Case Study

The objective of this case study is to construct a projection operator unknown input observer (UIO) for a continuous time (CT) system.

We design a UIO for the following model of a CT dynamical system:

$$
\begin{aligned}
\dot{x}(t) &= Ax(t) + B_1u_1(t) + B_2u_2(t) \\
&= \begin{bmatrix} 0.5 & 1 & 0 \\ 0 & 0.5 & 1 \\ 0 & 0 & 0.5 \end{bmatrix} x(t) + \begin{bmatrix} -2 \\ -3 \\ -4 \end{bmatrix} u_2(t) \\
y(t) &= Cx(t) + Dv(t) \\
&= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} v(t).
\end{aligned}
$$

Note that in the above model, we have $B_1 = O$.

Explanation: We prepared the following script to design the UIO:

```
clear all; clc;
% System data
A=[0.5 1 0;0 0.5 1;0 0 0.5];
B2=[-2 -3 -4]';
C=[1 0 0;0 1 0];
D=[1; 2];
disp('rank(obsv(A,C))')
rank(obsv(A,C))
% Dimensions
n = size(A,1);
m2=size(B2,2);
p = size(C,1);
r=size(D,2);
disp('rank(B2)')
rank(B2)
disp('rank(C*B2)')
rank(C*B2)
% Design process
%disp('M')
M=[B2 zeros(n,r)]*pinv([C*B2 D])
disp('rank(M)')
rank(M)
disp('(eye(n)-M*C)*B2')
(eye(n)-M*C)*B2
%disp('A1')
A1 = (eye(n)-M*C)*A
disp('rank(obsv(A1,C))')
rank(obsv(A1,C))
cvx_begin sdp quiet
variable P(n,n) symmetric
variable Y(n,p)
A1'*P + P*A1 - C'*Y'-Y*C +0.05*P <= 0
Y*D==0
P >= 0.01*eye(n)
cvx_end
%disp('L')
L = P\Y
% Checking if design objectives satisfied
disp('eig(A1-L*C)')
eig(A1-L*C)
disp('M*D')
M*D
disp('L*D')
L*D
```


The UIO has the form

$$\begin{aligned}\dot{z}(t) &= (I_n - MC)(Az(t) + AMy(t) + B_1u_1(t)) \\ &\qquad + L(y(t) - \hat{y}(t)) \\ \hat{x}(t) &= z(t) + My(t),\end{aligned}$$

where

$$M = \begin{bmatrix} 4.0000 & -2.0000 \\ 6.0000 & -3.0000 \\ 8.0000 & -4.0000 \end{bmatrix} \text{ and } L = \begin{bmatrix} 0.0082 & -0.0041 \\ -1.0041 & 0.5020 \\ -0.8000 & 0.4000 \end{bmatrix}.$$

The poles of the UIO are located in the open-left half complex plane at $\{-0.5102,\ -0.5000 + 2.6458i\ -0.5000 - 2.6458i\}$.

We also satisfy the conditions:

$$(I_3 - MC)B_2 = O, \ \ MD = O, \ \ \text{and} \ \ LD = O.$$

Therefore, the observation error dynamics are governed by the differential equation,

$$\dot{e}(t) = (A_1 - LC)e(t).$$
