Contents


```
clear
clc
```

Problem 1

```
disp('--Problem 1 ----------------------------------------------------------')
disp(' ')

T           =  [1 0 -2 -1;...
                0 1 2 0; ....
                0 0 1 1;...
                0 0 0 1];
Tinv        = inv(T)
```

```
--Problem 1 ----------------------------------------------------------


Tinv =

     1     0     2    -1
     0     1    -2     2
     0     0     1    -1
     0     0     0     1
```

Problem 2

Social Network A----------

```
Aa          =  [1/4 1/4 1/4 1/4;...
                1/3 1/3 1/3 0;
                1/4 1/4 1/4 1/4;
                1/3 0 1/3 1/3];
% Eigenvalues
lambda_a    = eig(Aa);

disp('--Problem 2a ---------------------------------------------------------')
disp(' ')

% Check Algebraic and Geometric Multiplcities
AlgMulta    = GetAlgebraicMultiplicity(lambda_a);
```

```
GeoMulta    = GetGeometricMultiplicity(lambda_a, Aa);
[Ta,Ja]     = CheckDiagonalizable(AlgMulta, GeoMulta, lambda_a, Aa)

% left evecs are rows of T^-1
Tainv       = inv(Ta)
```

Social Network B----------

```
Ab          =  [1/3 1/3 0 1/3;...
                1/3 1/3 1/3 0;
                0 1/3 1/3 1/3;
                1/3 0 1/3 1/3];
% Eigenvalues
lambda_b    = eig(Ab);

disp('--Problem 2b ------------------------------------------------------')
disp(' ')

% Check Algebraic and Geometric Multiplcities
AlgMultb    = GetAlgebraicMultiplicity(lambda_b);
GeoMultb    = GetGeometricMultiplicity(lambda_b, Ab);
[Tb,Jb]     = CheckDiagonalizable(AlgMultb, GeoMultb, lambda_b, Ab)

% left evecs are rows of T^-1
Tbinv       = inv(Tb)
```

Social Network C----------

```
Ac          =  [1/3 1/3 0 1/3;...
                1/3 1/3 1/3 0;
                0 1/2 1/2 0;
                1/2 0 0 1/2];
% Eigenvalues
lambda_c    = eig(Ac);

disp('--Problem 2c ------------------------------------------------------')
disp(' ')

% Check Algebraic and Geometric Multiplcities
AlgMultc    = GetAlgebraicMultiplicity(lambda_c);
GeoMultc    = GetGeometricMultiplicity(lambda_c, Ac);
[Tc,Jc]     = CheckDiagonalizable(AlgMultc, GeoMultc, lambda_c, Ac)

% left evecs are rows of T^-1
Tcinv       = inv(Tc)
```

Social Network D----------

```
Ad          =  [1/2 0 0 1/2;...
                0 1/2 1/2 0;
                0 1/2 1/2 0;
                1/2 0 0 1/2];
% Eigenvalues
lambda_d    = eig(Ad);

disp('--Problem 2d ------------------------------------------------------')
disp(' ')
```

```
% Check Algebraic and Geometric Multiplcities
AlgMultd    = GetAlgebraicMultiplicity(lambda_d);
GeoMultd    = GetGeometricMultiplicity(lambda_d, Ad);
[Td,Jd]     = CheckDiagonalizable(AlgMultd, GeoMultd, lambda_d, Ad)

% left evecs are rows of T^-1
Tdinv       = inv(Td)
```

```
--Problem 2a -------------------------------------------------------

Eigenvalue of -0.1667 has Alg Mult of 1
Eigenvalue of -0.0000 has Alg Mult of 1
Eigenvalue of 0.3333 has Alg Mult of 1
Eigenvalue of 1.0000 has Alg Mult of 1
Matrix is diagonalizable

Ta =

    0.5000    0.4243   -0.7071    0.0000
    0.5000   -0.5657   -0.0000   -0.7071
    0.5000    0.4243    0.7071   -0.0000
    0.5000   -0.5657   -0.0000    0.7071


Ja =

    1.0000         0         0         0
         0   -0.1667         0         0
         0         0   -0.0000         0
         0         0         0    0.3333


Tainv =

    0.5714    0.4286    0.5714    0.4286
    0.5051   -0.5051    0.5051   -0.5051
   -0.7071    0.0000    0.7071    0.0000
   -0.0000   -0.7071    0.0000    0.7071

--Problem 2b -------------------------------------------------------

Eigenvalue of -0.3333 has Alg Mult of 1
Eigenvalue of 0.3333 has Alg Mult of 2
Eigenvalue of 1.0000 has Alg Mult of 1
Matrix is diagonalizable

Tb =

    0.5000    0.3627    0.6070    0.5000
   -0.5000   -0.6070    0.3627    0.5000
    0.5000   -0.3627   -0.6070    0.5000
   -0.5000    0.6070   -0.3627    0.5000


Jb =

   -0.3333         0         0         0
         0    0.3333         0         0
```

```
         0         0    0.3333         0
         0         0         0    1.0000


Tbinv =

    0.5000   -0.5000    0.5000   -0.5000
    0.3627   -0.6070   -0.3627    0.6070
    0.6070    0.3627   -0.6070   -0.3627
    0.5000    0.5000    0.5000    0.5000

--Problem 2c -----------------------------------------------------

Eigenvalue of -0.2287 has Alg Mult of 1
Eigenvalue of 0.1667 has Alg Mult of 1
Eigenvalue of 0.7287 has Alg Mult of 1
Eigenvalue of 1.0000 has Alg Mult of 1
Matrix is diagonalizable

Tc =

    0.5831   -0.3922    0.5000    0.2941
   -0.5831   -0.3922    0.5000   -0.2941
    0.4001    0.5883    0.5000   -0.6430
   -0.4001    0.5883    0.5000    0.6430


Jc =

   -0.2287         0         0         0
         0    0.1667         0         0
         0         0    1.0000         0
         0         0         0    0.7287


Tcinv =

    0.6527   -0.6527    0.2986   -0.2986
   -0.5099   -0.5099    0.5099    0.5099
    0.6000    0.6000    0.4000    0.4000
    0.4061   -0.4061   -0.5918    0.5918

--Problem 2d -----------------------------------------------------

Eigenvalue of 0.0000 has Alg Mult of 2
Eigenvalue of 1.0000 has Alg Mult of 2
Matrix is diagonalizable

Td =

         0    0.7071    0.7071         0
   -0.7071         0         0    0.7071
    0.7071         0         0    0.7071
         0   -0.7071    0.7071         0


Jd =

     0     0     0     0
     0     0     0     0
     0     0     1     0
```

```
     0     0     0     1


Tdinv =

         0   -0.7071    0.7071         0
    0.7071         0         0   -0.7071
    0.7071         0         0    0.7071
         0    0.7071    0.7071         0
```

Problem 3

```
disp('--Problem 3 ------------------------------------------------------')
disp(' ')

% Eigenvectors - from hand written
V1  = [-1; 0; 1]*1/sqrt(2);
V2  = [1;-1;1]*1/sqrt(3);
V3  = [1;1;1]*1/sqrt(3);

T   = [V1,V2,V3];
Tinv= inv(T)
```

Functions

```
% Function to find Alg Mult of each Eigenvalue
function am = GetAlgebraicMultiplicity(evals)
    % Initialize
    lam_uni = uniquetol(evals,1e-8); % unique evals
    am      = zeros(size(lam_uni));

    % Check to see how many times each eigenvalue repeats
    for i = 1:length(am)
        ind     = find(abs(lam_uni(i) - evals) <= 1e-8);
        am(i)   = length(ind);
        fprintf('Eigenvalue of %.4f has Alg Mult of %i \n',lam_uni(i), am(i))
    end
end

% Function to find Geo Mult of each Eigenvalue
function gm = GetGeometricMultiplicity(evals, A)
    % Initialize
    lam_uni = uniquetol(evals,1e-8); % unique evals
    gm      = zeros(size(lam_uni));

    % Check dimension of null space of lamda*I - A
    for i = 1:length(gm)
        gm(i) = size(null(lam_uni(i)*eye(size(A)) - A),2);
    end
end

function [T,J] = CheckDiagonalizable(AlgMult, GeoMult, lambda, A)
    % Check if Matrix is diagnolizable (n distinct evals or AlgMult = GeoMult)
    if (AlgMult == GeoMult)
        disp('Matrix is diagonalizable')
        [T,J] = eig(A);
    elseif ((length(uniquetol(lambda)) == length(lambda)) == 1)
```

```
        disp('Matrix is diagonalizable')
        [T,J] = eig(A);
    else
        disp('Matrix is not diagonalizable')
        [T,J] = jordan(A);
    end
end
```

```
--Problem 3 ----------------------------------------------------------


Tinv =

   -0.7071    0.0000    0.7071
    0.4330   -0.8660    0.4330
    0.4330    0.8660    0.4330
```

Published with MATLAB® R2021b