3.4.6

$$Q = \begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{14} & x_1 \\ 1/\sqrt{3} & 2/\sqrt{14} & x_2 \\ 1/\sqrt{3} & -3/\sqrt{14} & x_3 \end{pmatrix}$$

$$q_1 = \begin{pmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix} \quad q_2 = \begin{pmatrix} 1/\sqrt{14} \\ 2/\sqrt{14} \\ -3/\sqrt{14} \end{pmatrix} \quad q_3 = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$q_3^T q_1 = (x_1, x_2, x_3) \begin{pmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix} = 0$$

$$\frac{x_1}{\sqrt{3}} + \frac{x_2}{\sqrt{3}} + \frac{x_3}{\sqrt{3}} = 0$$

$$q_3^T q_2 = (x_1\ x_2, x_3) \begin{pmatrix} 1/\sqrt{14} \\ 2/\sqrt{14} \\ -3/\sqrt{14} \end{pmatrix} = 0$$

$$\frac{x_1}{\sqrt{14}} + \frac{2}{\sqrt{14}} x_2 \ \frac{-3}{\sqrt{14}} x_3 = 0$$

$$\|q_3\| = 1 = \sqrt{x_1^2 + x_2^2 + x_3^2}$$

$$x_3 = -x_1 - x_2$$

$$x_1 = -2x_2 + 3x_3$$

$$x_3 = -(-2x_2 + 3x_3) - x_2 \quad = \quad x_2 - 3x_3$$

$$x_3 = \frac{x_2}{4}$$

3.4.6

$$x_1^2 + x_2^2 + x_3^2 = 1^2$$

$$(-2x_2 + 3x_3)^2 + x_2^2 + \left(\frac{x_2}{4}\right)^2 = 1$$

$$\left(-2x_2 + \frac{3x_2}{4}\right)^2 + x_2^2 + \frac{x_2^2}{16} = 1$$

$$\frac{25}{16}x_2^2 + \frac{16x^2}{16} + \frac{x_2^2}{16} = 1$$

$$\frac{42}{16}x_2^2 = 1$$

$$x_2 = \sqrt{\frac{16}{42}} = \pm 4/\sqrt{42}$$

$$x_3 = \frac{\frac{4}{\sqrt{42}}}{4} = \pm\frac{1}{\sqrt{42}}$$

$$x_1 = -2\left(\frac{4}{\sqrt{42}}\right) + 3\left(\frac{1}{\sqrt{42}}\right) = \frac{-5}{\sqrt{42}}$$

$$\boxed{q_3 = \begin{pmatrix} -5/\sqrt{42} \\ 4/\sqrt{42} \\ 1/\sqrt{42} \end{pmatrix} \text{ or } \begin{pmatrix} 5/\sqrt{42} \\ -4/\sqrt{42} \\ -1/\sqrt{42} \end{pmatrix}}$$

There are $\boxed{\text{two degrees of freedom}}$ resulting from the positive & negative roots.

3.4.6

$$\left(1/\sqrt{3},\ 1/\sqrt{14},\ -5/\sqrt{42}\right)\begin{pmatrix}1/\sqrt{3}\\ 2/\sqrt{14}\\ 4/\sqrt{42}\end{pmatrix} = r_1^T r_2$$

$$= 1/3 + \frac{2}{14} - \frac{20}{42} = \frac{14}{42} + \frac{6}{42} - \frac{20}{42} = \underline{0} \therefore$$

$\underline{r_1 \perp r_2}$

$$\left(1/\sqrt{3},\ 1/\sqrt{14},\ -5/\sqrt{42}\right)\begin{pmatrix}1/\sqrt{3}\\ -3/\sqrt{14}\\ 1/\sqrt{42}\end{pmatrix} = 1/3 - \frac{3}{14} - \frac{5}{42} = \frac{14}{42} - \frac{9}{42} - \frac{5}{42}$$

$$\underline{= 0 = r_1^T r_3 \therefore r_1 \perp r_3}$$

$$\left(1/\sqrt{3},\ 2/\sqrt{14},\ 4/\sqrt{42}\right)\begin{pmatrix}1/\sqrt{3}\\ -3/\sqrt{14}\\ 1/\sqrt{42}\end{pmatrix} = \frac{1}{3} - \frac{6}{14} + \frac{4}{42} = \frac{14}{42} - \frac{18}{42} + \frac{4}{42}$$

$$\underline{= 0 = r_2^T r_3 \therefore r_2 \perp r_3}$$

3.4.12

$$a = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad b = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \quad c = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$q_1 = \frac{a}{\|a\|} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$a_2' = b - (q_1^T b) q_1$$

$$a_2' = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - \left[\begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right] \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$a_2' = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$q_2 = \frac{a_2'}{\|a_2'\|} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$a_3' = c - (q_1^T c) q_1 - (q_2^T c) q_2$$

$$a_3' = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \left[\begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right] \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} - \left[\begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right] \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$a_3' = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$q_3 = \frac{a_3'}{\|a_3'\|} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$Q = \begin{pmatrix} | & | & | \\ q_1 & q_2 & q_3 \\ | & | & | \end{pmatrix}$$

3.4.13

$$\boxed{Q = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}}$$

$$\boxed{A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}} = \begin{pmatrix} | & | & | \\ a & b & c \\ | & | & | \end{pmatrix}$$

$$R = Q^T A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\boxed{R = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}}$$

3.4.21

$a\cos x + b\sin x \quad [-\pi, \pi]$

$f = \sin 2x$

$$a = \frac{\langle \sin 2x, \cos x \rangle}{\langle \cos x, \cos x \rangle}$$

$$\int_{-\pi}^{\pi} \sin(2x)\cos x \, dx = \langle \sin 2x, \cos x \rangle$$

$\sin 2x = 2\sin x \cos x$

$$2\int_{-\pi}^{\pi} \sin x(\cos^2 x)\, dx$$

Let $u = \cos x \quad \therefore \quad du = -\sin x \, dx$

$$-2\int_{-1}^{-1} u^2 \, du = -2\left[\frac{u^3}{3} \Big|_{-1}^{-1} \right] = 0$$

$\therefore \quad a = 0$

$$b = \frac{\langle \sin 2x, \sin x \rangle}{\langle \sin x, \sin x \rangle}$$

$$\int_{-\pi}^{\pi} \sin(2x)\sin x \, dx = \langle \sin 2x, \sin x \rangle$$

$\sin(2x) = 2\cos x \sin x$

3.4.21

$$2\int_{-\pi}^{\pi} \cos x(\sin^2 x)\, dx$$

$U = \sin x \quad \therefore \quad du = \cos x\, dx$

$$2\int_{0}^{0} u^2\, dx = 2\left[\frac{u^3}{3}\Big|_0^0\right] = 0$$

$\therefore\ b = 0$

Closest function: $a\cos x + b\sin x = 0\cos x + 0\sin x = 0$

$\boxed{\text{Closest function} = 0}$

$$A = \begin{pmatrix} \langle 1,1\rangle & \langle 1,x\rangle \\ \langle x,1\rangle & \langle x,x\rangle \end{pmatrix}$$

$$\langle 1,1\rangle = \int_{-\pi}^{\pi} dx = 2\pi$$

$$\langle 1,x\rangle = \langle x,1\rangle = \int_{-\pi}^{\pi} x\, dx = \frac{x^2}{2}\Big|_{-\pi}^{\pi} = 0$$

$$\langle x,x\rangle = \int_{-\pi}^{\pi} x^2\, dx = \frac{x^3}{3}\Big|_{-\pi}^{\pi} = \frac{\pi^3}{3} + \frac{\pi^3}{3} = \frac{2\pi^3}{3}$$

3.4.21

$$A = \begin{pmatrix} 2\pi & 0 \\ 0 & \frac{2\pi^3}{3} \end{pmatrix}$$

$$b = \begin{pmatrix} \langle 1, \sin 2x \rangle \\ \langle x, \sin 2x \rangle \end{pmatrix}$$

$$\int_{-\pi}^{\pi} \sin(2x)\,dx \qquad u = 2x \qquad du = 2\,dx$$

$$\frac{1}{2}\int_{-2\pi}^{2\pi} \sin(u)\,du = \frac{1}{2}\left(-\cos(u)\Big|_{-2\pi}^{2\pi}\right) = -\frac{1}{2}(1-1) = 0$$

$$\langle x, \sin 2x \rangle = \int_{-\pi}^{\pi} x \sin 2x\,dx$$

Let $u = x$, $dv = \sin(2x)\,dx \therefore V = -\frac{1}{2}\cos(2x)$

$$\int_{-\pi}^{\pi} x \sin 2x\,dx = (x)\left(-\tfrac{1}{2}\cos(2x)\right)\Big|_{-\pi}^{\pi} - \frac{1}{2}\int_{-\pi}^{\pi} \cos(2x)\,dx$$

$$(\pi)(-\tfrac{1}{2}) - (-\pi)(-\tfrac{1}{2}) = -\pi$$

$$-\frac{1}{2}\int_{-\pi}^{\pi} \cos(2x)\,dx = -\frac{1}{4}\int_{-2\pi}^{2\pi} \cos(u)\,du$$

$$-\frac{1}{4}\left[\sin(u)\Big|_{-2\pi}^{2\pi}\right] = 0$$

3.4.21

$$\therefore \int_{-\pi}^{\pi} x\sin(2x)\,dx = -\pi$$

$$b = \begin{pmatrix} 0 \\ -\pi \end{pmatrix}$$

$$\hat{x} = \begin{pmatrix} C \\ D \end{pmatrix} = (A^TA)^{-1}A^Tb$$

$$\begin{pmatrix} 2\pi & 0 \\ 0 & \frac{2\pi^3}{3} \end{pmatrix}\begin{pmatrix} 2\pi & 0 \\ 0 & \frac{2\pi^3}{3} \end{pmatrix} = \begin{pmatrix} 4\pi^2 & 0 \\ 0 & \frac{4\pi^6}{9} \end{pmatrix}$$

$$(A^TA)^{-1} = \left(\frac{1}{\frac{16\pi^8}{9}}\right)\begin{pmatrix} \frac{4\pi^6}{9} & 0 \\ 0 & 4\pi^2 \end{pmatrix} = \begin{pmatrix} \frac{1}{4\pi^2} & 0 \\ 0 & \frac{9}{4\pi^6} \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{4\pi^2} & 0 \\ 0 & \frac{9}{4\pi^6} \end{pmatrix}\begin{pmatrix} 2\pi & 0 \\ 0 & \frac{2\pi^3}{3} \end{pmatrix} = \begin{pmatrix} \frac{1}{2\pi} & 0 \\ 0 & \frac{18}{12\pi^3} \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{2\pi} & 0 \\ 0 & \frac{3}{2\pi^3} \end{pmatrix}\begin{pmatrix} 0 \\ -\pi \end{pmatrix} = \begin{pmatrix} C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{-3}{2\pi^2} \end{pmatrix}$$

Closest line: $C + Dx$ $\quad C = 0,\ D = \frac{-3}{2\pi^2}$

$$\boxed{\text{Closest line: } \frac{-3}{2\pi^2}x}$$

3.4.24

$x^3 + ax^2 + bx + c = f$

$$v_4 = (x^3+ax^2+bx+c) - \frac{\langle 1, f\rangle}{\langle 1,1\rangle} 1 - \frac{\langle x, f\rangle}{\langle x, x\rangle} x$$

$$- \frac{\langle x^2-1/3, f\rangle}{\langle x^2-1/3, x^2-1/3\rangle} x^2-1/3$$

$$\int_{-1}^{1} x^3+ax^2+bx+c = \langle 1, f\rangle = 0$$

$$\frac{x^4}{4} + \frac{ax^3}{3} + \frac{bx^2}{2} + cx \Big|_{-1}^{1} = 0$$

$$\left(\cancel{\tfrac{1}{4}} + \frac{a}{3} + \cancel{\tfrac{b}{2}} + c\right) - \left(\cancel{1/4} - a/3 + \cancel{b/2} - c\right) = 0$$

$$\frac{2a}{3} + 2c = 0$$

$$\int_{-1}^{1} x^4+ax^3+bx^2+cx = \langle x, f\rangle = 0$$

$$= \frac{x^5}{5} + \frac{ax^4}{4} + \frac{bx^3}{3} + \frac{cx^2}{2} \Big|_{-1}^{1}$$

$$= \left(\frac{1}{5} + \cancel{\frac{a}{4}} + \frac{b}{3} + \cancel{\frac{c}{2}}\right) - \left(\frac{-1}{5} + \cancel{\frac{a}{4}} - \frac{b}{3} + \cancel{\frac{c}{2}}\right)$$

$$= \frac{2}{5} + \frac{2b}{3} = 0 \qquad b = -\frac{3}{5}$$

$$\int_{-1}^{1} (x^3+ax^2+bx+c)(x^2-1/3)\,dx = \langle x^2-1/3, f\rangle = 0$$

3.4.24

$$\int_{-1}^{1} x^5 + ax^4 + bx^3 + cx^2 \; - \frac{1}{3}\int_{-1}^{1} x^3 + ax^2 + bx + c$$

$$\left(\frac{x^6}{6} + \frac{ax^5}{5} + \frac{bx^4}{4} + \frac{cx^3}{3}\right)\Bigg|_{-1}^{1} - \frac{2a}{9} - \frac{2c}{3} = 0$$

$$\left(\cancel{\frac{1}{6}} + \frac{a}{5} + \cancel{\frac{b}{4}} + \frac{c}{3}\right) - \left(\cancel{\frac{1}{6}} - \frac{a}{5} + \cancel{\frac{1}{4}} - \frac{c}{3}\right) - \frac{2a}{9} - \frac{2c}{3} = 0$$

$$\frac{2a}{5} + \cancel{\frac{2c}{3}} - \frac{2a}{9} - \cancel{\frac{2c}{3}} = 0$$

$a = 0 \quad \therefore \quad c = 0$

$\therefore \quad \boxed{V_4 = x^3 - \frac{3}{5}x}$

3.5.8

$$F_4 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & w_4 & w_4^2 & w_4^3 \\ 1 & w_4^2 & w_4^4 & w_4^6 \\ 1 & w_4^3 & w_4^6 & w_4^9 \end{pmatrix}$$

$$w_4 = e^{2\pi i/4} = e^{\pi i/2} = \cos\left(\tfrac{\pi}{2}\right) + i\sin\left(\tfrac{\pi}{2}\right) = i$$

$$F_4 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix}$$

$$F_4^{-1} = \frac{\overline{F}}{4} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{pmatrix} \frac{1}{4}$$

$$c = F_4^{-1} y = \begin{pmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & -i/4 & -1/4 & i/4 \\ 1/4 & -1/4 & 1/4 & -1/4 \\ 1/4 & i/4 & -1/4 & -i/4 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

$$= \begin{pmatrix} 1/2 - 1/2 \\ 1/2 + 1/2 \\ 1/2 - 1/2 \\ 1/2 + 1/2 \end{pmatrix} = \boxed{\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{pmatrix}}$$

$$c_0 + c_1 e^{ix} + c_2 e^{2ix} + c_3 e^{3ix} = e^{ix} + e^{3ix}$$

$$e^{ix} + e^{3ix} \Big|_{x=0} = [\overset{1}{\cancel{\cos}(0)} + i\,\overset{0}{\cancel{\sin}(0)}] + [\overset{1}{\cancel{\cos}(0)} + i\,\overset{0}{\cancel{\sin}(0)}]$$

$$1 + 1 = 2$$

3.5.8

$$e^{ix}+e^{3ix} = 1+1 = \boxed{2}$$

$$e^{ix}+e^{3ix}\Big|_{x=\frac{\pi}{2}} = \left[\cancelto{0}{\cos\left(\tfrac{\pi}{2}\right)} + i\cancelto{1}{\sin\left(\tfrac{\pi}{2}\right)}\right] + \left[\cancelto{0}{\cos\left(\tfrac{3\pi}{2}\right)} + i\cancelto{-1}{\sin\left(\tfrac{3\pi}{2}\right)}\right]$$

$$e^{ix}+e^{3ix}\Big|_{x=\frac{\pi}{2}} = i - i = \boxed{0}$$

$$e^{ix}+e^{3ix}\Big|_{x=\pi} = \left[\cancelto{-1}{\cos(\pi)} + i\cancelto{0}{\sin(\pi)}\right] + \left[\cancelto{-1}{\cos(3\pi)} + i\cancelto{0}{\sin(3\pi)}\right]$$

$$e^{ix}+e^{3ix}\Big|_{x=\pi} = -1 + -1 = \boxed{-2}$$

$$e^{ix}+e^{3ix}\Big|_{x=\frac{3\pi}{2}} = \left[\cancelto{0}{\cos\left(\tfrac{3\pi}{2}\right)} + i\cancelto{-1}{\sin\left(\tfrac{3\pi}{2}\right)}\right] + \left[\cancelto{0}{\cos\left(\tfrac{9\pi}{2}\right)} + i\cancelto{1}{\sin\left(\tfrac{9\pi}{2}\right)}\right]$$

$$\sin\left(\tfrac{9\pi}{2}\right) = \sin\left(\tfrac{\pi}{2}\right)$$

$$e^{ix}+e^{3ix}\Big|_{x=\frac{3\pi}{2}} = 1 + -1 = \boxed{0}$$

3.5.4

$$y = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad c = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$F_4 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix}$$

$$F_4 c = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\boxed{F_4 c = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = y}$$

$$y = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad c = F_4^{-1} y$$

$$F_4^{-1} y = \begin{pmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ 1/4 & -i/4 & -1/4 & i/4 \\ 1/4 & -1/4 & 1/4 & -1/4 \\ 1/4 & i/4 & -1/4 & -i/4 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = c$$

$$\boxed{c = \begin{pmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{pmatrix}}$$

3.5.11

$c = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

$c' = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $c'' = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$y' = F_2 c'$

$F_2 = \begin{pmatrix} 1 & 1 \\ 1 & w_2 \end{pmatrix}$ $w_2 = e^{\frac{2\pi i}{2}} = e^{\pi i} = \cos(\pi) + i\sin(\pi)$

$w_2 = -1$

$F_2 = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

$y' = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

$y'' = F_2 c'' = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$y_j = y_j' + w_n^j y_j''$ $m = \frac{n}{2} = \frac{4}{2} = 2$

$y_{j+m} = y_j' - w_n^j y_j''$

$y_1 = y_1' + w_4^1 y_1''$

$y_2 = y_2' + w_4^2 y_2''$

$y_3 = y_1 - w_4^1 y_1''$

$y_4 = y_2' - w_4^2 y_2''$

3.5.11

$\omega_4 = i \quad , \quad \omega_4^2 = -1$

$y_1 = 2 + (i)(0) = 2$

$y_2 = 0 + (-1)(0) = 0$

$y_3 = 2 - (i)(0) = 2$

$y_4 = 0 - (-1)(0) = 0$

$$\boxed{y = \begin{pmatrix} 2 \\ 0 \\ 2 \\ 0 \end{pmatrix}}$$

A.2

a) y-axis, Sum is $\mathbb{R}^3$ consisting of x-y-z axes.

b) Intersection is line through (1,1,1). The sum is a plane containing (0,0,0), (1,0,0) & (0,1,1).

c) The intersection is the zero vector. The sum is $\mathbb{R}^3$.

D) $(1\ 1\ 0)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$ $\quad x + y = 0$

$(0\ 1\ 1)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$ $\quad y + z = 0$

Let $y = 1$ $\therefore$ $x = z = -1$

Intersection is line through $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$. The sum of the two planes is $\mathbb{R}^3$.

A.6

$$V = \operatorname{span}\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$2 + w_1 = 0$

$1 + w_2 = 0$

$2 + w_3 = 0$

$w_1 = w_3 = -2$

$w_2 = -1$

$$\boxed{W = \operatorname{span}\left\{ \begin{pmatrix} -2 \\ -1 \\ -2 \end{pmatrix} \right\}}$$

A.8

$\dim(V+W) = \operatorname{rank}(A+B)$

$A \perp B \quad \therefore \quad A \cap B = \{0\}$

$\dim(V) = \operatorname{rank}(A)$

$\dim(W) = \operatorname{rank} B$

$\dim(V+W) + \cancel{\dim(V \cap W)}^{\,0} = \dim(V) + \dim(W)$

$$\boxed{\operatorname{rank}(A+B) = \operatorname{rank}(A) + \operatorname{rank}(B)}$$