HW1

7.1.9)

$$3A = 3\begin{pmatrix} 0 & 2 & 4 \\ 6 & 5 & 5 \\ 1 & 0 & -3 \end{pmatrix} = \boxed{\begin{pmatrix} 0 & 6 & 12 \\ 18 & 15 & 15 \\ 3 & 0 & -9 \end{pmatrix} \quad - 4c}$$

$$0.5B = \frac{1}{2}\begin{pmatrix} 0 & 5 & 2 \\ 5 & 3 & 4 \\ -2 & 4 & -2 \end{pmatrix} = \boxed{\begin{pmatrix} 0 & 5/2 & 1 \\ 5/2 & 3/2 & 2 \\ -1 & 2 & -1 \end{pmatrix} \quad - 4c}$$

$$3A + 0.5B = \begin{pmatrix} 0 & 6 & 12 \\ 18 & 15 & 15 \\ 3 & 0 & -9 \end{pmatrix} + \begin{pmatrix} 0 & 5/2 & 1 \\ 5/2 & 3/2 & 2 \\ -1 & 2 & -1 \end{pmatrix} = \boxed{\begin{pmatrix} 0 & 17/2 & 13 \\ 41/2 & 33/2 & 17 \\ 2 & 2 & -10 \end{pmatrix} \quad - 4a}$$

$3A + 0.5B + C$ - $\boxed{\text{Not Defined, matrices not same size}}$

7.1.12)

$$(C+D)+E = \begin{pmatrix} 5 & 2 \\ -2 & 4 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} -4 & 1 \\ 5 & 0 \\ 2 & -1 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ 3 & 4 \\ 3 & -1 \end{pmatrix} = \boxed{\begin{pmatrix} 1 & 5 \\ 6 & 8 \\ 6 & -2 \end{pmatrix} \quad - 3b}$$

$$(D+E)+C = \begin{pmatrix} -4 & 1 \\ 5 & 0 \\ 2 & -1 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ 3 & 4 \\ 3 & -1 \end{pmatrix} + \begin{pmatrix} 5 & 2 \\ -2 & 4 \\ 1 & 0 \end{pmatrix} = \boxed{\begin{pmatrix} 1 & 5 \\ 6 & 8 \\ 6 & -2 \end{pmatrix} \quad - 3b}$$

7.1.12 cont)

$$O(C-E)+4D=\begin{pmatrix}0&0\\0&0\\0&0\end{pmatrix}+4\begin{pmatrix}-4&1\\5&0\\2&-1\end{pmatrix}=\boxed{\begin{pmatrix}-16&4\\20&0\\8&-4\end{pmatrix}}$$ - 4a

$A-OC$ - $\boxed{\text{Not defined, A \& C aren't same size}}$

7.2.12)

$$AA^T=\begin{pmatrix}4&-2&3\\-2&1&6\\1&2&2\end{pmatrix}\begin{pmatrix}4&-2&1\\-2&1&2\\3&6&2\end{pmatrix}=\begin{pmatrix}16+4+9&-8-2+18&4-4+6\\-8-2+18&4+1+36&-2+2+12\\4-4+6&-2+2+12&1+4+4\end{pmatrix}$$

$$\boxed{AA^T=\begin{pmatrix}29&8&6\\8&41&12\\6&12&9\end{pmatrix}}$$

$$A^2=\begin{pmatrix}4&-2&3\\-2&1&6\\1&2&2\end{pmatrix}\begin{pmatrix}4&-2&3\\-2&1&6\\1&2&2\end{pmatrix}=\begin{pmatrix}16+4+3&-8-2+6&12-12+6\\-8-2+6&4+1+12&-6+6+12\\4-4+2&-2+2+4&3+12+4\end{pmatrix}$$

$$\boxed{A^2=\begin{pmatrix}23&-4&6\\-4&17&12\\2&4&19\end{pmatrix}}$$

$$BB^T=\begin{pmatrix}1&-3&0\\-3&1&0\\0&0&-2\end{pmatrix}\begin{pmatrix}1&-3&0\\-3&1&0\\0&0&-2\end{pmatrix}=\begin{pmatrix}1+9&-3-3&0\\-3-3&9+1&0\\0&0&4\end{pmatrix}$$

$$\boxed{BB^T=\begin{pmatrix}10&-6&0\\-6&10&0\\0&0&4\end{pmatrix}}$$

$$B^2 = \begin{pmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}\begin{pmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} = \boxed{\begin{pmatrix} 10 & -6 & 0 \\ -6 & 10 & 0 \\ 0 & 0 & 4 \end{pmatrix}}$$

7.2.14)

$$3A - 2B = \begin{pmatrix} 12 & -6 & 9 \\ -6 & 3 & 18 \\ 3 & 6 & 6 \end{pmatrix} - \begin{pmatrix} 2 & -6 & 0 \\ -6 & 2 & 0 \\ 0 & 0 & -4 \end{pmatrix} = \boxed{\begin{pmatrix} 10 & 0 & 9 \\ 0 & 1 & 18 \\ 3 & 6 & 10 \end{pmatrix}}$$

$$(3A - 2B)^T = \begin{pmatrix} 10 & 0 & 9 \\ 0 & 1 & 18 \\ 3 & 6 & 10 \end{pmatrix}^T = \boxed{\begin{pmatrix} 10 & 0 & 3 \\ 0 & 1 & 6 \\ 9 & 18 & 10 \end{pmatrix}}$$

$$3A^T - 2B^T = \begin{pmatrix} 12 & -6 & 3 \\ -6 & 3 & 6 \\ 9 & 18 & 6 \end{pmatrix} - \begin{pmatrix} 2 & -6 & 0 \\ -6 & 2 & 0 \\ 0 & 0 & -4 \end{pmatrix} = \boxed{\begin{pmatrix} 10 & 0 & 3 \\ 0 & 1 & 6 \\ 9 & 18 & 10 \end{pmatrix}}$$

$$(3A - 2B)^T a^T = \begin{pmatrix} 10 & 0 & 3 \\ 0 & 1 & 6 \\ 9 & 18 & 10 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 10 \\ -2 \\ 9-36 \end{pmatrix} = \boxed{\begin{pmatrix} 10 \\ -2 \\ -27 \end{pmatrix}}$$

7.2.29) $A = \begin{pmatrix} 400 & 60 & 240 \\ 100 & 120 & 500 \end{pmatrix}$

$$\boxed{p = \begin{pmatrix} 35 \\ 62 \\ 30 \end{pmatrix}} \qquad v = Ap = \begin{pmatrix} 400 & 60 & 240 \\ 100 & 120 & 500 \end{pmatrix}\begin{pmatrix} 35 \\ 62 \\ 30 \end{pmatrix}$$

$$\boxed{V = \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} = \begin{pmatrix} 24920 \\ 25940 \end{pmatrix}}$$

7.3.3)

$$\begin{aligned} x + y - z &= 9 \\ 8y + 6z &= -6 \\ -2x + 4y - 6z &= 40 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 9 \\ 0 & 8 & 6 & -6 \\ -2 & 4 & -6 & 40 \end{array}\right) \underset{R_3 + 2R_1}{\Rightarrow} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 9 \\ 0 & 8 & 6 & -6 \\ 0 & 6 & -8 & 58 \end{array}\right) \Rightarrow R_3 - \tfrac{3}{4}R_2$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 9 \\ 0 & 8 & 6 & -6 \\ 0 & 0 & -25/2 & \frac{125}{2} \end{array}\right) \quad R_3 = R_3\left(-\tfrac{2}{25}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & -1 & 9 \\ 0 & 8 & 6 & -6 \\ 0 & 0 & 1 & -5 \end{array}\right)$$

$$\therefore \begin{aligned} x + y - z &= 9 \\ 8y + 6z &= -6 \\ z &= -5 \end{aligned}$$

$\therefore\ 8y - 30 = -6 \Rightarrow y = 3 \quad \therefore\ x + 3 + 5 = 9$

$\therefore\ x = 1$, $\boxed{x = 1,\ y = 3,\ z = -5}$

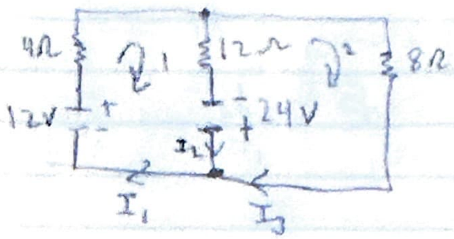
7.3.9)

$$\begin{aligned} -2y - 2z &= -8 \\ 3x + 4y - 5z &= 13 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 3 & 4 & -5 & 13 \\ 0 & -2 & -2 & -8 \end{array}\right) \quad \underset{\Rightarrow}{R_2 = -\tfrac{1}{2}R_2} \quad \left(\begin{array}{ccc|c} 3 & 4 & -5 & 13 \\ 0 & 1 & 1 & 4 \end{array}\right) \Rightarrow y + z = 4$$

$\boxed{y = 4 - z\ \therefore\ \text{Infinetly many solution due to arbitrary } z.}$

7.3.18)



$$\Sigma V_1 = 0 = V_1 - (I_1)(R_1) - (I_2)(R_2) + V_2 = 12 - 4I_1 - 12I_2 + 24$$

$$\Rightarrow -4I_1 - 12I_2 = -36$$

$$\Sigma V_2 = 0 = -(I_3)(R_3) + (I_2)(R_2) - 24 = -8I_3 + 12I_2 - 24 \Rightarrow$$

$$24 = -8I_3 + 12I_2$$

$$I_1 = I_2 + I_3 \Rightarrow -I_1 + I_2 + I_3 = 0$$

$$\left(\begin{array}{ccc|c} -1 & 1 & 1 & 0 \\ 0 & 12 & -8 & 24 \\ -4 & -12 & 0 & -36 \end{array}\right) \xrightarrow{R_3 = R_3 - 4R_1} \left(\begin{array}{ccc|c} -1 & 1 & 1 & 0 \\ 0 & 12 & -8 & 24 \\ 0 & -16 & -4 & -36 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} -1 & 1 & 1 & 0 \\ 0 & 3 & -2 & 6 \\ 0 & 4 & 1 & 9 \end{array}\right)$$

$$\xrightarrow{R_3 = R_3 - (4/3)R_2} \left(\begin{array}{ccc|c} -1 & 1 & 1 & 0 \\ 0 & 3 & -2 & 6 \\ 0 & 0 & 11/3 & 1 \end{array}\right) \Rightarrow \underline{I_3 = 3/11}$$

$$3I_2 - 2(3/11) = 6 \Rightarrow I_2 = 2 + 3/11 = \underline{24/11}$$

$$\therefore I_1 = \underline{27/11}$$

$$\boxed{\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 27/11 \\ 24/11 \\ 3/11 \end{pmatrix}}$$

7.4.1) $\begin{pmatrix} 4 & -2 & 6 \\ -2 & 1 & -3 \end{pmatrix} \xRightarrow{R_2 = R_2 + \frac{1}{2} R_1} \begin{pmatrix} \boxed{4} & -2 & 6 \\ 0 & 0 & 0 \end{pmatrix}$

$\boxed{\text{rank} = 1}$

$\boxed{\text{Row}(A) = \text{span}\{[-2 \;\; 1 \;\; -3]\}}$

$\boxed{\text{Col}(A) = \text{span}\left\{\begin{pmatrix} 4 \\ -2 \end{pmatrix}\right\}}$

7.4.2) $\begin{pmatrix} a & b \\ b & a \end{pmatrix} \quad R_2 = R_2 - \frac{b}{a} R_1 \Rightarrow \begin{pmatrix} \boxed{a} & b \\ 0 & \boxed{a - \frac{b^2}{a}} \end{pmatrix}$

$\boxed{\text{rank} = 2}$

$\boxed{\text{Row}(A) = \text{span}\{[a \;\; b], [b \;\; a]\}}$

$\boxed{\text{Col}(A) = \text{span}\left\{\begin{pmatrix} a \\ b \end{pmatrix}, \begin{pmatrix} b \\ a \end{pmatrix}\right\}}$

7.4.5) $\begin{pmatrix} 0.2 & -0.1 & 0.4 \\ 0 & 1.1 & -0.3 \\ 0.1 & 0 & -2.1 \end{pmatrix} \Rightarrow \begin{pmatrix} 0.2 & -0.1 & 0.4 \\ 0 & 1.1 & -0.3 \\ 0 & .05 & -2.3 \end{pmatrix} = \begin{pmatrix} \boxed{0.2} & -0.1 & 0.4 \\ 0 & \boxed{1.1} & -0.3 \\ 0 & 0 & \boxed{X} \end{pmatrix}$

$\boxed{\begin{array}{l} \text{rank} = 3, \quad \text{Row}(A) = \text{span}\left\{\begin{pmatrix} 0.2 \\ -0.1 \\ 0.4 \end{pmatrix}, \begin{pmatrix} 0 \\ 1.1 \\ -0.3 \end{pmatrix}, \begin{pmatrix} 0.1 \\ 0 \\ -2.1 \end{pmatrix}\right\} \\ \text{Col}(A) = \text{span}\left\{\begin{pmatrix} 0.2 \\ 0 \\ 0.1 \end{pmatrix}, \begin{pmatrix} -0.1 \\ 1.1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0.4 \\ -0.3 \\ -2.1 \end{pmatrix}\right\} \end{array}}$

7.4.7) $\begin{pmatrix} 8 & 0 & 4 & 0 \\ 0 & 2 & 0 & 4 \\ 4 & 0 & 2 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} \boxed{8} & 0 & 4 & 0 \\ 0 & \boxed{2} & 0 & 4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$\boxed{\text{rank} = 2, \quad \text{row}(A) = \text{span}\left\{ \begin{pmatrix} 8 \\ 0 \\ 4 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \\ 4 \end{pmatrix} \right\}, \quad \text{Col}(A) = \text{span}\left\{ \begin{pmatrix} 8 \\ 0 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} \right\}}$

7.4.9) $\begin{pmatrix} 9 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \xRightarrow{R_4 - R_2} \begin{pmatrix} 9 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 9 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 8/9 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} \boxed{9} & 0 & 1 & 0 \\ 0 & 0 & \boxed{1} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$\boxed{\text{Rank} = 2, \quad \text{Row}(A) = \text{span}\left\{ \begin{pmatrix} 9 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\}, \quad \text{Col}(A) = \text{span}\left\{ \begin{pmatrix} 9 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \right\}}$

7.4.12) $\text{rank}(B^T A^T) = \text{rank}(AB)$

Let $C = AB$, $\underline{\text{rank}(C) = \text{rank}(C^T)}$.

$\therefore C^T = (AB)^T = B^T A^T \quad \therefore \text{rank}(C) = \text{rank}(AB) = \text{rank}(C^T) = \text{rank}(B^T A^T)$

$\therefore \boxed{\text{rank}(B^T A^T) = \text{rank}(AB)}$

7.4.14) If $A_{m \times n}$ where $m > n$, max rank is m.

$\therefore$ either the rows or columns are linearly dependent.

7.4.15) If $A_{n \times n}$ rows are linearly independent, then A has rank$(A) = n$, $\therefore$ $\dim(\text{Row}(A)) = \dim(\text{Col}(A)) = n$
& columns of A must also be linearly independent.

7.4.17) $c_1 \begin{pmatrix} 3 \\ 4 \\ 0 \\ 2 \end{pmatrix} + c_2 \begin{pmatrix} 2 \\ -1 \\ 3 \\ 7 \end{pmatrix} + c_3 \begin{pmatrix} 1 \\ 16 \\ -12 \\ -22 \end{pmatrix} = 0$

$$\begin{aligned} 3c_1 + 2c_2 + c_3 &= 0 \\ 4c_1 - c_2 + 16c_3 &= 0 \\ 0c_1 + 3c_2 - 12c_3 &= 0 \\ 2c_1 + 7c_2 - 22c_3 &= 0 \end{aligned} \Rightarrow \left(\begin{array}{ccc|c} 3 & 2 & 1 & 0 \\ 4 & -1 & 16 & 0 \\ 0 & 3 & -12 & 0 \\ 2 & 7 & -22 & 0 \end{array}\right)$$

$$\Rightarrow \begin{pmatrix} 3 & 2 & 1 \\ 4 & -1 & 16 \\ 0 & 3 & -12 \\ 0 & 7.5 & -30 \end{pmatrix} \Rightarrow \begin{pmatrix} 3 & 2 & 1 \\ 4 & -1 & 16 \\ 0 & 3 & -12 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 3 & 2 & 1 \\ 0 & -3.67 & 14.67 \\ 0 & 3 & -12 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 3 & 2 & 1 \\ 0 & -3.67 & 14.67 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$\therefore$ $\boxed{c_1 = c_2 = c_3 = 0}$

7.4.32) $3v_1 - 2v_2 + v_3 = 0$ $\mathbb{R}^3$

$4v_1 + 5v_2 = 0$

$\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} + \begin{pmatrix} 4 \\ 5 \\ 0 \end{pmatrix} = \begin{pmatrix} 7 \\ 3 \\ 1 \end{pmatrix}$ ✓ ($u + v$ is in $\mathbb{R}^3$)

$(u + v = v + u)$ ✓, zero vector exists, inverse exists,

cu is in $\mathbb{R}^3$, $c\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} + c\begin{pmatrix} 4 \\ 5 \\ 0 \end{pmatrix} = \begin{pmatrix} 7c \\ 3c \\ c \end{pmatrix} = c(u+v)$,

$\therefore$ $\boxed{\text{Vector space with dim} = 3}$

7.4.34) $\mathbb{R}^n$ with $|v_j| = 1$

$|v_j| = 1$ $\therefore$ $\boxed{\text{Zero vector doesn't exist } (u + 0 \neq u)\text{, not a vector space}}$