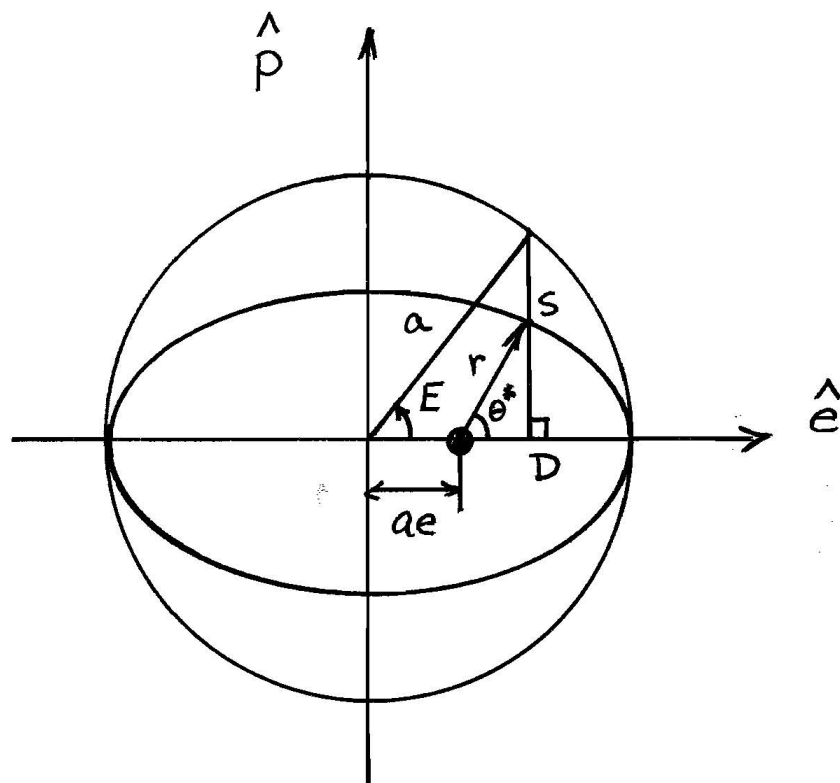


f and g Functions

E and H are most definitely useful for “time” relationships
But they are also useful in other ways.

—————→ new expressions for \bar{r}, \bar{v}

Begin with the elliptic case



$$\bar{r} = a(\cos E - e)\hat{e} + \overbrace{b \sin E}^{SD} \hat{p}$$

$$\frac{d}{dt}(M = nt = E - e \sin E) \rightarrow$$

$$\bar{v} = -\frac{a^2 n}{r} \sin E \hat{e} + \frac{abn}{r} \cos E \hat{p}$$

Evaluate \hat{e}, \hat{p} at $t = t_0$ (i.e., \bar{r}_0, \bar{v}_0)

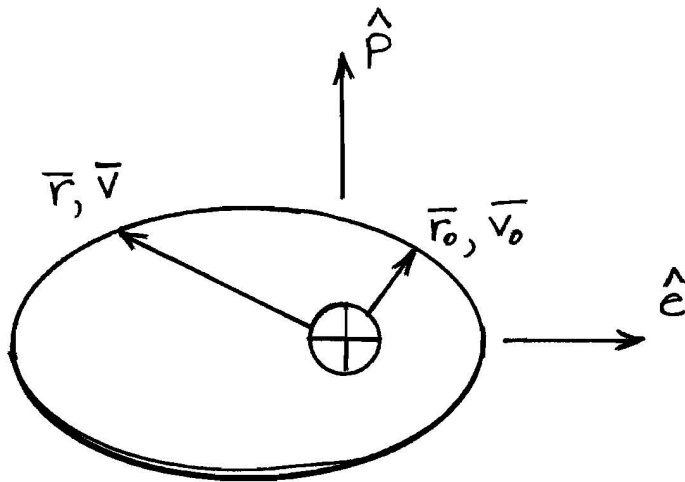
$$\hat{e} = \frac{1}{a(\cos E_0 - e)} \bar{r}_0 - \frac{b \sin E_0}{a(\cos E_0 - e)} \hat{p}$$

Substitute into \bar{v} equation

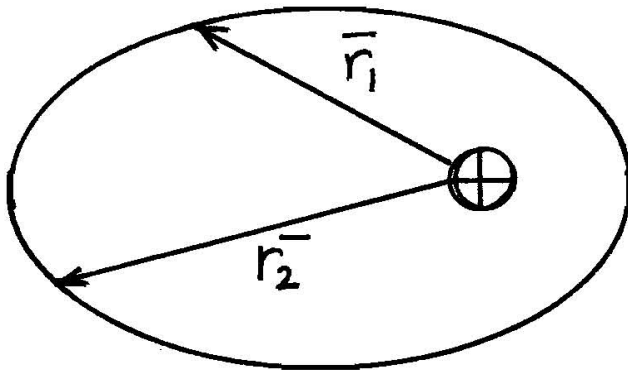
Substitute \hat{e}, \hat{p} into original expressions

$$\bar{r} = \left\{ 1 - \frac{a}{r_0} [1 - \cos(E - E_0)] \right\} \bar{r}_0 + \left\{ (t - t_0) + \left[\frac{\sin(E - E_0) - (E - E_0)}{n} \right] \right\} \bar{v}_0$$

$$\bar{v} = -\frac{na^2}{rr_0} \sin(E - E_0) \bar{r}_0 + \left\{ 1 - \frac{a}{r} [1 - \cos(E - E_0)] \right\} \bar{v}_0$$



Example:



$$\vec{r}_2 = f \vec{r}_1 + g \vec{v}_1 \quad \rightarrow$$

\rightarrow Do same in terms of θ^* ;
Do same in hyperbolic orbits

f and g Relationships

Any conic

$$\bar{r} = \left\{ 1 - \frac{r}{p} \left[1 - \cos(\theta^* - \theta_0^*) \right] \right\} \bar{r}_0 + \frac{r r_0}{\sqrt{\mu p}} \sin(\theta^* - \theta_0^*) \bar{v}_0$$

$$\bar{v} = \left\{ \frac{\bar{r}_0 \bar{v}_0}{p r_0} \left[1 - \cos(\theta^* - \theta_0^*) \right] - \frac{1}{r_0} \sqrt{\frac{\mu}{p}} \sin(\theta^* - \theta_0^*) \right\} \bar{r}_0 + \left\{ 1 - \frac{r_0}{p} \left[1 - \cos(\theta^* - \theta_0^*) \right] \right\} \bar{v}_0$$

Elliptic Orbits

$$\bar{r} = \left\{ 1 - \frac{a}{r_0} \left[1 - \cos(E - E_0) \right] \right\} \bar{r}_0 + \left\{ (t - t_0) - \sqrt{\frac{a^3}{\mu}} \left[(E - E_0) - \sin(E - E_0) \right] \right\} \bar{v}_0$$

$$\bar{v} = -\frac{\sqrt{\mu a}}{r r_0} \sin(E - E_0) \bar{r}_0 + \left\{ 1 - \frac{a}{r} \left[1 - \cos(E - E_0) \right] \right\} \bar{v}_0$$

Hyperbolic Orbits

$$\bar{r} = \left\{ 1 - \frac{|a|}{r_0} \left[\cosh(H - H_0) - 1 \right] \right\} \bar{r}_0 + \left\{ (t - t_0) - \sqrt{\frac{|a|^3}{\mu}} \left[\sinh(H - H_0) - (H - H_0) \right] \right\} \bar{v}_0$$

$$\bar{v} = -\frac{\sqrt{\mu |a|}}{r r_0} \sinh(H - H_0) \bar{r}_0 + \left\{ 1 - \frac{|a|}{r} \left[\cosh(H - H_0) - 1 \right] \right\} \bar{v}_0$$