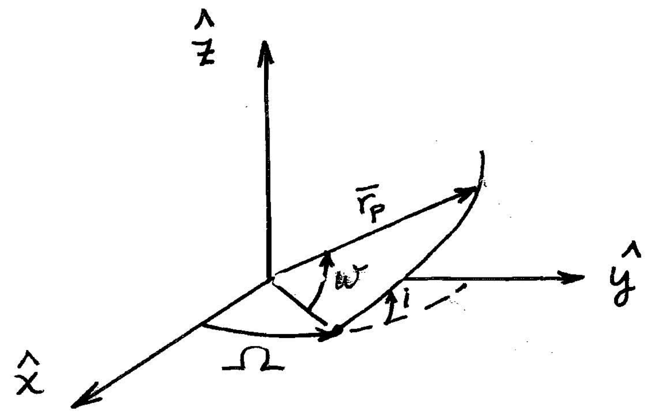
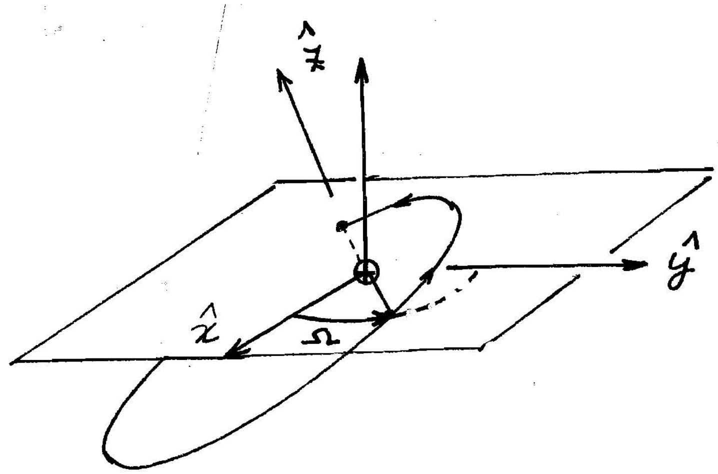
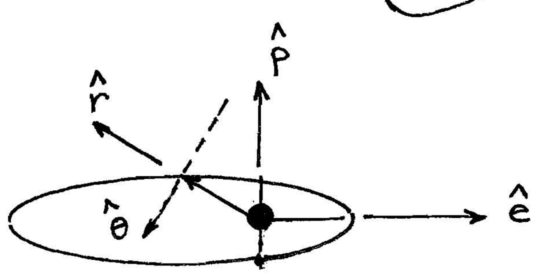
3D Example

Assume s/c moving in orbit about the Earth

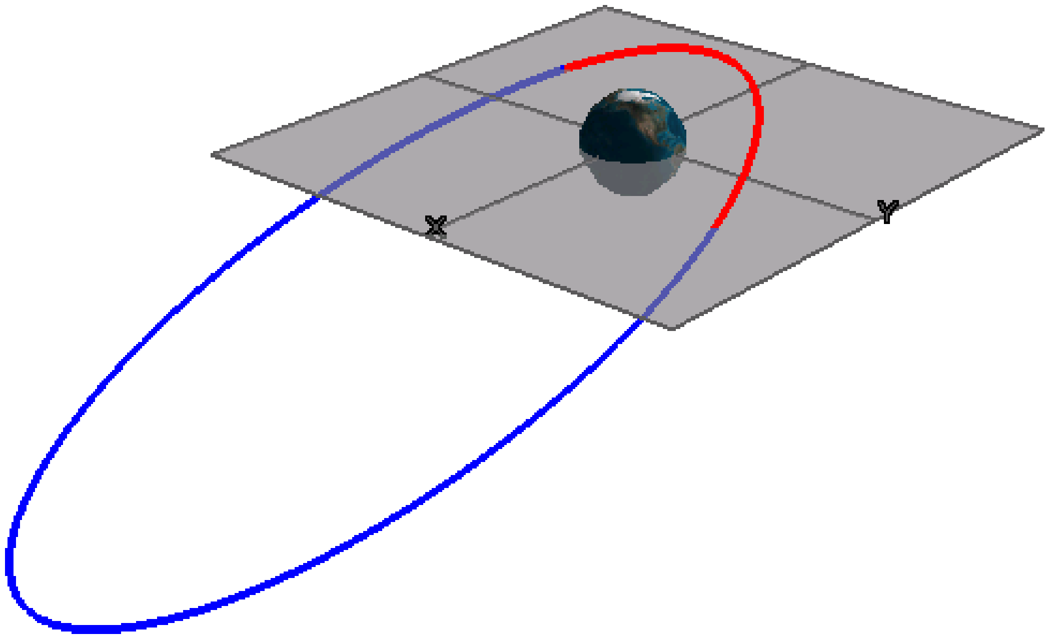
$$\left.\begin{array}{r} a = 8\,R_{\oplus} \\ e = .7 \\ i = 30^\circ \\ \Omega = 60^\circ \\ \omega = 90^\circ \end{array}\right\}\quad \text{wrt “Earth centered Mean J2000 coordinates”}$$







maneuver @ $\theta^* = 90^\circ$


X
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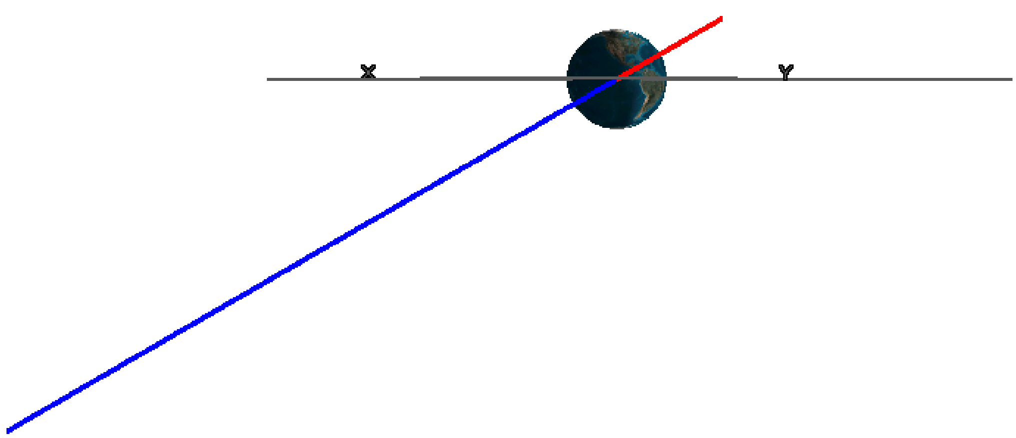
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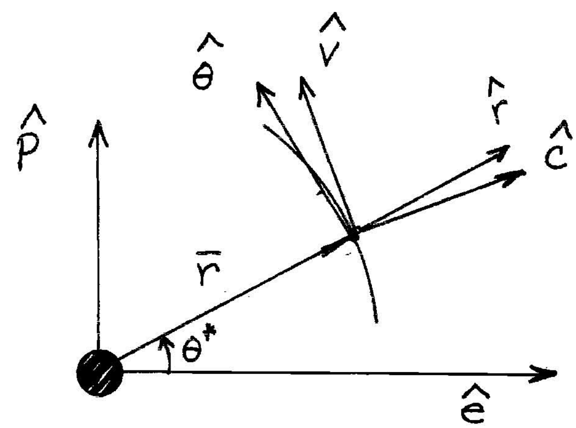
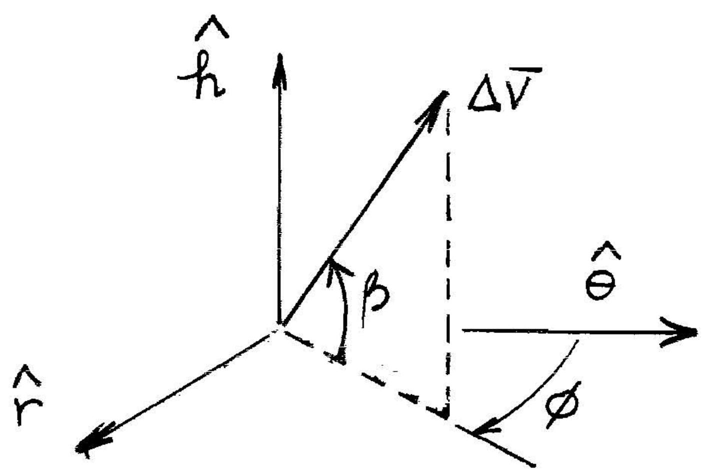
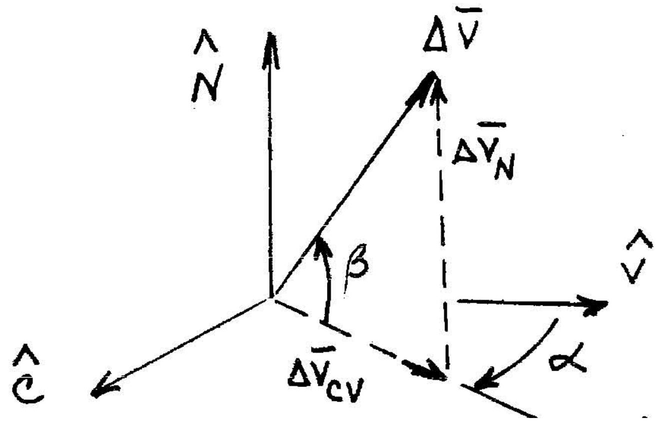


VNB coordinate frame
(useful for describing Δv's and maneuvers)

$\hat{V}$ parallel to velocity; tangent to path
$\hat{N}$ normal; out-of-plane
$\hat{B}$ bi-normal to curve; in plane of motion

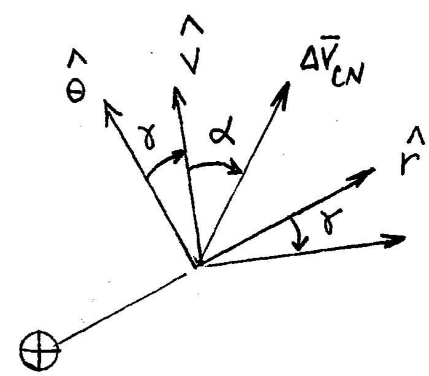




Assume a maneuver such that:

$\Delta v = 2\,\text{km/s} \qquad \alpha = 0^\circ \qquad \beta = 150^\circ$

$\Delta\bar{v} = -1.732\hat{V} + 1.0\hat{N}$ km/s



	$\hat{r}$	$\hat{\theta}$	$\hat{h}$
$\hat{x}$	$c_\Omega c_\theta - s_\Omega c_i s_\theta$	$-c_\Omega s_\theta - s_\Omega c_i c_\theta$	$s_\Omega s_i$
$\hat{y}$	$s_\Omega c_\theta + c_\Omega c_i s_\theta$	$-s_\Omega s_\theta + c_\Omega c_i c_\theta$	$-c_\Omega s_i$
$\hat{z}$	$s_i s_\theta$	$s_i c_\theta$	c_i

$$i = 30^\circ$$
$$\Omega = 60^\circ$$
$$\theta = \omega + \theta^* = 180^\circ$$

$$\begin{array}{c|ccc} & \hat{r} & \hat{\theta} & \hat{h} \\ \hline \hat{x} & -.5 & .75 & .433 \\ \hat{y} & -.866 & -.433 & -.25 \\ \hat{z} & 0 & -.5 & .866 \end{array}$$

$$\bar{r} = 26022.80\,\hat{r} \text{ km}$$

$$\bar{r} = -13011.40\,\hat{x} - 22536.40\,\hat{y} \text{ km/s}$$

$$v = 4.777328 \text{ km/s} \qquad \gamma = 34.992^\circ$$

$$\bar{v} = 2.739616\,\hat{r} + 3.91374\,\hat{\theta} \text{ km/s}$$
$$= \left(-1.36981\,\hat{x} - 2.37258\,\hat{y} \text{ km/s}\right)$$
$$+ \left(2.93573\,\hat{x} - 1.69465\,\hat{y} - 1.95687\,\hat{z} \text{ km/s}\right)$$

$$\bar{v} = 1.56550\,\hat{x} - 4.06728\,\hat{y} - 1.95687\,\hat{z} \text{ km/s}$$

$\Delta\bar{v}$ ← also in inertial coordinates

$$\Delta\bar{v} = -.134551\,\hat{x} + 1.22457\,\hat{y} + 1.57548\,\hat{z} \text{ km/s}$$

$$\bar{v}_{new} = \bar{v}_{old} + \Delta\bar{v}$$
$$\bar{v}_{new} =$$
$$\bar{r}_{new} = \bar{r}_{old}$$

Characteristics of new orbit?

$$\hat{h}_{new} =$$

$$\hat{h} = \sin\Omega\sin i\,\hat{x} - \cos\Omega\sin i\,\hat{y} + \cos i\,\hat{z}$$

from where?

$$\cos i = .989864407 \Longrightarrow$$

$$\left.\begin{array}{r} \sin\Omega\sin i = .122989 \\ -\cos\Omega\sin i = -.071008 \end{array}\right\} \quad \left.\begin{array}{l} \Omega = 60^\circ, 120^\circ \\ \Omega = \pm 60^\circ \end{array}\right\}$$

$$|\bar{v}| = 3.206887\,\text{km/s}$$

$$\frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a} \Longrightarrow \boxed{a = 19576.962\,\text{km}}$$

$$\underbrace{\bar{r} \bullet \bar{v}}_{r\dot{r} = r\,v\sin\gamma} = +45476\,\text{km}^2/\text{s} \longrightarrow \text{sign on } \gamma \text{ is positive}$$

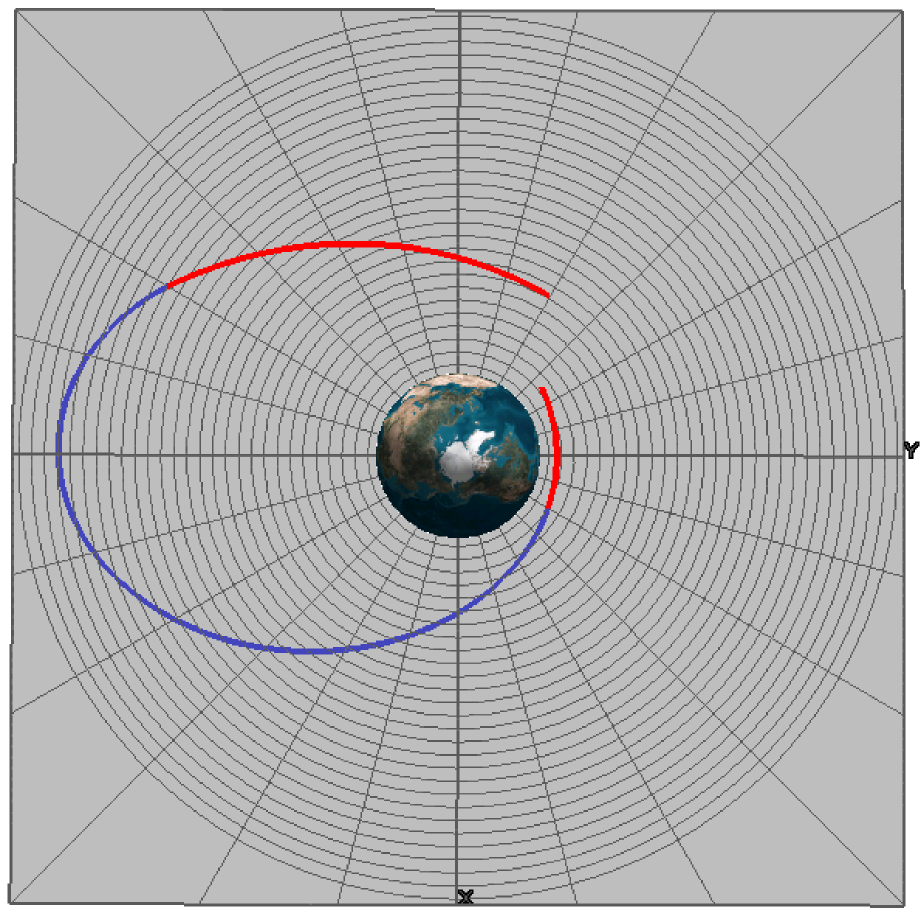
$$e^2 = \left(\frac{rv^2}{\mu} - 1\right)^2 \cos^2\gamma + \sin^2\gamma \Longrightarrow \boxed{e = .610802}$$

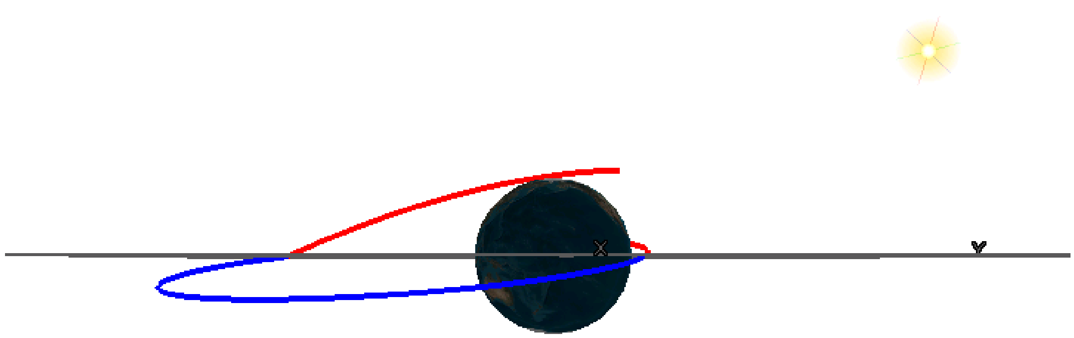
$$\tan\theta^* = \frac{\left(\dfrac{rv^2}{\mu}\right)\sin\gamma\cos\gamma}{\left(\dfrac{rv^2}{\mu}\right)\cos^2\gamma - 1} \Longrightarrow \theta^* = -30.14346^\circ,\ \boxed{149.88708^\circ}$$

$$\hat{r} \bullet \hat{z} = \sin i \sin\theta = 0 \longrightarrow \theta = 0^\circ, 180^\circ$$

$$\hat{r} \bullet \hat{x} = \cos\Omega\cos\theta - \cancel{\sin\Omega\cos i}\sin\theta$$

$$\omega_{new} = \theta_{new} - \theta^*_{new} \Longrightarrow \boxed{\omega_{new} = 30.11292^\circ}$$


Y
X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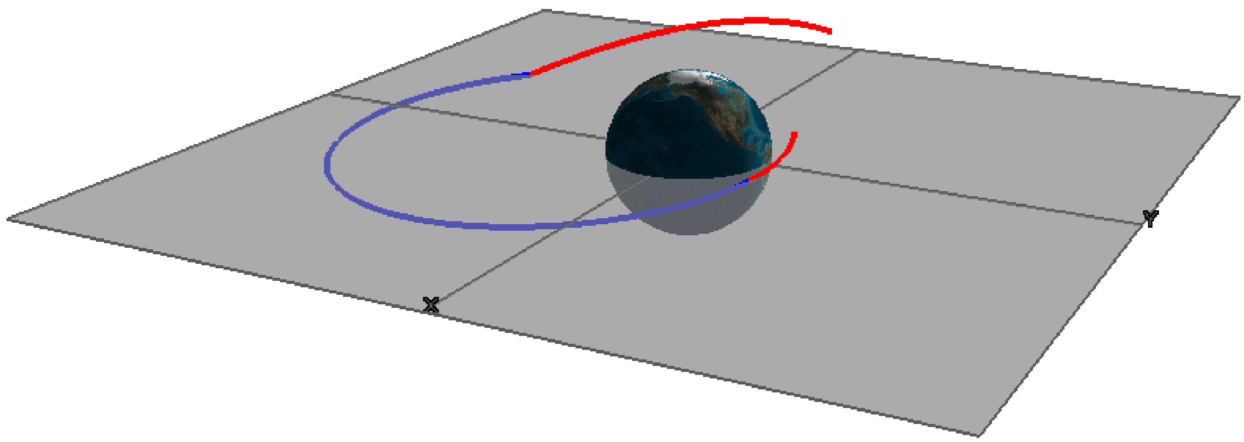
X
Y