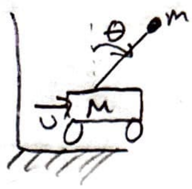
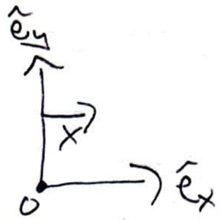
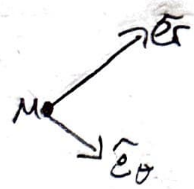
#1)

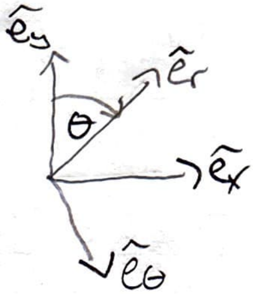






$q_1 = x$

$q_2 = \theta$



$$\hat{e}_z = -\hat{e}_3$$

$$\hat{e}_r = \cos\theta\,\hat{e}_y + \sin\theta\,\hat{e}_x$$

$$\hat{e}_\theta = \cos\theta\,\hat{e}_x - \sin\theta\,\hat{e}_y$$

$$\vec{r}_{M/O} = x\hat{e}_x \qquad {}^{I}\vec{v}_{M/O} = \dot{x}\hat{e}_x$$

$$\vec{r}_{m/O} = x\hat{e}_x + l\hat{e}_r = x\hat{e}_x + l\cos\theta\,\hat{e}_y + l\sin\theta\,\hat{e}_x$$

$$= (x + l\sin\theta)\hat{e}_x + l\cos\theta\,\hat{e}_y$$

$${}^{I}\vec{v}_{m/O} = (\dot{x} + l\dot{\theta}\cos\theta)\hat{e}_x + (-l\dot{\theta}\sin\theta)\hat{e}_y$$

$$T = \frac{1}{2}m\|{}^{I}\vec{v}_{m/O}\|^2 + \frac{1}{2}M\|{}^{I}\vec{v}_{M/O}\|^2$$

$$T = \frac{1}{2}m[\dot{x}^2 + l^2\dot{\theta}^2\cos^2\theta + 2l\dot{\theta}\dot{x}\cos\theta + l^2\dot{\theta}^2\sin^2\theta]$$

$$+ \frac{M}{2}\dot{x}^2$$

$$T = \frac{1}{2}m[\dot{x}^2 + l^2\dot{\theta}^2 + 2l\dot{\theta}\dot{x}\cos\theta] + \frac{M}{2}\dot{x}^2$$

$$U = -\int \vec{F}_g \cdot {}^{I}d\vec{r}_{m/O}$$

$F_g = -mg\,\hat{e}_y$

${}^{I}\vec{v}_{m/O} = \frac{{}^{I}dr_{m/O}}{dt} \Rightarrow dt\left({}^{I}\vec{v}_{m/O}\right) = {}^{I}dr_{m/O}$

${}^{I}dr_{m/O} = (dx + l\,d\theta\cos\theta)\,e_x + (-l\,d\theta\sin\theta)\,\hat{e}_y$

$F_g \cdot {}^{I}dr_{m/O} = mgl\sin\theta\,d\theta$

$U = -\int mgl\sin\theta\,d\theta = mgl\cos\theta$

$\mathcal{L} = T - U = \frac{1}{2}m[\dot{x}^2 + l^2\dot{\theta}^2 + 2l\dot{\theta}\dot{x}\cos\theta] + \frac{M}{2}\dot{x}^2 - mgl\cos\theta$

$\frac{\partial \mathcal{L}}{\partial q_1} = 0 \qquad \frac{\partial \mathcal{L}}{\partial \dot{q}_1} = m\dot{x} + M\dot{x} + l\dot{\theta}\cos\theta\, m$

$\frac{\partial \mathcal{L}}{\partial q_2} = -ml\dot{\theta}\dot{x}\sin\theta + mgl\sin\theta$

$\frac{\partial \mathcal{L}}{\partial \dot{q}_2} = ml^2\dot{\theta} + ml\dot{x}\cos\theta$

$Q_j = \Sigma\, \vec{F}_i^{(nc,a)} \cdot \frac{\partial \vec{r}_{i/O}}{\partial q_j} \qquad F_M = U$

$\frac{\partial \vec{r}_{M/O}}{\partial q_1} = 1 \qquad \frac{\partial \vec{r}_{M/O}}{\partial q_2} = 0$

$Q_x = U \cdot 1 = U \qquad Q_\theta = U \cdot 0 = 0$

$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_j}\right) - \frac{\partial \mathcal{L}}{\partial q_j} = Q_j$

$i = x$

$$\frac{d}{dt}\left(m\dot{x} + M\dot{x} + ml\dot{\theta}\cos\theta\right) = U$$

$$U = m\ddot{x} + M\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta \qquad (1)$$

$j = \theta$

$$\frac{d}{dt}\left(ml^2\dot{\theta} + ml\dot{x}\cos\theta\right) - \left(-ml\dot{\theta}\dot{x}\sin\theta + mgl\sin\theta\right) = 0$$

$$ml^2\ddot{\theta} + ml\ddot{x}\cos\theta - ml\cancel{\dot{x}}\dot{\theta}\sin\theta + ml\cancel{\dot{\theta}}\dot{x}\sin\theta - mgl\sin\theta = 0$$

$$ml^2\ddot{\theta} + ml\ddot{x}\cos\theta - mgl\sin\theta = 0 \qquad (2)$$

$$\ddot{\theta} = \frac{-\ddot{x}\cos\theta + g\sin\theta}{l}$$

$$\ddot{x} = \frac{U - ml\ddot{\theta}\cos\theta + ml\dot{\theta}^2\sin\theta}{(m+M)}$$

$$\ddot{x} = \frac{U + m\ddot{x}\cos^2\theta - mg\cos\theta\sin\theta + ml\dot{\theta}^2\sin\theta}{(m+M)}$$

$$\ddot{x} = \frac{U - mg\cos\theta\sin\theta + ml\dot{\theta}^2\sin\theta}{m+M-m\cos^2\theta} \qquad (3)$$

$$\ddot{\theta} = \frac{-U\cos\theta + mg\cancel{\cos^2\theta}\sin\theta - ml\dot{\theta}^2\sin\theta\cos\theta + (m+M-\cancel{m\cos^2\theta})g\sin\theta}{l(m+M-m\cos^2\theta)}$$

$$\ddot{\theta} = \frac{-U\cos\theta - ml\dot{\theta}^2\sin\theta\cos\theta + mg\sin\theta + Mg\sin\theta}{l(m+M-m\cos^2\theta)} \quad (4)$$

Non-linear state space Model:

$$\boxed{\begin{aligned}
\dot{X}_1 &= X_2 \\
\dot{X}_2 &= \frac{U - mg\sin(X_3)\cos(X_3) + mlX_4^2\sin(X_3)}{m+M-m\cos^2(X_3)} \\
\dot{X}_3 &= X_4 \\
\dot{X}_4 &= \frac{-U\cos(X_3) - mlX_4^2\sin(X_3)\cos(X_3) + mg\sin(X_3) + Mg\sin(X_3)}{l(m+M-m\cos^2(X_3))} \\
y_1 &= X_1 \\
y_2 &= X_3
\end{aligned}}$$

#2) $f_1 = \dot{X}_1$, $f_2 = \dot{X}_2$, $f_3 = \dot{X}_3$, $f_4 = \dot{X}_4$

$X^* = 0 = U^*$

$X = X^* + \delta X$, $U = U^* + \delta U$, $y = y^* + \delta y$

$\delta\dot{X} = A\,\delta X + B\,\delta U$

$\delta y = C\,\delta X + D\,\delta U$

$$A = \begin{pmatrix} \left.\frac{\partial f_1}{\partial x_1}\right|_{(x^*,u^*)} & \left.\frac{\partial f_1}{\partial x_2}\right|_{(x^*,u^*)} & \left.\frac{\partial f_1}{\partial x_3}\right|_{(x^*,u^*)} & \left.\frac{\partial f_1}{\partial x_4}\right|_{(x^*,u^*)} \\ \left.\frac{\partial f_2}{\partial x_1}\right|_{(x^*,u^*)} & \left.\frac{\partial f_2}{\partial x_2}\right|_{(x^*,u^*)} & \left.\frac{\partial f_2}{\partial x_3}\right|_{(x^*,u^*)} & \left.\frac{\partial f_2}{\partial x_4}\right|_{(x^*,u^*)} \\ \left.\frac{\partial f_3}{\partial x_1}\right|_{(x^*,u^*)} & \left.\frac{\partial f_3}{\partial x_2}\right|_{(x^*,u^*)} & \left.\frac{\partial f_3}{\partial x_3}\right|_{(x^*,u^*)} & \left.\frac{\partial f_3}{\partial x_4}\right|_{(x^*,u^*)} \\ \left.\frac{\partial f_4}{\partial x_1}\right|_{(x^*,u^*)} & \left.\frac{\partial f_4}{\partial x_2}\right|_{(x^*,u^*)} & \left.\frac{\partial f_4}{\partial x_3}\right|_{(x^*,u^*)} & \left.\frac{\partial f_4}{\partial f_4}\right|_{(x^*,u^*)} \end{pmatrix}$$

$$B = \begin{pmatrix} \left.\frac{\partial f_1}{\partial u}\right|_{(x^*,u^*)} \\ \left.\frac{\partial f_2}{\partial u}\right|_{(x^*,u^*)} \\ \left.\frac{\partial f_3}{\partial u}\right|_{(x^*,u^*)} \\ \left.\frac{\partial f_4}{\partial u}\right|_{(x^*,u^*)} \end{pmatrix}$$

$$y = \begin{pmatrix} x_1 \\ x_3 \end{pmatrix}$$

$$X = \cancel{X^*}^{\,0} + \delta X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} \delta x_1 \\ \delta x_2 \\ \delta x_3 \\ \delta x_4 \end{pmatrix}$$

$$y = \begin{pmatrix} x_1 \\ x_3 \end{pmatrix} = \begin{pmatrix} \delta x_1 \\ \delta x_3 \end{pmatrix}$$

$$\boxed{A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 11 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 0 \\ 1/10 \\ 0 \\ -1/10 \end{pmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad D = \begin{pmatrix} 0 \\ 0 \end{pmatrix}}$$

$$\operatorname{rank}[B \;\; AB \;\; A^2B \;\; A^3B] = 4$$

$$\operatorname{rank}\begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{bmatrix} = 4$$

System is <u>Controllable & observable</u>

#3)

Desired poles: $s_1 = -1$

$s_2 = -4$

$s_3 = -4 + i$

$s_4 = -4 - i$

$\delta u = -K\,\delta x \qquad \delta x = x - x^*$

$\delta\dot{x} = A\,\delta x + B\,\delta u = (A - BK)\,\delta x$

$u = u^* + \delta u = u^* - K\,\delta x = \cancel{u^*}^{\to 0} - K(x - \cancel{x^*}^{\to 0})$

$u = -Kx$

$$\boxed{K = [-68 \quad -117 \quad -788 \quad -247]}$$

#5) Desired observer poles: $s_1 = -3$

$z = \hat{x}^* + \delta\hat{x} = \cancel{\begin{pmatrix} x_1^* \\ x_2^* \\ x_3^* \\ x_4^* \end{pmatrix}}^{\to 0} + \begin{pmatrix} \delta\hat{x}_1 \\ \delta\hat{x}_2 \\ \delta\hat{x}_3 \\ \delta\hat{x}_4 \end{pmatrix}$

$s_2 = -12$

$s_3 = -12 + i$

$s_4 = -12 - i$

$u = u^* + \delta u$

$y = y^* + \delta y = \begin{pmatrix} x_1 \\ x_3 \end{pmatrix}$

$\dot{z} = Az + Bu + L(y - \hat{y}) = Az + B(\cancel{u^*}^{0} + \delta u) + L((\cancel{y^*}^{\to 0} + \delta y) - Cz)$

$\dot{z} = Az + B\,\delta u + L(\delta y - Cz)$

$\delta y = \begin{pmatrix} x_1 \\ x_3 \end{pmatrix} - \cancel{\begin{pmatrix} x_1^* \\ x_3^* \end{pmatrix}}^{\to 0} = \begin{pmatrix} x_1 \\ x_3 \end{pmatrix} = y$

$\delta u = u - \cancel{u^*}^{\to 0} = -Kz$

$$\dot{z} = (A - LC)z + Bu + Ly$$

$$\dot{z} = (A - LC)z - BKz + Ly$$

$$\boxed{\dot{z} = (A - LC - BK)z + Ly}$$ ← Observer dynamics for linearized system

where $y = \begin{pmatrix} x_1 \\ x_3 \end{pmatrix}$ & $z = \begin{pmatrix} \delta\hat{x}_1 \\ \delta\hat{x}_2 \\ \delta\hat{x}_3 \\ \delta\hat{x}_4 \end{pmatrix} = \begin{pmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \hat{x}_4 \end{pmatrix}$ as $\hat{x}^* = 0$

$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} x$

From pole placement of $(A - LC)$

$$\boxed{L = \begin{pmatrix} 15.4624 & 3.7298 \\ 43.7948 & 43.0396 \\ 0.6685 & 23.5376 \\ 16.4663 & 146.7505 \end{pmatrix}}$$

#6) $u = -Kz + r$

$\dot{x} = Ax + Bu = Ax - BKz + Br \qquad y = Cx$

$$\dot{z} = Az + Bu + L(y - \underbrace{\hat{y}}_{Cz}) = (A - BK - LC)z + LCx + Br$$

$$Y(s) = [C \;\; 0] \begin{pmatrix} sI - A + BK & -BK \\ 0 & sI - A + LC \end{pmatrix}^{-1} \begin{pmatrix} B \\ 0 \end{pmatrix} R(s)$$

$$Y(s) = C(sI - A + BK)^{-1} B R(s)$$

$$\frac{Y(s)}{R(s)} = C(sI - A + BK)^{-1}B$$

$$\frac{Y(s)}{R(s)} = \begin{pmatrix} \frac{Y_1(s)}{R(s)} \\ \frac{Y_2(s)}{R(s)} \end{pmatrix} = \begin{pmatrix} \frac{X_1(s)}{R(s)} \\ \frac{X_3(s)}{R(s)} \end{pmatrix}$$

$$\boxed{\begin{aligned} \frac{Y_1(s)}{R(s)} &= \frac{0.1s^2 - 1}{s^4 + 13s^3 + 61s^2 + 117s + 68} \\ \frac{Y_2(s)}{R(s)} &= \frac{-0.1s^2}{s^4 + 13s^3 + 61s + 117s + 68} \end{aligned}}$$

Contents


```
clear
close all
clc
```

Parameters

```
m       = 1;            % [kg]
l       = 1;            % [m]
g       = 10;           % [m/s^2]
M       = 10;           % [kg]
x_star  = zeros(4,1);   % State Operating Point
u_star  = 0;            % Input Operating Point
dt      = .005;         % Time Step
time    = (0:dt:5)';    % Time
```

Part 2 - Linearize Model about Operating Point x = 0, u = 0

```
syms x1 x2 x3 x4 u s real

% Create Symbolic Set of Equations
x       = [x1;x2;x3;x4];
xdot    = NonlinearCartPendulum([],x,u,m,M,g,l);

% Symbolic Jacobian Matrices
A_sym    = [diff(xdot(1),x1), diff(xdot(1),x2), diff(xdot(1),x3), diff(xdot(1),x4);...
            diff(xdot(2),x1), diff(xdot(2),x2), diff(xdot(2),x3), diff(xdot(2),x4);...
            diff(xdot(3),x1), diff(xdot(3),x2), diff(xdot(3),x3), diff(xdot(3),x4);...
            diff(xdot(4),x1), diff(xdot(4),x2), diff(xdot(4),x3), diff(xdot(4),x4)];

B_sym   = [diff(xdot(1),u);diff(xdot(2),u);diff(xdot(3),u);diff(xdot(4),u)];

disp('--Part 2 ---------------------------------------------------------')
disp(' ')
disp('Linearized State Space Model')

% Linearized State Space Model
A       = double(subs(A_sym,[x1;x2;x3;x4;u],[x_star;u_star]))
B       = double(subs(B_sym,[x1;x2;x3;x4;u],[x_star;u_star]))
C       = [1 0 0 0;0 0 1 0]    % y = [x1;x3]
D       = zeros(2,1)

% Verify System is Controllable and Observable
CO      = ctrb(A,B);
Obs     = obsv(A,C);

if rank(CO) == length(A)
```

```
    disp('System is Controllable');
end

if rank(Obs) == length(A)
    disp('System is Observable');
end
```

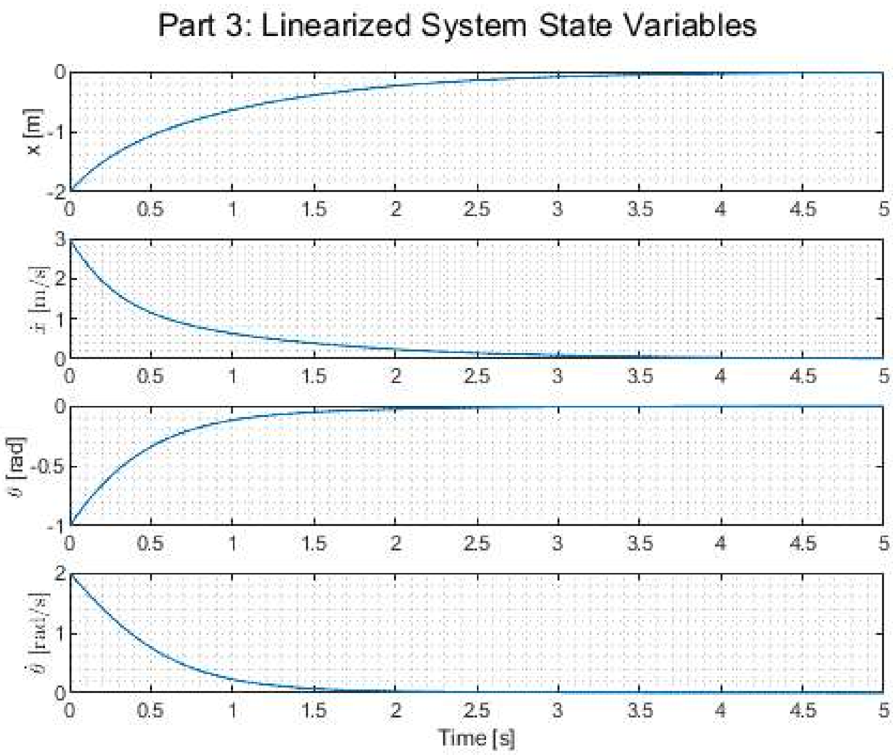
Part 3 Stabilizing Feedback Controller for Linearized Model, u = -Kx

del_xdot = A*del_x + B*del_u, del_u = -K*del_x, u = u_star + del_u, x = x_star + del_x, u = u_star - K*del_x => u = -K*x,

```
disp('--Part 3 --------------------------------------------------------')
disp(' ')

% desired poles
s_desired   = [-1;-4;-4 + 1i;-4 - 1i];

% Gain Matrix via ackerman formula
K           = acker(A,B,s_desired)

disp('Closed Loop Poles')
disp(eig(A - B*K))

% Simulate Controlled Linear System
x0          = [-2; 3;-1;2];     % Initial Conditions [m, ms/, rad, rad/s]

% ODE45 solver options
options     = odeset('AbsTol',1e-8,'RelTol',1e-8);

% ODE45 Function call
[T, X_lin]= ode45(@(t,x) ControlledLinearCartPendulum(t,x,A,B,K),time,x0,options);

figure
subplot(4,1,1)
sgtitle('Part 3: Linearized System State Variables')
plot(T,X_lin(:,1))
ylabel('x [m]')
grid minor
subplot(4,1,2)
plot(T,X_lin(:,2))
ylabel('$\dot{x}$ [m/s]','Interpreter','latex')
grid minor
subplot(4,1,3)
plot(T,X_lin(:,3))
grid minor
ylabel('\theta [rad]')
subplot(4,1,4)
plot(T,X_lin(:,4))
grid minor
ylabel('$\dot{\theta}$ [rad/s]','Interpreter','latex')
xlabel('Time [s]')
```

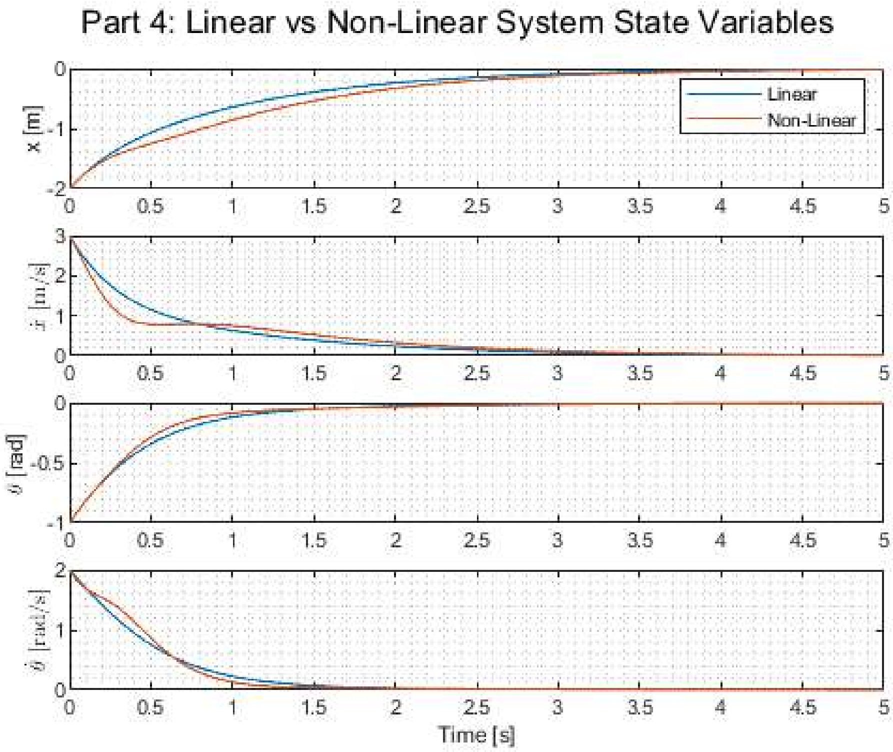
```
--Part 3 --------------------------------------------------------


K =

   -68  -117  -788  -247

Closed Loop Poles
  -4.0000 + 1.0000i
  -4.0000 - 1.0000i
```

```
-4.0000 + 0.0000i
-1.0000 + 0.0000i
```



Part 4 Feedback Controller on Nonlinear System

```
% ODE45 Function call
[T, X]      = ode45(@(t,x) ControlledCartPendulum(t,x,K,m,M,g,l),time,x0,options);

figure
subplot(4,1,1)
sgtitle('Part 4: Linear vs Non-Linear System State Variables')
plot(T,X_lin(:,1),T,X(:,1))
legend('Linear','Non-Linear')
ylabel('x [m]')
grid minor
subplot(4,1,2)
plot(T,X_lin(:,2),T,X(:,2))
ylabel('$\dot{x}$ [m/s]','Interpreter','latex')
grid minor
subplot(4,1,3)
plot(T,X_lin(:,3),T,X(:,3))
grid minor
ylabel('\theta [rad]')
subplot(4,1,4)
plot(T,X_lin(:,4),T,X(:,4))
grid minor
ylabel('$\dot{\theta}$ [rad/s]','Interpreter','latex')
xlabel('Time [s]')
```

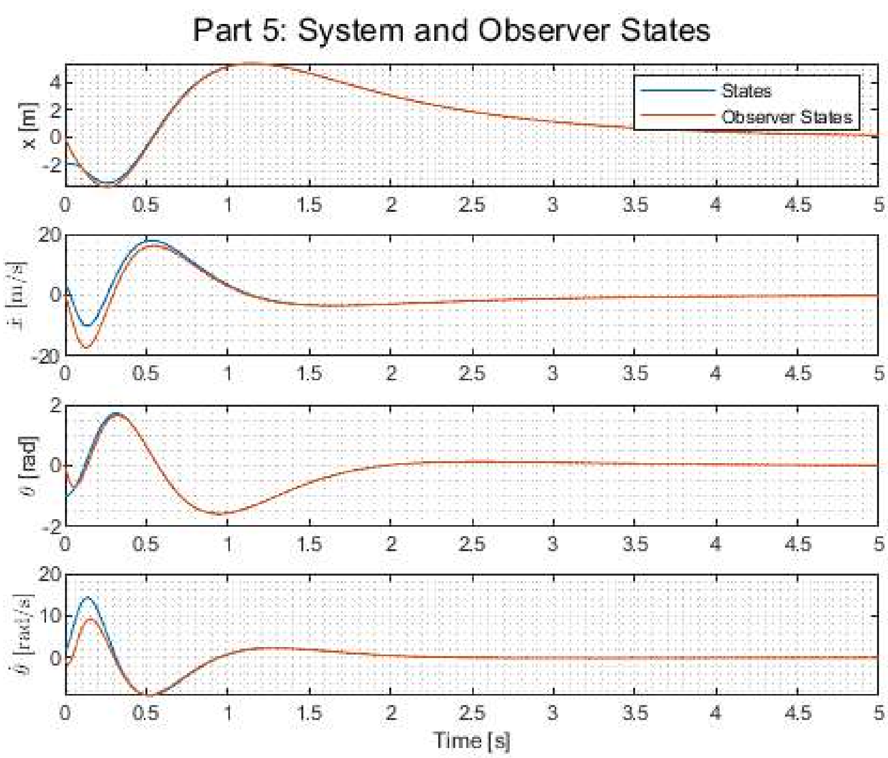


Part 5 State Observer Design with Linear System

```
disp('--Part 5 ---------------------------------------------------------')
disp(' ')

% desired observer poles
s_obs_desired   = [-3;-12;-12 + 1i;-12 - 1i];

% Observer Gain Matrix
L               = place(A',C',s_obs_desired)'

disp('Observer Poles')
disp(eig(A - L*C))

% Observer Initial Conditions
z0              = zeros(4,1);

% ODE45 Function call
[T, X_lin_obs]  = ode45(@(t,x) ControllerEstimatorLinearCartPendulum(t,x,A,B,C,K,L),time,[x0;z0],options);

figure
subplot(4,1,1)
sgtitle('Part 5: System and Observer States')
plot(T,X_lin_obs(:,1),T,X_lin_obs(:,5))
legend('States','Observer States')
ylabel('x [m]')
grid minor
subplot(4,1,2)
plot(T,X_lin_obs(:,2),T,X_lin_obs(:,6))
ylabel('$\dot{x}$ [m/s]','Interpreter','latex')
grid minor
subplot(4,1,3)
plot(T,X_lin_obs(:,3),T,X_lin_obs(:,7))
grid minor
ylabel('\theta [rad]')
subplot(4,1,4)
plot(T,X_lin_obs(:,4),T,X_lin_obs(:,8))
```

```
grid minor
ylabel('$\dot{\theta}$ [rad/s]','Interpreter','latex')
xlabel('Time [s]')
```

```
--Part 5 --------------------------------------------------------

L =

   15.4624    3.7298
   43.7948   43.0396
    0.6685   23.5376
   16.4663  146.7505

Observer Poles
  -3.0000 + 0.0000i
 -12.0000 + 1.0000i
 -12.0000 - 1.0000i
 -12.0000 + 0.0000i
```



Part 6 Closed Loop Transfer Function

```
disp('--Part 6 --------------------------------------------------------')
disp(' ')

% Closed Loop Transfer Function Equatiom: Y(s)/R(s) = C*(sI - A + BK)^-1*B
Y_R         = simplify(C*(inv(s*eye(size(A)) - A + B*K))*B);

% Closed loop matrices with states [x,e], where e is x - z
A_tilde     = [(A - B*K), B*K; zeros(size(A)), (A - L*C)];
B_tilde     = [B;zeros(size(B))];
C_tilde     = [C,zeros(size(C))];
```

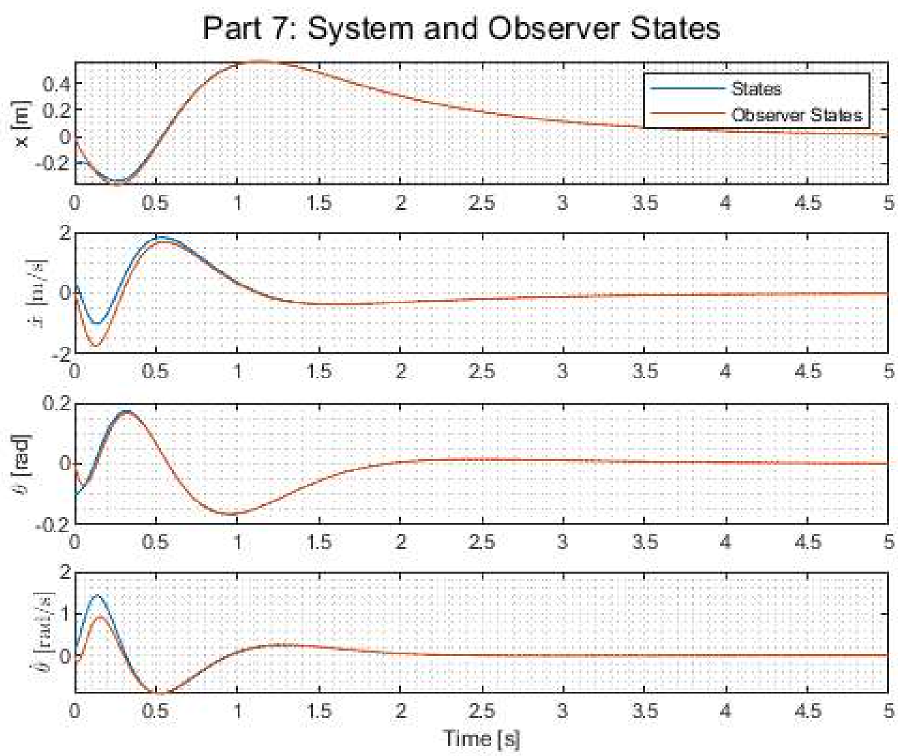
```
% Minimum realization for each output
[num,den]   = ss2tf(A_tilde,B_tilde,C_tilde,zeros(2,1),1);
Y1_R        = minreal(tf(num(1,:),den(1,:)));
Y2_R        = minreal(tf(num(2,:),den(1,:)));

% Verify both methods give same output
disp('Closed Loop Transfer Function: Y_1(s)/R(s): ss2tf output')
Y1_R
disp('Closed Loop Transfer Function: Y_1(s)/R(s): syms output')
disp(Y_R(1))

disp('Closed Loop Transfer Function: Y_2(s)/R(s): ss2tf output')
Y2_R
disp('Closed Loop Transfer Function: Y_2(s)/R(s): syms output')
disp(Y_R(2))
```

```
--Part 6 --------------------------------------------------------

Closed Loop Transfer Function: Y_1(s)/R(s): ss2tf output

Y1_R =

      0.1 s^2 - 1.732e-15 s - 1
  ----------------------------------
  s^4 + 13 s^3 + 61 s^2 + 117 s + 68

Continuous-time transfer function.

Closed Loop Transfer Function: Y_1(s)/R(s): syms output
(s^2 - 10)/(10*(s^4 + 13*s^3 + 61*s^2 + 117*s + 68))

Closed Loop Transfer Function: Y_2(s)/R(s): ss2tf output

Y2_R =

  -0.1 s^2 + 2.512e-16 s + 4.179e-30
  ----------------------------------
  s^4 + 13 s^3 + 61 s^2 + 117 s + 68

Continuous-time transfer function.

Closed Loop Transfer Function: Y_2(s)/R(s): syms output
-s^2/(10*(s^4 + 13*s^3 + 61*s^2 + 117*s + 68))
```

Part 7 State Observer with Non-Linear System

```
% ODE45 Function call
[T, X_obs]  = ode45(@(t,x) ControllerEstimatorCartPendulum(t,x,A,B,C,K,L,m,M,g,l),time,[-.2;.3;-.1;.2;z0],options);

figure
subplot(4,1,1)
sgtitle('Part 7: System and Observer States')
plot(T,X_obs(:,1),T,X_obs(:,5))
legend('States','Observer States')
ylabel('x [m]')
grid minor
subplot(4,1,2)
plot(T,X_obs(:,2),T,X_obs(:,6))
ylabel('$\dot{x}$ [m/s]','Interpreter','latex')
grid minor
subplot(4,1,3)
```

```
plot(T,X_obs(:,3),T,X_obs(:,7))
grid minor
ylabel('\theta [rad]')
subplot(4,1,4)
plot(T,X_obs(:,4),T,X_obs(:,8))
grid minor
ylabel('$\dot{\theta}$ [rad/s]','Interpreter','latex')
xlabel('Time [s]')
```



Functions

```
function xdot = NonlinearCartPendulum(t,state,u,m,M,g,l)
% Part 2 Function - nonlinear model

x1          = state(1,1);
x2          = state(2,1);
x3          = state(3,1);
x4          = state(4,1);

xdot(1,1)   = x2;
xdot(2,1)   = (u - m*g*cos(x3)*sin(x3) + m*l*x4^2*sin(x3))/(m + M - m*cos(x3)^2);
xdot(3,1)   = x4;
xdot(4,1)   = (-u*cos(x3) - m*l*x4^2*sin(x3)*cos(x3) + m*g*sin(x3) + M*g*sin(x3))/(l*(m + M - m*cos(x3)^2));

end

function [xdot]  = ControlledLinearCartPendulum(t,state,A,B,K)
% Part 3 Function - Linear Model with state feedback controller

% State Vector
x               = state(1:4,1);

% Feedback Control Law
u               = -K*x;

% State Space Equation
```

```
xdot            = A*x + B*u;
end

function [xdot] = ControlledCartPendulum(t,state,K,m,M,g,l)
% Part 4 Function - Non-linear model with state feedback controller

% State Vector
x1          = state(1,1);
x2          = state(2,1);
x3          = state(3,1);
x4          = state(4,1);
x           = state(1:4,1);

% Control Law
u           = -K*x;

% Non linear model
xdot(1,1)   = x2;
xdot(2,1)   = (u - m*g*cos(x3)*sin(x3) + m*l*x4^2*sin(x3))/(m + M - m*cos(x3)^2);
xdot(3,1)   = x4;
xdot(4,1)   = (-u*cos(x3) - m*l*x4^2*sin(x3)*cos(x3) + m*g*sin(x3) + M*g*sin(x3))/(l*(m + M - m*cos(x3)^2));

end

function [out] = ControllerEstimatorLinearCartPendulum(t,state,A,B,C,K,L)
% Part 5 Function - Linear Model with Luenberger Observer
% and Estimated State Feedback compensator

% State Vector
x       = state(1:4,1);

% Observer State Vector
z       = state(5:8,1);

% Control Law
u       = -K*z;

% Output, Y = Cx = [x1, x3]
Y       = [state(1,1);state(3,1)];

% Observer Dynamics -> dz/dt = Az + Bu + L(Y - C*z), u = -Kz
zdot    = (A - L*C - B*K)*z + L*Y;

% State Dynamics
xdot    = A*x + B*u;

out     = [xdot;zdot];
end

function [out] = ControllerEstimatorCartPendulum(t,state,A,B,C,K,L,m,M,g,l)
% Part 7 Function Non-Linear Model with Luenberger Observer
% and Estimated State Feedback compensator

% State Vector
x1          = state(1,1);
x2          = state(2,1);
x3          = state(3,1);
x4          = state(4,1);

% Observer State Vector
z           = state(5:8,1);

% Control Law
u           = -K*z;
```

```
% Output Y = [x1 x3]'
Y           = [x1;x3];

% Observer Dynamics
zdot        = (A - L*C - B*K)*z + L*Y;

% Non linear model
xdot(1,1)   = x2;
xdot(2,1)   = (u - m*g*cos(x3)*sin(x3) + m*l*x4^2*sin(x3))/(m + M - m*cos(x3)^2);
xdot(3,1)   = x4;
xdot(4,1)   = (-u*cos(x3) - m*l*x4^2*sin(x3)*cos(x3) + m*g*sin(x3) + M*g*sin(x3))/(l*(m + M - m*cos(x3)^2));

out         = [xdot;zdot];
end
```

```
--Part 2 ---------------------------------------------------------

Linearized State Space Model

A =

     0     1     0     0
     0     0    -1     0
     0     0     0     1
     0     0    11     0


B =

         0
    0.1000
         0
   -0.1000


C =

     1     0     0     0
     0     0     1     0


D =

     0
     0

System is Controllable
System is Observable
```

Published with MATLAB® R2021b