MA 527

Lecture Notes (section 7.3)

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7.3 Gaussian Elimination

(Ex) $\begin{cases} x - y = -3 & -① \\ 2x + y = 0 & -② \end{cases}$

(Matlab)

A, b

> X = A\b.

matrix form

$\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \end{bmatrix}$: $\left[\begin{array}{cc:c} 1 & -1 & -3 \\ 2 & 1 & 0 \end{array}\right]$

the augmented matrix.

② − 2×① :

$$\begin{array}{rl} & 2x + y = 0 \\ - \big) & 2x - 2y = -6 \\ \hline & 3y = 6 \end{array}$$

row2 − 2×row1 → row2

$\begin{cases} x - y = -3 \\ 3y = 6 & -③ \end{cases}$ $\left[\begin{array}{cc:c} 1 & -1 & -3 \\ 0 & 3 & 6 \end{array}\right]$

$\frac{1}{3}$row2 → row2

$\frac{1}{3}$×③ : $y = 2$ −④

$\left[\begin{array}{cc:c} 1 & -1 & -3 \\ 0 & 1 & 2 \end{array}\right]$ row Echelon form.

Elimination.

(Back substitution).

row1 + row2 → row1.

①: $x = -3 + y = -3 + 2$
$= -1$

$\begin{cases} x = -1 \\ y = 2 \end{cases}$

$\left[\begin{array}{cc:c} 1 & 0 & -1 \\ 0 & 1 & 2 \end{array}\right]$: reduced row echelon form.

Q1 $\begin{cases} x+2y=1 \\ 3x+6y=6. \end{cases}$: No solution.

$$\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 3 & 6 & 6 \end{array}\right] \xrightarrow{r_2-3r_1} \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 0 & 3 \end{array}\right]$$

$$x+2y=1$$
$$0 \neq 3.$$

Q2: $\begin{cases} x+2y=1 \\ 3x+6y=3 \end{cases}$: infinitely many solutions

$$\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 3 & 6 & 3 \end{array}\right] \xrightarrow{r_2-3r_1} \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 0 & 0 \end{array}\right] \leftarrow$$

row2: $0=0$

$$\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 3 & 6 & 3 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 0 & 0 \end{array}\right]$$

row - equivalent.

(Ex) $\begin{cases} \quad y+2z=1 \\ x \quad -z=2 \\ 2x+4y+3z=3 \end{cases}$ $\left[\begin{array}{ccc|c} 0 & 1 & 2 & 1 \\ 1 & 0 & -1 & 2 \\ 2 & 4 & 3 & 3 \end{array}\right]$

INF, NaN.

$$\xrightarrow{r_1 \leftrightarrow r_2} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & 1 \\ \underline{2} & 4 & 3 & 3 \end{array}\right] \xrightarrow{r_3-2\cdot r_1} \left[\begin{array}{cccc} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & \underline{4} & 5 & -1 \end{array}\right]$$

$$\xrightarrow{r_3 - 4r_2} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & -3 & -5 \end{array}\right] \xrightarrow{\frac{1}{-3} r_3 \to r_3} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 5/3 \end{array}\right]$$

(B.S.)

$$\xrightarrow{r_2 - 2r_3} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & 0 & -7/3 \\ 0 & 0 & 1 & 5/3 \end{array}\right] \xrightarrow{r_1 + r_3} \begin{array}{c} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 11/3 \\ 0 & 1 & 0 & -7/3 \\ 0 & 0 & 1 & 5/3 \end{array}\right] \end{array} \begin{array}{l} = x \\ = y \\ = z \end{array}$$

$$1 - 2 \cdot \frac{5}{3} = 1 - \frac{10}{3} = -\frac{7}{3}.$$