4.5 Nonlinear system of DEs

$$\mathbf{y}' = \mathbf{f}(\mathbf{y}) = \begin{bmatrix} f_1(y_1, y_2) \\ f_2(y_1, y_2) \end{bmatrix}$$

(Theorem 1)

$$\mathbf{y}' = A\mathbf{y}$$

(1) (Hypotheses)

$f_1(y_1, y_2)$ and $f_2(y_1, y_2)$ are **continuous** and have **continuous partial derivatives** in a neighborhood of the critical point (0, 0). det(A) is nonzero.
A is diagonalizable (: Case 1), or real part of complex eigenvalues of A is nonzero (: Case 2)

(Conclusion)
The nonlinear and linearized systems have the **same kind of critical points and the same stability.**

(2) If A has equal eigenvalues (**case 3**) or pure-imaginary eigenvalues (**case 2**), then the nonlinear system may have the same kind of critical points as the linearized system, or a spiral point.