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```
clear
close all
clc

% Earth Radius [km]
R_earth = 6378.1363;

% Earth Gravitational Parameter [km^3/s^2]
mu_earth= 398600.4415;

% Orbital Elements
a       = 66*R_earth;    % semi major axis [km]
e       = .9118;         % eccentricity
i       = 34;            % inclination [deg]
RAAN    = 60;            % RAAN [deg]
AOP     = 0;             % AOP
theta   = 235;           % True anoamly + AOP
```

Problem 3a)

$\underline{Find}: \bar{r}, \bar{v}, r, v, \gamma, \theta^*, M, E, (t - t_p)$

```
% True anomaly
ta      = theta - AOP;
fprintf('The current true anomaly is %.1f [deg]',ta)
```

```
The current true anomaly is 235.0 [deg]
```

$\theta = \omega + \theta^*$

$\theta^* = 235\ [deg] = -125\ [deg]$

```
% Semi-latus rectum
p       = a*(1 - e^2);

% Radial distance
r       = p/(1 + e*cosd(ta));
fprintf('The current orbit radius is %.2f Earth Radii',r/R_earth)
```

```
The current orbit radius is 23.33 Earth Radii
```

$p = a(1 - e^2)$

$r = \dfrac{p}{1 + e\cos\theta^*}$

$r = 23.33\ [R_{earth}]$

```
% Specific energy
energy  = -mu_earth/(2*a);

% Orbital speed
v       = sqrt(2*(energy + mu_earth/r));
```

```
fprintf('The current orbital speed is %.3f km/s', v)
```

```
The current orbital speed is 2.100 km/s
```

$$\epsilon = \frac{-\mu}{2a} = \frac{v^2}{2} - \frac{\mu}{r}$$

$$v = \sqrt{2(\epsilon + \frac{\mu}{r})} = 2.1 \;\; [km/s]$$

```
% Calculate eccentric anomaly
E            = 2*atan2d(tand(ta/2),sqrt((1+e)/(1-e)));
fprintf('The current eccentric anomaly is %.3f [deg] \n', E)
```

```
The current eccentric anomaly is -44.843 [deg]
```

$$\tan(\frac{\theta^*}{2}) = \sqrt{\frac{1+e}{1-e}} \tan(\frac{E}{2})$$

$$E = 2\tan^{-1}\left[\tan(\frac{\theta^*}{2}) / \sqrt{\frac{1+e}{1-e}}\right] = -44.84 \;\; [deg]$$

```
% Calculate mean anomaly
M            = E*(pi/180) - e*sind(E);
fprintf('The current mean anomaly is %.3f deg',M*180/pi)
```

```
The current mean anomaly is -8.003 deg
```

$$M = E - e\sin E$$

$$M = -8.0 \;\; [deg]$$

```
% Calculate time since periapsis
dtp          = M/(sqrt(mu_earth/a^3));

% Calculate period
tau          = 2*pi*sqrt(a^3/mu_earth);
fprintf('The time since last passage is %.2f days', (tau + dtp)/(24*3600))
```

```
The time since last passage is 30.76 days
```

$$Period = \sqrt{\frac{a^3}{\mu}} = 31.46 \;\; [days]$$

$$M = \sqrt{\frac{\mu}{a^3}}(t - t_p)$$

$$(t - t_p) = \frac{M}{\sqrt{\frac{\mu}{a^3}}}$$

$$(t - t_p) = -16.8 \;\; [hours] = 30.76 \;\; [days]$$

```
% Calculate angular momentum magnitude
h            = sqrt(mu_earth*p);

% True anomaly angular rate
tadot        = h/r^2;
```

```
% Radial component of velocity, use negative root due to sign of anomalys
rdot         = -sqrt(v^2 - r^2*tadot^2);

% Calculate flight path angle
gamma        = atan2d(rdot,r*tadot);
fprintf('The initial flight path angle is %.2f deg', gamma)
```

```
The initial flight path angle is -57.44 deg
```

$$h = \sqrt{\mu p} = r^2\dot{\theta}$$

$$v^2 = \dot{r}^2 + r^2\dot{\theta}^2$$

$$\tan\gamma = \frac{\dot{r}}{r\dot{\theta}} = \frac{v_r}{v_\theta}$$

$$\gamma = tan^{-1}\frac{\dot{r}}{r\dot{\theta}} = -57.44 \ [deg]$$

```
% Position and velocity vectors in rotating orbit frame
r_R           = [r;0;0];
v_R           = [rdot;r*tadot;0];
```

$$[r]^R = r\hat{r} = 23.33\hat{r} \ [R_{earth}]$$

$$[v]^R = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} = -1.77\hat{r} + 1.13\hat{\theta} \ [km/s]$$

```
% DCM rotating orbit frame to perifocal frame
P_DCM_R      = [ cosd(ta), -sind(ta), 0;...
                 sind(ta), cosd(ta), 0;...
                 0, 0, 1];

% Rotate position and velocity vectors from rotating orbit frame to perifocal inertial frame
r_P          = P_DCM_R*r_R;
v_P          = P_DCM_R*v_R;
```

$$[PR] = \begin{pmatrix} \cos\theta^* & -\sin\theta^* & 0 \\ \sin\theta^* & \cos\theta^* & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$[r]^P = [PR][r]^R$$

$$[v]^P = [PR][v]^R$$

$$\overline{r} = -13.38 \ \hat{e} - 19.11 \ \hat{p} + 0 \ \hat{h} \ [R_{earth}]$$

$$\overline{v} = 1.94 \ \hat{e} + 0.80 \ \hat{p} \ + 0 \ \hat{h} \ [km/s]$$

```
% DCM rotating orbit frame to perifocal frame
C            = @(x) cosd(x);
S            = @(x) sind(x);
```

```
I_DCM_R     = [C(RAAN)*C(theta) - S(RAAN)*C(i)*S(theta), -C(RAAN)*S(theta) - S(RAAN)*C(i)*C(the
              S(RAAN)*S(i); S(RAAN)*C(theta) + C(RAAN)*C(i)*S(theta), -S(RAAN)*S(theta) + C(the
              -C(RAAN)*S(i);S(i)*S(theta), S(i)*C(theta), C(i)];

% Rotate position and velocity vectors from rotating orbit frame to perifocal inertial frame
r_I         = I_DCM_R*r_R;
v_I         = I_DCM_R*v_R;
```

$$[IR] = \begin{pmatrix} C_\Omega C_\theta - S_\Omega C_i S_\theta & -C_\Omega S_\theta - S_\Omega C_i C_\theta & S_\Omega S_i \\ S_\Omega C_\theta + C_\Omega C_i S_\theta & -S_\Omega S_\theta + C_\theta C_i C_\Omega & -C_\Omega S_i \\ S_i S_\theta & S_i C_\theta & C_i \end{pmatrix}$$

$$[r]^I = [IR][r]^R$$

$$[v]^I = [IR][v]^R$$

$$\overline{r} = 7.03\ \hat{x} - 19.51\ \hat{y} - 10.69\ \hat{z}\ [R_{earth}]$$

$$\overline{v} = 0.395\ \hat{x} + 2.013\ \hat{y}\ + 0.448\ \hat{z}\ [km/s]$$

Problem 3b)

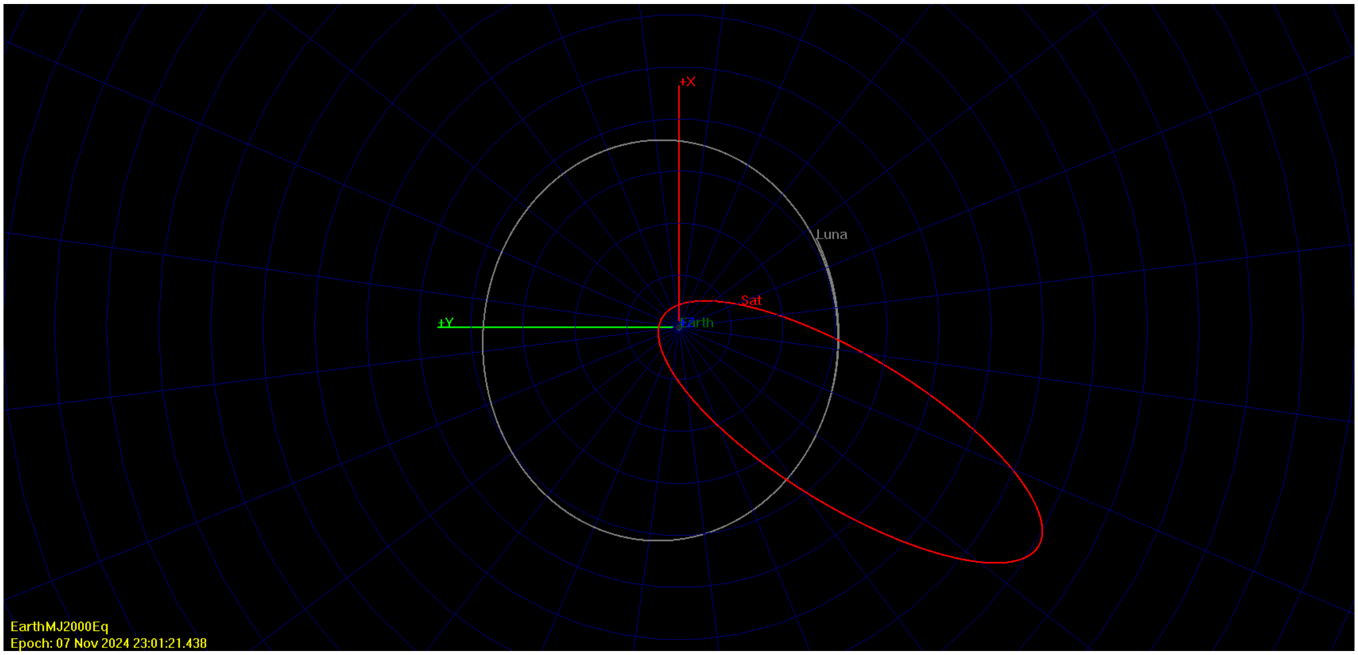


Figure 1: Orbit about Earth – Conic Propagator

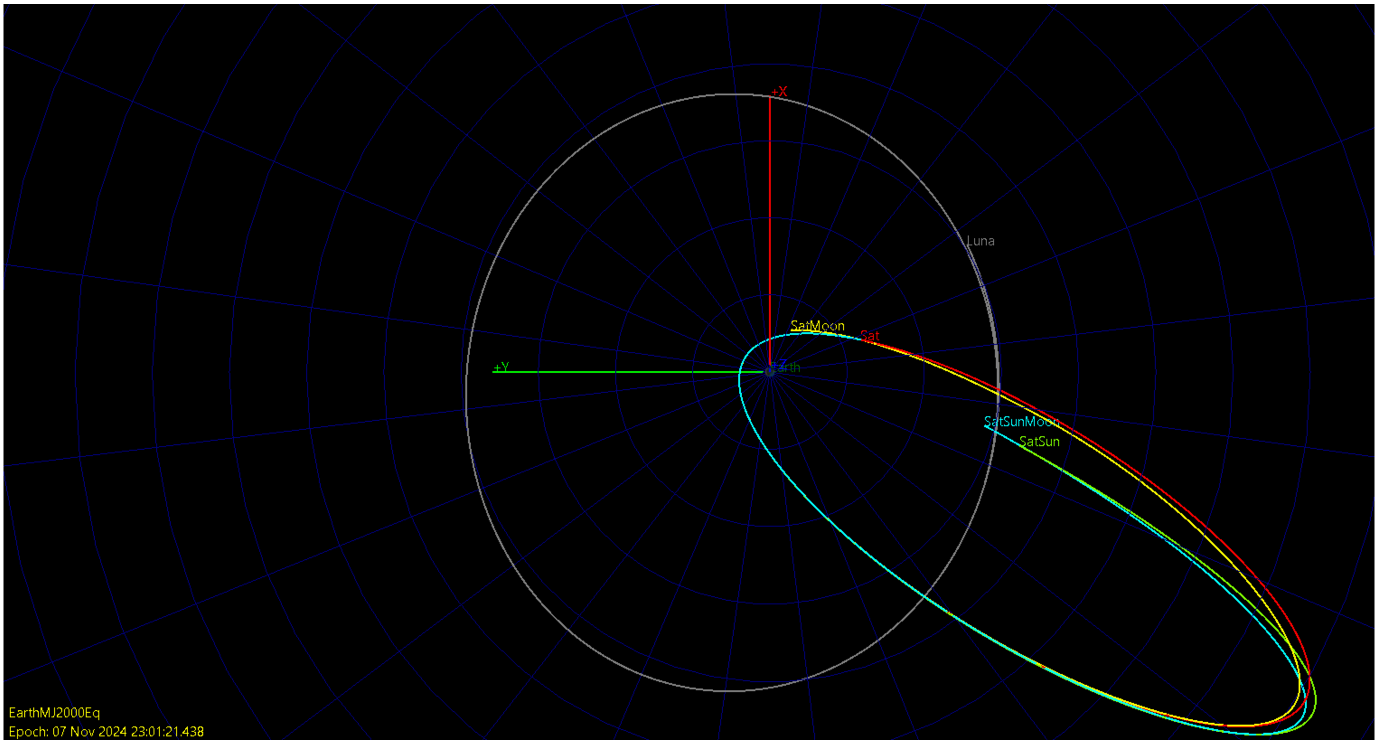
Below is a table with the orbit parameters from GMAT, where the true anomaly is 235 degrees (Problem 3).

Problem 3a/3b Data Comparison

Parameter	GMAT Output	MATLAB Output
True Anomaly [deg]	235	235
Flight Path Angle [deg]	147.435	-57.435
Orbital Distance [km]	148805.353	148805.353
Velocity Magnitude [km/s]	2.100	2.100
Mean Anomaly [deg]	351.996	-8.003 (351.996)
Eccentric Anomaly [deg]	315.157	-44.843 (315.157)
X Inertial Position [km]	44840.470	44840.470
Y Inertial Position [km]	-124443.785	-124443.785
Z Inertial Position [km]	-68162.376	-68162.376
X Inertial Velocity [km/s]	0.3951	0.3951
Y Inertial Velocity [km/s]	2.0133	2.0133
Z Inertial Velocity [km/s]	0.4482	0.4482

The values calculated in MATLAB and GMAT are consistent (the flight path angle is offset by -90 degrees, but this is due to definition).

Problem 3c)



All 4 vehicles were given the same starting condition and simulated for the same length of time. From the image above, we can see the Earth-spacecraft conic two-body relative model is NOT a good representative model for this problem. The sun (green line) significantly impacts the shape of the orbit (and hence the orbital period). The moon (yellow line) has a minor effect on the orbit but nothing of significance when compared to the impact of the sun. This orbit may be sensitive to the sun's gravity field due to how large the semi-major axis of the orbit is. When the satellite is at apoapsis (furthest point from the Earth), the Suns gravity field may be slowing down the satellite indicating that the satellite may be closest to the Sun at this location.