$\vec{r}_{P/O} = x\hat{e}_1 + y\hat{e}_2 + z\hat{e}_3$ (Basis)

Chapter 2, App C

$[\vec{r}_{P/O}]_I = \begin{pmatrix} x \\ y \\ z \end{pmatrix}_I$ (array)

expressed in I

Newtonian Mechanics

↳ Newton published Principia 1687

A law is a descriptive generalization about some aspect of how the natural world behaves. A law is not a theory, which is just an explanation.

LAW I: Everybody or object preserves its state of rest or uniform motion unless compelled to change its state under the action of forces

LAW II: The alteration of motion is ever proportional to the motive force impressed and it is made in the direction of the the right line in which that force is impressed.

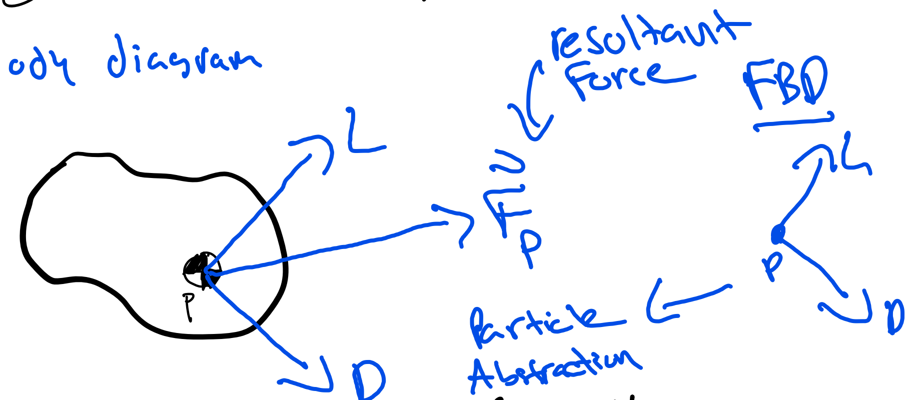
(N2L)

LAW III: To every action there is always an opposite and equal reaction or the mutual actions of the two bodies upon each other are always equal, and direct contrary.

Applying Newton's laws to a problem can help us to derive the <u>equations of motion</u> of the system.

One key step in Newton's process is drawing the

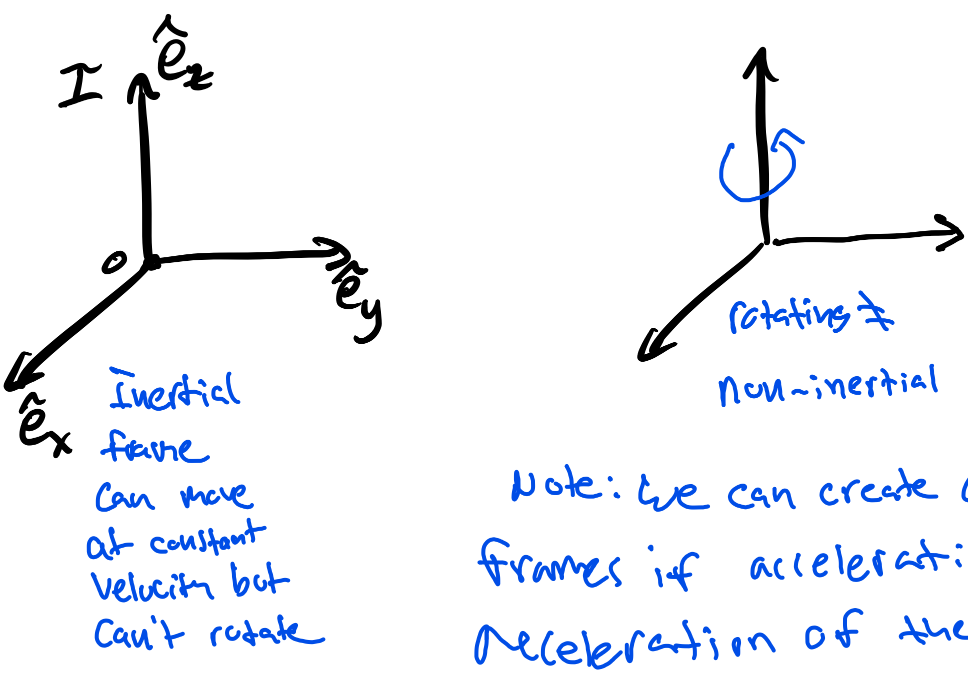
Free body diagram



<u>Key points to remember about N2L</u>

① N2L only applies to a point mass

② N2L only applies in an inertial frame
Absolute space, fixed
Non-accelerating



Note: we can create approximate inertial frames if acceleration of frame $\ll$ acceleration of the object in frame.

$$\vec{F}_p = {}^I\frac{d}{dt}\left({}^I\vec{p}_{p/o}\right)$$

N2L:

$$\vec{F}_p = m_p\, {}^I\vec{a}_{p/o} = m_p \frac{{}^Id}{dt}\left({}^I\vec{v}_{p/o}\right)$$

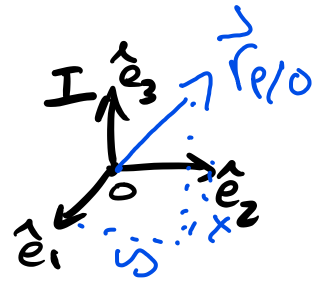
$${}^I\vec{v}_{p/o} = \frac{{}^Id}{dt}\left(\vec{r}_{p/o}\right)$$

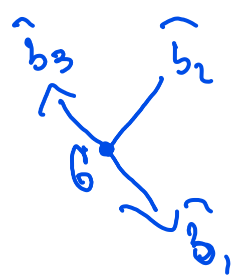
Cartesian (very simple) example

$$\vec{r}_{p/o} = x\hat{e}_1 + y\hat{e}_2 + z\hat{e}_3$$

$${}^I\vec{v}_{p/o} = \frac{{}^Id}{dt}\left(\vec{r}_{p/o}\right) = \frac{{}^Id}{dt}\left(x\hat{e}_1 + y\hat{e}_2 + z\hat{e}_3\right)$$

$$= \dot{x}\hat{e}_1 + \dot{y}\hat{e}_2 + \dot{z}\hat{e}_3$$





What if we had $\frac{{}^Bd}{dt}(\ )$?

$${}^B\vec{v}_{p/o} = \frac{{}^Bd}{dt}\left(x\hat{e}_1 + y\hat{e}_2 + z\hat{e}_3\right)$$

$$= \frac{{}^Bd}{dt}\left(x\hat{e}_1\right) + \frac{{}^Bd}{dt}\left(y\hat{e}_2\right) + \frac{{}^Bd}{dt}\left(z\hat{e}_3\right)$$

$$= \dot{x}\hat{e}_1 + x\frac{{}^Bd}{dt}\left(\hat{e}_1\right) + \dot{y}\hat{e}_2 + y\frac{{}^Bd}{dt}\left(\hat{e}_2\right) + \dot{z}\hat{e}_3$$

$$+ z\frac{{}^Bd}{dt}\left(\hat{e}_3\right)$$

N2L in scalar coordinate form:

Let,

$$\vec{F}_P = f_x \hat{e}_x + f_y \hat{e}_y + f_z \hat{e}_z$$

$$^I\vec{a}_{P/O} = \ddot{x}\hat{e}_x + \ddot{y}\hat{e}_y + \ddot{z}\hat{e}_z$$

$$\vec{F}_P = m\,^I\vec{a}_{P/O}$$

$$f_x \hat{e}_x + f_y \hat{e}_y + f_z \hat{e}_z = (\ddot{x}\hat{e}_x + \ddot{y}\hat{e}_y + \ddot{z}\hat{e}_z)\,m$$

$$\left.\begin{aligned} f_x &= m\ddot{x} \\ f_y &= m\ddot{y} \\ f_z &= m\ddot{z} \end{aligned}\right\}$$

Three scalar O.E.

ODE with IC's give IVP

Dfn: The <u>equations of motion</u> are (usually differential equations) whose solutions are the position and velocity of the particle or object.

There are many techniques to integrate or "solve" ordinary differential equations.

$$f(x, \dot{x}, \ddot{x}, \ldots, t) = 0 \xrightarrow{\text{"Solving"}} x(t)$$

Plus an IC's

Trajectory

Differential Equations Class

Separation of variables
Laplace transforms
Integrating factor method
Guess and check solution
Etc.

Numerical Methods Class

Forward and Backward Euler
Runge-Kutta methods
Etc.
ODE-45

A key step in numerical solution of an ODE is putting the equation in a <u>1st order form</u> (state-space form).

Ex.

$$\vec{\xi} = \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \\ z \\ \dot{z} \end{bmatrix} \qquad \dot{\xi} = \begin{bmatrix} \xi_2 \\ f_x/m \\ \xi_4 \\ f_y/m \\ \xi_6 \\ f_z/m \end{bmatrix}$$

1st order form of 3 EoM

State vector.

Characterizes system at given time so motion can predicted at next instant using E.O.M.