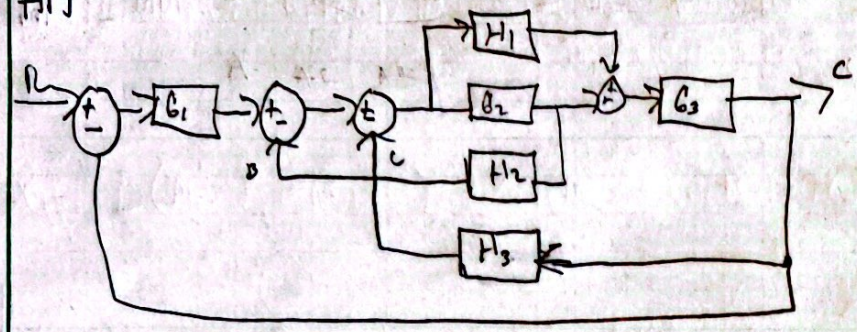
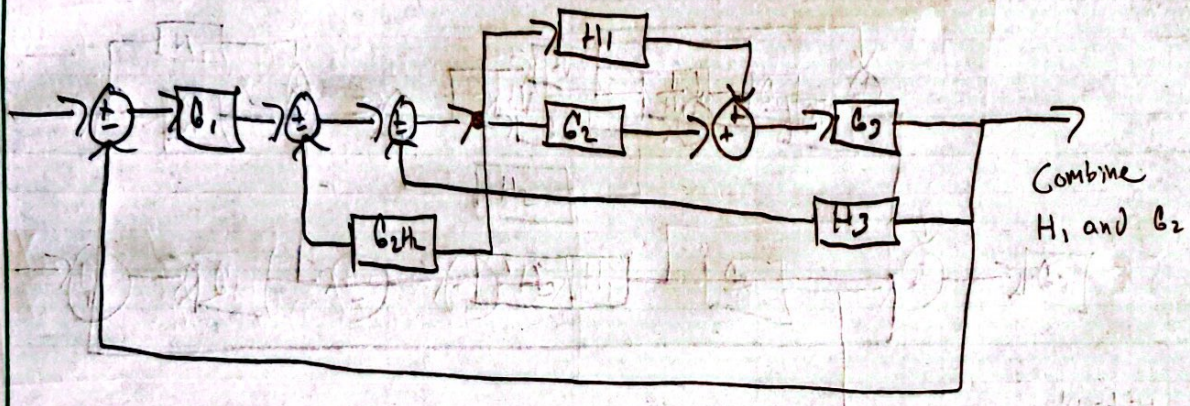
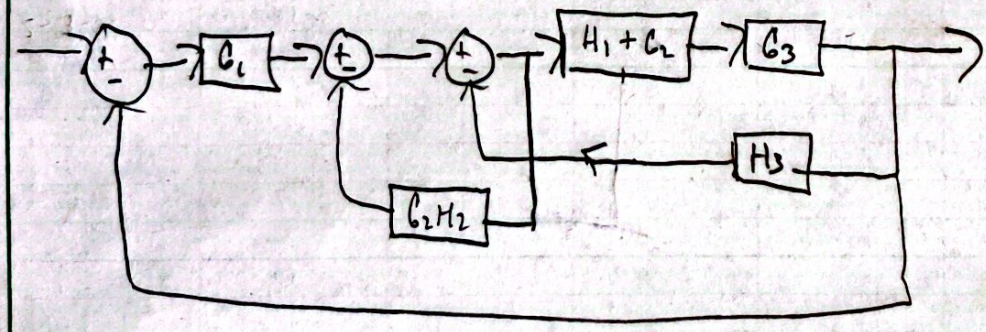
#1)



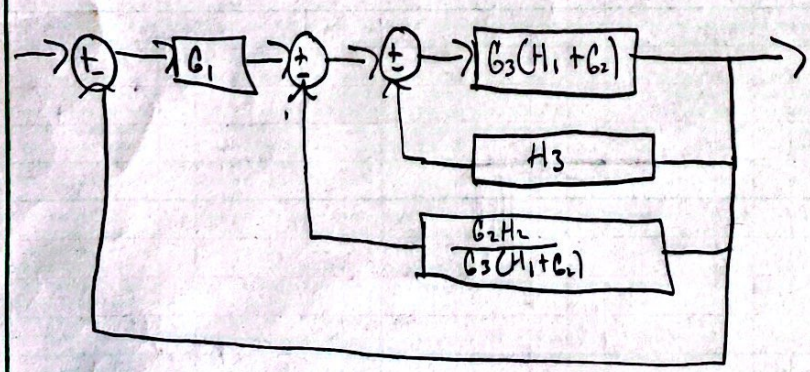
Move node



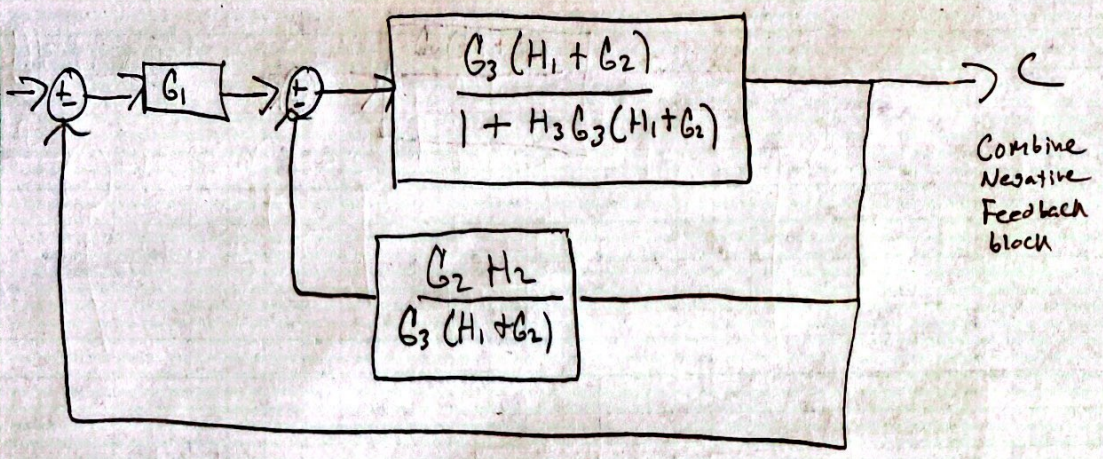
Combine H_1 and G_2



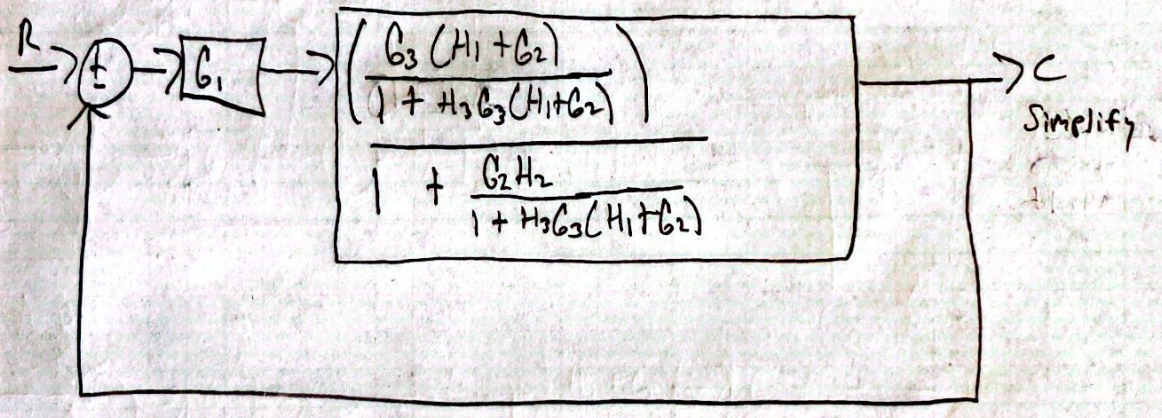
Move Node



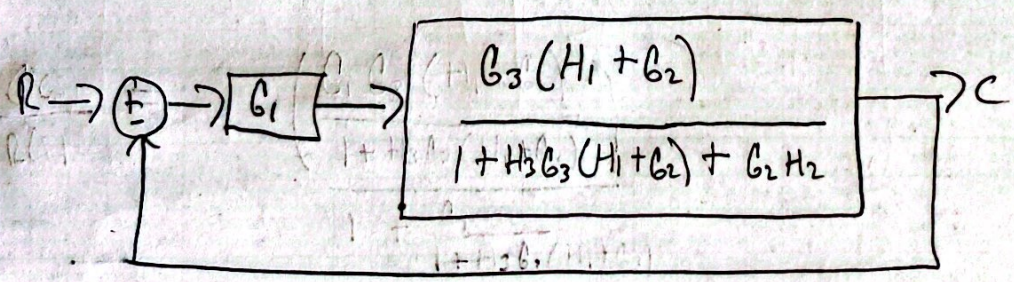
Combine Negative Feedback block



Combine Negative Feedback block



Simplify



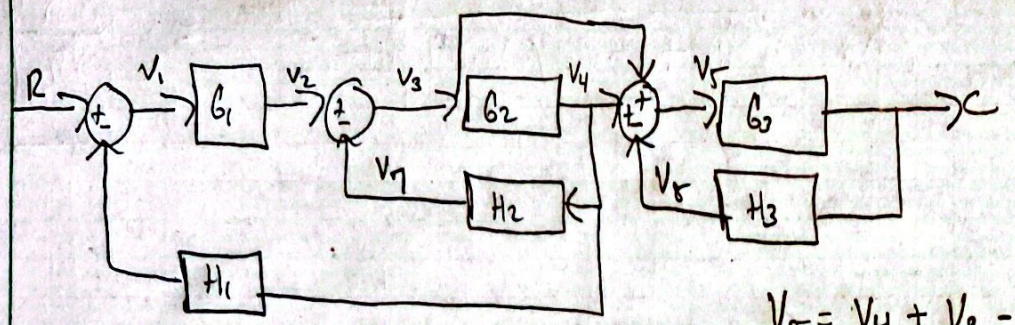
Combine cascaded blocks

$$\frac{C(s)}{R(s)} = \frac{\frac{G_1G_3(H_1+G_2)}{1+H_3G_3(H_1+G_2)+G_2H_2}}{1+\frac{G_1G_3(H_1+G_2)}{1+H_3G_3(H_1+G_2)+G_2H_2}}$$

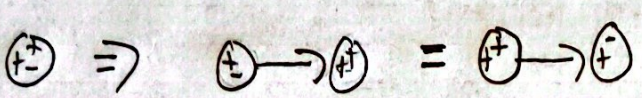
$$\frac{C(s)}{R(s)} = \frac{G_1 G_3 (H_1 + G_2)}{1 + H_3 G_3 (H_1 + G_2) + G_2 H_2 + G_1 G_3 (H_1 + G_2)}$$

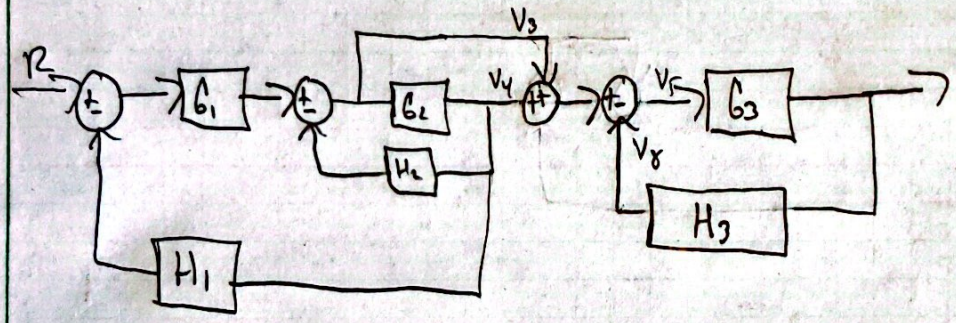
Figure 1 Input to output transfer function

#2)

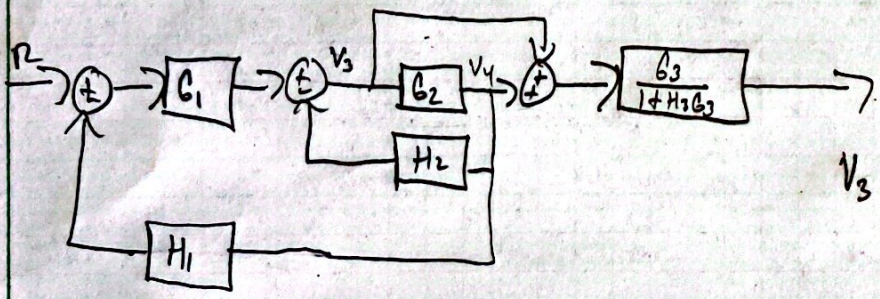


$V_5 = V_4 + V_3 - V_8$

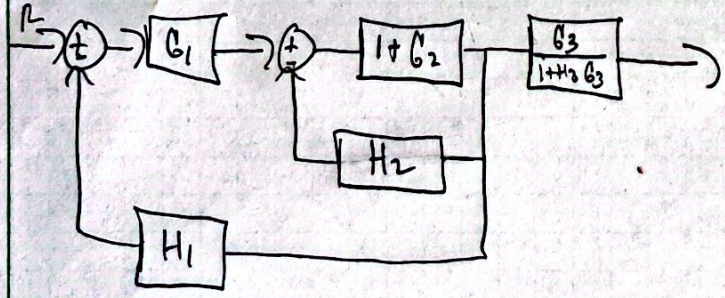
Seperate 



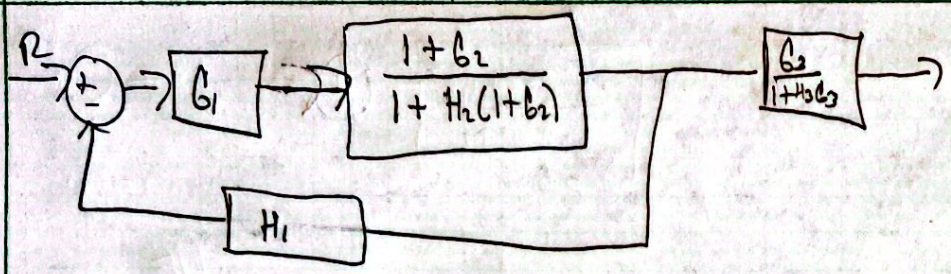
Combine Negative Feedback, $G_3 H_3$.



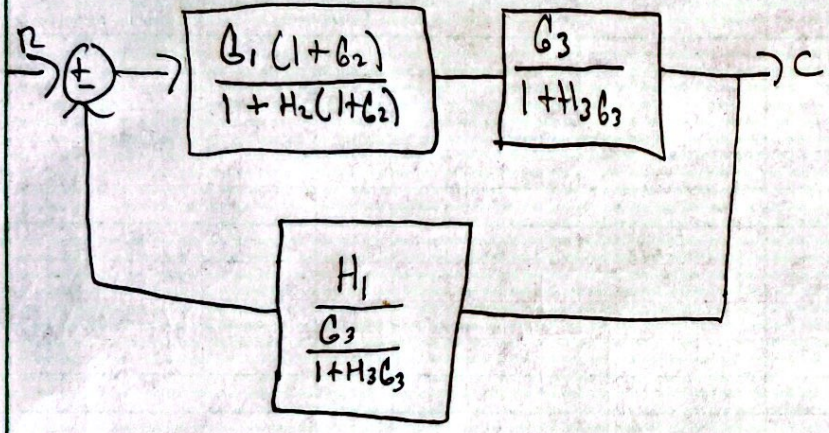
$V_3 + V_4 = V_3 + G_2 V_3$



Combine Negative Feedback



Move Node and cascade

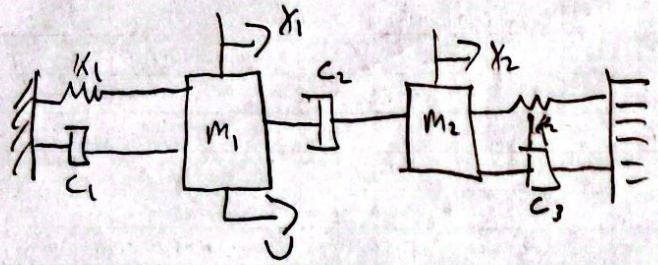


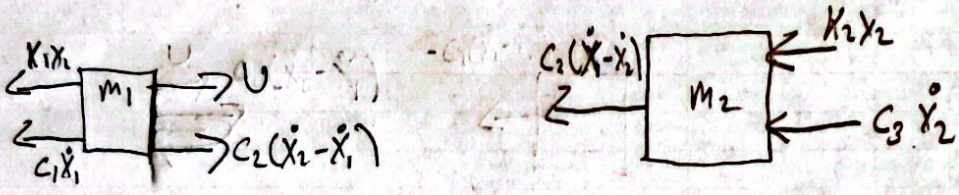
$$\frac{C}{R} = \frac{\left(\frac{G_3 G_1 (1+G_2)}{[1+H_2(1+G_2)][1+H_3G_3]}\right)}{\left(1 + \frac{G_3 G_1 (1+G_2)}{[1+H_2(1+G_2)][1+H_3G_3]} \cdot \frac{H_1}{\frac{G_3}{1+H_3G_3}}\right)}$$

$$\boxed{\frac{C}{R} = \frac{G_3 G_1 (1+G_2)}{[1+H_2(1+G_2)][1+H_3G_3] + G_1 H_1 (1+G_2)(1+H_3G_3)}}$$

Input / output transfer function to Figure 2

#3)





$$m_1\ddot{x}_1 = -K_1x_1 - C_1\dot{x}_1 + C\dot{x}_2 - C_2\dot{x}_1 + U$$

$$m_2\ddot{x}_2 = C_2\dot{x}_1 - C_2\dot{x}_2 - K_2x_2 - C_3\dot{x}_2$$

$$m_1\ddot{x}_1 + K_1x_1 + C_1\dot{x}_1 + C_2\dot{x}_1 = C_2\dot{x}_2 + U \qquad (1)$$

$$m_2\ddot{x}_2 + C_2\dot{x}_2 + K_2x_2 + C_3\dot{x}_2 = C_2\dot{x}_1 \qquad (2)$$

Assume zero I.C., take laplace of (1) & (2):

$$X_1(s)[m_1s^2 + K_1 + C_1s + C_2s] = C_2X_2(s) + U(s)$$

$$X_2(s)[m_2s^2 + C_2s + K_2 + C_3s] = C_2X_1(s)$$

$$X_2(s) = \frac{C_2X_1(s)}{m_2s^2 + C_2s + K_2 + C_3s} = \frac{C_2X_1(s)}{m_2s^2 + (C_2+C_3)s + K_2}$$

$$X_1(s)[m_1s^2 + (C_1+C_2)s + K_1] = \frac{C_2^2}{m_2s^2 + (C_2+C_3)s + K_2}X_1(s) + U(s)$$

$$X_1(s)\left[m_1 s^2 + (c_1+c_2)s + k_1\right] - \frac{c_2^2\, X_1(s)}{m_2 s^2 + (c_2+c_3)s + k_2} = U(s)$$

$$X_1(s)\left[(m_1 s^2 + (c_1+c_2)s + k_1)(m_2 s^2 + (c_2+c_3)s + k_2) - c_2^2\right] = (m_2 s^2 + (c_2+c_3)s + k_2)U(s)$$

$$\boxed{\frac{X_1(s)}{U(s)} = \frac{m_2 s^2 + (c_2+c_3)s + k_2}{(m_1 s^2 + (c_1+c_2)s + k_1)(m_2 s^2 + (c_2+c_3)s + k_2) - c_2^2}}$$

$$\boxed{\frac{X_2}{X_1} = \frac{c_2}{m_2 s^2 + (c_2+c_3)s + k_2}}$$