ECE 602: LUMPED LINEAR SYSTEMS

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Fundamental Matrices and State Transition Matrices for DT LTV Systems

Discrete-Time Autonomous LTV Systems

Discrete-Time LTV system

$$x[k+1] = A[k]x[k], \quad \text{with initial condition } x[0]$$

Solution $x[k]$ is $x[k] = \underbrace{A[k-1]A[k-2]\cdots A[0]}_{\Phi[k]} \cdot x[0], \quad k = 0, 1, \ldots$

$\Phi[k] = A[k-1]A[k-2]\cdots A[0]$, $k = 0, 1, \ldots$, is the fundamental matrix

- Describes how the state solution $x[k]$ propagates from time 0 to time k
- Unlike continuous-time case, $\Phi[k]$ may be singular
- For LTI system $x[k+1] = Ax[k]$, $\Phi[k] = A^k$

State Transition Matrix of DT LTV Systems

The state transition matrix for LTV system $x[k+1] = A[k]x[k]$ is

$$\Phi[k,\ell] = A[k-1]\cdots A[\ell], \quad k \geq \ell$$

- $\Phi[k,k] = I$ for all $k = 0, 1, \ldots$
- $\Phi[k_3, k_2]\Phi[k_2, k_1] = \Phi[k_3, k_1]$ for all $k_3 \geq k_2 \geq k_1$
- For a fixed ℓ, $\Phi[k,\ell]$ is the solution to the matrix difference equation:

  $$\Phi[k+1,\ell] = A[k]\Phi[k,\ell], \quad k = \ell, \ell+1, \ldots$$

  with the initial condition $\Phi[\ell,\ell] = I$
- May not be well defined for $k < \ell$