Transfers

Goal: Shift to an orbit that does NOT intersect the original orbit

→ To accomplish: use

Usually propellant is the limiting factor so use the transfer that requires the minimum total Δv

Approach transfer problems:

(1) **Define transfer geometry**

(2) **Define departure/arrival points** ← much more difficult

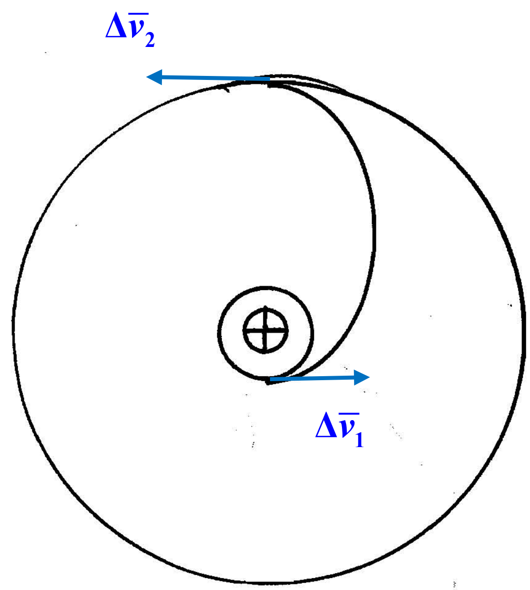
Since (2) more difficult, begin by considering some types from (1)

Simplest two-impulse transfer (also the minimum Δv two-impulse solution)

Walter Hohmann – first to draw attention to problem and compute mission times

1925 (Munich) "The Accessibility of the Heavenly Bodies"

Example $r_1 = 2R_\oplus$ $r_2 = 4R_\oplus$



Solution:

(a) Establish current orbit

$$a = r_1 = 2R_\oplus$$
$$e = 0$$

(b) Conditions at thrust point <u>before</u> maneuver

$$r_1 = 2R_\oplus$$
$$v_1 = 5.59\,\text{km/s}$$
$$\gamma_1 = 0^\circ$$

To calculate Δv requires conditions on the transfer ellipse so transfer ellipse must be defined

(c) Determine transfer ellipse

$$a_T = \frac{1}{2}\left(r_p + r_a\right) = 3R_\oplus$$

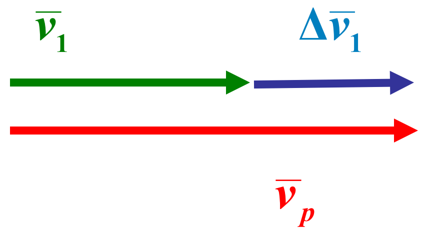
$$r_p = a(1-e) \longrightarrow$$

(d) Conditions at thrust point (on transfer) <u>after</u> maneuver

$$r = r_1$$
$$\frac{v_p^2}{2} = \frac{\mu}{r_1} - \frac{\mu}{2a_T} \longrightarrow$$
$$\gamma_1 = 0^\circ$$

(e) Vector Diagram for $\Delta\overline{v}_1$

ALWAYS sketch the vector diagram



(f) move to the next maneuver point

Conditions at thrust point <u>before</u> 2nd maneuver
(now in transfer orbit)

$$r_a = r_2 = 4\,R_\oplus$$
$$\frac{v_a^2}{2} = \frac{\mu}{r_2} - \frac{\mu}{2a_T}$$
$$\gamma_2 = 0^\circ \quad \text{(apogee)}$$

(g) Conditions required <u>after</u> maneuver in final orbit

$$r_2 = 4\,R_\oplus$$
$$v_2 = \sqrt{\frac{\mu}{r_2}} = 3.95\,\text{km/s}$$
$$\gamma = 0^\circ$$

(h) Vector diagram for $\Delta\overline{v}_2$

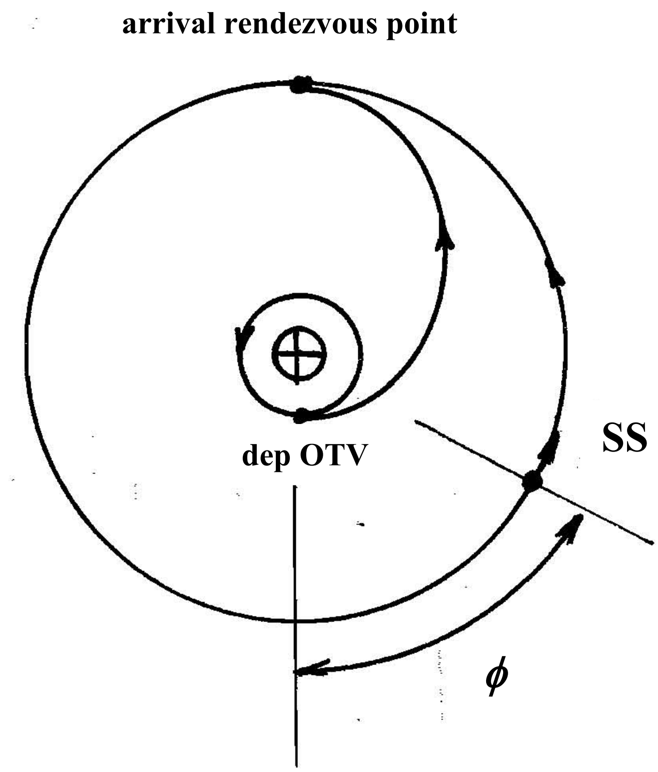
(i) Total $\Delta v = \left|\Delta\overline{v}_1\right| + \left|\Delta\overline{v}_2\right|$

Conditions for Rendezvous

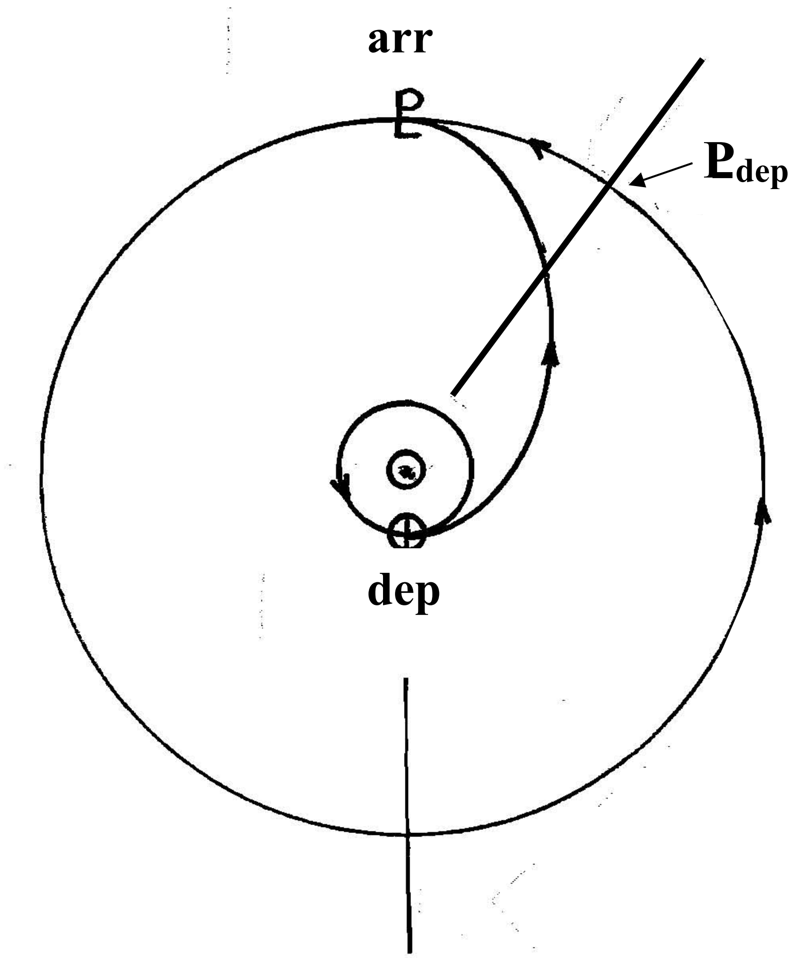
Transfers shift vehicles from one orbit to another

Additional complexity if rendezvous:
  Just reaching target orbit is not sufficient
  Timing becomes a critical factor

Example: ⊕ orbiting OTV departing low ⊕ orbit to rendezvous with a space station



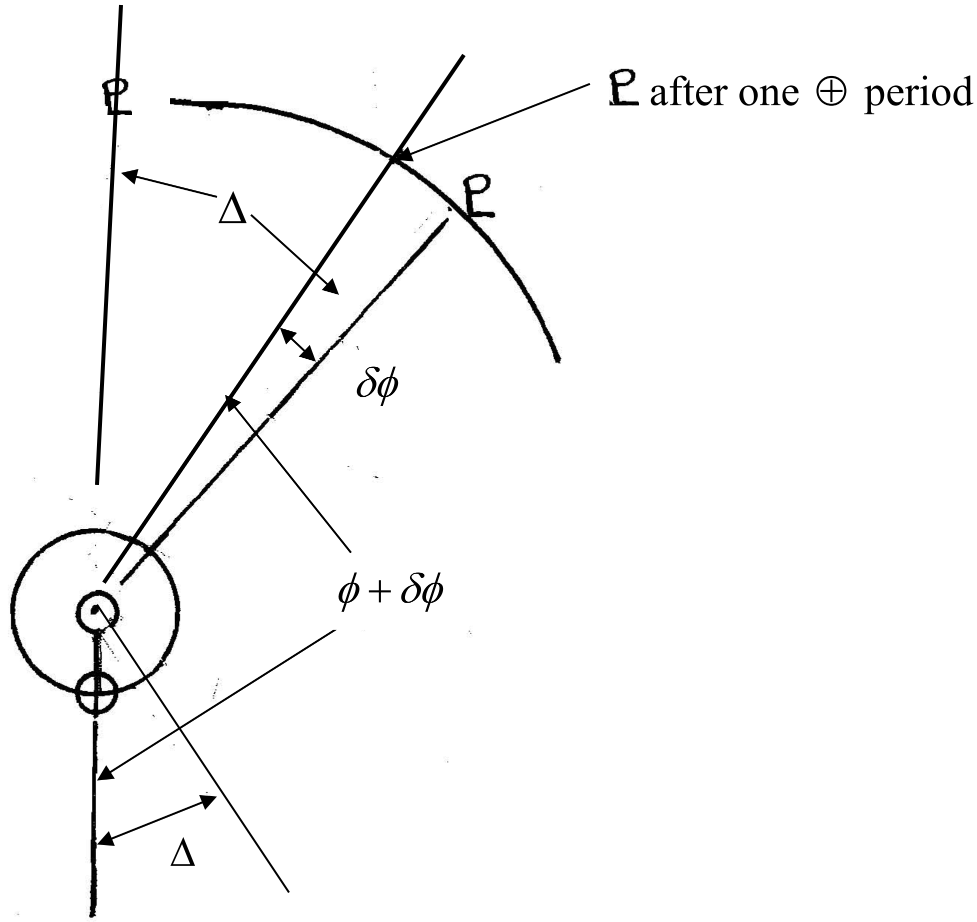
Example: Hohmann Earth-to-Pluto



Requirement for rendezvous/interception determines initial geometry

If this "launch" opportunity is missed, how long until proper alignment again available?

→ synodic period?



1. IP_{Pluto} = 247 yrs; ♇ does not move far in one Earth IP

2. After one IP_{Earth} , angle between Earth and Pluto = $\phi + \delta\phi$

3. Earth moves faster than Pluto, so if we let both move a little, Earth will "catch up"

   $$IP = \frac{2\pi}{n}$$

   Earth time to go one period plus a little = $IP + \Delta t = t_s$

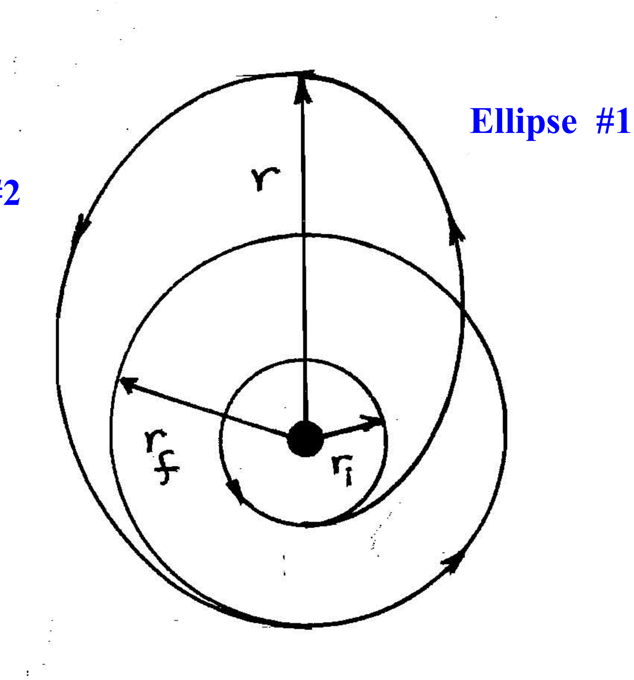
   $$\left.\begin{aligned} n_{Earth}\, t_s &= 2\pi + \Delta \\ n_{Pluto}\, t_s &= \Delta \end{aligned}\right\}$$

Bi-Elliptical Transfers

Hoelker-Silber

all tangential

Extension to Hohmann transfer that uses three impulses



Characteristics:

1. Initial orbit circular (?)
2. 1st impulse applies tangentially; shift to periapsis of transfer Ellipse #1 (E1)
3. Apogee on E1 = $r > r_f$

   2nd impulse applied tangentially; shifts from apoapsis of E1 tp apoapsis of transfer Ellipse #2 (E2)
4. Periapsis on E2 $= r_f$

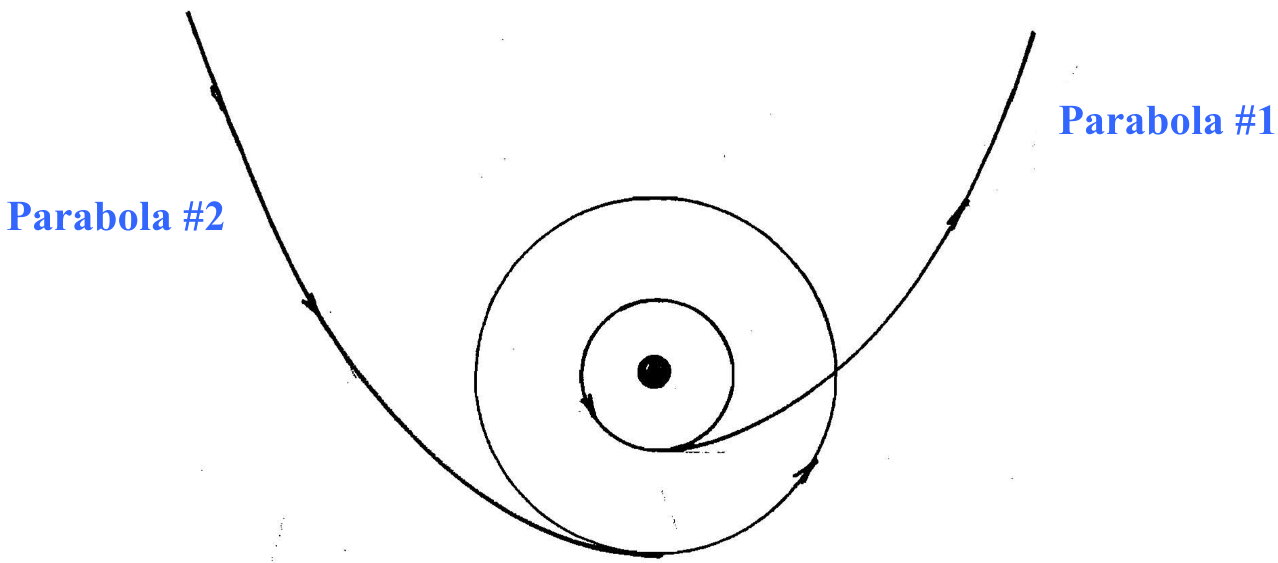
   3rd impulse applied tangentially; shifts into final circular (?) orbit
5. Total cost = $|\Delta\bar{v}_1| + |\Delta\bar{v}_2| + |\Delta\bar{v}_3|$

Bi-Parabolic Transfers

Move the intermediate radius out to infinity ($r \to \infty$)

⇒ Transfer paths become parabolic

2^{nd} impulse becomes infinitesimally small ($\Delta v_2 \approx 0$)



No practical value because duration infinite

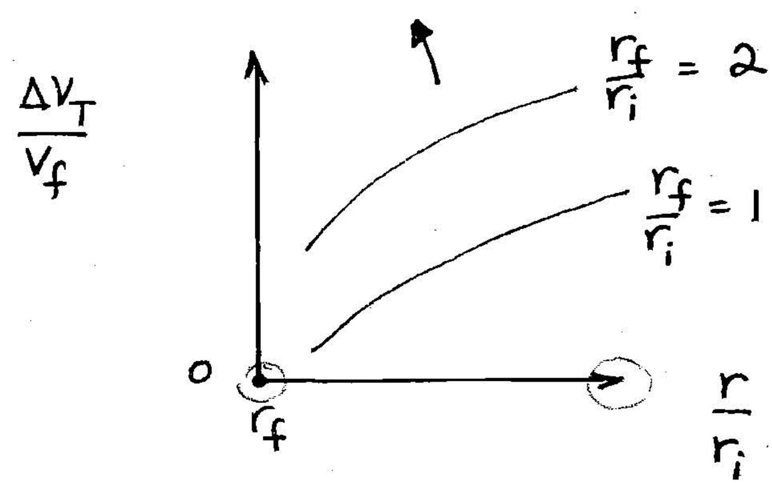
Gain achieved by use of bi-parabolic (in-plane) small (max ≈ 10%) so Hohmann preferred in practice

Return to Bi-elliptic

$$\Delta v_{Total} = |\Delta \bar{v}_1| + |\Delta \bar{v}_2| + |\Delta \bar{v}_3|$$

To clarify the relationship between Δv_{Total} and r, consider a plot for circle-to-circle bi-elliptic transfers

nondimensional cost



nondimensional intermediate radius

Next page: Find conditions for minimum cost

Check limits $r = r_f$ (two-impulse Hohmann)
$r \to \infty$ (bi-parabolic)

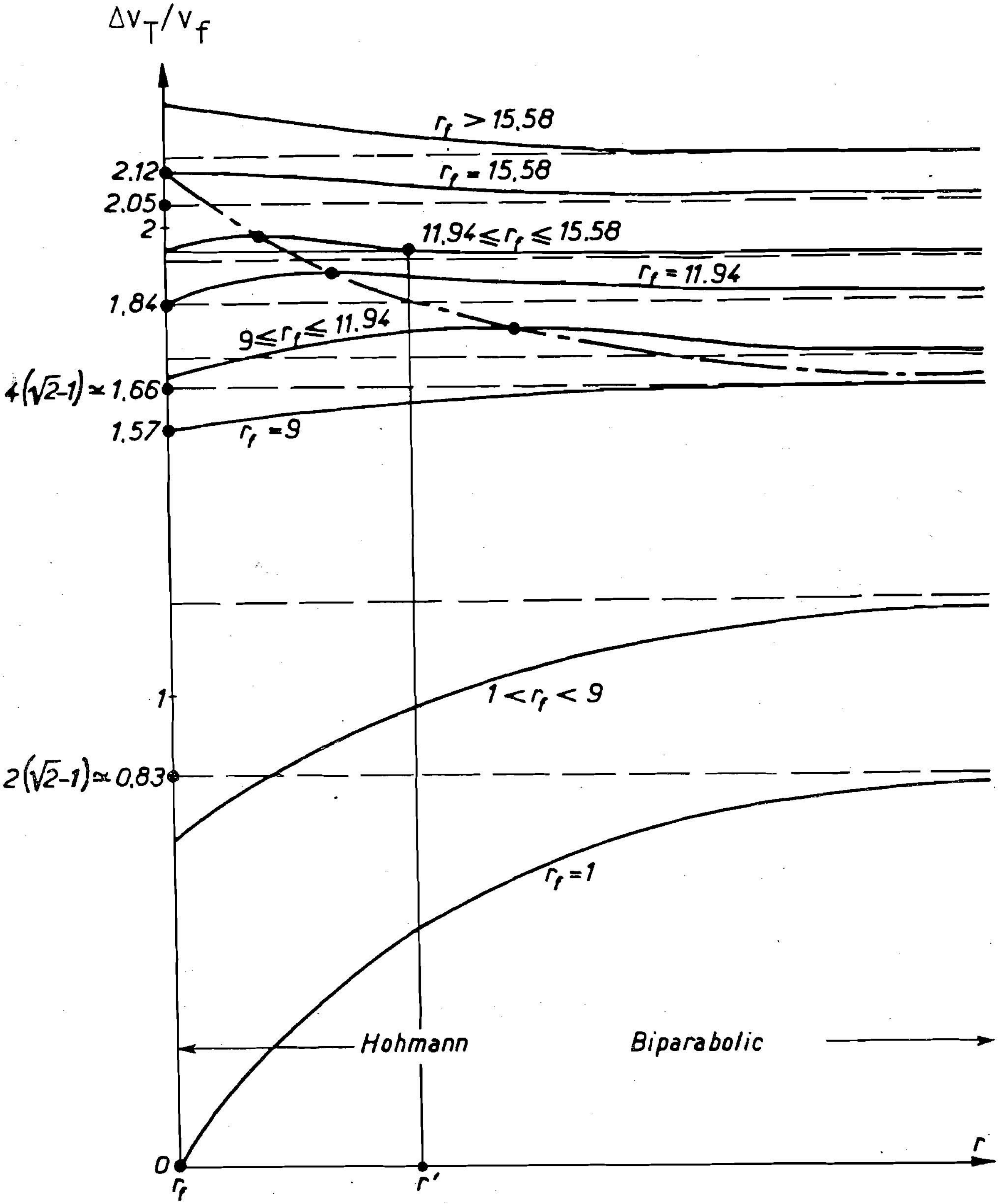
(a) $1 \le \boldsymbol{r_f} \le 9$

(b) $9 \le \boldsymbol{r_f} \le 15.58$

(i) $9 \le \boldsymbol{r_f} \le 11.94$

(ii) $11.94 \le \boldsymbol{r_f} \le 15.58$

(c) $\boldsymbol{r_f} \ge 15.58$



$\Delta v_T / v_f$
$r_f > 15.58$
$r_f = 15.58$
2.12
2.05
2
$11.94 \leq r_f \leq 15.58$
$r_f = 11.94$
1.84
$9 \leq r_f \leq 11.94$
$4(\sqrt{2}-1) \simeq 1.66$
1.57
$r_f = 9$
1
$1 < r_f < 9$
$2(\sqrt{2}-1) \simeq 0.83$
$r_f = 1$
Hohmann
Biparabolic
0
r_f
r'
r