Gabriel Colangelo

```
clear
close all
clc
```

Problem 2a)

```
% Earth Parameters
mu_earth        = 398600.4415;
R_earth         = 6378.1363;

% Initial Circular LEO Radius
r1              = 1.04*R_earth;

% Final Circular Orbit Radius
r2              = 18*R_earth;

fprintf('The shipment original orbit altitude is %.2f km ', r1 - R_earth);
```

```
The shipment original orbit altitude is 255.13 km
```

The shipment altitude of 255 km is reasonable, because it is outside the radius of the Earth. The orbit altitude is above 160 km and below 2000 km. Therefore it can be considered to be in LEO.

Problem 2b - i) Planar Hohmann Transfer

$\underline{Find}$: $a, e, Period$, for the intermediate tarnsfer orbit

```
% Calculate semi-major axis of Hohmann transfer orbit
a_hohmann       =  (r1 + r2)/2;
fprintf('The semi major axis of the intermediate orbit is %.2f Earth Radii',a_hohmann/R_earth)
```

```
The semi major axis of the intermediate orbit is 9.52 Earth Radii
```

$a = (r_1 + r_2)/2$

$a = 60719.8\ [km] = 9.52\ [R_{earth}]$

```
% Eccentricity of transfer orbit for Hohmann transfer
e_hohmann       = 1 - r1/a_hohmann;
fprintf('The eccentricity of the intermediate orbit is %.2f',e_hohmann)
```

```
The eccentricity of the intermediate orbit is 0.89
```

$r_1 = r_p = a(1 - e)$

$e = 1 - \frac{r_p}{a}$

$e = 0.89$

```
% Period of Hohmann transfer orbit
tau_hohmann     = 2*pi*sqrt(a_hohmann^3/mu_earth);
fprintf('The period of the intermediate orbit is %.2f hours',tau_hohmann/3600)
```

```
The period of the intermediate orbit is 41.36 hours
```

$Period = 2\pi \sqrt{\frac{a^3}{\mu}}$

$Period = 41.4 \ [hours] = 1.7 \ [days]$

To verify that the intermediate orbit intersects both the departure orbit and final orbit, we can check that the periapsis of the intermediate orbit equals the radius of the original circular orbit about Earth (departure orbit), and that the apoapsis of the intermediate orbit equals the radius of the final circular orbit about Earth.

$r_a = a(1+e) = 18 \ [R_{earth}] = r_1$

$r_p = a(1-e) = 1.04 \ [R_{earth}] = r_2$

Problem 2b - ii) Planar Hohmann Transfer

$\underline{Find}: \Delta v, \alpha, TOF, TA$

```
TOF_hohmann      = tau_hohmann/2;
fprintf('The time of flight is %.2f hours', TOF_hohmann/3600)
```

```
The time of flight is 20.68 hours
```

$TOF = Period/2$

$TOF = 20.7 \ [hours] = 0.86 \ [days]$

The transfer angle (TA) for a planar Hohmann transfer is known to be 180 degrees.

```
% LEO circular velocity
vc1              = sqrt(mu_earth/r1);

% Orbital speed post 1st maneuver - periapsis of transfer orbit
v1_plus_hohmann = sqrt(2*(mu_earth/r1 - mu_earth/(2*a_hohmann)));

% 1st maneuver deltaV - subtract vectors, along same direction
dv1_hohmann      = v1_plus_hohmann - vc1;
fprintf('The deltaV magnitude for the first maneuver is %.3f km/s', dv1_hohmann)
```

```
The deltaV magnitude for the first maneuver is 2.907 km/s
```

$v_1^- = \sqrt{\frac{\mu}{r_1}}$ - Circular Orbit

$v_1^+ = \sqrt{2(\frac{\mu}{r_1} - \frac{\mu}{2a_T})}$

$\Delta \bar{v}_1 = \bar{v}_1^+ - \bar{v}_1^-$

$|\Delta \bar{v}_1| = 2.9 \ [km/s]$

$\alpha = 0$, **no change in direction during manuever, applied tangentially at periapsis.**

```
% Final orbit circular velocity - post 2nd maneuver
vc2              = sqrt(mu_earth/r2);

% Orbital speed at apoapsis of transfer orbit, prior to 2nd maneuver
v2_minus_hohmann= sqrt(2*(mu_earth/r2 - mu_earth/(2*a_hohmann)));

% 2nd maneuver deltaV - subtract vectors, along same direction
dv2_hohmann      = vc2 - v2_minus_hohmann ;
fprintf('The deltaV magnitude for the second maneuver is %.3f km/s', dv2_hohmann)
```

```
The deltaV magnitude for the second maneuver is 1.247 km/s
```

$v_2^+ = \sqrt{\frac{\mu}{r_2}}$ - Circular Orbit

$v_2^- = \sqrt{2(\frac{\mu}{r_2} - \frac{\mu}{2a_T})}$

$\Delta \bar{v}_2 = \bar{v}_2^+ - \bar{v}_2^-$

$|\Delta \bar{v}_2| = 1.25 \ [km/s]$

$\alpha = 0$, **no change in direction during manuever, applied tangentially at apoapsis.**

```
% Total deltaV
dv_hohmann      = dv1_hohmann + dv2_hohmann;
fprintf('The planar Hohmann transfer total deltaV is %.3f km/s',dv_hohmann)
```

```
The planar Hohmann transfer total deltaV is 4.155 km/s
```

$|\Delta \bar{v}|_{total} = |\Delta \bar{v}_2| + |\Delta \bar{v}_1|$

$|\Delta \bar{v}|_{total} = 4.15 \ [km/s]$

Problem 2b - iv) Planar Hohmann Transfer

Find: Synodic period and phase angle at departue

```
% Mean motion for shipment in LEO
n_ship          = sqrt(mu_earth/r1^3);

% Mean motion for alpha station in final orbit
n_alpha         = sqrt(mu_earth/r2^3);

% Synodic period
ts              = 2*pi/(n_ship - n_alpha);
fprintf('The synodic period is %.3f Hours',ts/3600);
```

```
The synodic period is 1.515 Hours
```

$n = \sqrt{\frac{\mu}{a^3}}$

$t_s = \frac{2\pi}{n_{ship} - n_{alpha}}$

$t_s = 1.5 \ [hours] = 5452 \ [sec]$

```
% Calculate phase angle of station alpha at depature
phase_hohmann    = (pi - wrapTo2Pi(n_alpha*TOF_hohmann))*180/pi;
fprintf('The phase angle at departure is %.2f deg',phase_hohmann)
```

```
The phase angle at departure is 110.77 deg
```

$(n_{alpha})(T.O.F) = 180 - \phi$

$\phi = 110.8 \ [deg]$

Problem 2c - i) Planar Bielliptic Transfer

$\underline{Find} : \Delta v, \alpha, TOF, TA$

```
% Intermediate Radius
r_int            = 66*R_earth;

% Semi major axis of 1st ellipse
a1               = (r1 + r_int)/2;

% Semi major axis of 2nd ellipse
a2               = (r2 + r_int)/2;

% Period of 1st transfer ellipse
tau_1            = 2*pi*sqrt(a1^3/mu_earth);

% Period of 2nd transfer ellipse
tau_2            = 2*pi*sqrt(a2^3/mu_earth);

% Bielliptic transfer time of flight
TOF_bielliptic   = tau_1/2 + tau_2/2;
fprintf('The time of flight for the planar biellptic transfer is %.2f hours', TOF_bielliptic/3600)
```

```
The time of flight for the planar biellptic transfer is 328.28 hours
```

$a_1 = (r_i + r_1)/2$

$a_2 = (r_i + r_2)/2$

$\tau_1 = 2\pi\sqrt{\frac{a_1^3}{\mu}}$

$\tau_2 = 2\pi\sqrt{\frac{a_2^3}{\mu}}$

$TOF = \tau_1/2 + \tau_2/2$ (sum of TOF for two ellipses)

$TOF = 328.3\ [hours] = 13.68\ [days]$

For a planar biellptic transfer, the transfer angle is known to be 360 degress.

```
% Orbital speed post 1st maneuver - periapsis of 1st ellipse
v1_plus_be       = sqrt(2*(mu_earth/r1 - mu_earth/(2*a1)));

% 1st maneuver deltaV - subtract vectors along same direction
dv1_be           = v1_plus_be - vc1;
fprintf('The deltaV magnitude for the first maneuver is %.3f km/s', dv1_be)
```

```
The deltaV magnitude for the first maneuver is 3.126 km/s
```

$v_1^- = \sqrt{\frac{\mu}{r_1}}$ - Circular Orbit

$v_1^+ = \sqrt{2(\frac{\mu}{r_1} - \frac{\mu}{2a_1})}$

$\Delta\overline{v}_1 = \overline{v}_1^+ - \overline{v}_1^-$

$|\Delta\overline{v}_1| = 3.13\ [km/s]$

$\alpha = 0$, **no change in direction during manuever, applied tangentially at periapsis of 1st ellipse.**

```
% Orbital speed prior 2nd maneuver - apoapsis of 1st ellipse
v2_minus_be      = sqrt(2*(mu_earth/r_int - mu_earth/(2*a1)));

% Orbital speed post 2nd maneuver - apoapsis of 2nd ellipse
```

```
v2_plus_be      = sqrt(2*(mu_earth/r_int - mu_earth/(2*a2)));

% 2nd maneuver deltaV - subtract vectors along same direction
dv2_be          = v2_plus_be - v2_minus_be;
fprintf('The deltaV magnitude for the second maneuver is %.3f km/s', dv2_be)
```

```
The deltaV magnitude for the second maneuver is 0.466 km/s
```

$v_2^- = \sqrt{2(\frac{\mu}{r_i} - \frac{\mu}{2a_1})}$

$v_2^+ = \sqrt{2(\frac{\mu}{r_i} - \frac{\mu}{2a_2})}$

$\Delta \overline{v}_2 = \overline{v}_2^+ - \overline{v}_2^-$

$|\Delta \overline{v}_2| = 0.47 \ [km/s]$

$\alpha = 0$, **<u>no change in direction during manuever, applied tangentially at apoapsis of 1st and 2nd ellipse.</u>**

```
% Orbital speed at periapsis of 2nd ellipse, prior to 3rd maneuver
v3_minus_be     = sqrt(2*(mu_earth/r2 - mu_earth/(2*a2)));

% 3rd maneuver deltaV - subtract vectors along same direction
dv3_be          = vc2 - v3_minus_be;
fprintf('The deltaV magnitude for the third maneuver is %.3f km/s', abs(dv3_be))
```

```
The deltaV magnitude for the third maneuver is 0.472 km/s
```

$v_3^+ = \sqrt{\frac{\mu}{r_2}}$ - Circular Orbit

$v_3^- = \sqrt{2(\frac{\mu}{r_2} - \frac{\mu}{2a_2})}$

$\Delta \overline{v}_3 = \overline{v}_3^+ - \overline{v}_3^-$ Need to lose energy to get to circular orbit

$|\Delta \overline{v}_3| = 0.47 \ [km/s]$

$\alpha = 180$, **<u>applied in direction opposite of $\overline{v}_3^-$.</u>**

```
% Total deltaV
dv_be           = dv1_be + dv2_be + abs(dv3_be);
fprintf('The planar bielliptic transfer total deltaV is %.3f km/s',dv_be)
```

```
The planar bielliptic transfer total deltaV is 4.064 km/s
```

$|\Delta \overline{v}|_{total} = |\Delta \overline{v}_3| + |\Delta \overline{v}_2| + |\Delta \overline{v}_1|$

$|\Delta \overline{v}|_{total} = 4.06 \ [km/s]$

Problem 2c - iii) Planar Bielliptic Transfer

<u>Find</u>: Synodic period and phase angle at departue

```
% Calculate phase angle of station alpha at depature
phase_bielliptic= (2*pi - wrapTo2Pi(n_alpha*TOF_bielliptic))*180/pi;
fprintf('The phase angle at departure is %.2f deg', phase_bielliptic)
```

```
The phase angle at departure is 341.01 deg
```

$(n_{alpha})(T.O.F) = 360 - \phi$

TA for Bielliptic is 360°

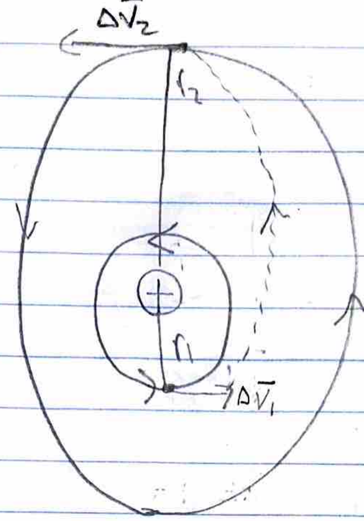
$\phi = 341.0 \ [deg]$

$n = \sqrt{\frac{\mu}{a^3}}$

$t_s = \frac{2\pi}{n_{ship} - n_{alpha}}$

$t_s = 1.5 \ [hours] = 5452 \ [sec]$ Unchanged from Hohmann transfer

2b)

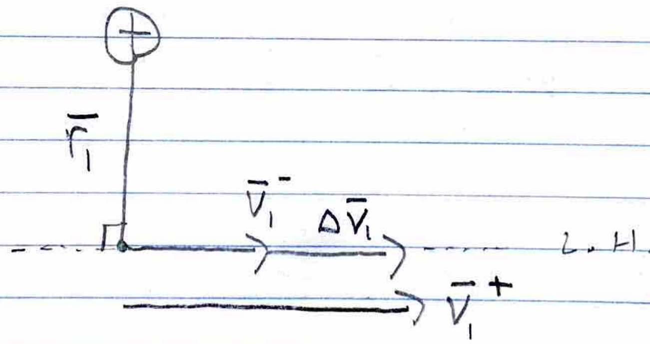


Circular orbit

Periapsis

$$\gamma_1^- = \gamma_1^+ = 0$$

$\boxed{\alpha = 0}$ (Applied tangential at periapsis)

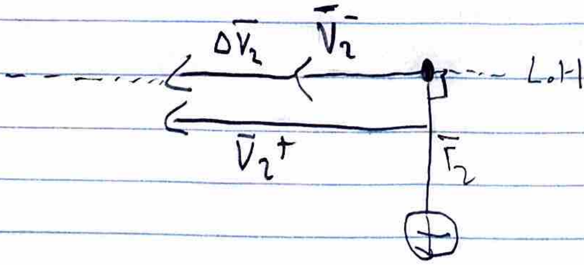


Apoapsis

Circular orbit

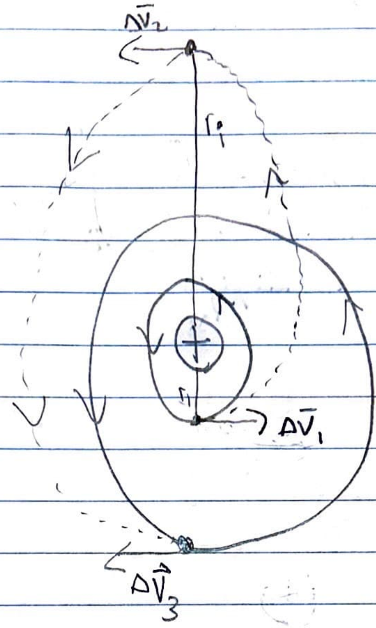
$$\gamma_2^- = \gamma_2^+ = 0$$

$\boxed{\alpha = 0}$ (Applied tangential at Apoapsis)



HW8

2c)

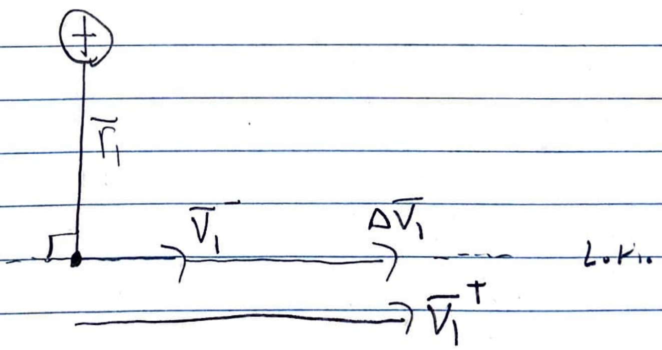


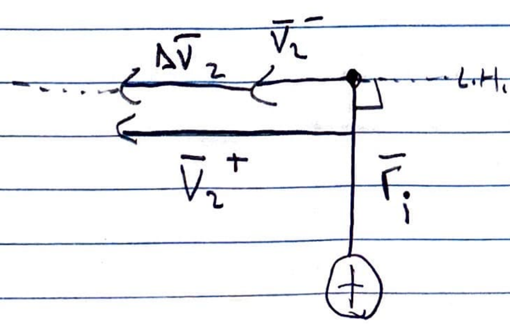
Circular orbit

periapsis

$\gamma_1^- = \gamma_1^+ = 0$

$\boxed{\alpha = 0}$, Applied tangentially at Periapsis

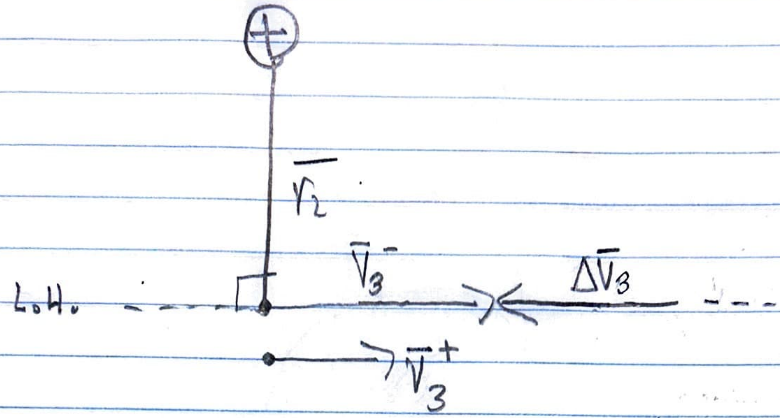




$\gamma_2^- = \gamma_2^+ = 0$

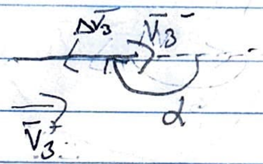
apoapsis apoapsis

$\boxed{\alpha = 0}$, applied tangentially at apoapsis



$$\boxed{\alpha = 180^\circ}$$

$\Delta\gamma = 180^\circ$



$\gamma_3^- = 0$

$\gamma_3^+ = 0$

Problem 2d)

To check the assumption of the two-body problem, one can analyze the net perturbation accelerations created by the Sun and Moon on the space station – Earth system. Assume the four bodies are aligned as such: Sun – Moon – SS-Earth, and the orbital radius is equal to the semi-major axis of each respective orbit. Defining the positive radial direction as outward from the Earth, we obtain the following position vectors.

$$\begin{aligned}
&\bar{r}_{earth,sun} = 149597898\ \hat{r}\ [km] \\
&\bar{r}_{earth,moon} = 384400\ \hat{r}\ [km] \\
&\bar{r}_{earth,ss} = 114806.4534\ \hat{r}\ [km] \\
&\bar{r}_{ss,sun} = \bar{r}_{earth,sun} - \bar{r}_{earth,ss} = 149483091.5466\ \hat{r}\ [km] \\
&\bar{r}_{ss,moon} = \bar{r}_{earth,moon} - \bar{r}_{earth,ss} = 269593.5466\ \hat{r}\ [km]
\end{aligned} \tag{1}$$

The pertubation accelerations for the sun and moon are given below.

$$\begin{aligned}
|\bar{a}_{pert,sun}| &= \mu_{sun}(\frac{\bar{r}_{ss,sun}}{r^3_{ss,sun}} - \frac{\bar{r}_{earth,sun}}{r^3_{earth,sun}}) = 9.11 \times 10^{-9}\ [km/s^2] \\
|\bar{a}_{pert,moon}| &= \mu_{moon}(\frac{\bar{r}_{ss,moon}}{r^3_{ss,moon}} - \frac{\bar{r}_{earth,moon}}{r^3_{earth,moon}}) = 3.43 \times 10^{-8}\ [km/s^2]
\end{aligned} \tag{2}$$

The dominant acceleration of the space station due to the Earth is given below.

$$|\bar{a}_{earth,ss}| = \mu_{earth}(\frac{\bar{r}_{earth,ss}}{r^3_{earth,ss}}) = 3.02 \times 10^{-5}\ [km/s^2] \tag{3}$$

The dominant acceleration is 3 orders of magnitude larger than the pertubring acceleration of the Moon, and 4 orders of magnitude larger than the perturbing acceleration of the Sun. Therefore, the effects of the Sun and Moon can be neglected, and the two-body problem stands to be a reasonable assumption, and should provide good results for a preliminary analysis. These findings are verified with a GMAT simulation shown on the following page, where three space station orbits are plotted over the course of 10 orbital periods about the Earth. The first being the space station-Earth system, the second being a space station-Earth-Moon system, and the final being a space station-Earth-Moon-Sun system. All three orbits are nearly identical.

Problem 2b)

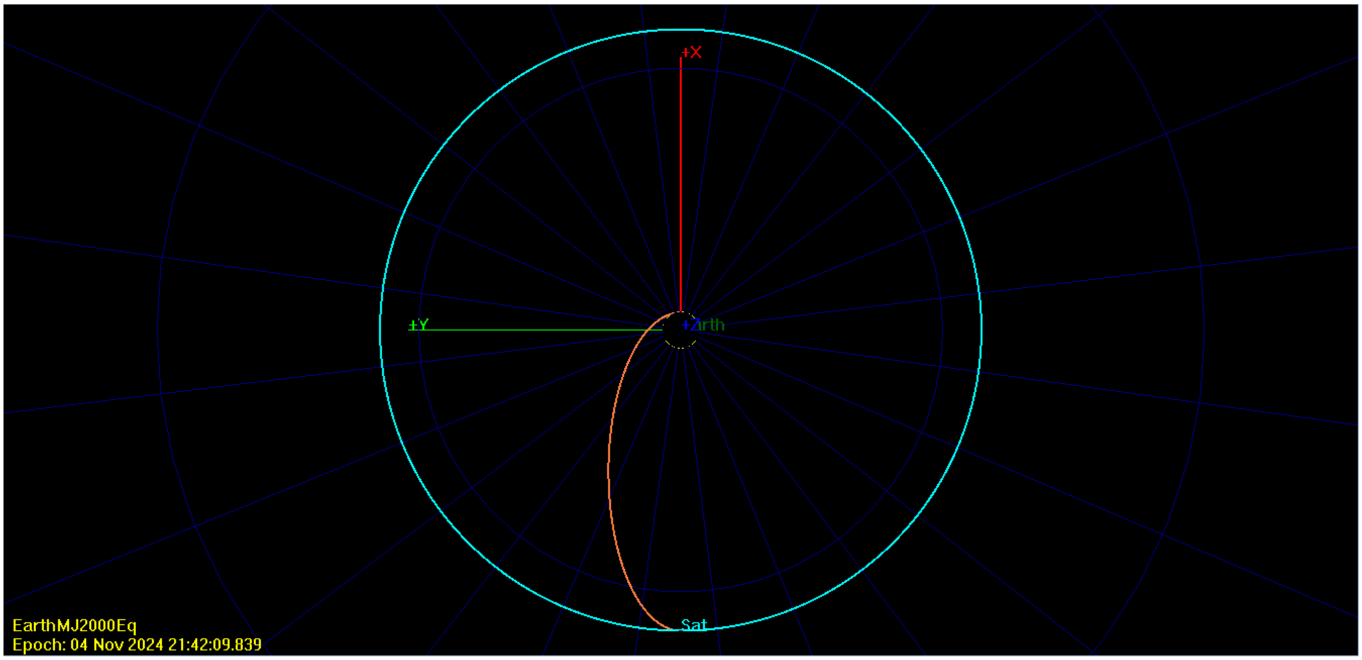


Figure 1: Problem 2b Hohmann Transfer

Faint yellow line is original shipment in LEO. Orange line is Hohmann transfer ellipse, and the blue line is the orbit when rendezvous with space station alpha.

Problem 2c)

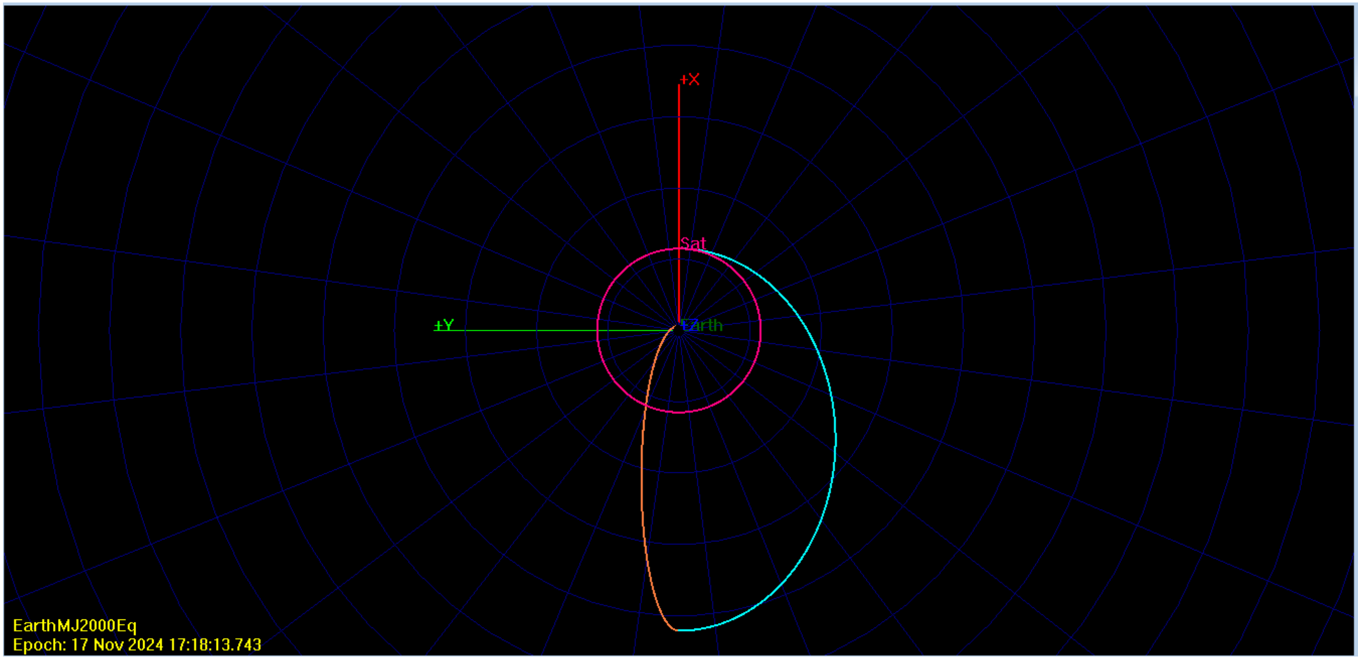
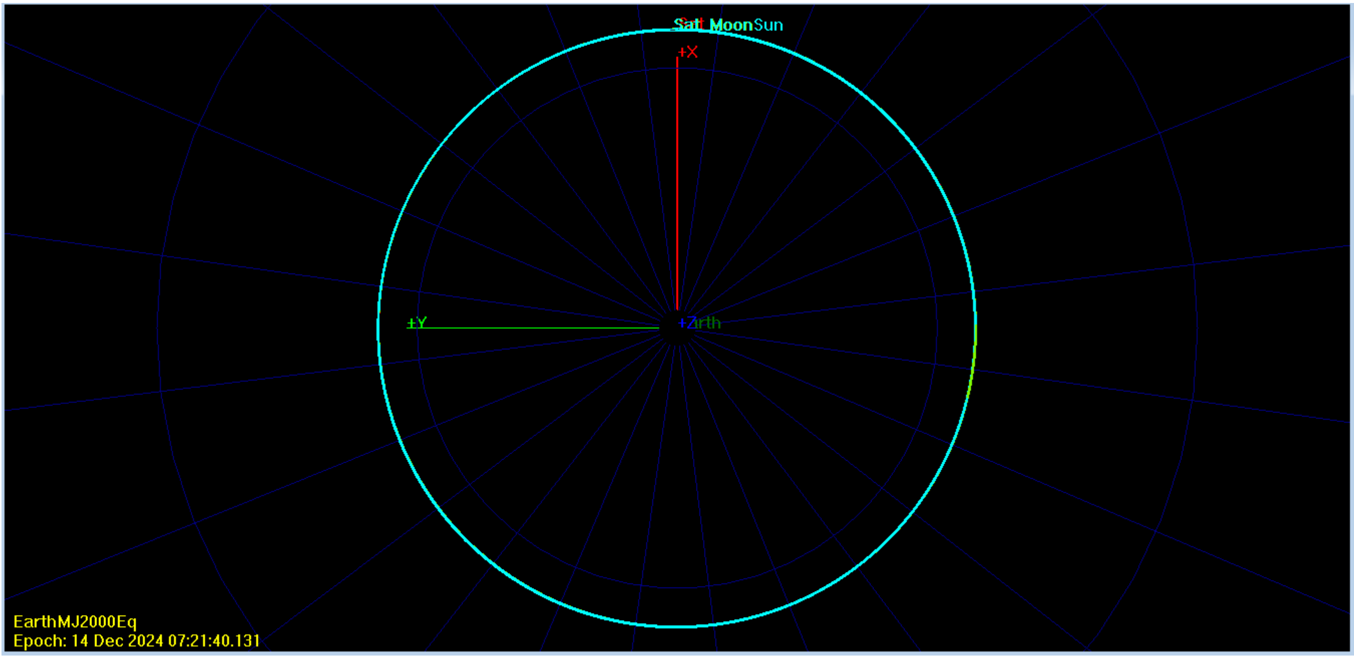


Figure 2: Problem 2c Bi-elliptic Transfer

Faint yellow line is original shipment in LEO. Orange line is 1st transfer ellipse of the bi-elliptic transfer, the blue line is the 2nd transfer ellipse of the bi-elliptic transfer. The pink line is the orbit when rendezvous with space station alpha.

Problem 2d)


Sat Moon Sun
+X
+Y
+Z
Earth
EarthMJ2000Eq
Epoch: 14 Dec 2024 07:21:40.131