Case Study

Determine the stability of the system

$$x[k+1] = \begin{bmatrix} -2 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & -2 & -2 \end{bmatrix} x[k]$$

in the sense of Lyapunov by solving the Lyapunov matrix equation using the Kronecker product. Take $Q = I_3$.

Verify your solution using the MATLAB function `dlyap`.

Please take a moment to work through this problem on your own. When you are ready, scroll down the page to view the solution.

Explanation: Represent the discrete Lyapunov matrix equation,

$$A^\top PA - P = -Q,$$

using the Kronecker product as

$$\left(A^\top \otimes A^\top - I_3 \otimes I_3\right) \operatorname{vec}(P) = -\operatorname{vec}(Q).$$

We then solve the above equation using MATLAB,

```
A=[-2 0 0;1 0 1;0 -2 -2];
nQ=-eye(3);
P_vec=(kron(A',A')-kron(eye(3),eye(3)))\nQ(:);
P=[P_vec(1:3) P_vec(4:6) P_vec(7:9)]
eig(P)
```

We obtain,

$$P = \begin{bmatrix} -2.2667 & -2.4000 & -1.4000 \\ -2.4000 & -3.8000 & -1.6000 \\ -1.4000 & -1.6000 & -1.2000 \end{bmatrix}.$$

The eigenvalues of P are $\{-6.4140,\ -0.2327,\ -0.6200\}$. They are all negative. So P is actually negative definite. By the theorem of Lyapunov the system is unstable. Indeed, the eigenvalues of A are:

$$\{-1.0000 + 1.0000i,\ -1.0000 - 1.0000i,\ -2.0000 + 0.0000i\}.$$

They are all outside the unit circle.

The same P is obtained when using the MATLAB function, `dlyap(A',eye(3))`.

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