ECE 68000: MODERN AUTOMATIC CONTROL

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Projection Operator Unknown Input Observer

Closed-Loop UIO Analysis

- Let $\boldsymbol{e} = \boldsymbol{x} - \tilde{\boldsymbol{x}}$
- We will show that

$$\dot{\boldsymbol{e}} = (\boldsymbol{I} - \boldsymbol{MC})(\boldsymbol{A} - \boldsymbol{LC})\boldsymbol{e}$$

  and $\boldsymbol{e}(t) \to \mathbf{0}$ as $t \to \infty$ under mild conditions
- Note that $(\boldsymbol{A} - \boldsymbol{LC})$ asymptotically stable does not guarantee that $(\boldsymbol{I} - \boldsymbol{MC})(\boldsymbol{A} - \boldsymbol{LC})$ is asymptotically stable
- It is possible for a product of a projection matrix and an asymptotically stable matrix to be unstable

Projection Operator UIO Structure

- We will analyze convergence properties of the proposed full-order observer
- We will show $\tilde{\boldsymbol{x}} \to \boldsymbol{x}$ as $t \to \infty$ for

$$\begin{aligned}
\dot{\boldsymbol{q}} &= (\boldsymbol{I} - \boldsymbol{M}\boldsymbol{C})((\boldsymbol{A}\boldsymbol{q} + \boldsymbol{A}\boldsymbol{M}\boldsymbol{y} + \boldsymbol{B}_1\boldsymbol{u}_1) \\
&\quad + \boldsymbol{L}(\boldsymbol{y} - \boldsymbol{C}\boldsymbol{q} - \boldsymbol{C}\boldsymbol{M}\boldsymbol{y})) \\
\tilde{\boldsymbol{x}} &= \boldsymbol{q} + \boldsymbol{M}\boldsymbol{y}
\end{aligned}$$

Projection Operator UIO Analysis

- Let $\boldsymbol{e}(t) = \boldsymbol{x}(t) - \tilde{\boldsymbol{x}}(t)$ be the estimation error
- Recall $(\boldsymbol{I} - \boldsymbol{M}\boldsymbol{C})\boldsymbol{B}_2 = \boldsymbol{O}$ and $\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x}$. Then we have

$$
\begin{aligned}
\frac{d\boldsymbol{e}}{dt} &= \frac{d}{dt}\left(\boldsymbol{x} - \tilde{\boldsymbol{x}}\right) \\
&= \frac{d}{dt}\left(\boldsymbol{x} - \boldsymbol{q} - \boldsymbol{M}\boldsymbol{C}\boldsymbol{x}\right) \\
&= \frac{d}{dt}\left((\boldsymbol{I} - \boldsymbol{M}\boldsymbol{C})\boldsymbol{x} - \boldsymbol{q}\right) \\
&= (\boldsymbol{I} - \boldsymbol{M}\boldsymbol{C})(\boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}_1\boldsymbol{u}_1 + \boldsymbol{B}_2\boldsymbol{u}_2) \\
&\quad\; -(\boldsymbol{I} - \boldsymbol{M}\boldsymbol{C})((\boldsymbol{A}\boldsymbol{q} + \boldsymbol{A}\boldsymbol{M}\boldsymbol{y} + \boldsymbol{B}_1\boldsymbol{u}_1) \\
&\quad +\boldsymbol{L}(\boldsymbol{y} - \boldsymbol{C}\boldsymbol{q} - \boldsymbol{C}\boldsymbol{M}\boldsymbol{y}))
\end{aligned}
$$

Projection Operator UIO Analysis—contd.

We continue

$$
\begin{aligned}
\frac{d\boldsymbol{e}}{dt} &= (\boldsymbol{I}-\boldsymbol{MC})(\boldsymbol{Ax}+\boldsymbol{B}_1\boldsymbol{u}_1)+(\boldsymbol{I}-\boldsymbol{MC})\boldsymbol{B}_2\boldsymbol{u}_2 \\
&\quad -(\boldsymbol{I}-\boldsymbol{MC})((\boldsymbol{Aq}+\boldsymbol{AMCx}+\boldsymbol{B}_1\boldsymbol{u}_1) \\
&\quad +\boldsymbol{L}(\boldsymbol{Cx}-\boldsymbol{Cq}-\boldsymbol{CMCx})) \\
&= (\boldsymbol{I}-\boldsymbol{MC})(\boldsymbol{A}-\boldsymbol{LC})(\boldsymbol{x}-\boldsymbol{q}-\boldsymbol{MCx}) \\
&= (\boldsymbol{I}-\boldsymbol{MC})\,(\boldsymbol{A}-\boldsymbol{LC})\,\boldsymbol{e}
\end{aligned}
$$

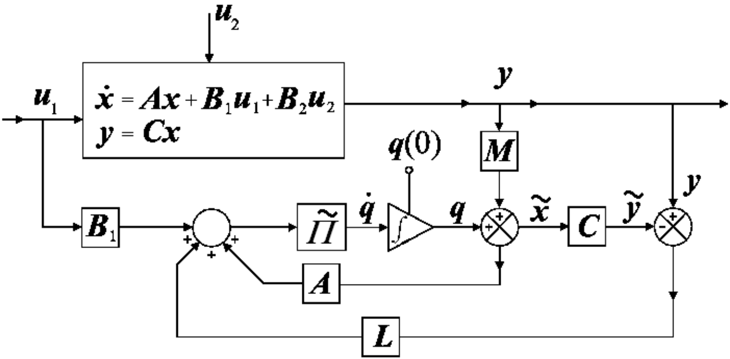
Projection Operator UIO Design

- Objective: Specify $\boldsymbol{M}$ and $\boldsymbol{L}$ and a set of initial conditions so that $\boldsymbol{e}(t) \to \mathbf{0}$ as $t \to \infty$
- A class of solutions to $(\boldsymbol{I} - \boldsymbol{M}\boldsymbol{C})\boldsymbol{B}_2 = \boldsymbol{O}$

$$\boldsymbol{M} = \boldsymbol{B}_2 \left((\boldsymbol{C}\boldsymbol{B}_2)^\dagger + \boldsymbol{H}_0 \left(\boldsymbol{I}_p - (\boldsymbol{C}\boldsymbol{B}_2)(\boldsymbol{C}\boldsymbol{B}_2)^\dagger \right) \right)$$

- $\dagger$ denotes the Moore-Penrose pseudo-inverse
- $\boldsymbol{H}_0 \in \mathbb{R}^{m_2 \times p}$ is a design parameter matrix
- We have $(\boldsymbol{C}\boldsymbol{B}_2)^\dagger(\boldsymbol{C}\boldsymbol{B}_2) = \boldsymbol{I}_{m_2}$ because rank $(\boldsymbol{C}\boldsymbol{B}_2) =$ rank $\boldsymbol{B}_2$ and $\boldsymbol{B}_2$ has full rank
- If $\boldsymbol{C}\boldsymbol{B}_2$ is square, $\boldsymbol{M}$ reduces to $\boldsymbol{B}_2(\boldsymbol{C}\boldsymbol{B}_2)^{-1}$
- $\boldsymbol{M}\boldsymbol{C}$ is a projection (not necessarily orthogonal): $(\boldsymbol{M}\boldsymbol{C})^2 = \boldsymbol{M}\boldsymbol{C}$
- $\tilde{\mathbf{\Pi}} = \boldsymbol{I} - \boldsymbol{M}\boldsymbol{C}$ is also a projection

Block diagram of the Full-Order UIO



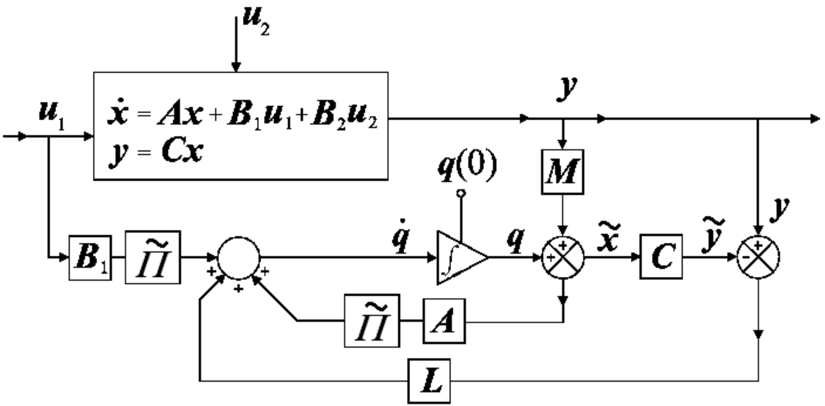
Projection Operator UIO—Second Method

- Adding the innovation term to obtain the closed-loop UIO without being premultiplied by $(\boldsymbol{I} - \boldsymbol{M}\boldsymbol{C})$:

$$
\begin{aligned}
\dot{\boldsymbol{q}} &= (\boldsymbol{I} - \boldsymbol{M}\boldsymbol{C})(\boldsymbol{A}\boldsymbol{q} + \boldsymbol{A}\boldsymbol{M}\boldsymbol{y} + \boldsymbol{B}_1\boldsymbol{u}_1) + \boldsymbol{L}(\boldsymbol{y} - \tilde{\boldsymbol{y}}) \\
&= (\boldsymbol{I} - \boldsymbol{M}\boldsymbol{C})(\boldsymbol{A}\boldsymbol{q} + \boldsymbol{A}\boldsymbol{M}\boldsymbol{y} + \boldsymbol{B}_1\boldsymbol{u}_1) \\
&\quad + \boldsymbol{L}(\boldsymbol{y} - \boldsymbol{C}\boldsymbol{q} - \boldsymbol{C}\boldsymbol{M}\boldsymbol{y}) \\
&= (\boldsymbol{I} - \boldsymbol{M}\boldsymbol{C})(\boldsymbol{A}\boldsymbol{q} + \boldsymbol{A}\boldsymbol{M}\boldsymbol{y} + \boldsymbol{B}_1\boldsymbol{u}_1) + \boldsymbol{L}\boldsymbol{C}(\boldsymbol{x} - \boldsymbol{q} - \boldsymbol{M}\boldsymbol{y})
\end{aligned}
$$

- $\tilde{\boldsymbol{x}} = \boldsymbol{q} + \boldsymbol{M}\boldsymbol{y}$

Block diagram of the second full-order UIO



Projection Operator UIO—Second Method Contd.

- Let $\boldsymbol{e} = \boldsymbol{x} - \tilde{\boldsymbol{x}}$
- Let $\boldsymbol{A}_1 = (\boldsymbol{I} - \boldsymbol{M}\boldsymbol{C})\boldsymbol{A}$
- Easy to show that

$$\dot{\boldsymbol{e}} = (\boldsymbol{A}_1 - \boldsymbol{L}\boldsymbol{C})\boldsymbol{e}$$

- $\boldsymbol{e}(t) \to \mathbf{0}$ as $t \to \infty \iff$ the pair $(\boldsymbol{A}_1, \boldsymbol{C})$ is detectable