2.1.2

a) $(0, b_2, b_3)$

$$c\begin{pmatrix}0\\b_2\\b_3\end{pmatrix}=\begin{pmatrix}0\\cb_2\\cb_3\end{pmatrix}$$

$$\begin{pmatrix}0\\b_2\\b_3\end{pmatrix}+\begin{pmatrix}0\\a_2\\a_3\end{pmatrix}=\begin{pmatrix}0\\b_2+a_2\\b_3+a_3\end{pmatrix}$$

0 is a 1x1 vector space

$\therefore$ $\boxed{\begin{pmatrix}0\\b_2\\b_3\end{pmatrix} \text{ is a subspace}}$

b) $\begin{pmatrix}1\\b_2\\b_3\end{pmatrix}$ is <u>not a subspace as 0 is not included</u>

c) $$\begin{pmatrix}b_1\\b_2\\0\end{pmatrix}+\begin{pmatrix}b_1\\0\\b_3\end{pmatrix}=\begin{pmatrix}2b_1\\b_2\\b_3\end{pmatrix}$$

$(b_2)(b_3) \neq 0$ $\therefore$ <u>Not a subspace</u>

d) $\begin{pmatrix}1\\1\\0\end{pmatrix}$ & $\begin{pmatrix}2\\0\\1\end{pmatrix}$ <u>are subspaces</u> as the combinations

under addition and subtraction can be shown as linear combinations.

e) $\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$ <u>is a subspace</u> as $b_3 - b_2 + b_1 = 0$

includes 0 and can be written as a linear combination

2.1.3

a) $A = \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}$

$$\begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} u-v \\ 0 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

Column space is $(u-v, 0)$ or just the x-axis

$$\begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} u-v \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Null space is $u = v$, or a line.

b) $$\begin{pmatrix} 0 & 0 & 3 \\ 1 & 2 & 3 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

$$\begin{pmatrix} w \\ u+2v+3w \end{pmatrix}$$

Column space is x-y plane

$w = 0$

$u + 2v = 0$ $\quad u = -2v$

Null space is line with $u = -2v$ & $w = 0$

c) $C = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

2.1.3

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Column space is $(0,0)$

Null space is $\mathbb{R}^3$

2.1.5

a)

$$x+y = \begin{pmatrix} x_1+y_1+1 \\ x_2+y_2+1 \end{pmatrix}$$

$$y+x = \begin{pmatrix} y_1+x_1+1 \\ y_2+x_2+1 \end{pmatrix} \qquad x+y = y+x$$

$$x+(y+z) = \begin{pmatrix} x_1+(y_1+z_1) \\ x_2+(y_2+z_2) \end{pmatrix}$$

$$(x+y)+z = \begin{pmatrix} (x_1+y_1)+z_1 \\ (x_2+y_2)+z_2 \end{pmatrix} \qquad x+(y+z) = (x+y)+z$$

$$x+(-x) = 0 = (x_1+1)+(-x-1) = 0$$

$$c(x+y) = \begin{pmatrix} cx_1+c+cy_1 \\ cx_2+c+cy_2 \end{pmatrix}$$

$$cx+cy = \begin{matrix} cx_1+cy_1+1 \\ cx_2+cy_2+1 \end{matrix} \qquad \therefore cx+cy \neq c(x+y)$$

$$(c_1+c_2)x = (c_1+c_2+1)x = c_1x+c_2x+x$$
$$c_1x+c_2x = c_1x+c_2x+1 \neq c_1x+c_2x+x$$

$\therefore$ $\boxed{\text{Rule 7 \& 8 are broken}}$

b) $x+y = xy \qquad y+x = yx$

$$x+y = xy = yx = y+x$$

2.1.5

$x+(y+z) = x+yz = xyz$

$(x+y)+z = xy+z = xyz = x+(y+z)$

$x+\vec{0} = x$

$x+\vec{0} = x\vec{0} = x \quad \therefore \quad \vec{0} = \vec{1}$

$\boxed{\text{"Zero" vector is a vector of } 1}$

$x+(-x) = 1$ - this is "zero"

Let $-x = b$

$x+b = 1$

$xb = 1 \qquad b = 1/x$

$1x = x$

$cx = x^c$

$1x = x$

$(c_1c_2)x = x^{c_1c_2}$

$c_1(c_2x) = x^{c_1c_2}$

$c(xy) = c(x+y)$

$cx+cy = c(x+y) = c(xy)$

$(c_1+c_2)x = x^{c_1+c_2}$

$c_1x+c_2x = x^{c_1}+x^{c_2} = x^{c_1+c_2}$

2.1.5

c) $X + y = \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \end{pmatrix} = \begin{pmatrix} x_1 + y_2 \\ x_2 + y_1 \end{pmatrix}$

$y + x = \begin{pmatrix} y_1 + x_1 \\ y_2 + x_2 \end{pmatrix} \neq x + y$

$x + (y + z) = \begin{pmatrix} x_1 + y_1 + z_1 \\ x_2 + y_2 + z_2 \end{pmatrix}$

$(x+y) + z = \begin{pmatrix} (x_1 + y_1) + z_1 \\ (x_2 + y_2) + z_2 \end{pmatrix} = \begin{pmatrix} (x_1 + y_2) + z_1 \\ (x_2 + y_1) + z_2 \end{pmatrix}$

$x + (y+z) \neq (x+y) + z$

Rules 3-6 are unaffected as no y vector is added.

$c(x+y) = c\begin{pmatrix} x_1 + y_2 \\ x_2 + y_1 \end{pmatrix}$

$cx + cy = c\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + c\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = c\begin{pmatrix} x_1 + y_2 \\ x_2 + y_1 \end{pmatrix}$

$\boxed{\text{Rules 1 \& 2 are broken}}$

2.1.25

Adding a column to A doesn't increase the column space if the added column exists in the column space

$A = \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} \quad b = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$A|b = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} u-v \\ w \end{pmatrix}$

Added to column space

$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad b = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$A|b = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} u+w \\ v+w \end{pmatrix}$

Column space remains the same.

The possible solution to $Ax = b$ exist only in the column space of A, therefore if b exists in column space, it can't change solution.

2.2.2

$$A = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{pmatrix} \xrightarrow{E_{31}(-1)} \begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{E_{12}(-2)} \begin{pmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \boxed{1} & 0 & -2 & 1 \\ 0 & \boxed{1} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{matrix} x_1 - 2x_3 + x_4 \\ x_2 + x_3 \end{matrix}$$

2 pivots, $\boxed{\text{rank} = 2}$. Pivots are x_1 & x_2, so free

Variables are x_3 and x_4

$$Ax = 0 \Rightarrow \begin{matrix} x_1 = 2x_3 - x_4 \\ x_2 = -x_3 \end{matrix}$$

$$\vec{x} = \begin{pmatrix} 2x_3 - x_4 \\ -x_3 \\ x_3 \\ x_4 \end{pmatrix} = \boxed{x_3 \begin{pmatrix} 2 \\ -1 \\ 1 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}}$$

special solution

$Ax = b$

$$\left(\begin{array}{cccc|c} 1 & 2 & 0 & 1 & b_1 \\ 0 & 1 & 1 & 0 & b_2 \\ 1 & 2 & 0 & 1 & b_3 \end{array}\right)$$

2.2.2

$$\xrightarrow{E_{31}(-1)} \left(\begin{array}{cccc|c} 1 & 2 & 0 & 1 & b_1 \\ 0 & 1 & 1 & 0 & b_2 \\ 0 & 0 & 0 & 0 & b_3 - b_1 \end{array}\right)$$

$$\xrightarrow{E_{12}(-2)} \left(\begin{array}{cccc|c} 1 & 0 & -2 & 1 & b_1 - 2b_2 \\ 0 & 1 & 1 & 0 & b_2 \\ 0 & 0 & 0 & 0 & b_3 - b_1 \end{array}\right)$$

$b_3 = b_1$ for solution

$$x_1 - 2x_3 + x_4 = b_1 - 2b_2$$
$$x_2 + x_3 = b_2$$

$$x_2 = b_2 - x_3$$
$$x_1 = b_1 - 2b_2 + 2x_3 - x_4$$

$$\boxed{\vec{x} = \begin{pmatrix} b_1 - 2b_2 \\ b_2 \\ 0 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 2 \\ -1 \\ 1 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}}$$

All solutions $A\vec{x} = b$

2.2.2

$$B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

$$\xrightarrow{E_{21}(-4)} \begin{pmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 7 & 8 & 9 \end{pmatrix} \xrightarrow{E_{31}(-7)} \begin{pmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -6 & -12 \end{pmatrix}$$

$$\xrightarrow{E_{32}(-2)} \begin{pmatrix} \boxed{1} & 2 & 3 \\ 0 & \boxed{-3} & -6 \\ 0 & 0 & 0 \end{pmatrix}$$ 2 pivots, $\boxed{\text{rank} = 2}$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{matrix} x_1 + 2x_2 + 3x_3 \\ -3x_2 - 6x_3 \end{matrix}$$

x_1 & x_2 are pivots so $\boxed{x_3 \text{ is free variable}}$

$$Bx = 0 \Rightarrow x_1 = -2x_2 - 3x_3$$

$$x_2 = -2x_3 \quad \therefore x_1 = x_3$$

$$\vec{x} = \begin{pmatrix} x_3 \\ -2x_3 \\ x_3 \end{pmatrix} = \boxed{x_3 \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}}$$

$Bx = b$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & b_1 \\ 4 & 5 & 6 & b_2 \\ 7 & 8 & 9 & b_3 \end{array}\right) \xrightarrow{E_{21}(-4)} \left(\begin{array}{ccc|c} 1 & 2 & 3 & b_1 \\ 0 & -3 & -6 & b_2 - 4b_1 \\ 7 & 8 & 9 & b_3 \end{array}\right)$$

2.2.2

$$\xrightarrow{E_{31}(-7)} \left(\begin{array}{ccc|c} 1 & 2 & 3 & b_1 \\ 0 & -3 & -6 & b_2-4b_1 \\ 0 & -6 & -12 & b_3-7b_1 \end{array}\right)$$

$$\xrightarrow{E_{32}(-2)} \left(\begin{array}{ccc|c} 1 & 2 & 3 & b_1 \\ 0 & -3 & -6 & b_2-4b_1 \\ 0 & 0 & 0 & b_3-7b_1-2b_2+8b_1 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} 1 & 2 & 3 & b_1 \\ 0 & -3 & -6 & b_2-4b_1 \\ 0 & 0 & 0 & b_3+b_1-2b_2 \end{array}\right)$$

To be solvable, $b_3+b_1-2b_2=0$

$$x_1+2x_2+3x_3=b_1 \Rightarrow x_1 = b_1-2x_2-3x_3$$

$$-3x_2-6x_3 = b_2-4b_1 \Rightarrow x_2 = \frac{6x_3+b_2-4b_1}{-3}$$

$$x_1 = b_1 - 2\left(-2x_3 - \frac{b_2}{3} + \frac{4}{3}b_1\right) - 3x_3$$

$$x_1 = -\frac{5}{3}b_1 + x_3 + \frac{2}{3}b_2$$

$$\boxed{\vec{x} = \begin{pmatrix} -\frac{5}{3}b_1 + \frac{2}{3}b_2 \\ 4/3\, b_1 - \frac{b_2}{3} \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}}$$

All solutions
$Bx=b$

2.2.5

$$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 4 & 5 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 2 & 4 & 5 & 4 \end{array}\right) \xrightarrow{E_{21}(-2)} \left(\begin{array}{ccc|c} \boxed{1} & 2 & 2 & 1 \\ 0 & 0 & \boxed{1} & 2 \end{array}\right)$$

V is a free variable

X_n: $u + 2v + 2w = 0$

$w = 0$

$u = -2v$

$$X_n = \begin{pmatrix} u \\ v \\ w \end{pmatrix} = v \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$$

X_p = $u + 2v + 2w = 1$

$w = 2$

$u = 1 - 2v - 4 = -2v - 3$

$$X_p = \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ 2 \end{pmatrix}$$

$$\boxed{X = \begin{pmatrix} -3 \\ 0 \\ 2 \end{pmatrix} + v \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}}$$

2.2.5

$$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 4 & 4 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 2 & 4 & 4 & 4 \end{array}\right) \xrightarrow{E_{21}(-2)} \left(\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{array}\right)$$

$0u + 0v + 0w \neq 2$ $\boxed{\text{no Solution}}$

2.2.6

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$\left(\begin{array}{cc|c} 1 & 0 & b_1 \\ 0 & 1 & b_2 \\ 2 & 3 & b_3 \end{array}\right) \xrightarrow{E_{31}(-2)} \left(\begin{array}{cc|c} 1 & 0 & b_1 \\ 0 & 1 & b_2 \\ 0 & 3 & b_3 - 2b_1 \end{array}\right) \xrightarrow{E_{32}(-3)}$$

$$\left(\begin{array}{cc|c} \boxed{1} & 0 & b_1 \\ 0 & \boxed{1} & b_2 \\ 0 & 0 & b_3 - 2b_1 - 3b_2 \end{array}\right)$$

$$b_3 - 2b_1 - 3b_2 = 0 \quad \Rightarrow \quad b_3 = 2b_1 + 3b_2$$

$$\boxed{b = b_1 \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + b_2 \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ 2b_1 + 3b_2 \end{pmatrix}}$$

$\boxed{\text{Rank} = 2}$ as there are two pivots

$$X_n = \begin{matrix} u = 0 \\ v = 0 \end{matrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$X_p = \begin{matrix} u = b_1 \\ v = b_2 \end{matrix}$$

$$\boxed{X_p = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}}$$

2.3.1

$$V_1 = \begin{pmatrix}1\\0\\0\end{pmatrix} \quad V_2 = \begin{pmatrix}1\\1\\0\end{pmatrix} \quad V_3 = \begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$C_1 V_1 + C_2 V_2 + C_3 V_3 = \vec{0}$$

$$C_1\begin{pmatrix}1\\0\\0\end{pmatrix} + C_2\begin{pmatrix}1\\1\\0\end{pmatrix} + C_3\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$\begin{aligned} C_1 + C_2 + C_3 &= 0 \\ C_2 + C_3 &= 0 \\ C_3 &= 0 \end{aligned}$$

$C_3 = 0 \quad \therefore \quad C_2 = -C_3 = 0 \quad \& \quad C_1 = -C_2 - C_3 = 0$

$\therefore$ $\boxed{\text{Linearly independent}}$

$$C_1 V_1 + C_2 V_2 + C_3 V_3 + C_4 V_4 = \vec{0}$$

$$C_1\begin{pmatrix}1\\0\\0\end{pmatrix} + C_2\begin{pmatrix}1\\1\\0\end{pmatrix} + C_3\begin{pmatrix}1\\1\\1\end{pmatrix} + C_4\begin{pmatrix}2\\3\\4\end{pmatrix} = 0$$

$$\begin{aligned} C_1 + C_2 + C_3 + 2C_4 &= 0 \\ C_2 + C_3 + 3C_4 &= 0 \\ C_3 + 4C_4 = 0 &\Rightarrow C_3 = -4C_4 \end{aligned}$$

$\therefore \quad C_2 = -C_3 - 3C_4 = 4C_4 - 3C_4 = C_4$

$\therefore \quad C_1 = -C_2 - C_3 - 2C_4 = -C_4 + 4C_4 - 2C_4 = -C_4$

2.3.1

C_1, C_2, & C_3 are all directly dependent on C_4 and $\neq 0$. $\therefore$ $\boxed{V_1, V_2, V_3, V_4 \text{ are linearly dependent}}$

2.3.7

$$c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$$

$$c_1 (w_2 - w_3) + c_2 (w_1 - w_3) + c_3 (w_1 - w_2) = 0$$

$$c_1 w_2 - c_1 w_3 + c_2 w_1 - c_2 w_3 + c_3 w_1 - c_3 w_2 = 0$$

$$w_1 (c_2 + c_3) + w_2 (c_1 - c_3) + w_3 (-c_1 - c_2) = 0$$

$w_1, w_2, w_3 \neq 0 \quad \therefore \quad (c_2 + c_3) = 0$

$(c_1 - c_3) = 0$

$(-c_1 - c_2) = 0$

$c_1 = -c_2, \quad c_1 = c_3, \quad c_2 = -c_3$

The coefficients are directly dependent on each other and $\neq 0$ $\therefore$ $\boxed{v_1, v_2, \text{ \& } v_3 \text{ are dependent}}$

$$(\cancel{w_2} - \cancel{w_3}) - \cancel{(}\cancel{w_1} - \cancel{w_3}) + (\cancel{w_1} - \cancel{w_2}) = 0$$

$$\boxed{v_1 - v_2 + v_3 = 0}$$

2.3.37

a) $A = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$

$A = a\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + b\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + c\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\boxed{\text{Basis: } \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}}$

b) $A = \begin{pmatrix} a & d & e \\ d & b & f \\ e & f & c \end{pmatrix}$

$A = a\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + b\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + c\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + d\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$+ e\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + f\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

$\boxed{\text{Basis: } \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}}$

c) $A = \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$ $A^T = \begin{pmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{pmatrix}$ $-A = \begin{pmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{pmatrix}$

2.3.37

$$A = a\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + b\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} + c\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$\boxed{\text{Basis: } \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}}$$