Root locus: locus of roots of CL system as parameter varies from 0

$a > 0$

$$\frac{Y}{R} = \frac{K}{s^2+as+K}$$

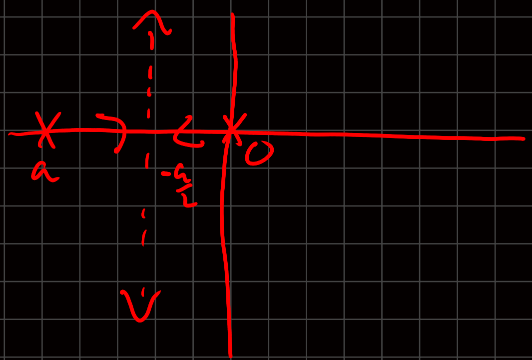
when $K=0$: $s = 0, -a$

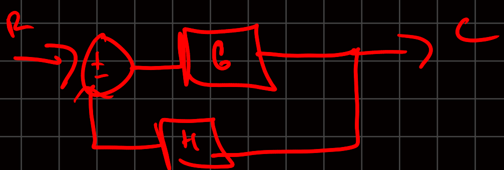
In general: $s = \dfrac{-a \pm \sqrt{a^2-4K}}{2} = -\dfrac{a}{2} \pm \sqrt{\left(\dfrac{a}{2}\right)^2 - K}$

$0 < K < \left(\frac{a}{2}\right)^2 \rightarrow$ negative real poles

$K = \left(\frac{a}{2}\right)^2 \rightarrow s = -\frac{a}{2}, -\frac{a}{2}$

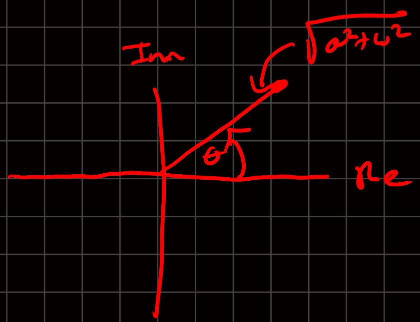
$K > \left(\frac{a}{2}\right)^2 \rightarrow s = -\frac{a}{2} \pm j\sqrt{K - \left(\frac{a}{2}\right)^2}$





$$\frac{C}{R} = \frac{G}{1+GH}$$

Characteristic equation $1 + GH = 0$, gives stability info



$s = \sigma + j\omega = \sqrt{\sigma^2+\omega^2}\, e^{j\theta}$

$\theta = \tan^{-1}\left(\frac{\omega}{\sigma}\right)$

Characteristic equation in polar form: $G(s)\,H(s) = -1$

$$|GH|\, e^{j\angle GH} = 1\, e^{\pm j\pi}$$

$|GH| = |-1| = 1$

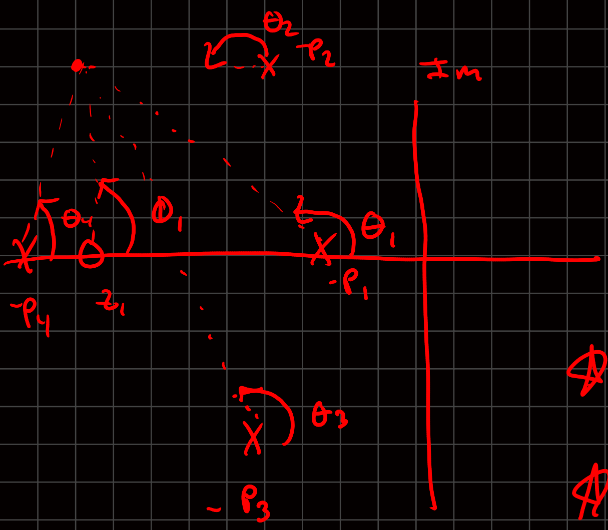
$\angle GH = \pm\pi(2k+1)$

Assume Characteristic equation of

$$1 + \underbrace{\frac{K(s+z_1)(s+z_2)\ldots(s+z_m)}{(s+p_1)(s+p_2)\ldots(s+p_n)}}_{GH} = 0$$

Consider $GH = \dfrac{K(s+z_1)}{(s+p_1)(s+p_2)(s+p_3)(s+p_4)} = \dfrac{K B_1 e^{j\phi_1}}{A_1A_2A_3A_4 e^{j(\theta_1+\theta_2+\theta_3+\theta_4)}}$

$$= \frac{K B_1}{A_1A_2A_3A_4} e^{j(\phi_1-\theta_1-\theta_2-\theta_3-\theta_4)}$$



$s+p_2 = A_2 e^{j\theta_2}$

$s+p_1 = A_1 e^{j\theta_1}$

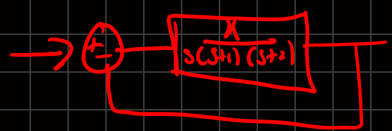
....

Conditions

$\left|\dfrac{K B_1}{A_1A_2A_3A_4}\right| = 1$

$\angle \phi_1 - \theta_1 - \theta_2 - \theta_3 - \theta_4 = \pi$

Ex)



poles at $s = 0, -1, -2$

Asymptotes as $K \to \infty$

$$\lim_{s\to\infty}(GH) = \lim_{s\to\infty}\left(\frac{K}{s(s+1)(s+2)}\right) = \lim_{s\to\infty}\left(\frac{K}{s^3}\right)$$

$\angle \dfrac{K}{s^3} = \pm\pi(2K+1)$ $\quad s = Ae^{j\theta}$ $\quad \therefore \angle\dfrac{K}{s^3} = \angle\dfrac{K}{(Ae^{j\theta})^3}$

$= \angle \dfrac{K}{A^3} e^{-3\theta j}$

$-3\theta = \pm\pi(2K+1)$

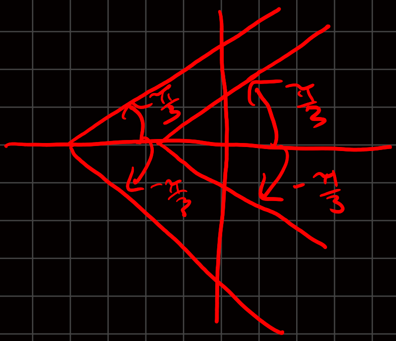
$\theta = \pm\dfrac{\pi}{3}(2K+1)$

$K=0, \ \theta = \pm\frac{\pi}{3}$

$K=1, \ \theta = \pm\pi$

$K=2, \ \theta = \pm\frac{\pi}{3}$

$K=3, \ \theta = \pm\frac{\pi}{3}$



$$GH = \frac{K}{s(s+1)(s+2)} = \frac{K}{s^3+3s^2+2s} \approx \frac{K}{(s+a)^3} \approx \frac{K}{s^3+3as^2+3a^2s+a^3} \approx \frac{K}{(s+1)^3}$$

$$\angle \frac{K}{(s+1)^3} = \pm\pi(2k+1) \qquad = -3\angle s+1 = \pm\pi(2k+1)$$

$$\angle s+1 = \pm\frac{\pi}{3}(2k+1) \qquad s = \sigma + i\omega$$

$$\angle \sigma + i\omega + 1 = \pm\frac{\pi}{3}(2k+1)$$

$$\tan\left(\tan^{-1}\left(\frac{\omega}{\sigma+1}\right)\right) = \pm\frac{\pi}{3}, \pm\pi$$

$$\frac{\omega}{\sigma+1} = \sqrt{3} \qquad \omega = \sqrt{3}\sigma + \sqrt{3}$$, line on complex plane