ECE 68000: MODERN AUTOMATIC CONTROL

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System definition, control problem formulation, examples of system modeling

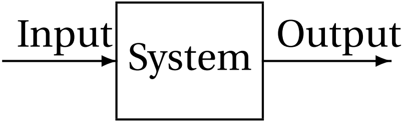
Outline

- What is a system?
- Simple examples of systems
- Systems modeling
- State-plane analysis
- Phase portraits
- The method of isoclines

System Definition (Text—page 1)

- A system is a combination of components that act together
- A system is a collection of objects that are related by interactions and produce various outputs in response to different inputs

Two Properties of a System



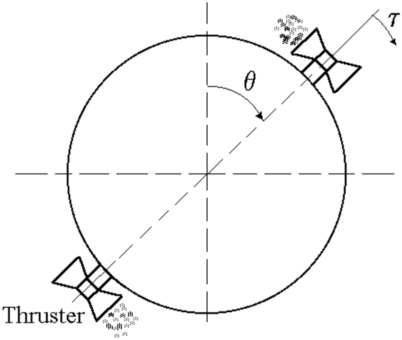
- the interrelations between the components that are contained within the system
- the system boundaries that separate the components within the system from the components outside

Control Problem—page 2 in Text

- A specified **objective** for the system
- A **model** of the system to be controlled
- A set of **admissible controllers**
- A means of measuring the performance of any given control strategy to evaluate its effectiveness—**optimal control**

Simple Examples of System Modeling

Example 2.2 on page 50 in Text—rigid satellite



Mechanical Systems

- Linear rotational systems are analogous to linear translational systems

Translational	Rotational
$F = ma$	$\tau = I\ddot{\theta}$

- Very simple model of the rigid satellite

$$\tau = I\ddot{\theta}$$

Rigid Satellite Model

- Define state variables

$$x_1 = \theta \quad \text{and} \quad x_2 = \dot{\theta} = \dot{x}_1$$

- Hence, $\dot{x}_2 = \ddot{\theta} = \frac{1}{I}\tau$
- State-space model

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{1}{I}\tau = u \end{aligned}$$

Satellite Model in Matrix Format

- $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$
- The above is a special case of
$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u},$$
- which is a special case of
$$\dot{\boldsymbol{x}} = \boldsymbol{f}(t, \boldsymbol{x}, \boldsymbol{u})$$

2D Model Analysis

- In our model

$$\boldsymbol{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \in \mathbb{R}^2$$

- We can plot x_1 vs. t and x_2 vs. t
- We can also plot x_2 vs. x_1 using t as a parameter
- The plane with coordinate axes x_1, x_2 is called the *state plane* or *phase plane*

State-Plane Analysis

- To each state $\boldsymbol{x}(t)$ of the system there corresponds a point in the state-space
- This point is called the *representative point* (RP)
- As t varies the RP describes a curve in the state plane, called a *trajectory*
- A family of trajectories is a *phase portrait*

Method of Isoclines—p. 53 in Text

- $$\begin{cases} \dot{x}_1 &= \frac{dx_1}{dt} = f_1(x_1, x_2) \\ \dot{x}_2 &= \frac{dx_2}{dt} = f_2(x_1, x_2) \end{cases}$$

- $$\frac{dx_2}{dx_1} = \frac{f_2(x_1, x_2)}{f_1(x_1, x_2)}$$

  We eliminated the independent variable t

- Consider the case when

$$\frac{dx_2}{dx_1} = m(x_1, x_2) = m = \text{constant}$$

What is Isocline?

- The set of points satisfying

$$\frac{dx_2}{dx_1} = m(x_1, x_2) = m = \text{constant}$$

  is called the *isocline*
- Another form of the eqn of the isocline corresponding to a specific m,

$$f_2(x_1, x_2) = m f_1(x_1, x_2)$$

- Example: $\ddot{y} + y = 0$

Constructing Isoclines

- Let
$$x_1 = y \quad \text{and} \quad x_2 = \dot{x}_1$$
- We represent $\ddot{y} = -y$ as
$$\begin{cases} \dot{x}_1 &= x_2 = f_1(x_1, x_2) \\ \dot{x}_2 &= -x_1 = f_2(x_1, x_2) \end{cases}$$
- Construct
$$\frac{dx_2}{dx_1} = -\frac{x_1}{x_2} = m$$

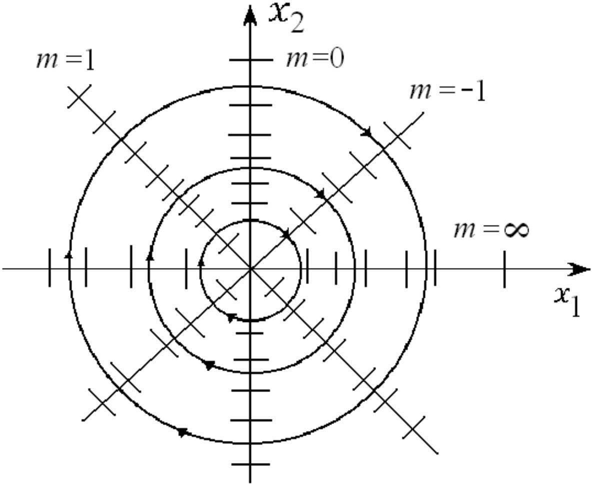
Isoclines' Equation

- $$x_2 = -\frac{1}{m}x_1$$
- The isoclines for this example are a family of straight lines that pass through the origin
- The line that satisfies the above equation is an isocline corresponding to the trajectories' slope m because a trajectory crossing the isocline will have its slope equal to m

The Isocline Method

- Construct several isoclines in the state plane
- Construct a field of local tangents m
- The trajectory passing through any given point in the state plane is obtained by drawing a continuous curve following the directions of the field

The Isocline Method—Example



Interactive Phase Portrait—Prep

```
t0=0;tf=20;tspan=tf-t0;
x0=[-4 -4]';
button=1;
p=4*[-1 0;1 0];
clf;plot(p(:,1),p(:,2))
hold on
plot(p(:,2),p(:,1))
axis(4*[-1 1 -1 1])
```

Interactive Phase Portrait

```
while(button==1)
[t,x]=ode45(@my_xdot,tspan,x0);
plot(x(:,1),x(:,2))
[x1,x2,button]=ginput(1);
x0=[x1 x2]';
end
```

Interactive Phase Portrait—ODE

```
function xdot=Diff_eq(t,x)
xdot=[x(2);-2*x(2)-x(1)];
```