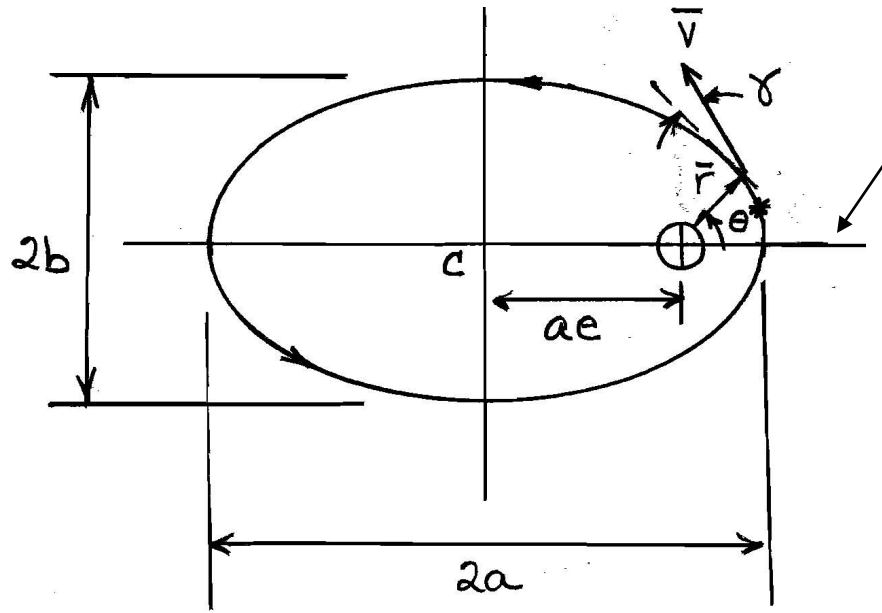


Conic Sections

Ellipse $0 \leq e \leq 1$ $a > 0$ $\mathcal{E} < 0$



$$r = \frac{p}{1 + e \cos \theta^*}$$

periapsis $\theta^* = 0$ $r_p = \frac{p}{1 + e} =$

apoapsis $\theta^* = 180^\circ$ $r_a = \frac{p}{1 - e} =$

$a = \frac{1}{2}(r_p + r_a)$ also known as mean distance

Circle (special case; $e = 0$)

$$a = r = p \quad \mathcal{E} = -\frac{\mu}{2a} = \frac{v^2}{2} - \frac{\mu}{r}$$

General Ellipse:

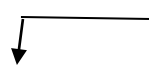
$$\frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}$$

$$v^2 = \frac{2\mu}{r} - \frac{\mu}{a} = 2\underbrace{v_c^2}_{\text{positive by definition}} - \frac{\mu}{a}$$

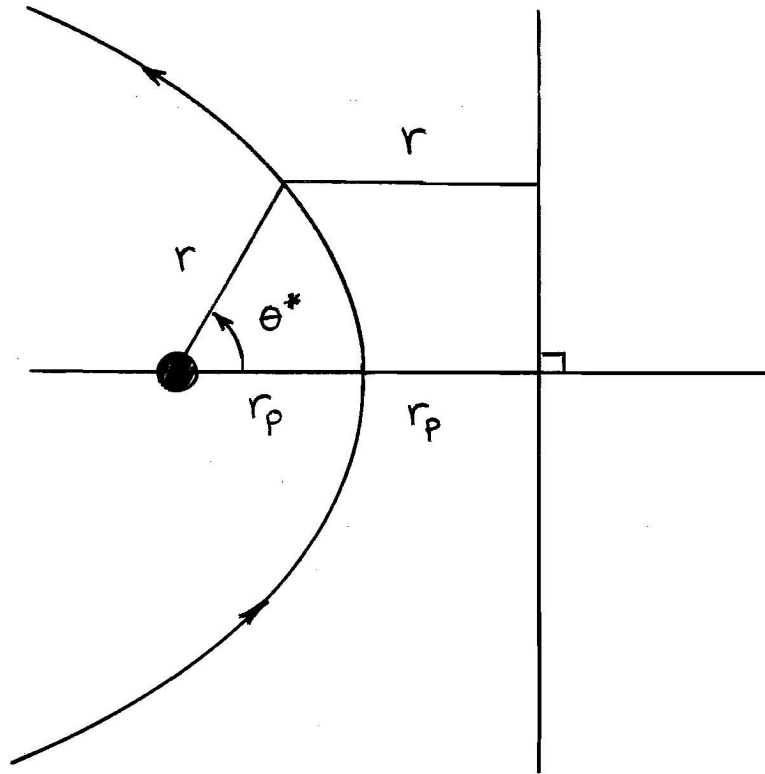
$$\frac{dA}{dt} = \frac{h}{2} \quad \rightarrow \quad dt = \frac{2}{h} dA$$

$$IP = \frac{2}{h} (\pi ab)$$

area of ellipse

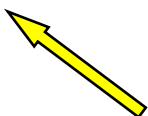


Parabola $e=1$ $a=\infty$ $\mathcal{E}=0$

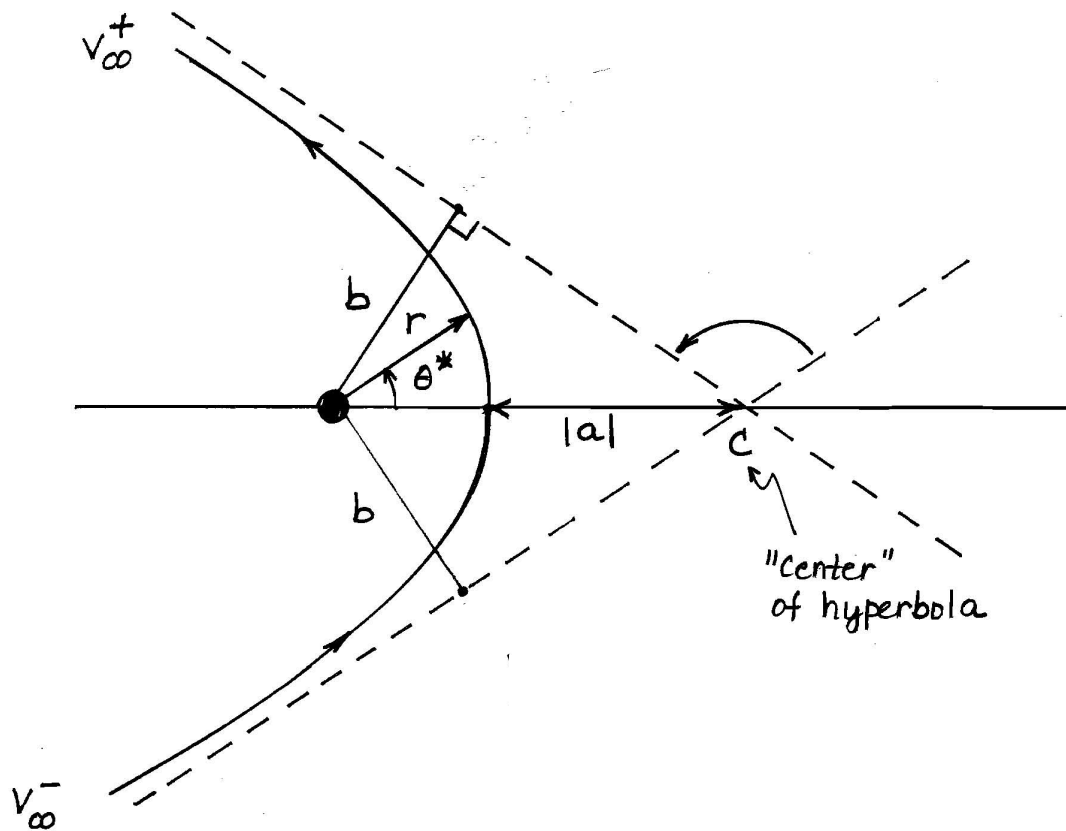


Orbit NOT closed; particle leaves vicinity of attracting body

$$\frac{v^2}{2} - \frac{\mu}{r} = 0 \quad \rightarrow$$

$$\frac{v_\infty^2}{2} - \frac{\mu}{r_\infty} = 0 \quad \rightarrow \quad v_\infty = 0$$


Hyperbola $e > 1$ $a < \infty$ (by convention) $\mathcal{E} > 0$
 (one branch)



$$\mathcal{E} = -\frac{\mu}{2a} = +\frac{\mu}{2|a|} = \frac{v_{\infty}^2}{2} - \frac{\mu}{r_{\infty}} = \frac{v_{\infty}^2}{2}$$

$$r_p = a(1 - e) = |a|(e - 1)$$

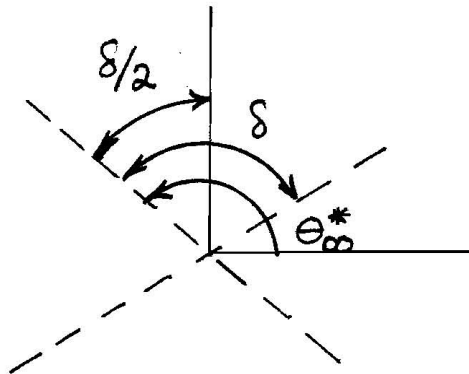
$$p = a(1 - e^2) = |a|(e^2 - 1)$$

parallel to asymptote $r_{\infty} = \frac{|a|(e^2 - 1)}{1 + e \cos \theta_{\infty}^*}$

OR

$$1 + e \cos \theta_{\infty}^* = \frac{|a|(e^2 - 1)}{r_{\infty}}$$

zero



$$\mathcal{E} = \frac{\mu}{2|a|} = \frac{v^2}{2} - \frac{\mu}{r}$$

$$v^2 = \frac{2\mu}{r} + \underbrace{\frac{\mu}{|a|}}_{\text{positive}} \quad \longrightarrow$$