Sec 7.4 / 7.9

(Ex) $V = (\mathbb{R}^2, +, \cdot)$

$W \overset{\text{let}}{=} \{ (x, y) \in V \mid \underline{2x - y = 0} \}$

Is W a subspace of V? Yes.

① Any $u = (x_1, y_1)$, $v = (x_2, y_2) \in W$,

$u + v = (x_1, y_1) + (x_2, y_2)$

$= \underline{(x_1 + x_2, y_1 + y_2)}$

$2x_1 - y_1 = 0$ & $2x_2 - y_2 = 0$

$\Rightarrow 2(x_1 + x_2) - (y_1 + y_2)$

$$= \underline{2x_1} + 2x_2 \ \underline{-y_1} - y_2$$

$$= \underbrace{2x_1 - y_1}_{=0} + \underbrace{2x_2 - y_2}_{=0} = 0 \checkmark$$

② Let any $\beta \in \mathbb{R}$, $u = (x_1, y_1) \in W$

$$\underline{\beta u = (\beta x_1, \beta y_1)}$$

$$2(\beta x_1) - \beta y_1 = \beta(2x_1 - y_1) = 0 \checkmark$$

(Row spaces / Column spaces)

Let $A = \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$ rank$A = 1$.

(1) $\mathrm{Row}(A) = \{ c_1 [1\ 2] + c_2 [4\ 8] \mid c_1, c_2 \in \mathbb{R} \}$

: the row space of A

= the set of all linear combinations of two rows of A

Notation: $\underline{\mathrm{Row}(A) = \mathrm{span}\{[1\ 2], [4\ 8]\}}$

: the set of all linear combinations

(2) $\mathrm{Col}(A) = \left\{ d_1 \begin{bmatrix} 1 \\ 4 \end{bmatrix} + d_2 \begin{bmatrix} 2 \\ 8 \end{bmatrix} \;\middle|\; d_1, d_2 \in \mathbb{R} \right\}$

= the column space of A.

= the set of all lin. combinations of

$\begin{bmatrix}1\\4\end{bmatrix}$ & $\begin{bmatrix}2\\8\end{bmatrix}$ of A.

Notation: $Col(A) = span\left\{\begin{bmatrix}1\\4\end{bmatrix}, \begin{bmatrix}2\\8\end{bmatrix}\right\}$

$A = \begin{bmatrix}1 & 2\\4 & 8\end{bmatrix}$.

(1) Is Row (A) a subspace of $\mathbb{R}^2$?

① $u = c_1[1\ 2] + c_2[4\ 8]$

$v = b_1[1\ 2] + b_2[4\ 8] \in Row(A)$

$u + v = (c_1+b_1)[1\ 2] + (c_2+b_2)[4\ 8] \in Row(A$

② $\beta \in \mathbb{R}$

$\beta u = \beta c_1 [1 \ 2] + \beta c_2 [4 \ 8] \in Row(A)$

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(2) $Col(A) = span\left\{ \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 8 \end{bmatrix} \right\}$

: a subspace of $\mathbb{R}^2$

Remark $A_{m \times n} = \begin{bmatrix} A_1 \\ \vdots \\ A_m \end{bmatrix} = [a_1, a_2 \cdots a_n]$

(1) $Row(A)$: a subspace of $\mathbb{R}^n$

(2) $Col(A)$: a subspace of $\mathbb{R}^m$

(Basis).

(Ex) $A = \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$

(1) $Row(A) = span\{[1\ 2], [4\ 8]\}$.

$[4\ 8] = 4[1\ 2]$

$\Rightarrow Row(A) = span\{[1\ 2]\}$.

$\{[1\ 2]\}$: a <u>basis for Row(A)</u>.

<u>Def</u> $(V, \oplus, \odot)$: a vector space.

~~[illegible]~~ .

$\{ v_1, v_2, \cdots, v_k \} \subset V$

is a basis for V

iff (1) $v_1, \cdots, v_k$ are linearly independent

(2) $V = \mathrm{span}\{ v_1, v_2, \cdots, v_k \}$.

$k = \dim V$: the dimension of V.

(Ex) $A = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 0 & 2 \end{bmatrix}$: $\mathrm{rank} A = 2$.

$A \xrightarrow{r_2 - r_1} \begin{bmatrix} 1 & 2 & 4 \\ 0 & -2 & -2 \end{bmatrix}$

2 pivots.

(1) Find a basis for Row (A)

[1 2 4], [1 0 2] : lin. indep.

$\mathbb{R}$ {[1 2 4], [1 0 2]}

: a basis for Row(A)

dim Row (A) = 2

(2) Find a basis for Col (A)

$\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right\}$: a basis for Col (A)

dim Col (A) = 2.