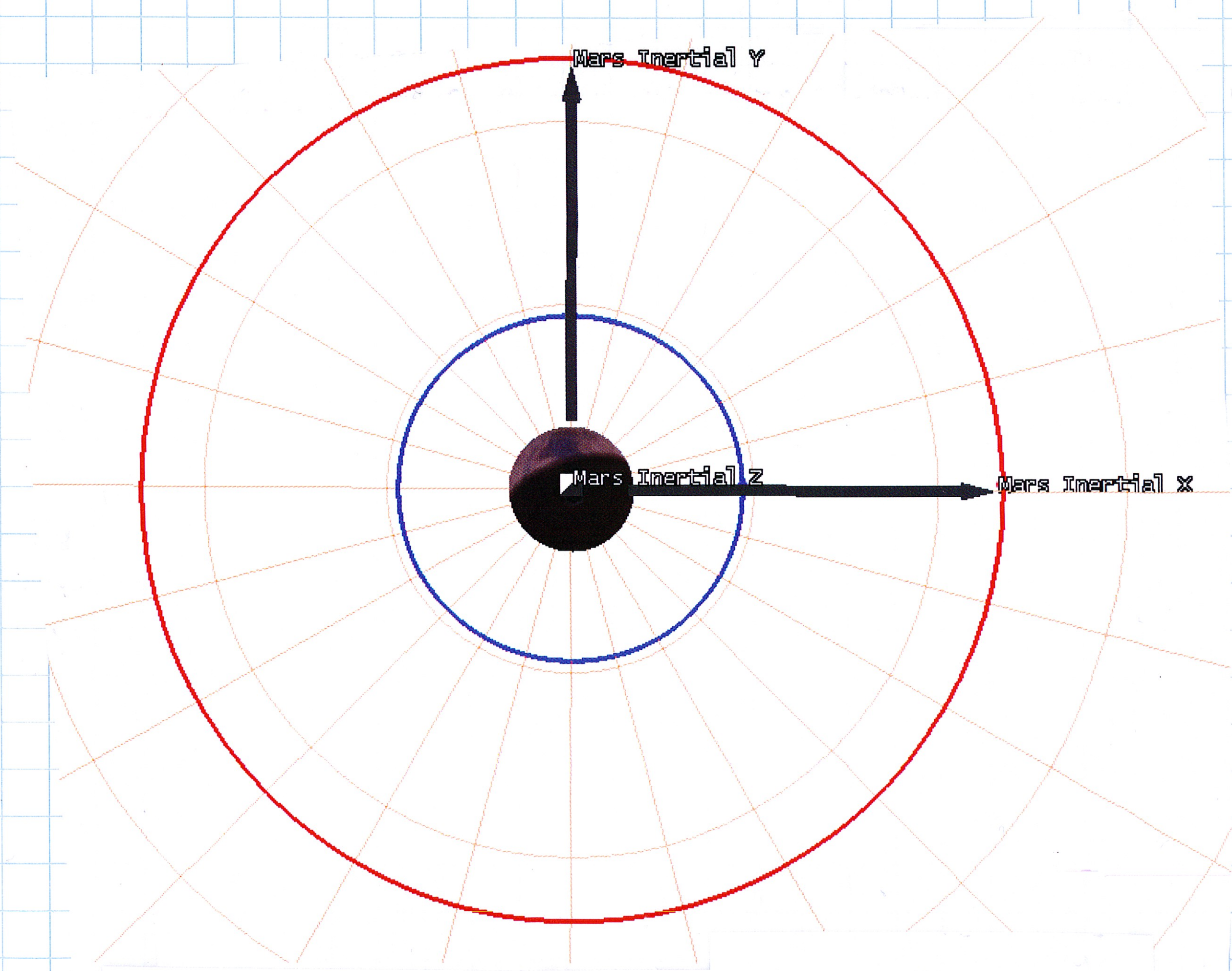
Mars Explorer

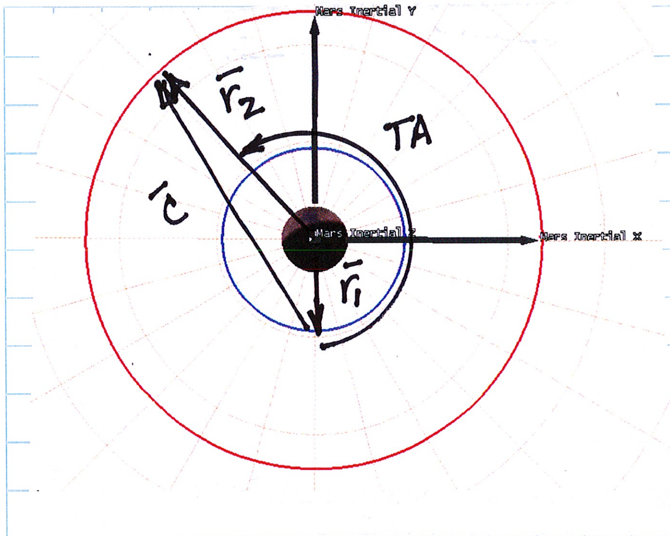
A small robotic explorer has been sent to the Martian system to observe and characterize the two moons – Phobos and Deimos, whose orbits are assumed to be circular and coplanar about Mars, with a radius equal to the semi-major axis listed in the constants table for moons and dwarfs. Let's assume that the spacecraft has completed its observations in the orbit of Phobos and must transfer to the orbit of Deimos. Consider only the gravity of Mars.

a) Assume that the spacecraft departs from Phobos to rendezvous with Deimos using an elliptical minimum energy transfer arc and a transfer angle of 240 degrees. Determine the following quantities: $a, TOF, p, e, \mathcal{E}, type$ (1 or 2), $v_D, v_A, \theta_D^*, \theta_A^*, \gamma_D, \gamma_A$.



$\mu_♂ = 42828.37\ Km^3/s^2$ $\quad R_♂ = 3397\ Km$

$a_{phobos} = 9376\ Km$ $\quad a_{Deimos} = 23{,}458\ Km$



Min Energy Transfer

$c = 2.9294 \times 10^4$ Km
$= 8.6 R_{♂}$

$s = 3.1064 \times 10^4$ Km
$= 9.144 R_{♂}$

$a_{min} = 15{,}532$ Km

$$\alpha_0 = 2\sin^{-1}\sqrt{\frac{s}{2a_m}} = 180°$$

$$\beta_0 = 2\sin^{-1}\sqrt{\frac{s-c}{2a_m}} = 27.62°$$

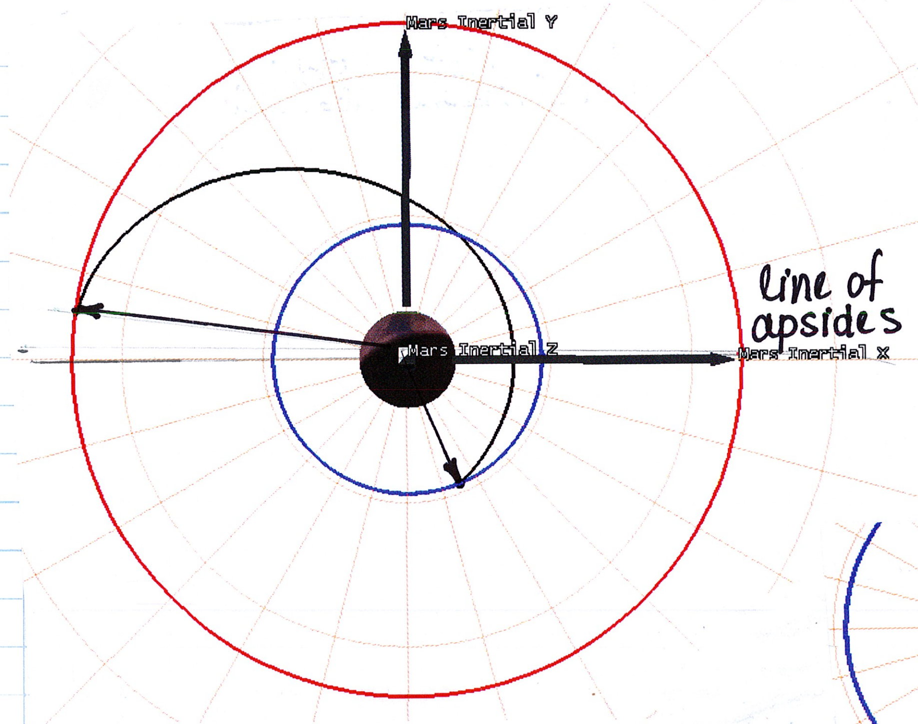
TA > 180° $\rightarrow$ Type

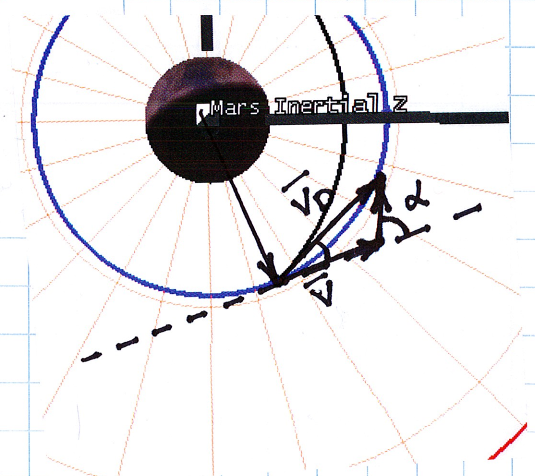
$p = 11{,}262$ Km

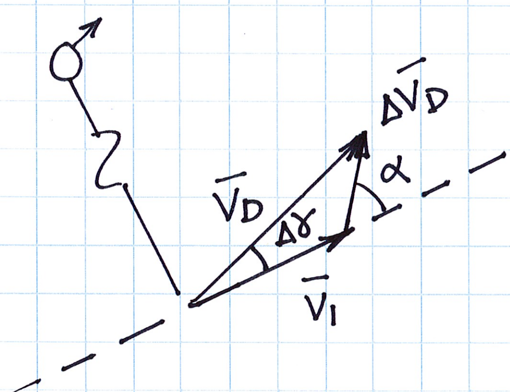
$p = a(1-e^2) \Rightarrow$

Departure

Arrival







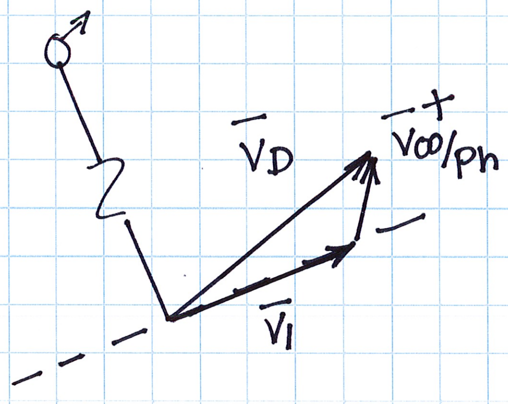
$$\bar{V}_1 = \sqrt{\frac{\mu_{♂}}{r_D}} = 2.137 \text{ Km/s}$$

$$V_D = 2.525 \text{ Km/s}$$

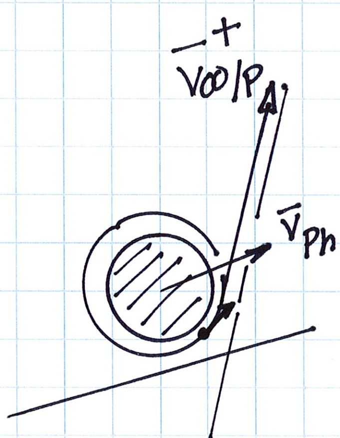
$$\Delta\gamma = \gamma_D = -21.954^\circ$$

Phobos $R_{Ph} = 11.1$ Km

$\mu_{Ph} = 7.112 \times 10^{-4}$ Km^3/s^2

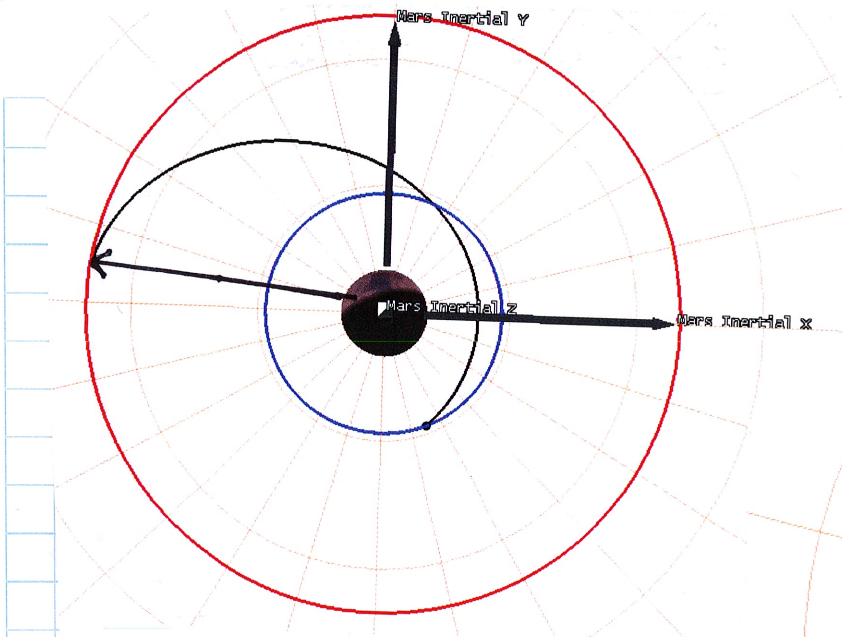


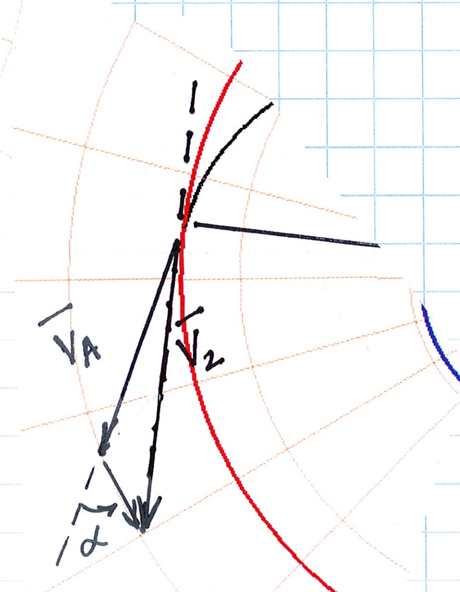
$$\bar{V}_D = \bar{V}^{+} = \bar{V}_1 + \bar{V}^{+}_{\infty/Ph}$$

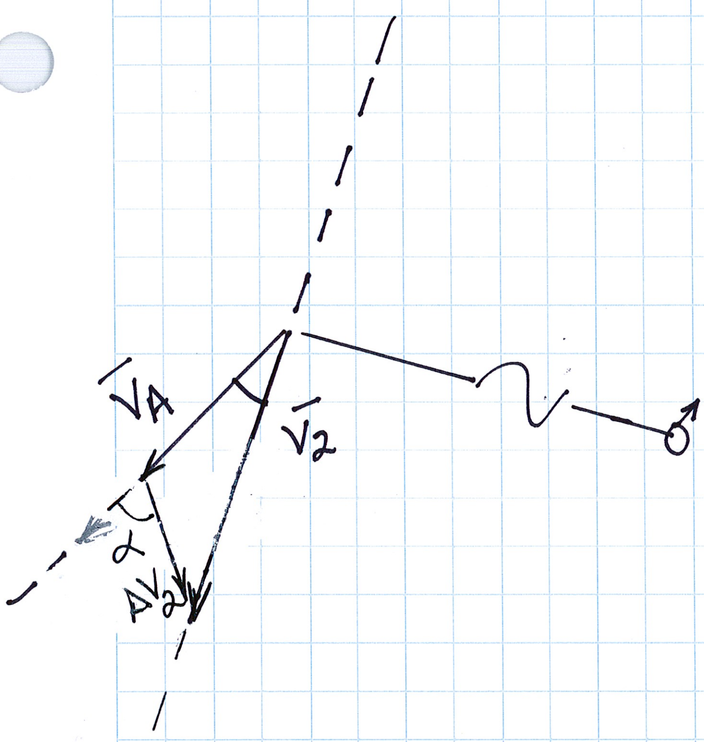


V_c at ~20 Km altitude

$V_c = 4.87 \times 10^{-3}$ Km/s





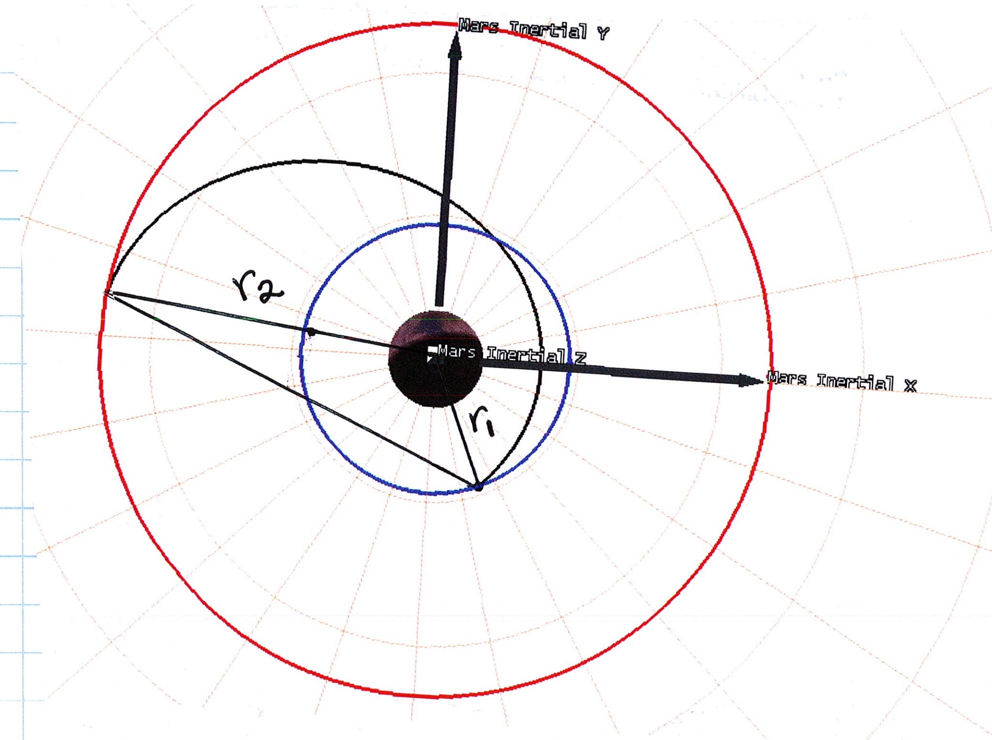


$$V_2 = \sqrt{\frac{\mu_{♂}}{r_{Deimos}}} = 1.3512 \ Km/s$$

$$V_A = .9455 \ Km/s$$

min Energy Transfer

$$|\Delta \bar{V}_{Total}| = |\Delta \bar{V}_D| + |\Delta \bar{V}_A| =$$



Min Energy Transfer

$TOF_{min} = 5.7\ hR$

$\Downarrow$

$r_p = 7388\ Km$

$= 2.18\ R_♂$

What happens if more time?

TA > 180° $\Longrightarrow$ remain Type 2

TOF >

$$\alpha = 2\pi - \alpha_0 \qquad \beta = -\beta_0$$

$$\alpha_0 = 2\sin^{-1}\sqrt{\frac{s}{2a}}$$

$$\beta_0 = 2\sin^{-1}\sqrt{\frac{s-c}{2a}}$$

$$TOF = \sqrt{\frac{a^3}{\mu}}\left[(\alpha - \sin\alpha) - (\beta - \sin\beta)\right]$$

Given $a_{min} = 15{,}532\ Km = 4.57\ R_{♂}$

Guess a ?

Try $a =$

$\Downarrow$

$\alpha_0 = 109.47°$ $\beta_0 = 22.48°$

$\alpha = 250.53°$ $\beta = -22.48°$

$TOF = 25.4\ hr$

$\Downarrow$

$p =$ $14945\ km = 4.40\ R_{♂}$
$8487.2\ km = 2.50\ R_{♂}$

2B

Departure

$V_D = 2.701\ Km/s$

$\theta^*_D = -7.308°$

$\gamma_D = -2.736°$

$|\Delta \bar{V}_D| = .576\ Km/s$

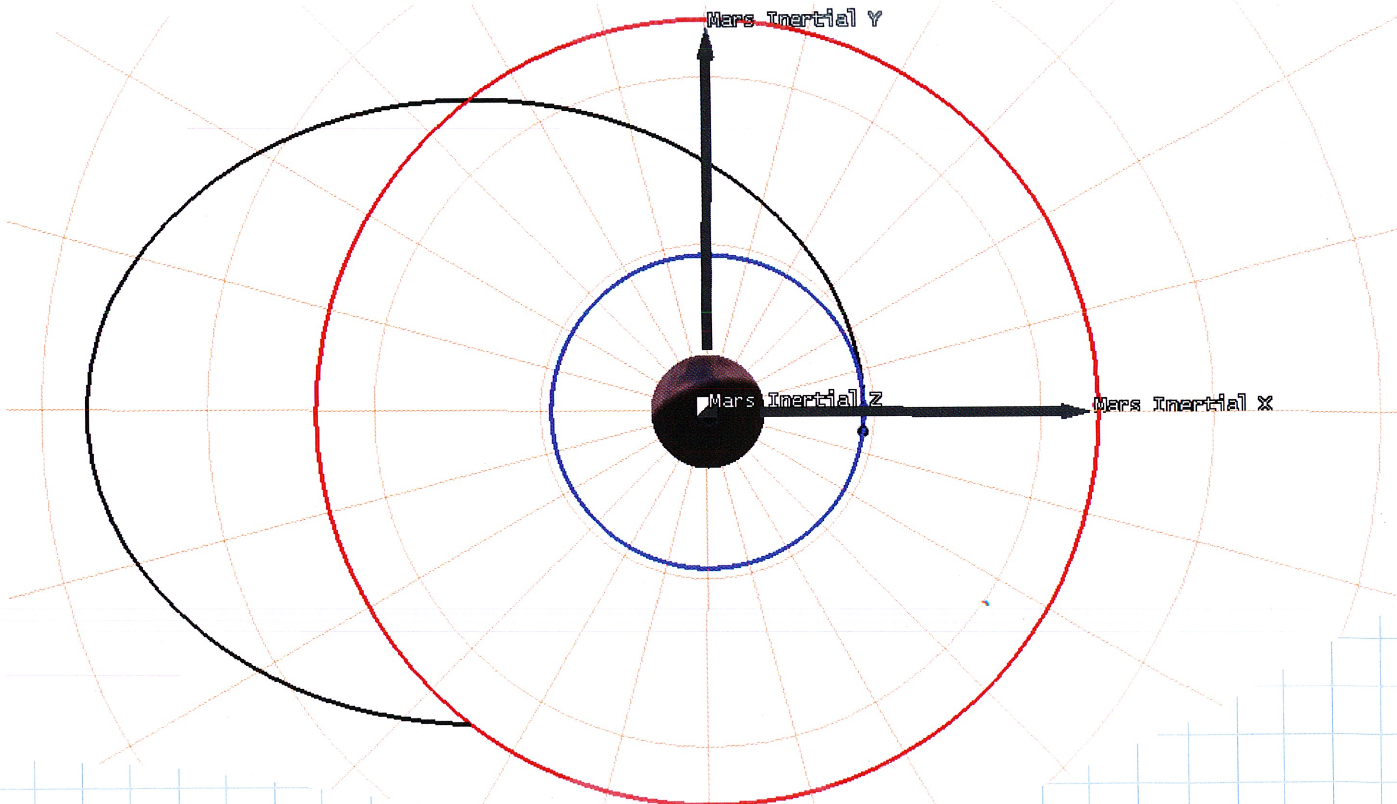
Arrival

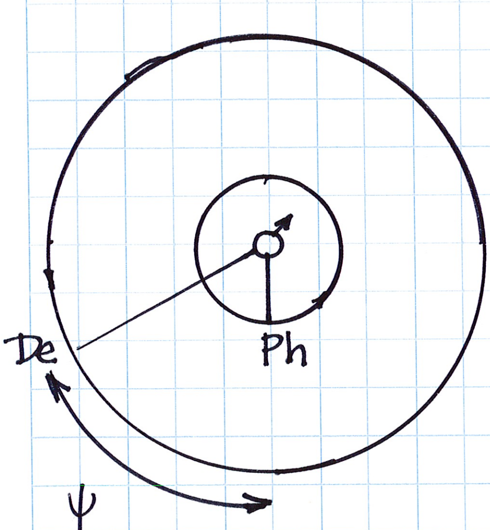
$V_A = 1.347\ Km/s$

$\theta^*_A = -127.31°$

$\gamma_A = -36.781°$

$|\Delta \bar{V}_A| = .851\ Km/s$





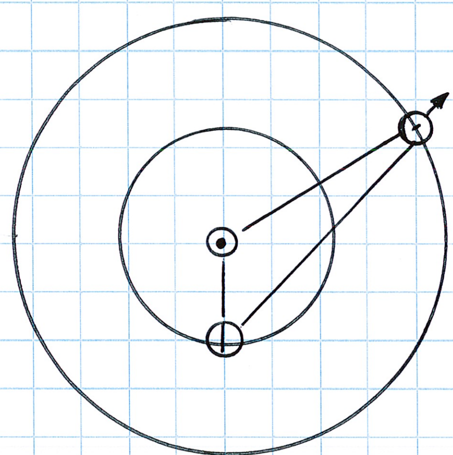
ψ - phase angle betwee Phobos and Deimos at departure

$$\underbrace{n_2 TOF}_{\text{angle Deimos travels during TOF}} = \phi - \psi$$

ϕ: angle s/c travels during TOF

ψ: required angle between Phobos/Deimos at departure

Now consider the trip Earth → Mars



Let's assume $\phi = 140°$

Perhaps we are considering the rescue of Mark Watney! Can we get there in 90 days?

$r_D = a_{\oplus} = 1.496 \times 10^8$ Km $= 1$ AU

$r_A = a_{♂} = 2.2795 \times 10^8$ Km $= 1.52$ AU

$c = 3.5589 \times 10^8$ Km $= 2.38$ AU

Type I $\Longrightarrow$ $TOF_{par} =$

$TOF < TOF_{par} \Longrightarrow$

$$\alpha_0' = 2\sinh^{-1}\sqrt{\frac{s}{2|a|}} \qquad \beta_0' = 2\sinh^{-1}\sqrt{\frac{s-c}{2|a|}}$$

$$\alpha' = \alpha_0' \qquad \beta' = \beta_0'$$

$$90 \text{ da} = TOF = \sqrt{\frac{|a|^3}{\mu_\odot}}\left[(\sinh\alpha' - \alpha') - (\sinh\beta' - \beta')\right]$$

Guess $|a| =$

$$a = -3.0203 \times 10^{8}\ Km =$$

$$p = \frac{4|a|(s - r_D)(s - r_A)}{c^2} \sinh^2\left(\frac{\alpha' \pm \beta'}{2}\right)$$

$$p = \begin{matrix} 2.6283 \times 10^{8}\ Km = \\ 1.0906 \times 10^{8}\ Km = \end{matrix}$$

$$\Longrightarrow$$

Departure

$v_D = 47.043\ Km/s$

$\theta_D^* = -56.43°$

$\gamma_D = -32.97°$

$|\Delta \bar{v}_D| =$

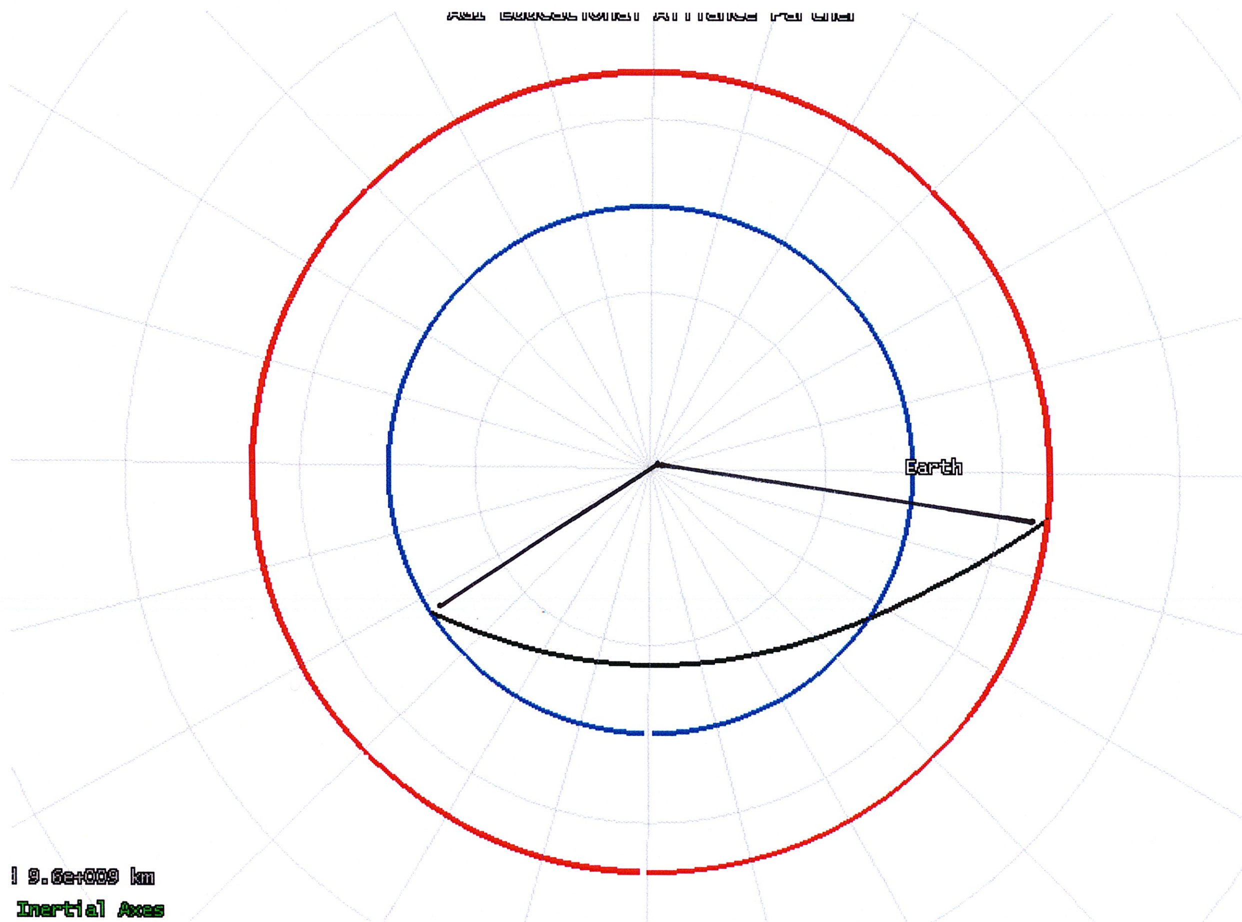
Arrival

$v_A = 40.047\ Km/s$

$\theta_A^* = 83.58°$

$\gamma_A = 49.69°$

$|\Delta \bar{v}_A| =$


Earth
9.6e+009 km
Inertial Axes