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```
clear
close all
clc
```

Problem 1a)

```
% Periapsis and Apoapsis
rp       = 1.8;  % Mars Radii
ra       = 8;    % Mars Radii

% Initial Mean Anomaly
M0       = -120; % [deg]
```

$\underline{Find}: a, e, p, period, \epsilon, r_o, v_0, \theta_0^*, E_0, \gamma_0, t_0 - t_p$

```
% Calculate Semi major axis
a        = (ra + rp)/2;
fprintf('The semi major axis %.1f Mars Radii', a)
```

```
The semi major axis 4.9 Mars Radii
```

$a = \frac{r_a + r_p}{2} = 4.9\ [R_{Mars}]$

```
% Calculate Eccentricity
e        = 1 - (rp/a);
fprintf('Eccentricity is %.3f', e)
```

```
Eccentricity is 0.633
```

$r_p = a(1 - e)$

$e = 1 - \frac{r_p}{a} = 0.63$

```
% Calculate semi latus rectum
p        = rp*(1+e);
fprintf('Semi latus rectum is %.3f Mars Radii', p)
```

```
Semi latus rectum is 2.939 Mars Radii
```

$r_p = \frac{p}{1 + e}$

$p = r_p(1 + e) = 2.94\ [R_{Mars}]$

```
% Mars gravitional constant
mu_mars = 42828.314258067; % [km^3/s^2]

% Mars Radius at equator
R_mars  = 3397 ;           % [km]
```

```
% Calculate period [s]
tau      = 2*pi*sqrt((a*R_mars)^3/mu_mars);
fprintf('The orbital period around mars is %.2f hours',tau/3600)
```

```
The orbital period around mars is 18.11 hours
```

$$period = 2\pi\sqrt{\frac{a^3}{\mu}} = 18.11\ [hrs]$$

```
% Calculate specific energy
energy   = -mu_mars/(2*a*R_mars);
fprintf('The specific energy is %.3f km^2/s^2', energy)
```

```
The specific energy is -1.286 km^2/s^2
```

$$\epsilon = \frac{-\mu}{2a} = -1.29\ [km^2/s^2]$$

```
% Calculate eccentric anomaly via newton raphson
E0       = CalcEccentricAnomaly(e,M0*pi/180)*180/pi;
fprintf('The initial eccentric anomaly is %.2f deg',E0)
```

```
The initial eccentric anomaly is -142.21 deg
```

$$M = E - e\sin E$$
$$E_0 = -142.2\ [deg]$$

```
% Calclate true anomaly from eccentric anomaly
ta0      = 2*atand(sqrt((1+e)/(1-e))*tand(E0/2));
fprintf('The initial true anomaly is %.2f deg',ta0)
```

```
The initial true anomaly is -161.56 deg
```

$$\tan\frac{\theta^*}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}$$
$$\theta_0^* = 2\tan^{-1}\left(\sqrt{\frac{1+e}{1-e}}\tan\frac{E_0}{2}\right) = -161.6\ [deg]$$

```
% Calculate Initial radius
r0       = a*(1 - e*cosd(E0));
fprintf('The initial radius is %.4f Mars radii',r0)
```

```
The initial radius is 7.3499 Mars radii
```

$$r_0 = a(1 - e\cos E_0) = 7.35\ [R_{Mars}]$$

```
% Calculate initial velocity
v0       = sqrt(2*(energy + mu_mars/(r0*R_mars)));
fprintf('The initial velocity is %.3f km/s', v0)
```

```
The initial velocity is 0.926 km/s
```

$$\epsilon = \frac{v^2}{2} - \frac{\mu}{r}$$

$$v_0 = \sqrt{2(\epsilon + \frac{\mu}{r_0})} = 0.93 \ [km/s]$$

```
% Calculate angular momentum magnitude
h       = sqrt(mu_mars*p*R_mars);

% Calculate angular velocity from angular momentum magntiude
ta0dot  = h/((r0*R_mars)^2);

% Calculate radial component of velocity, use negative root due to sign of anomalys
r0dot   = -sqrt(v0^2 - (r0*R_mars)^2*ta0dot^2);

% Calculate flight path angle
gamma0  = atan2d(r0dot,(r0*R_mars)*ta0dot);
fprintf('The initial flight path angle is %.2f deg', gamma0)
```

```
The initial flight path angle is -26.59 deg
```

$$h = \sqrt{\mu p} = r^2\dot{\theta}$$

$$v^2 = \dot{r}^2 + r^2\dot{\theta}^2$$

$$\tan\gamma = \frac{\dot{r}}{r\dot{\theta}} = \frac{v_r}{v_\theta}$$

$$\gamma_0 = \ tan^{-1}\frac{\dot{r}}{r\dot{\theta}} = -26.6 \ [deg]$$

```
% Calculate time since periapsis - convert M to degrees and convert M  = -120 to 240
dtp     = (M0*pi/180 + 2*pi)/(sqrt(mu_mars/(a*R_mars)^3));
fprintf('The time since the last periapsis is %.1f hours',dtp/3600)
```

```
The time since the last periapsis is 12.1 hours
```

$$M = \sqrt{\frac{\mu}{a^3}}(t - t_p)$$

$$(t_0 - t_p) = \frac{M_0}{\sqrt{\frac{\mu}{a^3}}} = 12.1 \ [hrs]$$

Problem 1b)

$\underline{Find}: \overline{r}_0, \overline{v}_0, \ in \ \hat{r}, \ \hat{\theta}, \ and \ \hat{e}, \hat{p}$

```
% Define r0 and v0 in orbit frame
v0_O    = [r0dot; (r0*R_mars)*ta0dot;0];
r0_O    = [r0;0;0];
```

$\overline{v}_0 = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} = -0.415\ \hat{r} + 0.828\ \hat{\theta}\ [km/s]$

$\overline{r}_0 = 7.35\ \hat{r}\ [R_{mars}]$

```
% DCM matrix from rotating orbit frame to perifocal frame
P_DCM_O = [cosd(ta0), -sind(ta0), 0;...
           sind(ta0), cosd(ta0), 0;...
           0, 0, 1];

% Rotate vectors into perifocal frame
r0_P    = P_DCM_O*r0_O;
v0_P    = P_DCM_O*v0_O;
```

$[r_0]^P = [PO][r_0]^O$

$[v_0]^P = [PO][v_0]^O$

$\overline{r}_0 = -6.97\ \hat{e} - 2.32\ \hat{p}\ [R_{mars}]$

$\overline{v}_0 = 0.655\ \hat{e} - 0.654\ \hat{p}\ [km/s]$

Problem 1c)

$\underline{Find}: \theta_1^*, r_1, v_1, E_1, \gamma_1, (t_1 - t_p)$

```
% Mean motion
n       = sqrt(mu_mars/(a*R_mars)^3);

% Change in time
dt      = 0.75*tau;

% Calculate mean anomaly for time equal to 75% of period from t0
M1      = M0*(pi/180) + n*dt;

% Calculate eccentric anomaly from mean anomaly
E1      = CalcEccentricAnomaly(e,M1)*180/pi;
fprintf('The eccentric anomaly at new time is %.2f deg',E1)
```

```
The eccentric anomaly at new time is 161.50 deg
```

$n = \sqrt{\frac{\mu}{a^3}}$

$M_1 = M_0 + n(t_1 - t_0)$

$M = E - e\sin E$

$E_1 = 161.5\ [deg]$

```
% Calclate true anomaly from eccentric anomaly
ta1     = 2*atand(sqrt((1+e)/(1-e))*tand(E1/2));
fprintf('The true anomaly at new time is %.2f deg',ta1)
```

```
The true anomaly at new time is 171.17 deg
```

$$\tan\frac{\theta^*}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}$$

$$\theta_1^* = 2\tan^{-1}\left(\sqrt{\frac{1+e}{1-e}}\tan\frac{E_1}{2}\right) = 171.2 \ [deg]$$

```
% Calculate Initial radius
r1      = a*(1 - e*cosd(E1));
fprintf('The radius at new time is %.4f Mars radii',r1)
```

```
The radius at new time is 7.8398 Mars radii
```

$$r_1 = a(1 - e\cos E_1) = 7.84 \ [R_{Mars}]$$

```
% Calculate initial velocity
v1      = sqrt(2*(energy + mu_mars/(r1*R_mars)));
fprintf('The velocity at new time is %.3f km/s', v1)
```

```
The velocity at new time is 0.802 km/s
```

$$\epsilon = \frac{v^2}{2} - \frac{\mu}{r}$$

$$v_1 = \sqrt{2(\epsilon + \frac{\mu}{r_1})} = 0.80 \ [km/s]$$

```
% Calculate angular velocity from angular momentum magntiude
ta1dot  = h/((r1*R_mars)^2);

% Calculate radial component of velocity, use positive root due to sign of anomalys
r1dot   = sqrt(v1^2 - (r1*R_mars)^2*ta1dot^2);

% Calculate flight path angle
gamma1  = atan2d(r1dot,(r1*R_mars)*ta1dot);
fprintf('The flight path angle at the new time is %.2f deg', gamma1)
```

```
The flight path angle at the new time is 14.53 deg
```

$$h = r^2\dot{\theta}$$

$$v^2 = \dot{r}^2 + r^2\dot{\theta}^2$$

$$\tan\gamma = \frac{\dot{r}}{r\dot{\theta}} = \frac{v_r}{v_\theta}$$

$$\gamma_1 = tan^{-1}\frac{\dot{r}_1}{r_1\dot{\theta}_1} = 14.5 \ [deg]$$

```
% Calculate time since periapsis - convert M to degrees
dtp1    = M1/(sqrt(mu_mars/(a*R_mars)^3));
fprintf('The time since the last periapsis is %.1f hours',dtp1/3600)
```

```
The time since the last periapsis is 7.5 hours
```

$$M = \sqrt{\frac{\mu}{a^3}}(t - t_p)$$

$$(t_1 - t_p) = \frac{M_1}{\sqrt{\frac{\mu}{a^3}}} = 7.5 \ [hrs]$$

```
% Calculate change in true anomaly and eccentric anomaly
dta     = ta1 - ta0;
fprintf('The change in true anomaly is %.1f degrees', dta)
```

```
The change in true anomaly is 332.7 degrees
```

$$\Delta\theta^* = \theta_1^* - \theta_0^* = 332.7 \ [deg]$$

```
dE      = E1 - E0;
fprintf('The change in eccentric anomaly is %.1f degrees', dE)
```

```
The change in eccentric anomaly is 303.7 degrees
```

$$\Delta E = E_1 - E_0 = 303.7 \ [deg]$$

Problem 1d)

$\underline{Find}: \bar{r}_1, \bar{v}_1$

```
% Calculate f, g, fdot, and gdot using eccentric anomaly
f1      = (1 - (a/r0)*(1 - cosd(E1 - E0)));
g1      = dt - sqrt((a*R_mars)^3/mu_mars)*(E1*(pi/180) - E0*(pi/180) - sind(E1 - E0));
f1dot   =  -sqrt(mu_mars*a*R_mars)*sind(E1 - E0)/(r1*r0*R_mars*R_mars);
g1dot   = (1 - (a/r1)*(1 - cosd(E1 - E0)));

% Position vector perifocal coordinates [R_mars]
r1_P    = f1*r0_P + g1*v0_P/R_mars;

% Position vector perifocal coordinates [R_mars]
v1_P    = f1dot*(r0_P*R_mars) + g1dot*v0_P;
```

Note: f and $\dot{g}$ are unitless, while g has units of [sec] and $\dot{f}$ has units of $[sec]^{-1}$

$$f = 1 - \frac{a}{r_0}\left[1 - \cos\left(E_1 - E_0\right)\right]$$

$$g = (t_1 - t_0) - \sqrt{\frac{a^3}{\mu}}\left[E_1 - E_0 - \sin\left(E_1 - E_0\right)\right]$$

$$\dot{f} = -\frac{\sqrt{\mu a}}{r_1 r_0}\sin\left(E_1 - E_0\right)$$

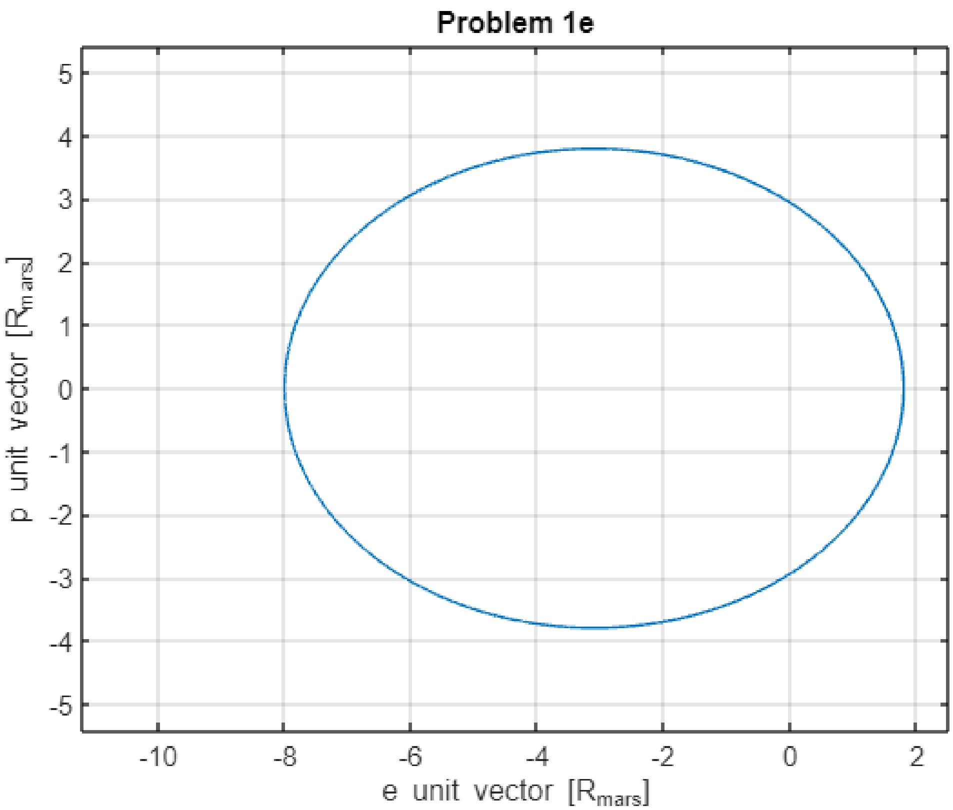
$$\dot{g} = 1 - \frac{a}{r_1}\left[1 - \cos\left(E_1 - E_0\right)\right]$$

$$\overline{r}_1 = f\ \ \overline{r}_0 + g\ \ \overline{v}_0 = 0.703\ \ \overline{r}_0\ \ [R_{mars}] - 1.474 \times 10^4\ \ \overline{v}_0\ \ [km/s] = -7.75\ \ \hat{e} + 1.20\ \ \hat{p}\ \ [R_{mars}]$$

$$\overline{v}_1 = \dot{f}\ \ \overline{r}_0 + \dot{g}\ \ \overline{v}_0 = 3.34 \times 10^{-5}\ \ \overline{r}_0\ \ [R_{mars}] + 0.722\ \ \overline{v}_0\ \ [km/s] = -0.32\ \ \hat{e} - 0.74\ \ \hat{p}\ \ [km/s]$$

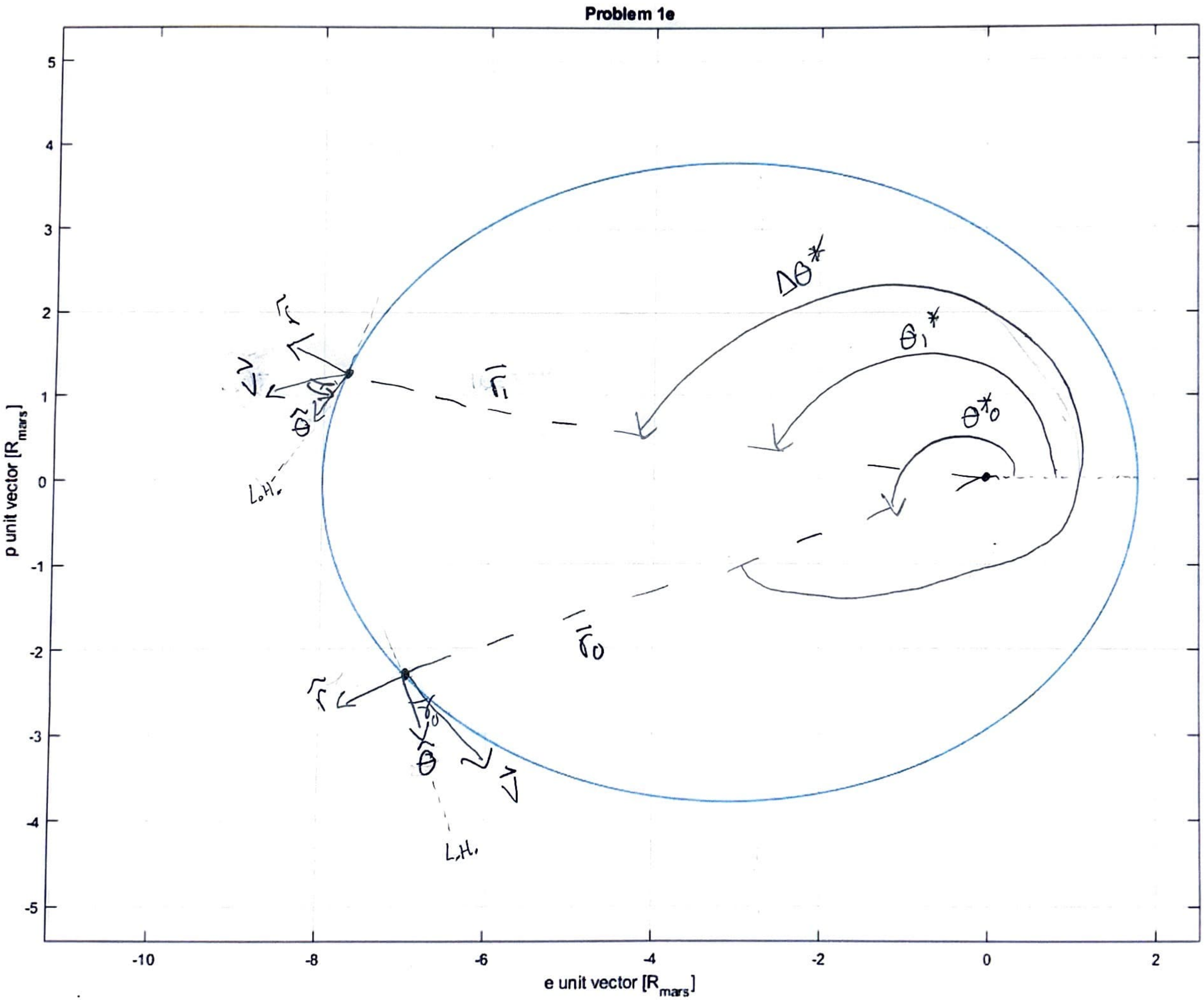
Problem 1e) Plot Orbit

```
% Define time - increment by 10 second
time    = 0:10:tau;

% Initialize Perifocal position and velocity vector
r_P     = zeros(3,length(time));

% Calculate vectors in perifocal coordinates for each time step using f & g
% relations
for i = 1:length(time)

    % Calculate Mean Anonamly
    M           = (M0*pi/180) + n*(time(i)- time(1));

    % Calculate Eccentric anomaly [rad]
    E           = CalcEccentricAnomaly(e, M);

    % Calculate f & g relations
    f       = (1 - (a/r0)*(1 - cosd(E*(180/pi) - E0)));
    g       = (time(i)- time(1)) - sqrt((a*R_mars)^3/mu_mars)*(E - E0*(pi/180) - sind(E*(180/p

    % Calculate perifocal position vector, convert r0 to km to be consistent with v0 in km/s
    r_P(:,i) = f*(r0_P*R_mars) + g*v0_P;
end

figure
plot(r_P(1,:)/R_mars,r_P(2,:)/R_mars)
xlim([-ra*1.4 rp*1.4])
axis equal
title('Problem 1e')
grid on
xlabel('e unit vector [R_{mars}]')
ylabel('p unit vector [R_{mars}]')
```




```
function E = CalcEccentricAnomaly(e,M)
    % Input M in rad and output E in rad

    % Define error tolerance
      etol = 1e-8;

    % Initialize change in Eccentric Anomaly
      dE   = 1;

    % Initialize Eccentric Anomaly and counter
      count= 0;
      E    = M;

      while dE > etol
          % Increase counter
          count = count + 1;

          % Newton Raphson
          Enp1 = E - (E - e*sin(E) - M)/(1 - e*cos(E));

          % Assign new values and calculate delta
          dE   = abs(Enp1 - E);
          E    = Enp1;

          if count > 1000
              disp('Max iterations reached');
              break
```

```
                end
            end
    end
```


Problem 1e
5
4
3
2
1
0
-1
-2
-3
-4
-5
-10
-8
-6
-4
-2
0
2
p unit vector [R_{mars}]
e unit vector [R_{mars}]
$\Delta\theta^*$
θ_1^*
θ_0^*
$\bar{r}_1$
$\bar{r}_0$
$\hat{r}$
$\hat{\theta}$
$\bar{v}$
γ_0
L.H.