Newtonian Mechanics

{Read Ch2, App. C in Kasdin & Paley}

→ Newton published Principia 1687

A <u>law</u> is a descriptive generalization about some aspect of how the natural world behaves. A law is not a theory, which is just an explanation.

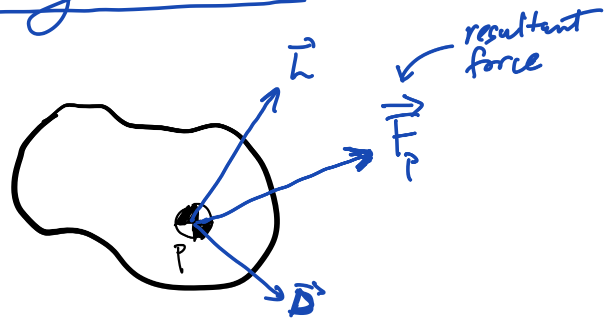
LAW I: Every body preserves its state of rest or uniform, straight-line motion, unless it is compelled to change its state under the action of forces.

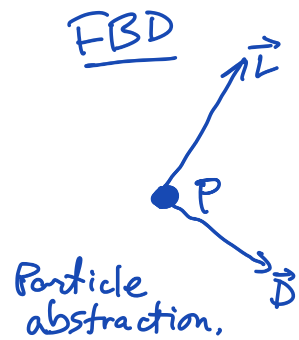
LAW II (N2L): The alteration of motion is ever proportional to the motive force impressed and it is made in the direction of the the right line in which that force is impressed.

LAW III: To every action, there is always opposed an equal reaction or the mutual actions of the two bodies upon each other are always equal, and direct contrary.

Applying Newton's laws to a problem can help us to derive the <u>equations of motion</u> of the system.

One key step in Newton's process is drawing the <u>Free Body Diagram</u>

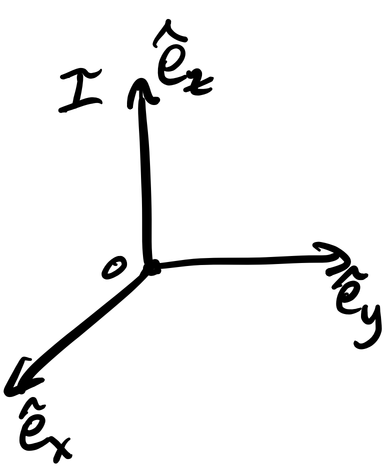


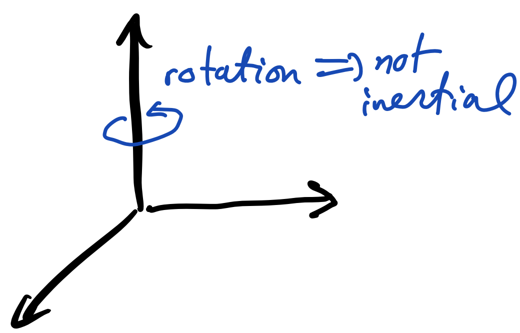


<u>Key points to remember about N2L</u>

① N2L only applies to a <u>point mass</u>

② N2L only applies in an <u>inertial frame</u>
↳ (a.k.a. absolute space)





(OR, an inertial frame can be moving at a <u>constant</u> velocity, but not rotating.)

Note: We can create approximate inertial frames to solve problems as long as the relative acceleration of the frame is very small relative to the acceleration involved in the system we're studying.

N2L:

$$\vec{F}_P = m_P \, {}^I\vec{a}_{P/O}$$

inertial acceleration

$$= m_P \frac{{}^I d}{dt}\left({}^I\vec{v}_{P/O}\right)$$

$${}^I\vec{v}_{P/O} = \frac{{}^I d}{dt}\left(\vec{r}_{P/O}\right)$$

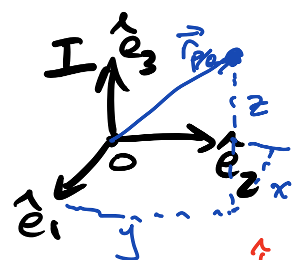
Cartesian (very simple) example

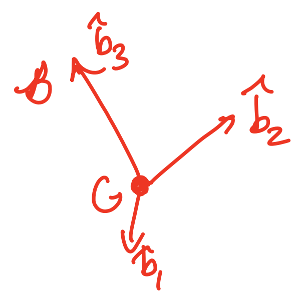
$$\vec{r}_{P/O} = x\hat{e}_1 + y\hat{e}_2 + z\hat{e}_3$$

$${}^I\vec{v}_{P/O} = \frac{{}^I d}{dt}\left(\vec{r}_{P/O}\right)$$

$$= \frac{{}^I d}{dt}\left(x\hat{e}_1 + y\hat{e}_2 + z\hat{e}_3\right)$$

$$= \dot{x}\hat{e}_1 + \dot{y}\hat{e}_2 + \dot{z}\hat{e}_3$$





What if we had $\frac{{}^B d}{dt}(\ )$?

$${}^B\vec{v}_{P/O} = \frac{{}^B d}{dt}\left(x\hat{e}_1 + y\hat{e}_2 + z\hat{e}_3\right)$$

$$= \frac{{}^B d}{dt}\left(x\hat{e}_1\right) + \frac{{}^B d}{dt}\left(y\hat{e}_2\right) + \frac{{}^B d}{dt}\left(z\hat{e}_3\right)$$

$$\frac{{}^B d}{dt}\left(\vec{r}_{P/O}\right) = \dot{x}\hat{e}_1 + x\underbrace{\frac{{}^B d}{dt}\hat{e}_1} + \dot{y}\hat{e}_2 + y\frac{{}^B d}{dt}\hat{e}_2 + \dot{z}\hat{e}_3 + z\frac{{}^B d}{dt}\left(\hat{e}_3\right)$$

We will see what this is soon!

N2L in scalar coordinate form:

Let,

$$\vec{F}_P = f_x \hat{e}_x + f_y \hat{e}_y + f_z \hat{e}_z$$

$${}^{I}\vec{a}_{P/O} = \ddot{x}\hat{e}_x + \ddot{y}\hat{e}_y + \ddot{z}\hat{e}_z$$

$$\Rightarrow \quad \vec{F}_P = m\,{}^{I}\vec{a}_{P/O}$$

$$f_x \hat{e}_x + f_y \hat{e}_y + f_z \hat{e}_z = m(\ddot{x}\hat{e}_x + \ddot{y}\hat{e}_y + \ddot{z}\hat{e}_z)$$

$$= m\ddot{x}\hat{e}_x + m\ddot{y}\hat{e}_y + m\ddot{z}\hat{e}_z$$

Equating coefficients,

$$\left.\begin{aligned} f_x &= m\ddot{x} \\ f_y &= m\ddot{y} \\ f_z &= m\ddot{z} \end{aligned}\right\}$$

Three, scalar, second-order differential equations.

Dfn: The <u>equations of motion</u> are (usually differential equations) whose solutions are the position and velocity of the particle or object.

ODE's - ordinary differential equations

when combined with an initial condition, an ODE gives us an <u>Initial-Value Problem</u>.

There are many techniques to integrate or "solve" ordinary differential equations.

$$f(x, \dot{x}, \ddot{x}, \ldots, t) = 0 \xrightarrow{\text{"Solving"}} x(t)$$

plus initial conditions

trajectory

Differential Equations Class

- Separation of variables
- Laplace transforms
- Integrating factor method
- Guess and check solution
- Etc.

Numerical Methods Class

- Forward and Backward Euler
- Runge-Kutta methods
- Etc.

ode45

A key step in numerical solution of an ODE is putting the equation in a <u>1st order form</u> (state-space form).

Ex.

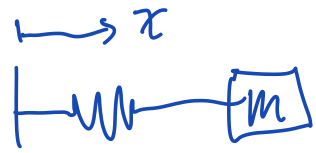
$$\vec{\xi} = \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \\ z \\ \dot{z} \end{bmatrix}$$

state vector

$$\dot{\vec{\xi}} = \left.\begin{bmatrix} \xi_2 \\ f_x/m \\ \xi_4 \\ f_y/m \\ \xi_6 \\ f_z/m \end{bmatrix}\right\}$$

1st order form for our 3 E.O.M!s

The state vector characterizes everything you need to know about a system at a given time so you can predict its motion at the next instant, using the equations of motion.



Need to know x and $\dot{x}$