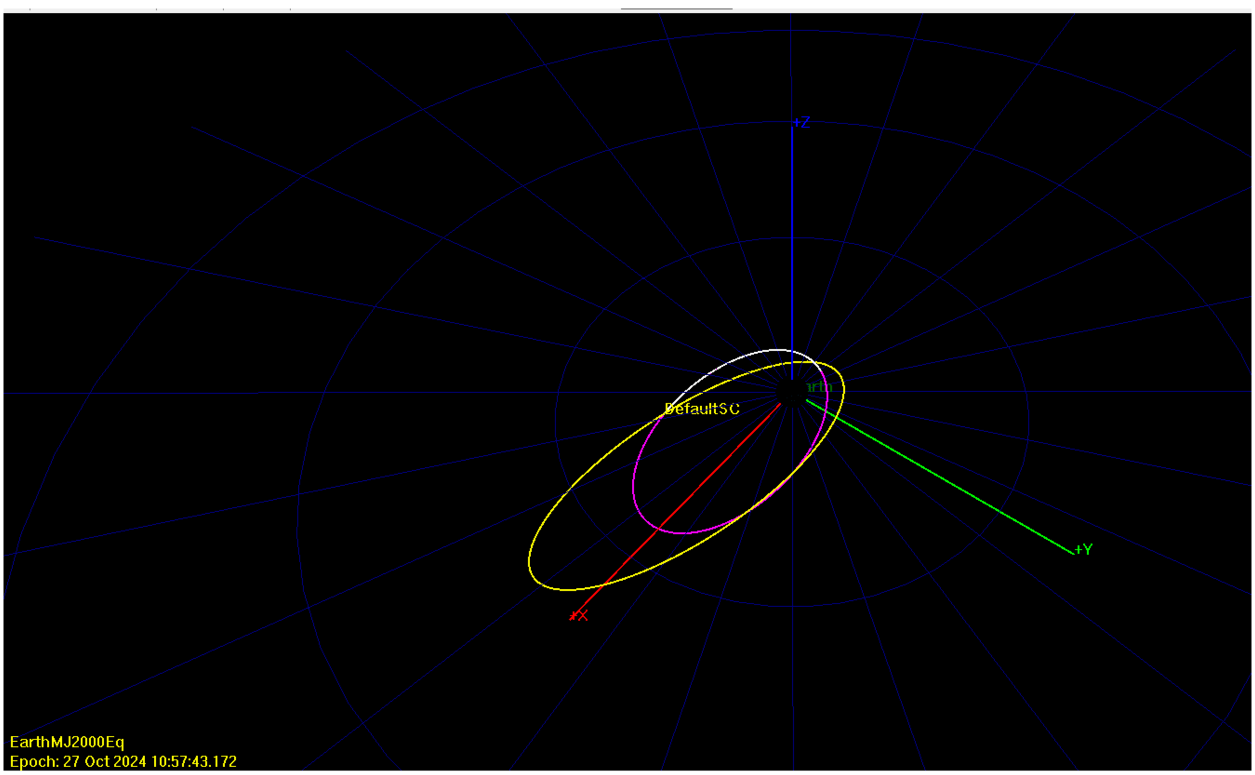

Figure 6: Problem 2c Orbits - Angled View

Below is tabulated data from the maneuver point. The first row is GMAT data at the maneuver point, prior to the maneuver in the initial orbit. The second row is GMAT data at the maneuver point, after the maneuver in the final orbit.

TA [deg]	SMA [km]	ECC	INC [deg]	RAAN [deg]	AOP [deg]	FPA [deg]	EA [deg]	MA [deg]	VX [km/s]	VY [km/s]	VZ [km/s]	HX [km^2/s]	HY [km^2/s]	HZ [km^2/s]
150	51025.0904	0.7	30	60	90	41.63121	114.9367	78.56872	1.79254237	-0.28462	-0.97843	44100.8036	-25461.61083	88201.6072
130.11815	74451.2146	0.725706	28.4673791	358.6799024	164.685138	46.18963	81.21165	40.11995	2.38850888	-0.88435	-0.44955	-1301.513745	-56479.19592	104190.7731

These values align with what was calculated in problem 2a and problem 2b, note that the flight path angle (FPA) was adjusted to reflect the 90 degree offset caused by the FPA definition in GMAT.