ECE 68000: MODERN AUTOMATIC CONTROL

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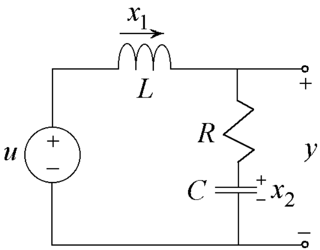
Unknown Input Observer---Example

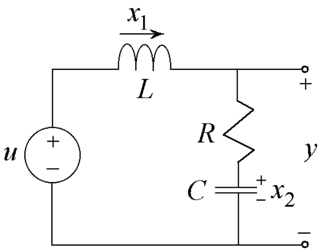
Full-Order UIO Example: Plant model



x_1 = Current through L
x_2 = Voltage across C

- By KVL, $L\frac{dx_1}{dt} + Rx_1 + x_2 = u$
- Hence, $\dot{x}_1 = -\frac{R}{L}x_1 - \frac{1}{L}x_2 + \frac{1}{L}u$
- The capacitor current, $x_1 = C\frac{dx_2}{dt}$
- Hence, $\frac{dx_2}{dt} = \dot{x}_2 = \frac{1}{C}x_1$

Circuit's state-space model



$$x_1 = \text{Current through } L$$
$$x_2 = \text{Voltage across } C$$

- Plant's model

$$\begin{aligned}\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} &= \begin{bmatrix}-\frac{R}{L} & -\frac{1}{L}\\ \frac{1}{C} & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}\frac{1}{L}\\ 0\end{bmatrix} u = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}_2\boldsymbol{u}_2\\ y &= \begin{bmatrix} R & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} = \boldsymbol{C}\boldsymbol{x}\end{aligned}$$

- Note that $\boldsymbol{u}_1 = 0$

Full-Order UIO Example: Numerical Values

- Let $R = 2$, $L = 2$, and $C = 1/2$.
- The model

$$\begin{aligned} \dot{\boldsymbol{x}} &= \begin{bmatrix} -1 & -0.5 \\ 2 & 0 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} u_2 \\ y &= \begin{bmatrix} 2 & 1 \end{bmatrix} \boldsymbol{x} \end{aligned}$$

UIO construction

- Since $\boldsymbol{CB}_2$ is square,

$$\boldsymbol{M} = \boldsymbol{B}_2(\boldsymbol{CB}_2)^{-1} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}.$$

- Then

$$\tilde{\boldsymbol{\Pi}} = \boldsymbol{I} - \boldsymbol{MC} = \begin{bmatrix} 0 & -0.5 \\ 0 & 1 \end{bmatrix}$$

- Use the second UIO architecture

$$\begin{aligned} \dot{\boldsymbol{q}} &= (\boldsymbol{I} - \boldsymbol{MC})(\boldsymbol{Aq} + \boldsymbol{AMy}) + \boldsymbol{L}(\boldsymbol{y} - \tilde{y}) \\ \tilde{y} &= \boldsymbol{C}\tilde{\boldsymbol{x}} \end{aligned}$$

- Note that $\boldsymbol{A}_1 = \tilde{\boldsymbol{\Pi}}\boldsymbol{A} = (\boldsymbol{I}_2 - \boldsymbol{MC})\boldsymbol{A}) = \begin{bmatrix} -1 & 0 \\ 2 & 0 \end{bmatrix}$
- We have

$$\boldsymbol{A}_1 - \boldsymbol{LC} = \begin{bmatrix} -1 & 0 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} -0 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

UIO dynamics

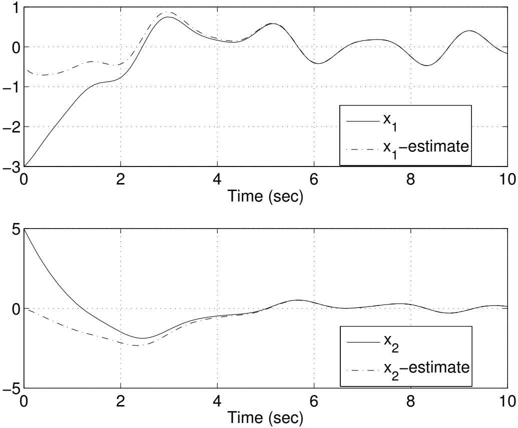
- Note that

$$(\boldsymbol{I}_2 - \boldsymbol{M}\boldsymbol{C})\boldsymbol{B}_2 = \begin{bmatrix} 0 & -0.5 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- The dynamics of the observer

$$\begin{aligned}
\dot{\boldsymbol{q}} &= (\boldsymbol{I} - \boldsymbol{M}\boldsymbol{C})(\boldsymbol{A}\boldsymbol{q} + \boldsymbol{A}\boldsymbol{M}\boldsymbol{y}) + \boldsymbol{L}(\boldsymbol{y} - \tilde{y}) \\
&= (\boldsymbol{I} - \boldsymbol{M}\boldsymbol{C})(\boldsymbol{A}\boldsymbol{q} + \boldsymbol{A}\boldsymbol{M}\boldsymbol{y}) + \boldsymbol{L}(\boldsymbol{y} - \boldsymbol{C}\boldsymbol{q} - \boldsymbol{C}\boldsymbol{M}\boldsymbol{y}) \\
&= \begin{bmatrix} -1 & 0 \\ 2 & 0 \end{bmatrix} \boldsymbol{q} + \begin{bmatrix} -0.5 \\ 1 \end{bmatrix} y + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (y - \tilde{y}) \\
\tilde{\boldsymbol{x}} &= \boldsymbol{q} + \boldsymbol{M}y = \boldsymbol{q} + \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} y \\
\tilde{y} &= \boldsymbol{C}\tilde{\boldsymbol{x}}
\end{aligned}$$

Full-Order UIO Numerical Example: Simulation Result



Unknown input:
$u_2(t) = \cos(5t) + 2\sin(3t)$

Initial conditions:
$\boldsymbol{x}(0) = [-3 \quad 5]^\top$
$\boldsymbol{q}(0) = \mathbf{0}$