AAE 666

Homework Three

Exercise 1 Determine whether or not the following functions are lpd.
(a)

$$V(x) = x_1^2 - x_1^4 + x_2^2$$

(b)

$$V(x) = x_1 + x_2^2$$

(c)

$$V(x) = 2x_1^2 - x_1^3 + x_1x_2 + x_2^2$$

Exercise 2 By appropriate choice of Lyapunov function, show that the origin is a stable equilibrium state for

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1^3 \end{aligned}$$

Note that the linearization of this system about the origin is unstable.

Exercise 3 By appropriate choice of Lyapunov function, show that the origin is a stable equilibrium state for

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 + x_1^3 \end{aligned}$$

Exercise 4 Show that the following system is stable about the zero state.

$$\begin{aligned} \dot{x}_1 &= x_2^3 \\ \dot{x}_2 &= -x_2^2x_1 \end{aligned}$$

Exercise 5 Show that the following system is GAS about zero

$$\dot{x} = -(2 + \cos x)x$$

Exercise 6 Show that the following system is GAS about 1.

$$\dot{x} = -(2 + \cos x)(x - 1)$$