HW1 Gabe Colangelo

```
clear
close all
warning off
clc
```

Problem 1 Lagrangian Derivation & E.O.M

```
% Lagrangian Derivation
% x1     = x
% x2     = theta1
% x3     = theta2
% x4     = x_dot
% x5     = theta_dot_1
% x6     = theta_dot_2

syms x(t) x_dot(t) theta1(t) theta_dot_1(t) theta2(t) theta_dot_2(t) M m1 m2...
     L1 L2 x1 x2 x3 x4 x5 x6 g theta1_int theta2_int theta_ddot_1 x_ddot ...
     theta_ddot_2 u s real

% Position vector to cart - inertial coordinates
r_M         = [x;0;0];

% Rotation Matrix from body frame 1 to inertial frame
I_DCM_B     = [sin(theta1), cos(theta1), 0;...
               cos(theta1), -sin(theta1),0;...
               0, 0 ,-1];

% Position vector to mass 1, inertial frame: r_m1 = x ix + l br
r_m1        = r_M + I_DCM_B*[L1;0;0];

% Define angle between body frame 1 and body frame 2
beta        = theta2 - theta1;

% Rotation Matrix from body frame 1 to inertial frame
B_DCM_C     = [cos(beta), -sin(beta), 0;...
               sin(beta), cos(beta), 0;...
               0, 0, 1];

% Position vector to mass 2, inertial frame: r_m2 = x ix + l br + l cr
r_m2        = r_M + I_DCM_B*[L1;0;0] + I_DCM_B*B_DCM_C*[L2;0;0];

% Inertial Velocities
v_M        = subs(diff(r_M),diff(x(t),t),x_dot);
```

```
v_m1       = simplify(subs(diff(r_m1),[diff(x(t),t), diff(theta1(t),t)]...
               ,[x_dot, theta_dot_1]));

v_m2       = simplify(subs(diff(r_m2),[diff(x(t),t), diff(theta1(t),t)...
               ,diff(theta2(t),t)],[x_dot, theta_dot_1, theta_dot_2]));

% Kinetic Energy
T          = simplify((1/2)*M*transpose(v_M)*v_M +
(1/2)*m1*transpose(v_m1)*v_m1...
                + (1/2)*m2*transpose(v_m2)*v_m2);

% Gravitational Forces
W_M        = [0;-M*g;0];
W_m1       = [0;-m1*g;0];
W_m2       = [0;-m2*g;0];

% Differential Displacements
dsM        = subs(diff(r_M),diff(x(t),t),1);
dsm1       = simplify(subs(diff(r_m1),[diff(x(t),t), diff(theta1(t),t)]...
               ,[1 1]));
dsm2        = simplify(subs(diff(r_m2),[diff(x(t),t), diff(theta1(t),t)...
               ,diff(theta2(t),t)],[1 1 1]));

% Potential Energies of mass M: V = -int(dot(F,ds))
V_M        = -int(transpose(W_M)*dsM);

% Potential Energy of mass 1
int_m1     = subs(transpose(W_m1)*dsm1,theta1,theta1_int);
V_m1       = subs(-int(int_m1,theta1_int),theta1_int,theta1);

% Potential Energy of mass 2
int_m2     = subs(transpose(W_m2)*dsm2,[theta1 theta2],[theta1_int,
theta2_int]);
V_m2       = subs(-int(subs(int_m2,theta2_int,0),theta1_int) +....
            -
int(subs(int_m2,theta1_int,0),theta2_int),[theta1_int,theta2_int],...
                [theta1,theta2]);

% Total potential energy
V          = simplify(V_M + V_m1 + V_m2);

% Lagrangian
disp('The Lagrangian is')
```

```
The Lagrangian is
```

```matlab
L            = simplify(T - V)
```

L(t) =

$$\frac{m_2\,\left(\dot{x}(t)+\sigma_1+L_2\cos(\theta_2(t))\,\dot{\theta}_2(t)\right)^2}{2}+\frac{m_1\,\left(\dot{x}(t)+\sigma_1\right)^2}{2}+\frac{m_2\,\left(L_1\sin(\theta_1(t))\,\dot{\theta}_1(t)+L_2\sin(\theta_2(t))\,\dot{\theta}_2(t)\right)^2}{2}+\frac{M\,\dot{x}(t)^2}{2}+\frac{{L_1}^2\,m_1\sin(\theta_1(t))^2\,\dot{\theta}_1(t)^2}{2}-L_1\,g\,m_1\cos(\theta_1(t))-L_1\,g\,m_2\cos(\theta_1(t))-L_2\,g\,m_2\cos(\theta_2(t))$$

where

$$\sigma_1=L_1\cos(\theta_1(t))\,\dot{\theta}_1(t)$$

```matlab
% Lagrange Equations of Motions: d/dt(del_L/del_qdot) - del_L/del_q = Q
disp('Lagranges Equations of Motion are:')
```

```
Lagranges Equations of Motion are:
```

```matlab
% : q = x, Q = u
eqn_x      = subs(simplify(diff(diff(L,x_dot),t) - diff(L,x)),[diff(x(t),t),
diff(theta1(t),t)...
              ,diff(theta2(t),t), diff(x_dot,t), diff(theta_dot_1(t), t),
diff(theta_dot_2(t), t)],...
              [x_dot, theta_dot_1, theta_dot_2, x_ddot, theta_ddot_1,
theta_ddot_2]) == u
```

eqn_x(t) =

$$M\,\ddot{x}+m_1\,\ddot{x}+m_2\,\ddot{x}-L_1\,m_1\sin(\theta_1(t))\,\dot{\theta}_1(t)^2-L_1\,m_2\sin(\theta_1(t))\,\dot{\theta}_1(t)^2-L_2\,m_2\sin(\theta_2(t))\,\dot{\theta}_2(t)^2+L_1\,m_1\,\ddot{\theta}_1\cos(\theta_1(t))+L_1\,m_2\,\ddot{\theta}_1\cos(\theta_1(t))+L_2\,m_2\,\ddot{\theta}_2\cos(\theta_2(t))=u$$

```matlab
% : q = theta1, Q = 0
eqn_theta1 = subs(simplify(diff(diff(L,theta_dot_1),t) -
diff(L,theta1)),[diff(x(t),t), diff(theta1(t),t)...
              ,diff(theta2(t),t), diff(x_dot,t), diff(theta_dot_1(t), t),
diff(theta_dot_2(t), t)],...
              [x_dot, theta_dot_1, theta_dot_2, x_ddot, theta_ddot_1,
theta_ddot_2]) == 0
```

eqn_theta1(t) =

$${L_1}^2\,m_1\,\ddot{\theta}_1+{L_1}^2\,m_2\,\ddot{\theta}_1-L_1\,g\,m_1\sin(\theta_1(t))-L_1\,g\,m_2\sin(\theta_1(t))+L_1\,m_1\,\ddot{x}\cos(\theta_1(t))+L_1\,m_2\,\ddot{x}\cos(\theta_1(t))+L_1\,L_2\,m_2\sin(\sigma_1)\,\dot{\theta}_2(t)^2+L_1\,L_2\,m_2\,\ddot{\theta}_2\cos(\sigma_1)=0$$

where

$$\sigma_1=\theta_1(t)-\theta_2(t)$$

```matlab
% : q = theta2, Q = 0
eqn_theta2 = subs(simplify(diff(diff(L,theta_dot_2),t) -
diff(L,theta2)),[diff(x(t),t), diff(theta1(t),t)...
              ,diff(theta2(t),t), diff(x_dot,t), diff(theta_dot_1(t), t),
diff(theta_dot_2(t), t)],...
              [x_dot, theta_dot_1, theta_dot_2, x_ddot, theta_ddot_1,
theta_ddot_2]) == 0
```

eqn_theta2(t) =

$$L_2^2\, m_2\, \ddot{\theta}_2 - L_2\, g\, m_2 \sin(\theta_2(t)) + L_2\, m_2\, \ddot{x} \cos(\theta_2(t)) - L_1\, L_2\, m_2 \sin(\theta_1(t) - \theta_2(t))\, \dot{\theta}_1(t)^2 + L_1\, L_2\, m_2\, \ddot{\theta}_1 \cos(\theta_1(t) - \theta_2(t)) = 0$$

```
% Solve system of equations for 2nd derivative of states
sys_eqn    =
solve([eqn_x,eqn_theta1,eqn_theta2],[x_ddot,theta_ddot_1,theta_ddot_2]);

% Lagrange Equation of motion, state space form
fprintf('\n')
disp('Lagranges Equation of motion in state space form are given by')
```

Lagranges Equation of motion in state space form are given by

```
x1_dot      = x4
```

x1_dot = x_4

```
x2_dot      = x5
```

x2_dot = x_5

```
x3_dot      = x6
```

x3_dot = x_6

```
x4_dot      = subs(simplify(sys_eqn.x_ddot),[x theta1 theta2 x_dot theta_dot_1
theta_dot_2],[x1 x2 x3 x4 x5 x6])
```

x4_dot =

$$\frac{2\, m_1\, u + m_2\, u - m_2\, u\, \sigma_1 - g\, m_1^2 \sin(2\, x_2) + 2\, L_1\, m_1^2\, x_5^2 \sin(x_2) - g\, m_1\, m_2 \sin(2\, x_2) + 2\, L_1\, m_1\, m_2\, x_5^2 \sin(x_2) + L_2\, m_1\, m_2\, x_6^2 \sin(x_3) + L_2\, m_1\, m_2\, x_6^2 \sin(2\, x_2 - x_3)}{2\, M\, m_1 + M\, m_2 + m_1\, m_2 - m_1^2 \cos(2\, x_2) + m_1^2 - m_1\, m_2 \cos(2\, x_2) - M\, m_2\, \sigma_1}$$

where

$$\sigma_1 = \cos(2\, x_2 - 2\, x_3)$$

```
x5_dot      = subs(simplify(sys_eqn.theta_ddot_1),[x theta1 theta2 x_dot
theta_dot_1 theta_dot_2],[x1 x2 x3 x4 x5 x6])
```

x5_dot =

$$-\frac{m_1\, u \cos(x_2) + \frac{m_2\, u \cos(x_2)}{2} - \frac{m_2\, u \cos(x_2 - 2\, x_3)}{2} - g\, m_1^2 \sin(x_2) - M\, g\, m_1 \sin(x_2) - \frac{M\, g\, m_2 \sin(x_2)}{2} - g\, m_1\, m_2 \sin(x_2) + \frac{L_1\, m_1^2\, x_5^2 \sin(2\, x_2)}{2} - \frac{M\, g\, m_2 \sin(x_2 - 2\, x_3)}{2} + \frac{L_2\, m_1\, m_2\, x_6^2 \sin(x_2 + x_3)}{2} + L_2\, M\, m_2\, x_6^2 \sin(x_2 - x_3) + \frac{L_2\, m_1\, m_2\, x_6^2 \sin(x_2 - x_3)}{2} + \frac{L_1\, m_1\, m_2\, x_5^2 \sin(2\, x_2)}{2} + \frac{L_1\, M\, m_2\, x_5^2 \sin(\sigma_1)}{2}}{L_1 \left(M\, m_1 + \frac{M\, m_2}{2} + \frac{m_1\, m_2}{2} - \frac{m_1^2 \cos(2\, x_2)}{2} + \frac{m_1^2}{2} - \frac{m_1\, m_2 \cos(2\, x_2)}{2} - \frac{M\, m_2 \cos(\sigma_1)}{2}\right)}$$

where

$$\sigma_1 = 2\, x_2 - 2\, x_3$$

```
x6_dot      = subs(simplify(sys_eqn.theta_ddot_2),[x theta1 theta2 x_dot
theta_dot_1 theta_dot_2],[x1 x2 x3 x4 x5 x6])
```

x6_dot =

$$\frac{\frac{m_1\,u\,\sigma_2}{2}-\frac{m_2\,u\cos(x_3)}{2}-\frac{m_1\,u\cos(x_3)}{2}+\frac{m_2\,u\,\sigma_2}{2}-\frac{M\,g\,m_1\,\sigma_1}{2}-\frac{M\,g\,m_2\,\sigma_1}{2}+\frac{M\,g\,m_1\sin(x_3)}{2}+\frac{M\,g\,m_2\sin(x_3)}{2}+L_1\,M\,m_1\,{x_5}^2\sin(x_2-x_3)+L_1\,M\,m_2\,{x_5}^2\sin(x_2-x_3)+\frac{L_2\,M\,m_2\,{x_6}^2\sin(\sigma_3)}{2}}{L_2\left(M\,m_1+\frac{M\,m_2}{2}+\frac{m_1\,m_2}{2}-\frac{{m_1}^2\cos(2\,x_2)}{2}+\frac{{m_1}^2}{2}-\frac{m_1\,m_2\cos(2\,x_2)}{2}-\frac{M\,m_2\cos(\sigma_3)}{2}\right)}$$

where

$\sigma_1 = \sin(2\,x_2 - x_3)$

$\sigma_2 = \cos(2\,x_2 - x_3)$

$\sigma_3 = 2\,x_2 - 2\,x_3$

Part 2 Linearized Model Derivation

```
% Numerical Parameters
m1_num       = 0.5;
L1_num       = 0.5;
m2_num       = 0.75;
L2_num       = 0.75;
M_num        = 1.5;
g_num        = 9.81;

% Define Non-linear system
f            = [x1_dot;x2_dot;x3_dot;x4_dot;x5_dot;x6_dot];
h            = [x1;x2;x3];

% Jacobian Matrices
df_dx        = jacobian(f,[x1;x2;x3;x4;x5;x6]);
df_du        = jacobian(f,u);
dh_dx        = jacobian(h,[x1;x2;x3;x4;x5;x6]);
dh_du        = jacobian(h,u);

% Equilibirum pair - origin
xe           = zeros(6,1);
ue           = 0;

fprintf('\n')
disp('The linearized state space model about the origin is')
```

```
The linearized state space model about the origin is
```

```
% Linearized Model
A            =
double(subs(df_dx,[x1;x2;x3;x4;x5;x6;ue;m1;L1;m2;L2;M;g],[xe;ue;m1_num;L1_num;m2_num;L2_num;M_num;g_num]))
```

```
A = 6×6
         0         0         0    1.0000         0         0
         0         0         0         0    1.0000         0
         0         0         0         0         0    1.0000
         0   -8.1750         0         0         0         0
```

```
         0    65.4000   -29.4300          0          0          0
         0   -32.7000    32.7000          0          0          0
```

```
B            = double(subs(df_du,[x1;x2;x3;x4;x5;x6;ue;m1;L1;m2;L2;M;g],[xe;ue;m1_num;L1_num;m2_num;L2_num;M_num;g_num]))
```

```
B = 6×1
         0
         0
         0
    0.6667
   -1.3333
         0
```

```
C            = double(dh_dx)
```

```
C = 3×6
     1     0     0     0     0     0
     0     1     0     0     0     0
     0     0     1     0     0     0
```

```
D            = double(dh_du)
```

```
D = 3×1
     0
     0
     0
```

Problem 3 Linearized Model

```
% Controllability Matrix
Co           = ctrb(A,B);

% Observability Matrix
Ob           = obsv(A,C);

% Number of states
n            = length(A);

% Verify system is reachable & observable
if rank(Co) == n
    disp('Pair (A,B) of the linearized model is reachable/controllable')
end
```

```
Pair (A,B) of the linearized model is reachable/controllable
```

```
if rank(Ob) == n
    disp('Pair (A,C) of the linearized model is observable')
end
```

```
Pair (A,C) of the linearized model is observable
```

```
% Inverse of Controllability Matrix
Co_inv       = inv(Co);

% Last row of inverse of Controllability Matrix
q1           = Co_inv(end,:);

% Transformation matrix used to get CCF
T_ccf        = [q1;q1*A;q1*A^2;q1*A^3;q1*A^4;q1*A^5];

% Transform System into Controller Canonical Form
disp('The Linearized Model in Controller Form is')
```

```
The Linearized Model in Controller Form is
```

```
A_ccf        = T_ccf*A*inv(T_ccf)
```

```
A_ccf = 6×6
10³ ×
         0    0.0010         0   -0.0000         0   -0.0000
         0         0    0.0010         0    0.0000         0
         0         0         0    0.0010         0    0.0000
         0         0    0.0000         0    0.0010         0
         0         0         0         0         0    0.0010
         0         0   -1.1762         0    0.0981         0
```

```
B_ccf        = T_ccf*B
```

```
B_ccf = 6×1
         0
         0
         0
   -0.0000
         0
    1.0000
```

```
C_ccf        = C*inv(T_ccf)
```

```
C_ccf = 3×6
  427.7160         0  -54.5000         0    0.6667         0
         0         0   43.6000         0   -1.3333         0
         0         0   43.6000         0    0.0000         0
```

```
D_ccf        = D
```

```
D_ccf = 3×1
     0
     0
     0
```

```
% Controllability Matrix of (A',C')
Co_dual      = ctrb(A',C');
```

```
% Row reduced echelon form of Controlability, get indice of pivots
[co_ref,p]  = rref(Co_dual);

% Form L martix from inspection - L = [c1',A'*c1', c2',A'*c2', c3', A'*c3']
L_obs       = [C(1,:)', A'*C(1,:)',C(2,:)', A'*C(2,:)',C(3,:)', A'*C(3,:)'];

% Inverse of L
inv_L       = inv(L_obs);

% Observability Matrices from inspection
d1_obs      = 2;
d2_obs      = 2;
d3_obs      = 2;

% Vectors needed for transformation matrix
q1          = inv_L(d1_obs,:);
q2          = inv_L(d2_obs + d1_obs,:);
q3          = inv_L(d3_obs + d2_obs + d1_obs,:);

% Form Transformation Matrix
T_obs       = [q1; q1*A'; q2; q2*A'; q3; q3*A'];

% Transform linear system into observer form
disp('The Linearized Model in Observer Form is')
```

```
The Linearized Model in Observer Form is
```

```
A_obs       = (T_obs*A'*inv(T_obs))'
```

```
A_obs = 6×6
         0         0         0   -8.1750         0         0
    1.0000         0         0         0         0         0
         0         0         0   65.4000         0  -29.4300
         0         0    1.0000         0         0         0
         0         0         0  -32.7000         0   32.7000
         0         0         0         0    1.0000         0
```

```
C_obs       = (T_obs*C')'
```

```
C_obs = 3×6
     0     1     0     0     0     0
     0     0     0     1     0     0
     0     0     0     0     0     1
```

```
B_obs       = (B'*inv(T_obs))'
```

```
B_obs = 6×1
    0.6667
         0
```

```
   -1.3333
         0
         0
         0
```

```
D_obs        = D
```

```
D_obs = 3×1
     0
     0
     0
```

Problem 4 Transfer Function

```
% Transfer Function Matrix Equation
Y_U          = C*inv(s*eye(size(A)) - A)*B + D;

disp('Transfer function for X to u:')
```

```
Transfer function for X to u:
```

```
disp(simplify(Y_U(1)))
```

$$\frac{4\ (500\, s^4 - 40875\, s^2 + 320787)}{3\, s^2\ (1000\, s^4 - 98100\, s^2 + 1176219)}$$

```
disp('Trasnfer Function for theta_1 to u:')
```

```
Transfer Function for theta_1 to u:
```

```
disp(Y_U(2))
```

$$-\frac{400\ (10\, s^2 - 327)}{3\ (1000\, s^4 - 98100\, s^2 + 1176219)}$$

```
disp('Transfer Function for theta_2 to u:')
```

```
Transfer Function for theta_2 to u:
```

```
disp(Y_U(3))
```

$$\frac{43600}{1000\, s^4 - 98100\, s^2 + 1176219}$$

Problem 5 - Part 1 Pendulum Simulation

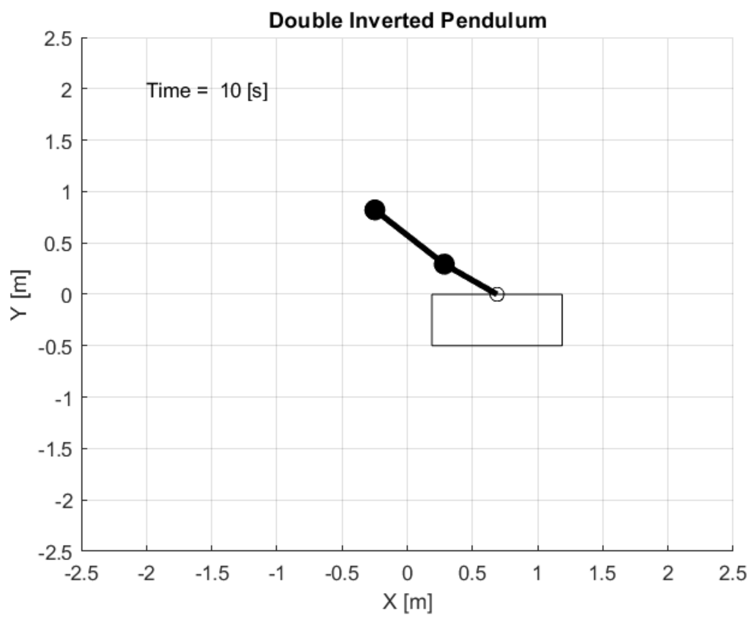
```
% Initial Conditions [m, rad, rad, m/s, rad/s, rad/s]
x0           = [0 .01 .02 0 0 0]';

% Time interval
dt           = 1/200;

% time and input
```

```
time         = (0:dt:10)';
u            = 0;

% ODE45 Function Call
[~, X] = ode45(@(t,x)
DIPC(t,x,u,m1_num,m2_num,M_num,L1_num,L2_num,g_num),time,x0);
```

Problem 5 Part - 2 Animation

```
% Cart width and height
w            = 1;
h            = .5;

% Graphics handle - cart
cart         = rectangle('position',[X(1,1) - w/2, -h, w, h]);

% Graphics handle - hinge
hinge       = line('xdata', X(1,1),'ydata',0,'marker','o','markersize',7);

% Graphics handle - mass 1
mass1        = line('xdata', X(1,1) + L1_num*sin(X(1,2)), 'ydata',
L1_num*cos(X(1,2)),...
               'marker','o','markersize',10,'MarkerFaceColor','k');

% Graphics handle - bar 1
bar1         = line('xdata', [X(1,1) X(1,1) + L1_num*sin(X(1,2))],'ydata',...
                [0 L1_num*cos(X(1,2))],'linewidth',3);

% Graphics handle - mass 2
mass2        = line('xdata', X(1,1) + L1_num*sin(X(1,2)) + L2_num*sin(X(1,3)),
'ydata',...

L1_num*cos(X(1,2))+L2_num*cos(X(1,3)),'marker','o','markersize',10,'MarkerFaceCo
lor','k');

% Graphics handle - bar 2
bar2         = line('xdata', [(X(1,1) + L1_num*sin(X(1,2))), (X(1,1) +
L1_num*sin(X(1,2)) + L2_num*sin(X(1,3)))],'ydata',...
                [(L1_num*cos(X(1,2)))
(L1_num*cos(X(1,2))+L2_num*cos(X(1,3)))],'linewidth',3);

h_txt        = text(-2,2,strcat(['Time = ',' ', num2str(time(1)), ' [s]']));

% Define axis limits
```

```
 axis([-2*(L1_num + L2_num), 2*(L1_num + L2_num),-2*(L1_num + L2_num), 2*(L1_num
+ L2_num)]);
 grid on
 xlabel('X [m]')
 ylabel('Y [m]')
 title('Double Inverted Pendulum')

 % Video stuff
 vidobj      = VideoWriter('DIPC.avi');
 open(vidobj);
 nframes     = length(X);
 frames      = moviein(nframes);

 for i = 2:nframes

     % Update handles
     set(cart,'position',[X(i,1) - w/2, -h, w, h]);
     set(hinge,'xdata', X(i,1),'ydata',0,'marker','o','markersize',7);
     set(mass1,'xdata', X(i,1) + L1_num*sin(X(i,2)), 'ydata',
L1_num*cos(X(i,2)),...
                   'marker','o','markersize',10,'MarkerFaceColor','k');
     set(bar1,'xdata', [X(i,1) X(i,1) + L1_num*sin(X(i,2))],'ydata',...
                    [0 L1_num*cos(X(i,2))],'linewidth',3);
     set(mass2,'xdata', X(i,1) + L1_num*sin(X(i,2)) + L2_num*sin(X(i,3)),
'ydata',...

L1_num*cos(X(i,2))+L2_num*cos(X(i,3)),'marker','o','markersize',10,'MarkerFaceCo
lor','k');
     set(bar2,'xdata', [(X(i,1) + L1_num*sin(X(i,2))), (X(i,1) +
L1_num*sin(X(i,2)) + L2_num*sin(X(i,3)))],'ydata',...
                    [(L1_num*cos(X(i,2)))
(L1_num*cos(X(i,2))+L2_num*cos(X(i,3)))],'linewidth',3);
     set(h_txt,'String',strcat(['Time = ',' ', num2str(time(i)), ' [s]']));

     drawnow;
     frames(:,i) = getframe(gcf);
     writeVideo(vidobj,frames(:,i));
 end
```




```
close(vidobj);
```

Function for Double Inverted Cart Pendulum

```
function xdot = DIPC(t,x,u,m1,m2,M,L1,L2,g)

% States and inputs
x1      = x(1,1); % x
x2      = x(2,1); % theta_1
x3      = x(3,1); % theta_2
x4      = x(4,1); % xdot
x5      = x(5,1); % theta_1_dot
x6      = x(6,1); % theta_2_dot


% Equations of Motion
x1dot   = x4;    % xdot
x2dot   = x5;    % theta_1_dot
x3dot   = x6;    % theta_2_dot

% x_ddot
x4dot   = (2*m1*u + m2*u - m2*u*cos(2*x2 - 2*x3) - g*m1^2*sin(2*x2) +...
          2*L1*m1^2*x5^2*sin(x2) - g*m1*m2*sin(2*x2) +...
          2*L1*m1*m2*x5^2*sin(x2) + L2*m1*m2*x6^2*sin(x3) +...
          L2*m1*m2*x6^2*sin(2*x2 - x3))/(2*M*m1 + M*m2 + m1*m2 -...
          m1^2*cos(2*x2) + m1^2 - m1*m2*cos(2*x2) - M*m2*cos(2*x2 - 2*x3));

% theta_1_ddot
```

```matlab
x5dot   = -(m1*u*cos(x2) + (m2*u*cos(x2))/2 - (m2*u*cos(x2 - 2*x3))/2 -...
           g*m1^2*sin(x2) - M*g*m1*sin(x2) - (M*g*m2*sin(x2))/2 -...
           g*m1*m2*sin(x2) + (L1*m1^2*x5^2*sin(2*x2))/2 - (M*g*m2*...
           sin(x2 - 2*x3))/2 + (L2*m1*m2*x6^2*sin(x2 + x3))/2 + ...
           L2*M*m2*x6^2*sin(x2 - x3) + (L2*m1*m2*x6^2*sin(x2 - x3))/2 +...
           (L1*m1*m2*x5^2*sin(2*x2))/2 + (L1*M*m2*x5^2*sin(2*x2 - 2*x3))/2)...
           /(L1*(M*m1 + (M*m2)/2 + (m1*m2)/2 - (m1^2*cos(2*x2))/2 + m1^2/2 ...
           - (m1*m2*cos(2*x2))/2 - (M*m2*cos(2*x2 - 2*x3))/2));

% theta_2_ddot
x6dot   = ((m1*u*cos(2*x2 - x3))/2 - (m2*u*cos(x3))/2 - (m1*u*cos(x3))/2 +...
          (m2*u*cos(2*x2 - x3))/2 - (M*g*m1*sin(2*x2 - x3))/2 - ...
          (M*g*m2*sin(2*x2 - x3))/2 + (M*g*m1*sin(x3))/2 + ...
          (M*g*m2*sin(x3))/2 + L1*M*m1*x5^2*sin(x2 - x3) +...
          L1*M*m2*x5^2*sin(x2 - x3) + (L2*M*m2*x6^2*sin(2*x2 - 2*x3))/2)/...
          (L2*(M*m1 + (M*m2)/2 + (m1*m2)/2 - (m1^2*cos(2*x2))/2 + m1^2/2 -...
          (m1*m2*cos(2*x2))/2 - (M*m2*cos(2*x2 - 2*x3))/2));

xdot    = [x1dot;x2dot;x3dot;x4dot;x5dot;x6dot];

end
```