Transfer Orbits: Lambert Arcs

Two approaches to mission planning:

(a) Given the transfer orbit → initial and final positions are specified; relate to the time of flight

(b) Given the initial (departure) and final (target) points → determine the orbit that passes through the points

Transfer Orbit Design
(special class of boundary value problem)

1. **Geometrical relationships**

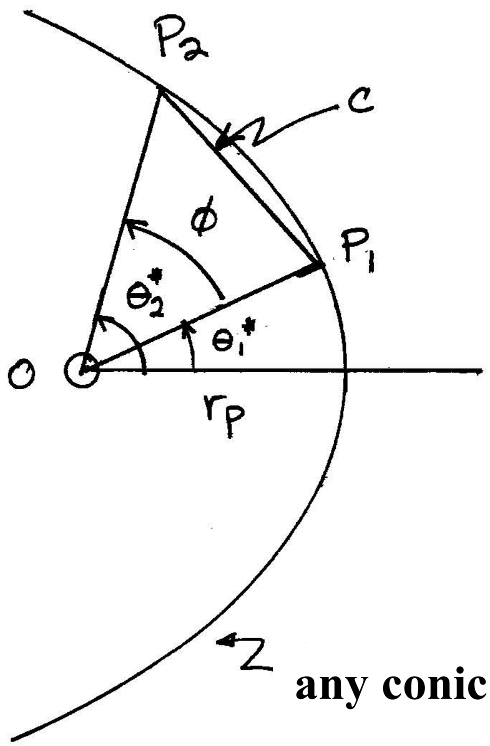
   Conic paths connecting two points that are fixed in space with focus at the attracting center

2. **Analytical Relationships**

3. **Lambert's Theorem**

Analytical Relationships

Objective: expression for *p*; *e*



any conic

$$r = \frac{p}{1 + e\cos\theta^*}$$

$$e\cos\theta_1^* = \frac{p}{r_1} - 1$$

$$e\cos\theta_2^* =$$

Also known:

$$a\,e^2 = a - p$$

$$c^2 = r_1^2 + r_2^2 - 2\,r_1\,r_2\cos\phi$$

Given the following trig identity

Sub above 5 expressions into trig identity and produce a quadratic in ***p***

$$\boxed{\begin{aligned} &a\,c^2\,p^2 + r_1\,r_2\left(1-\cos\phi\right)\left[-2\,a\left(r_1+r_2\right)+r_1\,r_2\left(1+\cos\phi\right)\right]p \\ &\qquad + a\,r_1^2\,r_2^2\left(-1+\cos\phi\right)^2 = 0 \end{aligned}}$$

I

Use $2s = r_1 + r_2 + c$ to rewrite term in brackets

$$\boxed{\left[-2a\left(r_1+r_2\right)+r_1 r_2\left(1+\cos\phi\right)\right]=2s\left(s-c-2a\right)+2ac} \qquad \textbf{A}$$

Also the last term

$$\boxed{r_1 r_2\left(1-\cos\phi\right)=2\left(s-r_1\right)\left(s-r_2\right)} \qquad \textbf{B}$$

AND add some new definitions:

IF transfer is <u>elliptic</u> arc

$$\left.\begin{aligned}
s-c-2a &= -2a\cos^2\left(\frac{\beta}{2}\right)\\
s &= 2a\sin^2\left(\frac{\alpha}{2}\right)\\
c &= 2a\left[\sin^2\left(\frac{\alpha}{2}\right)-\sin^2\left(\frac{\beta}{2}\right)\right]
\end{aligned}\right\} \qquad \textbf{C}$$

Sub A, B, C into I

Quadratic for *p*

$$c^4 p^2 - 4a(s-r_1)(s-r_2)\left[\sin^2\left(\frac{\alpha+\beta}{2}\right)+\sin^2\left(\frac{\alpha-\beta}{2}\right)\right]c^2 p$$

$$+4a^2(s-r_1)^2(s-r_2)^2\sin^2\left(\frac{\alpha+\beta}{2}\right)\sin^2\left(\frac{\alpha-\beta}{2}\right)=0$$

Roots

If know *a*, produces two possible paths;
Each path possesses different values of *p* and *e*

@ $a = a_{\min}$

$$2a_{\min} = s \quad \Rightarrow \quad \alpha = \pi$$

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IF transfer is <u>hyperbolic</u> arc

$$\left.\begin{aligned} s-c-2a &= 2|a|\cosh^2\left(\frac{\beta'}{2}\right) \\ s &= 2|a|\sinh^2\left(\frac{\alpha'}{2}\right) \\ c &= 2|a|\left[\sinh^2\left(\frac{\alpha'}{2}\right)-\sinh^2\left(\frac{\beta'}{2}\right)\right] \end{aligned}\right\}\ \mathbf{C}$$

Using this C in I

Roots

If known $|a|$, produces two possible hyperbolic paths