AAE 666

Homework Eight

Exercise 1 What are the positive limit sets of the following solutions?

(a) $x(t) = \sin(t^2)$

(b) $x(t) = e^t \sin(t)$

Exercise 2 Using LaSalle's Theorem, show that all solutions of the system

$$\begin{aligned} \dot{x}_1 &= x_2^2 \\ \dot{x}_2 &= -x_1 x_2 \end{aligned}$$

must approach the x_1 axis.

Exercise 3 Consider the scalar nonlinear mechanical system

$$\ddot{q} + c(\dot{q}) + k(q) = 0$$

If the term $-c(\dot{q})$ is due to damping forces it is reasonable to assume that $c(0) = 0$ and

$$c(\dot{q})\dot{q} > 0 \qquad \text{for all} \quad \dot{q} \neq 0$$

Suppose the term $-k(q)$ is due to conservative forces and define the potential energy by

$$P(q) = \int_0^q k(\eta)\, d\eta$$

Show that if $\lim_{q\to\infty} P(q) = \infty$, then all motions of this system must approach one of its equilibrium positions.

Exercise 4 Consider a nonlinear mechanical system described by

$$m\ddot{q} + c\dot{q} + k(q) = 0$$

where q is scalar, $m, c > 0$ and k is a continuous function which satisfies

$$\begin{aligned} k(0) &= 0 \\ k(q)q &> 0 \qquad \text{for all } q \neq 0 \\ \lim_{q\to\infty} \int_0^q k(\eta)\, d\eta &= \infty \end{aligned}$$

(a) Obtain a state space description of this system.

(b) Prove that the state space model is GAS about the state corresponding to the equilibrium position $q = 0$.

   (i) Use a La Salle type result.

   (ii) Do not use a La Salle type result.

Exercise 5 Consider an inverted pendulum $\mathcal{B}$ (or one link manipulator) subject to a control torque u. This system can be described by

$$\ddot{q} - a \sin q = bu$$

where q is the angle between the pendulum and a vertical line, $a = mgl/I$, $b = 1/I$,m is the mass of $\mathcal{B}$, I is the moment of inertia of $\mathcal{B}$ about its axis of rotation through O, l is the distance between O and the mass center of $\mathcal{B}$, and g is the gravitational acceleration constant of YFHB. We wish to stabilize this system about the position corresponding to $q = 0$ by a linear feedback controller of the form

$$u = -k_p q - k_d \dot{q}$$

Using the results of the last problem, obtain the least restrictive conditions on the controller gains k_p, k_d which assure that the closed loop system is GAS about the state corresponding to $q(t) \equiv 0$. Illustrate your results with numerical simulations.

Exercise 6 Consider the system described by

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 - x_2 + \theta \sin x_1 + u\end{aligned}$$

with control input u where θ is an unknown constant parameter. Obtain an adaptive feedback controller which guarantees that, for any initial conditions, $\lim_{t\to\infty} x(t) = 0$ and $u(\cdot)$ is bounded. (Hint: As a candidate Lyapunov function for the closed loop system, consider something of the form $V(x) + U(\hat{\theta} - \theta)$ where V is a Lyapunov function for the nominal uncontrolled nominal linear system.) Illustrate the effectiveness of your controller with simulations.