#1) $\begin{pmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 3 & 7 & k \end{pmatrix} = A$

$|A| = 1\begin{vmatrix} -1 & 3 \\ 7 & k \end{vmatrix} - 2\begin{vmatrix} 2 & 3 \\ 3 & k \end{vmatrix} - 1\begin{vmatrix} 2 & -1 \\ 3 & 7 \end{vmatrix} = 0$

$= (-k-21) - (2)(2k-9) - (14+3) = 0$

$= -5k - 21 + 18 - 17 = 20/_{-5} = -4$

$\boxed{D, -4}$

#2) $A = \begin{pmatrix} 1 & -3 & 1 & 4 \\ 2 & 4 & -2 & 6 \\ -3 & -21 & 9 & -6 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & -3 & 1 & 4 \\ 2 & 4 & -2 & 6 \\ 0 & -30 & 12 & 6 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 1 & 3 & 1 & 4 \\ 0 & -2 & -4 & -2 \\ 0 & 30 & 12 & 6 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 3 & 1 & 4 \\ 0 & -2 & -4 & -2 \\ 0 & 0 & \boxed{-48} & X \end{pmatrix}$

rank = 3, dim(Col(A)) = 3, rank + Null = 4

3 + Null = 4

Null = 1

$\therefore$ $\boxed{\text{A, dimension of column space is 3}}$

#3) λ's of symmetric matrix are real.

$A = A^T = \begin{pmatrix} a & b & c \\ b & a & d \\ c & d & a \end{pmatrix}$ iii) is true

$A = \begin{pmatrix} a & k \\ -k & a \end{pmatrix}$, $A^T = \begin{pmatrix} a & -k \\ k & a \end{pmatrix}$. $-A = \begin{pmatrix} -a & -k \\ k & -a \end{pmatrix}$

iv) is false , i) is false

$\boxed{\text{B, (ii) \& (iii) are true}}$

#4) $C = \begin{pmatrix} 1 & 4 & 5 \\ 0 & -2 & 3 \\ 0 & 0 & -2 \end{pmatrix}$

C is triangular matrix $\therefore \lambda(C) = \text{diag}(C)$

$\lambda_2 = \lambda_3 = -2$, non-distinct eigenvalues .

$\therefore$ $\boxed{\text{C is Not diagonalizable}}$

#5) C is subspace , $x^2 + y^2 + z^2 = 4$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix}^T \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 4$

$[-4, 2, 13]^T \begin{bmatrix} 5 \\ 0 \\ 3 \end{bmatrix} = -20 + 39 = 19 \neq 0$ $\therefore$ A is vector space

$\boxed{\text{A \& C}}$

#6) $y' = \begin{pmatrix} 2 & k \\ 1 & 4 \end{pmatrix} y$

$\lambda^2 - 6\lambda + (8-k)$, for unstable $\lambda > 0$

$\frac{6 \pm \sqrt{36-32+4k}}{2} = 3 \pm \frac{\sqrt{4(1+k)}}{2} = 3 \pm \sqrt{1+k} = c$

$3 - \sqrt{1+k} > 0 \Rightarrow -\sqrt{1+k} > -3$

$\sqrt{1+k} > -3^2$

$3 + \sqrt{1+k} > 0$ for all k

$\boxed{\text{E, every real \#}}$

#7) $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \Rightarrow \lambda^2 - 4\lambda + 5 \Rightarrow \frac{4 \pm \sqrt{16-(4)(20)}}{2}$ = spiral

$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \Rightarrow \lambda^2 - 5\lambda - 2 \Rightarrow \frac{5 \pm \sqrt{25+8}}{2}$ = diff sign

$\begin{pmatrix} 2 & -1 \\ 5 & 4 \end{pmatrix} \Rightarrow \lambda^2 - 6\lambda + 13 \Rightarrow \frac{6 \pm \sqrt{36-(4)(13)}}{2}$ = spiral

$\begin{pmatrix} 2 & 4 \\ -5 & -2 \end{pmatrix} \Rightarrow \lambda^2 + (-4+20) \Rightarrow \lambda = \pm 4i$ (center)

$\boxed{A}$

#8) $X' = \begin{pmatrix} 2 & -1 \\ 4 & 6 \end{pmatrix} X$

$\lambda^2 - 8\lambda + 16 = 0 \Rightarrow (\lambda - 4)^2 = 0$

$\lambda_1 = \lambda_2 = 4$

$Av = \lambda v$

$\begin{pmatrix} 2 & -1 \\ 4 & 6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 4x_1 \\ 4x_2 \end{pmatrix}$

$2x_1 - x_2 = 4x_1 \Rightarrow -x_2 = 2x_1$

Let $x_1 = 1 \therefore x_2 = -2$

$x(t) = c_1 v_1 e^{\lambda t} + c_2 (v_1 t + v_2) e^{\lambda t}$

$$\boxed{x(t) = \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{4t} c_1 + c_2 \left[\begin{pmatrix} 1 \\ -2 \end{pmatrix} t + \begin{pmatrix} 0 \\ -1 \end{pmatrix}\right] e^{4t}}$$

$\boxed{A}$

$\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ is linearly independent of $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$

$y_1' = 2y_1 + y_1 y_2$
$y_2' = 2\sin(y_1) + 3y_2$

#9) $\frac{\partial y_1'}{\partial y_1} = 2 + y_2 \Big|_{(0,0)} = 2$

$\frac{\partial y_1'}{\partial y_2} = y_1 \Big|_{(0,0)} = 0$

$\frac{\partial y_2'}{\partial y_1} = 2\cos(y_1) \Big|_{(0,0)} = 2$

$\frac{\partial y_2'}{\partial y_2} = 3$

$A = \begin{pmatrix} 2 & 0 \\ 2 & 3 \end{pmatrix}$ $|A - \lambda I| = \lambda^2 - 5\lambda + 6 = 0$

$(\lambda - 2)(\lambda - 3) = 0$

$\lambda_1 = 2, \ \lambda_2 = 3 > 0 \ \therefore$ $\boxed{\text{Unstable Node B}}$

#10) $f(t) = \begin{cases} e^t & 0 < t < 5 \\ 4 & t \geq 5 \end{cases}$

$f(t) = e^t [u(t) - u(t-5)] + 4u(t-5)$

$\mathcal{L}[e^t] = \frac{1}{s-1}$

$e^{(t-5)} = e^t e^{-5}$

$\mathcal{L}[e^{(t-5)} u(t-5)] = e^{-5} \frac{1}{s-1} e^{-5s}$

$\mathcal{L}[4u(t-5)] = \frac{4}{s}e^{-5s}$

$F(s) = \frac{4}{s}e^{-5s} + \frac{1}{s-1} - \frac{e^{-5}e^{-5s}}{s-1}$

$= \frac{1}{s-1} + e^{-5s}\left(\frac{4}{s} - \frac{e^{5}}{s-1}\right)$

$\boxed{B}$

11) $y - 2(y * \cos(t)) = \sin(t)$

$Y(s) - 2\left[Y(s)\frac{s}{s^2+1}\right] = \frac{1}{s^2+1}$

$Y(s) - \frac{2Y(s)s}{s^2+1} = \frac{1}{s^2+1}$

$Y(s)\left[1 - \frac{2s}{s^2+1}\right] = \left(\frac{s^2+1-2s}{s^2+1}\right)Y(s) = \frac{1}{s^2+1}$

$Y(s) = \frac{1}{s^2-2s+1} = \frac{1}{(s-1)^2}$ (repeated)

$\mathcal{L}^{-1}\left(\frac{1}{s-1}\right) = e^{t}$

$\boxed{C, \ y(t) = te^{t}}$

12) $y'' - 6y' + 5y = \delta(t-2)$

$Y(s)(s^2 - 6s + 5) = e^{-2s}$

$Y(s) = \dfrac{e^{-2s}}{s^2 - 6s + 5} = \left(\dfrac{1}{(s-1)(s-5)} e^{-2s}\right)$

$\dfrac{1}{(s-1)(s-5)} = \dfrac{A}{s-1} + \dfrac{B}{s-5}$

$0 = A + B \qquad A = -B$

$1 = -5A - B = -4A \qquad A = -1/4, \quad B = 1/4$

$Y(s) = \dfrac{-e^{-2s}}{4(s-1)} + \dfrac{e^{-2s}}{4(s-5)}$

$\tau = t-2$

$y(t) = \dfrac{u(t-2)}{4}\left(-e^{\tau} + e^{5\tau}\right) = \dfrac{u(t-2)}{4}\left(-e^{t-2} + e^{5t-10}\right)$

$\boxed{B} \quad \boxed{y(t) = \dfrac{u(t-2)}{4}\left(-e^{t-2} + e^{5t-10}\right)}$

#13) $F = \mathcal{L}[\sin^2(t)]$

$G = F' = -\mathcal{L}[t \sin^2 t]$

$g = \mathcal{L}^{-1}[G] = -\mathcal{L}^{-1}\mathcal{L}[t \sin^2 t]$

$\therefore \; g = -t \sin^2 t$

$g = -\frac{\pi}{3} \sin\left(\frac{\pi}{3}\right)^2 \qquad \sin\left(\pi/3\right) = 1/2$

$= \left(-\pi/3\right)\left(1/4\right) = -\pi/12$

$\boxed{C, -\pi/12}$

#14) A, $\frac{d}{dt}(3^t)$ is not defined contiously

$\boxed{A}$

#15) $\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -1 - 1 = -2$

$\boxed{\text{C is Not orthogonal}}$