Virtual Lab 1: Liquid Level Systems

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Contents

Part A

The transfer function relating the inflow, Q_i, to the head height, H, for a liquid level system is given by:

$$\frac{H(s)}{Q_i(s)} = \frac{R}{RCs+1} \tag{1}$$

Where R is the flow resistance, C is the flow capacitance, and s is the continuous time Laplace variable. The outflow can be related to the head via:

$$q_0 = \frac{h}{R} \tag{2}$$

Converting to the Laplace domain yields:

$$Q_0(s) = \frac{H(s)}{R} \tag{3}$$

Therefore the system inflow can be related to the system outflow through the transfer function:

$$\frac{Q_0(s)}{Q_i(s)} = \frac{Q_0(s)}{H(s)}\frac{H(s)}{Q_i(s)} = \frac{1}{R}\frac{R}{RCs+1} \tag{4}$$

Evaluating gives the solution for Part A.

$$\frac{Q_0(s)}{Q_i(s)} = \frac{1}{RCs+1} \tag{5}$$

Part B

The capacitance and resistance of the flow can be found through the time constant and the application of the final value theorem for a unit step input. Applying the final value theorem to $\frac{H(s)}{Q_i(s)}$ in response to a unit step input ($\frac{1}{s}$) gives:

$$f(\infty) = \lim_{s\to 0}[sF(s)] = \lim_{s\to 0}\left[s\left(\frac{R}{RCs+1}\frac{1}{s}\right)\right] = R \tag{6}$$

The pole of the transfer function $\frac{H(s)}{Q_i(s)}$ is found to be $s = \frac{-1}{RC}$. Therefore the time constant, τ, is given by $\tau = RC$.The time constant and final value for a unit step input was found from the ODE45 tankcontrol.p simulation. The system response from the ODE45 simulation is shown below (All relevant MATLAB code shown in appendix).

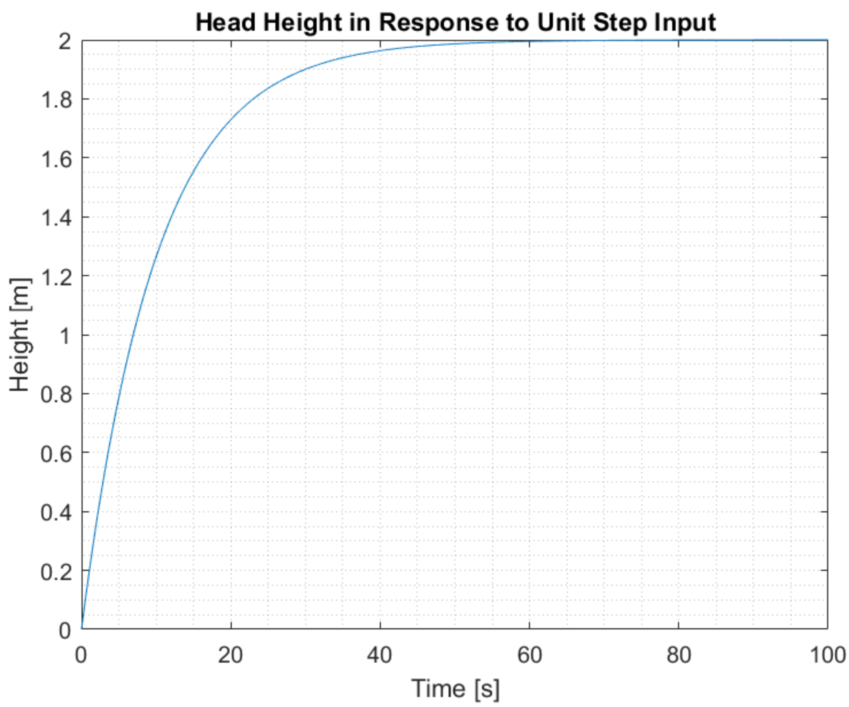

Figure 1: Tank Control.p ODE45 Simulation

From the response, the final steady state value, $f(\infty)$, was found to be 2 meters. The time for 63.2% of the final value to be reached was found to be 10 seconds, therefore $\tau = 10[s]$. With these two values now known, the solution for Part B is known. The coefficients for the resistance, R, and capacitance, C, are given by:

$$R = f(\infty) = 2 \tag{7}$$

$$C = \frac{\tau}{R} = 5 \tag{8}$$

PART C

With the coefficients of R and C determined, simulations of $\frac{Q_0(s)}{Q_i(s)}$ (equation 5) and $\frac{H(s)}{Q_i(s)}$ (equation 1) can be performed. The simulations for both transfer functions are performed using the input signal $u = e^{-t^2}\cos 3t$ for a time range of 0 to 2 seconds. The lsim function in MATLAB is used to simulate both transfer functions that came from the prior system identification, and the resulting responses are compared to the output for the tankcontrol.p ODE45 for both the height and flow outputs. The responses are shown below.

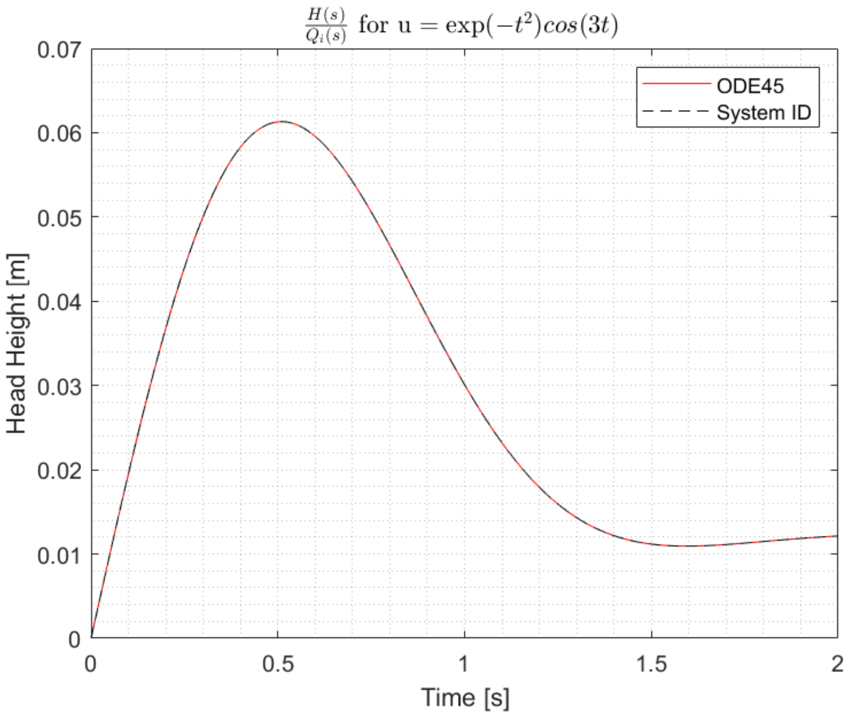

Figure 2: $\frac{H(s)}{Q_i(s)}$

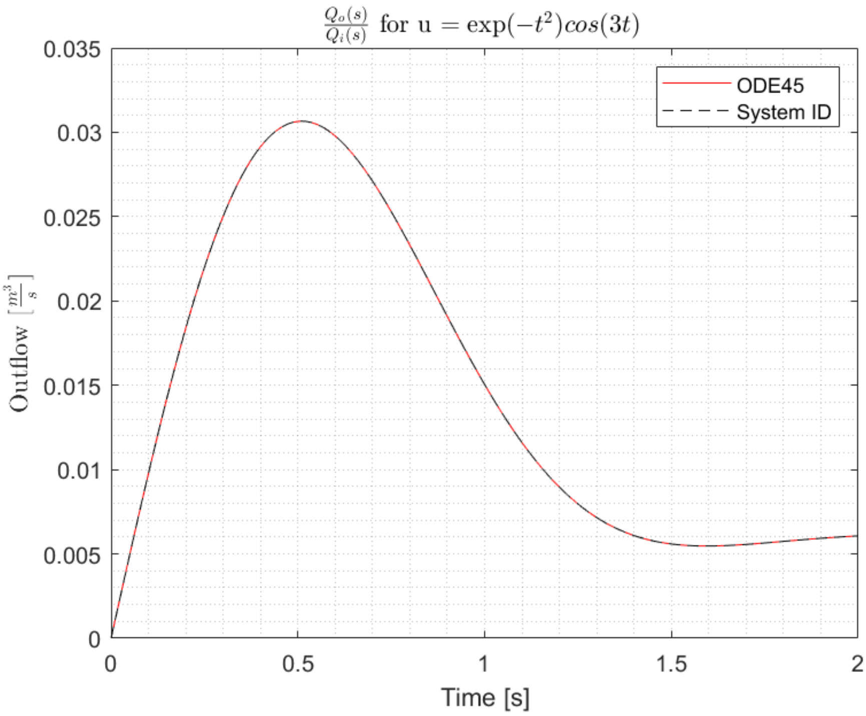

Figure 3: $\frac{Q_0(s)}{Q_i(s)}$

From the figures above, it is shown that both transfer functions generated from the system identification match up with the model used in ODE45 simulation to a very high degree of accuracy.

APPENDIX

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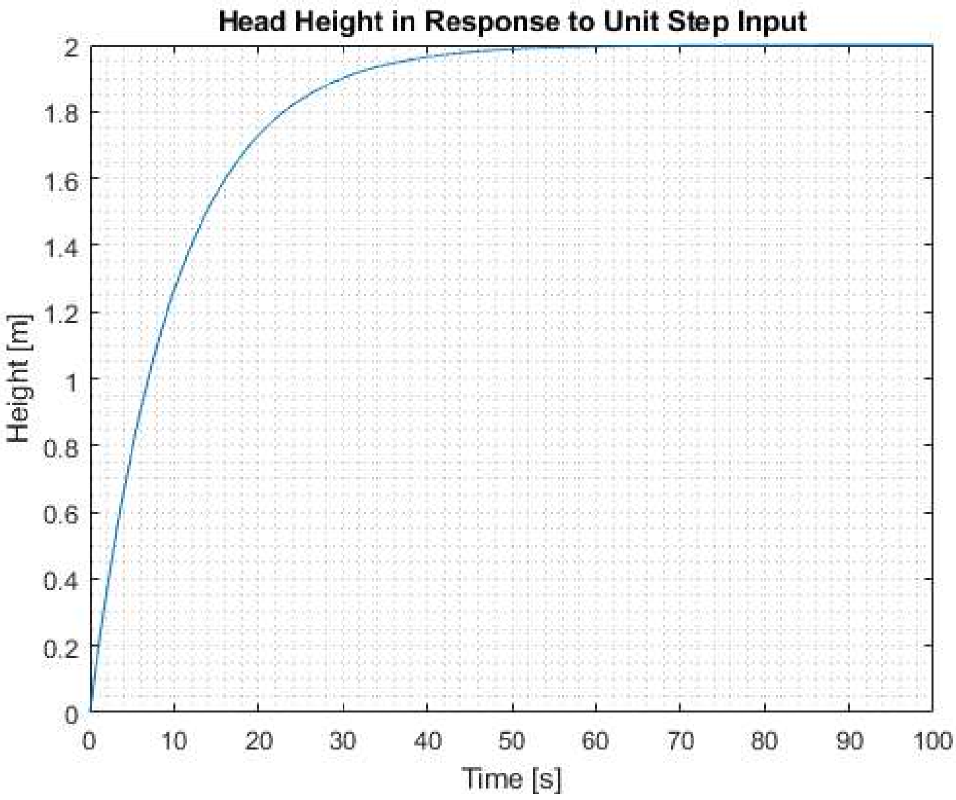
```
clear
close all
clc
```

Part B

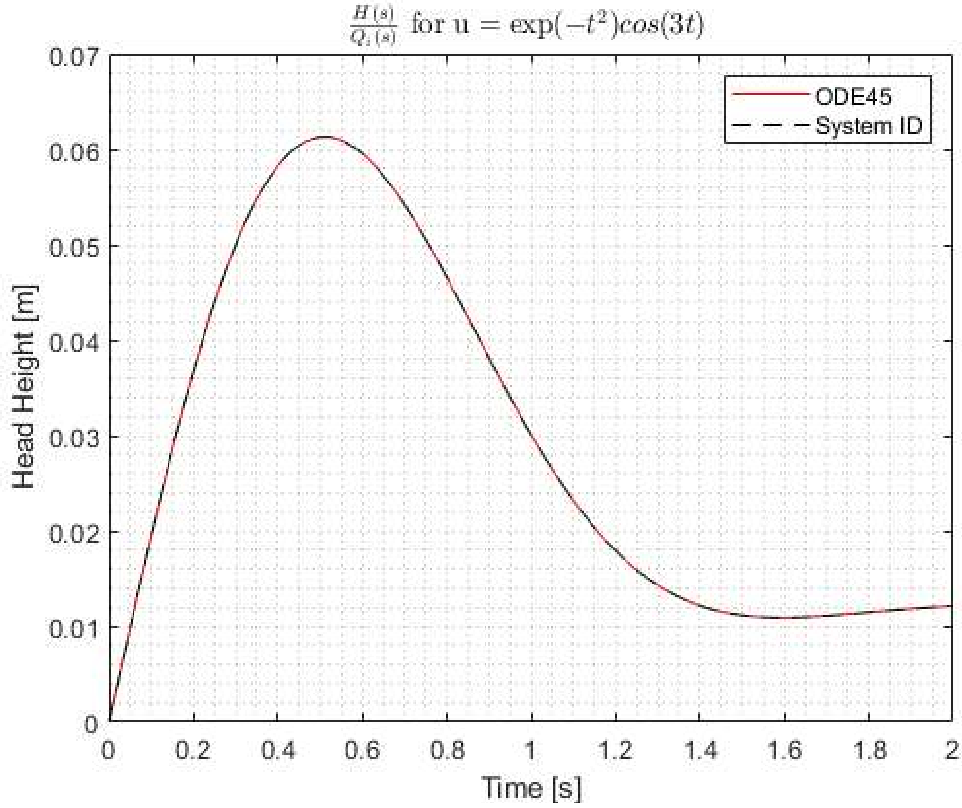
```
% Time Vector, 10 seconds
tvec                = (0:.01:100)';
% Unit Step Input
u                   = ones(length(tvec),1);

% Ode45 call for height
[t,height]          = ode45(@tank_control,tvec,[0],[],u,tvec,'height');

% Plot results
figure
plot(t,height)
xlabel('Time [s]')
ylabel('Height [m]')
grid minor
title('Head Height in Response to Unit Step Input')

% From Final Value Theorem for Unit Step Input f(infinity) = R [s/m^2]
R                   = height(end);

% Get Indices of Time Constant
ind_tau             = find(height >= 0.632*height(end),1);
% Time Costant
tau                 = t(ind_tau);

% From Calculated Time Constant, tau = RC
C                   = tau/R;

% Check with step function
s                   = tf('s');
sys                 = R/(R*C*s + 1);
opt                 = stepDataOptions;
opt.StepAmplitude   = 1;
y1                  = step(sys, tvec, opt);
```

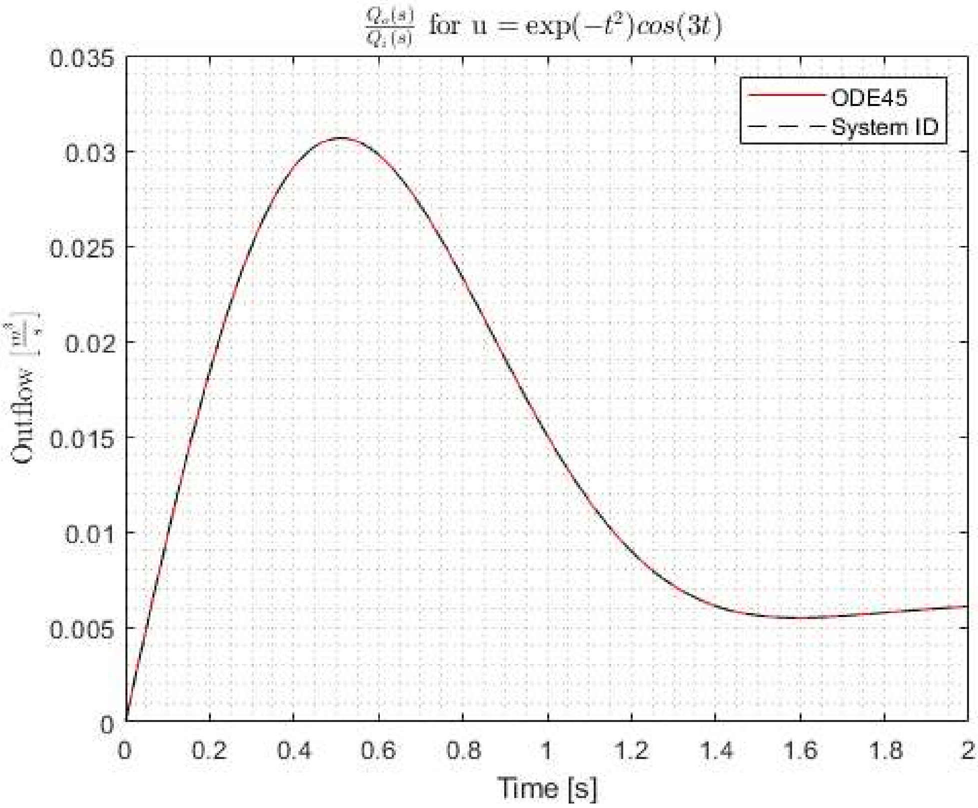


Part C

```
% Sim time
t                   = (0:.01:2)';
% nonlinear input signal
u                   = exp(-t.^2).*cos(3.*t);

% Ode45 call for height
[Th,height]         = ode45(@tank_control,t,[0],[],u,t,'height');
% Ode45 call for flow
[Tq,flow]           = ode45(@tank_control,t,[0],[],u,t,'flow');

% H(s)/Qi(s)
H_Qi                = R/(R*C*s + 1);
% Qo(s)/Qi(s)
Qo_Qi               = 1/(R*C*s + 1);

% lsim call for H/Qi
[yh, th]            = lsim(H_Qi,u,t);
% lsim call for Qo/Qi
[yq, tq]            = lsim(Qo_Qi,u,t);

% Plot Results
figure
plot(Th,height,'-r',th,yh,'--k')
xlabel('Time [s]')
ylabel('Head Height [m]')
grid minor
legend('ODE45','System ID')
title('$\frac{H(s)}{Q_i(s)}$ for u $= \exp(-t^2)cos(3t)$  ','interpreter','latex')

figure
plot(Tq,flow,'-r',tq,yq,'--k')
```

```
xlabel('Time [s]')
ylabel('Outflow [$\frac{m^3}{s}$]','interpreter','latex')
grid minor
legend('ODE45','System ID')
title('$\frac{Q_o(s)}{Q_i(s)}$ for u $= \exp(-t^2)cos(3t)$  ','interpreter','latex')
```


$\frac{H(s)}{Q_i(s)}$ for u $= \exp(-t^2)cos(3t)$
ODE45
System ID
Head Height [m]
Time [s]
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2


$\frac{Q_o(s)}{Q_i(s)}$ for u $= \exp(-t^2)cos(3t)$
ODE45
System ID
Outflow $[\frac{m^3}{s}]$
Time [s]
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2