ECE 602: LUMPED LINEAR SYSTEMS

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Stability of Discrete-Time LTI and LTV Systems

Stability of DT Autonomous Linear Systems

Discrete-time LTV system $x[k+1] = A[k]x[k], \quad k = 0, 1, \ldots$

Definition (Asymptotic Stability)

LTV system is asymptotically stable at time k_0 if its solution $x[k]$ starting from any initial state $x[k_0]$ at time k_0 satisfies

$$x[k] \to 0 \ \text{ as } k \to \infty$$

Definition (Exponential Stability)

LTV system is exponentially stable at time k_0 if its solution $x[k]$ starting from any initial state $x[k_0]$ at time k_0 satisfies

$$\|x[k]\| \leq K r^{k-k_0} \|x[k_0]\|, \quad \forall k = k_0, k_0+1, \ldots$$

for some constants $K > 0$, $0 \leq r < 1$

Stability of DT LTI Systems

Theorem

For LTI system $x[k+1] = Ax[k]$, the following statements are equivalent

1. *The LTI system is asymptotically stable*
2. *The LTI system is exponentially stable*
3. *All the eigenvalues of A are inside the open unit disk of $\mathbb{C}$*

- Asymptotic stability = exponential stability
- Starting time k_0 **does not** matter

Marginal Stability of DT LTI Systems

System $x[k+1] = Ax[k]$ is unstable if **either** of the following is true:

1. A has eigenvalues outside the closed unit disk of $\mathbb{C}$
2. A has defective eigenvalues on the unit circle of $\mathbb{C}$

System $x[k+1] = Ax[k]$ is marginally stable if **both** of following hold:

1. A has no eigenvalue outside the close unit disk of $\mathbb{C}$
2. A has eigenvalues on the unit circle of $\mathbb{C}$, each being non-defective

Stability of DT LTV Systems

LTV system $x[k+1] = A[k]x[k]$ has solution $x[k] = \Phi[k]x[0]$

Theorem

- *LTV system is asymptotically stable at time k_0 if*

$$\Phi[k, k_0] \to 0 \ \ \textit{as } k \to \infty$$

- *LTV system is exponentially stable at time k_0 if there exist $C > 0$, $0 \leq r < 1$, such that*

$$\|\Phi[k, k_0]\| \leq Cr^{k-k_0}, \quad \forall k \geq k_0$$

- Asymptotic stability $\neq$ exponential stability
- Starting time k_0 **does** matter