ECE 602: LUMPED LINEAR SYSTEMS

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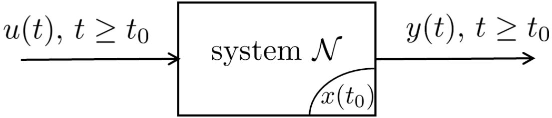
Lumped and Distributed Systems

System Representation Using State Variables

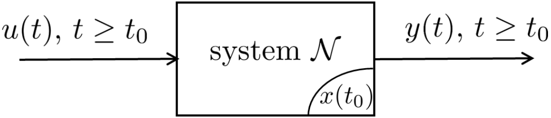


With the state variables, systems $\mathcal{N}$ can be represented as

$$y\big|_{t \geq t_0} = \mathcal{N}\left(u\big|_{t \geq t_0}, x(t_0)\right)$$

where $t_0 \in \mathcal{I}$ is an arbitrary time

A Relook at Linear Systems



System $\mathcal{N}$ is linear if for all $u_1, u_2 \in \mathcal{U}$, $\lambda_1, \lambda_2 \in \mathbb{R}$, and $x_1(t_0), x_2(t_0)$:

$$\mathcal{N}\left(\lambda_1 \cdot u_1\big|_{t \geq t_0} + \lambda_2 \cdot u_2\big|_{t \geq t_0},\, \lambda_1 \cdot x_1(t_0) + \lambda_2 \cdot x_2(t_0)\right)$$
$$= \lambda_1 \cdot \mathcal{N}\left(u_1\big|_{t \geq t_0}, x_1(t_0)\right) + \lambda_2 \cdot \mathcal{N}\left(u_2\big|_{t \geq t_0}, x_2(t_0)\right)$$

The response of a linear system $\mathcal{N}$ can be decomposed as:

$$\mathcal{N}\left(u\big|_{t \geq t_0}, x(t_0)\right) = \underbrace{\mathcal{N}\left(u\big|_{t \geq t_0}, 0\right)}_{\text{zero-state response}} + \underbrace{\mathcal{N}\left(0, x(t_0)\right)}_{\text{zero-input response}}$$

Lumped vs. Distributed Systems

System $\mathcal{N}$

- a lumped system if it has a finite number of state variables
- a distributed system if it has an infinite number of state variables

Examples

1. $\ddot{y}(t) + 2\dot{y}(t) + y(t) = u(t)$

2. $y(t) = \int_{t-1}^{t} u(s)\, ds$

3. $y(t) = u(t-1)$

4. $\frac{\partial y(z,t)}{\partial t} = \frac{\partial^2 y(z,t)}{\partial z^2} + u(z,t)$ (1D heat equation)