AAE 666

Homework Five

Exercise 1 Using linearization, determine (if possible) the stability properties of the following system about the zero solution.

$$\frac{d^4q}{dt^4} - \sin(q) = 0.$$

If not possible, explain why.

Exercise 2 Using linearization, determine (if possible) the stability properties of the following system about the zero solution.

$$\ddot{q} + \dot{q} - q^3 = 0$$

If not possible, explain why.

Exercise 3 If possible, use linearization to determine the stability properties of each of the following systems about the zero equilibrium state.

(i)

$$\begin{aligned} \dot{x}_1 &= (1 + x_1^2)x_2 \\ \dot{x}_2 &= -x_1^3 \end{aligned}$$

(ii)

$$\begin{aligned} \dot{x}_1 &= \sin x_2 \\ \dot{x}_2 &= (\cos x_1)x_3 \\ \dot{x}_3 &= e^{x_1}x_2 \end{aligned}$$

Exercise 4 If possible, use linearization to determine the stability properties of the following system about the zero equilibrium state.

$$\begin{aligned} x_1(k+1) &= x_1(k)^2 + \sin(x_2(k)) \\ x_2(k+1) &= 0.4\,\cos(x_2(k))\,x_1(k) \end{aligned}$$

Exercise 5 If possible, use linearization to determine the stability properties of the following system about the zero equilibrium state.

$$\begin{aligned} x_1(k+1) &= (1 + x_1(k)^3)x_2(k) \\ x_2(k+1) &= x_1(k)^3 + x_2(k)^5 \end{aligned}$$

Exercise 6 If possible, use linearization to determine the stability properties of the following system about the zero equilibrium state.

$$
\begin{aligned}
x_1(k+1) &= x_2(k) \\
x_2(k+1) &= \sin(x_1(k)) + x_2(k)^5
\end{aligned}
$$

Exercise 7 Recall the Lorenz system

$$
\begin{aligned}
\dot{x}_1 &= \sigma(x_2 - x_1) \\
\dot{x}_2 &= r x_1 - x_2 - x_1 x_3 \\
\dot{x}_3 &= -b x_3 + x_1 x_2
\end{aligned}
$$

with $\sigma, r, b > 0$. Prove that all solutions of this system are bounded. (Hint: Consider $V(x) = r x_1^2 + \sigma x_2^2 + \sigma(x_3 - 2r)^2$.)