Problem 2d)

To check the assumption of the two-body problem, one can analyze the net perturbation accelerations created by the Sun and Moon on the space station – Earth system. Assume the four bodies are aligned as such: Sun – Moon – SS-Earth, and the orbital radius is equal to the semi-major axis of each respective orbit. Defining the positive radial direction as outward from the Earth, we obtain the following position vectors.

$$\begin{aligned}
&\bar{r}_{earth,sun} = 149597898\ \hat{r}\ [km] \\
&\bar{r}_{earth,moon} = 384400\ \hat{r}\ [km] \\
&\bar{r}_{earth,ss} = 114806.4534\ \hat{r}\ [km] \\
&\bar{r}_{ss,sun} = \bar{r}_{earth,sun} - \bar{r}_{earth,ss} = 149483091.5466\ \hat{r}\ [km] \\
&\bar{r}_{ss,moon} = \bar{r}_{earth,moon} - \bar{r}_{earth,ss} = 269593.5466\ \hat{r}\ [km]
\end{aligned} \tag{1}$$

The pertubation accelerations for the sun and moon are given below.

$$\begin{aligned}
|\bar{a}_{pert,sun}| &= \mu_{sun}\left(\frac{\bar{r}_{ss,sun}}{r^3_{ss,sun}} - \frac{\bar{r}_{earth,sun}}{r^3_{earth,sun}}\right) = 9.11 \times 10^{-9}\ [km/s^2] \\
|\bar{a}_{pert,moon}| &= \mu_{moon}\left(\frac{\bar{r}_{ss,moon}}{r^3_{ss,moon}} - \frac{\bar{r}_{earth,moon}}{r^3_{earth,moon}}\right) = 3.43 \times 10^{-8}\ [km/s^2]
\end{aligned} \tag{2}$$

The dominant acceleration of the space station due to the Earth is given below.

$$|\bar{a}_{earth,ss}| = \mu_{earth}\left(\frac{\bar{r}_{earth,ss}}{r^3_{earth,ss}}\right) = 3.02 \times 10^{-5}\ [km/s^2] \tag{3}$$

The dominant acceleration is 3 orders of magnitude larger than the pertubring acceleration of the Moon, and 4 orders of magnitude larger than the perturbing acceleration of the Sun. Therefore, the effects of the Sun and Moon can be neglected, and the two-body problem stands to be a reasonable assumption, and should provide good results for a preliminary analysis. These findings are verified with a GMAT simulation shown on the following page, where three space station orbits are plotted over the course of 10 orbital periods about the Earth. The first being the space station-Earth system, the second being a space station-Earth-Moon system, and the final being a space station-Earth-Moon-Sun system. All three orbits are nearly identical.