Celestial Mechanics and Astrodynamics

Formal Astronomy

Phenomena apart from causes:

→ Divisions of time

→ Constellations

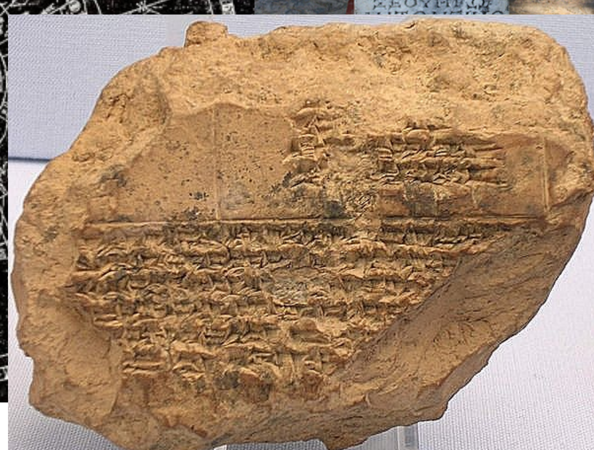
→ Planets

Dynamical Astronomy

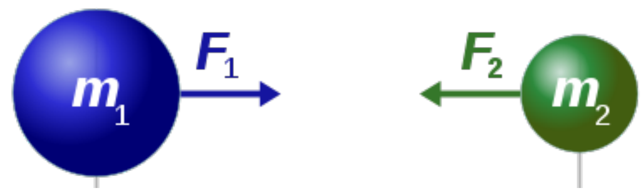
Physical aspects → natural phenomena

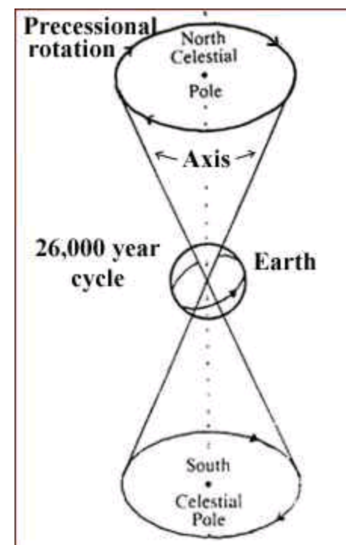
Fundamental properties →

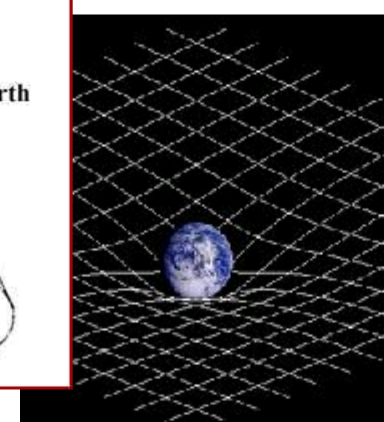
force, matter, space, time



Babylonian Tablet: eclipses 518 - 465







I. Formal Astronomy (phenomena apart from its causes)

1. Ancient Astronomers

- Noted natural divisions of time (day, month, year)
- Careful observations
- Time relationships – considerable accuracy
    - Babylonia
    - Egypt
    - Poor records

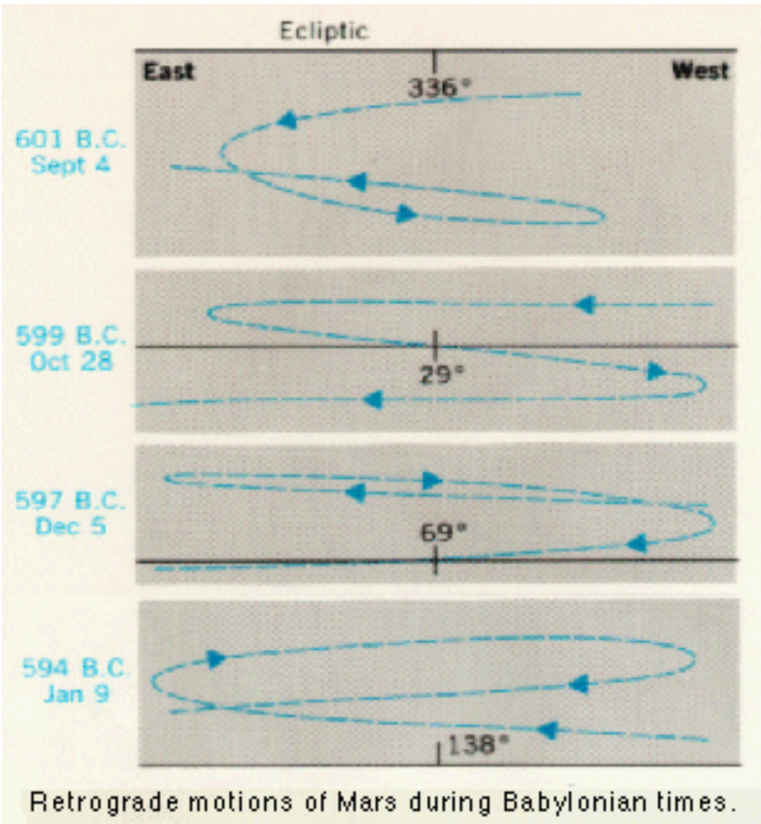
240 BC
Chinese astronomers
→ first confirmed perihelion passage of Halley's comet

Ancient map of the stars – appear as flat screen circling world

I. Formal Astronomy (phenomena apart from its causes)

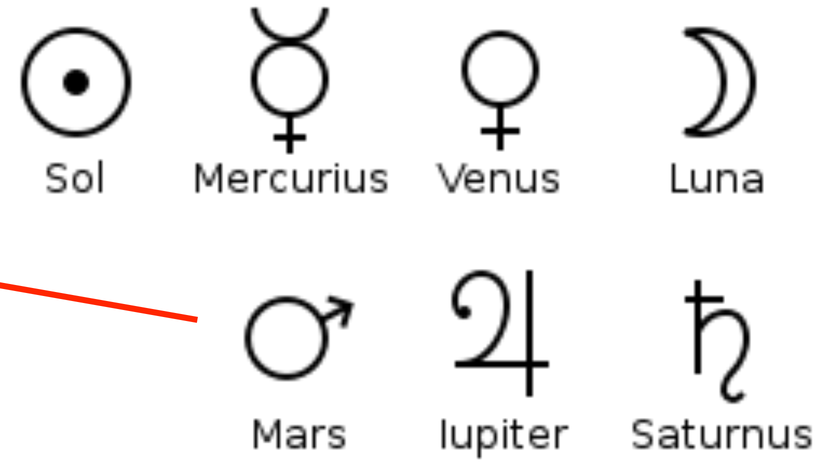
1. Ancient Astronomers

Wanderer



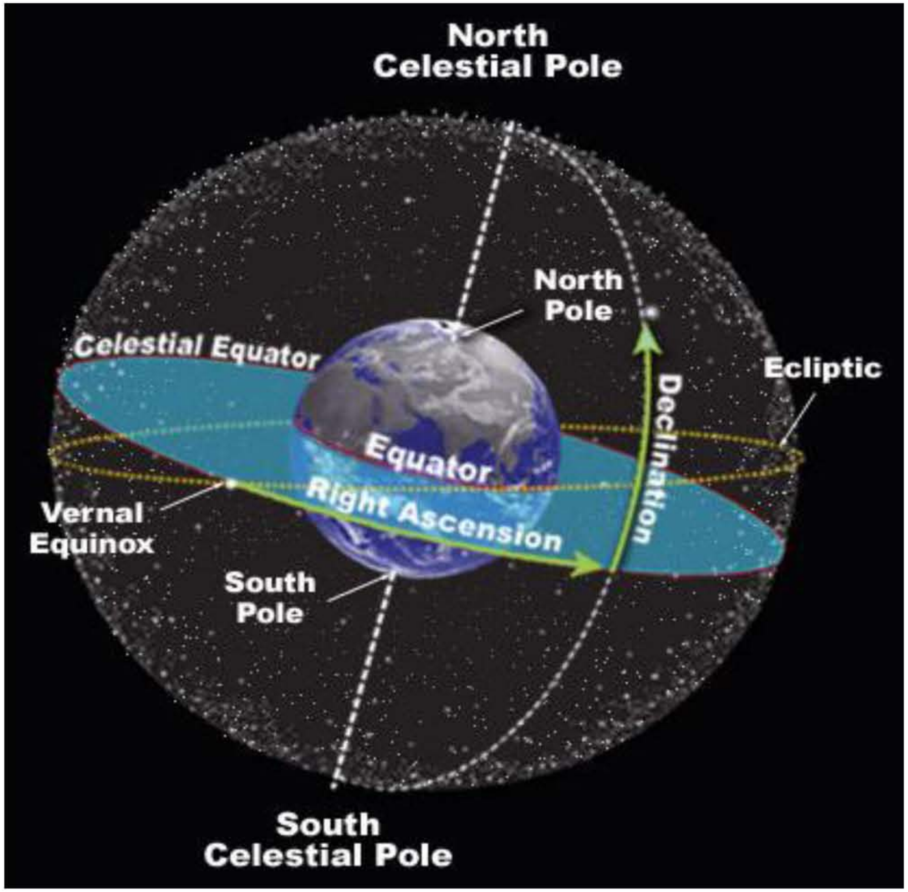
Retrograde motions of Mars during Babylonian times.

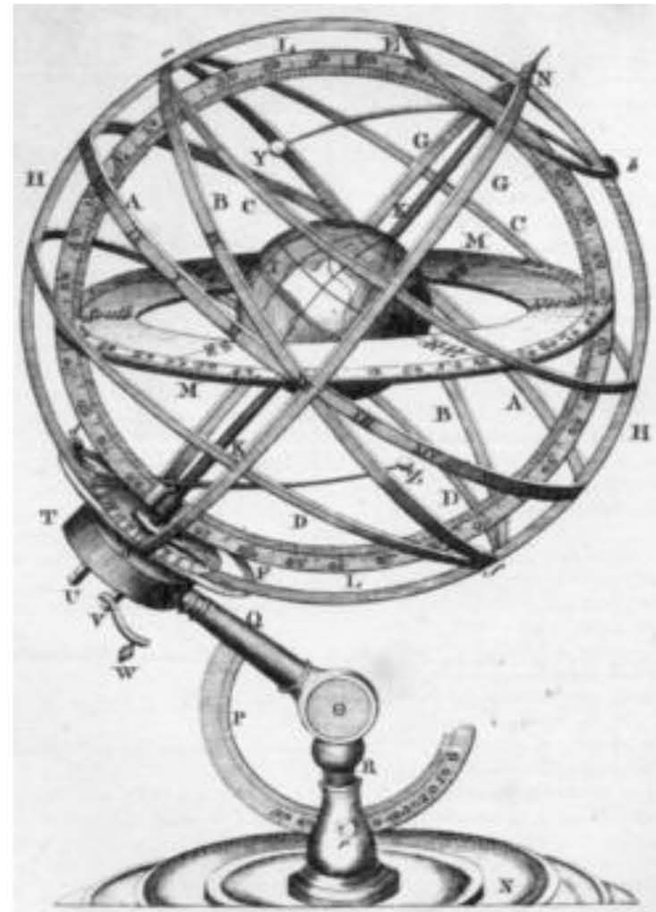
The 7 Planets of the Ancients



Ideas pursued by Greeks → First to think "**geometrically**"

Celestial Sphere





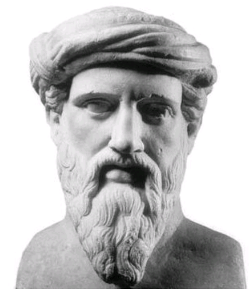
Imaginary sphere
- Arbitrarily large radius
- Concentric with Earth
- Rotates upon the same axis
- All objects projected upon celestial sphere

2. Thales (640 -546 BC) of Miletus in Asia

equator in different plane

Went to Egypt for instruction
Taught sphericity of Earth, obliquity of ecliptic
Determined length of year
Predicted solar eclipse of 585 BC

3. Pythagoras (~ 569-475 BC) Samos (off coast of Turkey)
Traveled in Egypt and Chaldea
Primarily a philosopher – no mathematical writings
Founded School of Astronomy and Philosophy in Sicily
Taught: Earth rotates; comets/planets in circular orbits



4. Meton (~ 465-385 BC)
Greek mathematician, astronomer, geometer, and engineer; lived in Athens
Accurate astronomical observations
Noted in 432 BC: the 19-year Metonic cycle → Earth axis nutates

$$19\,\text{years} = 19\left(\frac{365.2425\,\text{days}}{\text{year}}\right)\left(\frac{\text{lunar month}}{29.53059\,\text{days}}\right) = 234.997\,\text{lunar months}$$

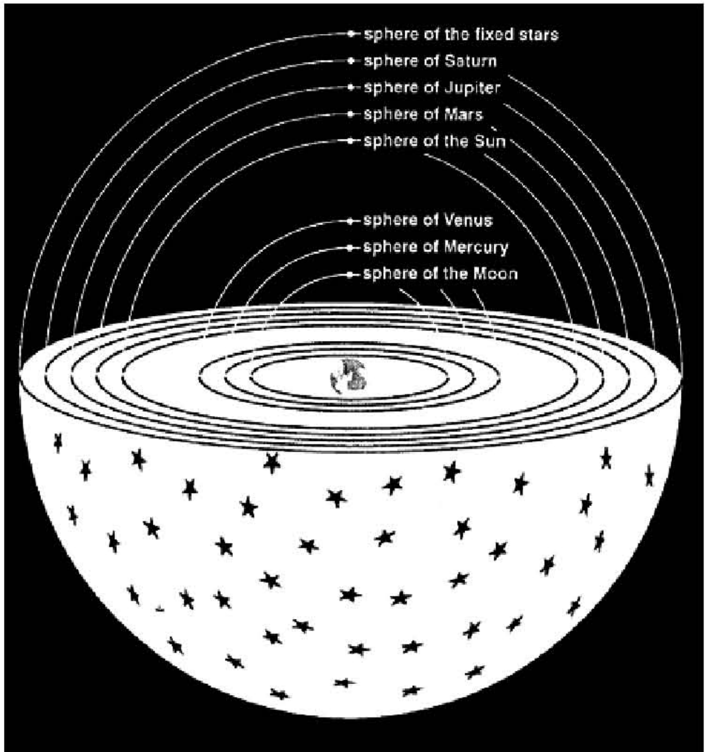
Phases of the Moon recur same days of year, same time of day
Period of 19 tropical years is almost exactly equal to 235 synodic months; rounded to full days counts 6940 days
Difference between two periods (of 19 tropical years and 235 synodic months) is only 2 hours
Thus, lunar periods repeat on the same day of the year as 19 years previous.

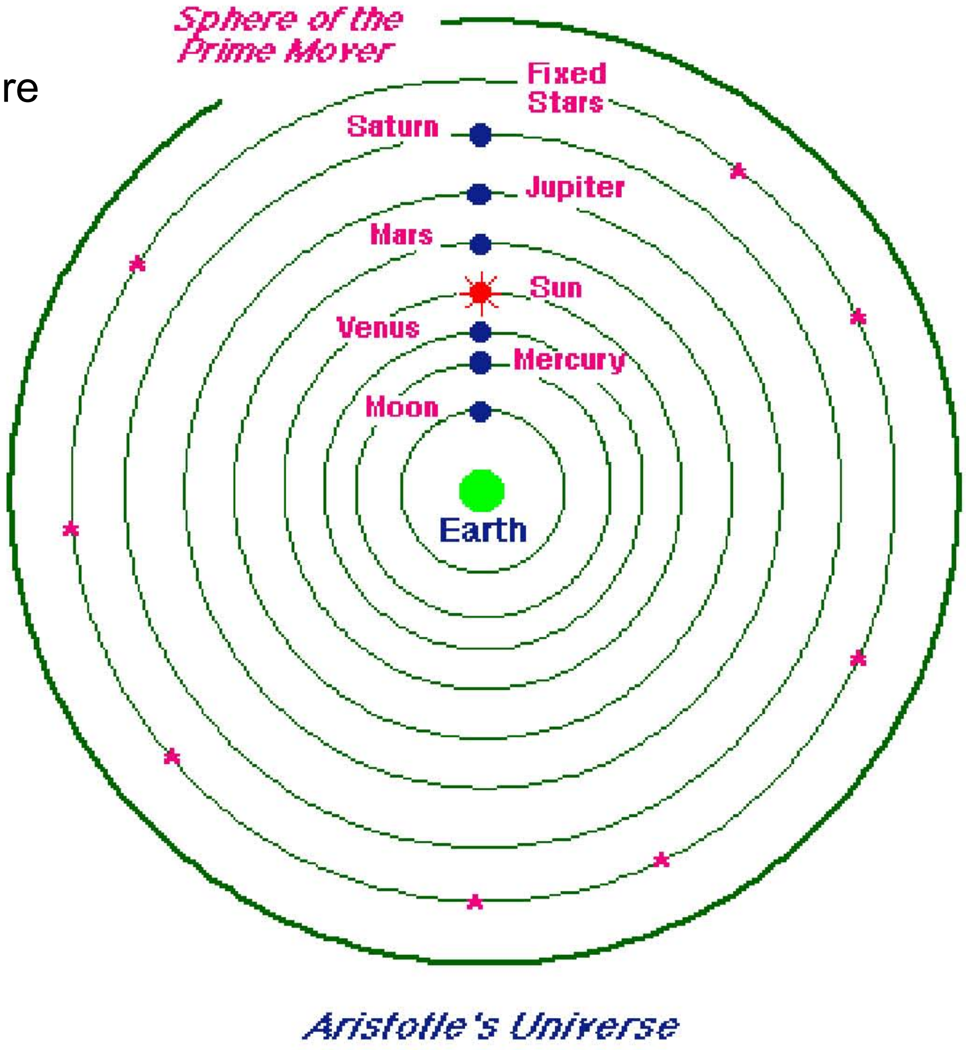
5. <u>Aristotle</u> (384 BC - 322 BC)

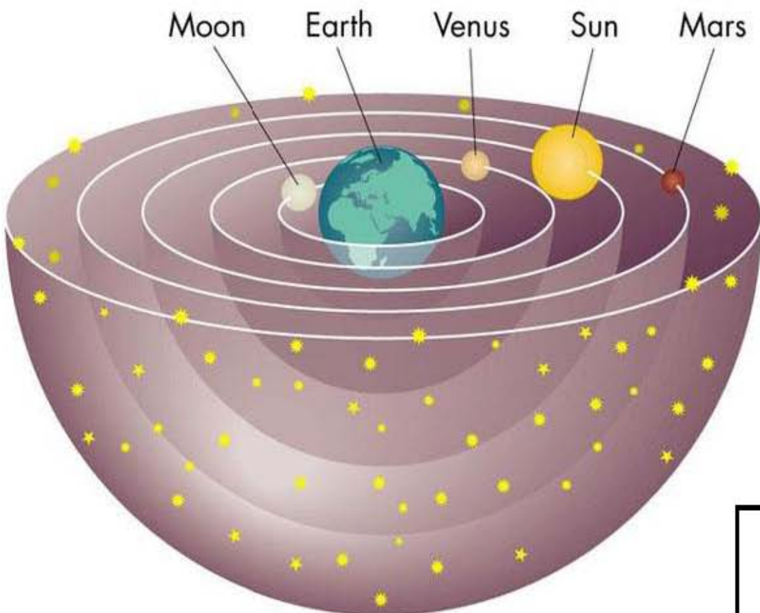
Universe spherical and finite, but eternal (no beginning and no end)
Earth spherical; much smaller than stars
Aristotle: three kinds of motion: rectilinear, circular and mixed

Aristotle's Universe

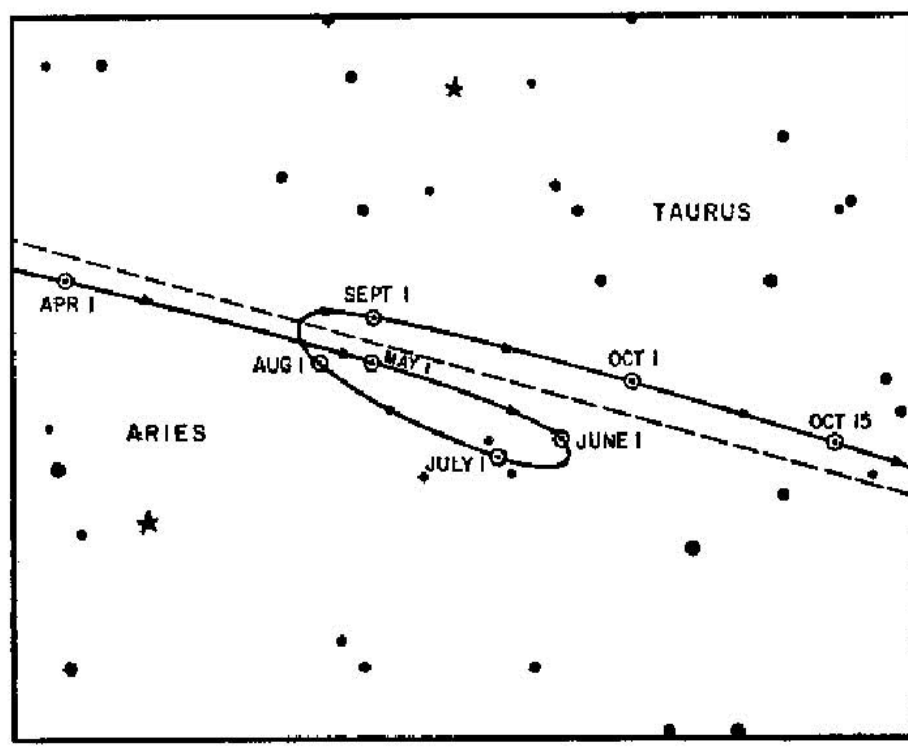
- 55 concentric, crystalline spheres
- Rotate at different velocities
- Angular velocity constant for given sphere
- Earth at the center



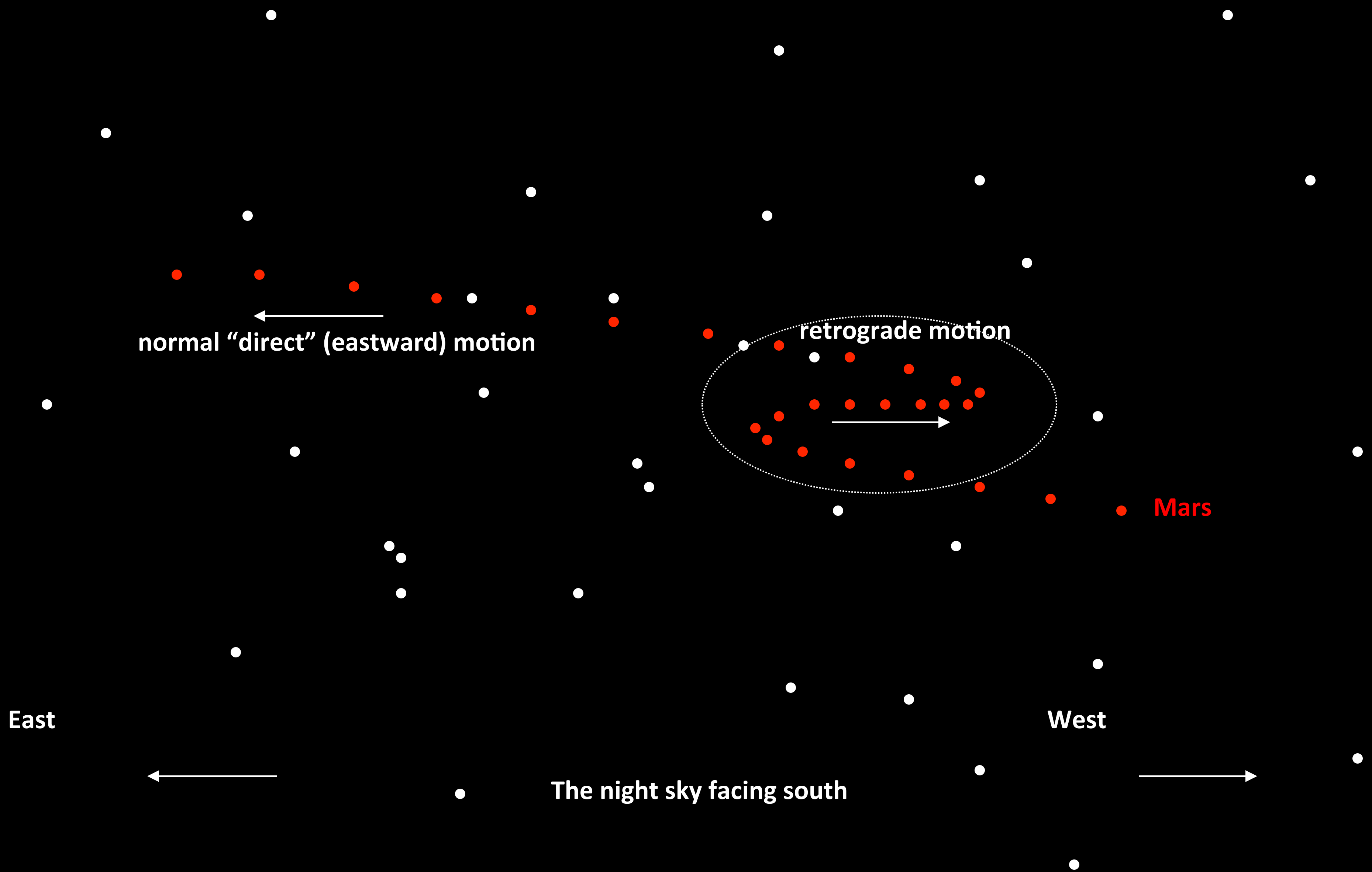




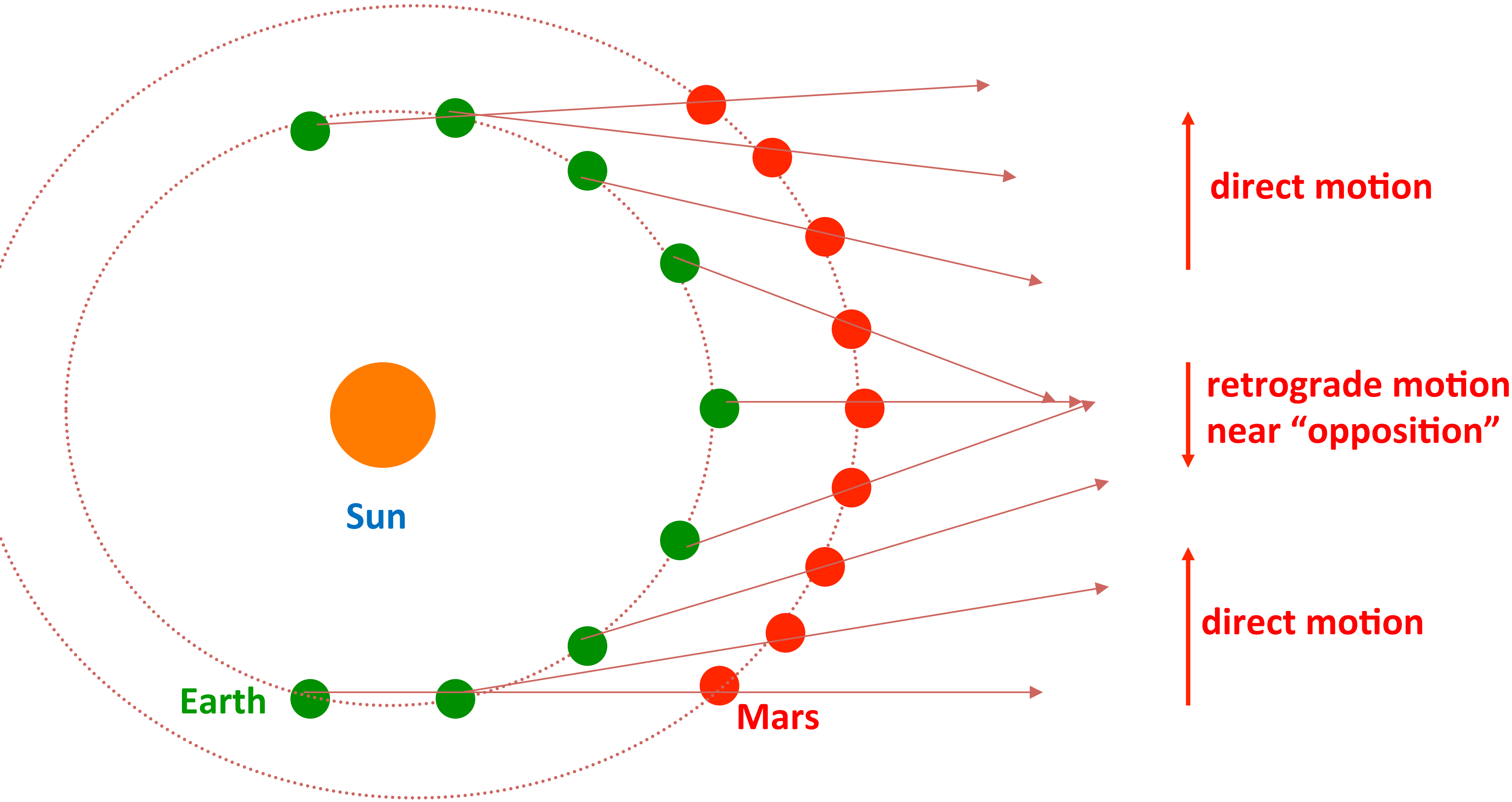
Concept does NOT produce observed motion



Motions of the "Wanderers" – The Planets



The Heliocentric Explanation



First proposed by Copernicus: ~1505
not published till *De Revolutionibus:* 1543

important work

6. <u>Aristarchus</u> (310-250 BC)
"Magnitudes and Distances"
Heliocentric Theory → 6 planets move in simple circles at uniform rates; rates differ for each planet; rates/distances determined from observations (strikingly similar to modern view)

Attempted to calculate the Sun-Earth distance → 4 million miles
(method pretty good!!)

Two other issues combine to discredit him:

#1 Other astronomers tried to use his model to predict the future position of Mars
Prediction / observation differ by → $\boxed{}$ ← Move to abandon theory

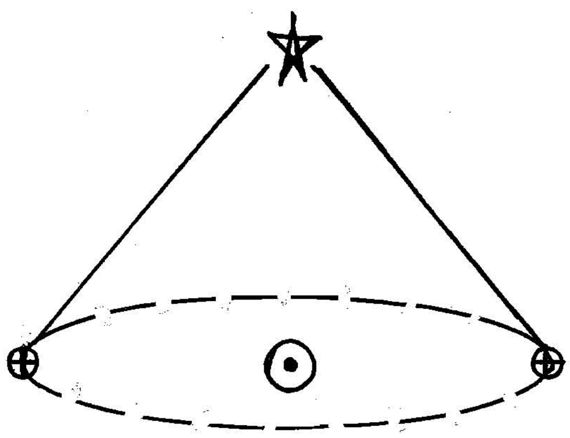
We know now that orbits only <u>slightly</u> different from circles produce large angle prediction errors!

#2 Arrival of Hipparchus

7. <u>Hipparchus</u> (190BC – 120 BC)

Born in Nicaea (now Iznik, Turkey)
Observer; greatest astronomer of antiquity
Developed science of spherical trigonometry
Located position of Earth by latitude and longitude
Located stars by right ascension and declination
Predicted the positions of planets, stars and constellations used by sailors for navigation
Most famous for discovery of precession of the equinoxes – due to slow change in direction of Earth axis of rotation
Estimated distance from Earth to Moon

> Argued against Aristarchus:
> If planets move around Sun, then the direction in which we observe a particular star should change during the year



In his observations, no such change occurs →
so Earth does NOT revolve around Sun!!

8. Eratosthenes (~276 BC - 196 BC)

Born Cyrene, now Libya, North Africa

Greek mathematician, astronomer, geographer, poet

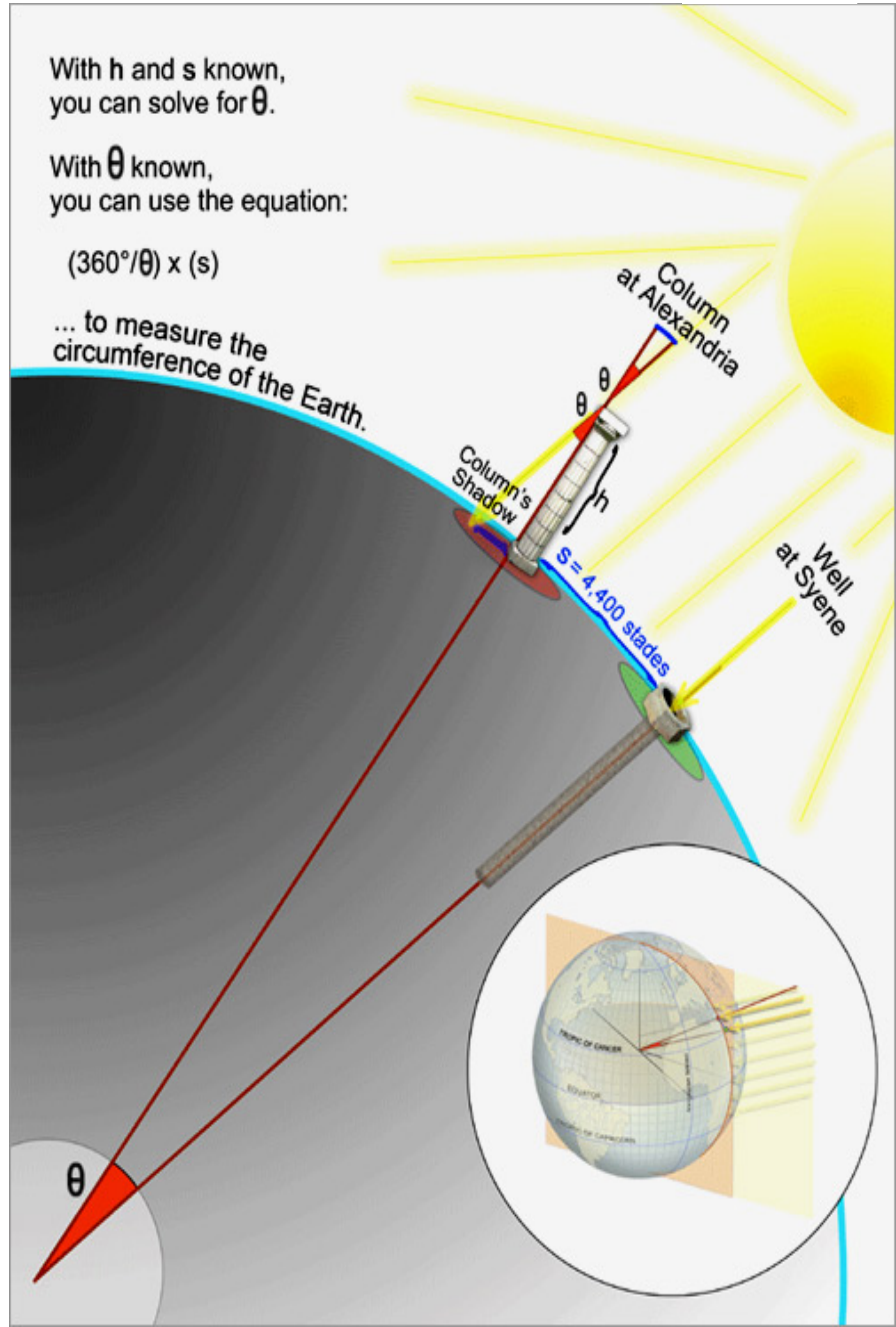
Credited – measuring circumference of Earth

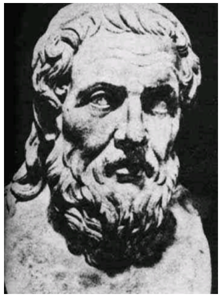
Astronomically determined differences in latitude between Egyptian cities of Syene (present day Aswan) and Alexandria; compared noon midsummer shadow between the two cities

Believed Sun so far away that rays parallel

Knowing distance between the two cities, formulated Earth circumference

Also contributed: calendar with leap years; star catalogue with 675 stars; distance to the Sun and Moon using data from lunar eclipses





(Now Muntina in Antalya, Turkey)
Lived in Perga, Pamphylia, Greek Ionia

9. **Apollonius of Perga** (262 -190 BC)

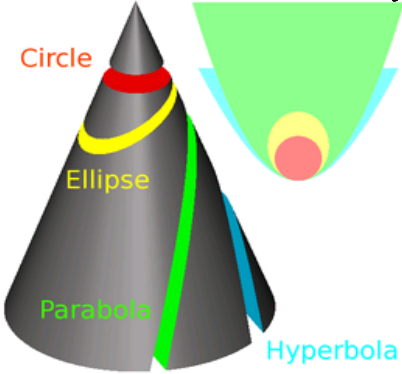
"The Great Geometer"

Little known of his life but his work has had great influence on development of mathematics, in particular, his book "Conics" – introduced terms ellipse, parabola, hyperbola

one of the greatest scientific works from the ancient world

Born in Perga, Pamphylia
Perga was a center of culture when Apollonius a young man. He went to Alexandria to study under followers of Euclid and later taught there
Apollonius visited Pergamum, where a university and library similar to Alexandria was built
(Pergamum is today the town of Bergama in modern province of Izmir in Turkey; then an ancient Greek city)
The two universities competed to amass the most complete collection of books in the world.
"Conics" was written as 8 books: only 1→ 4 survive in Greek
1→ 7 survive in Arabic

Conic sections are defined by Apollonius as the curves formed when plane intersects a cone



Books 1→ 4 info already known to Euclid and others
(elementary intro to basic properties of conics)
5→ 7 highly original; beyond results in Euclid's book "Elements" (normals to conics, center of curvature, higher order geometric)

12 regular pentagonal faces, 20 vertices and 30 edges

regular polyhedron with 20 identical equilateral triangular faces, 30 edges and 12 vertices

Other works by Apollonius: “Hypsicles” (compares dodecahedron and icosahedron)
“Quick Delivery” (better approximation for)
“On the Burning Mirror” (focal properties of parabolic mirror)

Apollonius: important founder of Greek mathematical astronomy – geometric models to explain planetary theory

Ptolemy later credits Apollonius with concept of eccentric and epicycle motion to explain apparent motion of planets across sky

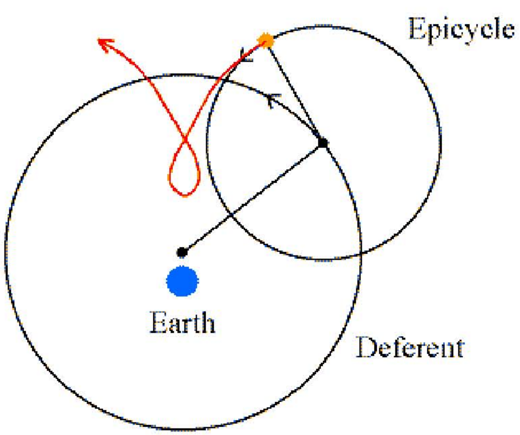
Not strictly true – epicycle concept predates Apollonius but he makes a big contribution because of his geometry skills

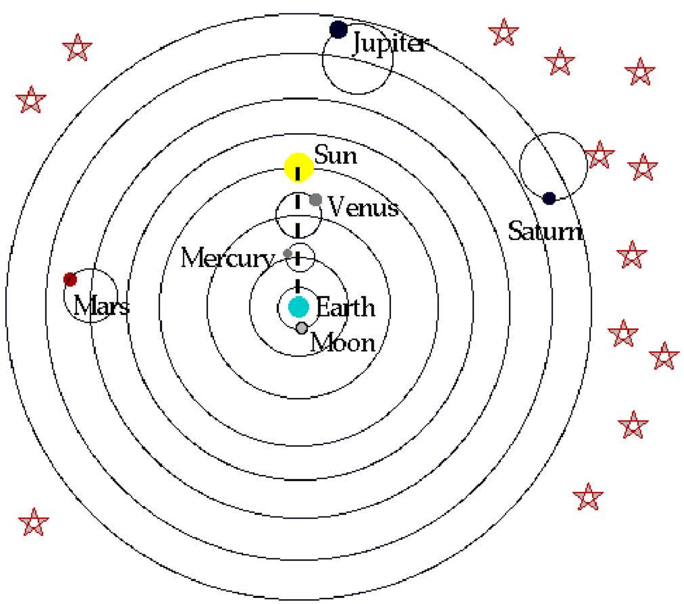
10. <u>Claudius Ptolemy</u> (100 – 170 AD)

Carried forward work of Hipparchus (circles)
Greatest discovery: perturbation of Moon's orbital motion (actually due to attraction of the Sun)
Discovered refraction → deflection of light ray from straight path as ray passes from one medium to another; change in apparent position of celestial body due to bending of light rays emanating from body as they pass through atmosphere

Most remembered for his planetary motion theory:
A complicated construction of circles predicted planetary positions with maximum error of only 1 deg !!

Epicycles



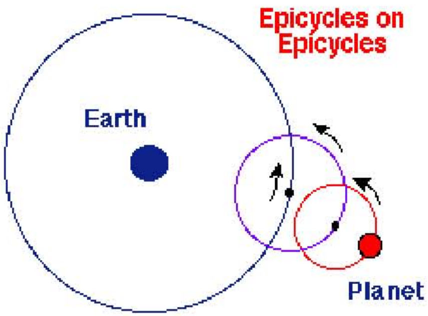


Planets → "epicycles"
Concentric spheres → "deferents"
Centers of epicycles → uniform circular motion
Epicycles → own uniform circular motion

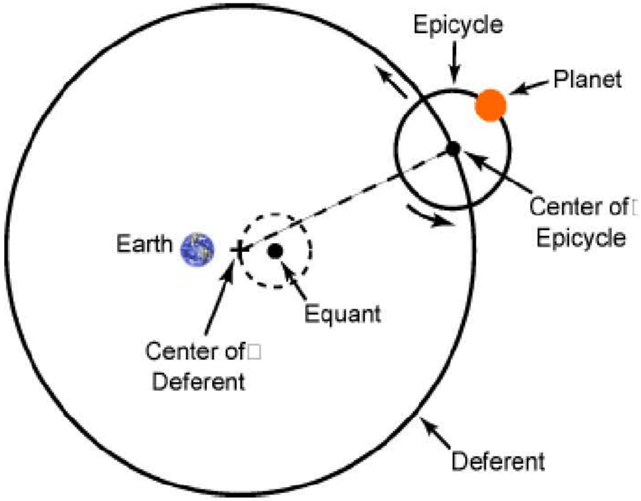
Ptolemy's Universe

Required refinements:

→ epicycles on epicycles



→ center of the epicycle uniform motion about offset point



"uniform circular motion" :

1. All motion in the heavens → uniform circular
2. Objects in heavens from perfect material →cannot change intrinsic properties (e.g. brightness)
3. Earth at center of Universe
4. VERY GOOD predictions

Ideas catalogued by Ptolemy in Book: "Almagest" (i.e., "The Greatest) 150 AD

→ "Ptolemaic Universe"

11. <u>Hypatia</u> (between 355→370 – 415) Alexandria, Egypt

Considered the first notable woman in mathematics who also taught philosophy and astronomy → mathematician, astronomer, physicist, philosopher

Not all aspects of her life are well documented….but

Daughter of Theon ← most educated man in Alexandria, Egypt; taught mathematics

She learned the fundamentals of teaching and became a profound orator

Hypatia dressed in the clothing of a scholar or teacher, rather than women's clothing; moved about freely, driving her own chariot → contrary to the norm for women's public behavior; she exerted considerable political influence in city

Astronomy at this time mostly observations

Hypatia known more for work in mathematics than astronomy – primarily :

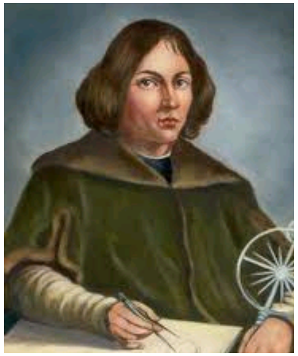
- ideas of conic sections introduced by Apollonius
- she developed concepts to be easier to understand; thus, work survived thru many centuries
- considered by many to be first woman with such profound impact onearly thought in mathematics
- discouraged mysticism while encouraging logical and mathematical studies
- from the little historical information about Hypatia that survives, it appears
  tshe invented the plane astrolabe, the graduated brass hydrometer,
  and the hydroscope (with Synesius of Greece, her student and later colleague.)

Hypatia was the last scientist to work at Library in Alexandria,
so she signifies the beginning of a 1000 year gap in scientific development:

Hypatia's life ended tragically (murdered in AD 415). However, her work remained. Later, Descartes, Newton, and Leibniz would expand on her work. She made extraordinary accomplishments for a woman in her time. However, her death would initiate a long, stationary period when science was not cultivated.

$\boxed{\textbf{1000-Year Scientific Dark Ages}}$

12. **<u>Nicolaus Copernicus</u>** (1473-1543)



Most remembered for the heliocentric theory:
BUT idea of circular motion still obsessed Copernicus;
Disliked the idea of rotating uniformly about a point off-center

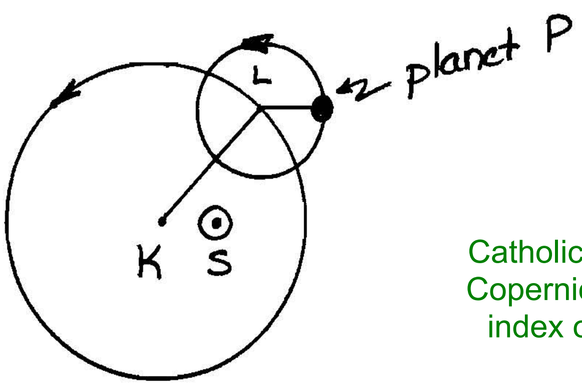
1529 – The *Commentariolis* manuscript written by Copernicus and circulated

Seven basic axioms:

1. The heavenly bodies do not all move around the same center.
2. The Earth is not at the center of the universe, center only of the Moon's orbit and terrestrial gravity.
3. The Sun is the center of the planetary system and hence the universe.
4. The stars are much farther from Sun than Earth is from Sun.
5. Daily motion is due to rotation of the Earth on its axis.
6. Annual motion of the Sun is due to Earth's motion around it (and other planets orbit in the same fashion).
7. Apparent motions of planets (like retrograde motion) due to combined motion of Earth and planets around Sun

Published in 1543 (year he died) -- founded new cosmology; established new theory of planetary motion.
Moving the 'center' of system from Earth to Sun,
Copernicus succeeded in obtaining for <u>all</u> planets,
quite simple orbits (circular to first approximation)
and no longer curves like complex figures from
epicycle theory.

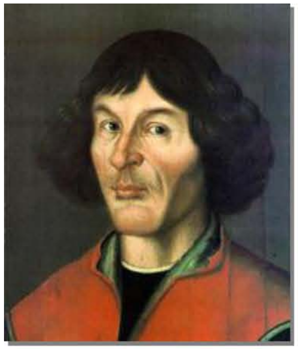
Still use epicycles →
KL and LP at uniform rates



Catholic Church put Copernicus work on index of prohibited books

Copernicus does give Earth a prime role, however.
Assumes in planetary theories that ‘center’ of Earth orbit rather than center of Sun-planet system is center of every motion → led to mistakes that required use of epicycles as correctives to agree to observation (albeit still rather inaccurate). Did not worry astronomers who were used to a bevy of epicycles.

Copernicus: Heliocentric Model



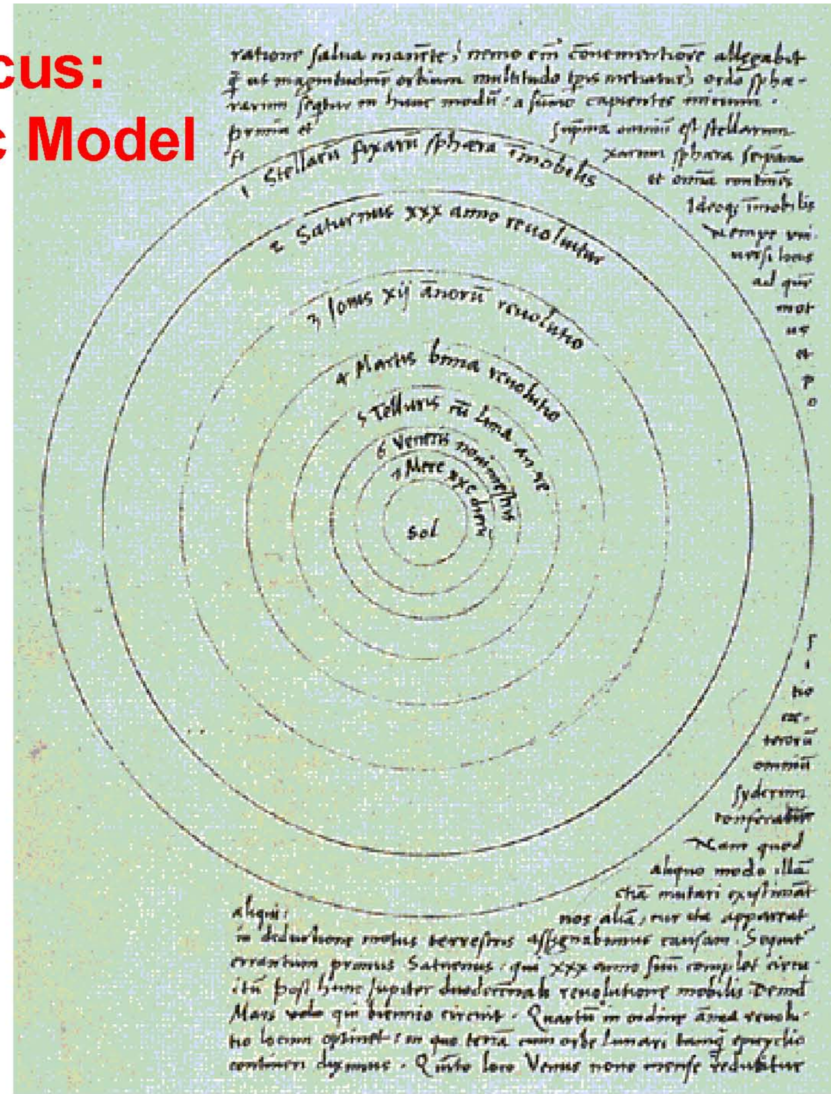
Copernicus (1473-1543)

Earth not fit to be center; Sun divine

Equant: betrayed concept of circles

Sun + Epicycles → no equant

Copernicus’ Model:
- No better results than Ptolemy
- Basic Info:
  - Sun at center
  - Stars far away
  - Earth rotates on axis
  - Earth rotates about Sun
- Simple Orbits



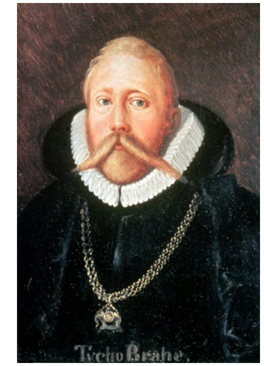
Copernicus: "And in this way, Mercury moves altogether by seven circles, Venus by five, the Earth by three and, around it, the Moon by four; at last Mars, Jupiter and Saturn, each five. As a consequence, 34 circles are sufficient to explain the entire structure of the universe, as well as the dance of the planets."

It was an all-time worst-seller
→ **especially for a book that "Changed the World"**

Virtually unreadable, the 1st edition never sold out. There were only 4 editions in 330 years (the first English translation was in 1952). Document did possess useful tables for computing.
Other problems: The center of orbits is near the Sun but each planet's orbit had a different center.
Eventually needed 48 circles (with epicycles) to get it to work (even more than Ptolemy, although it was also more accurate).

Still Copernicus marked **beginning of new era for astronomy.**
Less than 70 years to Kepler → Kepler is the high point of "the kinematics of celestial bodies" and sunset of epicycles.

Ironically, Copernicus is remembered for heliocentric theory but he really just wanted to avoid the complexity (epicycles) of using Earth as a center. In the end his model required a lot of epicycles anyway!



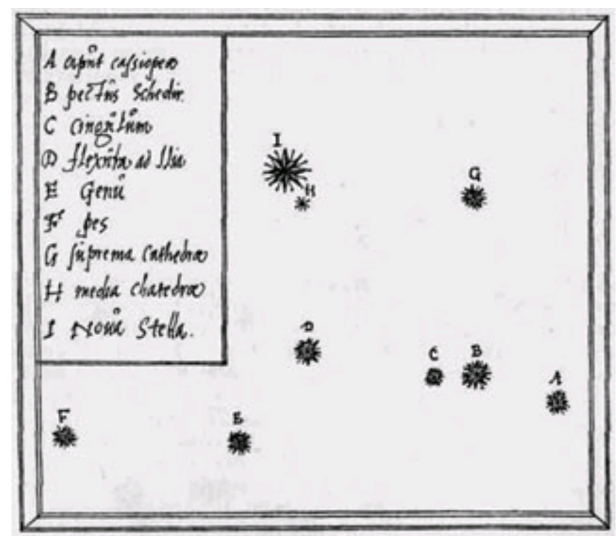
13. <u>Tycho Brahe</u> (1546-1601)

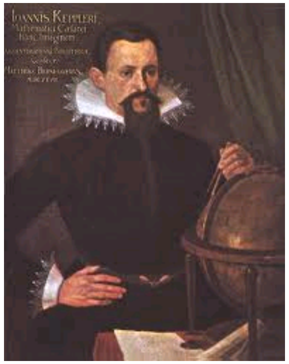
Painstaking observer; motivated by desire to disprove Copernicus
Worked in Denmark under patronage of King Frederick
After death of King, move to Prague
Kepler as disciple and assistant

[It is important to remember that Brahe lived **before** invention of telescope → Astronomical observations by naked eye; Galilei invents telescope **<u>9 years after Brahe's death</u>**
Devices Brahe used and constructed are therefore mainly devices for measuring angles and positions
Also, clocks very limited at the time; pendulum clock not yet invented either, so to measure time, Tycho usually chose to use movements of stars and planets, with admirably accurate results]

Evening of the Nov 11, 1572, Brahe for first time sees new star in constellation Cassiopeia

Tycho Brahe observes carefully, and publishes findings about the "new star", Stella Nova in Latin, and becomes known as a respected astronomer





14. <u>Johannes Kepler</u> (1571-1630)

Much adversity in life
Assistant to Brahe; access to meticulous collection and recording of accurate data on position of planets (supports heliocentric theory)
Willing to face unbelievable amount of calculation (nonlinear problem; no computer; no assistant)

After much fruitless work with circles, <u>re-evaluated initial assumption</u> (circles)
He succeeded by asking the right question:

38 years old

Kepler's Laws of Planetary Motion (From observations; trial-and-error solution)

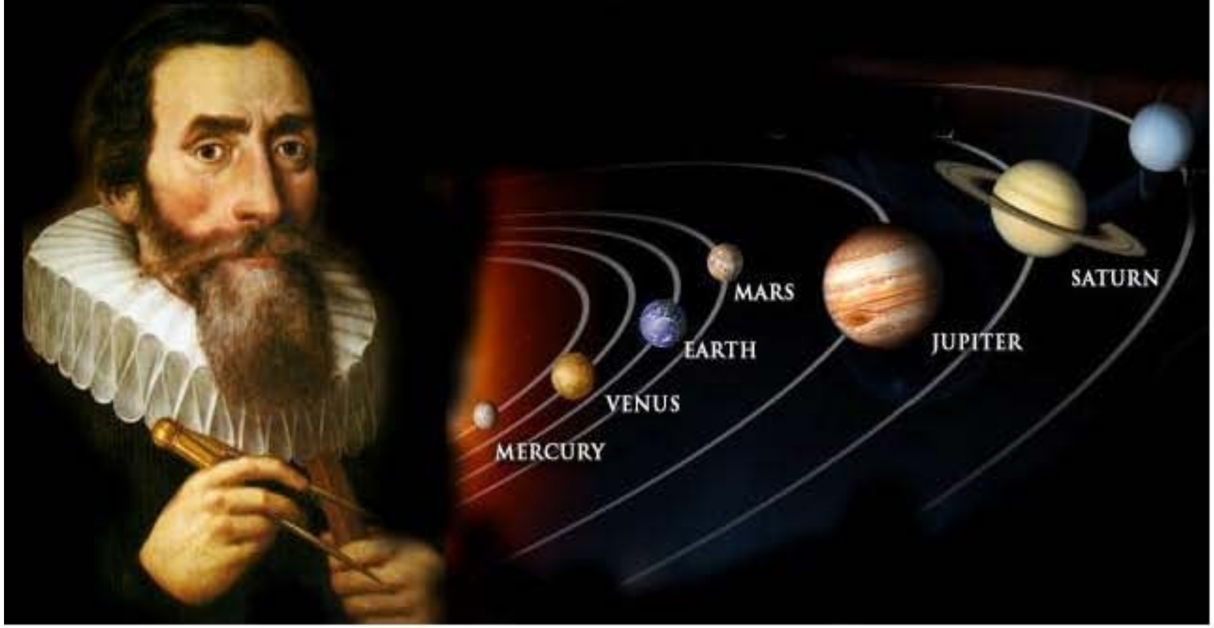
1609 I. The orbit of each planet is an ellipse, with the Sun at a focus.

1609 II. The line joining the planet to the Sun sweeps out equal areas in equal times. (This one contributed to the future development of calculus.)

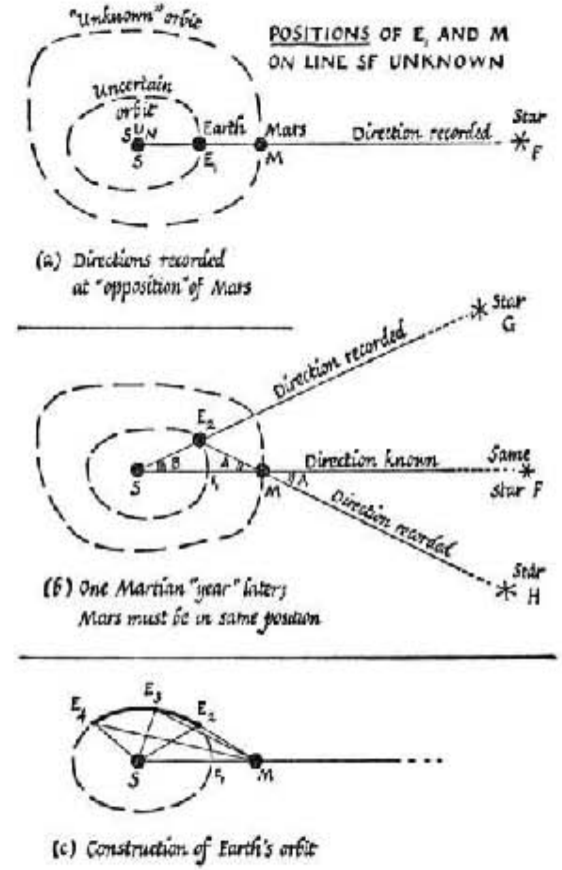
1619 III. The square of the period of a planet is proportional to the cube of its mean distance from the Sun.

Notes: Kepler's laws are only a description, NOT an explanation
[Newton later derives his law of gravitation from these; he explains why they are true.]

The actual planetary orbits are nearly all in one plane. This simplified the discovery of forms of the planetary orbits. IF they had been as variable as cometary orbits, we could still be searching!

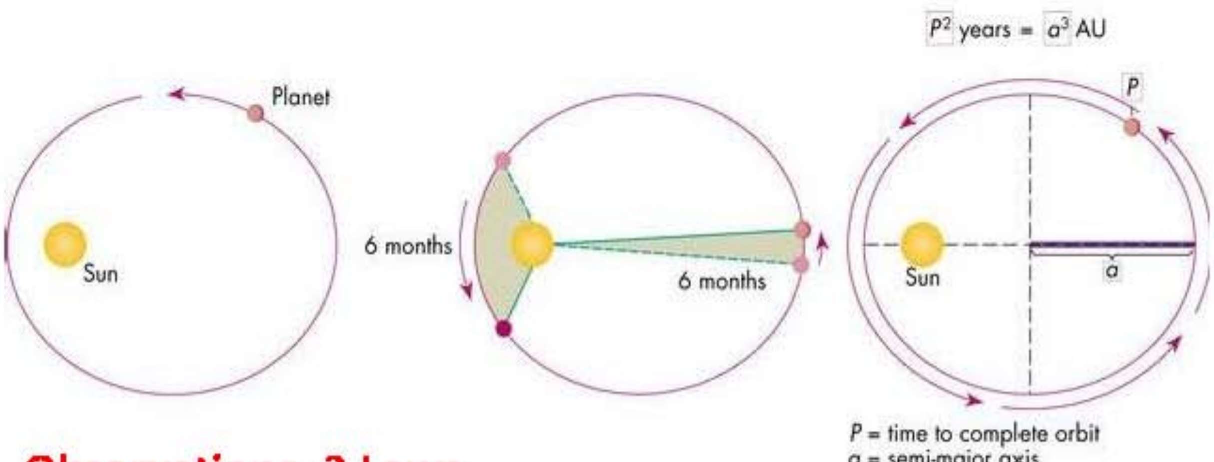


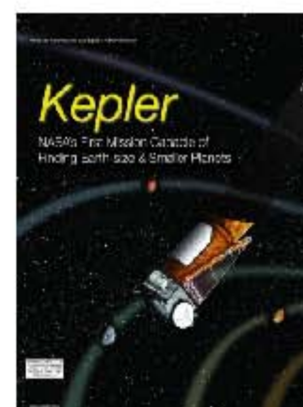
Johannes Kepler (1571-1630)



KEPLER'S SCHEME TO PLOT THE EARTH'S ORBIT

What do paths actually look like?





Observations: 3 Laws

15. <u>Galileo Galilei</u> (1564 – 1642)

Contemporary of Kepler, of greater genius and greater fame
Supporter of the heliocentric theory
Applied telescope to celestial objects
Discovered 4 satellites about Jupiter

First to start connecting astronomical observations with physical causes

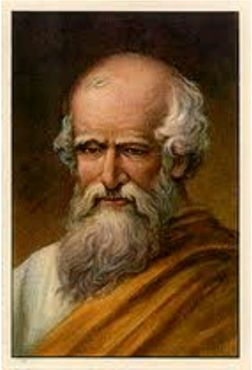
↓

Back up a moment to consider parallel development of mechanical laws

II. Dynamical Astronomy (Connecting of mechanical and physical causes with observed phenomena. Advances at rare intervals.)

1. **Archimedes** (287 – 212 BC) of Syracuse
   Greatest mathematician of his age: combined a genius for mathematics with a physical insight
   First sound ideas regarding mechanical laws
   Principles of the lever and meaning of a body's center of mass
   Results of Archimedes more remarkable when considering times in which he lived

   Archimedes made fundamental discoveries in several fields, then advanced them so far that his results not improved for many centuries

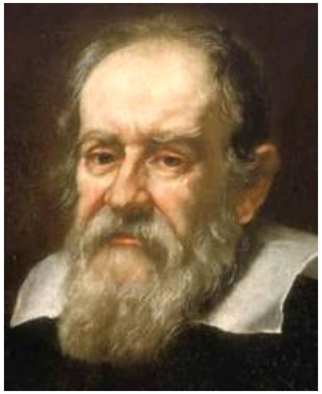


2. **Leonardo da Vinci** (1452 – 1519)

Best known as painter but thousands of scientific and technological observations
According to Leonardo's observations, the study of mechanics, with which he became quite familiar as architect and engineer, also reflected the workings of nature
Throughout his life Leonardo was inventive builder; thoroughly understood the principles of mechanics of his time and contributed in many ways to advancing them
Improved form and generality of Archimedes' treatment
Investigations of statistical moments ➔ statics of rigid bodies involves only application of the proper mathematics to fundamental principles

3. **Simon Stevinus** (1548 – 1620) Dutch
   Published extensively on mathematics, engineering (both military and civil), mechanics; also hydraulics, navigation, mechanical devices, instruments
   First to investigate mechanics of the inclined plane (1586)

4. **Galileo Galilei** (1564 – 1642)
   Italian physicist + astronomer
   First important advances in **kinetics**



   Fundamental error by most investigators:
   Assumed it required a continually acting force to keep a body in motion; assumed it was natural for a body to possess position rather than a state of motion

Galileo: "Founder of Dynamics" –
applied his principles to discover laws of falling bodies and motion of projectiles

Note: Kepler 1571 - 1630

5. **Christian Huyghens** (1629 – 1695)
Dutch Mathematician
Planetary orbits nearly circles → simplify problem and assume them to be circles

Since planets not in uniform motion in straight lines, must be acted on by a force

Circular motion → force acts radially inward

Like stone whirled around in sling, strength of force depends on size of stone, speed, length of string

But ellipses (planets) do not move on circles
Form of the actual force?

6. <u>Isaac Newton</u> (1642 – 1727)

Completed formulation of fundamental principles of mechanics and applied them with unparalleled success in solution of mechanical and astronomical problems

Education: Trinity College of Cambridge University BA 1665

1665 – 1666 Home in Woolsthrope, England → Invented differential and integral calculus; made fundamental discoveries on nature of light; developed foundation for theory of universal gravitation

1687 "Principia" ← Very difficult to read; Very complicated since he used geometry (actually used calculus)

Newton initially spent time explaining Galileo's experiments; produced three laws of motion:

I. Bodies continue in rest or uniform motion unless compelled to change by a force

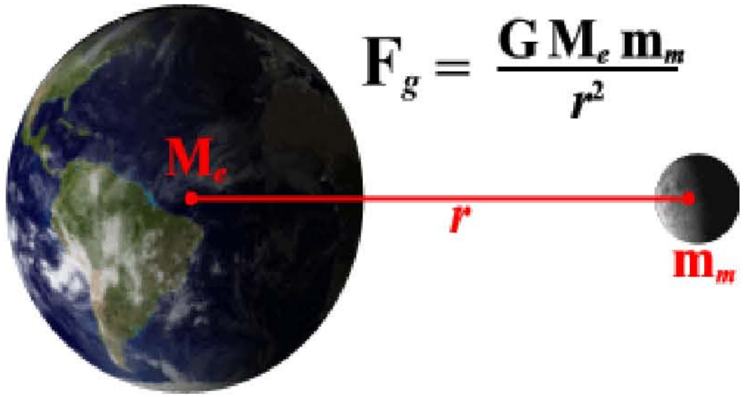
II. Rate of change of momentum is proportional to the force impressed and in the same direction

III. Every action is opposed by an equal and opposite reaction

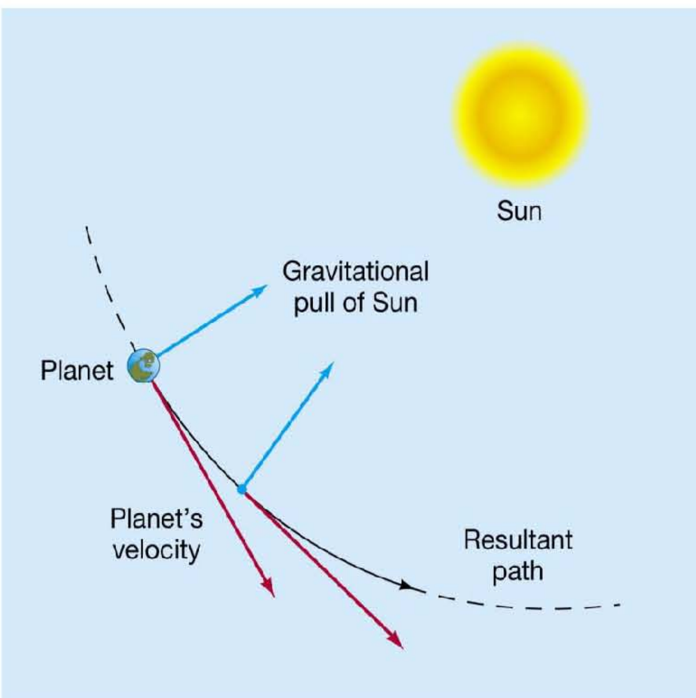
Newton → Law of Universal Gravitation

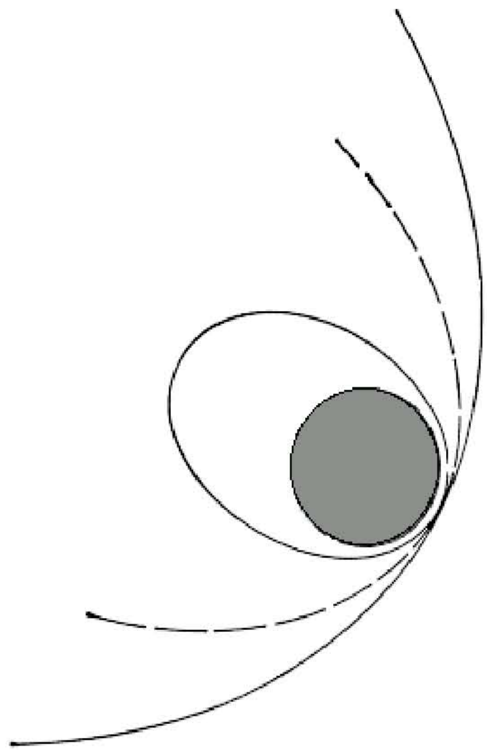
Dynamics of Celestial Bodies

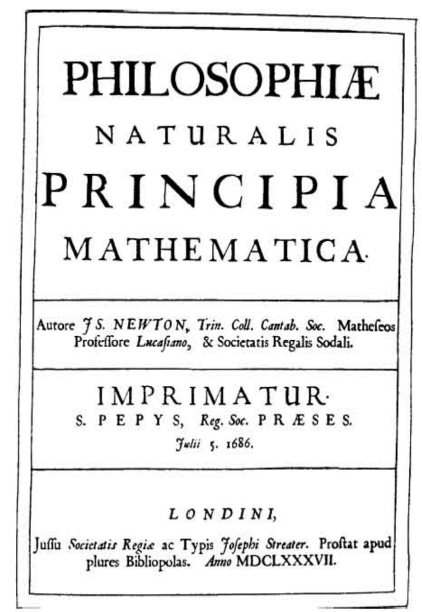
$$F_g = \frac{G M_e m_m}{r^2}$$



Isaac Newton
(1687 –1725)







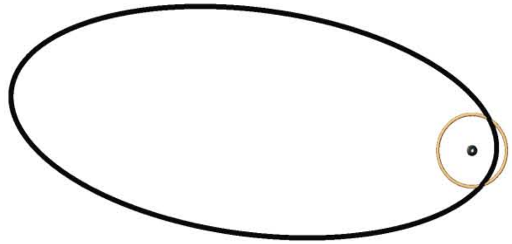
PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

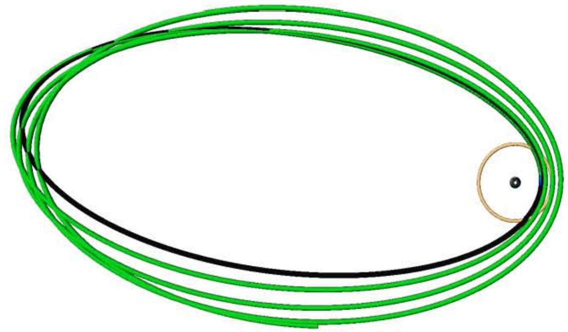
Autore J. S. NEWTON, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

LONDINI,
Jussu Societatis Regiæ ac Typis Josephi Streater. Prostat apud plures Bibliopolas. Anno MDCLXXXVII.

Newton + Two-Body Problem





Gravitational interaction of 2 bodies

Deviation due to other bodies in solar system

Dynamics of Celestial Bodies

SUBSTANTIAL CHANGE

Pre-Newton :

- Every orbit reproduced with approximation by combining circular motions
- Justified because the observed orbits could be obtained
- No acknowledgement of fact that the entire solar system was “held up” by the mutual interactions of the various components

With Newton:

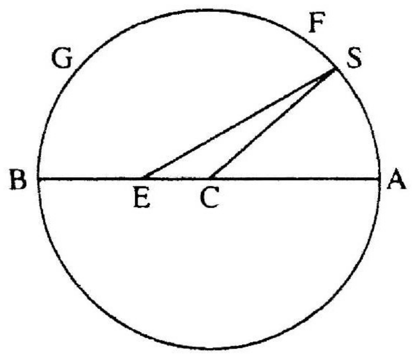
- Orbit followed by each body could be exactly calculated <u>in principle</u>
- Incorporate all bodies involved and mutual interactions
- Mathematical model could completely reproduce the physical phenomena
- Possible inadequacies did not derive from failures of the model but from <u>calculation</u> difficulties

New Phase:

→ Mathematical model can completely reproduce the physical phenomena
→ Possible inadequacies derive not from failures of model but computational difficulties.
→ (Principia) Newton applied only geometrical methods → evaluation of perturbations to Keplerian orbits very cumbersome
→Most important case: motion of the Moon
Influence of Sun on motion of Moon about Earth so important must be a 3-body problem
→ But Newton’s geometrical constructions complex; correspond roughly to first -order perturbation theories developed in next century

Summary: Motion of Celestial Bodies

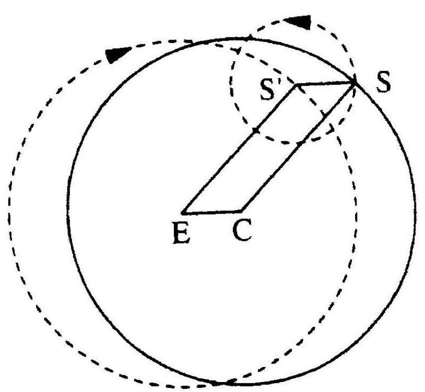
Greeks → their ideas captured motion of celestial bodies: ***theory of eccentrics*** and ***theory of epicycles***. Note: according to Greeks, celestial bodies could not move on any kind of curve unless it was a circle → the perfect curve! SO, this implies that motion could only be uniform circular motion or combinations of uniform circular motions.
Essentially, they renounced the possibility of seriously considering the true nature of physical systems because of their assumption and, thus, the causes of the motion. These are essentially descriptions of the motions. From Hipparchus (2nd century BC) through Ptolemy (2nd century AD) to Newton (1687), the attention remained focused on the **kinematics**.



Theory of Eccentrics:

Basis of theory: Earth sits at center of universe and Moon rotates around on circular orbit with period of 27 days. Sun rotates about Earth with period of one year.

- Center of orbits NOT center of Earth E; Sun S on circle with center C
- Inner planets Mercury and Venus move on circles with centers always on line joining Sun and Earth; Earth remains outside these circles. Outer planets (Mars, Jupiter, Saturn) also move on circles with centers on Sun-Earth line but circles so large that they encircle both Earth and Sun.
- S moves with uniform motion → angle ACS increases uniformly
- angle AES increases non-uniformly: slowly near A (apogee); quickly at B (perigee)



Theory of Epicycles:

Motion of Sun also measured by *epicycle* model (from Apollonius 3rd century BC)
•Body moving uniformly on a small circle (epicycle).
•Center of epicycle moves on second circle called deferent.
•Deferent is circle equal to 'eccentric' circle but with center at E
•Point S' such that ES' parallel to CS then S'S equal to EC
•Sun S (which moves uniformly on left-side eccentric) considered in the same way to be uniform motion on the circle of radius S'S with center S' that moves uniformly on the deferent.

Both models yield same result