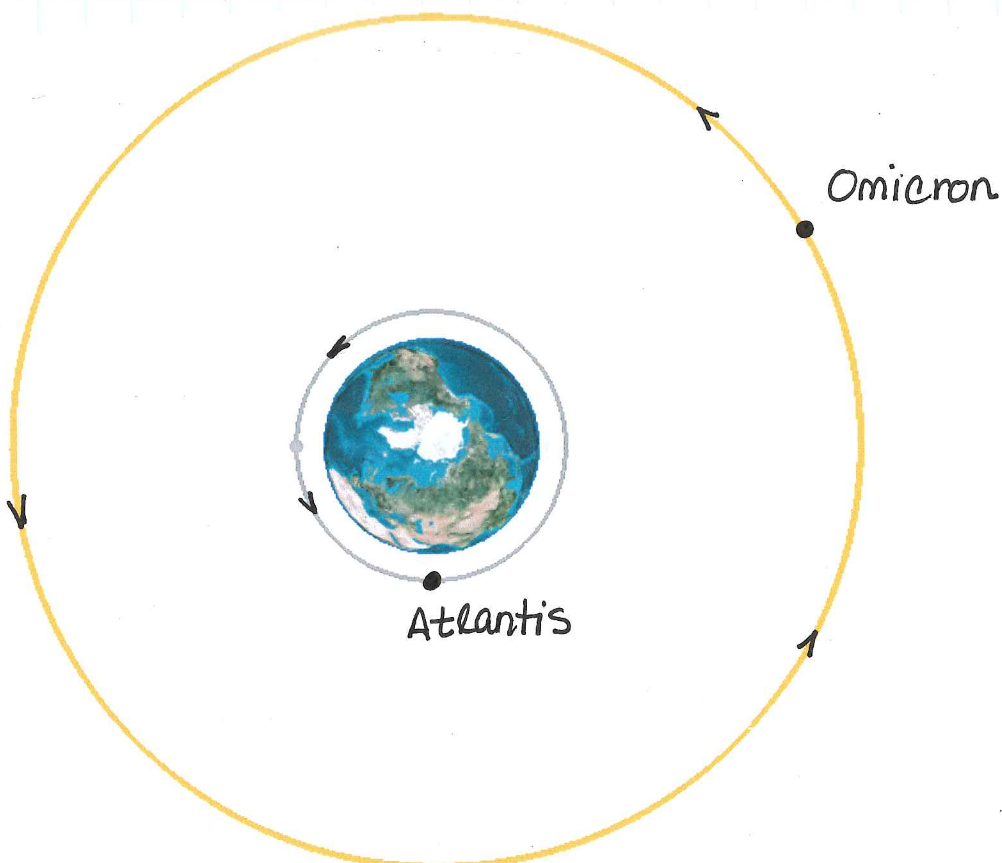
Example:

A space facility (Omicron) is currently in a circular Earth orbit at $4R_\oplus$. The spaceship Atlantis is currently in a coplanar, circular orbit at $1.25R_\oplus$. You are asked to provide information on possible transfer orbits for Atlantis to rendezvous with Omicron.

Consider the potential transfer angle: $120°$

Try TOF $= 15$ hr



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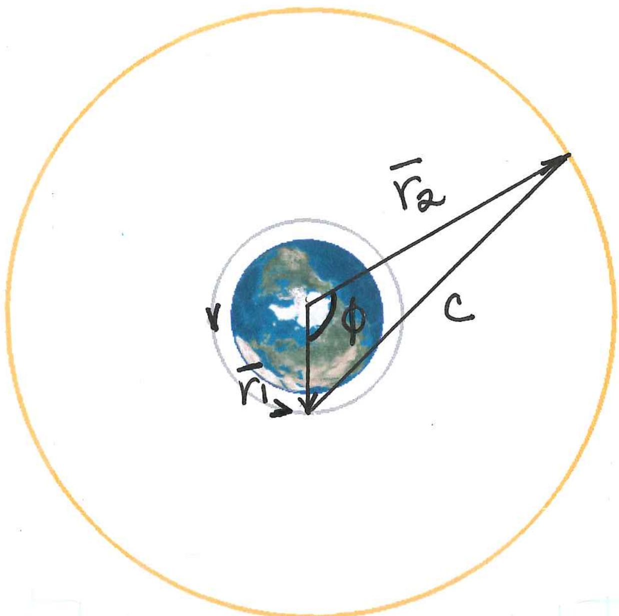
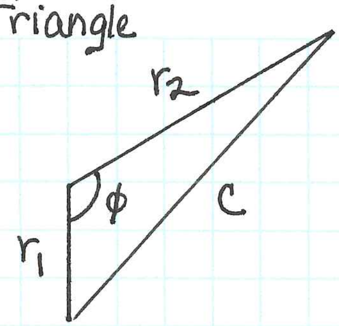


Space Triangle



(a) $r_1 = 1.25 R_\oplus$

$r_2 = 4 R_\oplus$

$TA = 120° = \phi$

(b) Conditions on original orbit/final orbit

$r_1 = 1.25 R_\oplus$

$v_1 = \sqrt{\frac{\mu}{r_1}} = 7.071$ Km/s   7.0707685

$\gamma_1 = 0°$

$r_2 = 4 R_\oplus$

$v_2 = \sqrt{\frac{\mu}{r_2}} = 3.953$ Km/s

$\gamma_2 = 0°$

(c) Transfer: elliptical or hyperbolic?

1. Check parabolic

$$TOF_{par} = \frac{1}{3}\sqrt{\frac{2}{\mu_\oplus}}\left[s^{3/2} - (s-c)^{3/2}\right]$$

2. 1A OR 1B $\Longrightarrow$ check minimum energy path

$$a_{min} = \frac{s}{2} =$$

$$\alpha_0 = 2\sin^{-1}\sqrt{\frac{s}{2a}} =$$

$$\beta_0 = 2\sin^{-1}\sqrt{\frac{s-c}{2a}} =$$

$$TOF = \sqrt{\frac{a^3}{\mu}}\left[(\alpha - S_\alpha) - (\beta - S_\beta)\right] =$$

$$TOF > TOF_{min} \Longrightarrow$$

3. Iterate on 'a'

$$1B \Longrightarrow \quad \alpha = 2\pi - \alpha_0 \quad \Longrightarrow$$
$$\beta = \beta_0$$

$$p = \frac{4a(s-r_1)(s-r_2)}{c^2}\sin^2\left(\frac{\alpha \pm \beta}{2}\right)$$

smaller or larger p ?

$\downarrow$

check

$$\phi = \theta_A^* - \theta_D^* = TA \quad \left\{ \begin{array}{l} \theta_D^* \\ \theta_A^* \end{array} \right.$$

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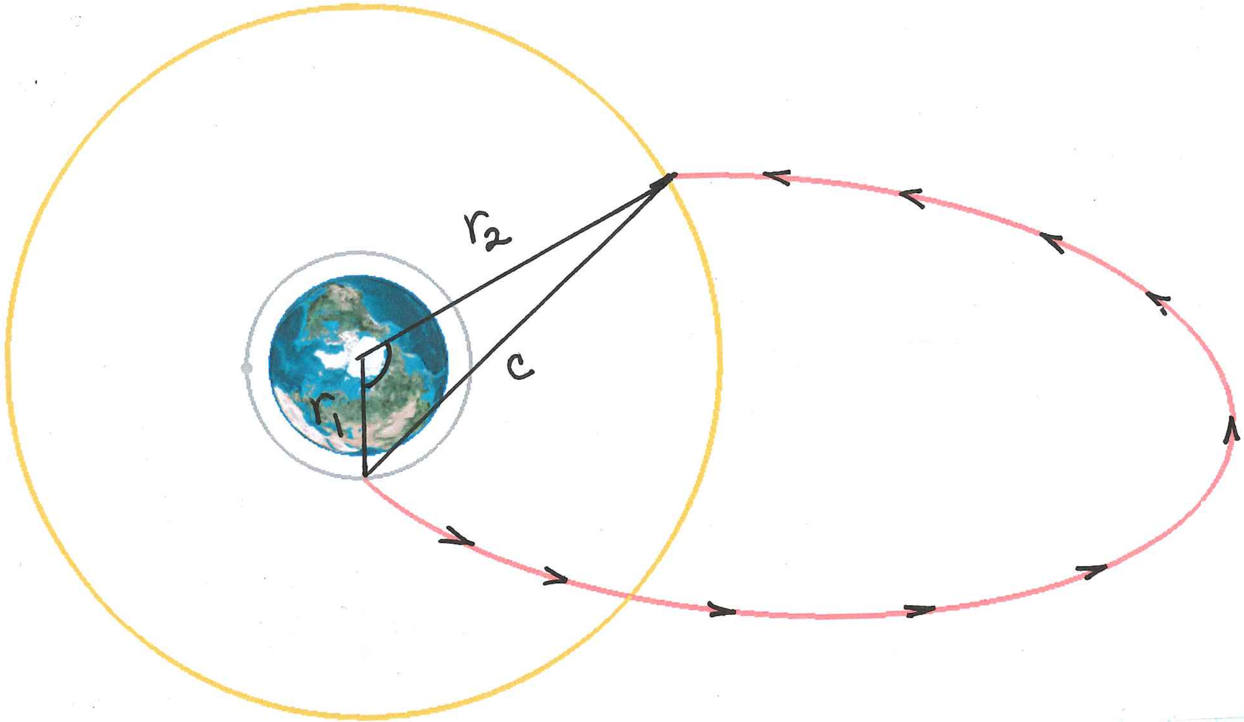

4. $\bar{r}_A = f\,\bar{r}_D + g\,\bar{v}_D \implies \bar{v}_D = \dfrac{\bar{r}_A - f\,\bar{r}_D}{g}$

$f = \left\{ 1 - \dfrac{r_A}{p}\left[1 - \cos TA\right] \right\} = -4.2726$

$g = \dfrac{r_D r_A}{\sqrt{\mu p}} \sin TA = 3274.98\,s$

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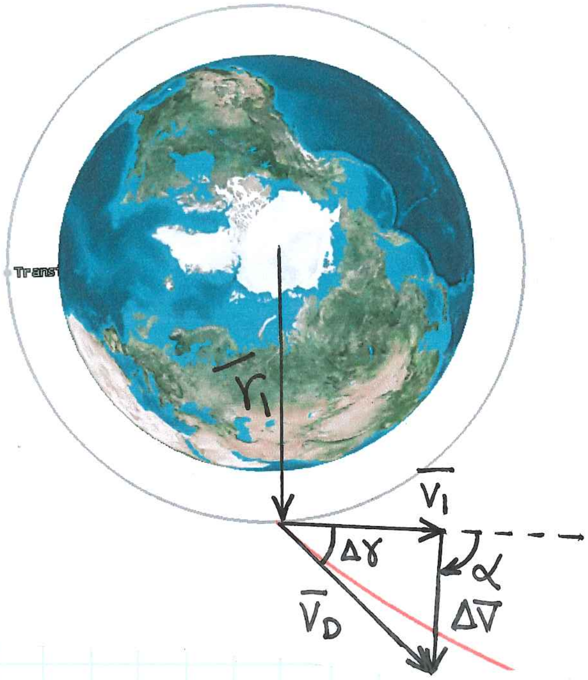

5. $\bar{V}_A = \dot{f}\,\bar{r}_D + \dot{g}\,\bar{V}_D$

$$\dot{f} = \frac{\bar{r}_D \cdot \bar{V}_D}{p\, r_D}(1-\cos TA) - \frac{1}{r_D}\sqrt{\frac{\mu}{p}}\,\sin TA =$$

$$\dot{g} = 1 - \frac{r_D}{p}(1-\cos TA) =$$

$\bar{V}_A = .088336\,\hat{x} - 4.3695\,\hat{y}$ Km/s $\quad V_A = 4.3704$ Km/s

$\Delta\bar{V}_A = -3.5115\,\hat{x} + 2.3931\,\hat{y}$ Km/s $\quad \Delta V_A$