pg 261

7.1.9 $3\underline{A} = \begin{bmatrix} 0 & 6 & 12 \\ 18 & 15 & 15 \\ 3 & 0 & -9 \end{bmatrix}$

$0.5\,\underline{B} = \begin{bmatrix} 0 & 2.5 & 1 \\ 2.5 & 1.5 & 2 \\ -1 & 2 & -1 \end{bmatrix}$

$\Rightarrow 3\underline{A} + 0.5\underline{B} = \begin{bmatrix} 0 & 8.5 & 13 \\ 20.5 & 16.5 & 17 \\ 2 & 2 & -10 \end{bmatrix}$

and

$3\underline{A} + 0.5\underline{B} + \underline{C}$ is <u>not</u> well-defined since $\underline{C}$ has different dimensions to $3\underline{A}$ and $0.5\underline{B}$.

pg 271

7.2.12 $\underline{A}\,\underline{A}^T = \begin{bmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 4 & -2 & 1 \\ -2 & 1 & 2 \\ 3 & 6 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 29 & 8 & 6 \\ 8 & 41 & 12 \\ 6 & 12 & 9 \end{bmatrix}$$

$$\underline{A}^2 = \begin{bmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 23 & -4 & 6 \\ -4 & 17 & 12 \\ 2 & 4 & 19 \end{bmatrix}$$

$$\underline{B}\,\underline{B}^T = \begin{bmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -6 & 0 \\ -6 & 10 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

Notice that $\underline{B}^T = \underline{B}$ i.e. $\underline{B}$ is symmetric.

$$\Rightarrow \underline{B}^2 = \underline{B}\,\underline{B}^T$$

$$= \begin{bmatrix} 10 & -6 & 0 \\ -6 & 10 & 0 \\ 0 & 0 & 4 \end{bmatrix} \quad \text{(from above)}$$

7.2.29 Want: $\underline{p}$ such that

$$\underline{A}\,\underline{p} = \underline{v}$$

$$= \begin{bmatrix} 400\times 35 + 60\times 62 + 240\times 30 \\ 100\times 35 + 120\times 62 + 500\times 30 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} 400 & 60 & 240 \\ 100 & 120 & 500 \end{bmatrix}}_{\underline{A}} \underbrace{\begin{bmatrix} 35 \\ 62 \\ 30 \end{bmatrix}}_{\underline{p}}$$

Pg 280

7.3.9 $\left[\begin{array}{ccc|c} 0 & -2 & -2 & -8 \\ 3 & 4 & -5 & 13 \end{array}\right]$ (Augmented matrix)

$\xrightarrow{R_2 + 2R_1 \to R_2}$

$\left[\begin{array}{ccc|c} 0 & -2 & -2 & -8 \\ 3 & 0 & -9 & -3 \end{array}\right]$

$\xrightarrow{R_1 \leftrightarrow R_3}$

$\left[\begin{array}{ccc|c} 3 & 0 & -9 & -3 \\ 0 & -2 & -2 & -8 \end{array}\right]$

$\xrightarrow{\frac{1}{3}R_1 \to R_1,\ -\frac{1}{2}R_2 \to R_2}$

$\left[\begin{array}{ccc|c} 1 & 0 & -3 & -1 \\ 0 & 1 & 1 & 4 \end{array}\right]$

$$\Rightarrow \boxed{\begin{aligned} x &= 3z - 1 \\ y &= -z + 4. \end{aligned}}$$

pg 287

7.4.2

Case	rank	row basis	col. basis
$a=b=0$	0	n/a	n/a
$a=0, b\neq 0$	2	$\{[1\ 0],[0\ 1]\}$	$\left\{\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}0\\1\end{bmatrix}\right\}$
$a\neq 0, b=0$	2	—— " ——	—— " ——
$a=b\neq 0$	1	$\{[a\ b]\}$	$\left\{\begin{bmatrix}a\\b\end{bmatrix}\right\}$
$a,b\neq 0, a\neq b$	2	$\{[a,b],[b,a]\}$	$\left\{\begin{bmatrix}a\\b\end{bmatrix},\begin{bmatrix}b\\a\end{bmatrix}\right\}$

clearly rows are lin indep.

Symmetric matrices have the same col. and row space (after transposing vectors)

7.4.9

$$\begin{bmatrix} 9 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$R_4 - R_2 \to R_4$

$R_3 - \frac{1}{9}R_1 \to R_3$

$$\begin{bmatrix} 9 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & \frac{8}{9} & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 9 & 0 & 1 & 0 \\ 0 & 1 & \frac{8}{9} & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 - R_3 \to R_1$

$R_2 - \frac{8}{9}R_3 \to R_2$

$$\begin{bmatrix} 9 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ Rank $= 3$ and

row space has basis

$\{[1\ 0\ 0\ 0], [0\ 1\ 0\ 1], [0\ 0\ 1\ 0]\}$

and since matrix is symmetric

the col. space has basis

$\{[1\ 0\ 0\ 0]^T, [0\ 1\ 0\ 1]^T, [0\ 0\ 1\ 0]^T\}.$

7.4.12 $\operatorname{rank}(B^T A^T) = \operatorname{rank}((AB)^T)$ $(B^T A^T = (AB)^T)$

$= \operatorname{rank}(AB)$

7.4.32 This set consists of the solns to

$$\begin{bmatrix} 3 & -2 & 1 \\ 4 & 5 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (*)$$

i.e. the nullspace a matrix

$\therefore$ This set is a vector space (nullspace is vector space).

Find solns to $(*)$:

$$\begin{bmatrix} 3 & -2 & 1 \\ 4 & 5 & 0 \end{bmatrix} \xrightarrow{R_2 - \frac{4}{3}R_1 \to R_2} \begin{bmatrix} 3 & -2 & 1 \\ 0 & \frac{23}{3} & -\frac{4}{3} \end{bmatrix} \xrightarrow{3R_2 \to R_2} \begin{bmatrix} 3 & -2 & 1 \\ 0 & 23 & -4 \end{bmatrix}$$

So $\quad$ rank $= 2 \quad \Rightarrow \quad$ nullity $= 1$

i.e. $\quad \dim(\text{null space}) = 1$

$$\begin{bmatrix} 3 & -2 & 1 \\ 0 & 23 & -4 \end{bmatrix} \to \begin{bmatrix} 3 & 0 & \frac{15}{23} \\ 0 & 23 & -4 \end{bmatrix}$$

So, v_1, v_2, v_3 are s.t.

$$\begin{bmatrix} 3v_1 + \frac{15}{23} v_3 \\ 23 v_2 - 4 v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(Gaussian elim. doesn't affect sol'ns)

$$\Leftrightarrow \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -\frac{5}{23} v_3 \\ \frac{4 v_3}{23} \\ v_3 \end{bmatrix}$$

$$\therefore \boxed{\left\{ \begin{bmatrix} -\frac{5}{23} \\ \frac{4}{23} \\ 1 \end{bmatrix} \right\} \text{ is } \underline{\text{a}} \text{ suitable basis and our space is of dimension } 1.}$$