HW 9

#1)

2460623.1041667 = 11/8/24 14:30 UTC

From JPL JD Date/Time Converter

14:30 UTC, After 11/3 so EST

EST = UTC - 5 $\therefore$ 14:30 UTC = 9:30 EST

$\boxed{\text{Due Date of 11/8/24 at 9:30 EST is Correct}}$

Birthdate: 8/10/99 at 15:00 ET $\therefore$ 19:00 UTC

Birthdate: 2451401.2916667 (JPL Converter)

Next due date: 11/15/24 14:30 UTC

Next due date: 2460630.1041667

Age at next due date

$\boxed{\text{Age: 9228.8125 JD}}$

Gabriel Colangelo

```
clear
close all
clc
```

Problem 2a) Hohmann Transfer from cicular parking orbit to circular final orbit

<u>*Find*</u> : $\Delta v, TOF, r^{-}, v^{-}, \gamma^{-}, \theta^{*}$at moon arrival, $a, e, r_p, r_a, period, \epsilon$, phase angle at departure

<u>Assume</u>: Moon in circular orbit about Earth, with radius equal to semi-major axis, account for moon local gravity field. Negelect moon mass during parking orbit and transfer due to mass of earth >>> mass of moon. All orbits are coplanar.

```
% Earth Parameters
mu_earth            = 398600.4415;
R_earth             = 6378.1363;

% Moon Parameters
R_moon              = 1738.2;
mu_moon             = 4902.8005821478;

% Parking orbit altitude
alt_park            = 175;

% Parking orbit radius
r_park              = R_earth + alt_park;

% Final orbit altitude
alt_final           = 120;

% Final orbit radius
r_final             = R_moon + alt_final;

% Geocentric parking orbit velocity of sc
v_sc_earth_park     = sqrt(mu_earth/r_park);

% Earth Moon Orbit Radius (assume circular)
r_earth_moon        = 384400;

% Transfer Orbit parameters
rp                  = r_park;
ra                  = r_earth_moon;
ta_arr              = 180;
```

$r_p = r_{park} = 6553.14 \ \ [km]$

$r_a = r_{earth,moon} = 384400 \ \ [km]$

$\theta^{*} = 180 \ \ [deg]$

<u>The periapsis distance of the transfer ellipse is equal to the parking orbit radius about Earth. The apoapsis of the transfer ellipse is equal to the orbit radius of the moon about Earth. By assuming a circular lunar orbit about Earth, we can neglect the final orbit radius about the moon as this assumption makes the variance in the Earth moon radial distance irrelevant. The true anomaly on the transfer ellipse at the moon arrival is 180 degrees, as it occurs at the apoapsis of the transfer ellipse.</u>

```
% Calculate semi-major axis of Hohmann transfer orbit
a_hohmann          = (ra + rp)/2;
fprintf('The semi major axis of the transfer ellipse is %.2f Earth Radii',a_hohmann)
```

```
The semi major axis of the transfer ellipse is 195476.57 Earth Radii
```

$a = (r_p + r_a)/2$

$a = 195476.6 \ \ [km]$

```
% Eccentricity of transfer orbit for Hohmann transfer
e_hohmann          = 1 - rp/a_hohmann;
fprintf('The eccentricity of the transfer ellipse is %.4f',e_hohmann)
```

```
The eccentricity of the transfer ellipse is 0.9665
```

$r_p = a(1 - e)$

$e = 1 - \frac{r_p}{a}$

$e = 0.9665$

```
% Period of Hohmann transfer orbit
tau_hohmann         = 2*pi*sqrt(a_hohmann^3/mu_earth);
fprintf('The period of the transfer ellipse is %.2f days',tau_hohmann/(24*3600))
```

```
The period of the transfer ellipse is 9.95 days
```

$Period = 2\pi\sqrt{\frac{a^3}{\mu}}$

$Period = 239\ [hours] = 9.95\ [days]$

```
% Transfer ellipse energy
energy_hohmann      = -mu_earth/(2*a_hohmann);
fprintf('The specific energy of the transfer ellipse is %.3f km^2/s^2', energy_hohmann)
```

```
The specific energy of the transfer ellipse is -1.020 km^2/s^2
```

$\epsilon = \frac{-\mu}{2a}$

$\epsilon = -1.02\ [km^2/s^2]$

```
% Geocentric velocity of sc at periapsis on transfer ellipse
v_sc_earth_p        = sqrt(2*(energy_hohmann + mu_earth/rp));

% First maneuver deltaV, vectors along same direction. Treat as scalars
dv_1                = v_sc_earth_p - v_sc_earth_park;

% Geocentric Velocity of sc at apoapsis on transfer ellipse
v_sc_earth_a        = sqrt(2*(energy_hohmann + mu_earth/ra));
fprintf('The geocentric velocity of the spacecraft at apoapsis on transfer ellipse to the moon is %.3f km/s',v_sc_earth_a);
```

```
The geocentric velocity of the spacecraft at apoapsis on transfer ellipse to the moon is 0.186 km/s
```

$v_{sc/earth,park} = \sqrt{\frac{\mu_{earth}}{r_{park}}}$ Geocentric speed of s/c in parking orbit

$v_{sc/earth,park} = 7.8\ [km/s]$

$v_{sc/earth,p} = \sqrt{2(\epsilon + \frac{\mu_{earth}}{r_{park}})}$ Geocentric speed of s/c at periapsis

$v_{sc/earth,p} = 10.94\ [km/s]$

$\Delta\bar{v}_1 = \bar{v}_{sc/earth,p} - \bar{v}_{sc/earth,park}$

$|\Delta\bar{v}_1| = 3.14\ [km/s]$

$v_{sc/earth,a} = \sqrt{2(\epsilon + \frac{\mu_{earth}}{r_a})}$ Geocentric speed of s/c at apoapsis (moon arrival)

Lunar Arrival Parameters, $\theta^* = 180$ [deg]

$v^- = v_{sc/earth,a} = 0.18\ [km/s]$

$r^- = r_a = 384400\ [km]$

$\gamma^- = 0\ [deg]$ <u>Velocity at apoapsis of transfer ellipse is tangential</u>

```
% Geocentric velocity of moon in circular orbit
v_moon_earth         = sqrt(mu_earth/r_earth_moon);

% Calculate excess velocity of s/c relative to the moon, vectors along same direction. Treat as scalars
v_inf_moon           = v_sc_earth_a - v_moon_earth;

% Velocity of sc wrt moon in final orbit about moon
v_sc_moon_plus       = sqrt(mu_moon/r_final);

% Energy of hyperbolic orbit about moon
energy_hyp_moon      = v_inf_moon^2/2;
```

```
% Calculate sc velocity wrt moon at lunar arrival
v_sc_moon_minus     = sqrt(2*(energy_hyp_moon + mu_moon/r_final));
fprintf('The moon relative velocity of the spacecraft at lunar arrival is %.3f km/s',v_sc_moon_minus);
```

```
The moon relative velocity of the spacecraft at lunar arrival is 2.443 km/s
```

$v_{moon/earth} = \sqrt{\frac{\mu_{earth}}{r_{moon}}}$ Geocentric speed of moon

$v_{moon/earth} = 1.02\ [km/s]$

$v^+_{sc/moon} = \sqrt{\frac{\mu_{moon}}{r_{final}}}$ spacecraft speed relative to moon, in final orbit

$v^+_{sc/moon} = 1.62\ [km/s]$

$\overline{v}_{\infty,moon} = \overline{v}_{sc/earth,a} - \overline{v}_{moon/earth}$ Excess velocity of s/c wrt moon

$|\overline{v}_{\infty,moon}| = 0.83\ [km/s]$

$\epsilon_{moon} = \frac{v^2_{\infty,moon}}{2}$

$v^-_{sc/moon} = \sqrt{2(\epsilon_{moon} + \frac{\mu_{moon}}{r_{final}})}$ spacecraft speed relative to moon, in hyperbolic orbit about moon at final orbit location

$v^-_{sc/moon} = 2.44\ [km/s]$

```
% Second maneuver deltaV, vectors along same direction. Treat as scalars
dv_2            = v_sc_moon_plus - v_sc_moon_minus;

% Total deltaV
dv              = abs(dv_1) + abs(dv_2);
fprintf('The total deltaV for a transfer to the moon is %.3f km/s',dv)
```

```
The total deltaV for a transfer to the moon is 3.956 km/s
```

$\Delta\overline{v}_2 = \overline{v}^+_{sc/moon} - \overline{v}^-_{sc/moon} = -0.82\ \hat{v}_\theta\ [km/s]$

$|\Delta\overline{v}_2| = 0.82\ [km/s]$

$\Delta v = |\Delta\overline{v}_2| + |\Delta\overline{v}_1|$

$\Delta v = 3.956\ [km/s]$

```
% Time of flight
TOF_hohmann     = tau_hohmann/2;
fprintf('The time of flight is %.2f days',TOF_hohmann/(24*3600))
```

```
The time of flight is 4.98 days
```

$T.O.F. = period/2$

$T.O.F. = 4.98\ [days] = 119.5\ [hours]$

```
% Mean motion for moon about earth
n_moon          = sqrt(mu_earth/r_earth_moon^3);

% Calculate phase angle of moon at depature
phase_moon      = (pi - wrapTo2Pi(n_moon*TOF_hohmann))*180/pi;
fprintf('The phase angle at departure is %.2f deg',phase_moon)
```

```
The phase angle at departure is 114.73 deg
```

$n_{moon} = \sqrt{\frac{\mu_{earth}}{r^3_{earth,moon}}}$

$(n_{moon})(T.O.F) = 180 - \phi$

$\phi = 114.7\ [deg]$

Problem 2b) Hohmann Transfer No Capture Maneuver

$\underline{Find}$: $r^+, v^+, \gamma^+, \theta^*$after lunar encounter, $a, e, r_p, r_a, period, \epsilon, \Delta\omega$

```
% Semi-major axis of hyperbolic orbit about moon
a_abs           = mu_moon/(2*energy_hyp_moon);
```

```
% Eccentricity of hyperbolic orbit
e_H             = r_final/a_abs + 1;

% Flyby angle
delta           = 2*asind(1/e_H);

% Geocentric velocity post flyby
v_plus_flyby    = sqrt(v_inf_moon^2 + v_moon_earth^2 - 2*abs(v_inf_moon)*v_moon_earth*cosd(delta));
fprintf('The geocentric velocity post flyby is %.3f km/s',v_plus_flyby)
```

```
The geocentric velocity post flyby is 1.470 km/s
```

$|a|_H = \frac{\mu_{moon}}{2\epsilon_{moon}}$

$e_H = \frac{r_{final}}{|a|_H} + 1$

$\sin\delta = 1/e_H$

$|\bar{v}^+_{\infty,moon}| = |\bar{v}^-_{\infty,moon}|$

$v^{+2} = v^2_{moon/earth} + v^{+2}_{\infty,moon} - 2v_{moon/earth}v^+_{\infty,moon}\cos\delta$ - Law of cosines

$v^+ = 1.47\ [km/s]$

$r^+ = r^- = r_a = 384400\ [km]$

```
% Flight path angle post flyby
gamma_plus        = asind(abs(v_inf_moon)*sind(delta)/v_plus_flyby);
fprintf('The new flight path angle is %.3f deg',gamma_plus)
```

```
The new flight path angle is 33.168 deg
```

$\frac{v^+}{\sin\delta} = \frac{v^+_\infty}{\sin\gamma^+}$

$\gamma^+ = 33.17\ [deg]$

```
% Calculate true anomaly post flyby in Heliocentric orbit
ta_plus            = atan2d((r_earth_moon*v_plus_flyby^2/mu_earth)*cosd(gamma_plus)*sind(gamma_plus),...
                      ((r_earth_moon*v_plus_flyby^2/mu_earth)*cosd(gamma_plus)^2 - 1));
fprintf('The true anomaly post flyby is %.3f deg', ta_plus)
```

```
The true anomaly post flyby is 64.246 deg
```

$\tan\theta^* = \frac{\frac{r^+v^{+2}}{\mu_{earth}}\cos\gamma^+\sin\gamma^+}{\frac{r^+v^{+2}}{\mu_{earth}}\cos^2\gamma^+ - 1}$

$\theta^{*+} = 64.25\ [deg]$ positive due to ascending, $\gamma > 0$

```
% Calculate eccentricity of heliocentric orbit
e_new              = sqrt((r_earth_moon*v_plus_flyby^2/mu_earth - 1)^2*cosd(gamma_plus)^2 + sind(gamma_plus)^2);
fprintf('The geocentric orbit eccentricity post flyby is %.4f ', e_new);
```

```
The geocentric orbit eccentricity post flyby is 1.0598
```

$e^2 = (\frac{r^+v^{+2}}{\mu_{earth}} - 1)^2\cos^2\gamma^+ + \sin^2\gamma^+$

$e = 1.06$

```
% Energy post flyby
energy_new          = v_plus_flyby^2/2 - mu_earth/r_earth_moon;
fprintf('The geocentric orbit energy post flyby is %.3f km^s/s^2',energy_new);
```

```
The geocentric orbit energy post flyby is 0.044 km^s/s^2
```

$\epsilon = \frac{v^{+2}}{2} - \frac{\mu_{earth}}{r^+}$

$\epsilon = 0.044\ [km^2/s^2]$

```
% Calculate semi major axis post flyby
a_new                = -mu_earth/(2*energy_new);
fprintf('The geocentric orbit semi major axis post flyby is %.5E km',a_new);
```

```
The geocentric orbit semi major axis post flyby is -4.55737E+06 km
```

$$\epsilon = -\frac{\mu_{earth}}{2a}$$

$a = -4.557 \times 10^6 \ [km]$

$|a| = 4.557 \times 10^6 \ [km]$

```
% Calculate distance to periapsis
rp_new               = abs(a_new)*(e_new - 1);
fprintf('The distance to periapsis post flyby is %.5E km',rp_new)
```

```
The distance to periapsis post flyby is 2.72556E+05 km
```

$r_p = |a|(e - 1)$

$r_p = 2.7256 \times 10^5 \ [km]$

```
% Change in argument of periapsis - original true anomaly is 180 (at apoapsis)
deltaAOP             = 180 - ta_plus;
fprintf('The change in argument of periapsis is %.3f deg ', deltaAOP)
```

```
The change in argument of periapsis is 115.754 deg
```

$\Delta\omega = \theta^{*-} - \theta^{*+}$

$\Delta\omega = 115.75 \ [deg]$

The orbit after the lunar encounter is a hyperbolic orbit, as decribed by the eccentricity > 1, energy being > 0, and the semi-major axis being < 0. Because of this, the orbital period and distance to apoapsis are undefined.

Due to the hyperbolic orbits nature, the new orbit will not come close to Earth. The periapsis distance (closest point in orbit to Earth) is 42 times larger than Earth's radius. Because of this, a crew onboard cannot return to Earth if a lunar capture maneuever can't be performed.

Problem 2c) Plot Orbits

Find: Δv_{eq}

```
% Change in FPA - original FPA is zero
deltaFPA     = gamma_plus;

% Calculate deltaV equivalent - law of cosines
dv_eq        = sqrt(v_plus_flyby^2 + v_sc_earth_a^2 - 2*v_sc_earth_a*v_plus_flyby*cosd(deltaFPA));
fprintf('The equivalent deltaV produced by the lunar flyby is %.4f km/s',dv_eq)
```

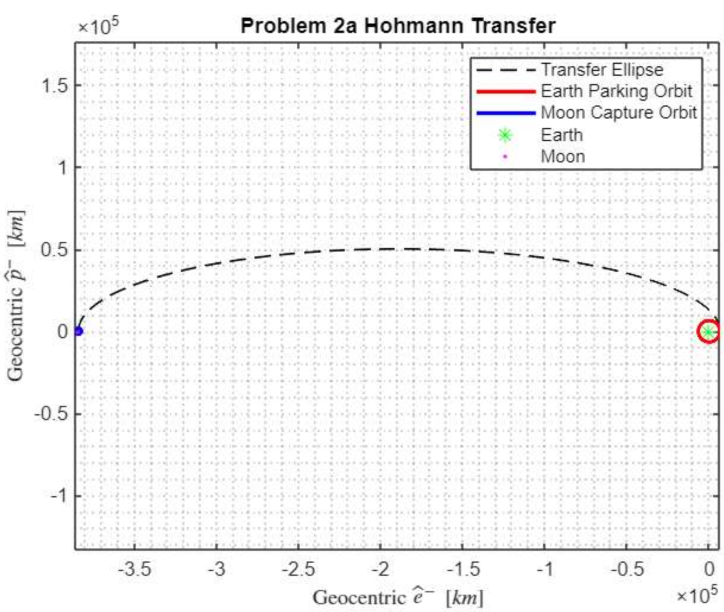
```
The equivalent deltaV produced by the lunar flyby is 1.3180 km/s
```

$\Delta v_{eq}^2 = v^{+2} + v^{-2} - 2v^{-}v^{+} + \cos\Delta\gamma$ - Law of cosines

$\Delta v_{eq} = 1.318 \ [km/s]$

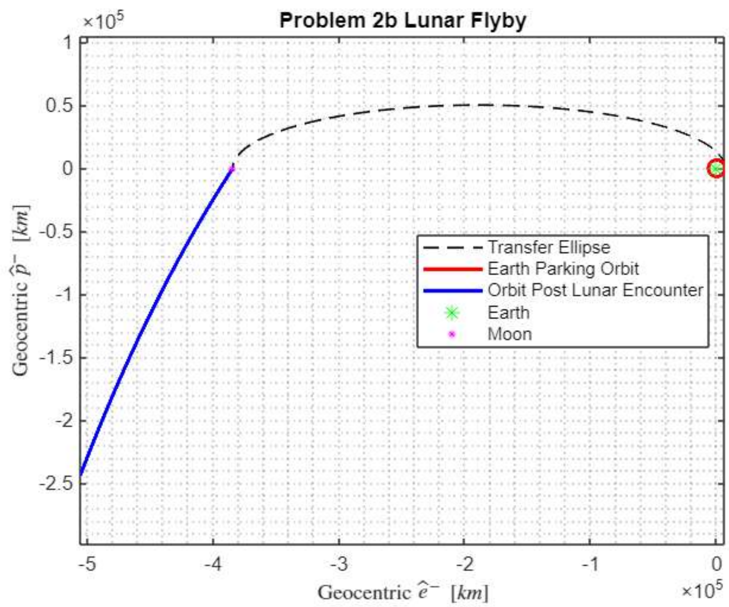
```
% True anomaly vector
ta_vec          = 0:.1:360;

% Initialize position vectors in perifocal frame
r_hohmann_P     = zeros(2,round(length(ta_vec)/2));
r_park_P        = zeros(2,length(ta_vec));
r_capture_P     = zeros(2,length(ta_vec));
r_nocapture_P   = zeros(2,round(length(ta_vec)/2));

% Semi-latus rectum of hohmann transfer
p_hohmann       = a_hohmann*(1 - e_hohmann^2);

% Semi-latus rectum of orbit post lunar encounter
p_nocapture     = a_new*(1 - e_new^2);

for i = 1:length(ta_vec)

    % DCM matrix from rotating orbit frame to perifocal frame
    P_DCM_R             = [cosd(ta_vec(i)), -sind(ta_vec(i));...
                           sind(ta_vec(i)), cosd(ta_vec(i))];
```

```
    if ta_vec(i) <= 180

        % Calculate transfer orbit radii
        r_hohmann         = p_hohmann/(1 + e_hohmann*cosd(ta_vec(i)));

        % Rotate transfer orbit radii position vector from orbit frame to perifocal frame
        r_hohmann_P(:,i)  = P_DCM_R*[r_hohmann;0];
    else
        % Calculate orbit radii post lunar encounter
        r_nocapture       = p_nocapture/(1 + e_new*cosd(ta_vec(i)));
    end

    % Parking orbit in perifocal coordinates
    r_park_P(:,i)       = P_DCM_R*[r_park;0];

    % Capture orbit post lunar encounter in perifocal coordinates
    r_capture_P(:,i)    = P_DCM_R*[r_final;0] + [-r_earth_moon;0];
end

figure
plot(r_hohmann_P(1,:), r_hohmann_P(2,:),'--k','LineWidth',1)
hold on
plot(r_park_P(1,:), r_park_P(2,:),'-r','LineWidth',2)
plot(r_capture_P(1,:), r_capture_P(2,:),'-b','LineWidth',2)
plot(0, 0,'g*','MarkerSize',8)
plot(-r_earth_moon, 0,'m*','MarkerSize',2)
grid minor
xlabel('Geocentric $\hat{e}^- \ [km]$', 'Interpreter','latex')
ylabel('Geocentric $\hat{p}^- \ [km]$', 'Interpreter','latex')
axis equal
title('Problem 2a Hohmann Transfer')
legend('Transfer Ellipse','Earth Parking Orbit','Moon Capture Orbit','Earth','Moon')
```



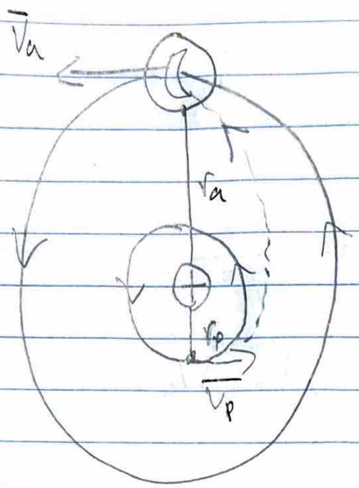

```
% Propogate hyperbolic orbit
ta_vec              = ta_plus:.01:90;

% Semi-latus rectum post lunar encounter
p_new               = a_new*(1 - e_new^2);

% Rotation matrix from new perifocal frame to original perifocal frame
Pminus_DCM_Pplus    = [cosd(deltaAOP), -sind(deltaAOP);...
                       sind(deltaAOP), cosd(deltaAOP)];

% Initialize
r_flyby_oldP        = zeros(2,length(ta_vec));

for i = 1:length(ta_vec)
    % Earth relative distance
    r_mag               =  p_new/(1 + e_new*cosd(ta_vec(i)));
```

```
    % Position Vector in Perifocal Frame - Centered at Earth
    r_flyby_newP        = [r_mag*cosd(ta_vec(i));r_mag*sind(ta_vec(i))];

    % Rotate into original perifocal frame
    r_flyby_oldP(:,i)    = Pminus_DCM_Pplus*r_flyby_newP;
end
figure
plot(r_hohmann_P(1,:), r_hohmann_P(2,:),'--k','LineWidth',1)
hold on
plot(r_park_P(1,:), r_park_P(2,:),'-r','LineWidth',2)
plot(r_flyby_oldP(1,:),r_flyby_oldP(2,:),'-b','LineWidth',2)
plot(0, 0,'g*','MarkerSize',8)
plot(-r_earth_moon, 0,'m*','MarkerSize',4)
grid minor
xlabel('Geocentric $\hat{e}^- \ [km]$', 'Interpreter','latex')
ylabel('Geocentric $\hat{p}^- \ [km]$', 'Interpreter','latex')
axis equal
title('Problem 2b Lunar Flyby')
legend('Transfer Ellipse','Earth Parking Orbit','Orbit Post Lunar Encounter','Earth','Moon','Location','best')
```

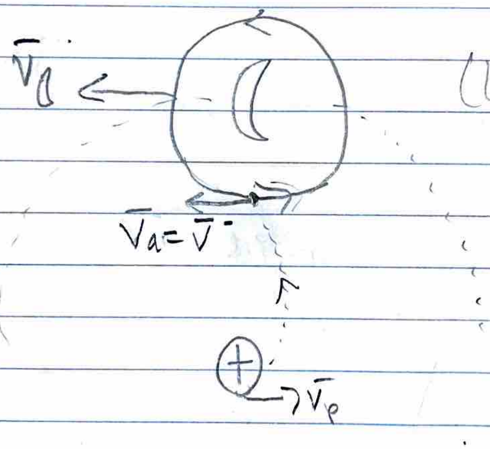


HW9

2)

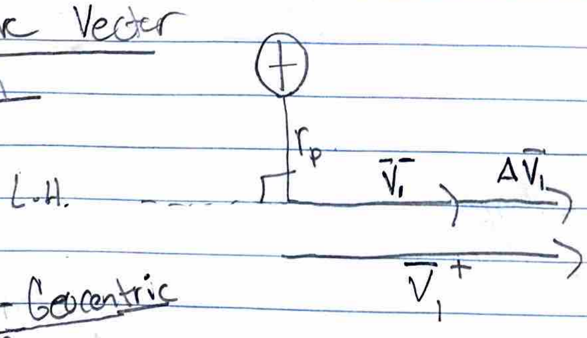


Hohmann transfer



(Light side Moon arrival)

Departure Vector Diagram

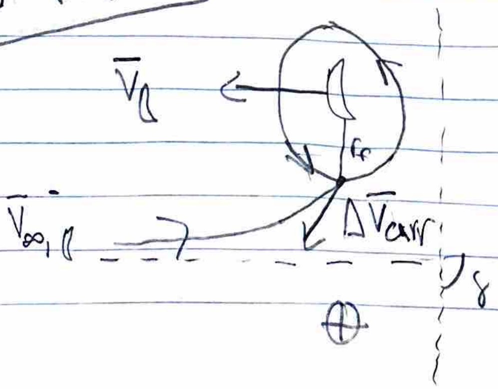


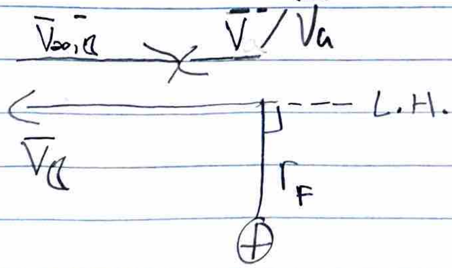
$\gamma_1 = 0$ (Circular orbit)

$\gamma_1^+ = 0$ (Periapsis)

$d = 0$

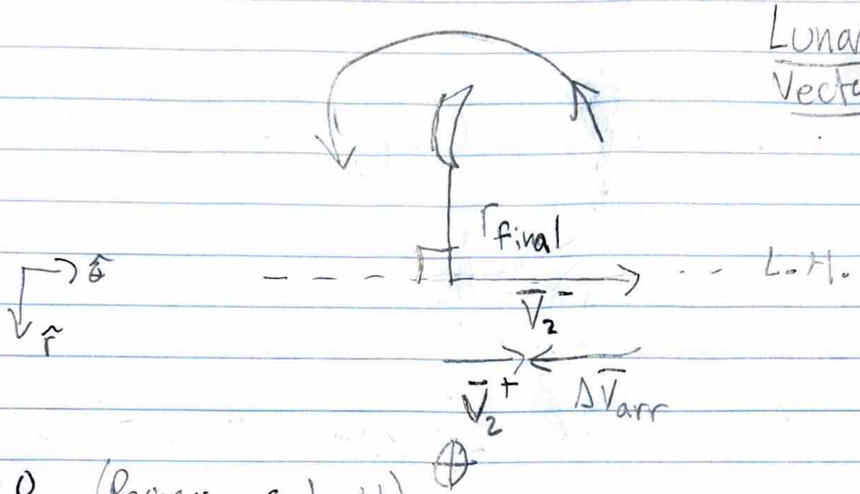
Arrival - Geocentric Vector Diagrams





2a)

Lunar Capture Vector Diagram

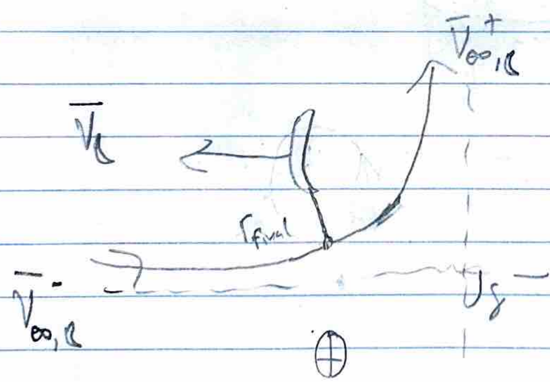


$\gamma^- = 0$ (Periapsis of hyperbola)

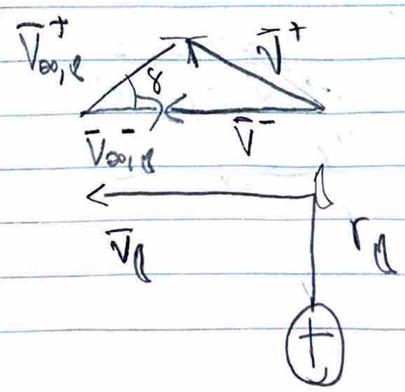
$\gamma^+ = 0$ (Circular orbit)

$\alpha = 0$

2b)

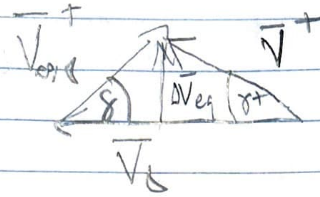


Lunar flyby Vecter Diagrams



HW9

2b)



$\gamma^- = 0$ (tangential)

$$V^{+2} = V_{\infty,\delta}^{+2} + V_\delta^2 - 2 V_{\infty,\delta} V_\delta \cos(\delta)$$

$$\frac{|\bar{V}^+|}{\sin(\delta)} = \frac{|\bar{V}_{\infty,\delta}^+|}{\sin(\gamma^+)}$$

$\delta\gamma = \gamma^+ - \cancel{\gamma^-}^{\,0} = \gamma^+$