#11) $\frac{Y}{U} = G = \frac{36}{s^2+2s+36}$

$G = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

$\therefore \; \omega_n^2 = 36 \Rightarrow \omega_n = 6$

$2\zeta\omega_n = 2 \Rightarrow \zeta = 1/6$

$\zeta < 1$, therefore underdamped

$\sigma = \zeta\omega_n \,, \; \omega_d = \omega_n\sqrt{1-\zeta^2}$

Rise time: $t_r = \frac{1}{\omega_d} \tan^{-1}\left(\frac{\omega_d}{-\sigma}\right)$

$\omega_d = 6\sqrt{1-\frac{1}{36}} = 5.9161$ rad/s

$\sigma = 1$ $\quad t_r = \frac{1}{5.9161} \tan^{-1}\left(-\frac{5.9161}{1}\right)$

$\therefore \; \boxed{t_r = 0.2938 \text{ sec}}$

Peak time: $t_p = \frac{\pi}{\omega_d} = \frac{\pi}{5.9161}$

$\boxed{t_p = 0.5310 \text{ sec}}$

Max overshoot: $M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100$

$\boxed{M_p = 58.80\,\%}$

2% Settling time : $t_s = \frac{4}{\zeta\omega_n} = \frac{4}{1}$

$$\boxed{t_s = 4 \text{ sec}}$$

See MATLAB Code for Computational results

#2)

$$G = \frac{Y(s)}{U(s)} = \frac{K}{ms^2+cs+K}$$

G ~~l~~ form of $\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

$\omega_n^2 = K$
$c = 2\zeta\omega_n$

ζ and ω_n can be found from step response

From plot: $t_p = 2.03$ $y(t_p) = 1.6$ $y(\infty) = 1$

Overshoot: $M_p = \frac{y(t_p)-y(\infty)}{y(\infty)} = 0.6$

$$M_p = 0.6 = e^{-(\zeta\pi/\sqrt{1-\zeta^2})}$$

$$-\ln(M_p) = \frac{-\zeta\pi}{\sqrt{1-\zeta^2}}$$

$$\ln^2(M_p) = \frac{\zeta^2\pi^2}{1-\zeta^2}$$

$$\ln^2(M_p) - \ln^2(M_p)\zeta^2 = \zeta^2\pi^2$$
$$\ln^2(M_p) = \zeta^2(\ln^2(M_p)+\pi^2)$$

$$\zeta = \sqrt{\frac{\ln^2(M_p)}{\ln^2(M_p)+\pi^2}} = 0.1605$$

From plot: $\frac{T}{2} = 4.05 - 2.03 = 2.02$ (Half Period)

$$\omega_d = \frac{2\pi}{T} = \omega_n\sqrt{1-\zeta^2} \Rightarrow \omega_n = 1.5757 \text{ rad/s}$$

$$G = \frac{2.483}{s^2 + 0.5058s + 2.483}$$

Compare Coefficients to $\frac{K}{ms^2 + cs + K}$:

(Assume $m=1$ to avoid indeterminate system)

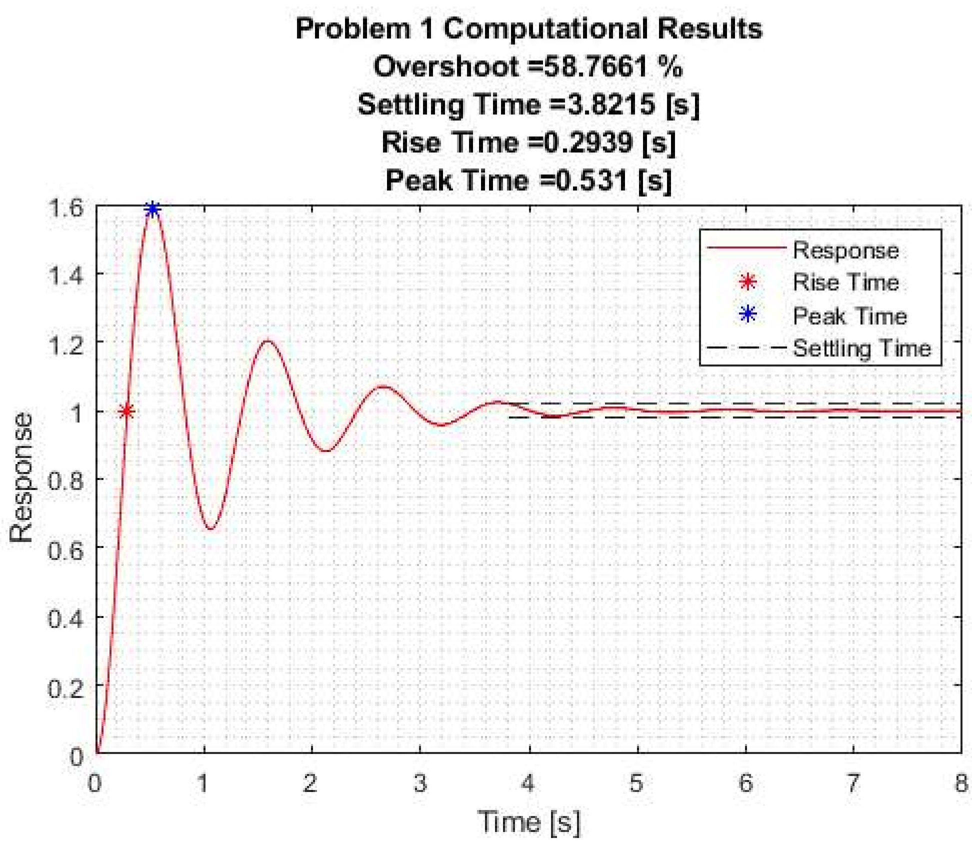
$$\boxed{\begin{aligned} m &= 1 \\ c &= 0.5058 \\ K &= 2.483 \end{aligned}}$$

Contents

MAE 543 HW3 Gabriel Colangelo

```
clear
clc
close all

% Transfer Function
s               = tf('s');
G               = 36/(s^2 + 2*s + 36);

% Simulation
time            = (0:.0001:8);                       % Time vector for sim
opt             = stepDataOptions('StepAmplitude',1);   % Unit Step Input
[y,t]           = step(G,time,opt);

% Find input step response characteristics
RiseTime        = t(find(y >= y(end),1));            % Find time of when y(inf) is first reached
PeakTime        = t(find(y == max(y),1));            % Find time when peak(y) is achieved
Overshoot       = ((max(y) - y(end))/(y(end)))*100;     % Mp = [peak(y) - y(inf)]/ y(inf) X 100

% Settling Time routine, 2% bounds on unit step input
indsettle       = find(y <= 1.02 & y >= 0.98);
filtind         = diff(indsettle)==1;
indzero         = find(filtind == 0);
SettlingTime    = t(indsettle(indzero(end)+1));

% plot results
plot(t,y,'-r',RiseTime,y(end),'*r',PeakTime,max(y),'*b')
line([SettlingTime, t(end)],[1.02, 1.02],'Color','black','LineStyle','--')
line([SettlingTime, t(end)],[.98, .98],'Color','black','LineStyle','--')
xlabel('Time [s]')
ylabel('Response')
grid minor
legend('Response','Rise Time','Peak Time','Settling Time')
titlestring     = strcat('Overshoot = ', num2str((max(y) - y(end))*100),' %');
titlestring2    = strcat('Settling Time = ', num2str(SettlingTime),' [s]');
titlestring3    = strcat('Rise Time = ', num2str(RiseTime),' [s]');
titlestring4    = strcat('Peak Time = ', num2str(PeakTime),' [s]');
title({'Problem 1 Computational Results';titlestring;titlestring2;titlestring3;titlestring4})

% Analytical Solutions
wn              = 6;                                 % wn^2 = 36
zeta            = 1/6;                               % 2*zeta*wn = 2

sigma           = wn*zeta;
wd              = wn*sqrt(1 - zeta^2);

tr              = (1/wd)*atan2(wd,-sigma)            % tr = wd*invtan(wd/-sigma)
tp              = pi/wd                              % tp = pi/wd
Mp              = exp(-zeta*pi/sqrt(1 - zeta^2)) *100   % MP = e^(-zeta*pi/(sqrt(1 - zeta^2))
ts              = 4/sigma                            % ts = 4/zeta*wn
```

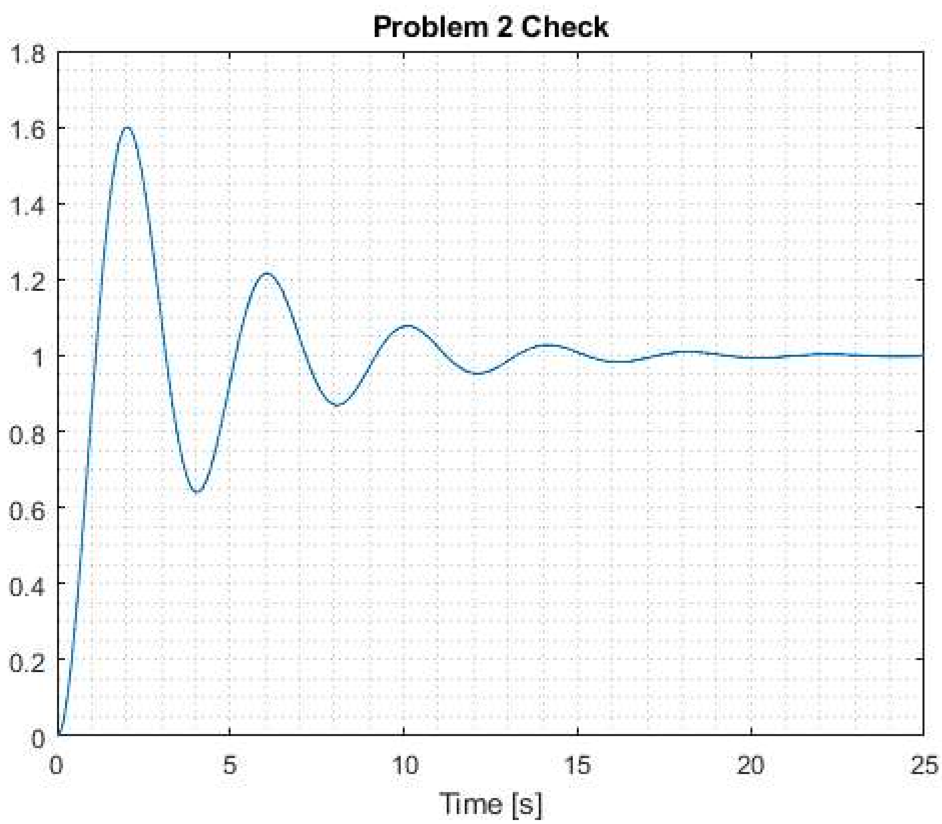
```
tr =

    0.2938


tp =

    0.5310


Mp =

   58.8001


ts =

     4
```



Problem 2 Check

```
zeta            = sqrt(log(0.6)^2/(log(0.6)^2 + pi^2));

HalfT           = 4.05 - 2.03;
T               = 2*HalfT;

wd              = 2*pi/T;

wn              = wd/sqrt(1 - zeta^2);
```

```
m               = 1;

k               = wn^2;
c               = 2*zeta*wn;

s               = tf('s');

sys             = k/(m*s^2 + c*s + k);

[X,T]           = step(sys,0:.01:25,opt);

figure
plot(T,X)
xlabel('Time [s]')
title('Problem 2 Check')
grid minor
```



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