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```
clear
close all
clc
```

Problem 2a)

$\underline{Find}$: $r_p, v_p, b, h, \delta, v_\infty, \epsilon, r, v, \gamma, H$, time to perilune

Assume passage altitude equals altitude at periapsis (i.e. r_p = 800 km)

```
% Magnitude of semi major axis for hyperbolic orbit
a_abs       = 7050; % [km]

% True anoamly at current time
ta0         = -90;  % [deg]

% Passage altitude (equals periapsis)
rp          = 800;  % [km]

% Moon gravitonal parameter
mu_moon     = 4902.8005821478; % [km^3/s^2]

% Velocity at periapsis
vp          = sqrt(2*mu_moon/rp + mu_moon/a_abs);
fprintf('The velocity at periapsis is %.2f km/s',vp)
```

```
The velocity at periapsis is 3.60 km/s
```

$$\epsilon = \frac{\mu}{2|a|} = \frac{v^2}{2} - \frac{\mu}{r}$$

$$v_p = \sqrt{\frac{2\mu}{r_p} + \frac{\mu}{|a|}} = 3.60 \ [km/s]$$

```
% Calculate enegry
energy      = mu_moon/(2*a_abs);
fprintf('The orbit specific energy is %.3f km^2/s^2', energy)
```

```
The orbit specific energy is 0.348 km^2/s^2
```

$$\epsilon = \frac{\mu}{2|a|} = 0.35 \ [km^2/s^2]$$

```
% Calculate eccentricity
e           = rp/a_abs + 1;

% Calculate semi minor axis
b           = a_abs*sqrt(e^2 -1);
fprintf('Semi minor axis is %.1f km ', b)
```

```
Semi minor axis is 3452.5 km
```

$r_p = |a|(e-1)$

$e = \frac{r_p}{|a|} + 1$

$b = |a|\sqrt{e^2-1} = 3452.5 \;\; [km]$

```
% Calculate semi-latus rectrum
p            = a_abs*(e^2 - 1);

% Calculate specific angular momentum
h            = sqrt(mu_moon*p);
fprintf('Specific angular momentum is %.2f km^2/s',h)
```

```
Specific angular momentum is 2879.16 km^2/s
```

$p = |a|(e^2-1)$

$h = \sqrt{\mu p} = 2879 \;\; [km^2/s]$

```
% Calculate fly-by angle
delta        = 2*asind(1/e);
fprintf('The fly by angle is %.2f deg', delta)
```

```
The fly by angle is 127.82 deg
```

$cos(\theta_\infty^*) = \frac{-1}{e} = -\sin\frac{\delta}{2}$

$\delta = 2\sin^{-1}(\frac{1}{e}) = 127.8 \;\; [deg]$

```
% Calculate excess velocity
vinf         = sqrt(mu_moon/a_abs);
fprintf('Excess velocity is %.3f km/s', vinf)
```

```
Excess velocity is 0.834 km/s
```

$v_\infty = \sqrt{\frac{2\mu}{r_\infty} + \frac{\mu}{|a|}} = \sqrt{\frac{\mu}{|a|}} = 0.83 \;\; [km/s]$

```
% Calculate current altitude
r            = p/(1 + e*cosd(ta0));
fprintf('Current altitude is %.2f km',r)
```

```
Current altitude is 1690.78 km
```

$r = \frac{p}{1 + e\cos\theta^*} = 1690.8 \;\; [km]$

```
% Calculate velocity magnitude
v            = sqrt(2*mu_moon/r + mu_moon/a_abs);
fprintf('The current speed is %.3f km/s', v)
```

```
The current speed is 2.549 km/s
```

$$v=\sqrt{\frac{2\mu}{r}+\frac{\mu}{|a|}}=2.55\ \ [km/s]$$

```
% Flight path angle (negative due to sign of true anomaly)
gamma        = sign(ta0)*acosd(h/(r*v));
fprintf('The current flight path angle is %.2f deg',gamma)
```

```
The current flight path angle is -48.07 deg
```

$$\cos\gamma=\frac{h}{rv}$$

$$\gamma=-48.1\ \ [deg]$$

```
% Calculate hyperbolic anomaly (negative due to sign of true anomaly)
H            = sign(ta0)*acosh((r/a_abs + 1)/e);
fprintf('The current hyperbolic anomaly is %.3f ', H)
```

```
The current hyperbolic anomaly is -0.472
```

$$r=|a|(e\cosh H-1)$$

$$H=\cosh^{-1}\frac{\frac{r}{|a|}+1}{e}=-0.47$$

```
% Time to perilune
dtp          = -(e*sinh(H) - H)/sqrt(mu_moon/a_abs^3);
fprintf('Time to perilune is %.2f hours',dtp/60);
```

```
Time to perilune is 10.33 hours
```

$$\sqrt{\frac{\mu}{|a|^3}}(t-t_p)=e\sinh H-H$$

$$(t_p-t)=-\frac{e\sinh H-H}{\sqrt{\frac{\mu}{|a|^3}}}=10.33\ \ [min]$$

Problem 2b)

Find: True anomaly and actual radius at aim point

```
% True anomaly at aim point
ta_aim       = -delta/2;
fprintf('True anomaly at aim point is %.2f deg', ta_aim)
```

```
True anomaly at aim point is -63.91 deg
```

$$\theta_b^*=-\frac{\delta}{2}$$

```
% Radius at aim point
r_aim        =  p/(1 + e*cosd(ta_aim));
fprintf('The radius at the aim point is %.2f km',r_aim)
```

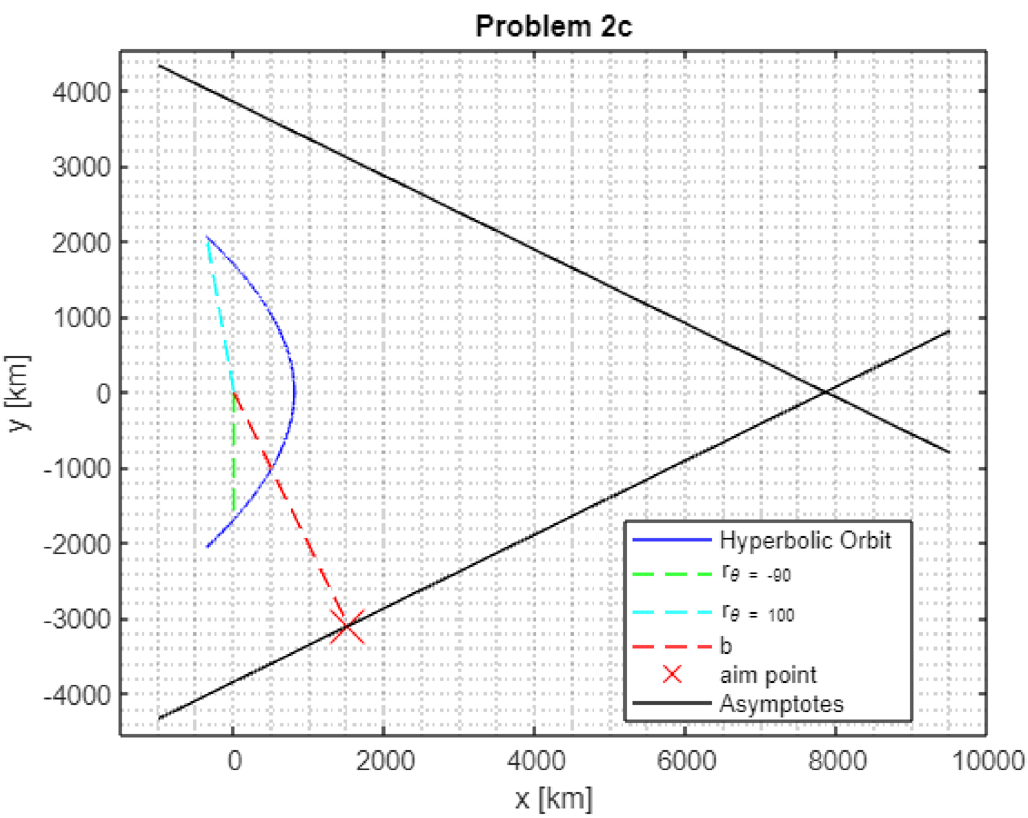
```
The radius at the aim point is 1134.96 km
```

$$r = \frac{p}{1 + e\cos\theta^*} = 1135.0\ [km]$$

Problem 2c) Plot Hyperbola

```
% True Anomaly Vector
ta_vec      = -100:.01:100;

% Initialize Perifocal position vector
r_P         = zeros(2,length(ta_vec));

for i = 1:length(ta_vec)

    % Distance to probe from moon
    r_mag     =  p/(1 + e*cosd(ta_vec(i)));

    % Position Vector in Perifocal Frame - Centered at Moon
    r_P(:,i)  = [r_mag*cosd(ta_vec(i));r_mag*sind(ta_vec(i))];

    if ta_vec(i) == -90
        indn90 = i;
    elseif ta_vec(i) == 100
        ind100 = i;
    end

    % % Calculate Hyperbolic anomaly
    % H         = 2*atanh(tand(ta_vec(i)/2)/sqrt((1+e)/(e - 1)));
    %
    % % Position Vector in Perifocal Frame
    % r_P2(:,i)  = [-a_abs*cosh(H) + (a_abs*e);b*sinh(H)];

end

% Calculate asymptote
ta_inf      = acosd(-1/e);

% Slope of line
slope       = tand(ta_inf - delta);

% Create asymptote
asym        = slope*((-1000:10:9500) - a_abs*e);

figure
plot(r_P(1,:),r_P(2,:),'-b')
%plot(r_P(1,:),r_P(2,:),'-b',r_P2(1,:),r_P2(2,:),'--k')
hold on
plot([0, r_P(1,indn90)], [0, r_P(2,indn90)],'Color','green','Linestyle','--')
plot([0, r_P(1,ind100)], [0, r_P(2,ind100)],'Color','cyan','Linestyle','--')
plot([0, b*cosd(ta_aim)], [0, b*sind(ta_aim)],'Color','red','Linestyle','--')
plot(b*cosd(ta_aim),b*sind(ta_aim),'rx','MarkerSize',18)
plot((-1000:10:9500), -asym,'-k')
```

```
plot((-1000:10:9500), asym,'-k')
title('Problem 2c')
axis equal
grid minor
hold on
xlabel('x [km]')
ylabel('y [km]')
legend('Hyperbolic Orbit','r_{\theta = -90}','r_{\theta = 100}','b','aim point','Asymptotes',...
       'Location', 'best')
hold off
xlim([-1500 10000])
```




```
% Calculate position for true anomaly 100 degrees
r_100       = p/(1 + e*cosd(100));
fprintf('The radius for a true anomaly of 100 degrees is %.1f km', r_100)
```

```
The radius for a true anomaly of 100 degrees is 2096.1 km
```

$$r = \frac{p}{1 + e\cos\theta^*} = 2096.1\ \ [km]$$

```
% Calculate velocity for true anomaly 100 degrees
v_100       = sqrt(2*mu_moon/r_100 + mu_moon/a_abs);
fprintf('The speed for a true anomaly of 100 degrees is %.3f km/s', v_100)
```

```
The speed for a true anomaly of 100 degrees is 2.318 km/s
```

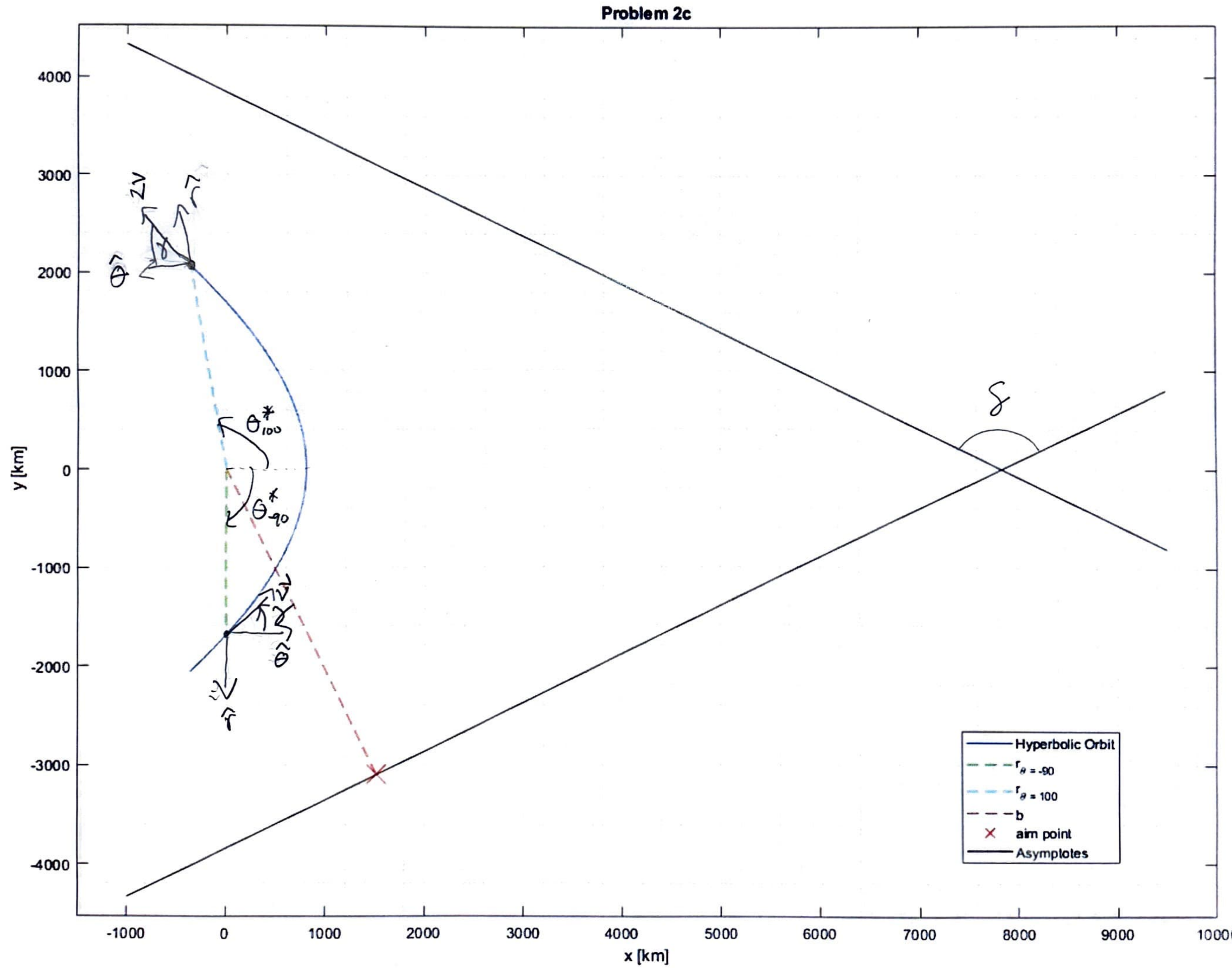
$$v = \sqrt{\frac{2\mu}{r} + \frac{\mu}{|a|}} = 2.32 \ [km/s]$$

```
% Calculate flight path angle - positive due to ascending
gamma_100   = acosd(h/(r_100*v_100));
fprintf('The flight path angle for a true anomaly of 100 degrees is %.2f deg', gamma_100)
```

```
The flight path angle for a true anomaly of 100 degrees is 53.66 deg
```

$$\cos\gamma = \frac{h}{rv}$$

$$\gamma = 53.66 \ [deg]$$



Problem 2c
y [km]
x [km]
4000
3000
2000
1000
0
-1000
-2000
-3000
-4000
-1000
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
Hyperbolic Orbit
$r_{\theta = -90}$
$r_{\theta = 100}$
b
aim point
Asymptotes
ΔV
$\hat{r}$
$\hat{\theta}$
γ
θ^*_{100}
θ^*_{-90}
δ
ΔV
$\hat{\theta}$
ΔV
$\hat{r}$