ECE 68000: MODERN AUTOMATIC CONTROL

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Introduction to Fuzzy Logic: Dealing with Uncertainty

Fuzzy Logic Control---Another Tool in Our Control Toolbox to Cope with Uncertainties

Fuzzy Logic=Computing With Words

- The term Fuzzy Set introduced by Lotfi A. Zadeh in his paper “Fuzzy Sets” published in *Information and Control,* Vol. 8, No. 3, pp. 338--353, June 1965
- The motivation for the notion of a fuzzy set---to provide a framework for “a natural way of dealing with problems in which the source of imprecision is the absence of sharply defined criteria of class membership rather than the presence of random variables.”

What is a Set?

- A set is a collection of objects. An element belongs to the set or it does not
- Example

$$X = \{x : x > 6\}$$

- If *x* greater than 6, then x belongs to the set *X*, otherwise x does not belong to the set
- Unambiguous set boundary

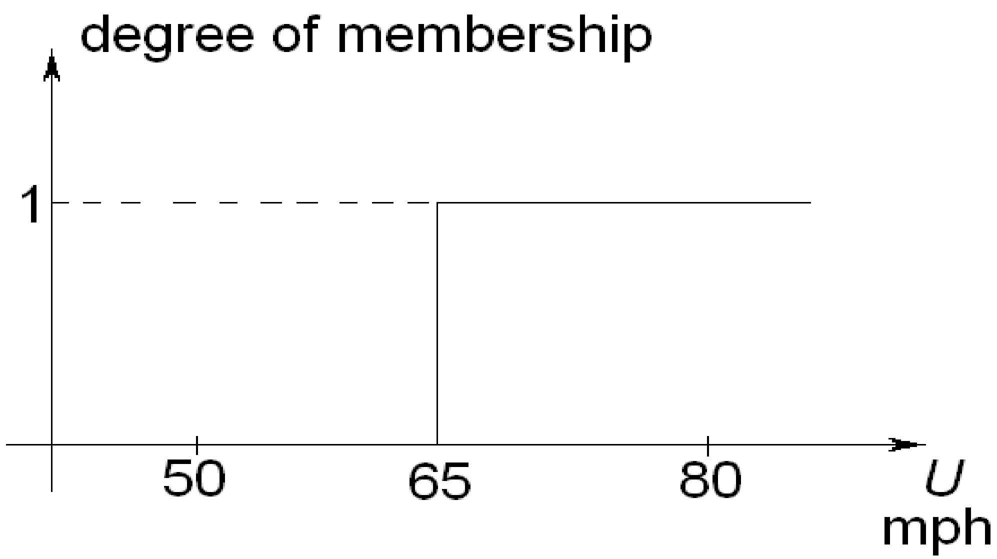
Characteristic Function

- Characteristic function of a set---If an element belongs to the set, then the function value is 1, if an element does not belong to the set, then the function is 0
- Can represent a set *X*, using its characteristic function, as a set of ordered pairs (*x*,0) or (*x*,1),
  where (*x*,0) means $x \notin X$
  while (*x*,1) means $x \in X$

☞ $\boxed{\text{An ordinary set can be represented as a collection of ordered pairs}}$

Crisp Set

- Crisp set ("ordinary" set)---a collection of objects. An element is a member of the set or it is not
- An example of a crisp set---fast cars

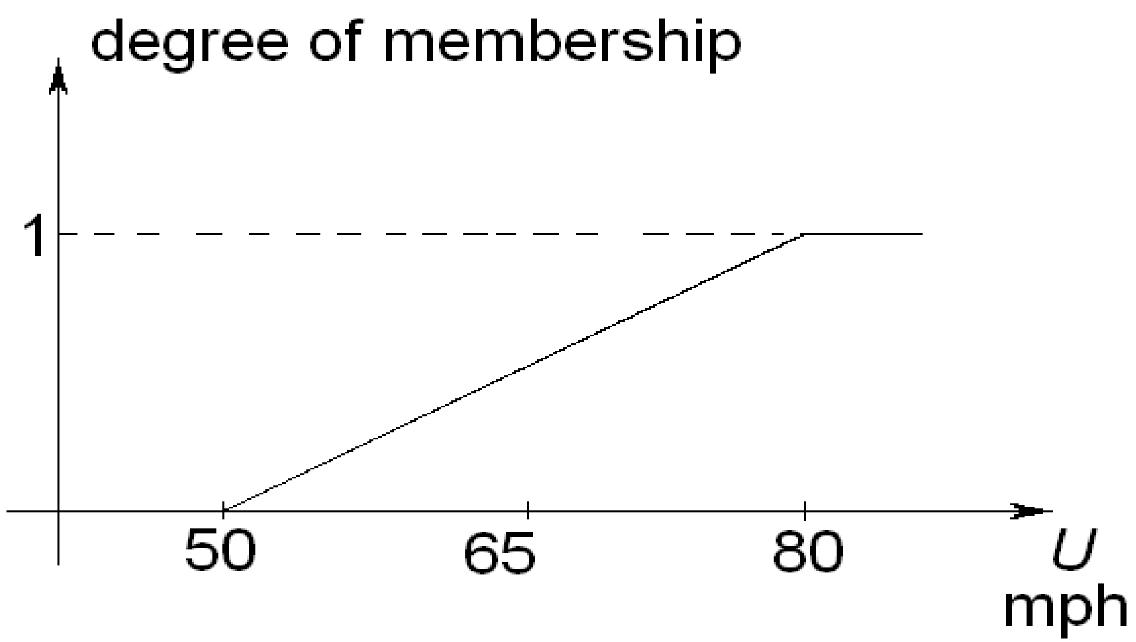


Zadeh's Definition of Fuzzy Set

"A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function which assigns to each object a grade membership ranging between zero and one."---L.A. Zadeh, 1965, p. 338

Example of Fuzzy Set

- Fuzzy sets allow for partial membership in a set
- Example of a fuzzy set---fast cars



Crisp versus Fuzzy

- Notation: U---collection of objects, universe
- μ ---membership function
- Crisp set

$$\mu : U \to \{0,1\}$$

- Fuzzy set

$$\mu : U \to [0,1]$$

Formal Definition of Fuzzy Set

- Denote u a generic element of the universe U
- A fuzzy set F in U is a set of ordered pairs:

$$F = \{(u, \mu_F(u)) : u \in U\}$$

Fuzzy Set Example

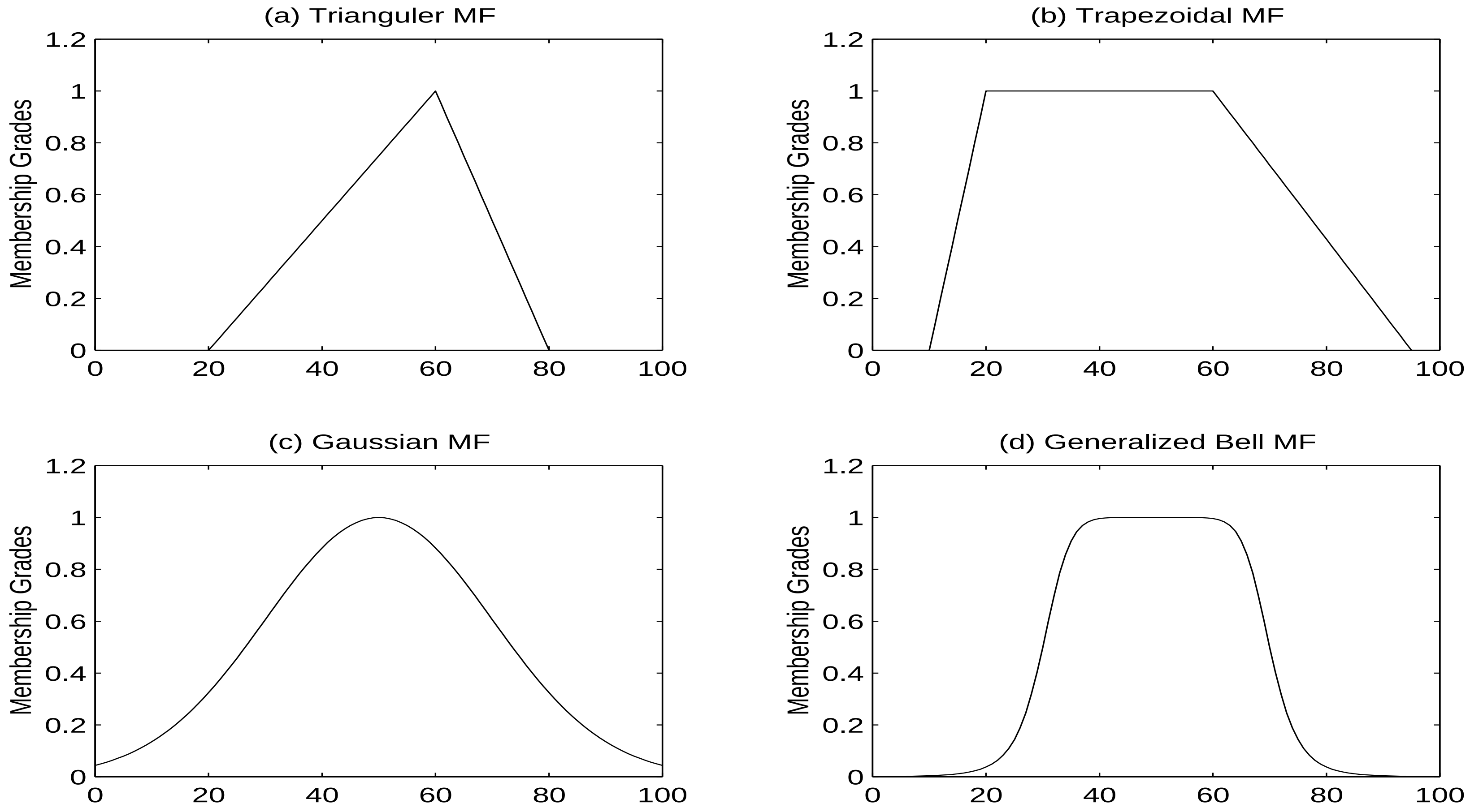
- Discrete universe

$$U = \{4,5,6,7,8\}$$

- Fuzzy set---”a number close to 6”

$$\{(4,0.0),(5,0.5),(6,1.0),(7,0.5),(8,0.0)\}$$

Possible Membership Functions



Special Types of Fuzzy Sets

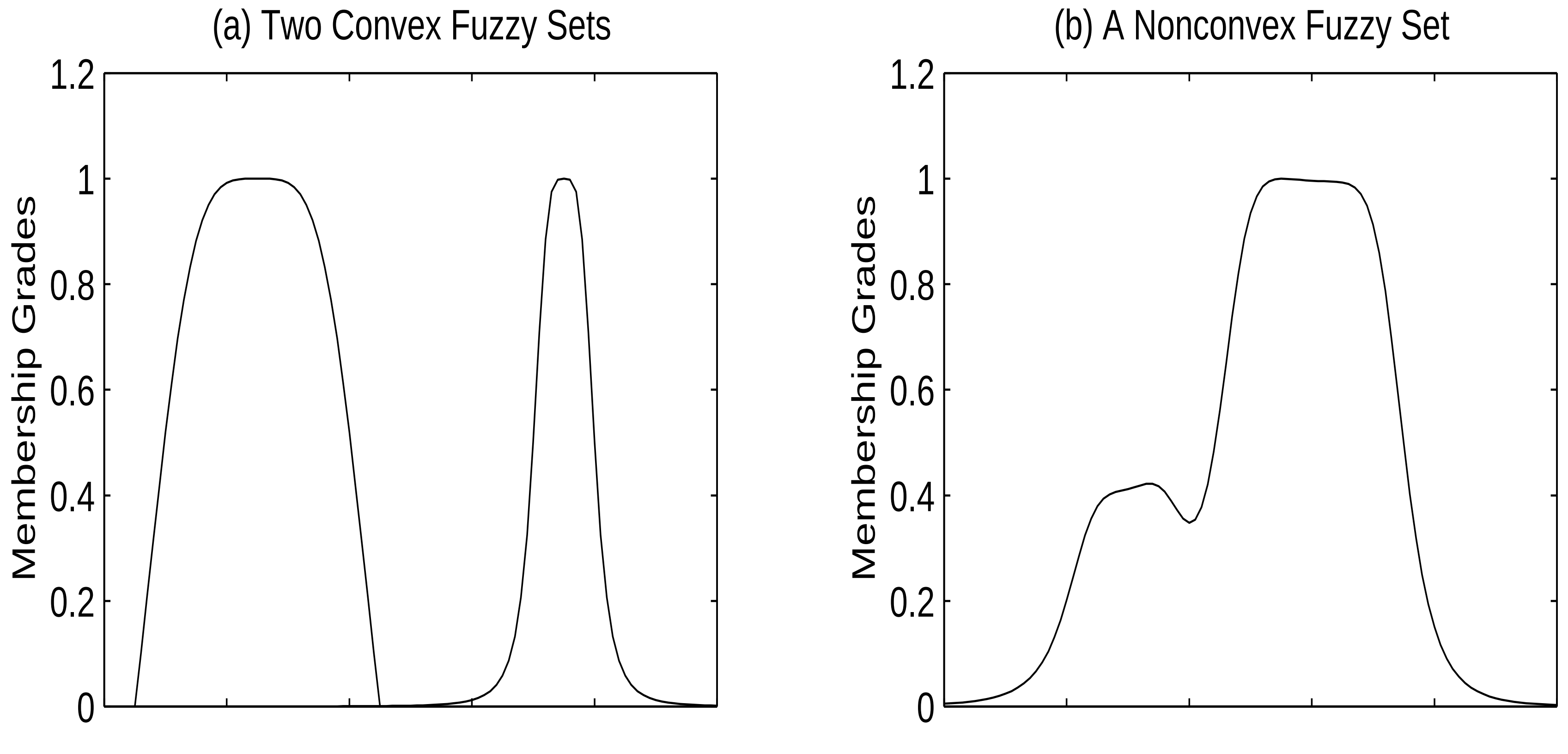
A fuzzy set *F* is **convex** if for any x, y from the universe X and for any

$$\lambda \in [0,1]$$

we have

$$\mu_F(\lambda x + (1-\lambda)y) \geq \min(\mu_F(x), \mu_F(y))$$

Convex and Nonconvex Fuzzy Sets



Normal Fuzzy Set and Fuzzy Number

- A fuzzy set F is **normal** if there is a point x in the universe X such that

$$\mu_F(x) = 1$$

- A fuzzy number is a fuzzy set that is both normal and convex

Linguistic Variable

- Linguistics variable---a variable whose values are fuzzy numbers
- Examples of linguistic variables:

  (i) SPEED---it can take its values from the set of fuzzy numbers

  {very slow, slow, medium, fast, very fast}

  (ii) TEMPERATURE---possible values

  {cold, cool, warm, hot}

More Examples of Linguistic Variables

ERROR

{LN, SN, ZE, SP, LP}

LN---large negative

SN---small negative

ZE---zero

SP---small positive

LP---large positive

Another Example of a Linguistic Variable

CHANGE-IN-ERROR

{LN, SN, ZE, SP, LP}

LN---large negative

SN---small negative

ZE---zero

SP---small positive

LP---large positive

Fuzzy Rules

- A fuzzy IF-THEN rule

IF *x* is *A* THEN *y* is *B*

- Examples:
  (i) IF speed is low THEN brake is zero
  (ii) IF error is large negative THEN control action is large positive

Linguistic Description of Control Action

- Instead of a math model of the plant we may have available fuzzy IF-THEN rules:

  RULE 1: IF error is ZE and change-in-error is SP then u is SN

  RULE 2: If error is SN and change-in-error is SN THEN u is SP

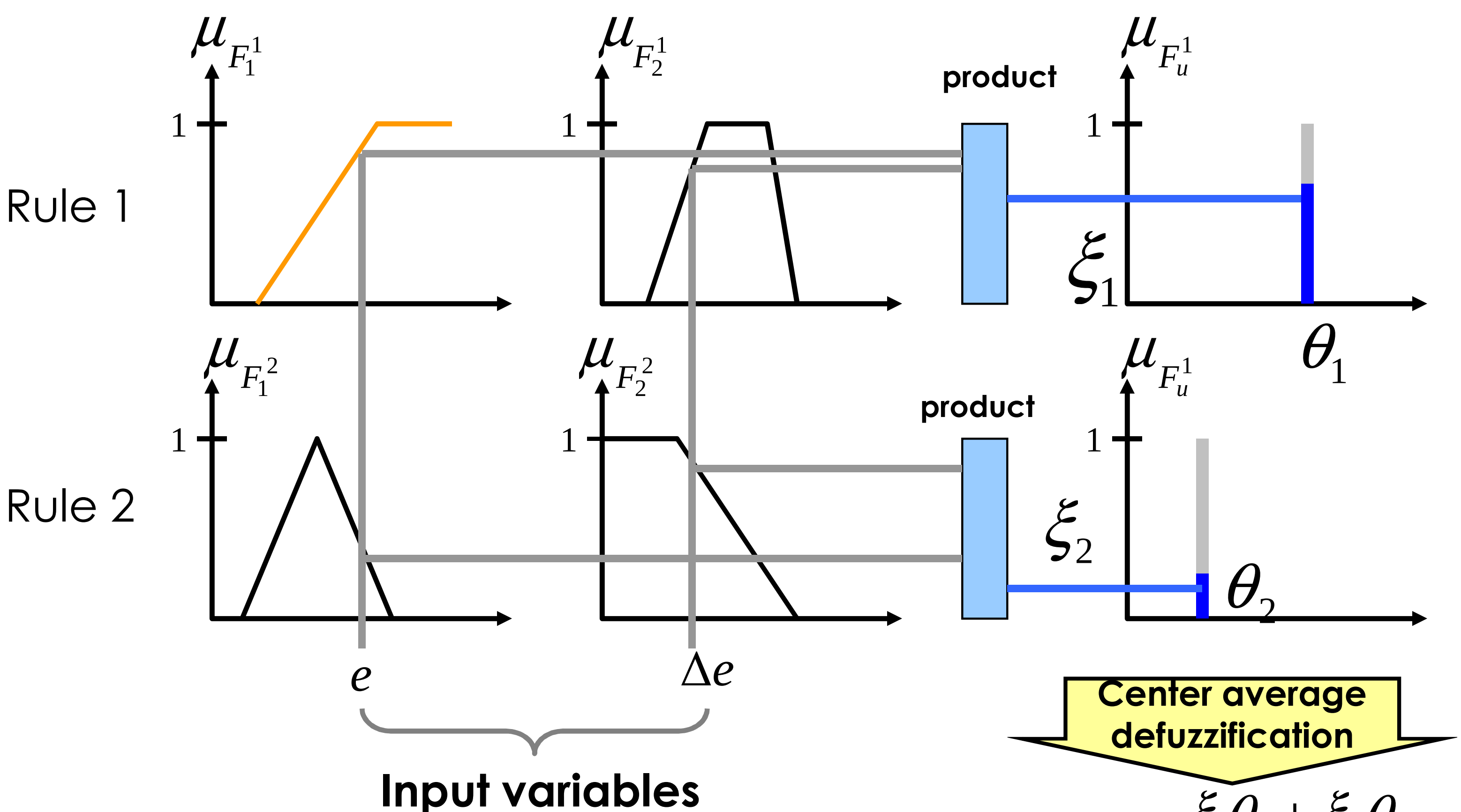
- Use the given linguistic description in the form of the IF-THEN rules to design a controller

Defuzzification: Moving From Fuzzy Rules to Numbers

- Computing with words--- Convert the given fuzzy IF-THEN rules into controller action
- Inference engine, or defuzzifier, uses fuzzy rules to generate crisp numbers
- Center average defuzzifier (product inference rule)

$$u = \frac{\sum_{l=1}^{M} \mu_{A_l}(e)\,\mu_{B_l}(\Delta e)\,\theta_c^l}{\sum_{l=1}^{M} \mu_{A_l}(e)\,\mu_{B_l}(\Delta e)}$$

Center average defuzzifier (product inference rule)



Control action $$u = \frac{\xi_1\theta_1 + \xi_2\theta_2}{\xi_1 + \xi_2}$$

Defuzzifier for *q* Inputs

Center average defuzzification for *q* input controller

$$u = \frac{\sum_{l=1}^{m}\prod_{j=1}^{q}\mu_{A_j^l}(v_j)\theta_l}{\sum_{l=1}^{m}\prod_{j=1}^{q}\mu_{A_j^l}(v_j)} = \sum_{l=1}^{m}\left(\frac{\prod_{j=1}^{q}\mu_{A_j^l}(v_j)}{\sum_{l=1}^{m}\prod_{j=1}^{q}\mu_{A_j^l}(v_j)}\right)\theta_l$$

$$= \boldsymbol{\theta}^T \boldsymbol{\xi}$$

where $\boldsymbol{\theta} = [\theta_1\ \theta_2 \cdots \theta_m]^T,\ \boldsymbol{\xi} = [\xi_1\ \xi_2 \cdots \xi_m]^T,$

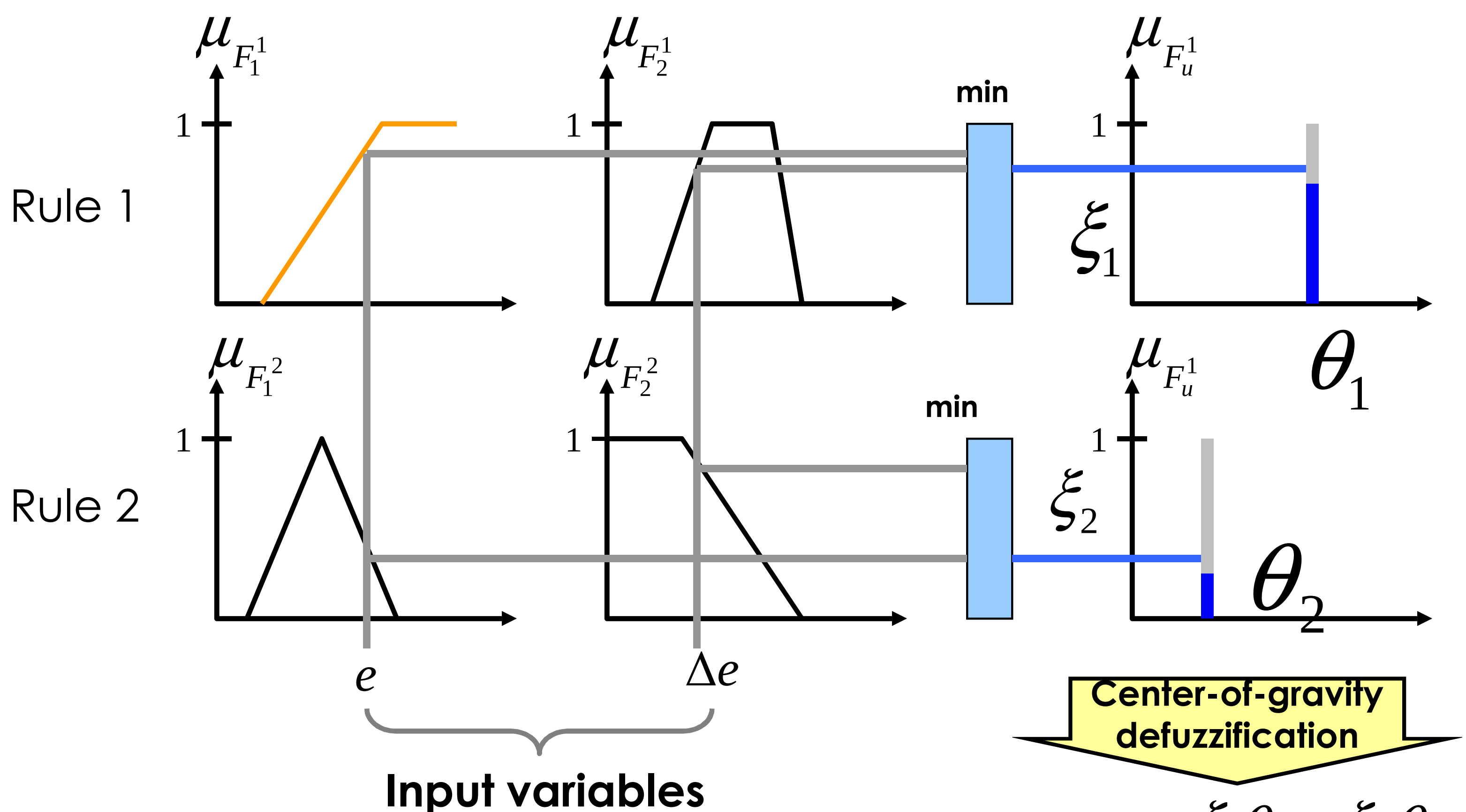
q is the number of inputs

m is the number of fuzzy rules

Center-of-Gravity Defuzzifier

$$u = \frac{\sum_{l=1}^{M} \min\left\{\mu_{A_l}(e), \mu_{B_l}(\Delta e)\right\} \times \theta_c^l}{\sum_{l=1}^{M} \min\left\{\mu_{A_l}(e), \mu_{B_l}(\Delta e)\right\}}$$

Center-of-Gravity Defuzzifier



Control action $$u = \frac{\xi_1\theta_1 + \xi_2\theta_2}{\xi_1 + \xi_2}$$

Computing With Words---Example

- Our example uses data from the paper by Y.F. Li and C.C. Lau, "*Development of fuzzy algorithms for servo systems*," IEEE Control Systems Magazine, pp. 65--72, April 1989
- Rule #1: IF the error is ZE and the error change is SP THEN the control is SN
- Rule #2: IF the error is ZE and the error change is ZE THEN the control is ZE
- Rule #3: IF the error is SN and the error change is SN THEN the control is SP
- Rule #4: IF the error is SN and the error change is ZE THEN the control is LP