ECE 602: LUMPED LINEAR SYSTEMS

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State-Space Models of Lumped Systems

State-Space Model of General Lumped Systems

State-space model of a lumped system $\mathcal{N}$ with state $x \in \mathbb{R}^n$:

- Continuous-time case:

$$\begin{cases} \frac{dx}{dt} = f\left(x(t), u(t), t\right) \\ y(t) = g\left(x(t), u(t), t\right) \end{cases}, \quad t \in \mathbb{R}$$

  where f and g are arbitrary (in general nonlinear) functions

- Discrete-time case:

$$\begin{cases} x[k+1] = f\left(x[k], u[k], k\right) \\ y[k] = g\left(x[k], u[k], k\right) \end{cases}, \quad k \in \mathbb{Z}$$

State-Space Model of CT Lumped Linear Systems

$\mathcal{N}$: lumped linear system with state $x \in \mathbb{R}^n$, input $u \in \mathbb{R}^m$, output $y \in \mathbb{R}^p$

State-space model of continuous-time linear time-invariant (LTI) systems:

$$\begin{cases} \frac{dx(t)}{dt} = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases}, \quad t \in \mathbb{R}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$, $D \in \mathbb{R}^{p\times m}$

State-space model of continuous-time linear time-varying (LTV) systems:

$$\begin{cases} \frac{dx(t)}{dt} = A(t)x(t) + B(t)u(t) \\ y(t) = C(t)x(t) + D(t)u(t) \end{cases}, \quad t \in \mathbb{R}$$

where $A(t) \in \mathbb{R}^{n\times n}$, $B(t) \in \mathbb{R}^{n\times m}$, $C(t) \in \mathbb{R}^{p\times n}$, $D(t) \in \mathbb{R}^{p\times m}$

State-Space Model of DT Lumped Linear Systems

$\mathcal{N}$: lumped linear system with state $x \in \mathbb{R}^n$, input $u \in \mathbb{R}^m$, output $y \in \mathbb{R}^p$

State-space model of discrete-time linear time-invariant (LTI) systems:

$$\begin{cases} x[k+1] = Ax[k] + Bu[k] \\ y[k] = Cx[k] + Du[k] \end{cases}, \quad k \in \mathbb{Z}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$, $D \in \mathbb{R}^{p\times m}$

State-space model of discrete-time linear time-varying (LTV) systems:

$$\begin{cases} x[k+1] = A[k]x[k] + B[k]u[k] \\ y[k] = C[k]x[k] + D[k]u[k] \end{cases}, \quad k \in \mathbb{Z}$$

where $A[k] \in \mathbb{R}^{n\times n}$, $B[k] \in \mathbb{R}^{n\times m}$, $C[k] \in \mathbb{R}^{p\times n}$, $D[k] \in \mathbb{R}^{p\times m}$

Deriving State-Space Model

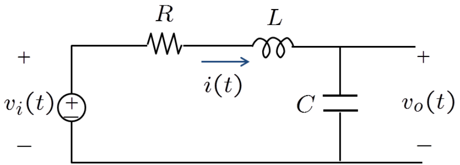
Given a physical process, the steps for deriving state-space model are:

1. Identify a set of state variables
2. Derive the dynamics of each state variable based on physical principles
3. Write output in terms of state variables
4. Assemble the obtained equations in standard space-space format

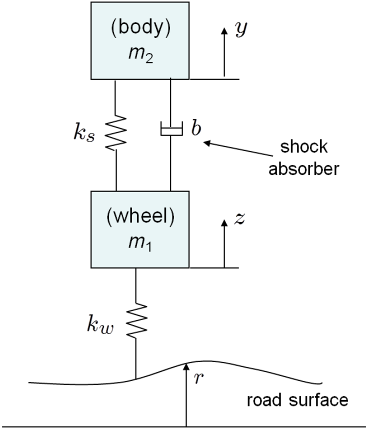
Example I: Systems given by ODEs

$$\ddot{y}(t) + 7\dot{y}(t) + 5y(t) = u(t)$$

Example II: Circuit Systems



Example III: Car Suspension System



General Linear Mechanical Systems

A mechanical system with n degrees of freedom:

$$M\ddot{q} + D\dot{q} + Kq = F$$

- q: vector of displacements
- M: mass matrix
- K: stiffness matrix
- D: damping matrix
- F: external force

Example IV: Digital Circuits

A digital circuit with two time-delay elements: