HW4

4.6.3) $y_1' = y_2 + e^{3t}$

$y_2' = y_1 - 3e^{3t}$

$$y' = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} y + \begin{pmatrix}1\\ -3\end{pmatrix} e^{3t}$$

$|A - \lambda I| = \lambda^2 - 1 = 0$ $\lambda_{1,2} = \pm 1$

$AV_1 = V_1$

$\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} = \begin{pmatrix}x_1\\ x_2\end{pmatrix}$ $x_2 = 1 \therefore x_1 = 1$ $V_1 = \begin{pmatrix}1\\ 1\end{pmatrix}$

$AV_2 = V_2$

$\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} = \begin{pmatrix}-x_1\\ -x_2\end{pmatrix}$ $x_2 = 1 \therefore x_1 = -1$ $V_2 = \begin{pmatrix}-1\\ 1\end{pmatrix}$

$y_h = c_1 \begin{pmatrix}1\\ 1\end{pmatrix} e^{t} + c_2 \begin{pmatrix}-1\\ 1\end{pmatrix} e^{-t}$

$y_p = ue^{3t}$ $y_p' = uAe^{3t}$

$3ue^{3t} = Aue^{3t} + \begin{pmatrix}1\\ -3\end{pmatrix} e^{3t} \Rightarrow 3u - Au = \begin{pmatrix}1\\ -3\end{pmatrix} \Rightarrow$

$\begin{matrix}3u_1 - u_2\\ 3u_2 - u_1\end{matrix} = \begin{matrix}1\\ -3\end{matrix} \Rightarrow \begin{pmatrix}3 & -1\\ -1 & 3\end{pmatrix}\begin{pmatrix}u_1\\ u_2\end{pmatrix} = \begin{pmatrix}1\\ -3\end{pmatrix} \Rightarrow \begin{pmatrix}u_1\\ u_2\end{pmatrix} = \begin{pmatrix}3/8 & 1/8\\ 1/8 & 3/8\end{pmatrix}\begin{pmatrix}1\\ -3\end{pmatrix} = \begin{pmatrix}0\\ -1\end{pmatrix}$

$$\boxed{y = c_1 \begin{pmatrix}1\\ 1\end{pmatrix} e^{t} + c_2 \begin{pmatrix}-1\\ 1\end{pmatrix} e^{-t} + \begin{pmatrix}0\\ -1\end{pmatrix} e^{3t}}$$

4.6.11) $\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} y_2 \\ y_1 \end{pmatrix} + \begin{pmatrix} 6e^{2t} \\ -e^{2t} \end{pmatrix}$ $y_1(0)=1$, $y_2(0)=0$

$y' = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} y + \begin{pmatrix} 6 \\ -1 \end{pmatrix} e^{2t}$

$|A-\lambda I| = 0 \Rightarrow \lambda_{1,2} = \pm 1$, $V_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $V_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

(See problem 4.6.3)

$y_h = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{t} + c_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-t}$

$y_p = ve^{2t}$ $\quad y_p' = 2ve^{2t}$

$\therefore 2v = Av + \begin{pmatrix} 6 \\ -1 \end{pmatrix} \Rightarrow \begin{pmatrix} 2v_1 \\ 2v_2 \end{pmatrix} - \begin{pmatrix} v_2 \\ v_1 \end{pmatrix} = \begin{pmatrix} 6 \\ -1 \end{pmatrix}$

$\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 6 \\ -1 \end{pmatrix} \Rightarrow \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{pmatrix} \begin{pmatrix} 6 \\ -1 \end{pmatrix} = \begin{pmatrix} 11/3 \\ 4/3 \end{pmatrix}$

$y = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{t} + c_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-t} + \begin{pmatrix} 11/3 \\ 4/3 \end{pmatrix} e^{2t}$

$y(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} c_1 \\ c_1 \end{pmatrix} + \begin{pmatrix} -c_2 \\ c_2 \end{pmatrix} + \begin{pmatrix} 11/3 \\ 4/3 \end{pmatrix}$

$\begin{pmatrix} -8/3 \\ -4/3 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \Rightarrow \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{pmatrix} \begin{pmatrix} -8/3 \\ -4/3 \end{pmatrix} = \begin{pmatrix} -12/6 \\ 4/6 \end{pmatrix}$

$$\boxed{y = \begin{pmatrix} -2 \\ -2 \end{pmatrix} e^{t} + \begin{pmatrix} -2/3 \\ 2/3 \end{pmatrix} e^{-t} + \begin{pmatrix} 11/3 \\ 4/3 \end{pmatrix} e^{2t}}$$

6.1.1) $3t + 12$

$$F(s) = \int_0^\infty e^{-st}(3t+12)\,dt$$

$$= 3\int_0^\infty te^{-st}\,dt + \underbrace{12\int_0^\infty e^{-st}\,dt}_{12\left[-\frac{e^{-st}}{s}\Big|_0^\infty\right] = 12\left[0 + \frac{1}{s}\right] = \frac{12}{s}}$$

$u = t \qquad dv = e^{-st}$

$du = dt \qquad v = -\frac{e^{-st}}{s}$

$$uv - \int v\,du = te^{-st}\Big|_0^\infty + \int_0^\infty \frac{e^{-st}}{s}\,dt = \cancel{te^{-st}}\Big|_0^\infty - \frac{1}{s}\left[e^{-st}\Big|_0^\infty\right]$$

$$= -\frac{1}{s^2}[0-1] = \frac{1}{s^2} = \int_0^\infty te^{-st}\,dt$$

$$\therefore \boxed{F(s) = \frac{3}{s^2} + \frac{12}{s}}$$

6.1.2) $(a-bt)^2 = (a-bt)(a-bt) = a^2 + b^2t^2 - 2abt$

$\mathcal{L}[a^2] = \frac{a^2}{s}$, $\mathcal{L}[-2abt] = \frac{-2ab}{s^2}$ (See 6.1.1)

$\mathcal{L}(b^2t^2) = b^2\int_0^\infty e^{-st}t^2$, $\quad u = t^2 \quad du = 2t\,dt$

$dv = e^{-st} \quad v = -\frac{e^{-st}}{s}$

$$VU - \int V\,dU = \frac{t^2}{s}\cancel{e^{-st}}\Big|_0^{\infty} \xrightarrow{} 0 + \int_0^{\infty} \frac{2}{s} t e^{-st}\,dt$$

$$\int_0^{\infty} te^{-st} = \frac{1}{s^2} \text{ (Problem 6.1.1)} \quad \therefore \quad \frac{2}{s}\int_0^{\infty} te^{-st}\,dt = \frac{2}{s^3}$$

$$\therefore \int_0^{\infty} e^{-st} t^2\,dt = \frac{2}{s^3}$$

$$\mathcal{L}[b^2t^2] = \frac{2b^2}{s^3}$$

$$\boxed{\mathcal{L}[(a-bt)^2] = \frac{2b^2}{s^3} - \frac{2ab}{s^2} + \frac{a^2}{s}}$$

6.1.5) $e^{2t}\sinh t$, $\sinh(t) = \frac{1}{2}(e^t - e^{-t})$

$$\frac{e^{2t}}{2}(e^t - e^{-t}) = \frac{e^{3t}}{2} - \frac{e^t}{2}$$

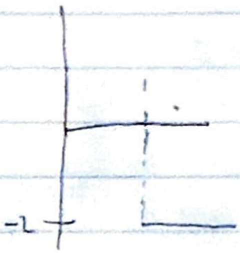
$$\mathcal{L}(e^{at}) = \frac{1}{s-a} \quad \therefore \quad \mathcal{L}\left[\frac{e^{3t}}{2}\right] = \left(\frac{1}{2}\right)\left(\frac{1}{s-3}\right)$$

$$\mathcal{L}\left[-\frac{e^t}{2}\right] = \left(-\frac{1}{2}\right)\left(\frac{1}{s-1}\right)$$

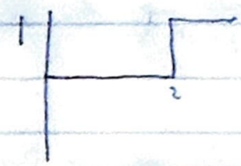
$$\boxed{\mathcal{L}[e^{2t}\sinh t] = \frac{1}{2}\left(\frac{1}{s-3} - \frac{1}{s-1}\right)} = \frac{1}{2}\left(\frac{(s-1)-(s-3)}{s^2-4s+3}\right) = \boxed{\frac{1}{s^2-4s+3}}$$

6.1.13) 

$= \frac{1}{s}$

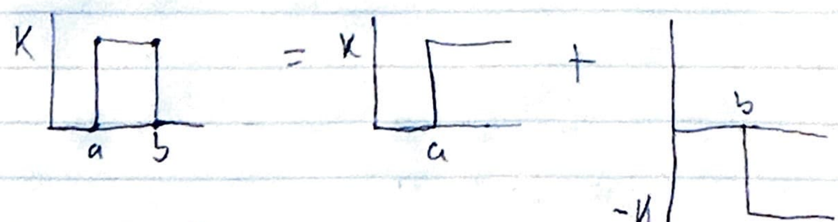


$= \frac{-2e^{-s}}{s}$ (Step delayed 1 second)



$= \frac{e^{-2s}}{s}$ (Step delayed 2 seconds)

$$\boxed{F(s) = \frac{1}{s} \cdot \frac{2e^{-s}}{s} + \frac{e^{-2s}}{s}} = \boxed{\frac{e^{-2s} - 2e^{-s} + 1}{s}}$$

6.1.14) 

$$F(s) = \frac{Ke^{-sa}}{s} - \frac{Ke^{-sb}}{s}$$

$$\boxed{F(s) = \frac{K}{s}\left(e^{-sa} - e^{-sb}\right)}$$

6.1.30) $\frac{4s+32}{s^2-16} = \frac{4(s+8)}{(s+4)(s-4)} = \frac{A}{(s+4)} + \frac{B}{(s-4)}$

$4s+32 = A(s-4) + B(s+4)$

$4s = As + Bs \Rightarrow 4 = A+B \quad , A = 4-B$

$32 = -4A + 4B \Rightarrow 32 = -4(4-B) + 4B = -16 + 8B$

$48 = 8B \quad , \quad B = 6 \quad , A = -2$

$\therefore \frac{4s+32}{s^2-16} = \frac{-2}{s+4} + \frac{6}{s-4}$

$\mathcal{L}^{-1}\left[\frac{-2}{s+4}\right] = -2e^{-4t}$

$\mathcal{L}^{-1}\left[\frac{6}{s-4}\right] = 6e^{4t}$

$$\boxed{\mathcal{L}^{-1}\left[\frac{4s+32}{s^2-16}\right] = -2e^{-4t} + 6e^{4t}}$$

6.1.32) $\frac{1}{(s+a)(s+b)} = \frac{A}{(s+a)} + \frac{B}{(s+b)}$

$1 = A(s+b) + B(s+a) \quad \therefore \quad 0 = (A+B)s \quad , A = -B$

$1 = Ab + Ba$

$1 = B(-b+a) \quad , \quad B = \frac{1}{-b+a} \quad , \quad A = \frac{1}{-a+b}$

$$\mathcal{L}^{-1}\left[\frac{\frac{1}{-a+b}}{(s+a)}\right] = \frac{1}{-a+b} e^{-at}$$

$$\mathcal{L}^{-1}\left[\frac{\frac{1}{-b+a}}{s+b}\right] = \frac{1}{a-b} e^{-bt}$$

$$\boxed{\mathcal{L}^{-1}\left[\frac{1}{(s+a)(s+b)}\right] = \frac{1}{-a+b} e^{-at} + \frac{1}{a-b} e^{-bt}}$$

6.2.4) $y'' + 9y = 10e^{-t}$ $\quad y(0) = y'(0) = 0$

$$(s^2Y - sy(0) - y'(0)) + 9Y = (s^2+9)Y = \mathcal{L}[10e^{-t}] = 10\left(\frac{1}{s+1}\right)$$

$$Y(s) = \frac{10}{(s^2+9)(s+1)} = \frac{A}{(s+1)} + \frac{Bs+C}{s^2+9}$$

$$10 = A(s^2+9) + (Bs+C)(s+1)$$
$$= As^2 + 9A + Bs^2 + Bs + Cs + C$$

$A + B = 0 \qquad A = -B$

$B + C = 0 \qquad \therefore\ C = A$

$9A + C = 10 \qquad \therefore\ A = 1,\ C = 1,\ B = -1$

$$Y(s) = \frac{1}{s+1} + \frac{-s+1}{s^2+9} = \frac{1}{s+1} - \frac{s}{s^2+9} + \frac{1}{s^2+9}$$

$$\mathcal{L}^{-1}\left[\frac{1}{s+1}\right] = e^{-t}$$

$$\mathcal{L}^{-1}\left[\frac{s}{s^2+9}\right] = \mathcal{L}^{-1}\left[\frac{s}{s^2+3^2}\right] = \cos(3t)$$

$$\mathcal{L}^{-1}\left[\frac{1}{s^2+9}\right] = \mathcal{L}^{-1}\left[\frac{1}{s^2+3^2}\right] = \frac{1}{3}\sin(3t) \quad \therefore$$

$$\boxed{y(t) = e^{-t} - \cos(3t) + \frac{1}{3}\sin(3t)}$$

6.2.5) $y'' - \frac{1}{4}y = 0$ $\quad y(0) = 12,\ y'(0) = 0$

$$\left(s^2Y - sy(0) - y'(0)\right) - \frac{1}{4}Y = 0$$

$$Y\left(s^2 - \frac{1}{4}\right) = 12s$$

$$Y(s) = \frac{12s}{s^2 - \frac{1}{4}} = 12\left(\frac{s}{(s+\frac{1}{2})(s-\frac{1}{2})}\right)$$

$$Y(s) = \frac{A}{s+\frac{1}{2}} + \frac{B}{s-\frac{1}{2}}$$

$$A(s-\tfrac{1}{2}) + B(s+\tfrac{1}{2}) = 12s$$

$A + B = 12$

$-\frac{1}{2}A + \frac{1}{2}B = 0 \quad \therefore \quad A = B, \quad A = B = 6$

$$Y(s) = \frac{6}{s+\frac{1}{2}} + \frac{6}{s-\frac{1}{2}}$$

$$\mathcal{L}^{-1}\left[\frac{6}{s+\frac{1}{2}}\right] = 6e^{-\frac{1}{2}t} \qquad \mathcal{L}^{-1}\left[\frac{6}{s-\frac{1}{2}}\right] = 6e^{\frac{1}{2}t}$$

$$\boxed{y(t) = 6e^{-\frac{1}{2}t} + 6e^{\frac{1}{2}t}} \qquad \cosh(t) = \frac{e^t + e^{-t}}{2}$$

$$\therefore \boxed{y(t) = 12\cosh(\tfrac{1}{2}t)}$$

6.2.17) $te^{-at} = y$

$\frac{d}{dt}(te^{-at}) = e^{-at} - ate^{-at}$

$y' = e^{-at} - ate^{-at} \Rightarrow y' + ay = e^{-at}$

$\mathcal{L}[y' + ay] = \mathcal{L}[e^{-at}] = (s+a)Y = \frac{1}{s+a}$

$$\boxed{Y(s) = \frac{1}{(s+a)(s+a)} = \frac{1}{(s+a)^2}}$$

6.2.19) $y = \sin^2(\omega t)$

$\frac{d}{dt}(\sin(\omega t)\sin(\omega t)) = 2\omega\cos(\omega t)\sin(\omega t) = y'$

$\frac{d}{dt}(2\omega\cos(\omega t)\sin(\omega t)) = -2\omega^2\sin^2(\omega t) + \underbrace{2\omega^2\cos^2(\omega t)}_{c^2+s^2=1} = y''$ $\quad c^2 = 1-s^2$

$y'' = -2\omega^2 y + 2\omega^2(1-y)$

$y'' = 2\omega^2 - 4\omega^2 y \Rightarrow \mathcal{L}[y'' + 4\omega^2 y] = \mathcal{L}[2\omega^2]$

$(s^2 + 4\omega^2)Y = \frac{2\omega^2}{s}$

$$\boxed{Y(s) = \frac{2\omega^2}{s(s^2+4\omega^2)}}$$

(6.2.26) $\frac{1}{s^4 - s^2} = \frac{1}{s^2(s^2-1)}$ $\int_0^t f(\tau)\,d\tau = \mathcal{L}^{-1}\left[\frac{1}{s} F(s)\right]$

$\frac{1}{s(s^2-1)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s-1}$

$1 = A(s^2-1) + Bs(s-1) + Cs(s+1)$

$1 = -A$ $\quad A = -1$

$0 = A + B + C$ $\quad A = -B - C$

$0 = -B + C$ $\quad C = B$

$\therefore A = -2B = -1, \; B = C = \frac{1}{2}$

$\frac{1}{s(s^2-1)} = \frac{-1}{s} + \frac{1}{2}\frac{1}{s+1} + \frac{1}{2}\frac{1}{s-1}$

$\mathcal{L}^{-1}\left[\frac{-1}{s}\right] = -1, \quad \mathcal{L}^{-1}\left[\frac{1}{s+1}\right] = e^{-t}, \quad \mathcal{L}^{-1}\left[\frac{1}{s-1}\right] = e^{t}$

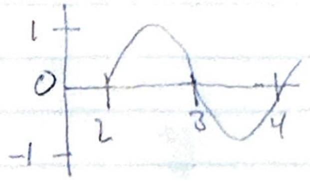
$\mathcal{L}^{-1}\left[\frac{1}{s(s^2-1)}\right] = -1 + \frac{e^{-t}}{2} + \frac{e^{t}}{2}$

$f(t) = \int_0^t -1 + \frac{e^{-\tau}}{2} + \frac{e^{\tau}}{2} = -\tau - \frac{e^{-\tau}}{2} + \frac{e^{\tau}}{2}\Big|_0^t$

$= -t - \frac{e^{-t}}{2} + \frac{e^{t}}{2} + \frac{1}{2} - \frac{1}{2}$

$\boxed{f(t) = \frac{e^{t}}{2} - \frac{e^{-t}}{2} - t}$

6.3.6)



$$f(t) = \begin{cases} 0 & t \le 2 \\ \sin(\pi t) & 2 < t < 4 \\ 0 & t \ge 4 \end{cases}$$

$$f(t) = \sin(\pi t)\left[u(t-2) - u(t-4)\right]$$

$$f(t-2) = \sin(\pi(t-2)) = \sin(\pi t - 2\pi) = \sin(\pi t)\cancelto{1}{\cos(-2\pi)} + \cos(\pi t)\cancelto{0}{\sin(-2\pi)}$$

$$f(t-2) = \sin(\pi t)$$

$$f(t-4) = \sin(\pi(t-4)) = \sin(\pi t - 4\pi) = \sin(\pi t)\cancelto{1}{\cos(-4\pi)} + \cos(\pi t)\cancelto{0}{\sin(-4\pi)}$$

$$f(t-4) = \sin(\pi t)$$

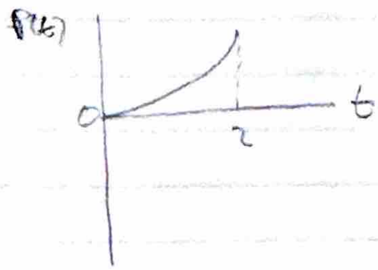
$$\mathcal{L}[f(t-a)u(t-a)] = e^{-as}F(s)$$

$$\mathcal{L}[\sin(\pi t)] = \frac{\pi}{s^2+\pi^2}, \quad \mathcal{L}[u(t-2)] = e^{-2s}, \quad \mathcal{L}[u(t-4)] = e^{-4s}$$

$$\mathcal{L}[f(t-2)u(t-2)] = \frac{\pi}{s^2+\pi^2}e^{-2s}, \quad \mathcal{L}[f(t-4)u(t-4)] = e^{-4s}$$

$$\boxed{\mathcal{L}[f(t)] = \frac{\pi}{s^2+\pi^2}\left(e^{-2s} - e^{-4s}\right)}$$

6.3.10) $f(t) = \begin{cases} \sinh(t) & 0<t<2 \\ 0 & t \geq 2 \end{cases}$



$$\sinh(t) = \frac{e^{t} - e^{-t}}{2}$$

$$f(t) = \frac{e^{t} - e^{-t}}{2}\left(u(t) - u(t-2)\right)$$

$$f(t-2) = \frac{e^{t-2} \cdot e^{-t+2}}{2} = \frac{e^{-2}e^{t} - e^{-t}e^{2}}{2}$$

$$\mathcal{L}[f(t-0)\,u(t)] = \mathcal{L}[f(t)] = \mathcal{L}[\sinh(t)] = \frac{1}{s^2-1}$$

$$\mathcal{L}[f(t-2)\,u(t-2)] = \frac{e^{-2}}{2}\mathcal{L}[e^{t}] - \frac{e^{2}}{2}\mathcal{L}[e^{-t}]$$

$$= \frac{1}{2}\left(\frac{e^{-2}}{s-1} - \frac{e^{2}}{s+1}\right)e^{-2s}$$

$$\boxed{\mathcal{L}[f(t)] = \frac{1}{s^2-1} - \frac{1}{2}\left[\frac{e^{-2}}{s-1} - \frac{e^{2}}{s+1}\right]e^{-2s}}$$

6.3.13) $\dfrac{6(1-e^{-\pi s})}{s^2+9} = \dfrac{6}{s^2+9} - \dfrac{6}{s^2+9}e^{-\pi s}$

$\dfrac{6}{s^2+9} = 2\left(\dfrac{3}{s^2+9}\right) = 2\left(\dfrac{3}{s^2+3^2}\right)$

$\mathcal{L}^{-1}\left[\dfrac{3}{s^2+3^2}\right] = \sin(3t)$, $\mathcal{L}^{-1}\left(\dfrac{6}{s^2+9}\right) = 2\sin(3t)$

$\mathcal{L}^{-1}[e^{-\pi s}] = u(t-\pi)$

$f(t-\pi) = \sin(3(t-\pi)) = \sin(3t-3\pi) = \sin(3t)\cancel{\cos(-3\pi)}^{-1} + \cos(3t)\cancel{\sin(-3\pi)}^{0}$

$f(t-\pi) = -\sin(3t)$

$\therefore \mathcal{L}^{-1}\left[\dfrac{6(1-e^{-\pi s})}{s^2+9}\right] = 2\sin(3t) - (-2\sin(3t))u(t-\pi)$

$= \boxed{2\sin(3t)(1+u(t-\pi))}$

6.3.16) $\dfrac{2(e^{-s}-e^{-3s})}{s^2-4}$

$\mathcal{L}^{-1}\left[\dfrac{2}{s^2-4}\right] = \mathcal{L}^{-1}\left[\dfrac{2}{s^2-(2)^2}\right] = \sinh(2t)$

$\mathcal{L}^{-1}[e^{-s}] = u(t-1)$

$\mathcal{L}^{-1}[e^{-3s}] = u(t-3)$

$f(t-1) = \sinh(2(t-1)) = \sinh(2t-2)$

$f(t-3) = \sinh(2(t-3)) = \sinh(2t-6)$

$f(t-1)u(t-1) = \sinh(2t-2)\,u(t-1)$

$f(t-3)u(t-3) = \sinh(2t-6)\,u(t-3)$

$\mathcal{L}^{-1} = \sinh(2t-2)\,u(t-1) - \sin(2t-6)\,u(t-3)$

$\sinh(t) = \dfrac{e^{t}-e^{-t}}{2}$

$\sinh(2t-2) = \dfrac{e^{2t-2}-e^{-2t+2}}{2}$

$\sinh(2t-6) = \dfrac{e^{2t-6}-e^{-2t+6}}{2}$

$$\boxed{\mathcal{L}^{-1} = \tfrac{1}{2}\left(e^{2t-2}-e^{-2t+2}\right)u(t-1) - \tfrac{1}{2}\left(e^{2t-6}-e^{-2t+6}\right)}$$

6.3.25) $y'' + y = f(t)$ $\quad f(t) = \begin{cases} t & 0<t<1 \\ 0 & t>1 \end{cases}$

$f(t) = t\,(u(t) - u(t-1))$

$f(t-1) = t-1$

$\mathcal{L}[t\,u(t)] = \dfrac{1}{s^2}$, $\quad \mathcal{L}[(t-1)u(t-1)] = \left(\dfrac{1}{s^2} - \dfrac{1}{s}\right)e^{-s} = \left(\dfrac{1-s}{s^2}\right)e^{-s}$

$\mathcal{L}[y'' + y] = (s^2+1)Y$

$\therefore\ Y(s) = \dfrac{1-(1-s)e^{-s}}{s^2(s^2+1)} = \dfrac{1}{s^2(s^2+1)} - \dfrac{(1-s)e^{-s}}{s^2(s^2+1)}$

$$\frac{1}{s^2(s^2+1)} = \frac{1}{s^2} - \frac{1}{s^2+1}$$

$$\mathcal{L}^{-1}\left[\frac{1}{s^2}\right] = t \quad , \quad \mathcal{L}^{-1}\left[\frac{1}{s^2+1}\right] = \sin(t)$$

$$\therefore \mathcal{L}^{-1}\left[\frac{1}{s^2(s^2+1)}\right] = t - \sin(t)$$

$$\frac{(1-s)}{s^2(s^2+1)} = \frac{As+B}{s^2} + \frac{Cs+D}{s^2+1}$$

$$(1-s) = As^3 + As + Bs^2 + B + Cs^3 + Ds^2$$

$A+C=0$, $A=-C$

$B+D=0$, $B=-D$

$A=-1$, $C=1$

$B=1$, $D=-1$

$$\frac{1-s}{s^2(s^2+1)} = \frac{-1}{s} + \frac{1}{s^2} + \frac{s}{s^2+1} - \frac{1}{s^2+1}$$

$$\mathcal{L}^{-1}\left[\frac{-1}{s}\right] = -1 \quad , \quad \mathcal{L}^{-1}\left(\frac{1}{s^2}\right] = t, \quad \mathcal{L}^{-1}\left[\frac{s}{s^2+1}\right] = \cos(t) \quad , \quad \mathcal{L}^{-1}\left[\frac{1}{s^2+1}\right] = \sin(t)$$

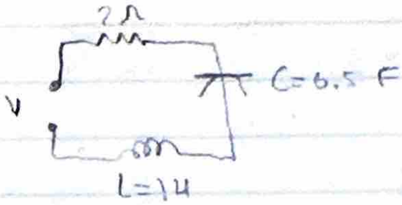
$$\mathcal{L}^{-1}\left[\frac{1-s}{s^2(s^2+1)}\right] = -1 + t + \cos(t) - \sin(t)$$

$$\mathcal{L}^{-1}\left[e^{-s}\left(\frac{1-s}{s^2(s^2+1)}\right)\right] = (-1 + (t-1) + \cos(t-1) - \sin(t-1))u(t-1)$$

$$\therefore \mathcal{L}^{-1}[Y(s)] = t - \sin(t) - (-1 + (t-1) + \cos(t-1) - \sin(t-1))u(t-1)$$

$$\boxed{y(t) = \begin{cases} t - \sin(t) & 0 < t < 1 \\ -\cos(t-1) + \sin(t-1) - \sin(t) + 2 & t > 1 \end{cases}}$$

6.3.39)



$$\Sigma V = V(t) - 2I - 2\int I\,dt - \dot{I} = 0$$

$$V(t) = \begin{cases} 1000 & 0<t<2 \\ 0 & t>2 \end{cases}$$

$$V(t) = (u(t) - u(t-2))\,1000$$

$$\mathcal{L}\left[u(t) - u(t-2) - 2I - 2\int I\,dt - \dot{I}\right] = 0$$

$$= \frac{1000}{s} - \left(\frac{e^{-2s}}{s}\right)(1000) - 2I(s) - \frac{2}{s}I(s) - sI(s) = 0$$

$$I(s)\left(-2 - \frac{2}{s} - s\right) = \left(\frac{e^{-2s}}{s} - \frac{1}{s}\right)1000$$

$$I(s) = \frac{\left(\frac{1}{s} - \frac{e^{-2s}}{s}\right)1000}{s + \frac{2}{s} + 2} = \frac{(1-e^{-2s})\,1000}{s^2+2s+2}$$

$$\frac{1}{s^2+2s+2} = \frac{1}{(s+1)^2+1} \qquad \mathcal{L}^{-1}\left[\frac{1}{(s+1)^2+1}\right] = \mathcal{L}^{-1}\left[\frac{\omega}{(s-a)^2+\omega^2}\right] = e^{at}\sin(\omega t)$$

$$\therefore \mathcal{L}^{-1}\left(\frac{1}{s^2+2s+2}\right) = e^{-t}\sin(t)$$

$$\mathcal{L}^{-1}\left[e^{-2s}\left(\frac{1}{s^2+2s+2}\right)\right] = \left(e^{-(t-2)}\sin(t-2)\right)u(t-2)$$

$$\boxed{I(t) = 1000\,e^{-t}\sin(t) - 1000\,u(t-2)\left[e^{-t+2}\sin(t-2)\right]}$$

6.4.3) $y'' + 4y = \delta(t-\pi)$

$\mathcal{L}[y'' + 4y] = (s^2Y - 8s + 4Y) = \mathcal{L}[\delta(t-\pi)] = e^{-\pi s}$

$(s^2+4)Y = e^{-\pi s} + 8s$

$$Y(s) = \frac{e^{-\pi s}}{s^2+4} + \frac{8s}{s^2+4}$$

$$\frac{1}{s^2+4} = \frac{1}{2}\left(\frac{2}{s^2+2^2}\right)$$

$\therefore \mathcal{L}^{-1}\left[\frac{1}{s^2+4}\right] = \frac{1}{2}\sin(2t)$

$\mathcal{L}^{-1}\left[\frac{s}{s^2+4}\right] = \cos(2t)$

$\mathcal{L}^{-1}\left[\frac{e^{-\pi s}}{s^2+4}\right] = \frac{1}{2}\sin(2(t-\pi))\,u(t-\pi) = \frac{1}{2}\sin(2t-2\pi)\,u(t-\pi)$

$\sin(2t-2\pi) = \sin(2t)$ (Problem 6.3.6)

$\therefore \boxed{Y(t) = 8\cos(2t) + \frac{1}{2}\sin(2t)\,u(t-\pi)}$

6.4.10) $y'' + 5y' + 6y = \delta(t - \frac{1}{2}\pi) + u(t-\pi)\cos(t)$

$\cos(t-\pi) = \cos(t)\underbrace{\cos(\pi)}_{-1} - \sin(t)\underbrace{\sin(\pi)}_{0} = -\cos(t)$

$\mathcal{L}[y'' + 5y' + 6y] = (s^2 + 5s + 6)Y$

$\mathcal{L}[\delta(t - \frac{\pi}{2})] = e^{-\pi/2\, s}$

$\mathcal{L}[u(t-\pi)\cos(t-\pi)] = -e^{-\pi}\frac{s}{s^2+1}$

$$Y(s) = \frac{e^{-\pi/2 s} - e^{-\pi s}\frac{s}{s^2+1}}{s^2+5s+6} = \frac{(s^2+1)e^{-\pi/2 s} - e^{-\pi s} s}{(s^2+1)(s+2)(s+3)}$$

$$Y(s) = \frac{As+B}{s^2+1} + \frac{C}{s+2} + \frac{D}{s+3}$$

$$s^2+1 = (As+B)(s+2)(s+3) + C(s^2+1)(s+3) + D(s+2)(s+1)$$

$$= As^3+3As^2+6As+6 + C(s^3+3s^2+s+3) + D(s^3+2s^2+s+2)$$
$$+Bs^2+5Bs+6B$$

$A+C+D=0$
$5A+B+3C+2D=1$
$6A+5B+C+D=0$
$6B+3C+2D=1$

$\therefore\ A=B=0,\ C=1,\ D=-1$

$$\therefore\ \frac{s^2+1}{(s^2+1)(s+2)(s+3)} = \frac{1}{s+2} - \frac{1}{s+3}$$

$A+C+D=0$
$5A+B+3C+2D=0$
$6A+5B+C+D=1$
$6B+3C+2D=0$

$\therefore\ A=\frac{1}{10},\ B=1/10,\ C=-4/10,\ D=3/10$

$$\therefore\ \frac{s}{(s^2+1)(s+2)(s+3)} = \frac{1/10(s+1)}{s^2+1} - \frac{4/10}{s+2} + \frac{3/10}{s+3}$$

$$\mathcal{L}^{-1}\left[\frac{1}{(s+2)(s+3)}\right] = e^{-2t} - e^{-3t}$$

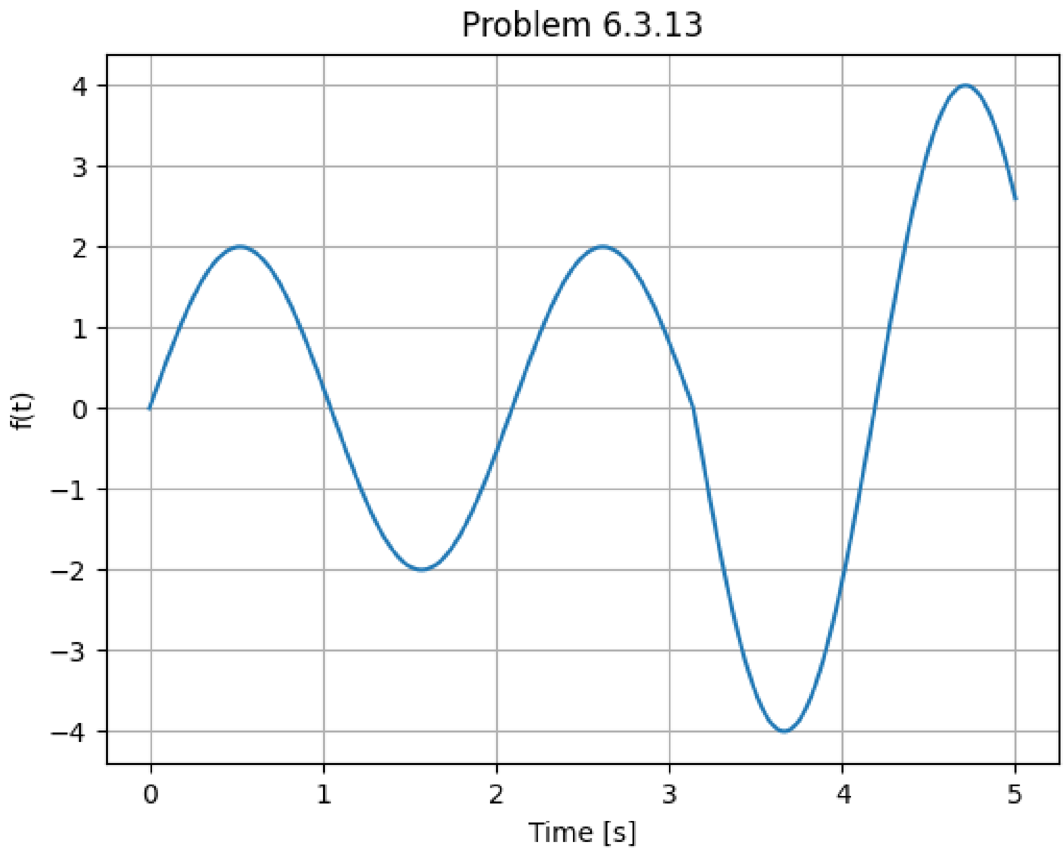
$$\mathcal{L}^{-1}\left[\frac{e^{-\frac{\pi}{2}s}}{(s+2)(s+3)}\right] = \left(e^{-2(t-\pi/2)} - e^{-3(t-\pi/2)}\right) u(t-\pi/2)$$

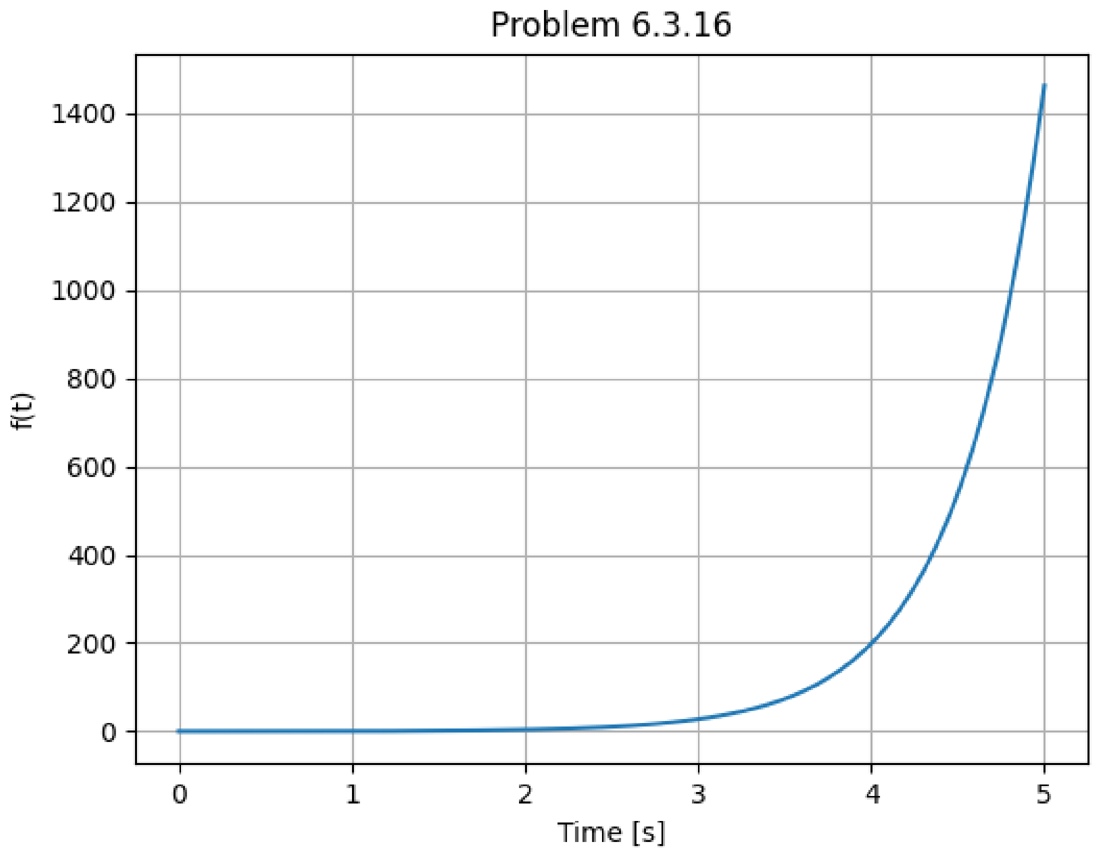
$$\mathcal{L}^{-1}\left[\frac{s}{(s^2+1)(s^2+5s+6)}\right] = \frac{1}{10}\mathcal{L}^{-1}\left[\frac{s}{s^2+1}\right] + \frac{1}{10}\mathcal{L}^{-1}\left[\frac{1}{s^2+1}\right] - 4/10\, e^{-2t} + 3/10\, e^{-3t}$$

$$= \frac{1}{10}\cos(t) + \frac{1}{10}\sin(t) - 4/10\, e^{-2t} + 3/10\, e^{-3t}$$

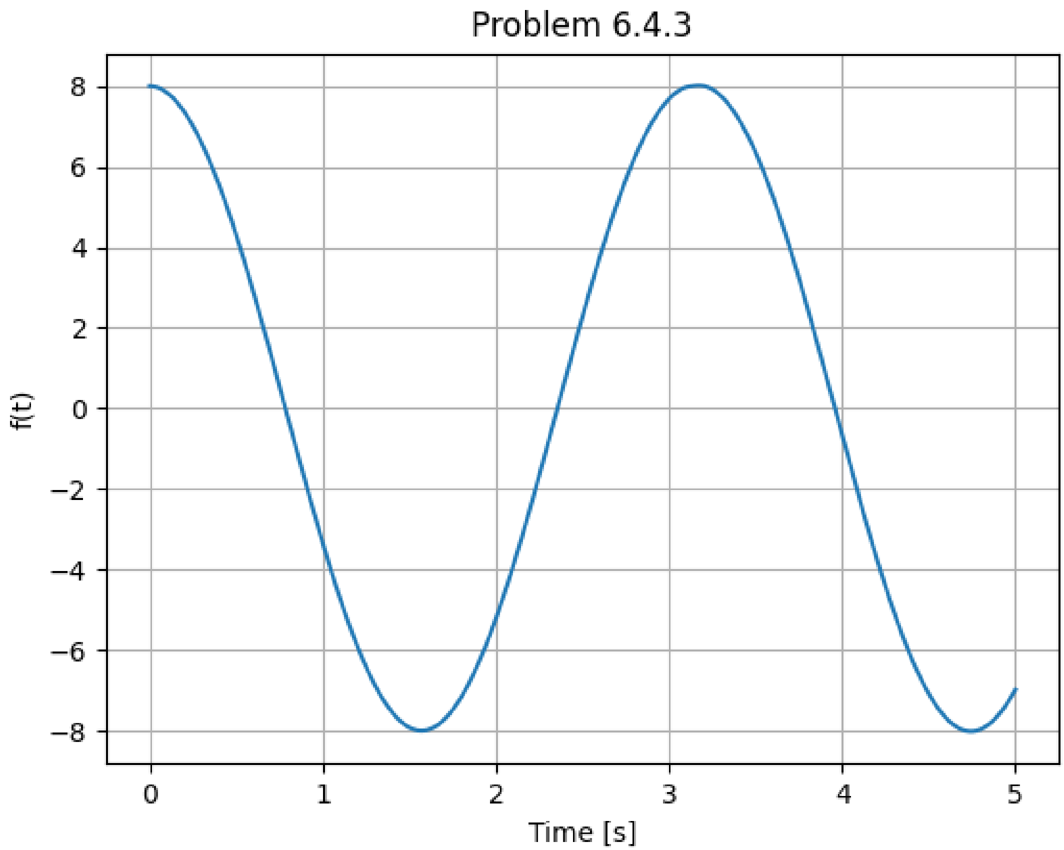
$$\mathcal{L}^{-1}\left[e^{-\pi s}\frac{s}{(s^2+1)(s^2+5s+6)}\right] = \left(\frac{1}{10}\cos(t-\pi) + 1/10\sin(t-\pi) - 4/10\, e^{-2(t-\pi)} + 3/10\, e^{-3(t-\pi)}\right) u(t-\pi)$$

$$\boxed{y(t) = \begin{cases} 0 & \leftarrow t < \pi/2 \\ e^{-2(t-\pi/2)} - e^{-3(t-\pi/2)} & \leftarrow \pi/2 < t < \pi \\ e^{-2(t-\pi/2)} - e^{-3(t-\pi/2)} - \frac{\cos(t-\pi)}{10} - \frac{\sin(t-\pi)}{10} + 4/10\, e^{-2(t-\pi)} - 3/10\, e^{-3(t-\pi)} & \leftarrow t > \pi \end{cases}}$$

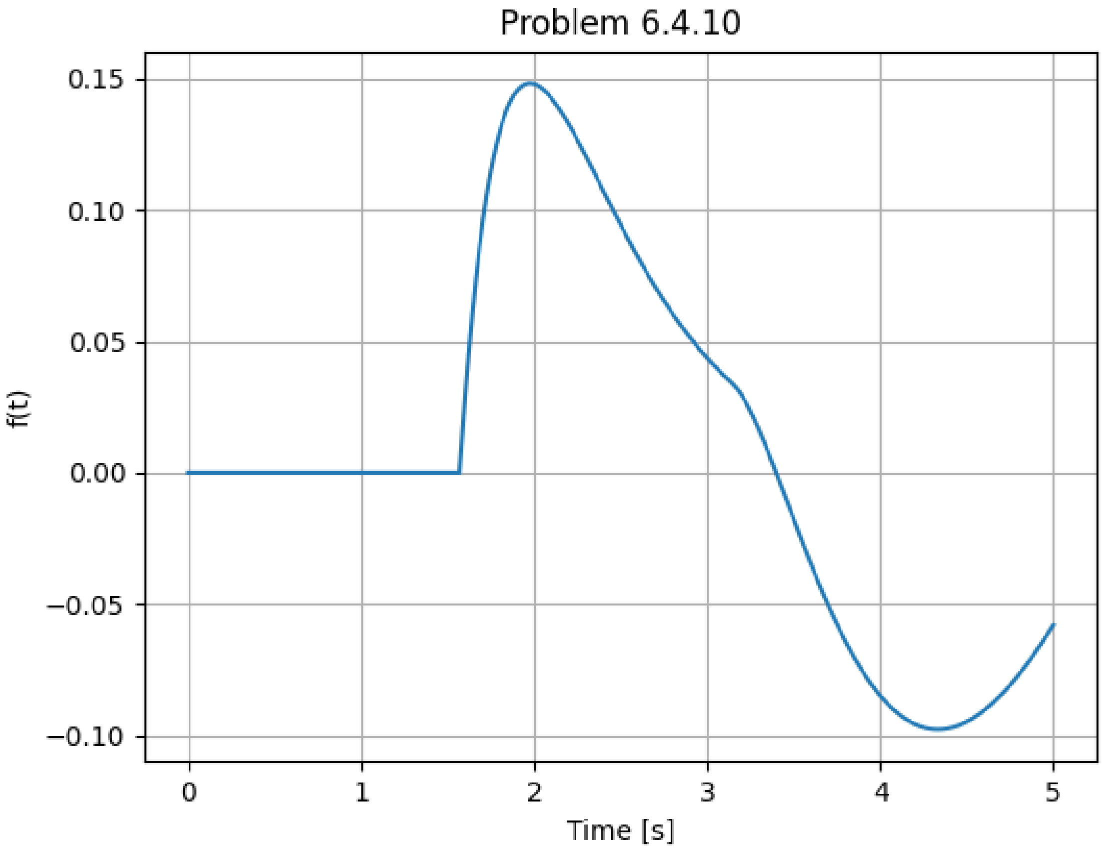

Problem 6.3.13
f(t)
Time [s]
4
3
2
1
0
−1
−2
−3
−4
0
1
2
3
4
5



Problem 6.3.16
1400
1200
1000
800
600
400
200
0
f(t)
0
1
2
3
4
5
Time [s]



Problem 6.4.3
f(t)
Time [s]
8
6
4
2
0
−2
−4
−6
−8
0
1
2
3
4
5



Problem 6.4.10
0.15
0.10
0.05
0.00
−0.05
−0.10
f(t)
0
1
2
3
4
5
Time [s]