MA 527

Lecture Notes (section 7.1 & 7.2)

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7.1 Matrices.

Def A matrix = a rectangular array of numbers or objects.

(Ex) $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3}$ $\quad X = \begin{bmatrix} x \\ y \end{bmatrix}_{2\times 1}$.

$AX = b$

1. $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\quad C = \begin{bmatrix} 1 & 5 \\ 2 & 4 \end{bmatrix}$: square matrices

$B = C$ iff $a=1,\ b=5$
$c=2,\ d=4$.

(Operations).

$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}_{2\times 3}$ $\quad B = \begin{bmatrix} 0 & 1 & 2 \\ 5 & 4 & 6 \end{bmatrix}_{2\times 3}$

$A+B = \begin{bmatrix} a+0 & b+1 & c+2 \\ d+5 & e+4 & f+6 \end{bmatrix}$

$D = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$: $A+D$. is not defined.

(Scalar multiplication)

$kA = \begin{bmatrix} ka & kb & kc \\ kd & ke & kf \end{bmatrix}$

Q $\begin{bmatrix} 2a & 3ab \\ 4a & -a \end{bmatrix} = a\begin{bmatrix} 2 & 3b \\ 4 & -1 \end{bmatrix}.$

7.2 Matrix multiplication.

(Motivation)

$$\begin{cases} 2x + 3y = 1 \\ 4x + 5y = 2 \end{cases} \rightarrow \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Def $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x+3y \\ 4x+5y \end{bmatrix}.$

Def $A = [a_{ij}]_{k\times m}$, $B = [b_{ij}]_{m\times n}$

$C = AB$: $C = [c_{ij}]_{k\times n}$

$$c_{ij} = \sum_{l=1}^{m} a_{il} b_{lj}$$

(Ex) $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}_{2\times 2}$ $B = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times 1}$

AB : not defined.

(Transpose).

$A = \begin{bmatrix} 1 & 5 \\ 4 & 3 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 4 \\ 5 & 3 \end{bmatrix}$

(Properties)

1. $(A^T)^T = A$ 2. $(A+B)^T = A^T + B^T$

2. $(cA)^T = cA^T$ 4. $(AB)^T = B^T A^T$

$(ABC)^T = C^T B^T A^T$.

Q $AB \overset{?}{\neq} BA$ in general.

(Ex) $A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}_{2\times1}$ $B = \begin{bmatrix} 3 & 4 \end{bmatrix}_{1\times2}$

$AB = \begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix}$ $BA = \begin{bmatrix} 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$= \begin{bmatrix} 3+8 \end{bmatrix} = \begin{bmatrix} 11 \end{bmatrix}$

Def 1. If $A^T = A$, A is called symmetric

2 If $A^T = -A$, A " skew-symmetric

(Ex) $A = \begin{bmatrix} 1 & 4 \\ 4 & 2 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 4 \\ 4 & 2 \end{bmatrix}$

$B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$: $B^T = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = (-1) \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(Ex) $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

: upper-triangular matrices.

$$C = \begin{bmatrix} 2 & 0 & 0 \\ 4 & 6 & 0 \\ 8 & 0 & 0 \end{bmatrix} \text{ : lower-triangular matrix.}$$

<u>Def</u> $A = [a_{ij}]$

1. A is called an upper-triangular matrix if $a_{ij} = 0$ for $i > j$.
2. A is called a lower-triangular matrix if $a_{ij} = 0$ for $i < j$

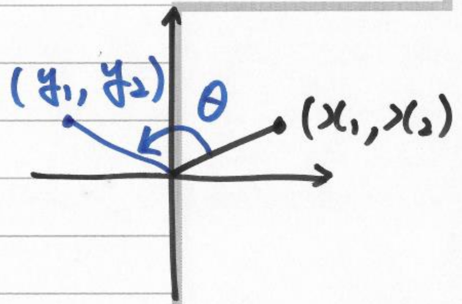
<u>Def</u> $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} = \mathrm{diag}(a, b, c)$

: a diagonal matrix.

$I = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$: the identity matrix.

(Application). Calculus.

<u>Rotate a point in $\mathbb{R}^2$</u>



$$y_1 = x_1 \cos\theta - x_2 \sin\theta$$

$$y_2 = x_1 \sin\theta + x_2 \cos\theta.$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}}_{A\text{ " Rotation matrix.}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$A^n = AA \cdots A.$

$$A^n = \begin{bmatrix} \cos(n\theta) & -\sin(n\theta) \\ \sin(n\theta) & \cos(n\theta) \end{bmatrix}$$

(Bonus problem: July 6 (Tue)).
4:00 pm

$AX = b.$

$$A = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{bmatrix}$$

$A = [a_{ij}]$

$a_{ij} = \left[\frac{1}{i+j-1}\right]$

$$A = \begin{bmatrix} 1 & \frac{1}{2} & \cdots & \frac{1}{n} \\ \frac{1}{2} & \frac{1}{3} & \cdots & \frac{1}{n+1} \\ \vdots & & & \\ \frac{1}{n} & \frac{1}{n+1} & \cdots & \frac{1}{2n-1} \end{bmatrix}$$

$$X_E = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \rightarrow \quad b = AX_E.$$

Solve $AX = b$

Check "$X_E - X$"