AAE 532 - ORBIT MECHANICS

PURDUE UNIVERSITY

PS8 Solutions

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Contents

Useful Constants

	Axial Rotaional Period (Rev/Day)	Mean Equatorial Radius (km)	Gravitational Parameter $\mu = Gm(km^3/sec^2)$	Semi-major Axis of orbit (km)	Orbital Period (sec)	Eccentricity of Orbit	Inclination of Orbit to Ecliptic (deg)
☉ Sun	0.0394011	695990	132712440017.99	-	-	-	-
☾ Moon	0.0366004	1738.2	4902.8005821478	384400 (around Earth)	2360592 27.32 Earth Days	0.0554	5.16
☾ Mercury	0.0170514	2439.7	22032.080486418	57909101	7600537 87.97 Earth Days	0.20563661	7.00497902
♀ Venus	0.0041149 (Retrograde)	6051.9	324858.59882646	108207284	19413722 224.70 Earth Days	0.00676399	3.39465605
⊕ Earth	1.0027378	6378.1363	398600.4415	149597898	31558205 365.26 Earth Days	0.01673163	0.00001531
♂ Mars	0.9747000	3397	42828.314258067	227944135	59356281 686.99 Earth Days	0.09336511	1.84969142
♃ Jupiter	2.4181573	71492	126712767.8578	778279959	374479305 11.87 Years	0.04853590	1.30439695
♄ Saturn	2.2522053	60268	37940626.061137	1427387908	930115906 29.47 Years	0.05550825	2.48599187
⛢ Uranus	1.3921114 (Retrograde)	25559	5794549.0070719	2870480873	2652503938 84.05 Years	0.04685740	0.77263783
Ψ Neptune	1.4897579	25269	6836534.0638793	4498337290	5203578080 164.89 Years	0.00895439	1.77004347
♇ Pluto	-0.1565620 (Retrograde)	1162	981.600887707	5907150229	7830528509 248.13 Years	0.24885238	17.14001206

- First three columns of the body data are consistent with GMAT 2020a default values, which are mainly from JPL's ephemerides file de405.spk

- The rest of the data are from JPL website(`https://ssd.jpl.nasa.gov/?planet_pos`, retrieved at 09/01/2020)
based on E.M. Standish's "Keplerian Elements for Approximate Positions of the Major Planets"

Problem 1

Problem Statement

The due dates for the problem sets are known to be Fridays at 9:30 am ET. What is the calendar date and time of day for the Julian Date given above? Well that is not correct!! What is the correct JD for the Problem Set 8 due date?

What is the Julian Date on the day YOU were born?

Solution

According to the JPL Julian date converter, the given Julian date of 2460615.5625000 corresponds to a calendar date of Nov 1 2024 1:30:00 UTC. Converting this to our local time zone at Purdue yields

$$\boxed{\text{Oct 31 2024 21:30:00 EDT}}$$

The due date for the assignment is Nov 1 2024 9:30:00 EDT, which we can convert to UTC and then Julian date:

$$\boxed{2460616.0625000}$$

The Julian date for someone born Jan 1 2000 at 00:00:00 UTC converted to Julian date would be:

$$\boxed{2451544.5000000}$$

Source: https://ssd.jpl.nasa.gov/tools/jdc//cd

Problem 2

Problem Statement

The year is 20??. Space Station Alpha is to receive a shipment of mercury for its heating system. The shipment of metal is in a low, circular Earth orbit of radius 1.04 $R_{\oplus}$, while Alpha is in a circular orbit at 18 $R_{\oplus}$. Since the supply ship can be launched well in advance, time is not a problem. Assume a relative two-body problem and consider possible transfers.

(a) The shipment originates in an orbit at what altitude? Is that reasonable?

(b) Consider first a planar Hohmann transfer:

   (i) Determine semi-major axis, eccentricity and period for the intermediate orbit that is required to produce the transfer. How can you confirm that the intermediate orbit that intersects both the departure orbit and planned final orbit? Do it!

   (ii) Determine the $|\Delta\bar{v}|, \alpha$ for each maneuver and the total Δv. Compute the total TOF, or the time-of flight, along the intermediate orbit. What is the transfer angle (TA)?

   (iii) In the same figure, plot the initial orbit; the arc for the intermediate orbit employed for the transfer; the final orbit. Input the orbits and the maneuvers to GMAT and print the plot.

   (iv) Of course, this transfer is a rendezvous problem; your transfer vehicle must rendezvous with the space station. What is the phase angle between the transfer vehicle and the rendezvous vehicle at departure? What is the synodic period?

(c) Determine the planar bielliptic transfer if the intermediate distance is 66 $R_{\oplus}$ (just outside the lunar orbital distance!).

   (i) Determine the $|\Delta\bar{v}|, \alpha$ for each maneuver and the total Δv. Compute the total TOF, or the time-of flight, along the two intermediate orbits. What is the transfer angle?

   (ii) In the same figure, plot the initial orbit; the arcs for the intermediate orbits employed for the transfer; the final orbit. Input the orbits and the maneuvers to GMAT and print the plot.

   (iii) It is still a rendezvous problem. Determine the departure phase angle? Synodic period?

(d) In evaluating your results, consider the location of Space Station Alpha. Speculate on the assumption of a relative two-body problem. Is it reasonable? Is it a good assumption for preliminary analysis? (You can also add the lunar and/or solar gravity in GMAT and see if it makes a difference!)

Part (a)

First we note that since the radius of the shipment orbit is $1.04R_{\oplus}$, that means the altitude at any given point in this orbit is $0.04R_{\oplus}$. Converting that to more intuitive units:

$$0.04R_{\oplus} = 255.1255\ km$$

This is an extremely low altitude for any spacecraft to be in. Generally, LEO (low Earth orbit) is considered to range from 300 km to about 2000 km, so while not theoretically impossible, the given orbit is not exactly reasonable. If implemented it would require excessive amounts of Δv to maintain.

Part (b)

First, we can visualize the Hohmann transfer where the initial orbit containing the shipment is at a radius of $1.04R_{\oplus}$ and the destination orbit where Space Station Alpha is located is at a radius of $18R_{\oplus}$:

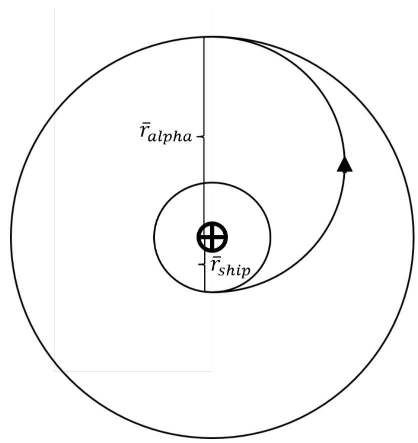

Figure 1: Hohmann transfer diagram between orbits.

i

Now we can analyze the transfer orbit:

$$a_t = \frac{r_p + r_a}{2} = 60719.857\ km\ \boxed{= 9.52R_{\oplus}}$$

$$r_p = a(1-e)$$

$$e_t = 1 - \frac{r_p}{a_t}\ \boxed{= 0.8908}$$

$$P = 2\pi\sqrt{\frac{a_t^3}{\mu}}\ \boxed{= 1.7234\ days}$$

For a Hohmann transfer the transfer orbit is defined such that the periapsis has the same r as the initial orbit and the apoapsis has the same r as the final orbit, and the rest of the problem is set up under this assumption.

ii

The vector diagrams for both maneuvers are shown below:

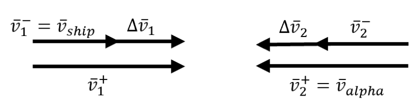

Figure 2: Vector diagrams for Hohmann transfer maneuvers.

For a Hohmann transfer we know the burns take place at the periapse and apoapsis of the transfer orbit and are tangential by definition. Therefore each Δv is just the difference between the initial and final velocities at the corresponding point and each velocity is computed as:

$$v = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}$$

$$\Delta v_1 = v_1^+ - v_1^- \boxed{= 2.9073\ km/s}$$

$$\Delta v_2 = v_2^+ - v_2^- \boxed{= 1.2474\ km/s}$$

And since both maneuvers are tangential and in the direction of motion:

$$\boxed{\alpha_1 = \alpha_2 = 0°}$$

The total Δv is just the sum of the two maneuvers:

$$\Delta v_{total} = \Delta v_1 + \Delta v_2 \boxed{= 4.1548\ km/s}$$

Finally, the definition of a Hohmann transfer is that the spacecraft traverses half of the total intermediate orbit. Therefore we can determine the transfer angle and transfer time by inspection:

$$\boxed{TA = 180°}$$

$$TOF = \frac{1}{2}P \boxed{= 0.8617\ days}$$

iii

We can plot the initial and final orbits as well as the transfer arc in GMAT using an arbitrary epoch of Oct 1 2024. The transfer arc is created by initializing a satellite with the same orbital characteristics as the shipment orbit and employing two maneuvers: Δv_1 at $\theta^* = 0°$ and Δv_2 at $\theta^* = 180°$. Note that the initial orbit is red, the transfer arc is blue, and the final orbit is green:

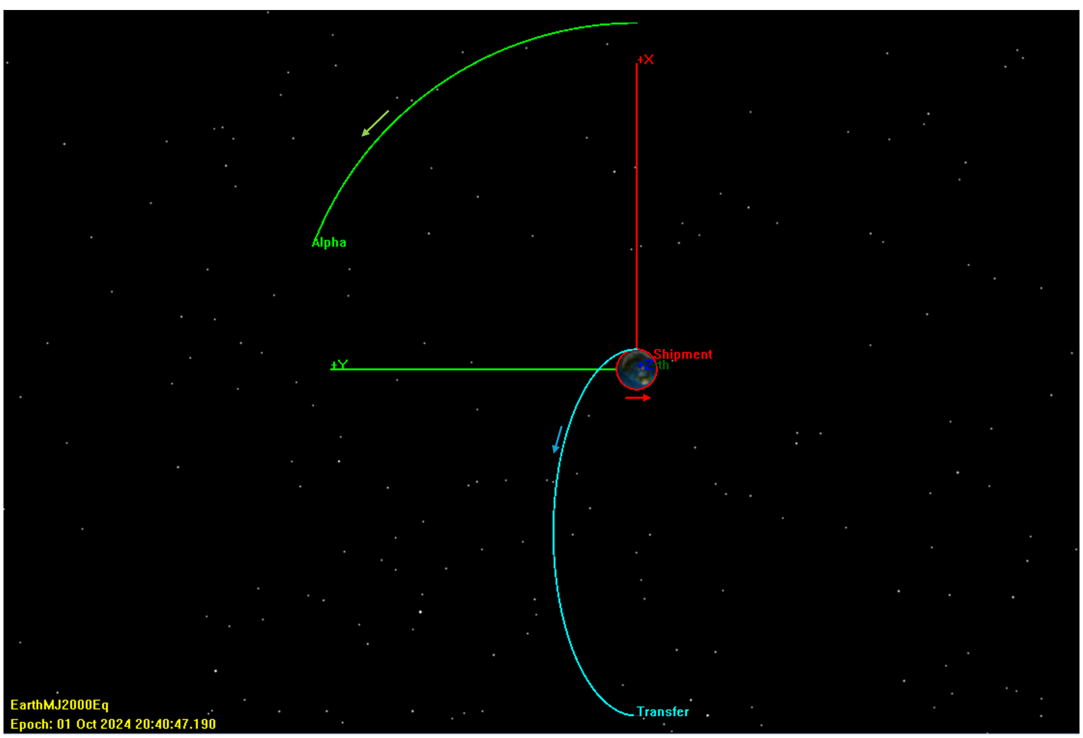

Figure 3: Orbits and transfer arc propagated using GMAT.

We can immediately see that at the time the spacecraft reaches the final orbit and performs the second maneuver Space Station Alpha has not yet completed a half orbit to meet it there. If we propagate each satellite a little longer, we can see that the phasing issue persists and the shipment is unable to rendezvous with Space Station Alpha:

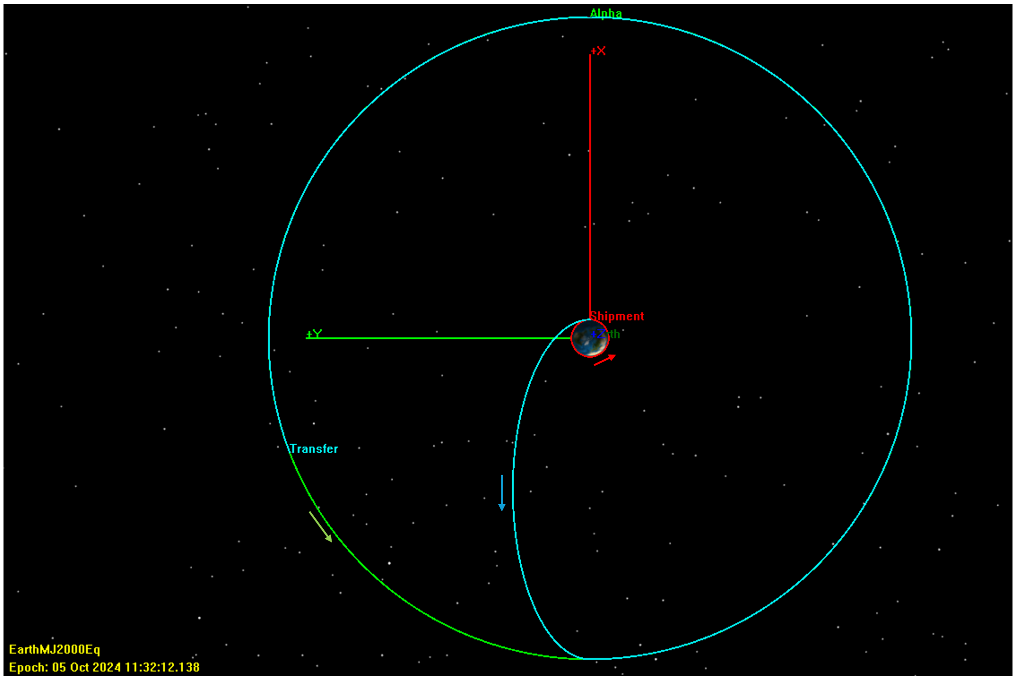

Figure 4: Orbits and transfer arc propagated using GMAT.

iv

To evaluate the phase angle:

$$180° - \phi = n_\alpha(TOF)$$

$$n_\alpha = 1.6230e - 5 \ rad/s$$

$$\boxed{\phi = 110.7660°}$$

The synodic period between the two is then:

$$n = \sqrt{\frac{\mu}{a^3}}$$

$$P_{synodic} = \frac{2\pi}{n_{ship} - n_\alpha} \boxed{= 1.5145 \ hr}$$

We can incorporate the phase angle into the GMAT simulation in order to ensure the shipment is able to rendezvous with the Space Station by setting the true anomaly in the initial conditions for Space Station Alpha to 110.7660°. Additionally, the synodic period we calculated represents the time it takes for any given alignment between objects in the initial and final orbits to reoccur. To test this, you can add a propagate segment before the first maneuver, and propagate for any amount of time that is a multiple of the synodic period. When this propagation segment completes and the maneuver occurs, the shipment will progress along the transfer trajectory and rendezvous with Alpha later in its orbit (depending on how long the propagate segment is). An example is shown below, where an initial propagation of $3P_{synodic}$ was used. Notice how now the shipments arrival now meets Alpha just passed the 180° mark.

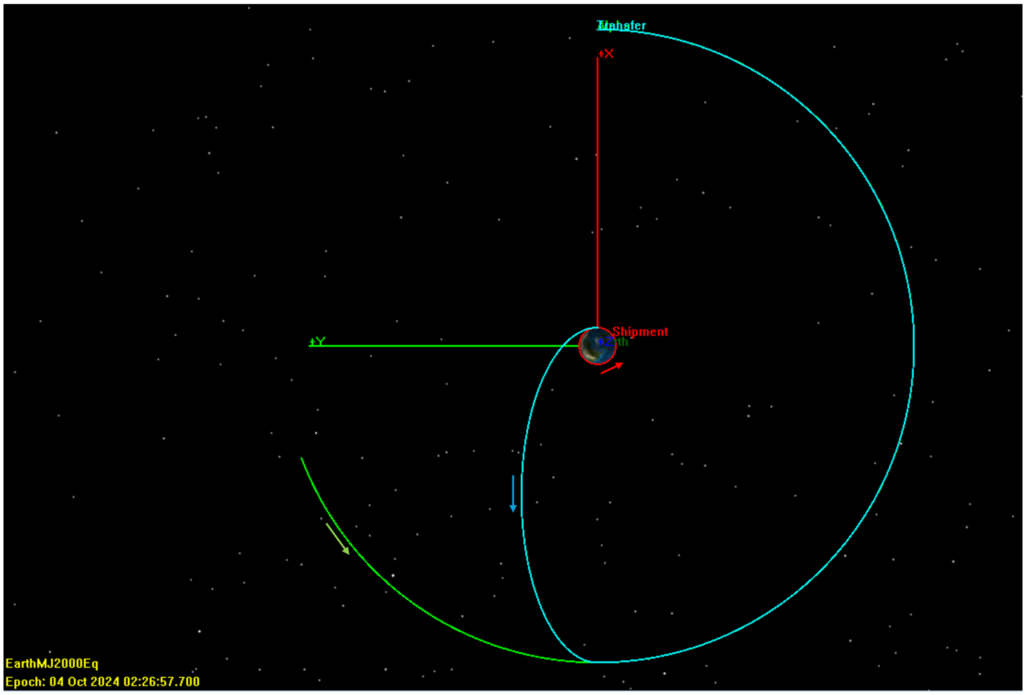

Figure 5: Orbits and transfer arc propagated using GMAT with phasing.

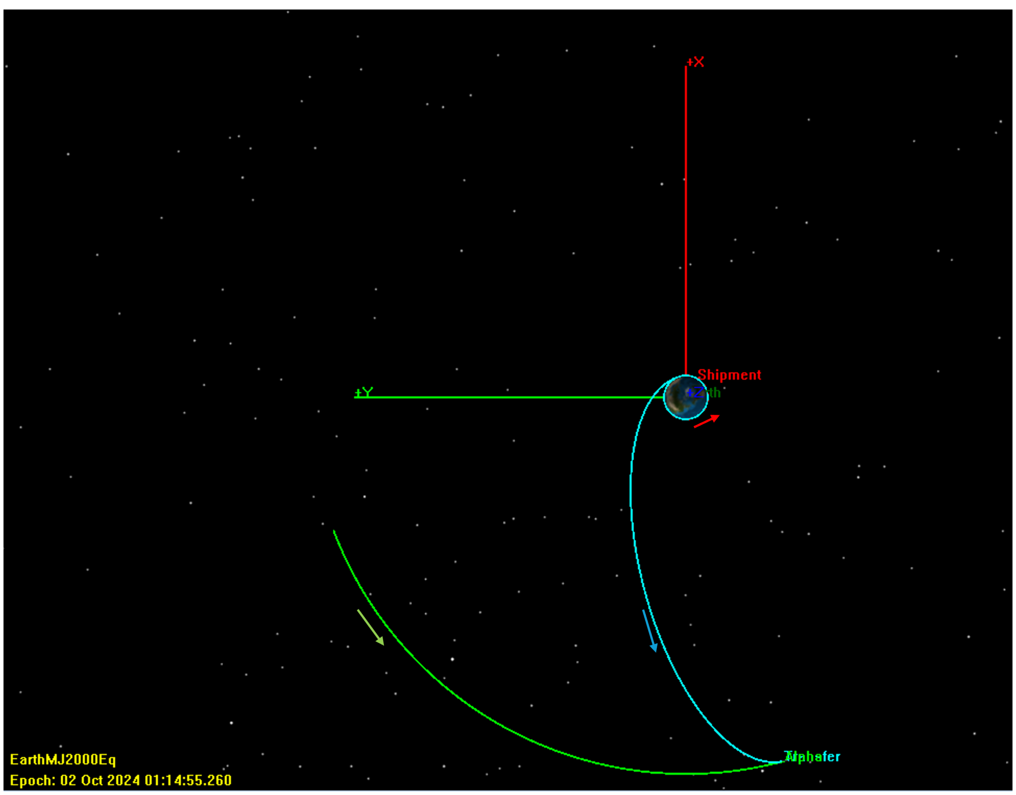

Figure 6: Orbits and transfer arc propagated using GMAT with phasing and $3P_{synodic}$ wait time.

Part (c)

We can also evaluate a bielliptic transfer to go from the initial to final orbits where the initial orbit containing the shipment is at a radius of $1.04R_{\oplus}$, the destination orbit where Space Station Alpha is located is at a radius of $18R_{\oplus}$, and the intermediate distance is $66R_{\oplus}$:

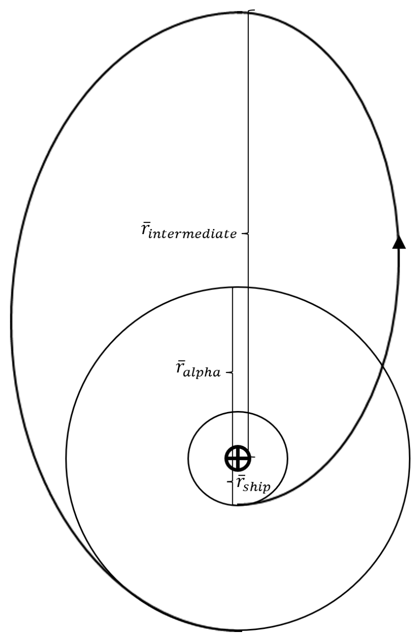

Figure 7: Bielliptic transfer diagram between orbits.

i

The vector diagrams for the three maneuvers are shown below:

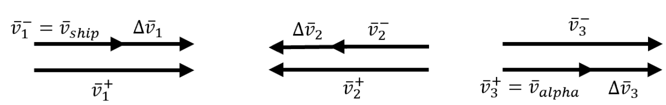

Figure 8: Vector diagrams for bielliptic transfer maneuvers.

Similarly to the Hohmann transfer, all maneuvers are tangential and performed at the apse points of the intermediate orbits. To determine the maneuvers we first must compute the semimajor axis of each transfer arc:

$$a_{t_1} = \frac{r_{p_1} + r_{a_1}}{2} = \frac{1.04R_{\oplus} + 66R_{\oplus}}{2} = 33.52R_{\oplus}$$

$$a_{t_2} = \frac{r_{p_2} + r_{a_2}}{2} = \frac{18R_{\oplus} + 66R_{\oplus}}{2} = 42R_{\oplus}$$

Once again, each Δv is found by taking the difference between the velocities at each point where each individual velocity is found using:

$$v = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}$$

Thus,

$$\Delta v_1 = v_1^+ - v_1^- \boxed{= 3.1256\ km/s}$$

$$\Delta v_2 = v_2^+ - v_2^- \boxed{= 0.4656\ km/s}$$

$$\Delta v_3 = v_3^+ - v_3^- \boxed{= 0.4725\ km/s}$$

Note that as with the Hohmann transfer the first two maneuvers are performed in the velocity direction to accelerate; however, the final maneuver is used to decelerate the spacecraft and lower its apoapsis to that of a circular orbit and therefore points opposite of the direction of motion.

$$\boxed{\alpha_1 = \alpha_2 = 0°}$$

$$\boxed{\alpha_3 = 180°}$$

This gives us a total Δv of:

$$\Delta v_{total} = \Delta v_1 + \Delta v_2 + \Delta v_3 \boxed{= 4.0637\ km/s}$$

which is about 0.0911 km/s less than what was required for the Hohmann transfer. We can also calculate the total time of flight:

$$TOF_1 = \pi\sqrt{\frac{a_{t_1}^3}{\mu}} = 5.6933\ days$$

$$TOF_2 = \pi\sqrt{\frac{a_{t_2}^3}{\mu}} = 7.9851\ days$$

$$TOF_{total} = TOF_1 + TOF_2 \boxed{= 13.6784\ days}$$

which is an additional 12.8167 *days* compared to the Hohmann! So although this method did save some Δv, it came at the cost of a significant increase in the time of flight. Finally, we can determine from inspection and the definition of the bielliptic transfer that the transfer angle for the first arc is 180° and the transfer angle for the second arc is 180°. We can combine these to get the transfer angle for the entire transfer:

$$\boxed{TA = 360°}$$

ii

Returning to GMAT we plot the new transfer along with the initial and final orbits. The coloring scheme is the same as in the previous part but this time a total of 3 maneuvers have been implemented. The orbit of the moon has been included for visualization but the effects of Lunar gravity are neglected in this simulation.

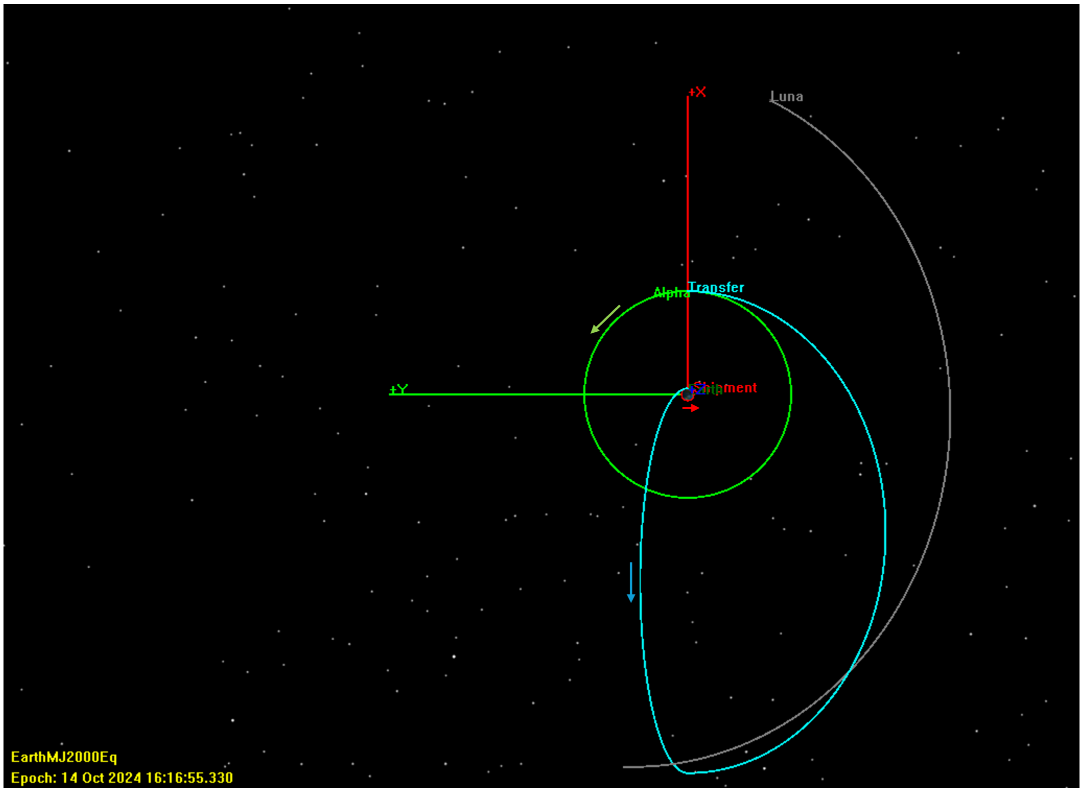

Figure 9: GMAT propagation of Bielliptic transfer between orbits.

iii

To determine the phase angle we first note that this time the transfer angle is 360° and we must incorporate the mean motion and time of flights in both transfer orbit legs.

To evaluate the phase angle:

$$360^\circ - \phi = n_\alpha(TOF)$$

$$n_\alpha = 1.6230e - 5 \ rad/s$$

$$\phi = -738.9855^\circ \ \boxed{= 341.0145^\circ}$$

The high phase angle simply means that due to the extended time of flight the Space Station completes multiple orbits in the time it takes to transfer (try viewing the GMAT animation to visualize this). We can wrap this value to make it more intuitive. The synodic period is the same as before:

$$\boxed{P_{synodic} = 1.5145 \ hr}$$

Similarly to the previous part, if we change the initial true anomaly to this calculated phase angle and rerun the simulation we can see that the shipment is now able to successfully rendezvous with Space Station Alpha.

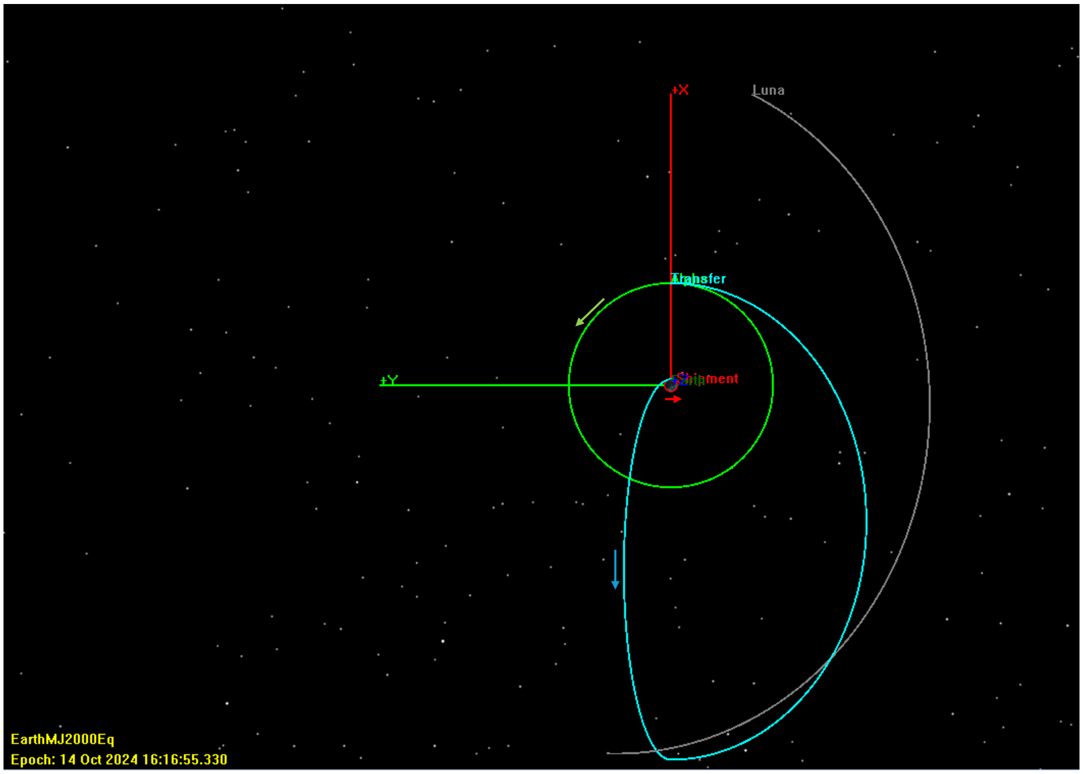

Figure 10: GMAT propagation of Bielliptic transfer between orbits including phasing.

Part (d)

We know that Space Station Alpha is in a circular orbit with a radius of $18R_{\oplus}$, while the moon orbits at a distance closer to $66R_{\oplus}$. Given this information, we can speculate that the reltive two-body model is a reasonable assumption for preliminary analysis of the Hohmann transfer problem, but might not be as well suited for the bielliptic transfer. First, lets add Lunar and Solar gravitational forces to the Hohmann transfer model and see how the results change:

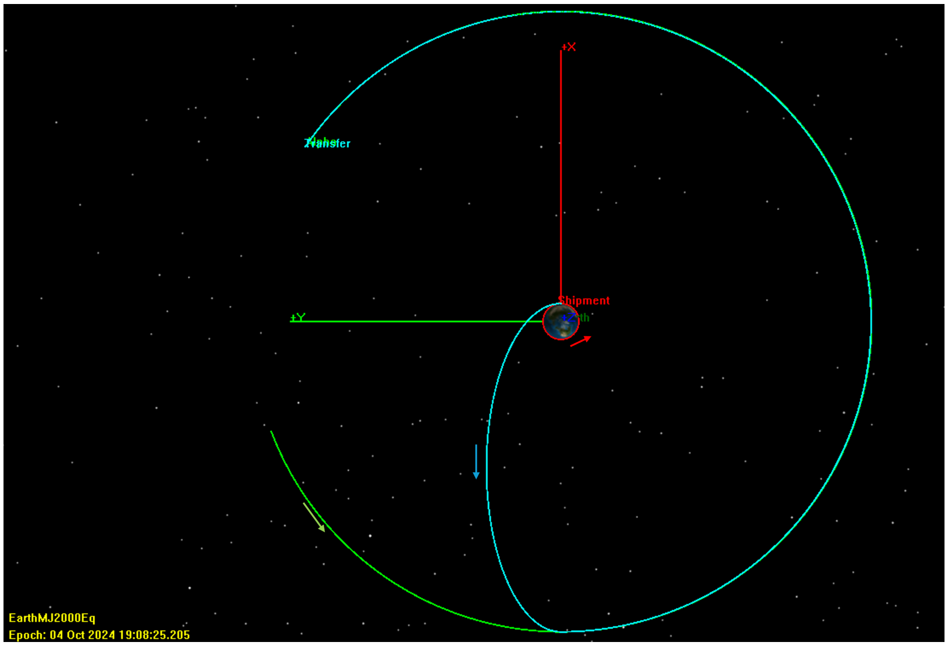

Figure 11: Orbits and Hohmann transfer arc propagated using GMAT including Lunar and Solar gravity.

We can clearly see that although there has been some effect on the orbits, the transfer and rendezvous geometry hold. The bielliptic transfer is a bit more complicated. We saw in the previous part that the trajectory of the bielliptic transfer crosses the path of the Lunar orbit. If we add the gravitational effects of only the moon and run the simulation again we can see how the transfer geometry changes:

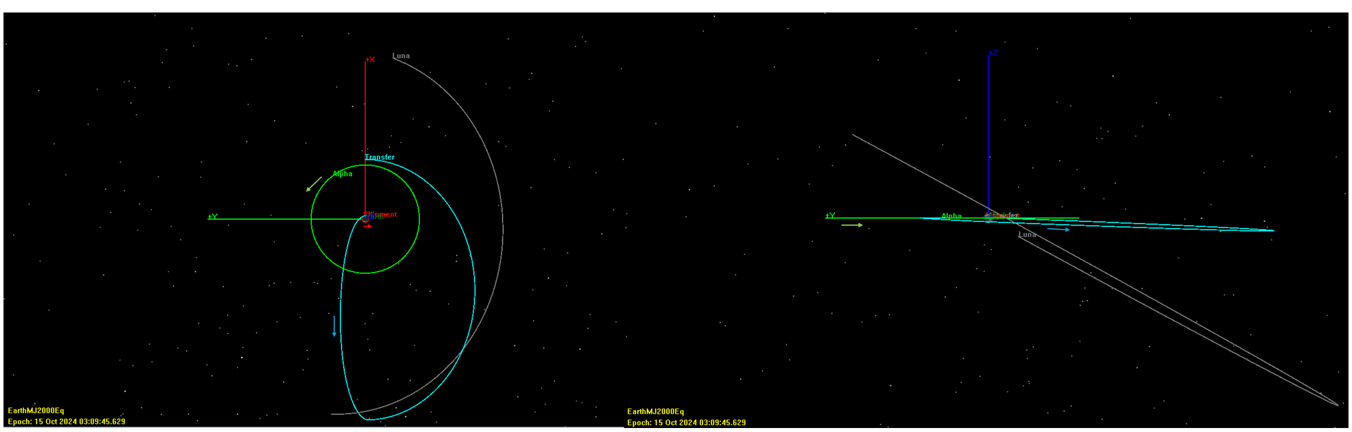

Figure 12: Orbits and bielliptic transfer arc propagated using GMAT including Lunar gravity.

Now when the shipment arrives at Space Station Alpha's orbit its periapse is slightly larger and therefore its final orbit will not be the same as Alpha's. Additionally, if we look at the yz plane projection on the right, we can see that the transfer trajectory and subsequently the final orbit of the shipment are now slightly out of plane. Despite this, the difference still does not appear too drastic especially considering how far of an excursion the shipment makes over the course of the transfer arc. We must also consider the phasing of the moon, however. If we adjust the epoch slightly, so that now the simulation begins on Sept 27 2024 and rerun the propagation:

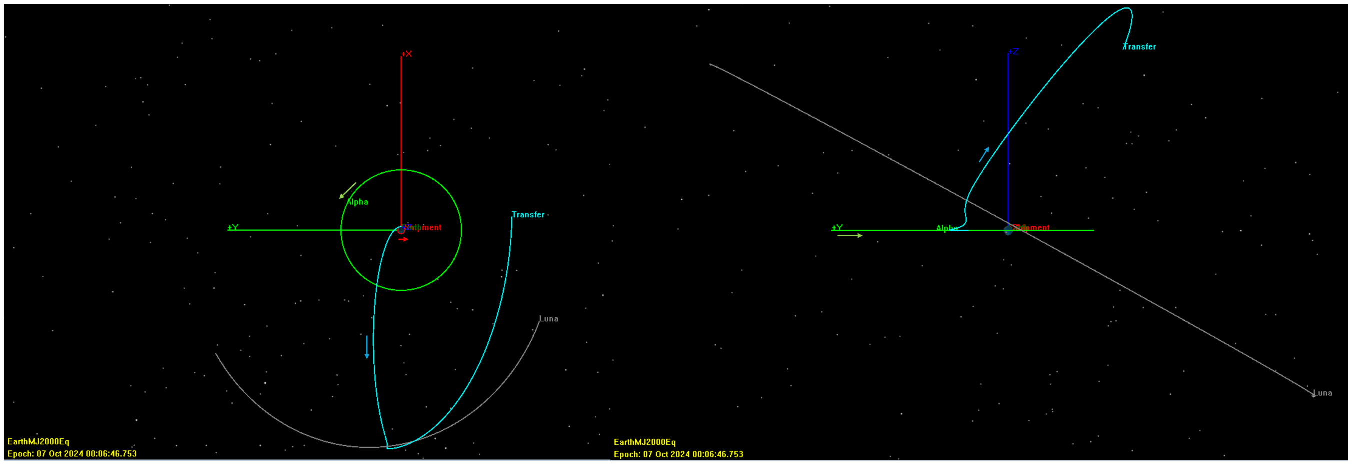

Figure 13: Orbits and bielliptic transfer arc propagated using GMAT including Lunar gravity.

We can now see that the entire geometry of the transfer trajectory has been ruined and there is no hope of the shipment being able to rendezvous with Space Station Alpha without a massive correction maneuver. This close encounter with the moon is essentially a gravity assist and permanently alters the energy and geometry of the orbit. These effects are further exacerbated by adding in the gravitational effects of the Sun:

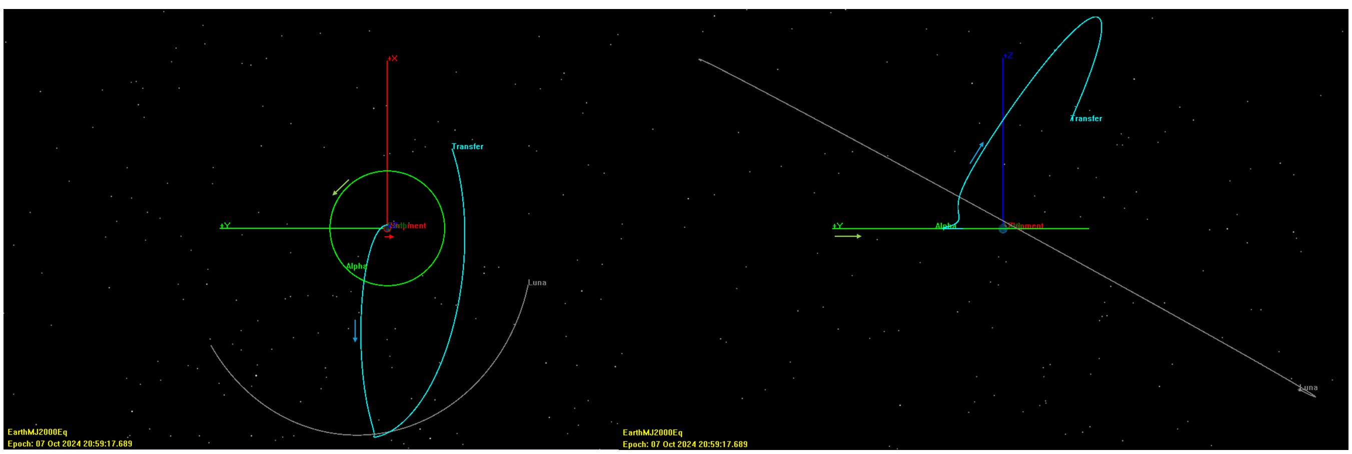

Figure 14: Orbits and bielliptic transfer arc propagated using GMAT including Lunar and Solar gravity.

The perturbing effects of the moon could be minimized by picking an epoch such that the moon is on the opposite side of its orbit at the time that the shipment is at the furthest point from Earth in its transfer arc; however, since the problem is very sensitive to changes in epoch we can conclude that in the case of the bielliptic transfer the two-body approximation is not reasonable.

Problem 3

Problem Statement

In NASA's original plan for a crewed lunar base (Orion), a ground facility near the Moon's south pole was envisioned, necessitating a polar orbit. Thus, the transfer trajectory design (both for arrival at the Moon and for the Earth return) included a 90° plane change. Consider the plane change maneuver. Assume that the spacecraft arrives in the plane of the lunar equator and is currently in a circular lunar orbit at 100 km altitude. Two options exist for the plane change to the polar orbit:

(i) A single maneuver at the current altitude to simply shift the orbit to an inclination of 90°;

(ii) A bi-elliptic strategy that includes three maneuvers: (1) a maneuver to raise apoapsis to $19,000$ km (close to the actual distance in the Orion bi-elliptic planning scenario), (2) a plane change maneuver at apoapsis, (3) a maneuver to insert back into the 100 km altitude polar orbit

(a) Compute and compare the cost, i.e., $|\Delta\bar{v}|_{total}$, for a 90° plane change accomplished with each of the two approaches. Vector diagrams are especially useful in this problem!! Is the bi-elliptic $|\Delta\bar{v}|$ savings substantial? The single plane change is accomplished instantaneously. How much time (TOF) is devoted to completion of the bi-elliptic option?

(b) Insert both cases into GMAT by adding maneuvers. The bi-elliptic option implies the addition of the three maneuvers. (Note that you need to use a propagator with the Moon as the central body.) Plot this scenario.

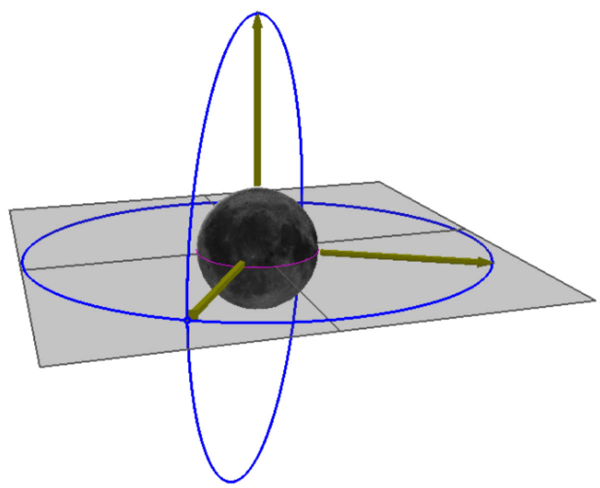

Figure 15: Lunar orbit diagram

Part (a)

For a circular orbit at an altitude of 100 km:

$$v = \sqrt{\frac{\mu}{r_0}} = 1.6331\ km/s$$

i

Starting with the single maneuver to shift the orbit to an inclination of 90°:

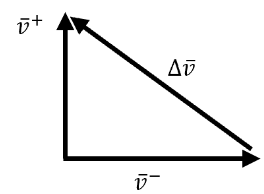

Figure 16: Vector diagram for single maneuver transfer.

We know that the only purpose of the maneuver is to shift the orbit by 90°, therefore the energy of the orbit does not change and the magnitudes of the velocity before and after the maneuver will be equivalent. The only purpose of the maneuver is to change the orientation of the velocity vector. Therefore the vector diagram above represents a 45° 45° 90° triangle and the $\Delta\bar{v}$ can be solved geometrically.

$$\alpha = 180 - 45 = 135°$$

$$\Delta v = \sqrt{v^{-2} + v^{+2}}\ \boxed{= 2.3096\ km/s}$$

ii

The bielliptic strategy involves three maneuvers:

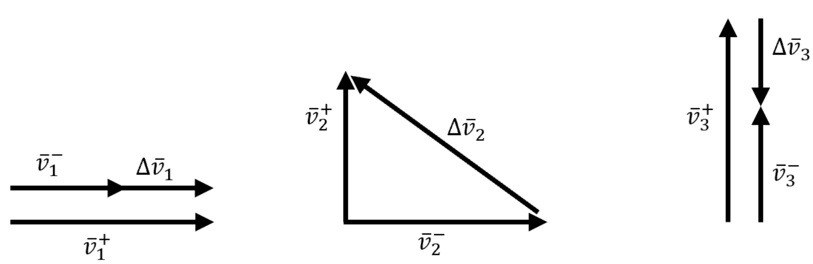

Figure 17: Vector diagram for maneuvers for bielliptic transfer.

Where each velocity is found using:

$$v = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}$$

The initial velocity is the same as the previous scenario:

$$v_1^- = 1.6331\ km/s$$

The first maneuver raises apoapsis to $19,000$ km. To compute it we need to find the semi-major axis for the intermediate transfer orbit:

$$a = \frac{r_0 + r_1}{2} = 10419.1184\ km$$

Then,

$$v_1^+ = 2.2054\ km/s,\ \ \alpha = 0°$$

$$\Delta v_1 = v_1^+ - v_1^- = 0.5723\ km/s$$

The second maneuver occurs at the apoapsis of the first transfer orbit to change the inclination of the orbit by 90°. Similarly to the previous part, the incoming and outgoing velocity magnitudes will be the same since the only goal of the maneuver is to change the orientation of the velocity vector.

$$|\bar{v}_2^-| = |\bar{v}_2^+| = v_{apoapsis} = 0.2134\ km/s$$

$$\alpha = 180 - 45 = 135°$$

$$\Delta v_2 = \sqrt{v_2^{-2} + v_2^{+2}} = 0.3017\ km/s$$

The return transfer orbit will bring us back to a circular lunar orbit with an altitude of 100 km and an inclination of 90°, therefore the ellipse we will follow for that transfer is exactly the same as the previous one but now with the added inclination from the second maneuver. The final maneuver will take place at the periapsis of the transfer ellipse to lower apoapsis and reinsert us into a circular orbit.

$$v_3^- = v_{periapsis} = 2.2054\ km/s$$

$$v_3^+ = v_1^- = 1.6331\ km/s$$

$$\Delta v_3 = v_3^+ - v_3^- = 0.5723\ km/s,\ \ \alpha = 180°$$

The total Δv associated with the bielliptic transfer is then:

$$|\Delta \bar{v}_{total}| = \Delta v_1 + \Delta v_2 + \Delta v_3 \boxed{= 1.4462\ km/s}$$

We can immediately see that the Δv savings associated with the bielliptic transfer is significant at about 0.8634 km/s. A reduction in cost of almost 40% is definitely substantial, but it comes at the price of an increase in time of flight. Since we have already shown that both legs of the transfer orbit are associated with one half of the same ellipse just oriented differently in space, we can determine the total time of flight of the bielliptic transfer by simply computing the period of the ellipse:

$$TOF = 2\pi\sqrt{\frac{a^3}{\mu}} \boxed{= 26.5094\ hrs = 1.1046\ days}$$

Part (b)

For the GMAT simulation we select an arbitrary epoch of Nov 1 2024 and set up the propagation to occur at the moon. Note that the Δv for each case must be split into components in order to implement. First we can show the single impulse maneuver which is applied after a full revolution about the initial planar orbit:

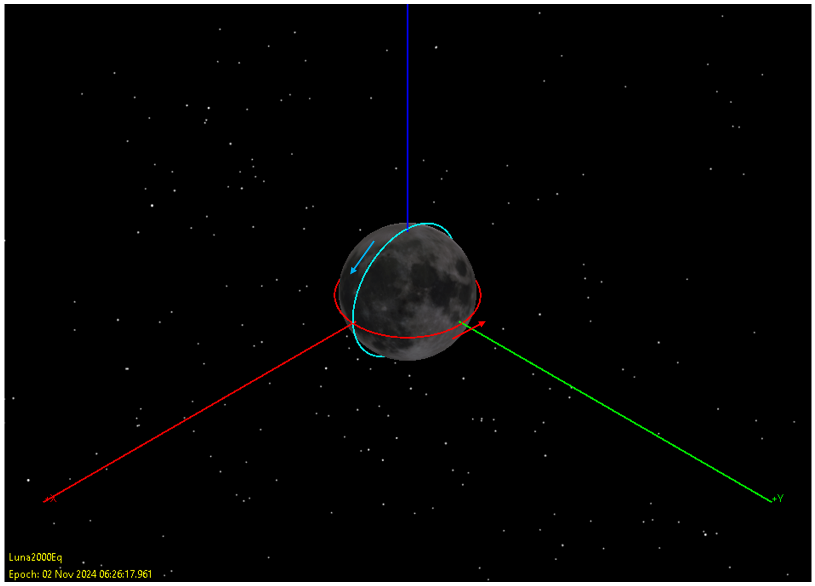

Figure 18: GMAT simulation for single impulse transfer.

The trajectory is plotted in red prior to the maneuver, and blue after the maneuver. For the bielliptic transfer, the trajectory is plotted in red initially, in green after the first maneuver, yellow after the second, and blue after the third. Once again the spacecraft makes one full revolution before the first maneuver is applied:

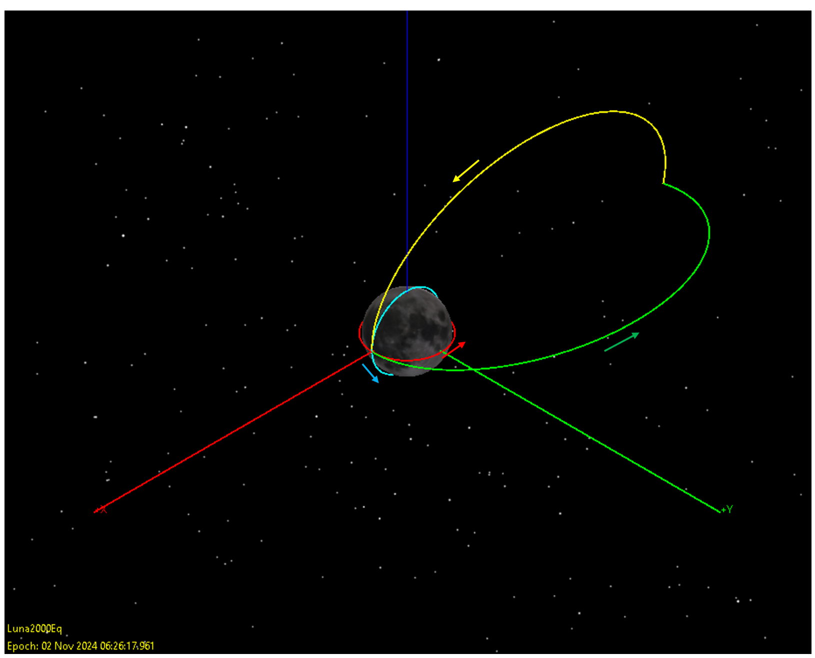

Figure 19: GMAT simulation for bielliptic transfer.

Since the geomerty of the second case is a bit more complex we can also show the three planar projections:

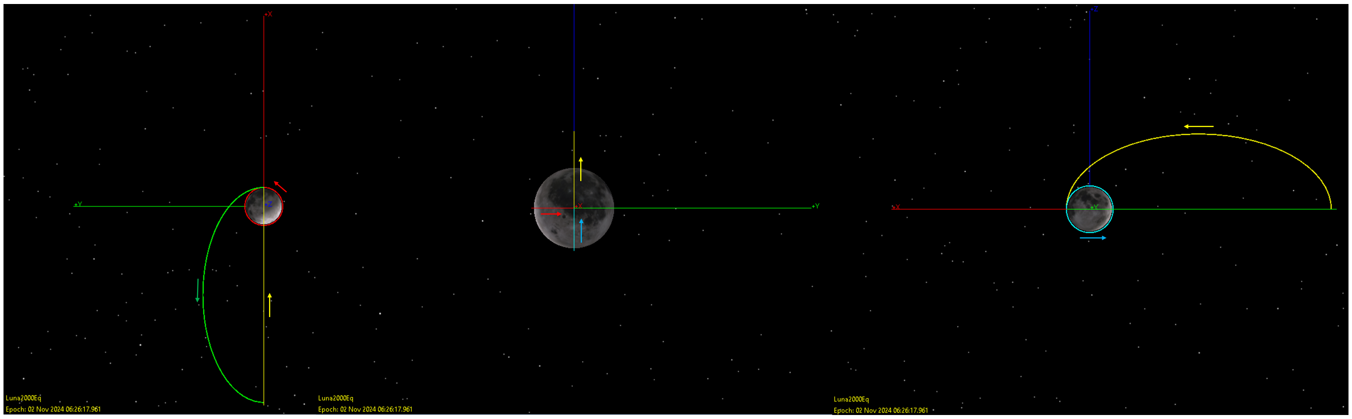

Figure 20: GMAT simulation for bielliptic transfer: plane projections.

In both simulations we note that the starting orbit (red) and final orbit(blue) are exactly the same. This confirms that both methods yield the same final orbit, but with the bielliptic transfer costing significantly less Δv.

Problem 4

Problem Statement

Recall Problem 3 in PS7. You computed the cost $||\Delta\bar{v}||$ and TOF associated with the Hohmann transfer to Pluto. But, in that preliminary analysis, you neglected the local gravity fields.

(a) Re-examine the Hohmann transfer but include the local fields. Assume the planetary orbits are circular; the Earth dark-side departure is from a 250 km altitude parking orbit. For arrival at Pluto, assume that the capture orbit is circular at $11R_{\text{P}}$ (approximately the distance of the New Horizons flyby). Of course, include all vector diagrams representing the local views. Compare the results with the cost Δv_{dep}, Δv_{arr}, and Δv_{total} as well as TOF in PS7. [The Earth departure maneuver is Δv_{dep}; then the maneuver to capture at Pluto is Δv_{arr}.]
Does adding the local fields impact the total $||\Delta\bar{v}||$? Does the inclusion of the local fields increase or decrease the cost? Does the maneuver cost increase or decrease at Earth? At Pluto?

(b) The cost will differ depending on the capture orbit at Pluto. As an alternative, assume a capture orbit that is an eccentric Plutonian orbit. Let capture orbit characteristics be $r_p = 11R_{\text{P}}$ and $e = 0.90$. Consider insertion into the capture orbit at periapsis and compute the insertion cost, that is, the $||\Delta\bar{v}_{arr}||$.
Does the total cost improve in terms of Δv_{arr} and Δv_{total}? Why do you think this difference occurs?
A spacecraft could first enter such an eccentric orbit at Pluto, then used a series of maneuvers to reduce the size and eventually reach the science orbit. Discuss: Given the information that you have here, it is likely better to enter an eccentric orbit at periapsis or apoapsis? Why?

(c) Reconsider the Pluto arrival. In New Horizons, the vehicle arrived at Pluto but did not capture. Instead, it was just a flyby. You should already have the arrival conditions in the heliocentric orbit: r^-, v^-, γ^-, θ^{*-} from (a).
If there is no capture maneuver, determine the characteristics of the new heliocentric orbit of the spacecraft after the Pluto encounter: r^+, v^+, γ^+, θ^{*+}. Compute the orbital characteristics of the new heliocentric orbit: a, e, r_p, r_a, *period*, *energy*, $\Delta\omega$. Did the spacecraft gain or lose energy?

(d) Plot the old and new heliocentric orbit of the spacecraft in Matlab.
Compute the equivalent Δv_{eq} and α. Will the spacecraft be on an elliptical heliocentric orbit or a hyperbolic orbit with respect to the Sun? Does it matter? As we know, New Horizons moved out into the solar system and encountered the Kuiper belt object Ultima Thule. To encounter the Kuiper belt object, the flyby of Pluto needed to be phased correctly. Do you think it possible to find a phase angle at departure so the spacecraft can encounter two objects, not just one? Describe in a few words, how might that be accomplished?

Part (a)

Let's begin by drawing the local view around Earth and the Heliocentric view:

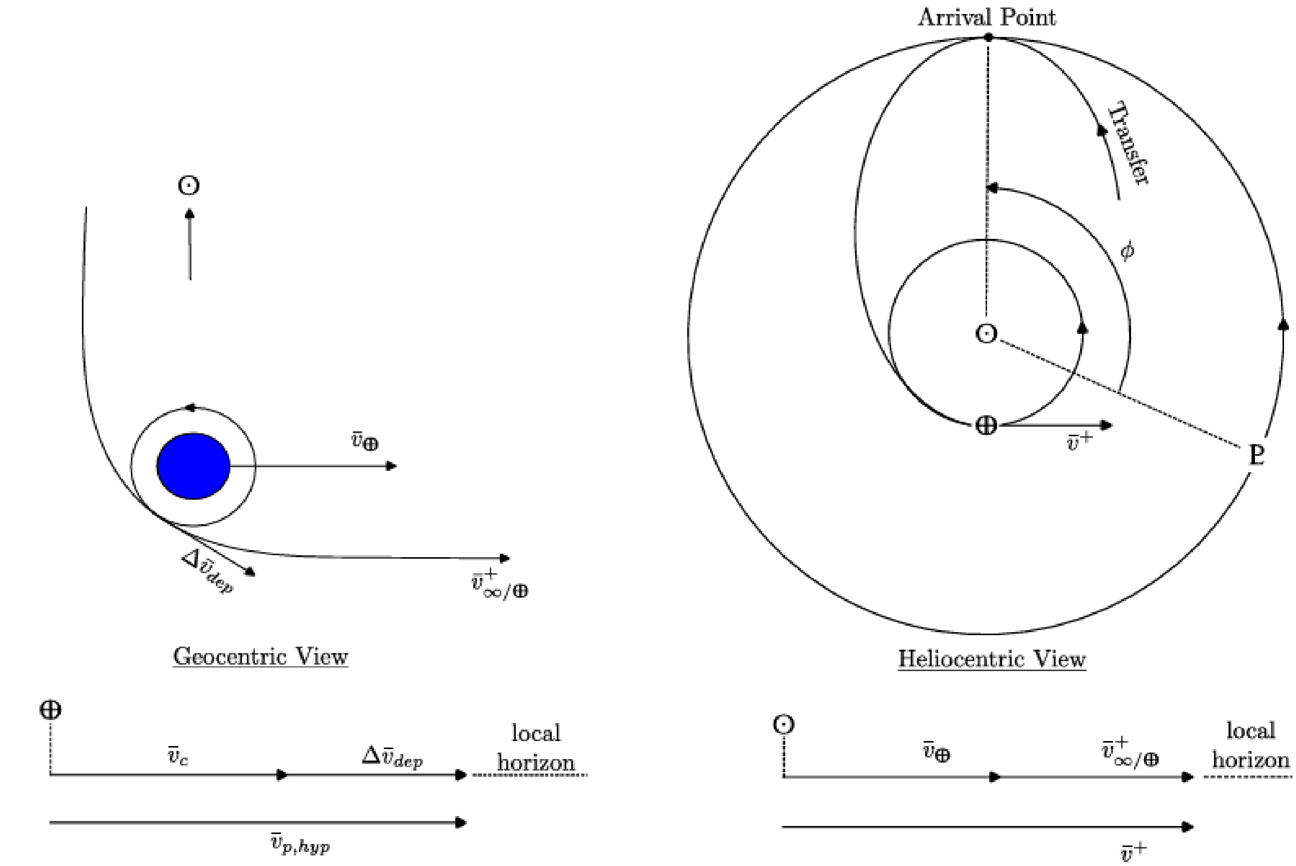

Figure 21: Geocentric and Heliocentric Views and vector diagrams at departure from Earth.

Note the collinear nature of the velocity vectors in both cases. So, by observation, the flight path angle, both before and after the departure maneuver, the maneuver angle, and True Anomaly for the transfer arc can be expressed as:

$$\gamma^- = \gamma^+ = 0° \tag{1}$$

$$\boxed{\alpha_{dep} = 0°} \tag{2}$$

$$\theta^{*+} = 0° \tag{3}$$

Then, treating Earth's orbit about the Sun as circular:

$$||\bar{v}_\oplus|| = \sqrt{\frac{\mu_\odot}{a_\oplus}} = 29.7847 \text{ km/s} \tag{4}$$

The speed of the spacecraft within the parking orbit about Earth is:

$$||\bar{v}_c|| = \sqrt{\frac{\mu_\oplus}{r_{p,\oplus}}} = 7.7548 \text{ km/s} \tag{5}$$

where $r_p = 6628.1363$ km. Now, let's consider the Hohmann Transfer orbit between the Earth and Pluto. Assuming that Pluto is in a circular orbit about the Sun:

$$\begin{aligned} a_{Hohmann} &= \frac{a_\oplus + a_{P}}{2} \\ &= 3.0284 \cdot 10^9 \text{ km} \\ &= 20.2434 \text{ AU} \end{aligned} \tag{6}$$

which means that the specific energy of the transfer orbit is:

$$\mathcal{E}_{Hohmann} = -\frac{\mu_\odot}{2a_{Hohmann}} = -21.9115 \text{ km}^2/\text{s}^2 \tag{7}$$

So, the speed of the spacecraft in the transfer orbit at periapsis is:

$$||\bar{v}^+|| = \sqrt{2\left(\mathcal{E} + \frac{\mu_\odot}{a_\oplus}\right)} = 41.5985 \text{ km/s} \tag{8}$$

Noting again, from the velocity vector diagrams, that the velocities are collinear, then:

$$||\bar{v}^+_{\infty/\oplus}|| = ||\bar{v}^+|| - ||\bar{v}_\oplus|| = 11.8138 \text{ km/s} \tag{9}$$

Then, again noting collinearity within the local view velocity vector diagrams, the required maneuver at departure can be expressed as:

$$\boxed{\begin{aligned} ||\Delta\bar{v}_{dep}|| &= ||\bar{v}_{p,hyp}|| - ||\bar{v}_c|| \\ &= \sqrt{||\bar{v}^+_{\infty/\oplus}||^2 + \frac{2\mu_\oplus}{r_{p,\oplus}}} - ||\bar{v}_c|| \\ &= 8.3647 \text{ km/s} \end{aligned}} \tag{10}$$

Now, for the second maneuver at Pluto, one can begin by examining the local view around Pluto and the Heliocentric view:

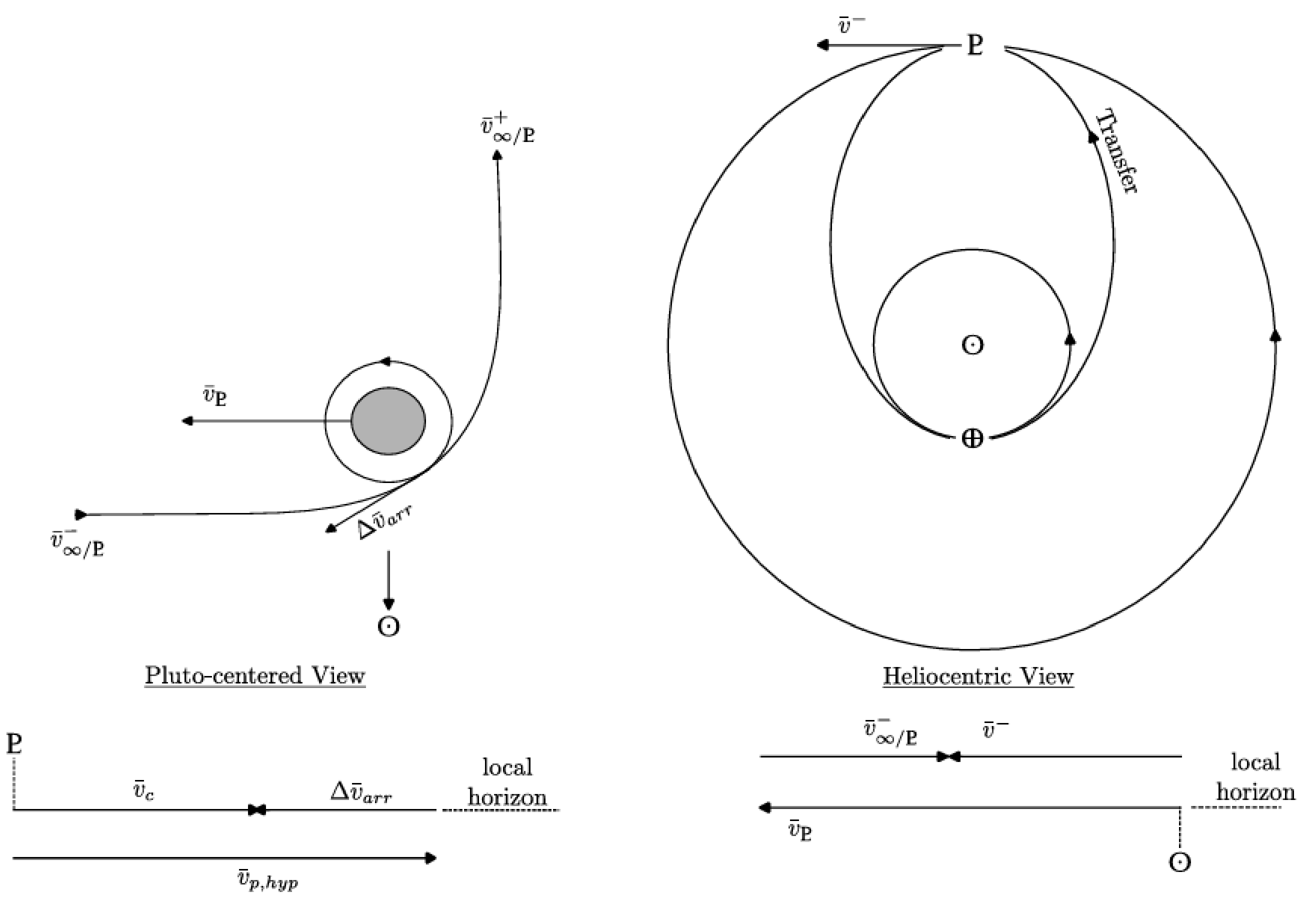

Figure 22: Pluto-centered and Heliocentric Views and vector diagrams at arrival at Pluto.

Again, note the collinear nature of the velocity vectors in both cases. So, by observation, the flight path angle, both before and after the arrival maneuver, the maneuver angle, and True Anomaly for the transfer arc can be expressed as:

$$\gamma^- = \gamma^+ = 0° \tag{11}$$

$$\boxed{\alpha_{arr} = 180°} \tag{12}$$

$$\theta^{*-} = 180° \tag{13}$$

Then, treating Pluto's orbit about the Sun as circular:

$$||\bar{v}_{♇}|| = \sqrt{\frac{\mu_{\odot}}{a_{♇}}} = 4.7399 \text{ km/s} \tag{14}$$

The speed of the spacecraft within the parking orbit about Earth is:

$$||\bar{v}_c|| = \sqrt{\frac{\mu_{♇}}{r_{p,♇}}} = 0.2771 \text{ km/s} \tag{15}$$

where $r_p = 12782$ km. Given we already know the specific energy of the transfer orbit, one can obtain the speed of the spacecraft in the Hohmann transfer at apoapsis:

$$||\bar{v}^-|| = \sqrt{2\left(\mathcal{E} + \frac{\mu_{\odot}}{a_{♇}}\right)} = 1.0535 \text{ km/s} \tag{16}$$

Noting again, from the velocity vector diagrams, that the velocities are collinear, then:

$$||\bar{v}^-_{\infty/♇}|| = ||\bar{v}_{♇}|| - ||\bar{v}^-|| = 3.6864 \text{ km/s} \tag{17}$$

Then, again noting collinearity within the local view velocity vector diagrams, the required maneuver at arrival to be captured in a parking orbit around Pluto can be expressed as:

$$\boxed{\begin{aligned} ||\Delta\bar{v}_{arr}|| &= ||\bar{v}_{p,hyp}|| - ||\bar{v}_c|| \\ &= \sqrt{||\bar{v}^-_{\infty/♇}||^2 + \frac{2\mu_{♇}}{r_{p,♇}}} - ||\bar{v}_c|| \\ &= 3.4301 \text{ km/s} \end{aligned}} \tag{18}$$

Thus, the total cost for the maneuvers is:

$$\boxed{\Delta v_{total} = ||\Delta\bar{v}_{dep}|| + ||\Delta\bar{v}_{arr}|| = 11.7948 \text{ km/s}} \tag{19}$$

Lastly, one can obtain the time of flight (TOF) for the transfer:

$$\boxed{\begin{aligned} TOF &= \frac{\mathcal{P}}{n} \\ &= 1.4372 \cdot 10^9 \text{ s} \\ &= 1.6634 \cdot 10^4 \text{ days} \\ &= 45.5412 \text{ years} \end{aligned}} \tag{20}$$

where 1 yr = 365.25 days and:

$$n = \sqrt{\frac{\mu_{\odot}}{a^3_{Hohmann}}} = 2.1860 \cdot 10^{-9} \text{ rad/s} \tag{21}$$

Recall from Problem Set 7 that the following was obtained:

$$\begin{aligned} ||\Delta\bar{v}_{dep}|| &= 11.8138 \text{ km/s} \\ ||\Delta\bar{v}_{arr}|| &= 3.6864 \text{ km/s} \\ \Delta v_{total} &= 15.5002 \text{ km/s} \\ TOF &= 45.5412 \text{ years} \end{aligned} \tag{22}$$

As one can see, the $||\Delta\bar{v}_{dep}||$, $||\Delta\bar{v}_{arr}||$, and Δv_{total}, when including local gravity fields, are all less than their comparable maneuver values when not including local gravity fields. In this case, the inclusion of local gravity fields has reduced the cost of the maneuvers at both the Earth and Pluto. Note, however, this may not always be the case as it depends on several variables, one of which is your periapsis radius of your parking orbit that you are departing or arriving. Also, since we are still employing a Hohmann transfers and the time spent within the local frames is regarded as negligible, the TOF for both schemes is the same.

Part (b)

Now, let's consider an elliptic orbit with $r_p = 11R_{♇}$ and $e = 0.90$. Noting that the $||\Delta\bar{v}_{dep}||$ should not change, one can just examine the local view of Pluto as well as the Heliocentric view: Because the arrival is at periapsis for both

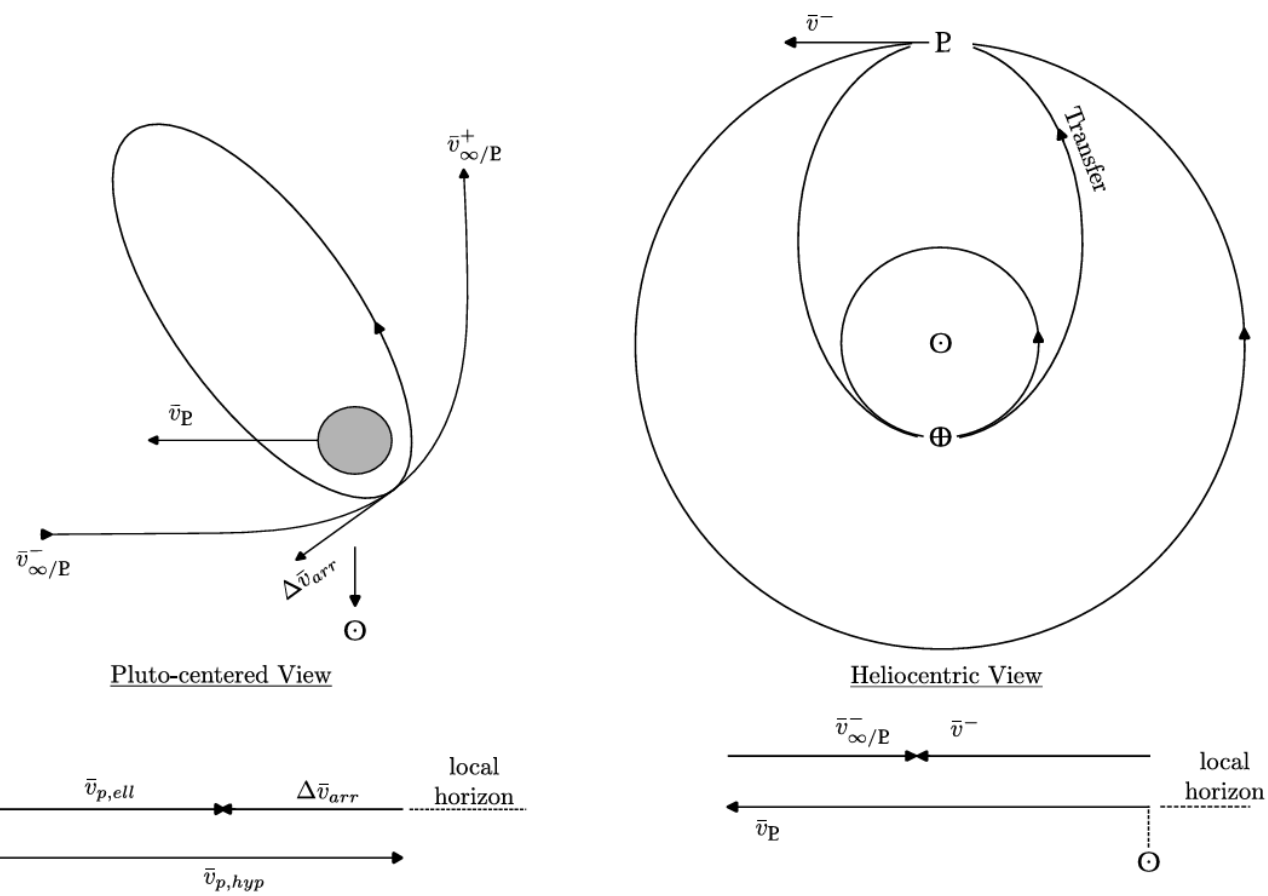

Figure 23: Pluto-centered and Heliocentric Views and vector diagrams at arrival at Pluto.

the incoming hyperbola and the resulting elliptic parking orbit, the flight path angles for both before and after the arrival maneuver and the maneuver angle are:

$$\gamma^- = \gamma^+ = 0° \tag{23}$$

$$\boxed{\alpha = 180°} \tag{24}$$

Now, let's obtain the speed at periapsis for the capture orbit, where the semi-major axis is:

$$a = \frac{r_p}{1-e} = 1.2782 \cdot 10^5 \text{ km} \tag{25}$$

where the specific energy is:

$$\mathcal{E} = -\frac{\mu_{♇}}{2a} = -0.003840 \text{ km}^2/\text{s}^2 \tag{26}$$

Then, the speed at periapsis within the capture orbit is:

$$||\bar{v}_{p,cap,♇}|| = \sqrt{2\left(\mathcal{E} + \frac{\mu_{♇}}{r_{p,cap,♇}}\right)} = 0.3820 \text{ km/s} \tag{27}$$

Note that the speed of Pluto, the speed at apoapsis of the Hohmann transfer, and $||\bar{v}^-_{\infty/♇}||$ should all remain unaffected, but $||\Delta\bar{v}_{arr}||$ will change:

$$\boxed{\begin{aligned} ||\Delta\bar{v}_{arr}|| &= ||\bar{v}_{p,hyp}|| - ||\bar{v}_{p,cap,♇}|| \\ &= \sqrt{||\bar{v}^-_{\infty/♇}||^2 + \frac{2\mu_♇}{r_{p,hyp,♇}}} - ||\bar{v}_{p,cap,♇}|| \\ &= 3.3252 \text{ km/s} \end{aligned}} \tag{28}$$

Thus, the new total cost for the maneuvers is:

$$\boxed{\Delta v_{total} = ||\Delta\bar{v}_{dep}|| + ||\Delta\bar{v}_{arr}|| = 11.6899 \text{ km/s}} \tag{29}$$

The $||\Delta\bar{v}_{arr}||$ and Δv_{total} do slightly improve. The difference results because the spacecraft does not have to expend more energy to transfer from a hyperbolic orbit to a circular orbit. Instead, it transitions to a highly elliptic orbit.

There are two items at play here. First, recall that the specific energy $\mathcal{E}$ is inversely proportional to the semi-major axis of the orbit, which means, for an orbit with high eccentricity, the semi-major axis will also be significantly large and so, the corresponding energy of the orbit will be small. This then translates into less energy required to expend to obtain a low energy orbit when transitioning from a hyperbolic trajectory to a closed elliptic orbit. In addition, insertion into an elliptic orbit will be cheaper than using a circular orbit with the same altitude as periapsis within the eccentric orbit since the speed of the spacecraft at periapsis will be larger than the speed of the spacecraft within the circular orbit. This means that the maneuver requires less energy or change in velocity from the hyperbolic orbit velocity at periapsis to insert into the orbit. Thus, it would be better to enter an eccentric orbit at periapsis than at apoapsis as the maneuver would require less energy to change velocity of the spacecraft at periapsis than at apoapsis.

Note, however, one must be careful in a real world scenario as the mean distance between Pluto and Charon is roughly 19640 km, which is well within the orbit radius of our spacecraft. This would imply that one may also have to account for its gravitational influence in addition to that from Pluto.

Part (c)

Let's now consider a flyby of Pluto by first examining the local view around Pluto as well at the Heliocentric view: Note, now, that, within the local view, the velocity vector diagram will no longer be collinear. Thus, one can begin

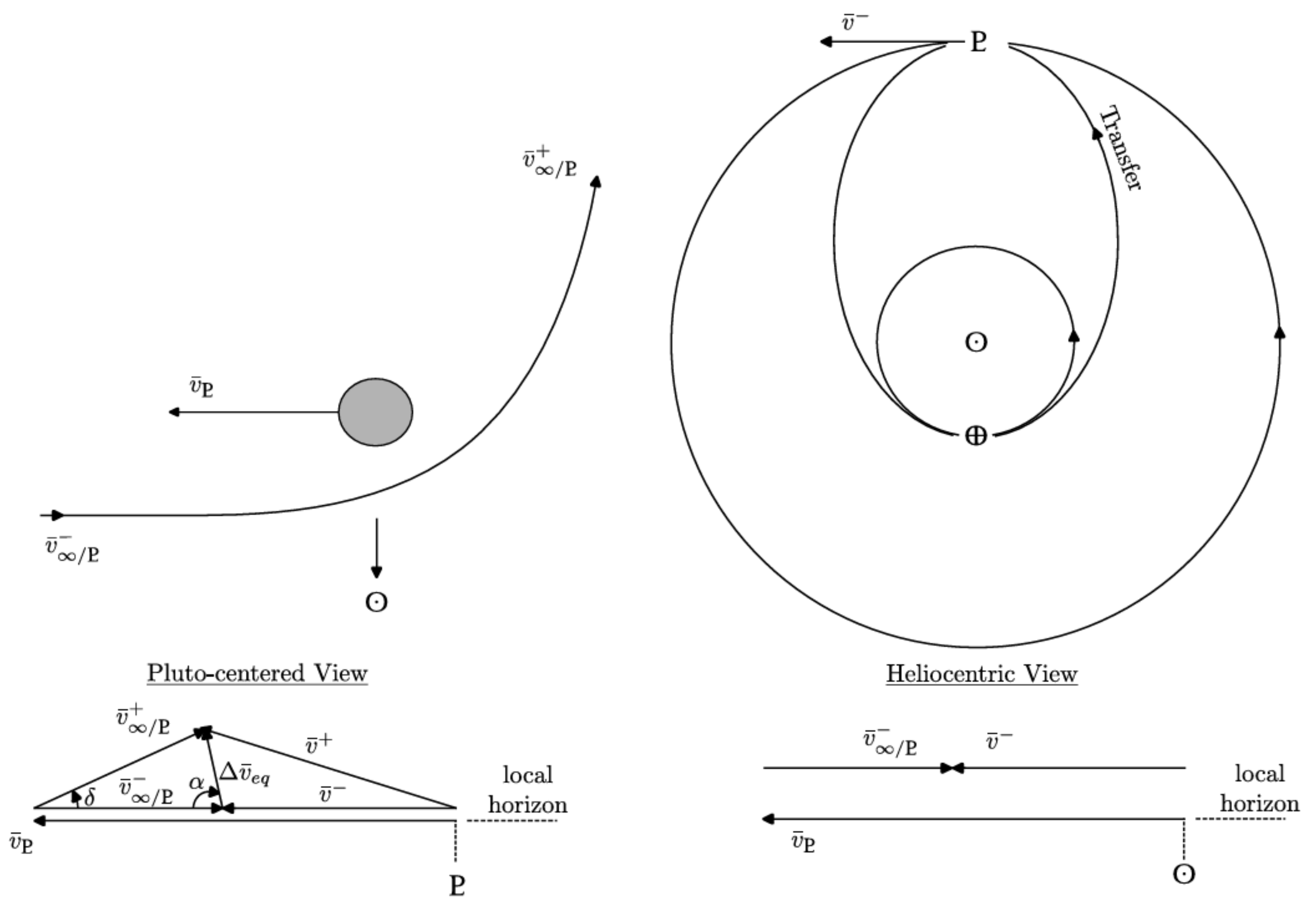

Figure 24: Pluto-centered and Heliocentric Views and vector diagrams at arrival at Pluto.

by solving for the turn angle for the flyby. But first, let's define the specific energy for the hyperbolic orbit:

$$\mathcal{E}_{hyp} = \frac{||\bar{v}^{-}_{\infty/\text{♇}}||^2}{2} = 6.7948 \text{ km}^2/\text{s}^2 \tag{30}$$

which means our semi-major axis and eccentricity are then:

$$a_{hyp} = -\frac{\mu_{\text{♇}}}{2\mathcal{E}} = -72.2322 \text{ km} \tag{31}$$

$$e_{hyp} = 1 - \frac{r_{p,hyp}}{a_{hyp}} = 177.9572 \tag{32}$$

Thus, the turn angle for our flyby is:

$$\delta = 2 \arcsin\left(\frac{1}{e_{hyp}}\right) = 0.6439° \tag{33}$$

Now, one can solve for the characteristics of the new Heliocentric orbit after the flyby:

$$\boxed{\begin{aligned} r^{+} = r^{-} &= a_{\text{♇}} \\ &= 5.9072 \cdot 10^9 \text{ km} \\ &= 39.4869 \text{ AU} \end{aligned}} \tag{34}$$

Using cosine law and the velocity vector diagram within the local view, one can solve for the magnitude of the velocity vector $\bar{v}^+$ after the flyby:

$$\boxed{\begin{aligned} ||\bar{v}^+|| &= \sqrt{||\bar{v}^+_{\infty/\text{P}}||^2 + ||\bar{v}_{\text{P}}||^2 - 2||\bar{v}^+_{\infty/\text{P}}||||\bar{v}_{\text{P}}||\cos\delta} \\ &= 1.0545 \text{ km/s} \end{aligned}} \tag{35}$$

where $||\bar{v}^+_{\infty/\text{P}}|| = ||\bar{v}^-_{\infty/\text{P}}||$. Now, because this flyby occurs on the light-side, by convention, the flight path angle will be positive and in the $+\hat{\theta}$ direction. So, using sine law and noting that $\gamma^- = 0°$, then:

$$\boxed{\gamma^+ = \arcsin\left(\frac{||\bar{v}^+_{\infty/\text{P}}||\sin\delta}{||\bar{v}^+||}\right) = 2.2516°} \tag{36}$$

Next, before one determines the true anomaly, one can determine the other orbital characteristics of the new heliocentric orbit, beginning with specific energy:

$$\boxed{\mathcal{E}^+ = \frac{||\bar{v}^+||^2}{2} - \frac{\mu_\odot}{r^+} = -21.9104 \text{ km }^2/\text{s}^2} \tag{37}$$

which means that semi-major axis is then:

$$\boxed{\begin{aligned} a^+ &= -\frac{\mu_\odot}{2\mathcal{E}^+} \\ &= 3.0285 \cdot 10^9 \text{ km} \\ &= 20.2444 \text{ AU} \end{aligned}} \tag{38}$$

Note that the spacecraft **gained** energy from this flyby. Next, one can determine the specific angular momentum:

$$h^+ = r^+||\bar{v}^+||\cos\gamma^+ = 6.2244 \cdot 10^9 \text{ km}^2/\text{s} \tag{39}$$

Then, the semi-latus rectum is:

$$p^+ \frac{(h^+)^2}{\mu_\odot} = 2.9193 \cdot 10^8 \text{ km} \tag{40}$$

which can be used to determine eccentricity:

$$\boxed{e^+ = \sqrt{1 = \frac{p^+}{a^+}} = 0.9506} \tag{41}$$

Now, one can determine the periapsis and apoapsis radii for this new Heliocentric orbit:

$$\boxed{r_p^+ = a^+(1 - e^+) = 1.4967 \cdot 10^8 \text{ km}} \tag{42}$$

$$\boxed{r_a^+ = a^+(1 + e^+) = 5.9074 \cdot 10^9 \text{ km}} \tag{43}$$

Next one can also determine the true anomaly for the new orbit. recall that our flight path angle was positive, meaning that the true anomaly should also be positive as the spacecraft is still ascending in its orbit:

$$\boxed{\theta^{*+} = \arccos\left(\frac{p^+}{r^+e^+} - \frac{1}{e^+}\right) = 179.8829°} \tag{44}$$

Then, one can also obtain the period of the new orbit:

$$\boxed{\begin{aligned} \mathcal{P}^+ &= \frac{2\pi}{n^+} \\ &= 2.8746 \cdot 10^9 \text{ s} \\ &= 3.3270 \cdot 10^4 \text{ days} \\ &= 91.0893 \text{ years} \end{aligned}} \tag{45}$$

where 1 yr = 365.25 days and:

$$n^+ = \sqrt{\frac{\mu_\odot}{(a^+)^3}} = 2.1858 \cdot 10^{-9} \text{ rad/s} \tag{46}$$

Lastly, one can determine the change in the argument of periapsis. Recalling that $\theta^{*-} = 180°$, then:

$$\boxed{\Delta\omega = \theta^{*-} - \theta^{*+} = 0.1171°} \tag{47}$$

Part (d)

As one can see with the new specific energy and eccentricity in part (c) above, the new heliocentric orbit will still be elliptical, although with a very high eccentricity.

One way to consider how to encounter two objects (assuming that all objects are in relatively circular, co-planar orbit) is to work backwards. First, one must determine the corresponding v_∞ values for departure and the flyby such that the spacecraft would have enough energy to get to Ultima Thule. Then, one can obtain the trajectory from Pluto to Ultima Thule, post flyby, in order to obtain the TOF for that leg of the journey. This would provide one with the geometry between Pluto and Ultima Thule. Then, one can obtain the trajectory between Earth and Pluto to obtain the TOF there. This would now provide the geometry between Earth and Pluto. In a simple world, one can then add these trajectories together to get the total TOF and thus, the phase angle for departure. However, in the real world, the objects may not have this geometry for some time, which, depending on the restrictions on each trajectory, may require a substantial wait time to be phased properly.

Lastly, lets determine the equivalent Δv_{eq} and α and plot the old and new heliocentric orbits of the spacecraft in MATLAB. Let's first obtain expressions for $\bar{v}^-$ and $\bar{v}^+$. Recall that, because the flyby occurs in-plane, for the heliocentric view, the $\hat{r}$ and $\hat{\theta}$ remain relatively the same:

$$\begin{aligned} \bar{v}^- &= v^- \sin\gamma^- \hat{r} + v^- \cos\gamma^- \hat{\theta} \\ &= 1.0535\,\hat{\theta} \text{ km/s} \end{aligned} \tag{48}$$

$$\begin{aligned} \bar{v}^+ &= v^+ \sin\gamma^+ \hat{r} + v^+ \cos\gamma^+ \hat{\theta} \\ &= 0.04143\,\hat{r} + 1.0537\,\hat{\theta} \text{ km/s} \end{aligned} \tag{49}$$

Then, since the $\hat{r}$ and $\hat{\theta}$ remain the same, the $\Delta\bar{v}_{eq}$ can be expressed in terms of the $\hat{r}$ and $\hat{\theta}$ directions (albeit, shifted to the end of $\bar{v}^-$):

$$\boxed{\Delta\bar{v}_{eq} = \bar{v}^+ - \bar{v}^- = 0.04143\,\hat{r} + 2.3281 \cdot 10^{-4}\,\hat{\theta} \text{ km/s}} \tag{50}$$

where the magnitude is:

$$\boxed{||\Delta\bar{v}_{eq}|| = 0.04143 \text{ km/s}} \tag{51}$$

One can then determine the angle α with respect to the original velocity vector $\bar{v}^-$ using sine law:

$$\frac{\sin\beta}{v^+} = \frac{\sin\gamma^+}{||\Delta\bar{v}_{eq}||} \tag{52}$$

where β is the supplementary angle to α. Thus, since $(v^+)^2 > (v^-)^2 + (\Delta v_{eq})^2$, $\beta > 90°$:

$$\beta = \arcsin\left[\frac{v^+ \sin\gamma^+}{||\Delta\bar{v}_{eq}||}\right] = 90.3220° \tag{53}$$

Since the angle α is pointing away from (clockwise direction) Pluto, it will be a positive angle ($\alpha > 0$). :

$$\boxed{\alpha = +(180° - \beta) = 89.6780°} \tag{54}$$

One can then plot the old and new Heliocentric orbits:

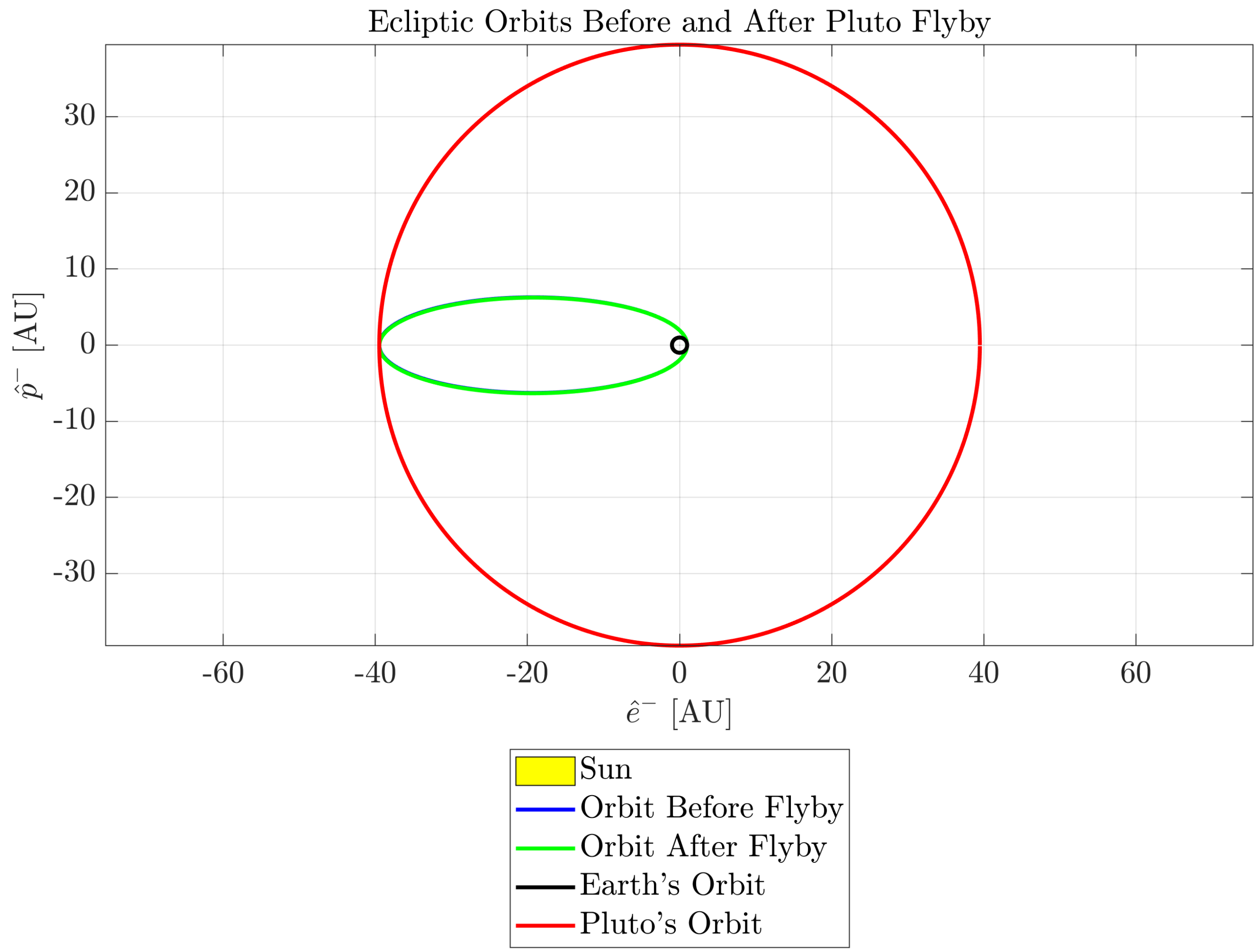

Figure 25: Old and New Heliocentric orbits with the orbits of Earth and Pluto for reference.

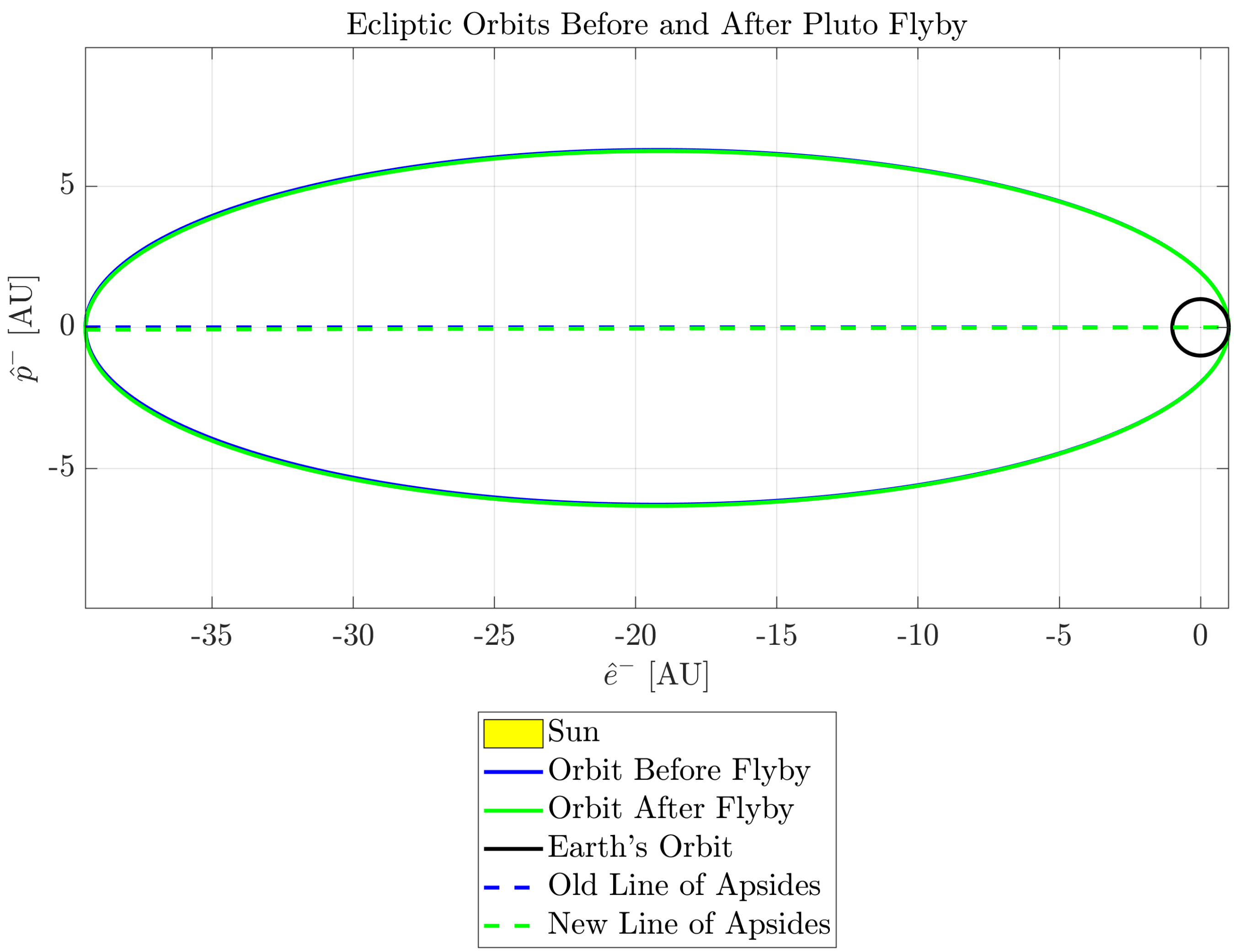

Figure 26: Old and New Heliocentric orbits with their corresponding line of apsides and Earth's orbit for reference.