(3) Set $Y_p(t) = V_1 \underline{t} e^{-2t} + V_2 e^{-2t}$ (Rule 2)

Ⓛ $Y_p' = V_1\left(e^{-2t} + (-2)te^{-2t}\right) - 2V_2 e^{-2t}$

Ⓡ $= AY_p + \begin{bmatrix}1\\2\end{bmatrix} e^{-2t}$

$= AV_1 te^{-2t} + AV_2 e^{-2t} + \begin{bmatrix}1\\2\end{bmatrix}e^{-2t}$ =

$$V_1 \cancel{e^{-2t}} - 2V_1 t\cancel{e^{-2t}} - 2V_2\cancel{e^{-2t}}$$

$$= AV_1 t\cancel{e^{-2t}} + AV_2\cancel{e^{-2t}} + \begin{bmatrix}1\\2\end{bmatrix}\cancel{e^{-2t}}$$

① $AV_1 = -2V_1$ ② $V_1 - 2V_2 = AV_2 + \begin{bmatrix}1\\2\end{bmatrix}$

① $(A-(-2)I)V_1 = 0$: V_1 = an eigenvector of A ~ $\lambda = -2$

Set $V_1 = a\begin{bmatrix}1\\-3\end{bmatrix}$: $A = \begin{bmatrix}1 & 1\\3 & -1\end{bmatrix}$

② $AV_2 + 2V_2 = V_1 - \begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}a-1\\-3a-2\end{bmatrix}$

$\begin{bmatrix}3 & 1\\3 & 1\end{bmatrix}V_2 = \begin{bmatrix}a-1\\-3a-2\end{bmatrix}$: $V_2 \overset{let}{=} \begin{bmatrix}v_1\\v_2\end{bmatrix}$

$$\begin{cases}3v_1 + v_2 = a-1\\3v_1 + v_2 = -3a-2\end{cases} :$$

$\underset{+3a}{a-1} = \underset{+3a}{-3a-2}$ iff $4a = -1$

$a = -\frac{1}{4}$.

$V_1 = \left(-\frac{1}{4}\right)\begin{bmatrix}1\\-3\end{bmatrix} = \begin{bmatrix}-\frac{1}{4}\\ \frac{3}{4}\end{bmatrix}$

$3v_1 + v_2 = -\frac{1}{4} - 1 = -\frac{5}{4}$

$v_2 = -\frac{5}{4} - 3v_1$: $v_1 = 0$: $v_2 = -\frac{5}{4}$

$v_1 = 1$: $v_2 = -\frac{17}{4}$, ...

$V_2 = \begin{bmatrix}v_1\\v_2\end{bmatrix} = \begin{bmatrix}0\\-\frac{5}{4}\end{bmatrix}$

$\therefore Y_p(t) = \begin{bmatrix}-\frac{1}{4}\\ \frac{3}{4}\end{bmatrix}te^{-2t} + \begin{bmatrix}0\\-\frac{5}{4}\end{bmatrix}e^{-2t}$

$Y(t) = c_1\begin{bmatrix}1\\1\end{bmatrix}e^{2t} + c_2\begin{bmatrix}1\\-3\end{bmatrix}e^{-2t} + Y_p(t)$