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<u>Find</u>: a,e,p,i,ω, Ω, r ,v, γ, θ^*,M,E, (t - tp)

```
clear
close all
clc

% Initial Position vector in Inertial Coordinates [R_earth]
r1      = [2.12; 2.73; -0.6];

% Initial Velocity vector in Inertial Coordinates [km/s]
v1      = [-3.4;1.62;2.9];

% Earth Radius [km]
R_earth = 6378.1363;

% Earth Gravitational Parameter [km^3/s^2]
mu_earth= 398600.4415;

% Calculate energy
energy  = norm(v1)^2/2 - (mu_earth/(R_earth*norm(r1)));

% Calculate semi major axis [km]
a       = -mu_earth/(2*energy);
fprintf('The semi major axis is %.2f Earth Radii',a/R_earth);
```

```
The semi major axis is 4.79 Earth Radii
```

$$\epsilon = \frac{-\mu}{2a} = \frac{v_1^2}{2} - \frac{\mu}{r_1}$$

$$a = \frac{-\mu}{2\epsilon} = 4.79 \ \ [R_{earth}]$$

```
% Calculate angular momentum vector
h_vec   = cross(r1*R_earth,v1);

% Angular momentum magntiude
h       = norm(h_vec);

% Calculate eccentricity
e       = sqrt(1 + 2*energy*h^2/mu_earth^2);
fprintf('Eccentricity is %.3f ',e)
```

```
Eccentricity is 0.375
```

$$h = |\overline{r}_1 \times \overline{v}_1|$$

$$e = \sqrt{1 + \frac{2\epsilon h^2}{\mu^2}} = 0.375$$

```
% Calculate semi latus rectrum
p       = a*(1 -e^2);
```

```
fprintf('Semi latus rectum is %.3f Earth Radii',p/R_earth);
```

```
Semi latus rectum is 4.122 Earth Radii
```

$p = a(1 - e^2) = 4.12 \ [R_{earth}]$

```
% Angular momentum unit vector
h_hat   = h_vec/h;

% Calculate inclination angle, choose positive sign
i       = acosd(h_hat(3));
fprintf('The inclination angle is %.2f deg',i)
```

```
The inclination angle is 37.60 deg
```

$$\hat{h} = \frac{\overline{r}_1 \times \overline{v}_1}{|\overline{r}_1 \times \overline{v}_1|} = \sin\Omega \sin i \ \hat{x} - \cos\Omega \sin i \ \hat{y} + \cos i \ \hat{z} = .5538\hat{x} - 0.2560\hat{y} + 0.7923\hat{z} \ [km^2/s]$$

$i = \cos^{-1} 0.7923 = 37.6 \ [deg]$

Note: The positive inclination angle was chosen (inverse cosine yields a +/- result)

```
% Calculate RAAN
RAAN    = atan2d(h_hat(1),-h_hat(2));
fprintf('The RAAN is %.3f deg', RAAN)
```

```
The RAAN is 65.196 deg
```

$$\tan\Omega = \frac{\sin\Omega \sin i}{\cos\Omega \sin i} = \frac{0.5538}{0.2560}$$

$\Omega = 65.2 \ [deg]$

Note: The sign check was taken care of by the atan2() function. This value of the RAAN satisifies both $\sin\Omega \sin i$ = 0.5538 & $-\cos\Omega \sin i$ = -0.2560.

```
% Calculate the distance/radius
fprintf('The current orbital radius is %.3f Earth Radii', norm(r1))
```

```
The current orbital radius is 3.508 Earth Radii
```

$r_1 = |\overline{r}_1| = 3.5 \ [R_{earth}]$

```
% Calculate the speed
fprintf('The current orbital speed is %.3f km/s', norm(v1))
```

```
The current orbital speed is 4.753 km/s
```

$v_1 = |\overline{v}_1| = 4.75 \ [km/s]$

```
% Calculate true anomaly
ta      = sign(r1'*v1)*acosd(((p/norm(r1*R_earth)) - 1)/e);
fprintf('The current true anomaly is %.3f deg',ta)
```

```
The current true anomaly is -62.163 deg
```

$$r_1 = \frac{p}{1 + e\cos\theta^*}$$

$$\theta^* = sgn(\bar{r}_1 \cdot \bar{v}_1)\cos^{-1}\frac{\frac{p}{r_1} - 1}{e} = -62.16 \ [deg]$$

Note: Sign of true anomaly determined from the sign of the dot product of the position and velocity vector

```
% Calculate rhat and thetahat unit vectors
r_hat    = r1/norm(r1);
theta_hat= cross(h_hat,r_hat);

% Calculate theta  = true anomaly + AOP
theta    = atan2d(r_hat(3),theta_hat(3));

% Calculate argument of periapsis
AOP      = theta - ta;
fprintf('The argument of periapsis is %.3f deg', AOP)
```

```
The argument of periapsis is 45.884 deg
```

$$\hat{r}_1 = \frac{\bar{r}_1}{r_1} = (\cos\Omega\cos\theta - \sin\Omega\cos i\sin\theta)\hat{x} + (\sin\Omega\cos\theta + \cos\Omega\cos i\sin\theta)\hat{y} + (\sin i\sin\theta)\hat{z}$$

$$\hat{r}_1 = 0.6043\hat{x} + 0.7782\hat{y} - 0.1710\hat{z}$$

$$\hat{\theta}_1 = \hat{h} \times \hat{r}_1 = (-\cos\Omega\sin\theta - \sin\Omega\cos i\cos\theta)\hat{x} + (-\sin\Omega\sin\theta + \cos\Omega\cos i\cos\theta)\hat{y} + (\sin i\cos\theta)\hat{z}$$

$$\hat{\theta}_1 = -0.5728\hat{x} + 0.5735\hat{y} + 0.5857\hat{z}$$

$$\tan\theta = \frac{\sin i\sin\theta}{\sin i\cos\theta} = \frac{-0.171}{0.5857}$$

$$\omega = \theta - \theta^* = -16.28 - -62.16$$

$$\omega = 45.88 \ [deg]$$

The quadrant check of θ was done using the atan2() function. This value of the θ satisifies both $\sin\theta\sin i$ = -0.171 & $\cos\theta\sin i$ = 0.5857.

```
% Rotation matrix from rotating orbit to cartesian inertial frame
I_DCM_R     = [r_hat, theta_hat, h_hat];

% Rotate velocity vector to rotating orbit frame
v1_R        = I_DCM_R'*v1;

% Calculate flight path angle - same sign as true anomaly
gamma       = atan2d(v1_R(1),v1_R(2));
fprintf('The current flight path angle is %.3f deg',gamma)
```

```
The current flight path angle is -15.746 deg
```

$[v]^R = [RI][v]^I = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} = -1.29\hat{r} + 4.575\hat{\theta}\ \ [km/s]$

$\gamma = tan^{-1}\dfrac{\dot{r}}{r\dot{\theta}}$

$\gamma = -15.75\ \ [deg]$

```
% Calculate eccentric anomaly
E           = 2*atan2d(tand(ta/2),sqrt((1+e)/(1-e)));
fprintf('The current eccentric anomaly is %.3f [deg] \n', E)
```

```
The current eccentric anomaly is -44.251 [deg]
```

$\tan\left(\dfrac{\theta^*}{2}\right) = \sqrt{\dfrac{1+e}{1-e}}\tan\left(\dfrac{E}{2}\right)$

$E\ = 2\tan^{-1}\left[\tan\left(\dfrac{\theta^*}{2}\right)/\sqrt{\dfrac{1+e}{1-e}}\right] = -44.25\ \ [deg]$

```
% Calculate mean anomaly
M           = E*(pi/180) -e*sind(E);
fprintf('The mean anomaly is %.3f deg',M*180/pi)
```

```
The mean anomaly is -29.273 deg
```

$M = E - e\sin E$

$M = -29.3\ \ [deg]$

```
% Calculate time since periapsis
dtp         = M/(sqrt(mu_earth/a^3));

% Calculate period
tau         = 2*pi*sqrt(a^3/mu_earth);
fprintf('The time since last passage is %.1f hours', (tau + dtp)/3600)
```

```
The time since last passage is 13.6 hours
```

$Period = \sqrt{\dfrac{a^3}{\mu}} = 14.8\ \ [hours]$

$M\ = \sqrt{\dfrac{\mu}{a^3}}(t - t_p)$

$(t - t_p) = \dfrac{M}{\sqrt{\dfrac{\mu}{a^3}}}$

$(t - t_p) = -1.2\ \ [hours] = 13.6\ \ [hours]$

This orbit is an ellipse, we know this because the total energy is less than 0, the semi major axis is greater than 0, and the eccentricity is greater than 0 but less than 1. These are all properties of an elliptical orbit.