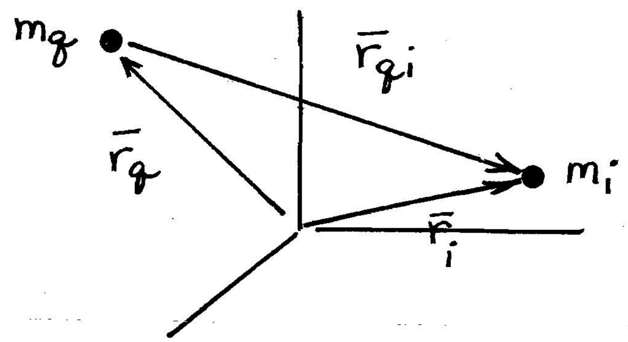
n – Body Problem

Problem: only 10 integrals of motion
12 are required to solve the 2BP

Observation: Do we really care about



We really want to know how it moves relative to some other point such as ⊕ ; ⊙

⇨

Redo the problem:

How to get the EOM governing $\bar{r}_{qi}$? ($\ddot{\bar{r}}_{qi}$)

For any acceleration
still necessary to consider {

To apply Newton's Law of Motion MUST differentiate in inertial frame and base point of the position vector must be fixed in that frame

→ CANNOT use $\bar{F} = m\,\bar{A}$ directly with $\bar{r}_{qi}$

But $\bar{r}_{qi}$ can be written <u>in terms of</u> appropriate vectors

$$\bar{r}_i = \bar{r}_{qi} + \bar{r}_q$$
$$\ddot{\bar{r}}_i = \ddot{\bar{r}}_{qi} + \ddot{\bar{r}}_q$$

Apply Newton's law of motion validly to $\bar{r}_i$ and $\bar{r}_q$

$$\ddot{\bar{r}}_i = -G\sum_{\substack{j=1\\ j\neq i}}^{n} \frac{m_j}{r_{ji}^3}\bar{r}_{ji}$$

$$\ddot{\bar{r}}_q = -G\sum_{\substack{j=1\\ j\neq q}}^{n} \frac{m_j}{r_{jq}^3}\bar{r}_{jq}$$

Sub into $\ddot{\bar{r}}_{qi} + \ddot{\bar{r}}_q = \ddot{\bar{r}}_i$

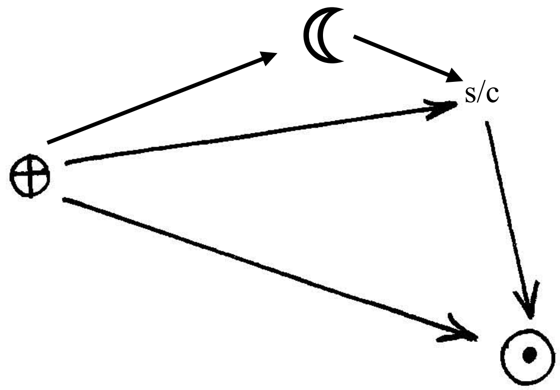
$$\ddot{\bar{r}}_{qi} - \underbrace{G\sum_{\substack{j=1\\ j\neq q}}^{n} \frac{m_j}{r_{jq}^3}\bar{r}_{jq}}_{\text{remove } i \text{ term}} = \underbrace{-G\sum_{\substack{j=1\\ j\neq i}}^{n} \frac{m_j}{r_{ji}^3}\bar{r}_{ji}}_{\text{remove } q \text{ term}}$$

Note: only relative positions appear

$$\ddot{\bar{r}}_{qi} - G\frac{m_i}{r_{iq}^3}\bar{r}_{iq} \underbrace{-G\sum_{\substack{j=1\\ j\neq i,q}}^{n} \frac{m_j}{r_{jq}^3}\bar{r}_{jq}}_{\text{move to right}} = \underbrace{-G\frac{m_q}{r_{qi}^3}\bar{r}_{qi}}_{\text{move to left}} - G\sum_{\substack{j=1\\ j\neq i,q}}^{n} \frac{m_j}{r_{ji}^3}\bar{r}_{ji}$$

Equation for motion of m_i relative to m_q:

Example: $\oplus \quad \odot \quad ☾ \quad$ s/c



How does s/c move relative to $\oplus$?

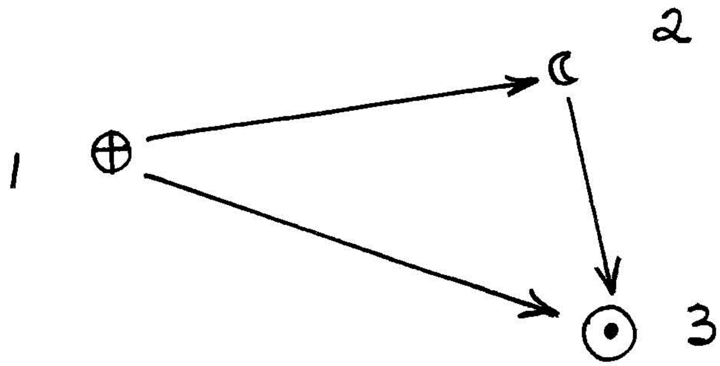
Motion of mass 2 relative to mass 1; perturbed by masses 3,4

$\therefore\ q = \qquad\qquad i = \qquad\qquad j =$

$$\ddot{\bar{r}}_{12} + \underbrace{G\frac{(m_1+m_2)}{r_{12}^3}\bar{r}_{12}}_{} = G\,m_3\left(\underbrace{\frac{\bar{r}_{23}}{r_{23}^3}}_{} - \underbrace{\frac{\bar{r}_{13}}{r_{13}^3}}_{}\right) + G\,m_4\left(\frac{\bar{r}_{24}}{r_{24}^3} - \frac{\bar{r}_{14}}{r_{14}^3}\right)$$

$$\ddot{\bar{r}}_{\oplus s/c} + G\frac{(m_\oplus + m_{s/c})}{r_{\oplus s/c}^3}\bar{r}_{\oplus s/c} = G\,m_\odot\left(\frac{\bar{r}_{s/c\odot}}{r_{s/c\odot}^3} - \frac{\bar{r}_{\oplus\odot}}{r_{\oplus\odot}^3}\right) + G\,m_{☾}\left(\frac{\bar{r}_{s/c☾}}{r_{s/c}^3} - \frac{\bar{r}_{\oplus☾}}{r_{\oplus☾}^3}\right)$$

Example: $\oplus \quad \odot \quad ☾$



How does ☾ move relative to $\oplus$?

Motion of mass 2 relative to mass 1;
perturbed by mass 3

➔ $\ddot{\bar{r}}_{12}$

$\therefore \; q = 1 \qquad i = 2 \qquad j = 3$

$$\ddot{\bar{r}}_{12} + \underbrace{G\frac{(m_1 + m_2)}{r_{12}^3}\bar{r}_{12}}_{} = G\, m_3 \left(\underbrace{\frac{\bar{r}_{23}}{r_{23}^3}}_{} - \underbrace{\frac{\bar{r}_{13}}{r_{13}^3}}_{} \right)$$

$$\ddot{\bar{r}}_{\oplus\leftmoon} + G\frac{(m_\oplus + m_\leftmoon)}{r_{\oplus\leftmoon}^3}\bar{r}_{\oplus\leftmoon} = G\, m_\odot \left(\frac{\bar{r}_{\leftmoon\odot}}{r_{\leftmoon\odot}^3} - \frac{\bar{r}_{\oplus\odot}}{r_{\oplus\odot}^3} \right)$$

Careful – indirect effects frequently forgotten but can be significant!!!

Given the equation of motion, do we now have an equation that we can solve?

$\Rightarrow$ Still can't solve if $n \geq 3$

$n = 3$ **:** requires position of ☉ relative to ⊕ or ☾ **;** an additional vector EOM is necessary

$\Downarrow$

to solve two 2nd-order vector DE requires 12 integrals of the motion**;** we only know 10 !!

$n = 2$ **:** no m_j perturbing bodies
a 2nd-order vector DE in only one unknown position vector!

$\Downarrow$

6 scalar EOMs **;** 6 dependent scalar variables **;** requires only 6 integrals of motion (we have 10 !!)

$\rightarrow$ <u>Relative</u> motion of <u>two</u> bodies solvable

$$\ddot{\bar{r}}_{12} + G\frac{(m_1 + m_2)}{r_{12}^3}\bar{r}_{12} = \bar{0}$$ solvable but nontrivial

OR

$$\ddot{\bar{r}} + \frac{\mu}{r^3}\bar{r} = \bar{0} \qquad \text{where} \qquad \mu = G(m_1 + m_2)$$