4.2.1

$\det(A) = \frac{1}{2}$ $\quad A_{4\times4}$

$\det(2A) = 2^4 \det(A) = (16)(\frac{1}{2})$

$\boxed{\det(2A) = 8}$

$\det(-A) = (-1)^4 \det(A) = (1)\det(A)$

$\boxed{\det(-A) = 1/2}$

$\det(A^2) = \det(A)\det(A) = (1/2)(1/2)$

$\boxed{\det(A^2) = 1/4}$

$\det(A^{-1}) = \frac{1}{\det(A)} = \frac{1}{\frac{1}{2}}$

$\boxed{\det(A^{-1}) = 2}$

4.2.4

$$\begin{pmatrix} 1 & 2 & -2 & 0 \\ 2 & 3 & -4 & 1 \\ -1 & -2 & 0 & 2 \\ 0 & 2 & 5 & 3 \end{pmatrix} \xrightarrow{E_{21}(-2)} \begin{pmatrix} 1 & 2 & -2 & 0 \\ 0 & -1 & 0 & 1 \\ -1 & -2 & 0 & 2 \\ 0 & 2 & 5 & 3 \end{pmatrix}$$

$$\xrightarrow{E_{31}(1)} \begin{pmatrix} 1 & 2 & -2 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -2 & 2 \\ 0 & 2 & 5 & 3 \end{pmatrix} \xrightarrow{E_{42}(2)} \begin{pmatrix} 1 & 2 & -2 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 5 & 5 \end{pmatrix}$$

$$\xrightarrow{E_{43}(5/2)} \begin{pmatrix} \boxed{1} & 2 & -2 & 0 \\ 0 & \boxed{-1} & 0 & 1 \\ 0 & 0 & \boxed{-2} & 2 \\ 0 & 0 & 0 & \boxed{10} \end{pmatrix}$$

$$\det \begin{pmatrix} 1 & 2 & -2 & 0 \\ 2 & 3 & -4 & 1 \\ -1 & -2 & 0 & 2 \\ 0 & 2 & 5 & 3 \end{pmatrix} = (1)(-1)(-2)(10) = \boxed{20}$$

$$\begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & -2 \end{pmatrix} \xrightarrow{E_{21}(1/2)} \begin{pmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & -2 \end{pmatrix}$$

$$\xrightarrow{E_{32}(2/3)} \begin{pmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & 0 & 4/3 & -1 \\ 0 & 0 & -1 & -2 \end{pmatrix} \xrightarrow{E_{43}(3/4)} \begin{pmatrix} \boxed{2} & -1 & 0 & 0 \\ 0 & \boxed{3/2} & -1 & 0 \\ 0 & 0 & \boxed{4/3} & -1 \\ 0 & 0 & 0 & \boxed{-11/4} \end{pmatrix}$$

4.2.4

$$\det\begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & -2 \end{pmatrix} = (2)(3/2)(4/3)(-11/4) = \boxed{-11}$$

$$\begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & -2 \\ 0 & -1 & 2 & -1 \end{pmatrix} \xrightarrow{E_{21}(1/2)} \begin{pmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & 0 & -1 & -2 \\ 0 & -1 & 2 & -1 \end{pmatrix}$$

$$\xrightarrow{E_{42}(2/3)} \begin{pmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 4/3 & -1 \end{pmatrix} \xrightarrow{E_{43}(4/3)} \begin{pmatrix} \boxed{2} & -1 & 0 & 0 \\ 0 & \boxed{3/2} & -1 & 0 \\ 0 & 0 & \boxed{-1} & -2 \\ 0 & 0 & 0 & \boxed{-\frac{11}{3}} \end{pmatrix}$$

$$\det\begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & -2 \\ 0 & -1 & 2 & -1 \end{pmatrix} = (2)(3/2)(-1)(-11/3) = \boxed{11}$$

4.2.7

a) $A = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} [2 \ -1 \ 2] = \begin{pmatrix} 2 & -1 & 2 \\ 8 & -4 & 8 \\ 4 & -2 & 4 \end{pmatrix}$

$\det(A) = 2\begin{vmatrix} -4 & 8 \\ -2 & 4 \end{vmatrix} - (-1)\begin{vmatrix} 8 & 8 \\ 4 & 4 \end{vmatrix} + 2\begin{vmatrix} 8 & -4 \\ 4 & -2 \end{vmatrix}$

$= (2)(\cancel{-16+16}^{0}) + 0 + (2)(\cancel{-16+16}^{0})$

$\boxed{\det(A) = 0}$

b) $U = \begin{pmatrix} \boxed{4} & 4 & 8 & 8 \\ 0 & \boxed{1} & 2 & 2 \\ 0 & 0 & \boxed{2} & 6 \\ 0 & 0 & 0 & \boxed{2} \end{pmatrix}$

$\det(U) = (4)(1)(2)(2)$

$\boxed{\det(U) = 16}$

c) $U^T = \begin{pmatrix} \boxed{4} & 0 & 0 & 0 \\ 4 & \boxed{1} & 0 & 0 \\ 8 & 2 & \boxed{2} & 0 \\ 8 & 2 & 6 & \boxed{2} \end{pmatrix}$

$\det(U^T) = (4)(1)(2)(2)$

$\boxed{\det(U^T) = 16}$

4.2.7

d)

$$\det(U^{-1}) = \frac{1}{\det(U)} = \frac{1}{16}$$

$$\boxed{\det(U^{-1}) = 1/16}$$

e) $M = \begin{pmatrix} 0 & 0 & 0 & 2 \\ 0 & 0 & 2 & 6 \\ 0 & 1 & 2 & 2 \\ 4 & 4 & 8 & 8 \end{pmatrix}$

$P = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$ $\leftarrow$ swaps rows (1-4) & row (2-3)

$PM = U = \begin{pmatrix} 4 & 4 & 8 & 8 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 2 \end{pmatrix}$

$M = P^{-1}U$

$P^{-1} = P^T = P$

$M = PU$

$\underline{\det(U) = 16}$

$\det(P)\det(M) = \det(U) \Rightarrow$

$\det(M) = \det(P)\det(U) = \det(P^{-1})\det(U)$

Two row exchanges so $\underline{\det(P)} = (-1)(-1) = \underline{1}$

$\therefore \boxed{\det(M) = 16}$

4.3.2

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$a_{11} = 0 \quad C_{11} = (-1)^{1+1} \det M_{11}$

$a_{12} = 1 \quad C_{12} = (-1)^{1+2} \det M_{12}$

$a_{13} = 0 \quad C_{13} = (-1)^{1+3} \det M_{13}$

$a_{14} = 0 \quad C_{14} = (-1)^{1+4} \det M_{14}$

$$M_{11} = \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = -(1)\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} = 0$$

$$M_{12} = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = (1)\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} - (1)\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = -1$$

$$M_{13} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{vmatrix} = (1)\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} = 0$$

$$M_{14} = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = (1)\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + (1)\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 1$$

$$\boxed{C_{11} = 0}$$

4.3.2

$C_{12} = (-1)(-1) = 1$

$\boxed{C_{12} = 1}$

$\boxed{C_{13} = 0}$

$C_{14} = (-1)(1)$

$\boxed{C_{14} = -1}$

$\det(A) = a_{12}C_{12} = (1)(1)$

$\boxed{\det(A) = 1}$

$$B = \begin{pmatrix} 0 & 0 & 1 & 2 \\ 0 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$a_{11} = 0$

$a_{12} = 0$

$a_{13} = 1$

$a_{14} = 2$

$$\det(M_{41}) = \begin{vmatrix} 3 & 4 & 5 \\ 7 & 8 & 9 \\ 0 & 0 & 1 \end{vmatrix} = 3\begin{vmatrix} 8 & 9 \\ 0 & 1 \end{vmatrix} - 4\begin{vmatrix} 7 & 9 \\ 0 & 1 \end{vmatrix} + 5\begin{vmatrix} 7 & 8 \\ 0 & 0 \end{vmatrix} = -4$$

4.3.2

$$M_{12} = \begin{vmatrix} 0 & 4 & 5 \\ 6 & 8 & 9 \\ 0 & 0 & 1 \end{vmatrix} = -4\begin{vmatrix} 6 & 9 \\ 0 & 1 \end{vmatrix} + 5\begin{vmatrix} 6 & 8 \\ 0 & 0 \end{vmatrix} = -24$$

$$M_{13} = \begin{vmatrix} 0 & 3 & 5 \\ 6 & 7 & 9 \\ 0 & 0 & 1 \end{vmatrix} = -3\begin{vmatrix} 6 & 9 \\ 0 & 1 \end{vmatrix} + 5\begin{vmatrix} 6 & 7 \\ 0 & 0 \end{vmatrix} = -18$$

$$M_{14} = \begin{vmatrix} 0 & 3 & 4 \\ 6 & 7 & 8 \\ 0 & 0 & 0 \end{vmatrix} = -3\begin{vmatrix} 6 & 8 \\ 0 & 0 \end{vmatrix} + 4\begin{vmatrix} 6 & 7 \\ 0 & 0 \end{vmatrix} = 0$$

$C_{11} = (1)(-4)$

$\boxed{C_{11} = -4}$

$C_{12} = (-1)(-24)$

$\boxed{C_{12} = 24}$

$C_{13} = (1)(-18)$

$\boxed{C_{13} = -18}$

$\boxed{C_{14} = 0}$

$\boxed{\det(B) = -18} = a_{13}C_{13} = (1)(-18)$

4.3.3

a) $\det(S^{-1}) = \frac{1}{\det(S)}$

$\frac{1}{\det(S)} \det(A) \det(S) = \det(A)$

$\boxed{\text{True}}$

b) Let A be 3x3 $\begin{pmatrix} 0 & 0 & 0 \\ 2 & 3 & 4 \\ 5 & 6 & 7 \end{pmatrix}$

$\det(A) = a_{11} C_{11} + a_{12} C_{12} + a_{13} C_{13}$

$C_{11} = (1) \begin{vmatrix} 3 & 4 \\ 6 & 7 \end{vmatrix}$ $\quad C_{12} = (-1) \begin{vmatrix} 2 & 4 \\ 5 & 7 \end{vmatrix}$

$C_{13} = (1) \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix}$

All cofactors $\neq 0$ $\therefore$ $\boxed{\text{False}}$

& $\det(A) = 0$

c) $\begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 2$

$\boxed{\text{False}}$

4.3.7

$$\begin{vmatrix} 4 & 4 & 4 & 4 \\ 1 & 2 & 0 & 1 \\ 2 & 0 & 1 & 2 \\ 1 & 1 & 0 & 2 \end{vmatrix}$$

$a_{11} = a_{12} = a_{13} = a_{14} = 4$

$$M_{11} = \begin{vmatrix} 2 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 2 \end{vmatrix} = (2)\begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} - 0 + 1\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = 3$$

$$M_{12} = \begin{vmatrix} 1 & 0 & 1 \\ 2 & 1 & 2 \\ 1 & 0 & 2 \end{vmatrix} = (1)\begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} - 0 + 1\begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} = 1$$

$$M_{13} = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 0 & 2 \\ 1 & 1 & 2 \end{vmatrix} = 1\begin{vmatrix} 0 & 2 \\ 1 & 2 \end{vmatrix} - 2\begin{vmatrix} 2 & 2 \\ 1 & 2 \end{vmatrix} + 1\begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = -4$$

$-2 \quad - 4 \quad + 2$

$$M_{14} = \begin{vmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = 1\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} = 1$$

$-1 \quad +2$

$\det = (4)(1)(3) + (4)(-1)(1) + (4)(1)(-4) + (4)(-1)(1)$

$\boxed{\det = -12}$

4.3.7

Subtract Column 1:

$$\begin{vmatrix} 4 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 \\ 2 & -2 & -1 & 0 \\ 1 & 0 & -1 & 1 \end{vmatrix}$$

$$= 4\begin{vmatrix} 1 & -1 & 0 \\ -2 & -1 & 0 \\ 0 & -1 & 1 \end{vmatrix} = 4\left[(1)\begin{vmatrix} -1 & 0 \\ -1 & 1 \end{vmatrix} - (-1)\begin{vmatrix} -2 & 0 \\ 0 & 1 \end{vmatrix}\right]$$

$$= 4[(-1) - (2)] = (4)(-3) = \boxed{-12}$$

4.3.26

$$B_4 = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$$

$$C_{41} = (-1)^{4+1} |M_{41}| = (-1) \begin{vmatrix} -1 & 0 & 0 \\ 2 & -1 & 0 \\ -1 & 2 & -1 \end{vmatrix} = (-1)\left[-1 \begin{vmatrix} -1 & 0 \\ 2 & -1 \end{vmatrix}\right]$$

$C_{41} = 1 \qquad a_{41} = 0$

$$C_{42} = (1) \begin{vmatrix} 1 & 0 & 0 \\ -1 & -1 & 0 \\ 0 & 2 & -1 \end{vmatrix} = (1)\left[(1) \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix}\right] = 1$$

$C_{42} = 1 \qquad a_{42} = 0$

$$C_{43} = (-1) \begin{vmatrix} 1 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & -1 \end{vmatrix} = (-1)\left[(1) \begin{vmatrix} 2 & 0 \\ -1 & -1 \end{vmatrix} - (-1) \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix}\right]$$

$C_{43} = (-1)[-2+1] = 1$

$a_{43} = -1$

$$C_{44} = (1) \begin{vmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{vmatrix} = \left[\underbrace{\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}}_{3} - \underbrace{(-1) \begin{vmatrix} -1 & -1 \\ 0 & 2 \end{vmatrix}}_{2}\right]$$

$C_{44} = 1 \qquad a_{44} = 2$

$|B_4| = a_{43} C_{43} + a_{44} C_{44} = (-1)(1) + (2)(1)$

$|B_4| = 1$

4.3.26

$$B_3 = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$

$$|B_3| = (1)\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ 0 & 2 \end{vmatrix}$$

$$|B_3| = 3 - 2 = 1$$

$$B_2 = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

$$|B_2| = 2 - 1 = 1$$

$$2|B_3| - |B_2| = 2(1) - 1 = 1$$

$$\therefore \boxed{|B_4| = 2|B_3| - |B_2| = 1}$$

4.4.2

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$

$$C_{11} = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} = 3$$

$$C_{12} = (-1) \begin{vmatrix} -1 & -1 \\ 0 & 2 \end{vmatrix} = 2$$

$$C_{13} = \begin{vmatrix} -1 & 2 \\ 0 & -1 \end{vmatrix} = 1$$

$$C_{21} = (-1) \begin{vmatrix} -1 & 0 \\ -1 & 2 \end{vmatrix} = 2$$

$$C_{22} = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4$$

$$C_{23} = (-1) \begin{vmatrix} 2 & -1 \\ 0 & -1 \end{vmatrix} = 2$$

$$C_{33} = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} = 3$$

$$C = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{pmatrix} \qquad C^T = C$$

$$\det(A) = a_{11} C_{11} + a_{12} C_{12} + a_{13} C_{13}$$

$$\det(A) = (2)(3) + (-1)(2) + 0 = 4$$

4.4.2

$$A^{-1} = \frac{C^T}{\det(A)} = \boxed{\begin{pmatrix} 3/4 & 1/2 & 1/4 \\ 1/2 & 1 & 1/2 \\ 1/4 & 1/2 & 3/4 \end{pmatrix}}$$

$$B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}$$

$$C_{11} = \begin{vmatrix} 2 & 2 \\ 2 & 3 \end{vmatrix} = 2$$

$$C_{12} = (-1)\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = -1$$

$$C_{13} = \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = 0$$

$$C_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = 2$$

$$C_{23} = (-1)\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = -1$$

$$C_{33} = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1$$

$$C = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix} = C^T$$

4.4.2

$$\det(B) = b_{11}C_{11} + b_{12}C_{12} + b_{13}C_{13}$$

$$\det(B) = (1)(2) + (1)(-1) + (1)(0)$$

$$\det(B) = 1$$

$$B^{-1} = \frac{C^T}{\det(B)}$$

$$\boxed{B^{-1} = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}}$$

4.4.14

$ax + by = 1$

$cx + dy = 0$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$x = A^{-1} b = \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$y = x_2 = \frac{\det(B_2)}{\det(A)}$$

$$B_2 = \begin{pmatrix} a & 1 \\ c & 0 \end{pmatrix}$$

$\det(B_2) = -c$

$\det(A) = ad - bc$

$$\boxed{y = \frac{-c}{ad-bc}}$$

$ax + by + cz = 1$

$dx + ey + fz = 0$

$gx + hy + iz = 0$

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \quad x = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad b = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

4.4.14

$$y = x_2 = \frac{\det(B_2)}{\det(A)} = \frac{\det(B_2)}{D}$$

$$B_2 = \begin{pmatrix} a & 1 & c \\ d & 0 & f \\ g & 0 & i \end{pmatrix}$$

$$\det(B_2) = a \overset{0}{\cancel{\begin{vmatrix} 0 & f \\ 0 & i \end{vmatrix}}} - (1)\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \overset{0}{\cancel{\begin{vmatrix} d & 0 \\ g & 0 \end{vmatrix}}}$$

$$\det(B_2) = -di + fg$$

$$\boxed{y = \frac{fg - di}{D}}$$

4.4.22

$$AC^T = \det(A)\, I$$

$$\det(C^T) = \det(C)$$
$$\det(I) = 1$$

$$\det(A)\det(C^T) = \det(AC^T) = \det(A)\det(C)$$

$$\det(A)\det(C) = \det(\det(A)\, I)$$

$I_{n\times n}$: $\det(\det(A) I_{n\times n}) = \det(A)^n \det(I)$

$$\det(\det(A) I_{n\times n}) = \det(A)^n$$

$$\det(A)^n = \det(A)\det(C)$$

$$\det(C) = \frac{\det(A)^n}{\det(A)}$$

$$\therefore \boxed{\det(C) = \det(A)^{n-1}}$$