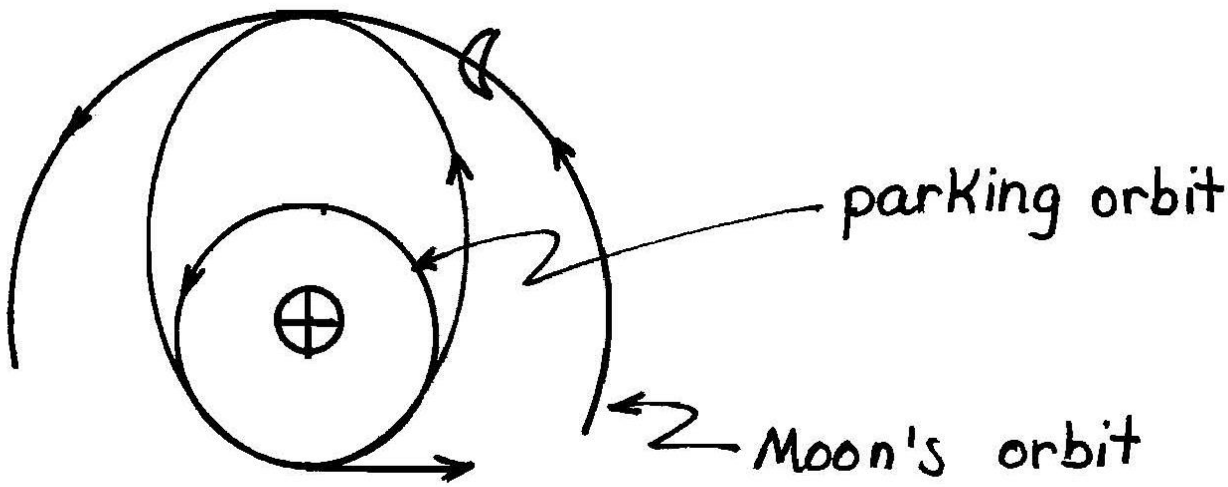
Example: Free-Return Trajectories

Consider circumlunar trajectories
Assume

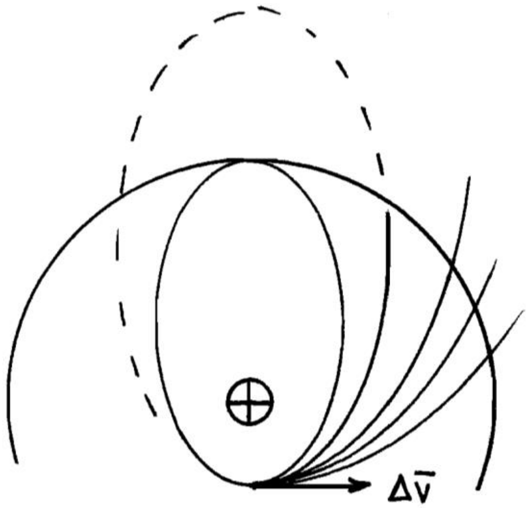
Note: patched-conic approach less accurate in this problem than for interplanetary



If ☾ has no gravity

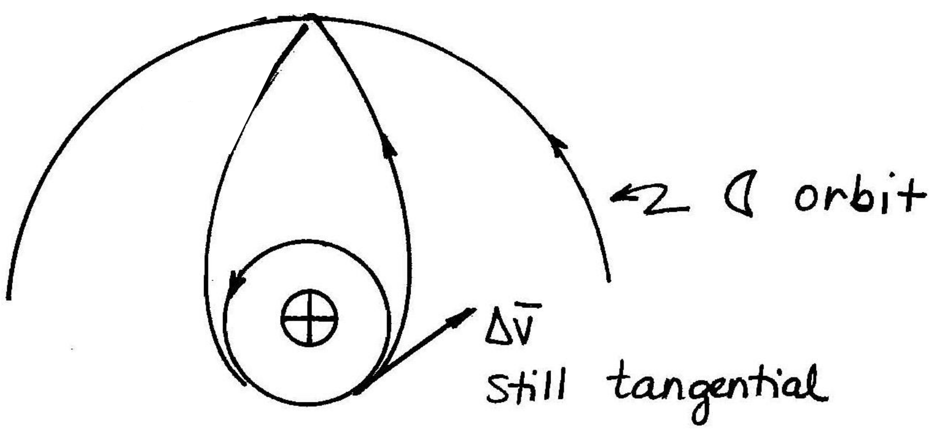
1. Jump to ellipse from parking orbit
2. At ☾, with no $\Delta\bar{v}$, remains on transfer ellipse
3. Returns to ⊕ at radius of parking orbit

☾ <u>does</u> possess gravity



Consider:

1. make transfer ellipse larger
2. $\Delta\bar{v}$ still tangential (most efficient)
3. apogee $\geq r_☾$
4. reach ☾ sooner at different angle

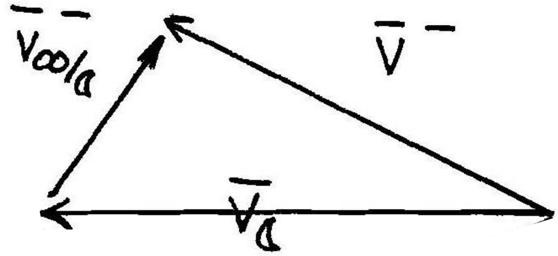


If pass ☾ such that

$$\boxed{\begin{aligned} v_r^+ &= -v_r^- \\ v_\theta^+ &= +v_\theta^- \end{aligned}}$$

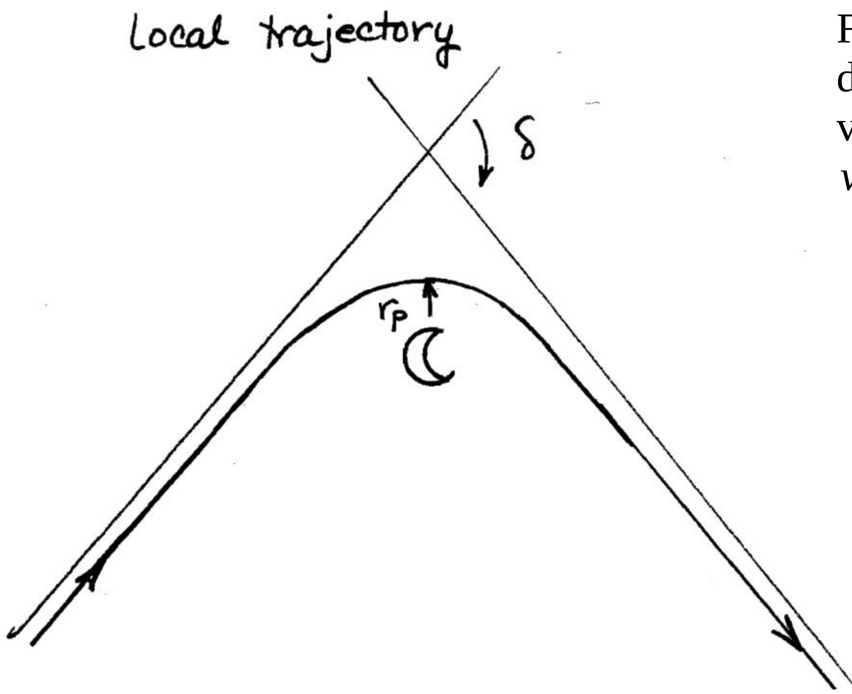
→ end up on same trajectory for ⊕ return

Vector Diagram



Same relative velocity equation:

$$\bar{v}^{-} = \bar{v}_{\infty/☾}^{-} + \bar{v}_{☾}$$



Flyby angle determined by value of v_{∞} and r_p

Notes:

1. Early Apollo flights $\longrightarrow$ free-return
   Typical 3-day outbound leg
   Pass ahead of ☾ so s/c could enter 3-day return leg if failure occurs
   Apollo 11 (for landing) 3 day out; 2.5 day return (if insertion did not occur would not return to vicinity of Earth)

2. Apollo 17 altered its initial free-return translunar trajectory to get a more precise landing
   $r_{p/☾} = 1849\,(111\,\text{altitude})$ / passed ahead

   Entered lunar orbit

3. Apollo 13
   Had made a mid-course correction to leave free-return path before experiencing failure that aborted ☾ landing
   Lunar module engines used after explosion to modify trajectory and return to ⊕

   Apollo 13 → same trajectory / failure enroute to ☾ / lunar module descent engines did fire
   Lunar approach: $v_r = +.244 \quad v_\theta = 0$