Gabriel Colangelo

```
clear
close all
clc
```

Problem 2a-i)

<u>Find</u>: Wait time till the manuever (apogee)

```
% Given orbit parameters
rp          = 7000;         % [km]
ra          = 45000;        % [km]
mu_earth    = 398600.4415;  % [km^3/s^2]

% Calculate semi major axis of old orbit
a           = (ra + rp)/2;  % [km]

% Calculate eccentricity of old orbit
e           = ra/a - 1;

% Calculate wait till manuever
dt          = pi/sqrt(mu_earth/a^3);
fprintf('The wait time till the manuever at apogee is %.2f hours',dt/3600)
```

```
The wait time till the manuever at apogee is 5.79 hours
```

$M = \sqrt{\frac{\mu}{a^3}}(t - t_p)$

Currently at perigee. At apogee, M $= \pi$

$(t - t_p) = \frac{\pi}{\sqrt{\frac{\mu}{a^3}}}$

$(t - t_p) = 5.8 \ [hours]$

Problem 2a - ii)

$\underline{Find}: \bar{r}_1, \bar{v}_1^-, \gamma_1^-, \bar{v}_1^+, \gamma_1^+$ for circularize at apogee

```
% Calculate specific enegry of old orbit - pre manuever
energy      = -mu_earth/(2*a);

% Calculate speed at apogee - pre manuever
v1_m        = sqrt(2*(energy + mu_earth/ra));
```

$\epsilon = \frac{v^2}{2} - \frac{\mu}{r}$

$v_1^- = \sqrt{2(\epsilon + \frac{\mu}{r_a})} = 1.5443 \ [km/s]$

$\bar{r}_1 = 45000\ \hat{r}\ [km] = -45000\ \hat{e}\ [km]$

$\bar{v}_1^- = 1.5443\ \hat{\theta}\ [km/s] = -1.5443\ \hat{p}\ [km/s]$

<u>Note:</u> At apogee, the velocity vector is directly aligned in the $\hat{\theta}$ direction which is aligned in the negative $\hat{p}$ direction.

$\tan\gamma = \dfrac{v_r}{v_\theta}$

$\tan\gamma_1^- = 0$

$\gamma_1^- = 0\ [deg]$

```
% Calculate circular velocity (circle with radius equal to distance to apogee) - post maneuver
v1_p         = sqrt(mu_earth/ra);

% Calculate new specific angular momentum - sami-latus rectum = radius for circle - post maneuv
h_p          = sqrt(mu_earth*ra);

% Calculate new flight path angle - post maneuver
gamma1_p     = acosd(h_p/(ra*v1_p));
fprintf('The flight path angle post maneuver is %.1f deg ', gamma1_p)
```

```
The flight path angle post maneuver is 0.0 deg
```

<u>Note:</u> For circular orbit, the semi-latus rectum is equal to the semi-major axis which equals the constant orbit radius.

$v_1^+ = \sqrt{\mu/r_1} = 2.9762\ [km/s]$

$h^+ = \sqrt{\mu p^+} = r_1 v_1^+ \cos(\gamma_1^+)$

$\gamma_1^+ = \cos^{-1}\dfrac{h^+}{r_1 v_1^+}$

$\gamma_1^+ = 0\ [deg]$

$\bar{v}_1^+ = v_1^+ \sin\gamma_1^+\ \hat{r} + v_1^+ \cos\gamma_1^+\ \hat{\theta}$

$\bar{v}_1^+ = 2.9762\ \hat{\theta}\ [km/s]$

$\Delta\bar{v} = \bar{v}_1^+ - \bar{v}_1^- = 1.4319\ \hat{\theta}\ [km/s]$

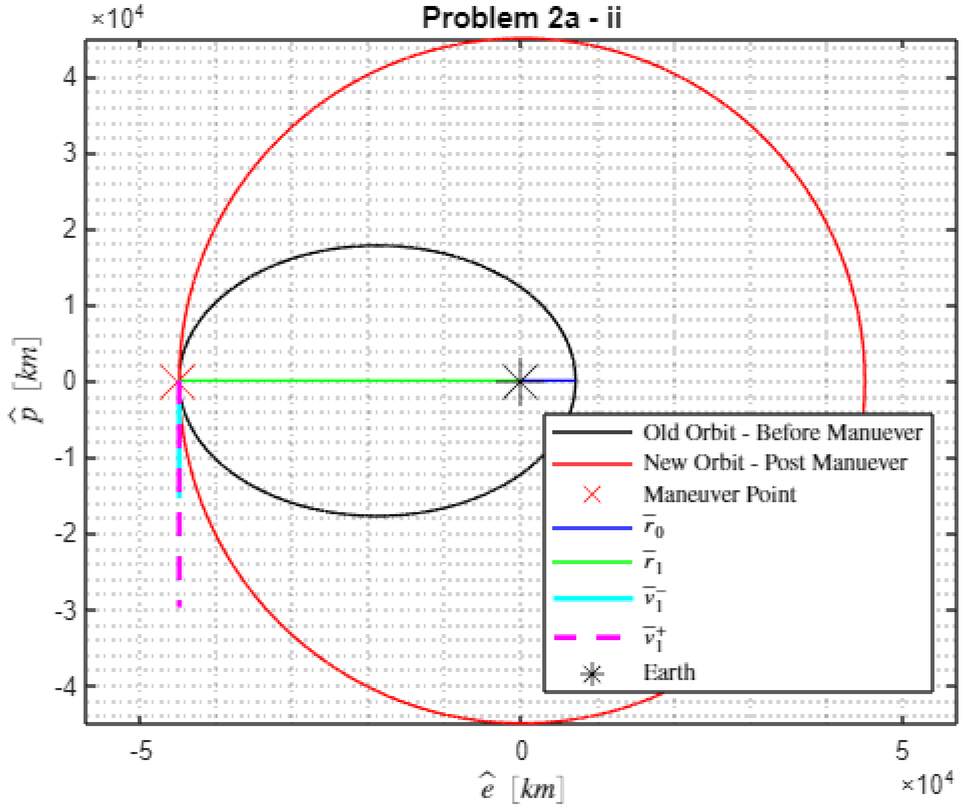
$\Delta v = 1.4319\ [km/s]$

<u>No change in direction, $\alpha\ = 0$</u>

Problem 2a - iii) Plot Orbits

```
% True anomaly vector
```

```
ta_vec  = 0:.01:360;

% Semi-latus rectum of old orbit
p       = a*(1 - e^2);

% Initialize position vectors - in perifocal coordinates
r_P_old = zeros(2,length(ta_vec));
r_P_new = r_P_old;

for i = 1:length(ta_vec)

    % Calculate old orbit radii
    r_old           = p/(1 + e*cosd(ta_vec(i)));

    % DCM matrix from rotating orbit frame to perifocal frame
    P_DCM_R         = [cosd(ta_vec(i)), -sind(ta_vec(i));...
                       sind(ta_vec(i)), cosd(ta_vec(i))];

    % Rotate vectors from orbit frame to perifocal frame
    r_P_old(:,i)    = P_DCM_R*[r_old;0];
    r_P_new(:,i)    = P_DCM_R*[ra;0];
end

figure
plot(r_P_old(1,:), r_P_old(2,:),'-k')
hold on
plot(r_P_new(1,:), r_P_new(2,:),'-r')
plot(-ra, 0,'rx','MarkerSize',18)
plot([0 rp],[0 0],'-b')
plot([-ra 0],[0 0],'-g')
plot([-ra -ra],[0 -v1_m*10000],'-c','LineWidth',2)
plot([-ra -ra],[0 -v1_p*10000],'--m','LineWidth',2)
plot(0, 0,'k*','MarkerSize',18)
grid minor
xlabel('$\hat{e} \  [km]$', 'Interpreter','latex')
ylabel('$\hat{p} \ [km]$', 'Interpreter','latex')
axis equal
title('Problem 2a - ii')
legend('Old Orbit - Before Manuever','New Orbit - Post Manuever','Maneuver Point',...
       '$\bar{r}_0$','$\bar{r}_1$','$\bar{v}_1^-$','$\bar{v}_1^+$','Earth',...
       'Interpreter','latex','Location','best')
```



Problem 2a - iv)

<u>100% of the maneuver magnitude is used to shift energy, as there is no change in direction for this maneuver, only a change in the shape of the orbit.</u>

Problem 2b - i)

<u>Find</u>: Determine the true anomaly in the original orbit

```
% New circular orbit radius
rc      = 25000;    % [km]

% True anomaly at maneuver location
ta_man  = acosd(((p/rc) - 1)/e);
fprintf('The true anomaly at the manuever location is %.3f deg (choose positive value)',ta_man)
```

```
The true anomaly at the manuever location is 134.851 deg (choose positive value)
```

$$r = \frac{p}{1 + e\cos\theta^*}$$

$$\theta^* = \pm\cos^{-1}\frac{\frac{p}{r} - 1}{e}$$

$$\theta^* = 134.8 \; [deg]$$

Problem 2b - ii) Wait Time Till Manuever

Find: Wait time till the manuever

```
% Calculate eccentric anomaly at manuever location
E            = 2*atan2d(tand(ta_man/2),sqrt((1+e)/(1-e)));

% Calculate Mean anomaly at Orbit location
M            = E*pi/180 - e*sind(E);

% Wait time till manuever
dt2          = M/sqrt(mu_earth/a^3);
fprintf('The wait time till the manuever at apogee is %.2f hours',dt2/3600)
```

```
The wait time till the manuever at apogee is 1.45 hours
```

$$\tan(\frac{\theta^*}{2}) = \sqrt{\frac{1+e}{1-e}}\tan(\frac{E}{2})$$

$$E = 2\tan^{-1}[\tan(\frac{\theta^*}{2}) / \sqrt{\frac{1+e}{1-e}}]$$

$$M = E - e\sin E$$

$$M = \sqrt{\frac{\mu}{a^3}}(t - t_p)$$

Currently at perigee

$$(t - t_p) = \frac{M}{\sqrt{\frac{\mu}{a^3}}}$$

$$(t - t_p) = 1.45 \; [hours]$$

Problem 2b - iii) Plot Orbits

$\underline{Find}$: $\overline{r}_1, \overline{v}_1^-, \gamma_1^-, \overline{v}_1^+, \gamma_1^+$ for circurlarizing manuever at 25000 km

```
% Calculate speed at manuever point - pre manuever
v1_m2        = sqrt(2*(energy + mu_earth/rc));

% Specific angular momentum of old orbit
h            = sqrt(mu_earth*p);

% Calculate flight path angle - choose positive sign due to sign of true anomaly
gamma1_m     = acosd(h/(rc*v1_m2));
```

```
fprintf('The initial flight path angle is %.2f deg', gamma1_m)
```

```
The initial flight path angle is 46.91 deg
```

$\epsilon = \frac{v^2}{2} - \frac{\mu}{r}$

$v_1^- = \sqrt{2(\epsilon + \frac{\mu}{r_1})} = 4.07 \ [km/s]$

$h^- = \sqrt{\mu p^-} = r_1 v_1^- \cos(\gamma_1^-)$

$\gamma_1^- = \cos^{-1} \frac{h^-}{r_1 v_1^-}$

$\gamma_1^- = 46.9 \ [deg]$ Positive due to ascending based off of θ^*

```
% Velocity vector pre-manuever - rotate from orbit frame to perifocal
v1_m_O      = [v1_m2*sind(gamma1_m) ; v1_m2*cosd(gamma1_m)];
v1_m_P      = [cosd(ta_man), -sind(ta_man);...
               sind(ta_man), cosd(ta_man)]*v1_m_O;

% Position vector at manuever point
r1_P        = [cosd(ta_man), -sind(ta_man);...
               sind(ta_man), cosd(ta_man)]*[rc;0];
```

$\bar{r}_1 = 25000 \ \hat{r} \ [km] = -17631.6 \ \hat{e} + 17723.6 \ \hat{p} \ [km]$

$\bar{v}_1^- = 2.97 \ \hat{r} + 2.78 \ \hat{\theta} \ [km/s] = -4.06 \ \hat{e} + 0.14 \ \hat{p} \ [km/s]$

```
% Calculate circular velocity (circle with radius equal to distance to apogee) - post maneuver
v1_p2       = sqrt(mu_earth/rc);

% Calculate new specific angular momentum - sami-latus rectum = radius for circle - post maneuv
h_p2        = sqrt(mu_earth*rc);

% Calculate new flight path angle - post maneuver
gamma1_p2   = acosd(h_p2/(rc*v1_p2));
fprintf('The flight path angle post maneuver is %.1f deg ', gamma1_p2)
```

```
The flight path angle post maneuver is 0.0 deg
```

<u>Note:</u> For circular orbit, the semi-latus rectum is equal to the semi-major axis which equals the constant orbit radius.

$v_1^+ = \sqrt{\mu / r_1} = 3.99 \ [km/s]$

$h^+ = \sqrt{\mu p^+} = r_1 v_1^+ \cos(\gamma_1^+)$

$\gamma_1^+ = \cos^{-1} \frac{h^+}{r_1 v_1^+}$

$\gamma_1^+ = 0 \ [deg]$

```
% Velocity vector in orbit frame - post manuever
v1_p_O      = [0;v1_p2];

% Rotate velocity vector into perifocal frame - post manuever
v1_p_P      = [cosd(ta_man), -sind(ta_man);...
               sind(ta_man), cosd(ta_man)]*v1_p_O;
```

$\bar{v}_1^+ = v_1^+ \sin \gamma_1^+ \ \hat{r} + v_1^+ \cos \gamma_1^+ \ \hat{\theta}$

$\bar{v}_1^+ = 3.99 \ \hat{\theta} \ [km/s] = -2.83 \ \hat{e} - 2.82 \ \hat{p} \ [km/s]$

```
% Calculate deltaV vector
dv_vec      = v1_p_O - v1_m_O;

% Calculate deltaV
dv          = sqrt(v1_p2^2 + v1_m2^2 - 2*v1_p2*v1_m2*cosd(gamma1_p - gamma1_m));
fprintf('The change in velocity magnitude is %.3f km/s', norm(dv_vec))
```

```
The change in velocity magnitude is 3.210 km/s
```

$\Delta \bar{v} = \bar{v}_1^+ - \bar{v}_1^-$

$\Delta \bar{v} = -2.97 \ \hat{r} + 1.21 \ \hat{\theta} \ [km/s]$

$\Delta v = |\Delta \bar{v}| = \sqrt{v_1^+ + v_1^- - 2v_1^+ v_1^- \cos \Delta \gamma} = 3.2 \ [km/s]$

```
% Calculate beta
beta        = asind(v1_p2*sind(gamma1_m - gamma1_p2)/dv);

% Calculate alpha - decrease in FPA, negative alpha
alpha       = sign(gamma1_p2 - gamma1_m)*(180 - beta);
fprintf('The alpha for the manuever is %.3f deg', alpha)
```

```
The alpha for the manuever is -114.701 deg
```

$\frac{\Delta v}{\sin \Delta \gamma} = \frac{v^+}{\sin \beta}$

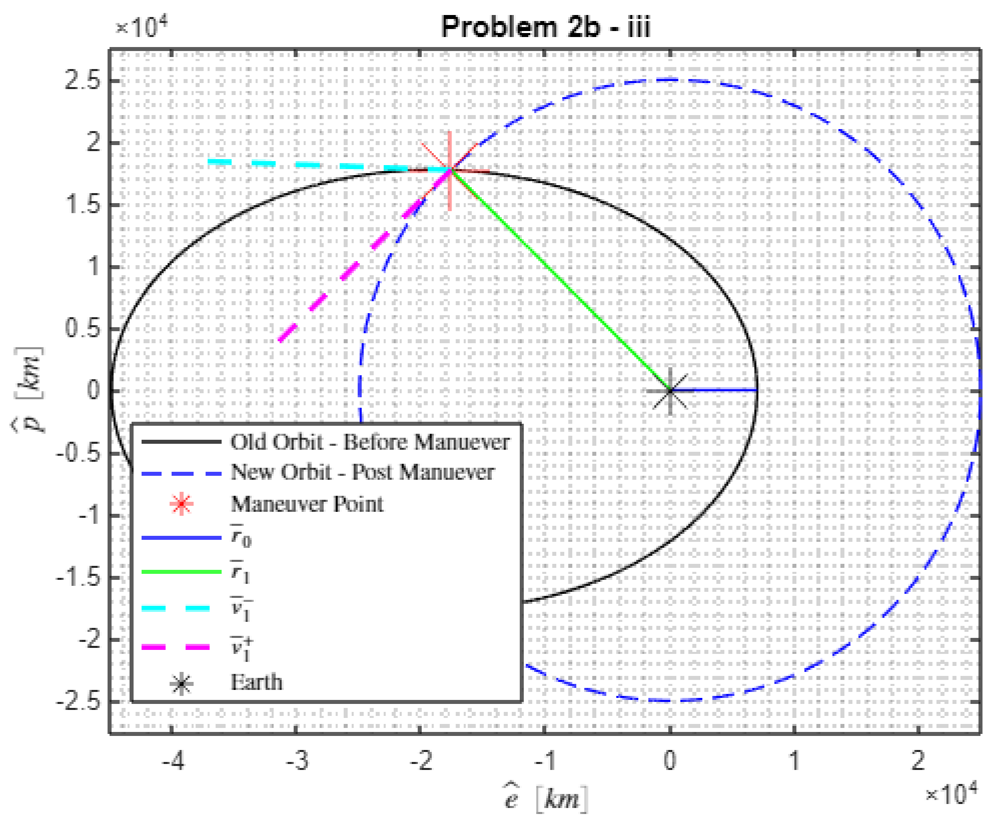
$\beta = \sin^{-1} \frac{v^+ \sin \Delta \gamma}{\Delta v}$

$\alpha = 180 - \beta$

$\alpha = -114.7 \ [deg]$

```
% Initialize position vectors - in perifocal coordinates
r_P_old = zeros(2,length(ta_vec));
r_P_new = r_P_old;

for i = 1:length(ta_vec)

    % Calculate old orbit radii
    r_old           = p/(1 + e*cosd(ta_vec(i)));

    % DCM matrix from rotating orbit frame to perifocal frame
```

```
    P_DCM_R         = [cosd(ta_vec(i)), -sind(ta_vec(i));...
                       sind(ta_vec(i)), cosd(ta_vec(i))];

    % Rotate vectors from orbit frame to perifocal frame
    r_P_old(:,i)    = P_DCM_R*[r_old;0];
    r_P_new(:,i)    = P_DCM_R*[rc;0];
end

figure
plot(r_P_old(1,:), r_P_old(2,:),'-k')
hold on
plot(r_P_new(1,:), r_P_new(2,:),'--b')
plot(r1_P(1), r1_P(2),'r*','MarkerSize',30)
plot([0 rp],[0 0],'-b')
plot([0 r1_P(1)],[0 r1_P(2)],'-g')
plot([r1_P(1) (r1_P(1)+ v1_m_P(1)*5000)],[r1_P(2) (r1_P(2) + v1_m_P(2)*5000)],'--c','LineWidth
plot([r1_P(1) (r1_P(1)+ v1_p_P(1)*5000)],[r1_P(2) (r1_P(2) + v1_p_P(2)*5000)],'--m','LineWidth
plot(0, 0,'k*','MarkerSize',18)
grid minor
xlabel('$\hat{e} \  [km]$', 'Interpreter','latex')
ylabel('$\hat{p} \ [km]$', 'Interpreter','latex')
axis equal
title('Problem 2b - iii')
legend('Old Orbit - Before Manuever','New Orbit - Post Manuever','Maneuver Point',...
       '$\bar{r}_0$','$\bar{r}_1$','$\bar{v}_1^-$','$\bar{v}_1^+$','Earth',...
       'Interpreter','latex','Location','best')
```



Problem 2b - iv)

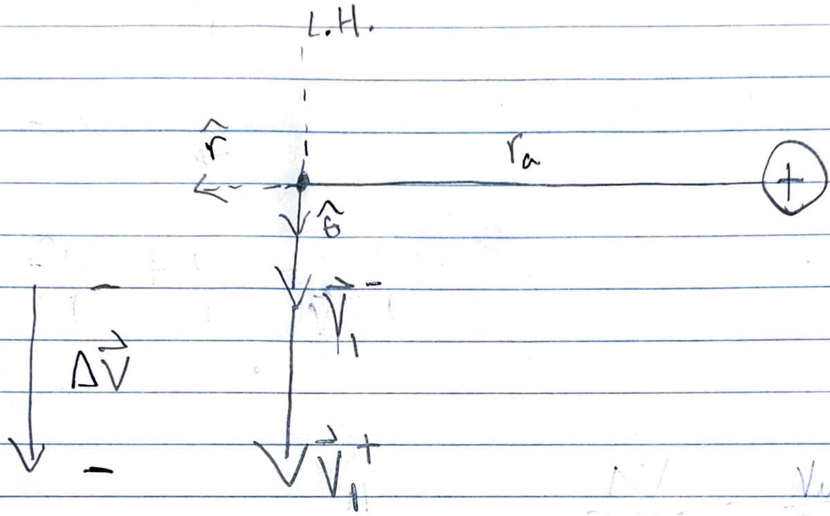
The difference in velocity magnitude between $\bar{v}^+$ and v^- is -0.076 km/s. Therefore, only 2.4 % of the Δv is used to shift energy. Therefore, the remaining 97.6% is used to shift direction.

Problem 2c)

The Δv required for the second manuever is more than double (3.2 km/s compared to 1.4 km/s) than what is required for the first manuever. Both manuevers produce circular orbits, but the second manuever produces a circular orbit of a much smaller size. However, it is not fair to compare the manuevers as they occur at different locations and have different desired final orbit properties, which both have a direct effect of the required Δv.

HW6

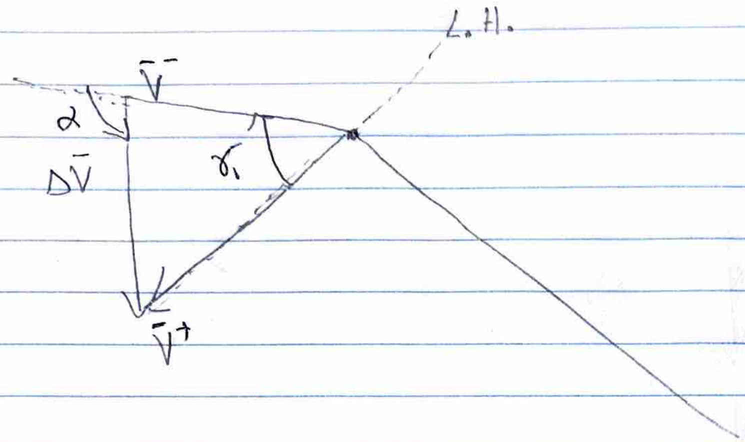
2a-ii)

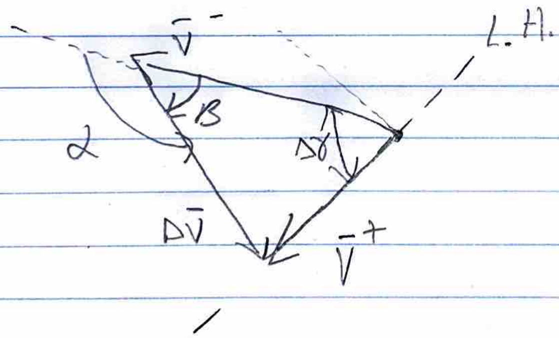


$\boxed{\Delta d = 0}$

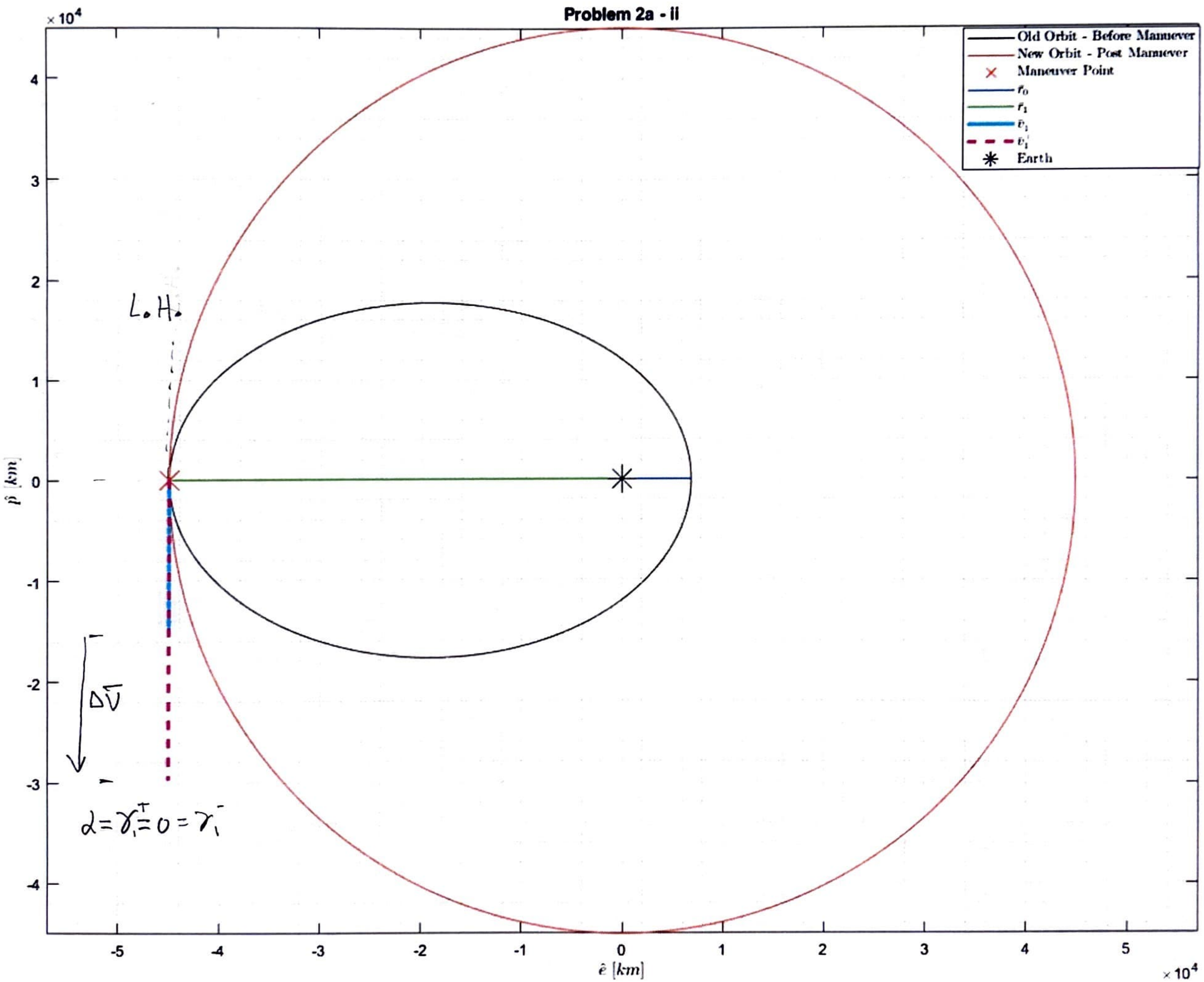
HW6

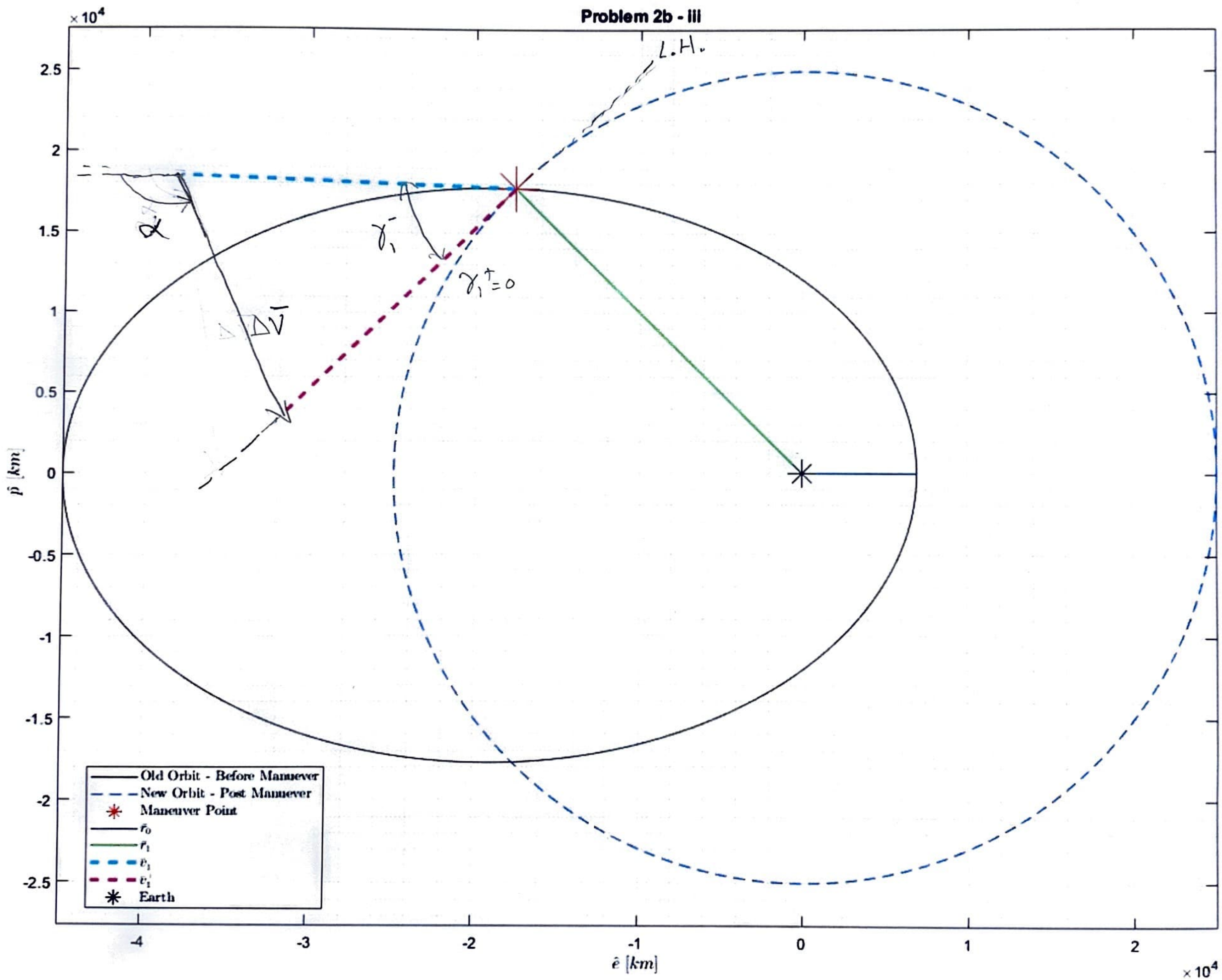
2b-iii)





$$\frac{\Delta\bar{V}}{\sin(\Delta\gamma)} = \frac{\bar{V}^+}{\sin(\beta)}$$


Problem 2a - ii
Old Orbit - Before Manuever
New Orbit - Post Manuever
Maneuver Point
$\bar{r}_0$
$\bar{r}_1$
$\bar{v}_1$
$\bar{v}_1'$
Earth
L.H.
$\Delta \bar{v}$
$\alpha = \gamma_1^+ = 0 = \gamma_1^-$
$\hat{p}$ [km]
$\hat{e}$ [km]
$\times 10^4$
$\times 10^4$



Problem 2b - iii
$\times 10^4$
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-4
-3
-2
-1
0
1
2
$\times 10^4$
$\hat{e}$ [km]
$\hat{p}$ [km]
L.H.
α
γ_1^-
$\gamma_1^+ = 0$
$\Delta\bar{V}$
Old Orbit - Before Manuever
New Orbit - Post Manuever
Maneuver Point
$\bar{r}_0$
$\bar{r}_1$
$\bar{v}_1$
$\bar{v}_1'$
Earth