7.7.4) $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = (a_{11})(a_{22}) - (a_{12})(a_{21}) = 2$ multiplications

$2! = 2$, $3! = 6$ multiplications

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \underbrace{\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}}_{2\times} - a_{12} \underbrace{\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}}_{2\times} + a_{13} \underbrace{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}}_{2\times}$$

$|A_{3\times3}| = 6$ multiplications

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix} = a_{11} \underbrace{\begin{vmatrix} a_{22} & a_{23} & a_{33} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{vmatrix}}_{6\times} - a_{12} \underbrace{\left|\quad\right|}_{6\times} + a_{12} \underbrace{\left|\quad\right|}_{6\times} + a_{14} \underbrace{\left|\quad\right|}_{6\times}$$

$|A_{4\times4}| = 24$ multiplications $= 4!$

$\therefore \boxed{|A_{n\times n}| = n! \text{ multiplications}}$

HW2

7.7.7) $\begin{vmatrix} \cos(\alpha) & \sin(\alpha) \\ \sin(\beta) & \cos(\beta) \end{vmatrix}$

$\cos(x)\cos(y) = \frac{1}{2}\cos(x+y) + \frac{1}{2}\cos(x-y)$
$\sin(x)\sin(y) = \frac{1}{2}\cos(x-y) - \frac{1}{2}\cos(x+y)$

$\cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) = \cos(\alpha+\beta)$ (Identity)

$\boxed{\begin{vmatrix} c\alpha & s\beta \\ s\beta & c\beta \end{vmatrix} = \cos(\alpha+\beta)}$

7.7.12) $\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} = a\begin{vmatrix} a & b \\ c & a \end{vmatrix} - b\begin{vmatrix} c & b \\ b & a \end{vmatrix} + c\begin{vmatrix} c & a \\ b & c \end{vmatrix}$

$= a(a^2 - bc) - b(ca - b^2) + c(c^2 - ba) = a^3 - bca + b^3 - bca + c^3 - bca$

$= \boxed{a^3 + b^3 + c^3 - 3bca}$

7.7.22) $2x - 4y = -24$
$5x + 2y = 0$
$\Rightarrow \begin{pmatrix} 2 & -4 \\ 5 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -24 \\ 0 \end{pmatrix}$

Cramers:

$x = \dfrac{\begin{vmatrix} -24 & -4 \\ 0 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & -4 \\ 5 & 2 \end{vmatrix}} = \dfrac{-48}{4+20} = \boxed{-2}$

$y = \dfrac{\begin{vmatrix} 2 & -24 \\ 5 & 0 \end{vmatrix}}{\begin{vmatrix} 2 & -4 \\ 5 & 2 \end{vmatrix}} = \dfrac{120}{24} = \boxed{5}$

$\boxed{\begin{matrix} x = -2 \\ y = 5 \end{matrix}}$

Gauss: $\left(\begin{array}{cc|c} 2 & -4 & -24 \\ 5 & 2 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{cc|c} 2 & -4 & -24 \\ 0 & 12 & 60 \end{array}\right) \Rightarrow \left(\begin{array}{cc|c} 2 & -4 & -24 \\ 0 & \boxed{1} & 5 \end{array}\right)$

$\therefore\ y = 5, \quad 2x - 20 = -24 \quad \therefore\ x = -2$

$\boxed{\begin{array}{c} x = -2 \\ y = 5 \end{array}}$ ← Matches results from cramers rule

7.8.2) $\begin{pmatrix} \cos(2\theta) & \sin(2\theta) \\ -\sin(2\theta) & \cos(2\theta) \end{pmatrix} \triangleq A$

$\cos^2(2\theta) + \sin^2(2\theta) = 1 = \begin{vmatrix} \cos(2\theta) & \sin(2\theta) \\ -\sin(2\theta) & \cos(2\theta) \end{vmatrix} \quad \therefore\ A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} \cos(2\theta) & -\sin(2\theta) \\ \sin(2\theta) & \cos(2\theta) \end{pmatrix}$

$\therefore\ \boxed{A^{-1} = \begin{pmatrix} \cos(2\theta) & -\sin(2\theta) \\ \sin(2\theta) & \cos(2\theta) \end{pmatrix}}$

$AA^{-1} = I = \begin{pmatrix} \cos(2\theta) & \sin(2\theta) \\ -\sin(2\theta) & \cos(2\theta) \end{pmatrix} \begin{pmatrix} \cos(2\theta) & -\sin(2\theta) \\ \sin(2\theta) & \cos(2\theta) \end{pmatrix}$

$= \begin{pmatrix} \cos^2(2\theta) + \sin^2(2\theta) & -\cos(2\theta)\sin(2\theta) + \cos(2\theta)\sin(2\theta) \\ -\sin(2\theta)\cos(2\theta) + \cos(2\theta)\sin(2\theta) & \sin^2(2\theta) + \cos^2(2\theta) \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ✓

7.8.5) $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 4 & 1 \end{pmatrix}$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 5 & 4 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 0 & 4 & 1 & -5 & 0 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 4 & 1 & -5 & 0 & 1 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 1 & 0 & 3 & -4 & 1 \end{array}\right)$$

$$\boxed{A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & -4 & 1 \end{pmatrix}} \qquad AA^{-1} = I = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 4 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & -4 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 2-2 & 1 & 0 \\ 5-8+3 & 4-4 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \checkmark$$

7.8.20) $\begin{pmatrix} -4 & 0 & 0 \\ 0 & 8 & 13 \\ 0 & 3 & 5 \end{pmatrix} = A$

$\det(A) = -4(40 - 39) = -4$

$C_{11} = 1$

$C_{22} = \begin{vmatrix} -4 & 0 \\ 0 & 5 \end{vmatrix} = -20$

$C_{33} = \begin{vmatrix} -4 & 0 \\ 0 & 8 \end{vmatrix} = -32$

$C_{23} = (-1)\begin{vmatrix} -4 & 0 \\ 0 & 3 \end{vmatrix} = 12$

$C_{32} = (-1)\begin{vmatrix} -4 & 0 \\ 0 & 13 \end{vmatrix} = 52$

$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -20 & 12 \\ 0 & 52 & -32 \end{pmatrix} \quad \therefore A^{-1} = \frac{1}{\det(A)} C^T$

$$\boxed{A^{-1} = \begin{pmatrix} -1/4 & 0 & 0 \\ 0 & 5 & -13 \\ 0 & -3 & 8 \end{pmatrix}}$$

$$\left(\begin{array}{ccc|ccc} -4 & 0 & 0 & 1 & 0 & 0 \\ 0 & 8 & 13 & 0 & 1 & 0 \\ 0 & 3 & 5 & 0 & 0 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{4} & 0 & 0 \\ 0 & 8 & 13 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{8} & 0 & -\frac{3}{8} & 1 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/4 & 0 & 0 \\ 0 & 8 & 13 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -3 & 8 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/4 & 0 & 0 \\ 0 & 8 & 0 & 0 & 40 & -104 \\ 0 & 0 & 1 & 0 & -3 & 8 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/4 & 0 & 0 \\ 0 & 8 & 0 & 0 & 40 & -104 \\ 0 & 0 & 1 & 0 & -3 & 8 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/4 & 0 & 0 \\ 0 & 1 & 0 & 0 & 5 & -13 \\ 0 & 0 & 1 & 0 & -3 & 8 \end{array}\right)$$

$$\boxed{A^{-1} = \begin{pmatrix} -1/4 & 0 & 0 \\ 0 & 5 & -13 \\ 0 & -3 & 8 \end{pmatrix}}$$

7.9.3)

$$-V_1 + 2V_2 + 3V_3 = 0$$
$$-4V_1 + V_2 + V_3 = 0$$

$$\begin{pmatrix} -1 & 2 & 3 \\ -4 & 1 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} \boxed{-1} & 2 & 3 \\ 0 & \boxed{-7} & -11 \end{pmatrix}$$

Let $V_3 = 1$ $\therefore$ $-7V_2 - 11 = 0$ $V_2 = -11/7$

$-V_1 + 2V_2 + 3V_3 = -V_1 - 22/7 + 3 = 0 \Rightarrow V_1 = -1/7$

$$\boxed{\text{Basis} = \text{span}\left\{\begin{pmatrix} -1 \\ -11 \\ 1 \end{pmatrix}\right\}, \ \dim = 1}$$

⟵ Vecter space

7.9.4) $\mathbb{R}^3$ Skew symmetric : $A = \begin{bmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & a & 0 \end{bmatrix}$

$$A = a\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} + b\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} + c\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Skew-symmetric matrices are vector spaces with dim=3 and basis:

$$\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

7.9.9) $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ $\quad a_{11} + a_{22} = 0 \quad \therefore \quad a_{22} = -a_{11}$

$$A = a_{11}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + a_{12}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + a_{21}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

Vecter space with dim = 3 & basis $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

7.9.12) $y_1 = 3x_1 + 2x_2$ $\quad y = Ax$

$y_2 = 4x_1 + x_2$

$$A = \begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix} \qquad A^{-1} = \frac{1}{3-8}\begin{pmatrix} 1 & -2 \\ -4 & 3 \end{pmatrix} = \begin{pmatrix} -1/5 & 2/5 \\ 4/5 & -3/5 \end{pmatrix}$$

$$x = \begin{pmatrix} -1/5 & 2/5 \\ 4/5 & -3/5 \end{pmatrix} y$$

7.9.22) $\begin{bmatrix} 2 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 2x_1 + x_3 = 0$

$$V = \text{span}\left\{ \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \right\}$$

8.1.4) $A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$

$|A - \lambda I| = 0 = \lambda^2 - 5\lambda = \lambda(\lambda - 5)$

$\boxed{\lambda_1 = 0 \quad \lambda_2 = 5}$

$A v_1 = \lambda_1 v_1$

$\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ Let $x_2 = 1 \therefore x_1 = -2$

$\boxed{v_1 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}}$

$A v_2 = \lambda_2 v_2 \Rightarrow \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 5v_1 \\ 5v_2 \end{pmatrix}$

Let $x_2 = 1 \therefore x_1 = 1/2$

$\boxed{v_2 = \begin{pmatrix} 1/2 \\ 1 \end{pmatrix}}$

8.4.11) $A = \begin{pmatrix} 6 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & 7 \end{pmatrix}$ $A - \lambda I = \begin{pmatrix} 6-\lambda & 2 & -2 \\ 2 & 5-\lambda & 0 \\ -2 & 0 & 7-\lambda \end{pmatrix}$

$$|A-\lambda I| = (6-\lambda)(5-\lambda)(7-\lambda) - 2(2)(7-\lambda) + -2(2)(5-\lambda)$$

$$= (30 - 6\lambda - 5\lambda + \lambda^2)(7-\lambda) - 28 + 4\lambda - 20 + 4\lambda$$

$$= (30 - 11\lambda + \lambda^2)(7-\lambda) - 48 + 8\lambda$$

$$= 210 - 30\lambda - 77\lambda + 11\lambda^2 + 7\lambda^2 - \lambda^3 - 48 + 8\lambda$$

$$= -\lambda^3 + 18\lambda^2 - 99\lambda + 162$$

$$= (\lambda-3)(-\lambda^2 + a\lambda + b) = -\lambda^3 + 18\lambda^2 - 99\lambda + 162$$

$$= \lambda^3 + a\lambda^2 + b\lambda + 3\lambda^2 - 3a\lambda - 3b$$

$\therefore$ $(a+3)\lambda^2 = 18\lambda^2$ $\quad\therefore a = 15$

$(b - 3a)\lambda = -99\lambda$ $\quad\therefore b = -54$

$$\therefore -\lambda^3 + 18\lambda^2 - 99\lambda + 162 = (\lambda-3)(-\lambda^2 + 15\lambda - 54) = (\lambda-3)(\lambda-6)(\lambda-9)$$

$$\therefore \boxed{\lambda_1 = 3, \ \lambda_2 = 6, \ \lambda_3 = 9}$$

$Av = \lambda v \Rightarrow (A-\lambda I)v = 0$

$$\begin{pmatrix} 3 & 2 & -2 \\ 2 & 2 & 0 \\ -2 & 0 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 3 & 2 & -2 & 0 \\ 2 & 2 & 0 & 0 \\ -2 & 0 & 4 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 3 & 2 & -2 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 2 & 4 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 3 & 2 & -2 & 0 \\ 0 & 2/3 & 4/3 & 0 \\ 0 & 2 & 4 & 0 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{ccc|c} \boxed{3} & 2 & -2 & 0 \\ 0 & \boxed{2} & 4 & 0 \\ 0 & 2 & 4 & 0 \end{array}\right)$$

Choose $x_3 = 1$

$2x_2 + 4x_3 = 0 \quad \therefore \quad x_2 = -4/2 = -2$

$3x_1 + 2x_2 \cdot 2x_3 = 0$

$3x_1 - 4 - 2 = 0 \quad \therefore \quad x_1 = 2$

$$\boxed{V = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}}$$

8.1.12) $\begin{pmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{pmatrix}$ → Triangular matrix, diagonals are eigenvalues

$\therefore \quad \boxed{\lambda_1 = 3, \ \lambda_2 = 4, \ \lambda_3 = 1}$

$(A - \lambda_1 I) V_1 = 0 \qquad \begin{pmatrix} 0 & 5 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 0 & 5 & 3 \\ 0 & 0 & 27/5 \\ 0 & 0 & -2 \end{pmatrix} \Rightarrow \left(\begin{array}{ccc|c} 0 & \boxed{5} & 3 & 0 \\ 0 & 0 & \boxed{27/5} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$, choose $x_1 = 1$

$x_3 = 0, \quad 5x_2 + 3\cancel{x_3}^{0} = 0$

$x_2 = 0$

$$\boxed{V_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}$$

$(A-\lambda_2 I)V_2 = \vec{0}$

$$\left(\begin{array}{ccc|c} -1 & 5 & 3 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} \boxed{-1} & 5 & 3 & 0 \\ 0 & 0 & \boxed{6} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Choose $X_2 = 1$: $\quad 6X_3 = 0 \Rightarrow X_3 = 0$

$-x_1 + 5x_2 + 3x_3 = -x_1 + 5 = 0 \qquad x_1 = 5$

$$\boxed{V_2 = \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix}}$$

$(A-\lambda_3 I)V_3 = 0$

$$\begin{pmatrix} \boxed{2} & 5 & 3 \\ 0 & \boxed{3} & 6 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad \text{Let } x_3 = 1$$

$\therefore 3x_2 + 6 = 0 \qquad x_2 = -2$

$2x_1 + 5x_2 + 3x_3 = 2x_1 - 10 + 3 = 0 \therefore x_1 = 7/2$

$$\boxed{V_3 = \begin{pmatrix} 7/2 \\ -2 \\ 1 \end{pmatrix}}$$

8.1.24) A^{-1} exits if $|A| \neq 0$

$|A-\lambda I|$, if $\lambda = 0 \Rightarrow$ $\boxed{|A| = 0 \therefore A \text{ is not Invertible}}$

invert.

$A^{-1}A = I$

$|A^{-1} - \lambda I| = |A^{-1} - \lambda(A^{-1}A)| = = |A^{-1}(I - \lambda A)| = 0$

$|A^{-1}||I - \lambda A| = |A^{-1}(I - \lambda A)| = 0$

$|A^{-1}| = \frac{1}{|A|} \therefore \frac{1}{|A|}|I - \lambda A| = 0$ (Don't know where to go from here)

8.3.3) $\begin{pmatrix} 2 & 8 \\ -8 & 2 \end{pmatrix}$ $\lambda^2 - 4\lambda + 68 = 0$

$\lambda_{1,2} = \frac{2 \pm \sqrt{16 - (68)4}}{2} = 2 \pm 8i$

$\boxed{\lambda_{1,2} = 2 \pm 8i \text{, Not symmetric, skew-symmetric, or orthogonal}}$

8.3.6) $\begin{pmatrix} a & k & k \\ k & a & k \\ k & k & a \end{pmatrix}$ $|A - \lambda I| = \begin{vmatrix} a-\lambda & k & k \\ k & a-\lambda & k \\ k & k & a-\lambda \end{vmatrix}$

$$|A-\lambda I| = (a-\lambda)\begin{vmatrix} a-\lambda & k \\ k & a-\lambda \end{vmatrix} - k\begin{vmatrix} k & k \\ k & a-\lambda \end{vmatrix} + k\begin{vmatrix} k & a-\lambda \\ k & k \end{vmatrix}$$

$$= (a-\lambda)((a-\lambda)^2 - k^2) - k(k(a-\lambda) - k^2) + k(k^2 - k(a-\lambda)$$

$$= (a-\lambda)((a-\lambda)^2 - k^2) - 2k^2(a-\lambda) + 2k^3$$

$$= (a-\lambda)^3 - 3k^2(a-\lambda) + 2k^3$$

$$= (\lambda-(a-k))^2 + (\lambda-(a+2k))$$

$$\boxed{\begin{array}{l} \lambda_1 = \lambda_2 = a-k \\ \lambda_3 = a+2k \\ \text{Symmetric Matrix} \end{array}}$$

Assume a & k are real

3.6.8) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$

$$|A-\lambda I| = \begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & \cos\theta-\lambda & -\sin\theta \\ 0 & \sin\theta & \cos\theta-\lambda \end{vmatrix} = (1-\lambda)[((\cos\theta-\lambda)^2 + \sin\theta^2]$$

$$|A-\lambda I| = (1-\lambda)[\cos^2\theta + \lambda^2 - 2\lambda\cos\theta + \sin^2\theta]$$

$$= (1-\lambda)(\lambda^2 - 2\cos\theta\lambda + 1) = 0$$

$$\boxed{\lambda_1 = 1}$$

$$\lambda_{2,3} = \frac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2} = \frac{2\cos\theta \pm \sqrt{4}\sqrt{\cos^2\theta - 1}}{2}$$

$$= \cos\theta \pm \sqrt{-\sin^2\theta} = \boxed{\cos\theta \pm \sin\theta\, i}$$

$$|\lambda_{2,3}| = \sqrt{\cos^2\theta + \sin^2\theta} = \sqrt{1} = 1$$

$$|\lambda_1| = |\lambda_{2,3}| = 1 \quad \therefore \quad \boxed{\text{orthogonal matrix}}$$