Validating the Observer-Based Feedback Controller Design

- **Objective**: Validate the combined observer-controller compensator design to control one-link manipulator controlled by a DC motor via a gear
- Design steps:
  1. Construct simulation and design models
  2. Construct stabilizing state-feedback using the design model
  3. Construct observer using the design model
  4. Combine the observer and controller into the combined observer-controller compensator
  5. Test the compensator on the non-linear simulation model

One-link robot state-space model

- Preparing to construct a third-order state-space model of the one-link robot
- Select the following state variables:

$$x_1 = \theta_p, \quad x_2 = \frac{d\theta_p}{dt} = \omega_p, \quad x_3 = i_a$$

- The model in state-space format

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{g}{l} \sin x_1 + \frac{NK_m}{ml^2} x_3 \\ -\frac{K_b N}{L_a} x_2 - \frac{R_a}{L_a} x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_a} \end{bmatrix} u$$

One-link robot state-space model simulation model

- Reasonable parameters of the robot are: $l = 1\,\text{m}$, $m = 1\,\text{kg}$, $N = 10$, $K_m = 0.1\,\text{Nm/A}$, $K_b = 0.1\,\text{Vs/rad}$, $R_a = 1\,\Omega$, $L_a = 100\,\text{mH}$
- With the above parameters the robot model takes the form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ 9.8 \sin x_1 + x_3 \\ -10\,x_2 - 10\,x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} u$$

- Assume the output, $y = x_1$

Design model

- Target equilibrium state of the robot

$$\boldsymbol{x}_e = \begin{bmatrix} 30\pi/180 \\ 0 \\ 0 \end{bmatrix}$$

- Compute the resulting $u_e = -4.9$
- The linearized model about the operating pair $\begin{bmatrix} \boldsymbol{x}_e^\top & u_e \end{bmatrix}^\top$ is

$$\frac{d}{dt}\delta\boldsymbol{x} = \boldsymbol{A}\delta\boldsymbol{x} + \boldsymbol{b}\delta u$$

where

$$\delta\boldsymbol{x} = \boldsymbol{x} - \boldsymbol{x}_e, \quad \delta u = u - u_e$$

and

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 \\ 8.487 & 0 & 1 \\ 0 & -10 & -10 \end{bmatrix} \quad \text{and} \quad \boldsymbol{b} = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

State-feedback control law

- Design a state feedback control law $\delta u = -\boldsymbol{k}\,\delta\boldsymbol{x}$ such that the closed-loop poles are located at $-2, -2 \pm j$
- Can use MATLAB's functions `acker` or `place`
- The resulting controller's gain

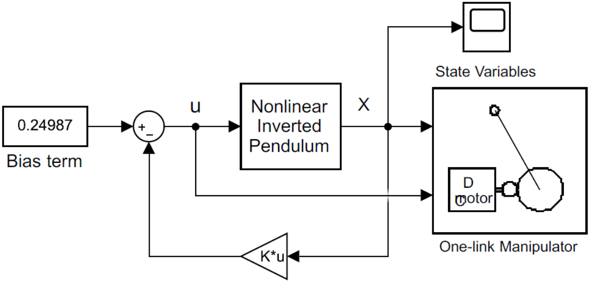
$$\boldsymbol{k} = \left[\begin{array}{ccc} 6.0922 & 1.1487 & -0.4000 \end{array}\right]$$

- Represent the control law $\delta u = -\boldsymbol{k}\,\delta\boldsymbol{x}$ as

$$u = -\boldsymbol{k}\boldsymbol{x} + \boldsymbol{k}\boldsymbol{x}_e + u_e = -\boldsymbol{k}\boldsymbol{x} + 0.24987$$

- Simulation—the initial state $\boldsymbol{x}(0) = \left[\begin{array}{ccc} -\pi & 0 & 0 \end{array}\right]^\top$

Simulink animation



K = [6.0922   1.1487 -0.4]

Double click on 'Nonlinear Inverted Pendulum' block
to change initial conditions.

If the animation if too fast or too slow, double click on
'Inverted Pendulum Animation' block
to change the sampling time of animation.

Combined observer-controller compensator

- Design the state observer using the linearized model
- The observer's poles are to be located at $-5, -5 \pm j$
- The observer's gain vector is

$$\boldsymbol{l} = \begin{bmatrix} 5 \\ 24.487 \\ -80 \end{bmatrix}$$

- The asymptotic state observer

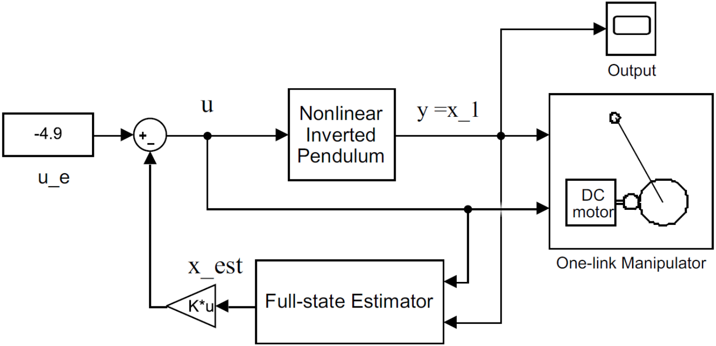
$$\dot{\boldsymbol{z}} = (\boldsymbol{A} - \boldsymbol{l}\boldsymbol{c})\boldsymbol{z} + \boldsymbol{b}\delta u + \boldsymbol{l}\delta y$$

  where $y = x_1$ and $\delta y = y - x_{1e}$

- The combined observer-controller compensator

$$\begin{cases} u &= u_e - \boldsymbol{k}\boldsymbol{z} \\ \dot{\boldsymbol{z}} &= (\boldsymbol{A} - \boldsymbol{l}\boldsymbol{c})\boldsymbol{z} + \boldsymbol{b}\left(u - u_{1e}\right) + \boldsymbol{l}\left(y - x_{1e}\right) \end{cases}$$

Simulink animation



Double click on 'Nonlinear Inverted Pendulum' block to change initial conditions.

k = [6.0922 1.1487 -0.4]

l = [5 24.487 -80]'