Virtual Lab 2: Liquid Level Systems

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MAE 543

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Contents

PART A

In the last lab it was shown that for a single tank, the transfer function relating the inflow, $Q_i(s)$, to the tank height, H(s), is given below where R is the flow resistance and C is the flow capacitance:

$$G(s) = \frac{H(s)}{Q_i(s)} = \frac{R}{RCs+1} \tag{1}$$

From the plant transfer function above, this is clearly a type 0 system, as there are no open loop poles at the origin. Therefore, for a unit step response, the steady state error for a closed loop system will be non-zero. To fix this, a PI controller will be used to place an open loop pole at the origin, which will allow for zero steady state error in the closed loop step response. The transfer function for a PI controller is given by:

$$C(s) = \frac{U(s)}{E(s)} = K_p + \frac{K_i}{s} \tag{2}$$

The open loop transfer function is given by:

$$G_{OL} = G(s)C(s) = \frac{R(K_p s + K_i)}{s(RCs+1)} \tag{3}$$

It is shown that the PI controller has added an open loop pole at the origin, creating a type 1 system. This causes the static error position coefficient to go to infinity, in turn causing the steady state error to go to zero. With the controller selected, the closed loop transfer function is now given by:

$$G_{CL} = \frac{Y(s)}{R(s)} = \frac{R(K_p s + K_i)}{RCs^2 + (RK_p+1)s + K_i R} \tag{4}$$

The system is held to design constraints such that the system overshoot must be less than 10% and the settling time must be less than 4 seconds in response to a unit step input, while having no steady state error. The controller selected should remove any steady state error, so now K_p and K_i must be chosen such that the design constraints are satisfied. The system will be designed around a chosen overshoot of 7.5% and a settling time of 3 seconds. The second order damping ratio and natural frequency that correspond to these values are $\zeta = 0.6362$ and $\omega_n = 2.0959$ [rad/s]. The transfer function for a general second order is given by:

$$D = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{5}$$

Comparing the coefficients of the denominator of equation 5 to the coefficients of the denominator of equation 4, yields the following two equations.

$$\omega_n^2 = \frac{K_i}{C} \tag{6}$$

$$2\zeta\omega_n = \frac{RK_p + 1}{RC} \tag{7}$$

Using values of R = 2 and C = 5 gives the control gain values of $K_p = 12.833$ and $K_i = 21.964$.

Part B

A simulink model was created to simulate the system dynamics in response to a unit step input. The layout of the simulink model is given below.

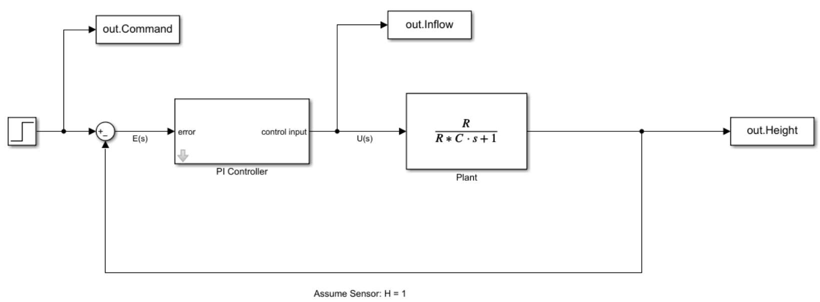


Figure 1: Single Tank Simulink Model

Running the model with the control gains gives the response shown in figure 2

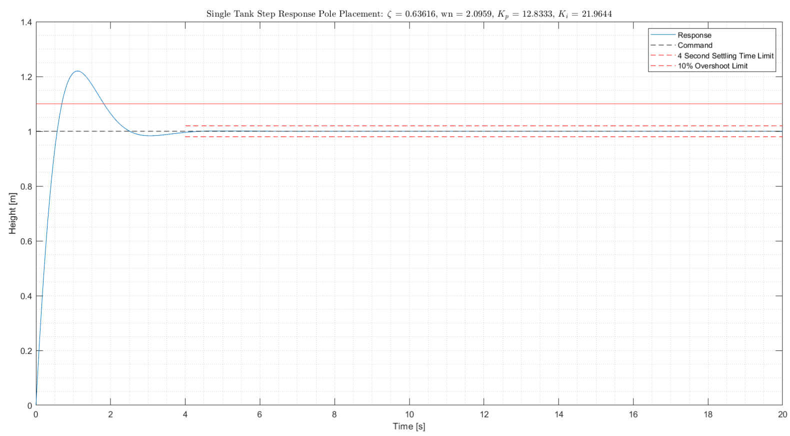


Figure 2: Tank Height with Original Tuning

From the response, it is observed that there is zero steady state error and the system settles within 4 seconds. However, the overshoot limit of 10% is exceeded. This is likely due to the effects of the zero that is the closed loop transfer function of equation 4. Therefore new control gains must be found that account for the differences in the numerators of the transfer functions shown in equation 4 and 5, so that the system satisfies all the requirements. After multiple iterations of tweaking the original control gains, a new tuning was found that satisfied all three requirements. The tuned control gains are K_p = 40.317 and K_i = 13.983. The system response is shown in figure 3. The system response to a unit step input has zero steady state error, a 2% settling time of 1.5 seconds, and an overshoot of 2.5%, thus successfully meeting all design constraints. The control input to the plant, the inflow, is shown in figure 4. The inflow spikes to 40 $[m^3/s]$ then settles to 0.5 $[m^3/s]$ after 1 second, when the system settles.

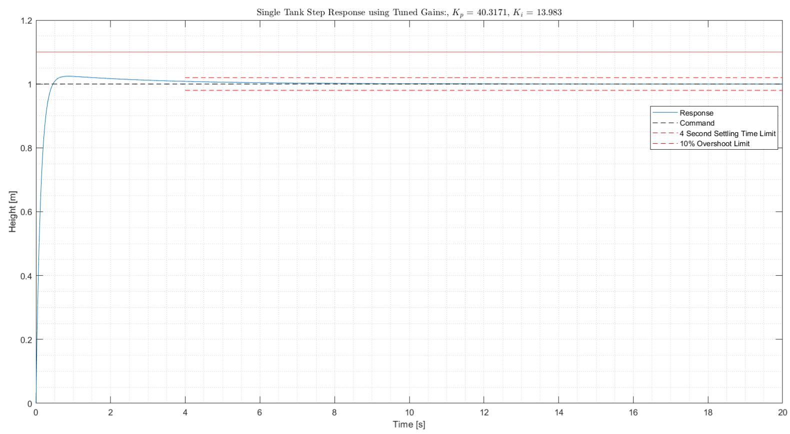


Figure 3: Tank Height with Final Tuning

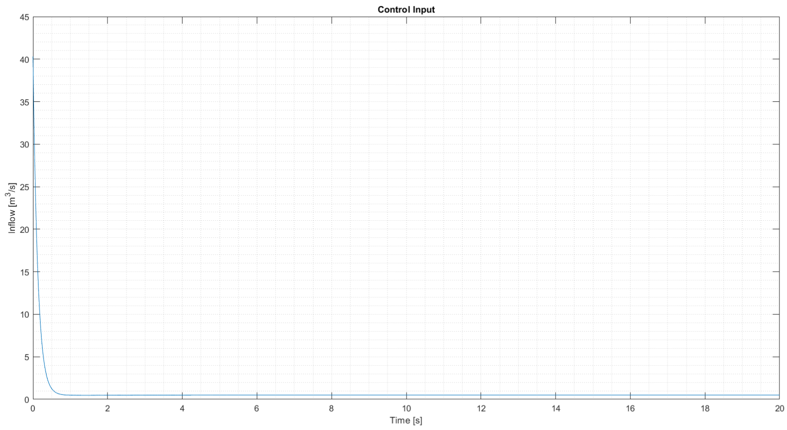


Figure 4: Inflow Control Input

PART C

A controller was developed to control the height of a tank in a two tank system. The transfer function relating the inflow to the outflow of the first tank is given by:

$$\frac{Q_{o1}(s)}{Q_{i1}(s)} = \frac{1}{RCs+1} \tag{8}$$

The height of the second tank is related to its inflow by the transfer function in equation 1. The height of the first tank can be found similarly. The inflow to the second tank is equal to the outflow of the first tank $Q_{o1}(s) = Q_{i2}(s)$. Therefore the transfer function relating the height of the second tank to the inflow of the first tank is given by:

$$\frac{H_2(s)}{Q_{i1}(s)} = \frac{Q_{o1}(s)}{Q_{i1}(s)}\frac{H_2(s)}{Q_{i2}(s)} = \frac{R}{R^2C^2s^2+2CRs+1} \tag{9}$$

A controller was to be designed such that there was no steady state error in a unit step response for the plant in equation 9. Looking at equation, it is clear that this is a type 0, as again there are no poles at the origin. Therefore, a PI controller in the form of equation 2 was again chosen. The Ziegler-Nichols method was used to determine the control gains for the controller. The controller K_i was set to 0 such that there was proportional control only. K_p was then increased until there was steady state oscillation for a unit step input. The gain at which this occurred was was called K_c and the period of the oscillation was called P_c. The critical gain value was found to be 175, and the period of oscillation was found to be 3.5 seconds. The Ziegler-Nichols step response is shown below.

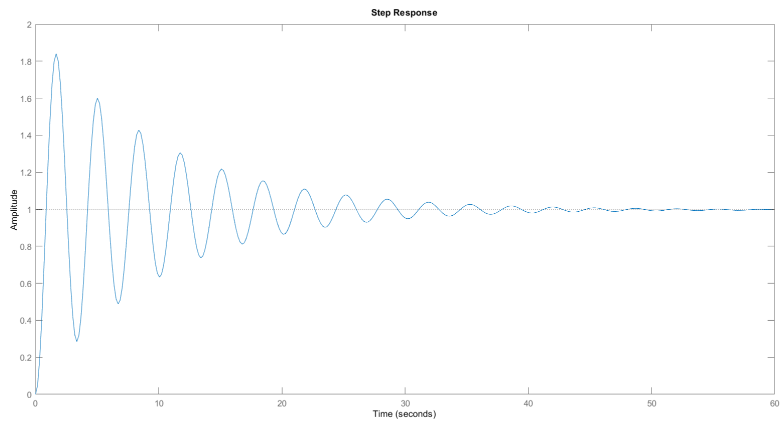


Figure 5: Ziegler-Nichols

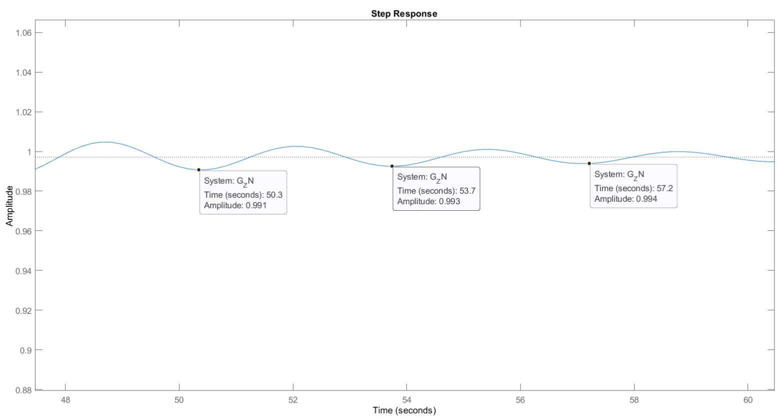


Figure 6: Ziegler-Nichols Zoomed In

The controller form assumed using the Ziegler-Nichols method is shown below.

$$G_c(s) = K_p(1 + \frac{1}{T_r s} + \frac{T_d s}{\tau_d s + 1}) = K_P + \frac{K_i}{s} + \frac{K_d s}{\tau_d s + 1} \tag{10}$$

For a PI only controller, the values for K_p and T_r can be found with the following relations (let $T_d = 0$).

$$K_p = 0.45 K_c \tag{11}$$

$$T_r = \frac{P_c}{1.2} \tag{12}$$

Using the above relations with the critical period and gain values discussed prior, gives the following step response.

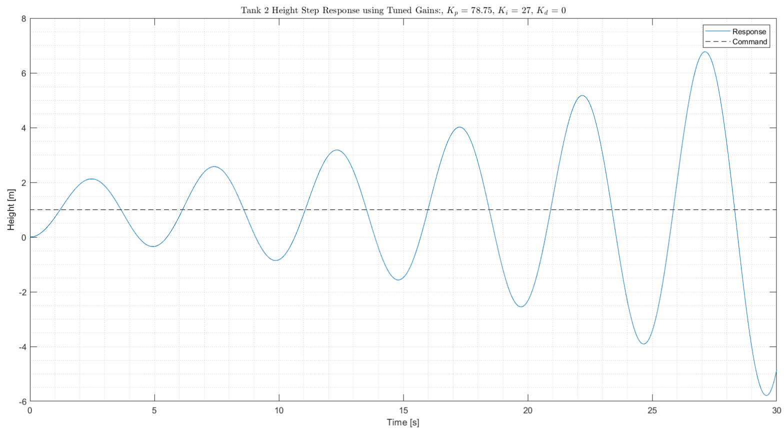


Figure 7: Unstable Tuning

From figure 7, it is clear that this tuning provides an unstable response. Inspection of the closed loop poles, shows that a pair of poles reside in the right hand plane. Decreasing the control gains causes the system to have a stable but sluggish and oscillatory response. To help reduce oscillation, the derivative term in equation 10 was set to a non-zero value. The new control gain values were calculated with the following equations.

$$K_p = 0.6K_c \tag{13}$$

$$T_r = 0.5P_c \tag{14}$$

$$T_d = \frac{P_c}{8} \tag{15}$$

$$\tau_d = 0.15T_d \tag{16}$$

$$K_i = \frac{K_p}{T_r} \tag{17}$$

$$K_d = K_P T_d \tag{18}$$

The calculated control gains have the values of $K_P = 105$, $K_i = 60$, and $K_d = 45.9$. It should be noted that the derivative portion of the PID controller in equation 10 is not a pure derivative, rather it is a filtered derivative that helps reduce high frequency noise. The time constant of this filtered derivative is $\tau_d = 0.066$. The height of tank 2 in response to an unit step input is shown below.

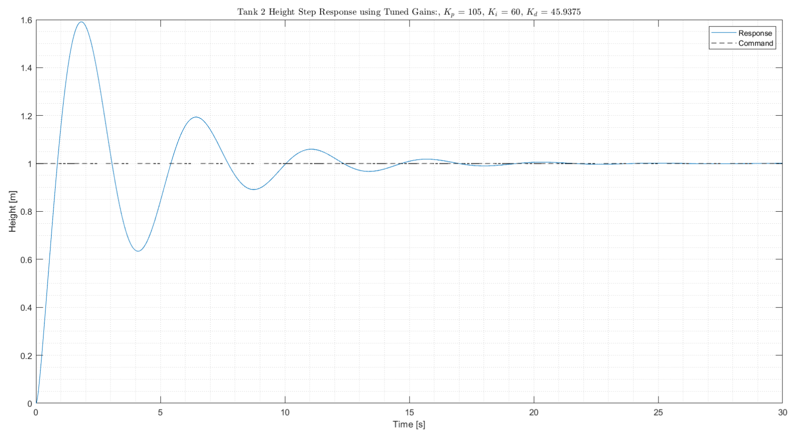


Figure 8: Tank 2 Height

From figure 8, it is clear that this tuning provides a stable response with zero steady state error. Thus satisfying the design constraint, and verifying the selection and tuning of the PID controller. The height of tank 1 is shown below, where it is observed that the height of tank 1 stabilizes when the height of tank 2 achieves steady state.

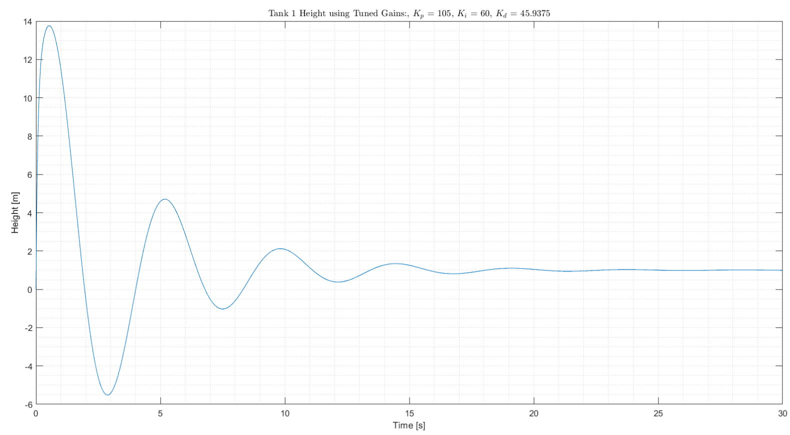


Figure 9: Tank 1 Height

APPENDIX

The layout of the two tank simulink model is shown below. It should be noted that the height of tank 1 is found using the transfer function in equation 1.

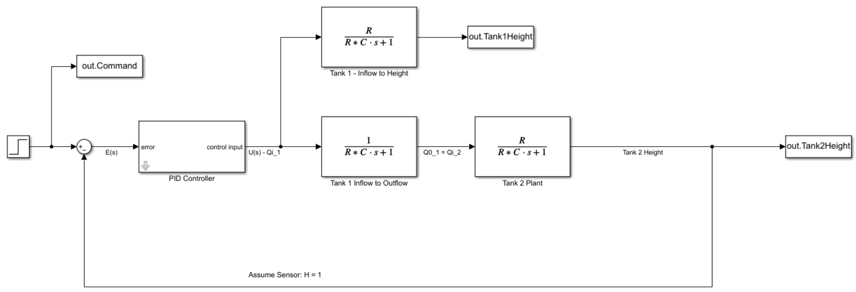


Figure 10: Two Tank Simulink Model

The PID controller layout is shown in the next figure.

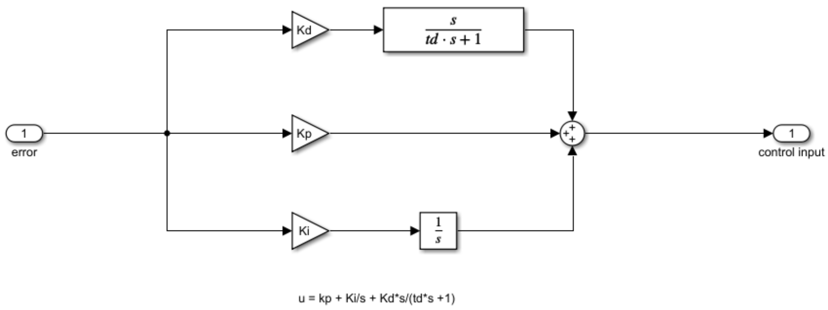


Figure 11: PID Controller Simulink Model

All relevant MATLAB code is shown on the next page.

Contents


```
clear
close all
clc
```

Part A

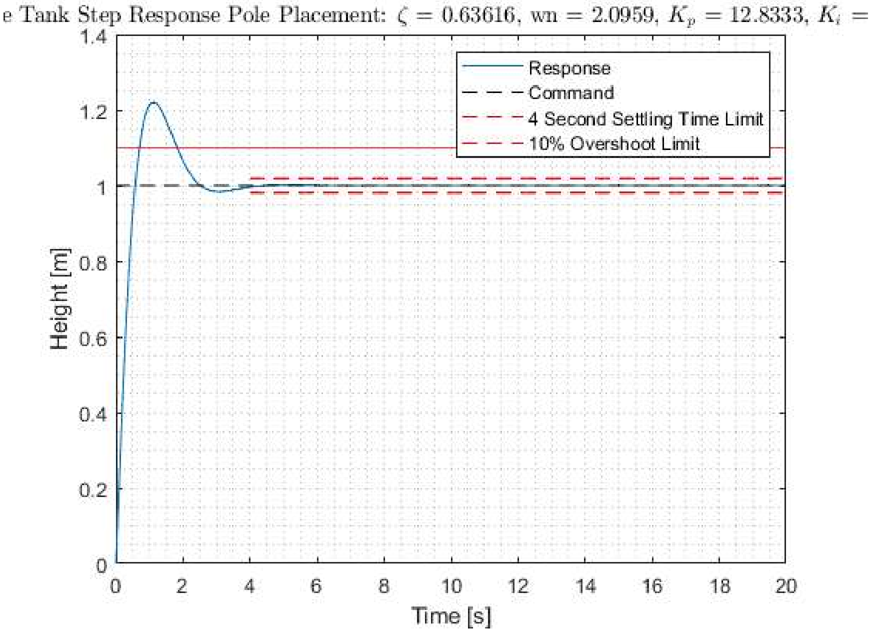
Parameters

```
R           = 2;
C           = 5;

% Target Response Parameters
ts          = 3;
Mp          = .075;

% 2nd order coefficients corresponding to target response parameters
zeta        = sqrt(log(Mp)^2 / (log(Mp)^2 +pi^2));
wn          = 4/(zeta*ts);

% Initial PI Control Gains from Pole Placement
Ki1          = C*wn^2;
Kp1          = ((2*zeta*wn)*R*C - 1)/R;

% Laplace Variable
s           = tf('s');

% Plant Transfer Function - Tank height to inlet flow
H_Qi          = R/(R*C*s + 1);

% PI Controller
C1          = Kp1 + Ki1/s;

% Open-Loop Transfer Function - Type 1
G_OL1       = H_Qi*C1;

% Closed Loop Transfer Function, assume H = 1
G_CL1       = minreal(G_OL1/(1 + G_OL1));
```

Pole Placement Gains

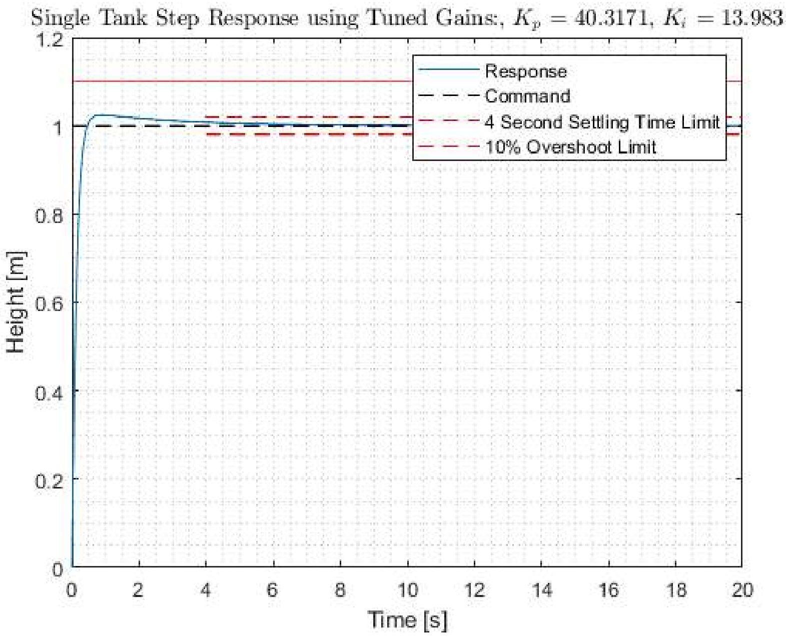
```
% Run Simulation
time        = 0:.01:20;
SingleTank  = sim('SingleTankSystem.slx',time);

% Design Requirments
Mp_Limit    = SingleTank.Height(end) + .1;
ts_lim_up   = SingleTank.Height(end) + .02;
ts_lim_low  = SingleTank.Height(end) - .02;

ind_ts      = find(time >= 4);
ts_limit    = [ts_lim_up*ones(1,length(time(ind_ts)));ts_lim_low*ones(1,length(time(ind_ts)))];

figure
plot(time,SingleTank.Height,time,SingleTank.Command,'--k',...
     time(ind_ts),ts_limit(1,:),'--r',time(ind_ts),ts_limit(2,:),'--r')
yline(Mp_Limit,'r')
xlabel('Time [s]')
legend('Response','Command','4 Second Settling Time Limit','10% Overshoot Limit')
grid minor
ylabel('Height [m]')
titlestr    = ['Single Tank Step Response Pole Placement: $\zeta$ = ', num2str(zeta),', wn = ',num2str(wn) ', $K_p$ = ', num2str(Kp1),...
```

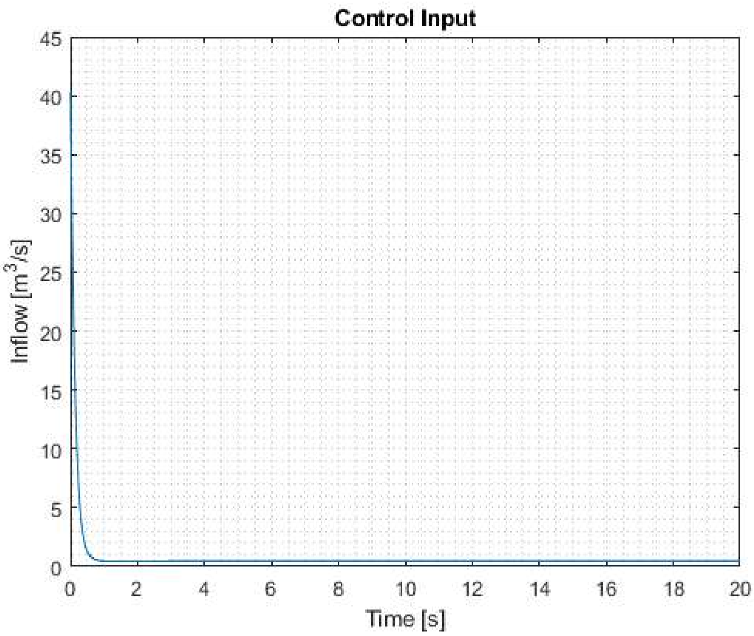
```
               ', $K_i$ = ',num2str(Ki1)];
title(titlestr,'Interpreter','latex')
```



Part B New Tuning

```
% Retune
Ki1         = Ki1/(pi/2);
Kp1         = Kp1*pi;

SingleTank  = sim('SingleTankSystem.slx',time);

Mp_Limit    = SingleTank.Height(end) + .1;
ts_lim_up   = SingleTank.Height(end) + .02;
ts_lim_low  = SingleTank.Height(end) - .02;

% Tank Height vs time
figure
plot(time,SingleTank.Height,time,SingleTank.Command,'--k',...
     time(ind_ts),ts_limit(1,:),'--r',time(ind_ts),ts_limit(2,:),'--r')
yline(Mp_Limit,'r')
xlabel('Time [s]')
legend('Response','Command','4 Second Settling Time Limit','10% Overshoot Limit')
grid minor
ylabel('Height [m]')
titlestr    = ['Single Tank Step Response using Tuned Gains:',', $K_p$ = ', num2str(Kp1),...
               ', $K_i$ = ',num2str(Ki1)];
title(titlestr,'Interpreter','latex')

% Control Input
figure
plot(time,SingleTank.Inflow)
xlabel('Time [s]')
ylabel('Inflow [m^3/s]')
grid minor
title('Control Input')
```





Part C - Multi Tank System

```
% Tank 1 inlet to outlet flow transfer function
Q01_Qi1     = 1/(R*C*s + 1);

% Tank 2 height to tank 1 outlet flow transfer function
H2_Q01      = R/(R*C*s + 1);

% Tank 2 height to tank 1 inlet flow transfer function
H2_Qi1      = H2_Q01*Q01_Qi1;
```

Ziegeler Nichols

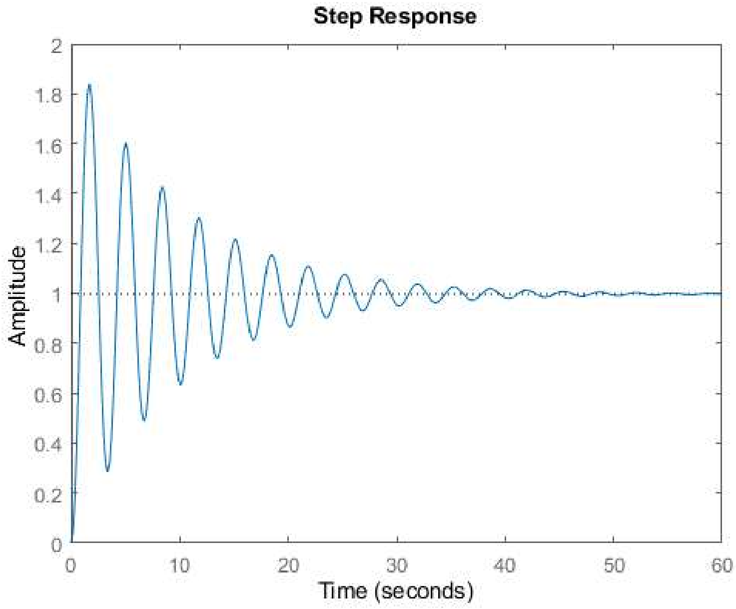
```
Kc          = 175;
CZN         = Kc ;
G_ZN        = H2_Qi1*CZN;
G_ZN        = minreal(G_ZN/(1 + G_ZN));

opt                 = stepDataOptions;
opt.StepAmplitude   = 1;
```

```
figure
step(G_ZN,opt)

time2        = (0:.01:30)';

Pc          = 3.5;
Tr          = Pc/2;
Td          = Pc/8;
taud        = .15*Td;
```

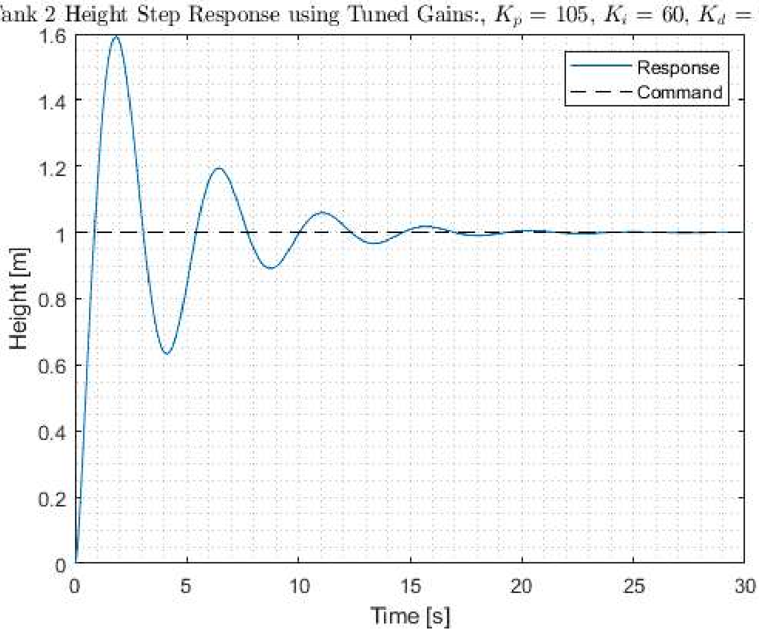
Tuned Controller

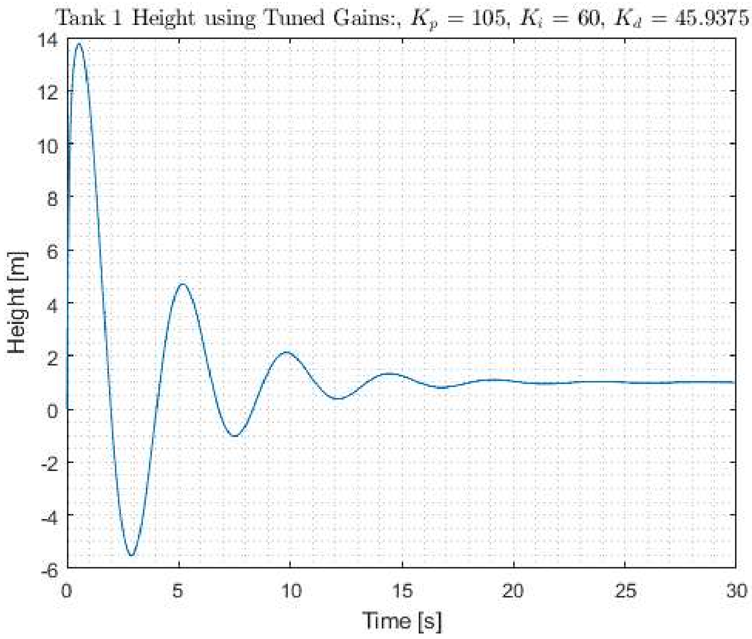
```
Kp2          = Kc*.6;
Ki2          = Kp2/Tr;
Kd2          = Kp2*Td;

% Kp2         = .45*Kc;
% Ki2         = Kp2/(Pc/1.2);
% Kd2         = 0;

C2           = Kp2 + Ki2/s + Kd2*s/(taud*s + 1);
G_OL2        = minreal(H2_Qi1*C2);
poles_OL     = pole(G_OL2)
G_CL2        = minreal(G_OL2/(1 + G_OL2));
poles_CL     = pole(G_CL2)

TwoTank      = sim('TwoTankSystem.slx',time2);

% Tank 2 Height vs time
figure
plot(time2,TwoTank.Tank2Height,time2,TwoTank.Command,'--k')
xlabel('Time [s]')
legend('Response','Command')
grid minor
ylabel('Height [m]')
titlestr    = ['Tank 2 Height Step Response using Tuned Gains:',', $K_p$ = ', num2str(Kp2),...
               ', $K_i$ = ',num2str(Ki2),  ', $K_d$ = ',num2str(Kd2),];
title(titlestr,'Interpreter','latex')

% Tank 1 Height vs time
figure
plot(time2,TwoTank.Tank1Height)
xlabel('Time [s]')
grid minor
ylabel('Height [m]')
titlestr    = ['Tank 1 Height using Tuned Gains:',', $K_p$ = ', num2str(Kp2),...
               ', $K_i$ = ',num2str(Ki2),  ', $K_d$ = ',num2str(Kd2),];
title(titlestr,'Interpreter','latex')
```

```
poles_OL =

         0
  -15.2381
   -0.1000
   -0.1000


poles_CL =

 -14.2518 + 0.0000i
  -0.2564 + 1.3561i
  -0.2564 - 1.3561i
  -0.6736 + 0.0000i
```



Tank 2 Height Step Response using Tuned Gains:, $K_p = 105$, $K_i = 60$, $K_d = 45.937$





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