PURDUE UNIVERSITY

ECE 602: LUMPED LINEAR SYSTEMS

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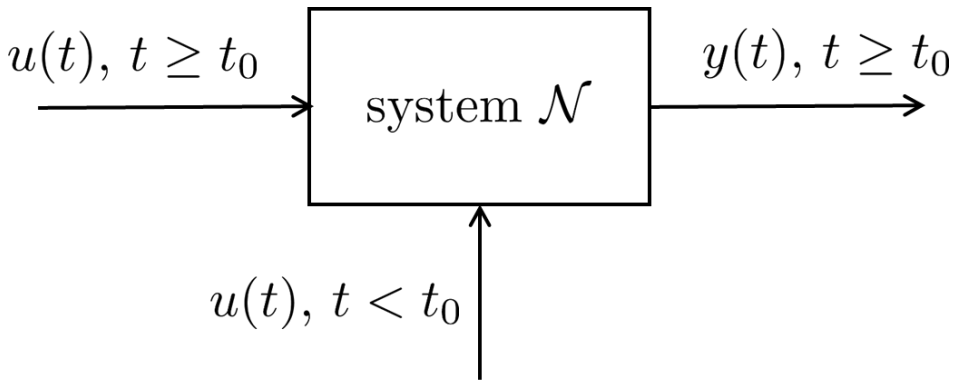
State Variables

State Variables: Motivation

Most systems $\mathcal{N}$ have memory: given a current time t_0, the output $y(t)$ after time t_0 depends on both **past input** before time t_0 and **future input** after time t_0.

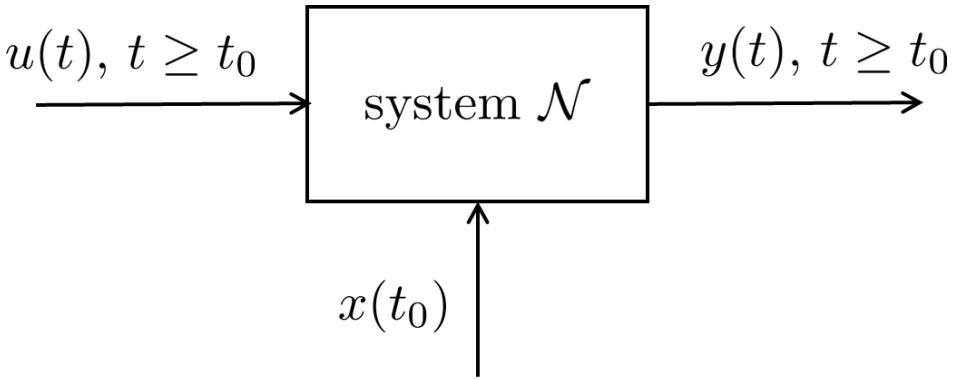


Example:

1. $\dot{y}(t) = u(t), \quad t \in \mathbb{R}$
2. $y[k] = y[k-1] - 2y[k-2] - u[k], \quad k \in \mathbb{Z}$

Definition of State Variables

State variables of a system $\mathcal{N}$ is a set of variables $x = (x_1, \ldots, x_n)$ whose values at any time t_0 together with future input after are sufficient to determine the system's future output $y(t)$, $t \geq t_0$.



- State variables summarize the effect of past input
- In certain contexts also called "initial conditions"

Examples of State Variables

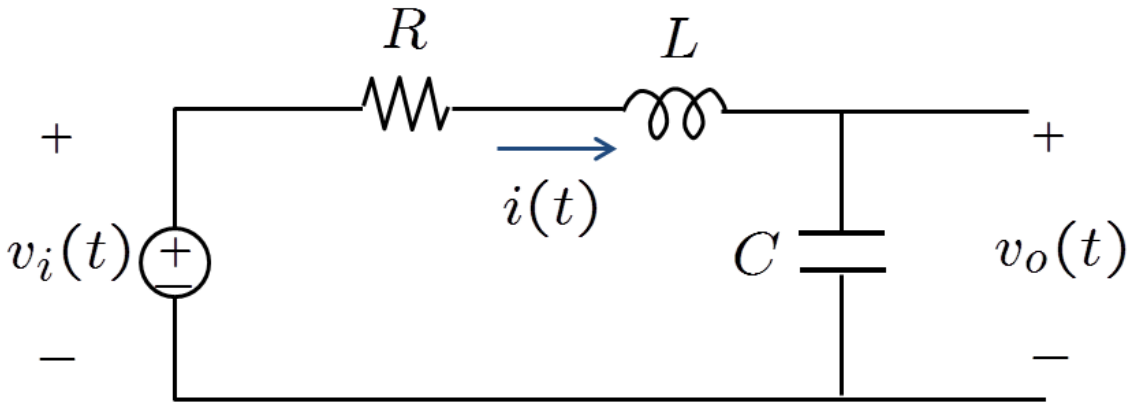
1. $\ddot{y}(t) + 2\dot{y}(t) + y(t) = u(t),\ \forall t \in \mathbb{R}$

2. $y(t) = \int_{t-1}^{t} u(s)\, ds,\ \forall t \in \mathbb{R}$

3. $y(t) = u(t) - u(t-1),\ \forall t \in \mathbb{R}$

4. $y[k] = u[k] - u[k-1],\ \forall k \in \mathbb{Z}$

Circuit Example



Mechanical Example