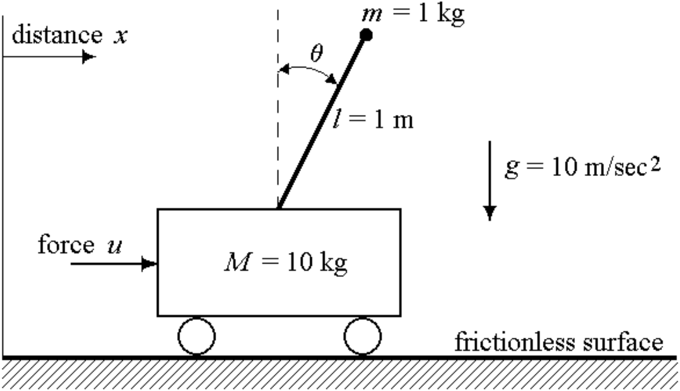
Stick balancer stabilizing feedback control law testing



Point mass on a mass-less shaft moving on a cart

- Modeling equations in matrix format

$$\begin{bmatrix} M+m & ml\cos\theta \\ \cos\theta & l \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} ml\dot{\theta}^2\sin\theta + u \\ g\sin\theta \end{bmatrix}$$

- Solve for $\begin{bmatrix} \ddot{x} & \ddot{\theta} \end{bmatrix}^\top$ using MATLAB

```
D=sym('[M+m m*l*cos(theta);cos(theta) l]');
v=sym('[u+m*l*thetadot^2*sin(theta);g*sin(theta)]');
D_inv=inv(D);
g=symmul(D_inv,v);
simplify(g);
pretty(ans)
```

Non-linear state-space model

- Let $\Delta = M + m - m\cos^2\theta$
- Then

$$\begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} u + ml\dot{\theta}^2\sin\theta - mg\cos\theta\sin\theta \\ \frac{1}{l}(-u\cos\theta - ml\dot{\theta}^2\cos\theta\sin\theta + gM\sin\theta + gm\sin\theta) \end{bmatrix}$$

- Non-linear state-space model, which is our simulation model,

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{mlx_4^2\sin x_3 - mg\cos x_3\sin x_3 + u}{M + m - m\cos^2 x_3} \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{-mlx_4^2\cos x_3\sin x_3 + gM\sin x_3 + gm\sin x_3 - \cos x_3 u}{l(M + m - m\cos^2 x_3)} \end{cases}$$

The linearized model about $\boldsymbol{x} = \mathbf{0}$, $u = 0$

- The linearized model, which is our design model,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 11 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} u$$

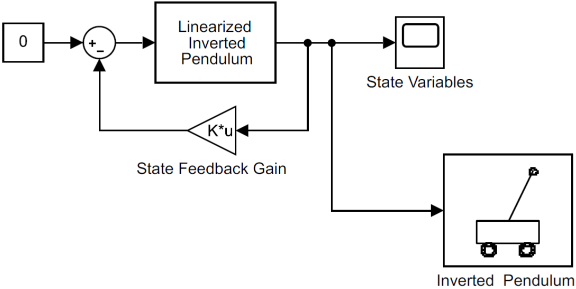
$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

- The state-feedback control law $u = -\boldsymbol{kx}$ such that the closed-loop poles are located at $\{-1, -2, -1 \pm j\}$ is

$$u = -\boldsymbol{kx} = -\begin{bmatrix} -0.4 & -1 & -21.4 & -6 \end{bmatrix} \boldsymbol{x}$$

- Can use MATLAB's functions `acker` or `place` to compute the feedback gain

Simulink animation



Double click on 'Linearized Inverted Pendulum'
to change the initial conditions

If the animation if too fast or too slow,
double click on 'Inverted Pendulum Animation'
to change the sampling time of animation.

A snapshot of the animation using SIMULINK