Contents


```
clear
close all
clc
```

Problem 1

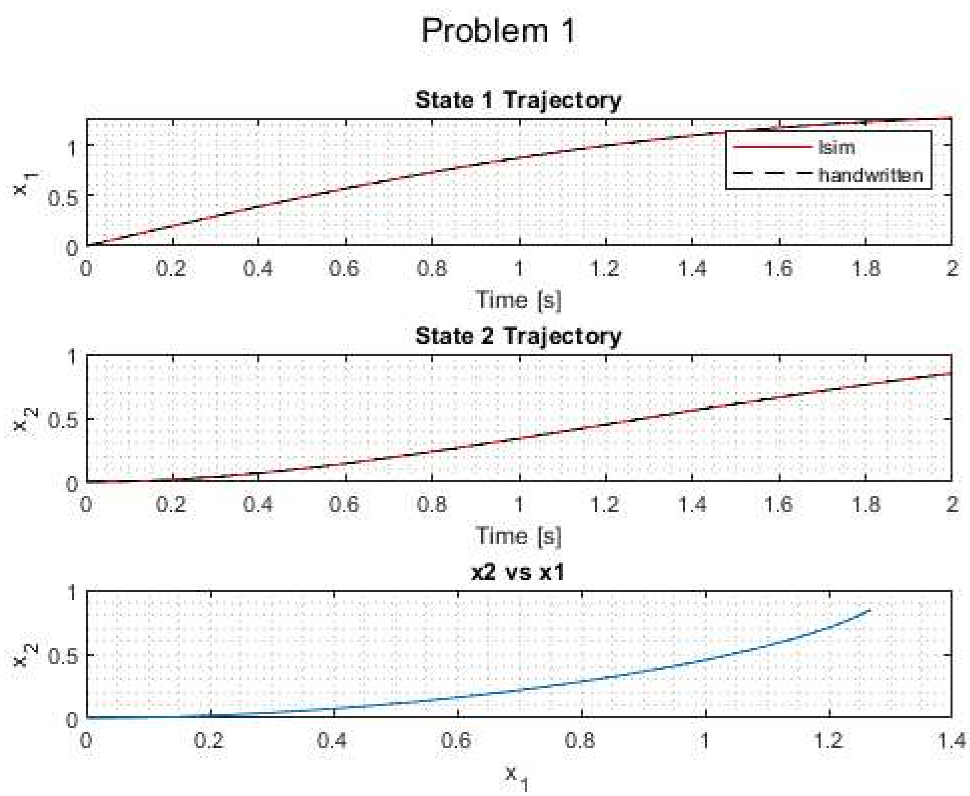
State Space Matrices

```
A       = [0 -1; 1 -1];
B       = [1; 0];
C       = eye(2);               % Output both states
D       = zeros(2,1);

t       = (0:.005:2)';          % time vector
x0      = zeros(2,1);           % Initial contiosn x(0) = 0
u       = ones(length(t),1);    % Step Input

sys     = ss(A,B,C,D);          % System state space object

[~,T,X] = lsim(sys,u,t,x0);     % lsim output
```

Hand Written Solutions for State Trajectories

```
x1      = exp(-t/2).*(-cos((sqrt(3)*t)/2) + (1/sqrt(3))*sin((sqrt(3)*t)/2)) + 1;
x2      = 1 - exp(-t/2).*(1/sqrt(3) * sin((sqrt(3)*t)/2) + cos((sqrt(3)*t)/2));

figure
subplot(3,1,1)
plot(T,X(:,1),'-r',t,x1,'--k')
xlabel('Time [s]')
title('State 1 Trajectory')
legend('lsim','handwritten')
grid minor
ylabel('x_1')
subplot(3,1,2)
plot(T,X(:,2),'-r',t,x2,'--k')
title('State 2 Trajectory')
xlabel('Time [s]')
grid minor
ylabel('x_2')
subplot(3,1,3)
plot(x1,x2)
title('x2 vs x1')
grid minor
xlabel('x_1')
```

```
ylabel('x_2')
sgtitle('Problem 1')
```



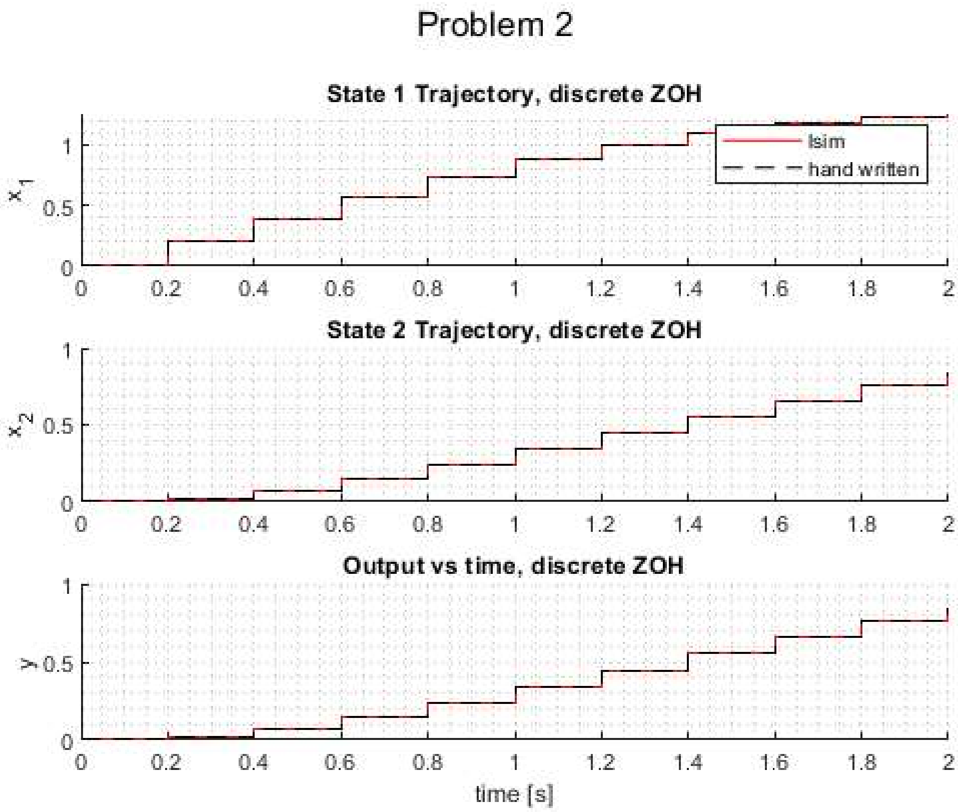
Problem 2

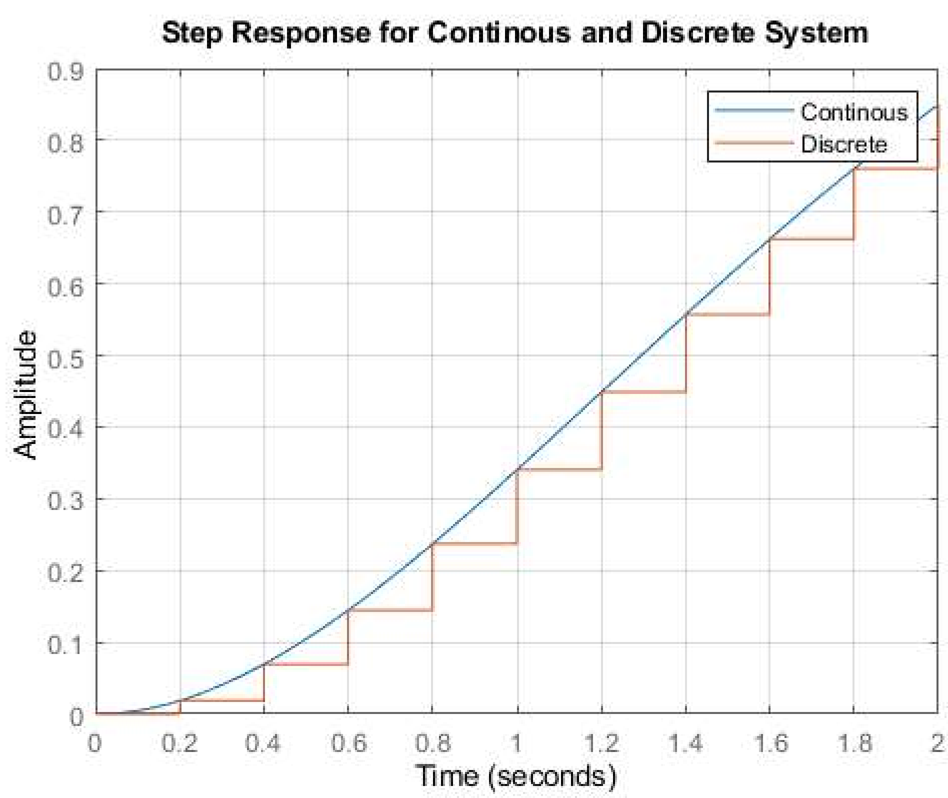
```
Ts       = 0.2;  % step size [s]

disp('Hand Written Discrete State Space Model')

% Hand Written discrete A matrix
Ad       = exp(-.2/2)*[cos(.2*sqrt(3)/2)+ (1/sqrt(3)*sin(.2*sqrt(3)/2)),...
           -2*sin(.2*sqrt(3)/2)/sqrt(3);2*sin(.2*sqrt(3)/2)/sqrt(3),...
           cos(.2*sqrt(3)/2)- (1/sqrt(3)*sin(.2*sqrt(3)/2))]

% Hand Written discrete B matrix
Bd       = [exp(-.2/2)*(-cos(.2*sqrt(3)/2) + sin(.2*sqrt(3)/2)/sqrt(3)) + 1;...
           -exp(-.2/2)*(sin(.2*sqrt(3)/2)/sqrt(3) + cos(.2*sqrt(3)/2)) + 1]

% Hand Written discrete C matrix
Cd       = [0 1]

% Hand Written discrete D matrix
Dd       = 0

% Continous time ss object
sysc     = ss(A,B,[0 1],0);

disp('c2d Discrete State Space Model')
sysd             = c2d(sysc, Ts, 'zoh')          % c2d discrete ss object

time             = (0:Ts:2)';                    % Time Vector
K                = round(time/Ts);               % Integer Steps
```

```
x               = zeros(2,length(K));           % Initialize State Vector
x(:,1)          = x0;
u               = ones(1,length(K));            % Step Input
zeroStateSum    = 0;
y               = zeros(1,length(K));

% x[k] = A^k*x0 + sum(Ad^(k-1-i))*B*u[i] + D*u[k]
for count = 2:length(K)
    k               = K(count);
    zeroStateSum    = Ad^(k - 1)*Bd*u(k) + zeroStateSum;
    x(:,count)      = (Ad^k)*x0 + zeroStateSum;
    y(:,count)      = Cd*x(:,count);
end

% Discrete lsim output
[Y,T,X]     = lsim(sysd,u,time,x0);

figure
subplot(3,1,1)
hold on
stairs(T,X(:,1),'-r')
stairs(time,x(1,:),'--k')
hold off
grid minor
title('State 1 Trajectory, discrete ZOH')
ylabel('x_1')
legend('lsim','hand written')
subplot(3,1,2)
hold on
stairs(T,X(:,2),'-r')
stairs(time,x(2,:),'--k')
hold off
grid minor
title('State 2 Trajectory, discrete ZOH')
ylabel('x_2')
subplot(3,1,3)
hold on
stairs(T,Y,'-r')
stairs(time,y,'--k')
hold off
grid minor
title('Output vs time, discrete ZOH')
ylabel('y')
xlabel('time [s]')
sgtitle('Problem 2')

figure
hold on
step(sysc,2)
step(sysd,2)
grid minor
title('Step Response for Continous and Discrete System')
legend('Continous','Discrete')
```

```
Hand Written Discrete State Space Model

Ad =

    0.9813   -0.1801
```

```
    0.1801    0.8013


Bd =

    0.1987
    0.0187


Cd =

     0     1


Dd =

     0

c2d Discrete State Space Model

sysd =

  A =
            x1       x2
   x1   0.9813  -0.1801
   x2   0.1801   0.8013

  B =
            u1
   x1   0.1987
   x2  0.01867

  C =
       x1  x2
   y1   0   1

  D =
       u1
   y1   0

Sample time: 0.2 seconds
Discrete-time state-space model.
```





Problem 5

```
% Continous Time LTI A Matrix
A_CT        = [-2 0 0; 1 0 1; 0 -2 -2];
```

```
% Symmetric Positive Definite Matrix
Q           = eye(size(A_CT));

% Solve Lyapunov Equation for P
P_CT         = lyap(A_CT',Q)

% Check Eigenvalues of P for Positive Definite
lambda_CT    = eig(P_CT)

if (lambda_CT > 0)
    disp('P is positive definite, Continous LTI System is asymptotically stable')
else
    disp('P is not positive definite, Continous LTI System is not asymptotically stable')
end
```

```
P_CT =

    0.4750    0.4500    0.1750
    0.4500    1.2500    0.2500
    0.1750    0.2500    0.3750


lambda_CT =

    0.2279
    0.3379
    1.5342

P is positive definite, Continous LTI System is asymptotically stable
```

Problem 6

```
% Discrte Time LTI A matrix
A_DT        = [-.8 0 0; .4 0 .4; 0 -.8 -.8];

% Symmetric Positive Definite Matrix
Q           = eye(size(A_DT));

% Solve Discrete Lyapunov Equation for P
P_DT        = dlyap(A_DT',Q)

% Check Eigenvalues of P for Positive Definite
lambda_DT    = eig(P_DT)

if (lambda_DT > 0)
    disp('P is positive definite, Discrete LTI System is asymptotically stable')
else
    disp('P is not positive definite, Discrete LTI System is not asymptotically stable')
end
```

```
P_DT =

    4.0571   -0.1428    0.2721
   -0.1428    2.3073    0.9904
    0.2721    0.9904    2.0426
```

```
lambda_DT =

    1.1451
    3.1686
    4.0933

P is positive definite, Discrete LTI System is asymptotically stable
```
