Orbits in Three Dimensions

Previously, we considered everything in 2D
Now, try 3D problems ← some background necessary to define an orbit in space

First, define coordinate systems (3D) to help

many available

Two basic types for us to use:

$x_\varepsilon, y_\varepsilon$

(1) **Ecliptic System** – fundamental plane is the plane of the ⊕'s orbit about the Sun (latitude, longitude)

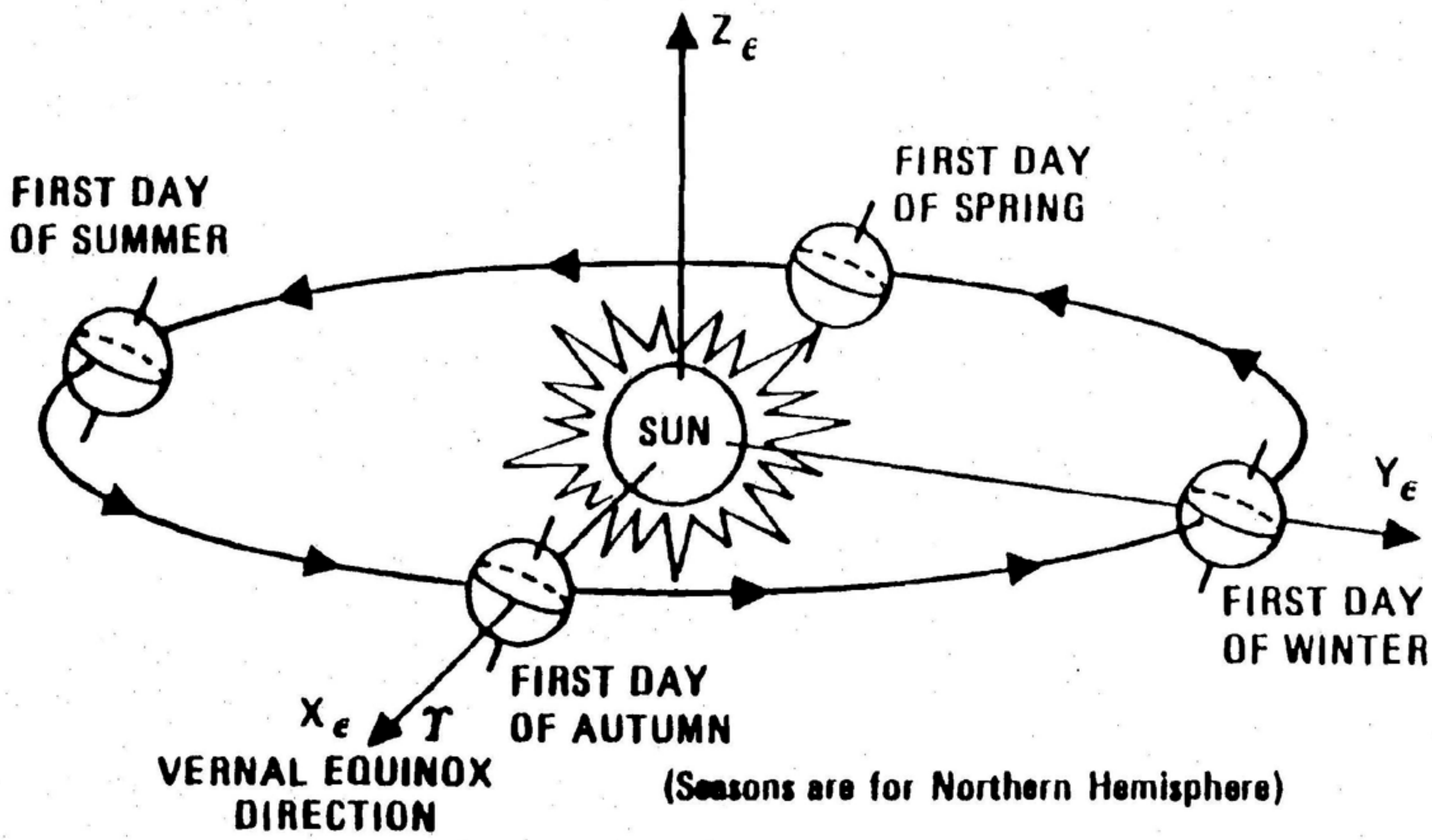
x, y

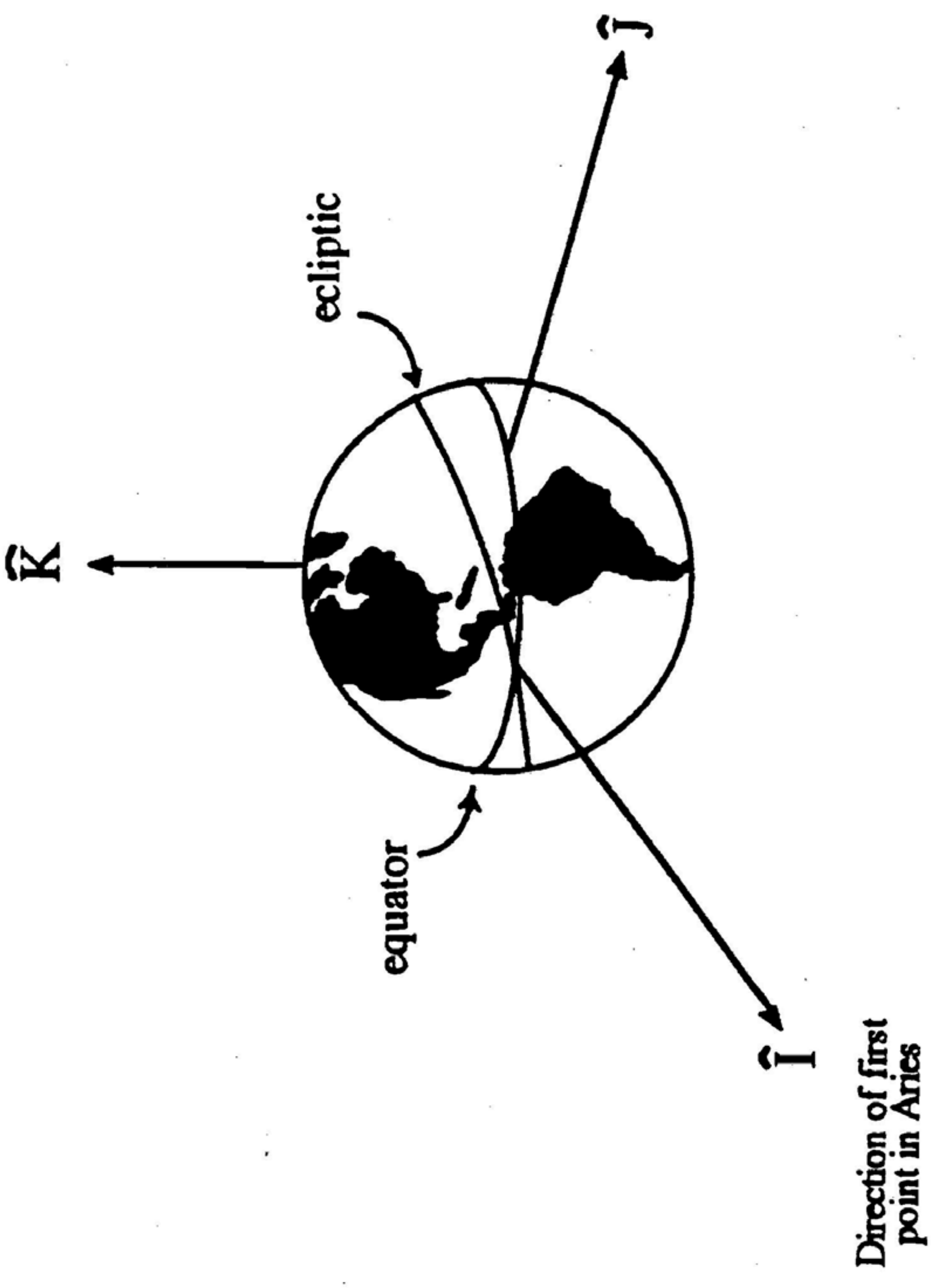
(2) **Equatorial System** – Fundamental plane is the plane of the body's equator (right ascension, declination)

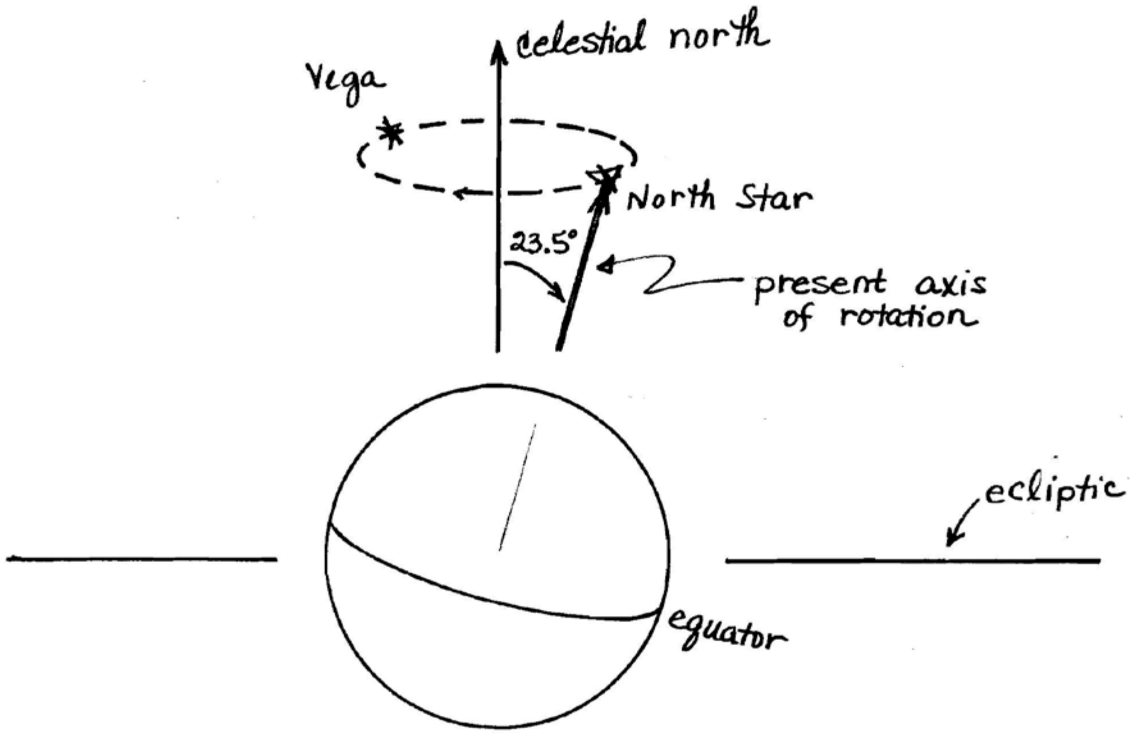
Obliquity of ecliptic (ε) –

To effectively use a coordinate system, reference directions must be known and understood; we need a fixed reference direction in the fundamental plane from which measurements are made

→


Z_ϵ
FIRST DAY
OF SPRING
FIRST DAY
OF SUMMER
SUN
Y_ϵ
FIRST DAY
OF WINTER
FIRST DAY
OF AUTUMN
X_ϵ ♈
VERNAL EQUINOX
DIRECTION
(Seasons are for Northern Hemisphere)


ecliptic
$\hat{K}$
$\hat{J}$
equator
$\hat{I}$
Direction of first point in Aries



"precession of the equinoxes" –

Caused by perturbing forces on its attitude, i.e., ☉ and ☾ gravity forces
These apply a precessing motion (same as the precessing motion of a spinning top or a torque-free rigid body)

Known as early as 2^{nd} century BC to Greek astronomer Hipparchus

Time for complete precession is 26,000 years

Consequence

1. Cataloging of celestial objects must refer to a specific date → epoch → currently 0.0 hrs

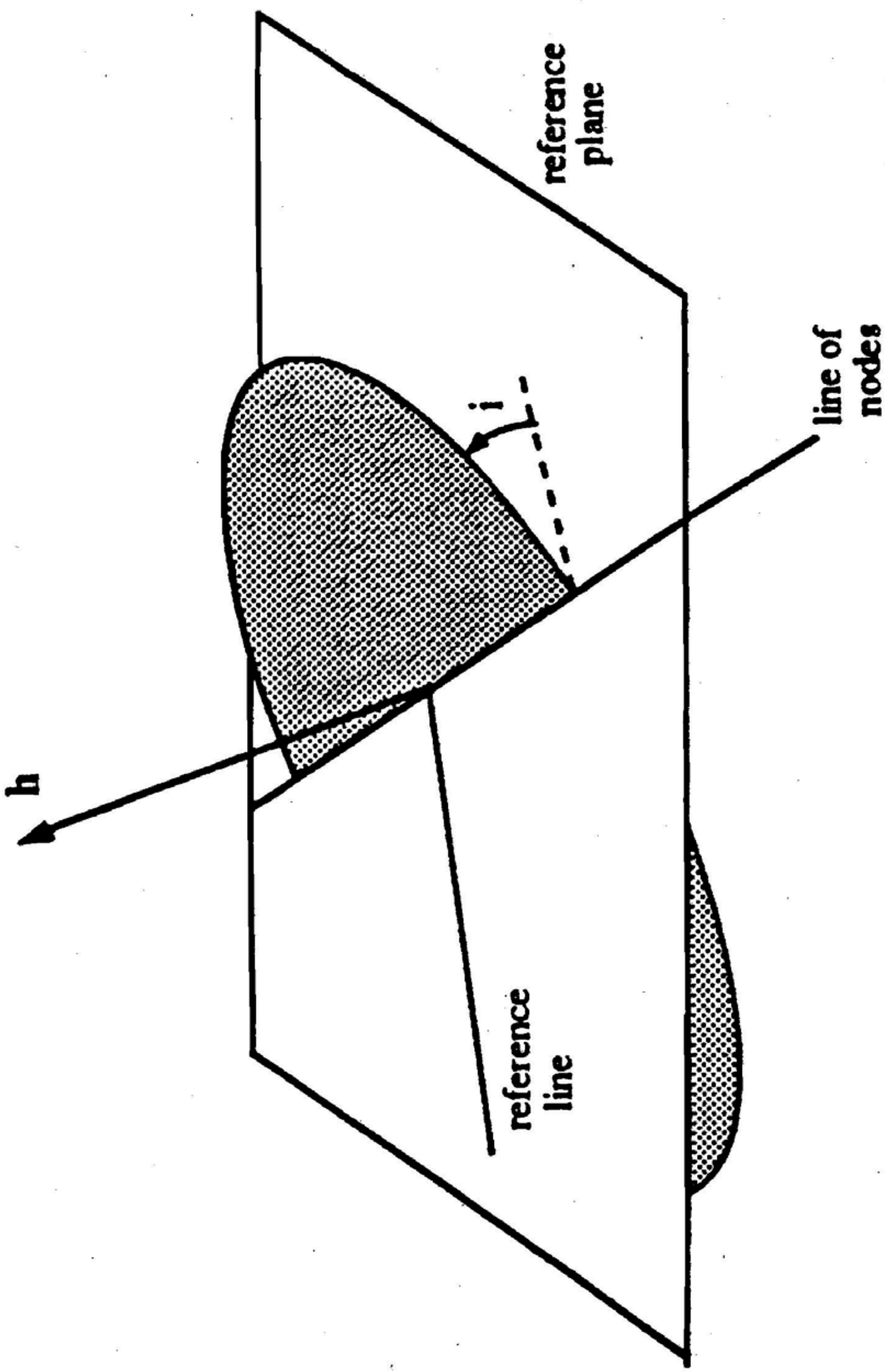
2. We will assume Υ fixed; reasonable over the relatively short intervals of interest

Reference System

$\hat{x}$ direction of the vernal equinox
$\hat{z}$ normal to fundamental plane; + north
$\hat{y}$ $= \hat{z} \times \hat{x}$

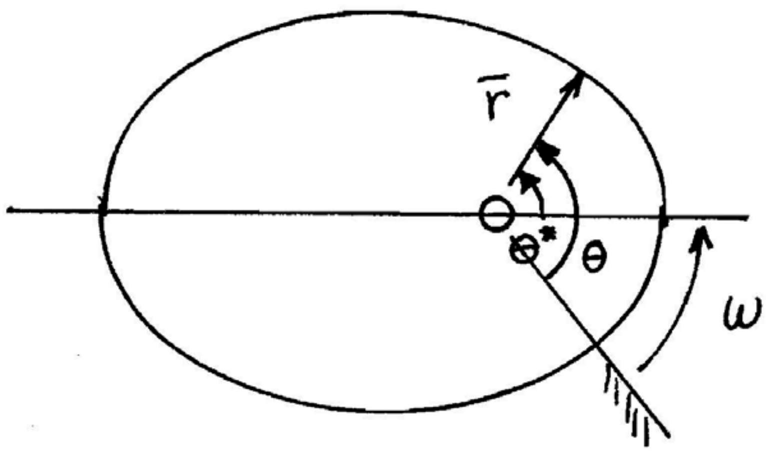
So, to locate s/c in space :

(1) locate s/c in orbit (θ^*, E, M)
(2) identify orientation of orbit within orbit plane (ω);
size and shape of orbit (a, e)
(3) identify orientation of orbit plane in space (Ω, i)

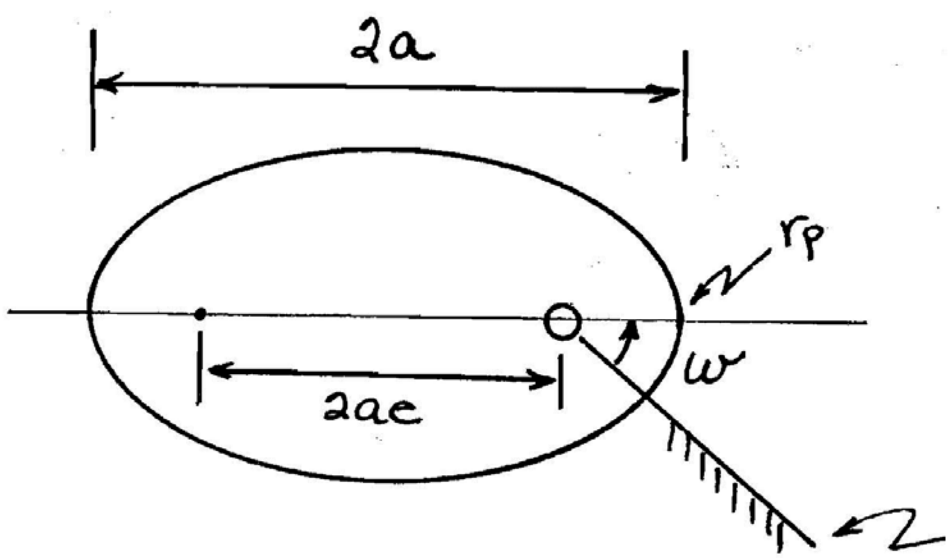


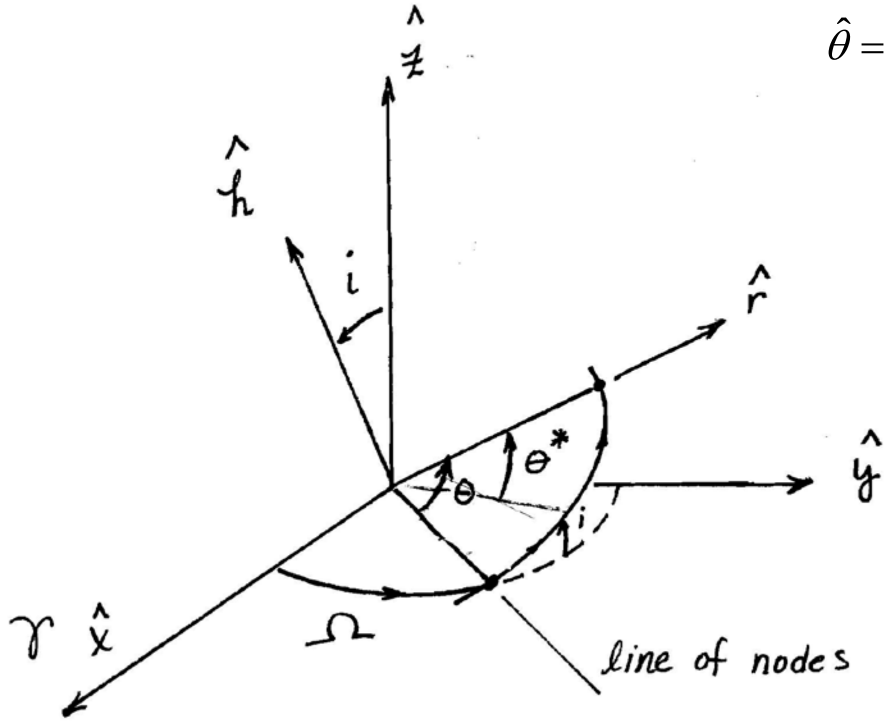
reference plane
line of nodes
i
h
reference line

(1) Locate s/c in orbit: time ←



(2) Within orbit plane : orbit size and shape
orbit orientation within plane





$\hat{\theta} = \hat{h} \times \hat{r}$

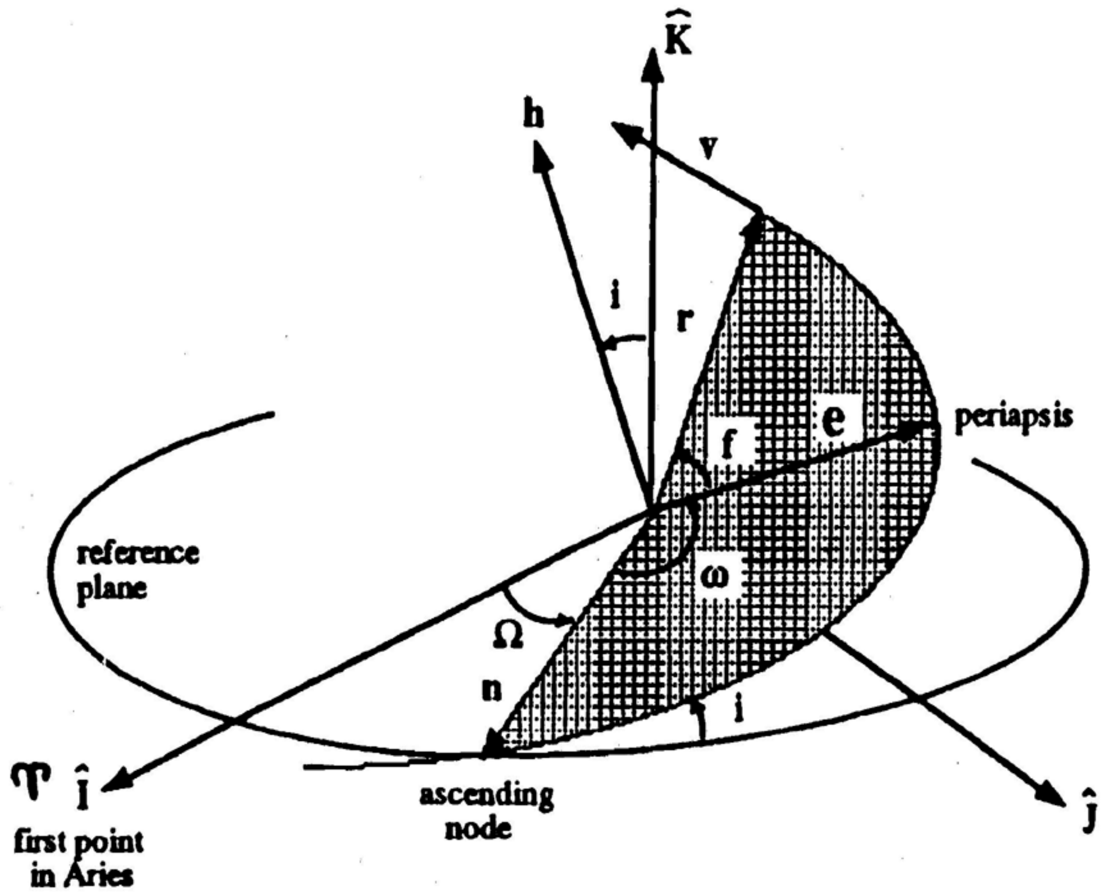
$\omega \quad \{$

$\Omega + \omega = \varpi \quad \{$

$\varpi + \theta^* = L \quad \{$

3-1-3 (body-two) Euler sequence $\Longrightarrow$ transformation matrix

	$\hat{r}$	$\hat{\theta}$	$\hat{h}$
$\hat{x}$	$c_\Omega c_\theta - s_\Omega c_i s_\theta$	$-c_\Omega s_\theta - s_\Omega c_i c_\theta$	$s_\Omega s_i$
$\hat{y}$	$s_\Omega c_\theta + c_\Omega c_i s_\theta$	$-s_\Omega s_\theta + c_\Omega c_i c_\theta$	$-c_\Omega s_i$
$\hat{z}$	$s_i s_\theta$	$s_i c_\theta$	c_i



Ω : longitude of the ascending node

ω : argument of periapsis

$\tilde{\omega} = \Omega + \omega$: longitude of periapsis

f : true anomaly

$L = \tilde{\omega} + f$: true longitude