ECE 602: LUMPED LINEAR SYSTEMS

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Continuous-Time and Discrete-Time Systems,
Linear and Nonlinear Systems

Continuous-Time vs. Discrete-Time Systems

Systems $\mathcal{N}$ are called

- continuous-time (CT) systems if input u and output y are continuous-time signals
- discrete-time (DT) systems if input u and output y are discrete-time signals

Examples:

1. $\ddot{y}(t) + 2\dot{y}(t) + y(t) = \dot{u}(t) - u(t)$ for $t \in (-\infty, \infty)$
2. $y[k+1] = 2y[k] - u[k]$ for $k = 0, 1, \ldots$

Linear vs. Nonlinear Systems

Systems $\mathcal{N}$ are linear systems if for all $u_1, u_2 \in \mathcal{U}$ and all $\lambda_1, \lambda_2 \in \mathbb{R}$,

$$\mathcal{N}(\lambda_1 u_1 + \lambda_2 u_2) = \lambda_1 \mathcal{N}(u_1) + \lambda_2 \mathcal{N}(u_2)$$

Or equivalently, $\mathcal{N}$ have the following two properties:

1. Homogeneity: $\mathcal{N}(\lambda u) = \lambda \mathcal{N}(u)$ for all $\lambda \in \mathbb{R}$ and all $u \in \mathcal{U}$
2. Additivity: $\mathcal{N}(u_1 + u_2) = \mathcal{N}(u_1) + \mathcal{N}(u_2)$ for all $u_1, u_2 \in \mathcal{U}$

Systems $\mathcal{N}$ are nonlinear systems if otherwise

Examples

1. $y(t) = [u(t)]^2$

2. $y(t) = t^2 u(t)$

3. $y(t) = \int_{t-1}^{t+2} u(s)\,ds$

4. $y(t) = u(t) - u(t-1)$

5. $y(t) = \begin{cases} t & \text{if } |u(t)| \leq 1 \\ 0 & \text{if } |u(t)| > 1 \end{cases}$

6. $y[k] = \begin{cases} 3u[k-1] & \text{if } k = 0, 1, \dots, \\ 0 & \text{if } k = -1, -2, \dots \end{cases}$