#1)

$f(t) = t \cos t$

$$F(s) = \int_0^{\infty} f(t)\, e^{-st}\, dt \quad = \mathcal{L}[f(t)]$$

$$F(s) = \int_0^{\infty} t \cos t\, e^{-st}\, dt$$

Let $u = t$ $\quad dv = \cos t\, e^{-st}\, dt$ (Interoration by Parts)

$du = dt$ $\quad V = ?$ (Intersrat by parts)

Let $f = e^{-st}$, $dg = \cos t\, dt$

$df = -s e^{-st} dt$, $g = \sin t$

$$V = fg - \int g\, df = e^{-st} \sin t - \int \sin t (-s e^{-st})\, dt$$

$$V = e^{-st} \sin t + \int s e^{-st} \sin t\, dt$$ (Intergrate by parts)

$a = s e^{-st}$ $\qquad dB = \sin t\, dt$

$da = -s^2 e^{-st}\, dt$ $\qquad B = -\cos t$

$$\int s e^{-st} \sin t\, dt = aB - \int B\, da = -\cos t\, s e^{-st} - \int \cos t\, s^2 e^{-st}\, dt$$

Plug into V:

$$V = \int \cos t\, e^{-st}\, dt = e^{-st} \sin t - \cos t\, e^{-st} s - \int \cos t\, s^2 e^{-st}\, dt$$

$$\int \cos t e^{-st} + s^2 \cos t e^{-st}\, dt = e^{-st}(\sin t - s \cos t)$$

$$\int \cos t e^{-st}(1+s^2)\, dt = e^{-st}(\sin t - s\cos t)$$

$$\int \cos t\, e^{-st}\, dt = \frac{e^{-st}(\sin t - s\cos t)}{s^2+1} = V$$

$$F(s) = UV - \int v\,dU \Big|_0^\infty$$

$$F(s) = (t)\left(\frac{e^{-st}[\sin t - s\cos t]}{s^2+1}\right) - \int \frac{e^{-st}(\sin t - s\cos t)}{s^2+1}\,dt \Big|_0^\infty$$

$$-\int \frac{e^{-st}\sin t}{s^2+1}dt + \int \frac{e^{-st}s\cos t}{s^2+1}\,dt = -\int \frac{e^{-st}(\sin t - s\cos t)}{s^2+1}\,dt$$

$$\int -\frac{e^{-st}\sin t}{s^2+1}\,dt = UV - \int v\,dU$$

$$U = \frac{-e^{-st}}{s^2+1} \qquad dU = \frac{se^{-st}}{s^2+1}\,dt \qquad \text{(Intergration by parts)}$$

$$dv = \sin t\,dt \qquad V = -\cos t$$

$$\int -\frac{e^{-st}\sin t\,dt}{s^2+1} = \frac{\cos t\, e^{-st}}{s^2+1} + \int \frac{\cos t\; s e^{-st}}{s^2+1}\,dt$$

Let $f = \frac{se^{-st}}{s^2+1}$ $\quad df = \frac{-s^2e^{-st}}{s^2+1}dt$ $\quad dg = \cos t\,dt$

$g = \sin t$

$$\int -\frac{e^{-st}\sin t\,dt}{s^2+1} = \frac{\cos t\, e^{-st}}{s^2+1} + \frac{se^{-st}\sin t}{s^2+1} + \int \frac{\sin t\; s^2 e^{-st}}{s^2+1}\,dt$$

$$\Rightarrow \int \frac{-e^{-st}\sin t - s^2 \sin t\, e^{-st}}{s^2+1}\,dt = \int \frac{-e^{-st}\sin t\,(1+s^2)}{s^2+1}\,dt$$

$$= \frac{e^{-st}(\cos t + s\sin t)}{s^2+1} = s^2+1 \int \frac{-e^{-st}\sin t}{s^2+1}\,dt$$

$$\Rightarrow \int \frac{-e^{-st}\sin t\,dt}{s^2+1} = \frac{e^{-st}(\cos t + s\sin t)}{(s^2+1)^2}$$

Previously it was shown: $\int \cos t\, e^{-st} dt = \dfrac{e^{-st}(\sin t - s\cos t)}{s^2+1}$

$\therefore \int \dfrac{e^{-st} s \cos t}{s^2+1} dt = \left(\dfrac{s}{s^2+1}\right)\left(\dfrac{e^{-st}[\sin t - s\cos t]}{s^2+1}\right)$

$= \dfrac{se^{-st}(\sin t - s\cos t)}{(s^2+1)^2}$

$F(s) = \dfrac{t e^{-st}(\sin t - s\cos t)}{s^2+1} + \dfrac{e^{-st}(\cos t + s\sin t)}{(s^2+1)^2} + \dfrac{se^{-st}(\sin t - s\cos t)}{(s^2+1)^2}\Bigg|_0^\infty$

$\lim_{t\to\infty}(e^{-st}) = 0 \therefore e^{-\infty} = 0$

$F(s) = [0 + 0 + 0] - [0 + \dfrac{1(1)}{(s^2+1)^2} + \dfrac{s(1)(-s)}{(s^2+1)^2}]$

$F(s) = \dfrac{-1+s^2}{(s^2+1)^2}$

$\boxed{F(s) = \dfrac{s^2-1}{(s^2+1)^2}} = \mathcal{L}[t\cos t]$

$f(t) = te^{-t} + t^3$

$\mathcal{L}[f(t)] = \int_0^\infty F(t) e^{-st} dt = \int_0^\infty te^{-t}e^{-st} + t^3 e^{-st} dt$

$\int_0^\infty te^{-t}e^{-st} dt + \int_0^\infty t^3 e^{-st} dt$

$\int_0^\infty te^{-t(s+1)} dt$ (Integrate by parts)

Let $u = t$ $\quad dv = e^{-t(s+1)} dt$

$du = dt$ $\quad V = \frac{-1}{(s+1)} e^{-t(s+1)}$

$uv - \int v du = \frac{-te^{-t(s+1)}}{s+1} + \int \frac{e^{-t(s+1)}}{(s+1)} dt$

$= \frac{-te^{-t(s+1)}}{s+1} + \left(\frac{1}{s+1}\right)\left(\frac{-1}{s+1} e^{-t(s+1)}\right)$

$= \frac{-te^{-t(s+1)}}{s+1} - \frac{e^{-t(s+1)}}{(s+1)^2} = -e^{-(s+1)t}\left[\frac{t}{s+1} + \frac{1}{(s+1)^2}\right]$

$= -e^{-(s+1)t}\left[\frac{t(s+1)+1}{(s+1)^2}\right] = \int te^{-t(s+1)} dt$

Let $f = t^3$ $\quad dv = e^{-st}$

$df = 3t^2 dt$ $\quad V = \frac{-1}{s} e^{-st}$

$\int t^3 e^{-st} dt = (t^3)\left(\frac{-1}{s} e^{-st}\right) + \int \frac{e^{-st}}{s} (3t^2) dt$

Let $a = 3t^2$ $\quad db = \frac{e^{-st}}{s} dt$

$da = 6t\ dt$ $\quad b = \frac{-1}{s^2} e^{-st}$

$ab - \int b\,da = (3t^2)(-\frac{1}{s^2}e^{-st}) + \int \frac{e^{-st}}{s^2} 6t\,dt$

Let $c = 6t$ $\quad de = \frac{e^{-st}}{s^2}dt$

$dc = 6\,dt$ $\quad e = \frac{-1}{s^3}e^{-st}$

$ce - \int e\,dc = (6t)(\frac{-1}{s^3}e^{-st}) + \int \frac{1}{s^3}e^{-st}(6dt)$

$= \frac{-6t}{s^3}e^{-st} + \frac{6}{s^3}\int e^{-st} = \frac{-6t}{s^3}e^{-st} - \frac{6}{s^4}e^{-st}$

$\int t^3 e^{-st}dt = \frac{-t^3}{s}e^{-st} - \frac{3t^2}{s^2}e^{-st} - \frac{6t}{s^3}e^{-st} - \frac{6}{s^4}e^{-st}$

$= e^{-st}(\frac{-t^3}{s} - \frac{3t^2}{s^2} - \frac{6t}{s^3} - \frac{6}{s^4})$

$= -\frac{e^{-st}}{s^4}(s^3t^3 + 3st^2 + 6st + 6)$

$\int_0^\infty f(t)e^{-st}dt = -e^{-(s+1)t}[\frac{t(s+1)+1}{(s+1)^2}] - \frac{e^{-st}}{s^4}(s^3t^3 + 3st^2 + 6st + 6)\Big|_0^\infty$

$e^{-\infty} = 0$, $e^0 = 1$

$0 - [-1(\frac{1}{(s+1)^2}) - \frac{1}{s^4}(6)]$

$\boxed{F(s) = \frac{1}{(s+1)^2} + \frac{6}{s^4}} = \mathcal{L}[te^{-t} + t^3]$

$f(t) = te^{-t}\sin t$

$$\int_0^\infty te^{-t}\sin t\, e^{-st}\,dt = \int_0^\infty te^{-(s+1)t}\sin t\,dt$$

Let $u = t$ $\qquad dv = e^{-(s+1)t}\sin t\,dt$

$du = dt$

Let $f = e^{-(s+1)t}$ $\qquad dg = \sin t\,dt$

$df = -(s+1)e^{-(s+1)t}dt \qquad g = -\cos t$

$$\int dv = \int e^{-(s+1)t}\sin t\,dt = \underline{(e^{-(s+1)t})(-\cos t)} - \int(\cos t)(s+1)e^{-(s+1)t}dt$$

Let $a = (s+1)e^{-(s+1)t}$ $\qquad db = \cos t\,dt$

$da = -(s+1)^2e^{-(s+1)t}dt \qquad b = \sin t$

$$ab - \int b\,da = \int \cos t\,(s+1)e^{-(s+1)t}dt \Rightarrow$$

$$(s+1)e^{-(s+1)t}(\sin t) + \int(s+1)^2e^{-(s+1)t}\sin t\,dt$$

$$\int e^{-(s+1)t}\sin t\,dt = (e^{-(s+1)t})(-\cos t) - (s+1)e^{-(s+1)t}\sin t - (s+1)^2\int e^{-(s+1)t}\sin t\,dt$$

$$\int e^{-(s+1)t}\sin t(1+(s+1)^2)\,dt = -e^{-(s+1)t}(\cos t + (s+1)\sin t)$$

$$\int e^{-(s+1)t}\sin t\,dt = \frac{-e^{-(s+1)t}[\cos t + (s+1)\sin t]}{s^2+2s+2}$$

$$\int te^{-(s+1)t}\sin t\,dt = \frac{(t)(-e^{-(s+1)t}[\cos t + (s+1)\sin t])}{s^2+2s+2} + \int\frac{e^{-(s+1)t}[\cos t + (s+1)\sin t]\,dt}{s^2+2s+2}$$

$$\int \frac{e^{-(s+1)t} \cos t \, dt}{s^2+2s+2} + \int \frac{e^{-(s+1)t} (s+1) \sin t \, dt}{s^2+2s+2}$$

It was shown earlier that:

$$\int e^{-(s+1)t} \sin t \, dt = \frac{-e^{-(s+1)t} [\cos t + (s+1) \sin t]}{s^2+2s+2}$$

$$\therefore \int \frac{e^{-(s+1)t} (s+1) \sin t \, dt}{s^2+2s+2} = \frac{-e^{-(s+1)t} (s+1) [\cos t + (s+1) \sin t]}{(s^2+2s+2)^2}$$

$U = e^{-(s+1)t}$ $\qquad dV = \cos t \, dt$

$dU = -(s+1) e^{-(s+1)t} dt$ $\qquad V = \sin t$

$$(\sin t)(e^{-(s+1)t}) + \int (\sin t)(s+1) e^{-(s+1)t} dt$$

$\therefore$

$$\int \frac{e^{-(s+1)} \cos t \, dt}{s^2+2s+2} = \left(\frac{1}{s^2+2s+2}\right) \left[(\sin t)(e^{-(s+1)t}) + (s+1)\left(\frac{-e^{-(s+1)t} [\cos t + (s+1) \sin t]}{s^2+2s+2}\right)\right]$$

$$= \frac{e^{-(s+1)t}}{(s^2+2s+2)^2} \left[(s^2+2s+2)(\sin t) - (s+1) \cos t - (s+1)^2 \sin t \right]$$

$$= \frac{e^{-(s+1)t}}{(s^2+2s+2)^2} \left[\sin t - (s+1) \cos t \right]$$

$$\frac{e^{-(s+1)t}}{(s^2+2s+2)^2} [\sin t - (s+1) \cos t] - \frac{e^{-(s+1)t}}{(s^2+2s+2)^2} [(s+1) \cos t + (s+1)^2 \sin t]$$

Scanned with CamScanner

$$= \frac{e^{-(s+1)t}}{(s^2+2s+2)^2}\left[(s^2+2s+2)\sin t \quad - 2(s+1)\cos t\right]$$

$$\int_0^\infty t e^{-(s+1)t}\sin t\,dt = \frac{e^{-(s+1)t}}{(s^2+2s+2)^2}\Big[(-t)(s^2+2s+2)[\cos t + (s+1)\sin t]$$

$$+ (s^2+2s+2)\sin t \quad - 2(s+1)\cos t\Big]\Big|_0^\infty$$

$$= 0 - \left[\frac{1}{(s^2+2s+2)^2}\left(0 + 0 - 2s - 2\right)\right]$$

$$\boxed{F(s) = \frac{2(s+1)}{(s^2+2s+2)^2}} = \mathcal{L}\left[t e^{-t}\sin t\right]$$

Scanned with CamScanner

#2)

$$y = \frac{y_2 - y_1}{t_2 - t_1} t + b$$

$$y = \frac{3-1}{2-0} t + b = t + b$$

$$y(0) = 1 = 0 + b$$

$$y = t + 1 \quad (0 \leq t \leq 2)$$

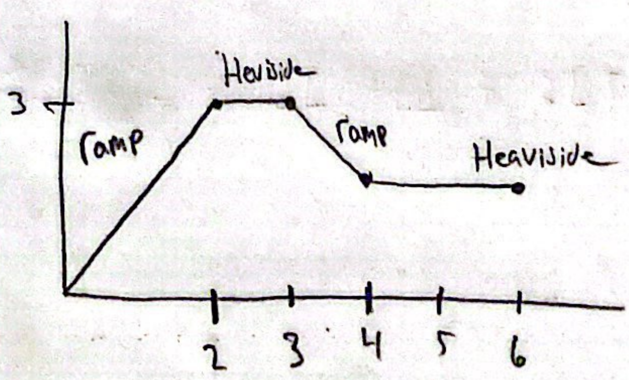
$$y = \frac{2-3}{4-3} t + b$$

$$y = -t + b$$

$$2 = -4 + b \qquad b = 6$$

$$y = -t + 6 \quad (3 \leq t \leq 4)$$

$$f(t) = \begin{cases} 0, & t < 0 \\ t+1, & 0 \leq t < 2 \\ 3, & 2 \leq t < 3 \\ -t+6, & 3 \leq t < 4 \\ 2, & t \geq 4 \end{cases}$$



$$F(s) = \int_0^\infty f(t) e^{-st} dt$$

$$F(s) = \underbrace{\int_0^2 (t+1) e^{-st} dt}_{A} + \underbrace{\int_2^3 3e^{-st} dt}_{B} + \underbrace{\int_3^4 (6-t) e^{-st} dt}_{C} + \underbrace{\int_4^\infty 2e^{-st} dt}_{D}$$

$$\int_0^2 te^{-st} dt + \int_0^2 e^{-st} dt = \int_0^2 (t+1) e^{-st} dt$$

Let $u = t \quad dv = e^{-st} dt \qquad uv - \int v du$

$du = dt \quad v = \frac{-e^{-st}}{s}$

$$(t)\left(-\frac{e^{-st}}{s}\right) + \int \frac{e^{-st}}{s} dt = \frac{-te^{-st}}{s} - \frac{1}{s^2} e^{-st} = \int te^{-st} dt$$

$$\int_0^2 (t+1) e^{-st} dt = e^{-st}\left[\frac{-t}{s} - \frac{1}{s^2} - \frac{1}{s}\right] \Big|_0^2$$

$$e^{-2s}\left[\frac{-2}{s} - \frac{1}{s^2} - \frac{1}{s}\right] - e^0\left[-\frac{1}{s^2} - \frac{1}{s}\right]$$

$$e^{-2s}\left[\frac{-2s - 1 - s}{s^2}\right] + \frac{1}{s^2} + \frac{s}{s^2} = \underbrace{\frac{s+1 - e^{-2s}(3s+1)}{s^2}}_{A} = \int_0^2 (t+1) e^{-st} dt$$

$$\int_2^3 3e^{-st} dt = \frac{-3e^{-st}}{s} \Big|_2^3 = \underbrace{\frac{-3}{s}\left[e^{-3s} - e^{-2s}\right]}_{B} = \frac{-3}{s^2}\left[se^{-3s} - se^{-2s}\right]$$

$$\int_3^4 (6-t) e^{-st} dt = \int 6e^{-st} dt - \int te^{-st} dt$$

$$= \frac{-6e^{-st}}{s} + \frac{te^{-st}}{s} + \frac{1}{s^2} e^{-st} = \frac{e^{-st}}{s^2}\left[-6s + ts + 1\right] \Big|_3^4$$

$$= \underbrace{\frac{e^{-4s}}{s^2}\left[-6s + 4s + 1\right] - \frac{e^{-3s}}{s^2}\left[-6s + 3s + 1\right]}_{C} = \frac{e^{-4s}}{s^2}\left[-2s + 1\right] - \frac{e^{-3s}}{s^2}\left[-3s + 1\right]$$

$$A+B+C = \frac{s+1}{s^2} - \frac{3se^{-2s}}{s^2} - \frac{e^{-2s}}{s^2} + \frac{3se^{-2s}}{s^2} - \frac{3se^{-3s}}{s^2}$$
$$+ \frac{3se^{-3s}}{s^2} - \frac{e^{-3s}}{s^2} - \frac{2se^{-4s}}{s^2} + \frac{e^{-4s}}{s^2}$$

$$A+B+C = \frac{1}{s^2}\left[s+1 - e^{-2s} - e^{-3s} + e^{-4s} - \frac{2se^{-4s}}{s^2} \right]$$

$$\int_4^{\infty} 2e^{-st} = \frac{-2}{s} e^{-st} \Big]_4^{\infty} = 0 + \frac{2}{s} e^{-4s} = \underbrace{\frac{2s}{s^2} e^{-4s}}_{D}$$

$$A+B+C+D = \frac{1}{s^2}\left[s+1 - e^{-2s} - e^{-3s} + e^{-4s} \right]$$

$$\therefore \boxed{F(s) = \mathcal{L}[f(t)] = \frac{1}{s^2}\left[s+1 - e^{-2s} - e^{-3s} + e^{-4s} \right]}$$

#3)

$$F(t) = \begin{cases} 1 & 0 \leq t < 2 \\ t^2 - 4t + 3 & t \geq 2 \end{cases}$$

$$F(s) = \int_0^\infty f(t)\, e^{-st}\, dt$$

$$= \int_0^2 e^{-st}\, dt + \int_2^\infty (t^2 - 4t + 3) e^{-st}\, dt$$

$$\int_0^2 e^{-st} dt = -\frac{1}{s} e^{-st} \Big]_0^2 = \frac{-1}{s}[e^{-2s} - 1]$$

$$\int t^2 e^{-st} dt = uv - \int v\, du \qquad \text{Let } \begin{matrix} u = t^2 \\ du = 2t\,dt \end{matrix} \qquad \begin{matrix} dv = e^{-st} dt \\ v = -\frac{1}{s} e^{-st} \end{matrix}$$

$$= (t^2)(-\tfrac{1}{s} e^{-st}) + \frac{2}{s} \int t e^{-st} dt$$

$$\int t e^{-st} = -e^{-st}\left[\frac{t}{s} + \frac{1}{s^2}\right]$$

$$\therefore \int t^2 e^{-st} dt = \frac{-t^2}{s} e^{-st} - \frac{2}{s} e^{-st}\left[\frac{t}{s} + \frac{1}{s^2}\right]$$

$$= \frac{e^{-st}}{s^3}\left[-2st - 2 - t^2 s^2\right]$$

$$\int -4t e^{-st} dt = 4e^{-st}\left[\frac{t}{s} + \frac{1}{s^2}\right]$$

$$\int 3e^{-st} dt = -\frac{3}{s} e^{-st}$$

$$\int_2^\infty (t^2 - 4t + 3) e^{-st}\, dt = e^{-st}\left[-\frac{2t}{s^2} - \frac{2}{s^3} - \frac{t^2}{s} + \frac{4t}{s} + \frac{4}{s^2} - \frac{3}{s}\right]\Bigg|_2^\infty$$

$e^{-\infty} = 0$

$\therefore$

$$\int_2^{\infty} (t^2 - 4t + 3) e^{-st} \, dt = 0 - e^{-2s}\left[-\frac{4}{s^2} - \frac{2}{s^3} - \frac{4}{s} + \frac{8}{s} + \frac{4}{s^2} - \frac{3}{s} \right]$$

$$= -e^{-2s}\left[\frac{-2}{s^3} + \frac{1}{s} \right]$$

$$F(s) = -\frac{1}{s}\left[e^{-2s} - 1 \right] - e^{-2s}\left[\frac{-2}{s^3} + \frac{1}{s} \right]$$

$$\boxed{F(s) = \frac{2e^{-2s}}{s^3} - \frac{2}{s} e^{-2s} + \frac{1}{s}} = \mathcal{L}[f(t)]$$

#4) $\ddot{x} + \dot{x} + 2x = e^{-t}$ $x(0) = 2$, $\dot{x}(0) = 0$

$\mathcal{L}[\ddot{x} + \dot{x} + 2x] = [s^2 X - s x(0) - \dot{x}(0)] + [s X - x(0)] + 2X$

$= (s^2 + s + 2) X(s) - s x(0) - \dot{x}(0) - x(0)$

$= (s^2 + s + 2) X(s) - 2s - 0 - 2 = (s^2 + s + 2) X - 2(s+1)$

$\mathcal{L}[e^{-at}] = \frac{1}{s+a}$ $\therefore$ $\mathcal{L}[e^{-t}] = \frac{1}{s+1}$

$\mathcal{L}[\ddot{x} + \dot{x} + 2x = e^{-t}]$ with $x(0) = 2$ & $\dot{x}(0) = 0$ $\Rightarrow$

$(s^2 + s + 2) X(s) - 2(s+1) = \frac{1}{s+1}$

$X(s)[s^2 + s + 2] = \frac{1}{s+1} + 2(s+1) = \frac{1 + 2(s+1)^2}{s+1}$

$X(s) = \frac{1 + 2(s+1)^2}{(s+1)(s^2+s+2)}$

$X(s) = \frac{1 + 2(s+1)^2}{(s+1)(s^2+s+2)} = \frac{a_1}{s+1} + \frac{a_2 s + a_3}{s^2+s+2}$

a_1: $\frac{1 + 2(s+1)^2}{s^2+s+2} = a_1 + \left(\frac{a_2 s + a_3}{s^2+s+2}\right)(s+1) \Big|_{s=-1}$

$\frac{1}{1-1+2} = \underline{a_1 = \frac{1}{2}}$

a_2: $1 + 2(s+1)^2 = a_1(s^2+s+2) + a_2 s(s+1) + a_3(s+1)$

$2s^2 + 4s + 3 = \frac{s^2}{2} + \frac{s}{2} + 1 + a_2 s^2 + a_2 s + a_3(s+1)$

$2s^2 + 4s + 3 = (\frac{1}{2} + a_2)s^2 + (\frac{1}{2} + a_2 + a_3) + (a_3 + 1)$

$2 = \frac{1}{2} + a_2 \qquad a_2 = \frac{3}{2}$

$4 = a_2 + a_3 + \frac{1}{2} \qquad a_3 = 4 - \frac{3}{2} - \frac{1}{2} = 2$

$$X(s) = \frac{2s^2+4s+3}{(s+1)(s^2+s+2)} = \frac{1}{2(s+1)} + \frac{\frac{3}{2}s+2}{s^2+s+2}$$

$$X(s) = \frac{1}{2(s+1)} + \frac{\frac{3}{2}(s+\frac{1}{2}) + \frac{5}{4}}{(s+\frac{1}{2})^2 + \frac{7}{4}}$$

$$X(s) = \frac{1}{2(s+1)} + \frac{3}{2}\left(\frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+\frac{7}{4}}\right) + \frac{\frac{5}{4}}{(s+\frac{1}{2})^2+\frac{7}{4}}$$

$\omega^2 = \frac{7}{4} \Rightarrow \omega = \frac{\sqrt{7}}{2} \Rightarrow (\gamma)\left(\frac{\sqrt{7}}{2}\right) = \frac{5}{4} \Rightarrow \gamma = \frac{10}{4\sqrt{7}}$

$$X(s) = \frac{1}{2(s+1)} + \frac{3}{2}\left(\frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+\frac{7}{4}}\right) + \frac{10}{4\sqrt{7}}\left(\frac{(\frac{\sqrt{7}}{2})}{(s+\frac{1}{2})^2+\frac{7}{4}}\right)$$

$a=1$

$\mathcal{L}\left[\frac{1}{2}\left(\frac{1}{s+1}\right)\right] = \frac{1}{2}e^{-t}$

$a = \frac{1}{2},\ \omega = \sqrt{\frac{7}{4}} = \frac{\sqrt{7}}{2}$

$\mathcal{L}\left[\frac{3}{2}\left(\frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+\frac{7}{4}}\right)\right] = \frac{3}{2}e^{-t/2}\cos\left(\frac{\sqrt{7}}{2}t\right)$

$\mathcal{L}\left[\frac{10}{4\sqrt{7}}\left(\frac{\frac{\sqrt{7}}{2}}{(s+\frac{1}{2})^2+\frac{7}{4}}\right)\right] = \frac{10}{4\sqrt{7}}e^{-t/2}\sin\left(\frac{\sqrt{7}}{2}t\right)$

$$\boxed{x(t) = \frac{e^{-t}}{2} + \frac{3}{2}e^{-t/2}\cos\left(\frac{\sqrt{7}}{2}t\right) + \frac{10}{4\sqrt{7}}e^{-t/2}\sin\left(\frac{\sqrt{7}}{2}t\right)}$$