#1)

a) $\sin(t^2)$

$$\lim_{t_k \to \infty} = \infty$$

$$\lim_{k \to \infty} (\sin(t_k^2)) = \sin(\infty) = [-1 \quad 1]$$

$\therefore$ $\boxed{\text{Positive limit set is } [-1, 1]}$

b) $e^t \sin(t)$

Positive limit set of e^t is empty

$$\lim_{t \to \infty} (e^t \sin(t)) = \infty$$

$\therefore$ $\boxed{\text{Positive limit set is empty}}$

#2) $\dot{x}_1 = x_2^2$

$\dot{x}_2 = -x_1 x_2$

If V is radially unbounded such that $\dot{V} \le 0$ for all x, all solutions converse to larsest invariant set, M in S.

Choose $V(x) = x_1^2 + x_2^2 - x_1 = V_1 + V_2$

For radially unbounded: $\lim_{\|x\| \to \infty} V(x) = \infty$

$x_1^2 - x_1$ is radially unbounded & x_2^2 is radially unbounded. The sum of 2 radially unbounded functions is radially unbounded $\therefore$ $V(x)$ is radially unbounded. ($V_1 = x_1^2 - x_1$ & $V_2 = x_2^2$)

Check: $\lim_{x \to (0,\infty)} V(x) = \infty$ ✓ $\quad \|x\| = \infty$ if $x = (0, \infty)$ or $x = (\infty, 0)$ or $x = (\infty, \infty)$

$\lim_{x \to (\infty, 0)} V(x) = \infty$ ✓

$$\dot{V} = \frac{\partial V}{\partial x} \dot{x} = [2x_1 - 1 \quad 2x_2] \begin{bmatrix} x_2^2 \\ -x_1 x_2 \end{bmatrix} = 2x_1 x_2^2 - x_2^2 - 2x_1 x_2^2$$

$\dot{V} = -x_2^2 \le 0$ for all x

$\dot{V} = 0$ only if $x_2 = 0$ $\therefore$ $S = \{(x_1, 0) : x_1 \in \mathbb{R}\}$ where

$S = \{x \in \mathbb{R}^n \mid \dot{V} = 0\}$

If $x_2 = 0$, $\dot{x}_2 = 0$ $\therefore$ $\dot{x}_2 = 0 = (-x_1)(0)$, x_1 can be any real number $\therefore$ $M = \{(x_1, 0) : x_1 \in \mathbb{R}\}$ which is the x_1 axis. All solutions approach M.

#3) $\ddot{q} + c(\dot{q}) + K(q) = 0$

$P(q) = \int_0^q K(\eta)\,d\eta$ $\qquad c(\dot{q})\dot{q} > 0$ for $\dot{q} \neq 0$

$KE = \frac{1}{2}\dot{q}^2$ $\qquad \lim_{q \to \infty} P(q) = \infty$ (Potential Energy is radially unbounded).

$c(0) = 0$

$x_1 = q \qquad x_2 = \dot{q}$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ -c(x_2) - K(x_1) \end{pmatrix}$$

$$V(x) = \frac{1}{2}x_2^2 + \int_0^{x_1} K(\eta)\,d\eta = KE + PE$$

$$\dot{V} = \frac{\partial V}{\partial x}\dot{x}$$

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x) \quad \therefore \quad \frac{\partial V}{\partial x_1}\int_0^{x_1} K(\eta)\,d\eta = K(x_1)$$

$$\therefore \dot{V} = [K(x_1) \quad x_2]\begin{bmatrix} x_2 \\ -c(x_2) - K(x_1) \end{bmatrix} = -c(x_2)x_2$$

$c(q)\dot{q} > 0 \quad \therefore -c(x_2)x_2 < 0$ for all $x_2 \neq 0$

& $c(0) = 0 \quad \therefore \underline{\dot{V} \leq 0 \text{ for all } x}$

For all solutions to converse to Equilibrium states, $\dot{V} \leq 0$ for all x & V is radially unbounded.

$\lim_{q \to \infty} P(q) = \infty$, $\lim_{\dot{q} \to \infty} (\frac{1}{2}\dot{q}^2) = \infty$, $\therefore$ if $x_1 = q$ & $x_2 = \dot{q}$

$\underline{V(x) \text{ is radially unbounded}}$, sum of 2 radially unbounded functions is radially unbounded.

$\dot{V} \leq 0$ for all x, $\dot{V} = 0$ only if $x_2 = 0$

$x_2 \equiv 0 \Rightarrow \dot{x}_2 = 0 = -c(x_2) - K(x_1)$

$\dot{x}_2 = 0 = -c(0) - K(x_1) = 0 - K(x_1)$

$\therefore K(x_1) = 0$

$\boxed{M = \{ (x_1, 0) \mid K(x_1) = 0 \} \text{, all solutions converse to } M \text{, which is a set of equilibrium states}}$

#4) $m\ddot{q} + c\dot{q} + K(q) = 0$

$K(q)q > 0$ for $q \neq 0$ $\quad K(0) = 0$

$\lim_{q \to \infty} \int_0^q K(\eta)d\eta = \infty$

a) $x_1 = q \qquad x_2 = \dot{q} \qquad \ddot{q} = -\frac{c\dot{q}}{m} - \frac{K(q)}{m}$

$$\boxed{\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{cx_2}{m} - \frac{K(x_1)}{m} \end{aligned}}$$

b)

i) For GAS about origin: $\dot{V} \leq 0$ for all x where $\dot{V} \equiv 0$ only for zero solution & V is P.D.

Potential Energy: $U = \int_0^{x_1} K(\eta)d\eta$

Kinetic Energy: $T = \frac{1}{2}mx_2^2$

Canidate Lyapunov: $V(x) = T + U = \frac{1}{2}mx_2^2 + \int_0^{x_1} K(\eta)d\eta$

$$\dot{V} = \frac{\partial V}{\partial x}\dot{x} = [K(x_1) \quad mx_2]\begin{bmatrix} x_2 \\ -\frac{cx_2}{m} - \frac{K(x_1)}{m} \end{bmatrix}$$

$\dot{V} = K(x_1)x_2 - cx_2^2 - K(x_1)x_2 = \underline{-cx_2^2 \leq 0 \quad \text{for all } x}$

$\frac{1}{2}mx_2^2 > 0$ for all x_2 as $m > 0$, $\therefore T$ is P.D.

$K(0)=0$, $K(q)q>0$ (for $q\neq 0$), & $\lim_{q\to\infty} \int_0^q K(\eta)d\eta = \infty$ $\therefore$

$U = \int_0^{x_1} K(\eta)d\eta$ is P.D. as a function is P.D.

if $f(0)=0$, $F(x)>0$ for $x\neq 0$, & $\lim_{\|x\|\to\infty} F(x) = \infty$.

U & T are both P.D., $\therefore$ V(x) is P.D. as a sum of 2 P.D. functions is P.D.

$\dot{V} = -Cx_2^2 \leq 0$, $\dot{V}\equiv 0$ only if $x_2 \equiv 0$

$x_2 \equiv 0 \Rightarrow \dot{x}_2 = 0$

$\dot{x}_2 = 0 = -\underbrace{\cancel{\frac{C x_2}{m}}}_{x_2=0}^{\to 0} - \frac{K(x_1)}{m}$ $\therefore$ $K(x_1)=0$ as $m>0$

$K(x_1)=0$ only for $x_1=0$ $\therefore$ $x_1 \equiv 0$

$\boxed{\text{V(x) is Positive definite \& } \dot{V}\equiv 0 \text{ only if } x\equiv 0\ (x_1=x_2=0),\text{ where } \dot{V}\leq 0 \text{ for all } x. \text{ The equilibrium state } q=\dot{q}=0 \text{ is then G.A.S.}}$

Lasalle type Result

ii) For G.A.S. (Non-Lasalle type); V is Positive definite & $\dot{V}<0$ for all $x\neq 0$

Candidate Lyapunov function: $V(x) = \frac{1}{2}m x_2^2 + \int_0^{x_1} K(\eta)d\eta + \frac{1}{2}\lambda\left(\frac{c}{m}\right)^2 x_1^2 + \lambda \frac{C}{m} x_1 x_2$ =

$$V = x^T P x + \int_0^{x_1} K(\eta)d\eta, \quad P = \frac{1}{2}\begin{pmatrix} \lambda \frac{c^2}{m^2} & \lambda\frac{c}{m} \\ \lambda\frac{c}{m} & m \end{pmatrix}$$

where $0<\lambda<m$, $m>0$

$\frac{\lambda c^2}{m^2} > 0$ as $\lambda > 0$ & $(\frac{c}{m})^2 > 0$

$\det(P) = \frac{1}{4}\left[\frac{\lambda c^2}{m} - \frac{\lambda^2 c^2}{m^2}\right] = \frac{1}{4}\frac{\lambda c^2}{m^2}(m - \lambda) > 0$

as $0 < \lambda < m$ & $m > 0$.

$P(1,1)$ & $\det(P) > 0$ $\therefore$ $\underline{P = P^T \succ 0 \text{ via sylvester criteria}}$

$V(x) = x^T P x + \int_0^{x_1} K(\eta)d\eta \geq x^T P x$ as $\int_0^{x_1} K(\eta)d\eta \geq 0$

$\therefore$ $\underline{V(x) \text{ is positive definite}}$.

$\dot{V} = \frac{\partial V}{\partial x}\dot{x} = \left[K(x_1) + \lambda (\frac{c}{m})^2 x_1 + \lambda \frac{c}{m} x_2 \quad m x_2 + \lambda \frac{c}{m} x_1\right] \begin{bmatrix} x_2 \\ -\frac{c x_2}{m} - \frac{K(x_1)}{m} \end{bmatrix}$

$\dot{V} = \cancel{K(x_1)}x_2 + \lambda \cancel{(\frac{c}{m})^2} x_2 x_1 + \lambda \frac{c}{m} x_2^2 - c x_2^2 - \cancel{K(x_1)} x_2 - \cancel{\frac{\lambda c^2}{m^2}} x_1 x_2 - \lambda \frac{c}{m^2} K(x_1) x_1$

$\dot{V} = -c x_2^2 (1 - \frac{\lambda}{m}) - \frac{\lambda c}{m^2} K(x_1) x_1$

$0 < \frac{\lambda}{m} < 1$ as $0 < \lambda < m$ $\therefore$ $-c x_2^2 (1 - \frac{\lambda}{m}) < 0$ for all $x_2 \neq 0$

$K(q)q > 0$ for all $q \neq 0$ & $\frac{\lambda c}{m^2} > 0$ as $m, c, \& \lambda > 0$

$\therefore$ $-\frac{\lambda c}{m^2} K(x_1) x_1 < 0$ for all $x \neq 0$, $K(x_1) x_1 > 0$

$-cx_2^2\left(1-\frac{\lambda}{m}\right) < 0$ & $-\frac{\lambda c}{m^2}\, k(x_1)x_1 < 0$ for all $x \neq 0$

$\therefore\ \dot{V} < 0$ for all $x \neq 0$

$V(x)$ is positive definite & $\dot{V} < 0$ for all $x \neq 0$
$\therefore\ q = \dot{q} = 0$ is a G.A.S. equilibrium state.

#5)

$$\ddot{q} - a\sin(q) = bu \qquad a = \frac{mgl}{I} \quad b = \frac{1}{I}$$

$$U = -K_p q - K_d \dot{q}$$

$$\ddot{q} = a\sin(q) - bK_p q - bK_d\dot{q} = \frac{mgl}{I}\sin(q) - \frac{1}{I}K_p q - \frac{1}{I}K_d\dot{q}$$

$x_1 = q \quad x_2 = \dot{q}$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = a\sin(x_1) - bK_p x_1 - bK_d x_2$$

KE: $T = \frac{1}{2} I x_2^2$

PE: $U = -\int_0^{x_1} mgl\sin(n)\,dn - \int_0^{x_1} -K_p n\,dn$

$$U = -mgl\left[-\cos(n)\Big|_0^{x_1}\right] + \frac{K_p x_1^2}{2}$$

$$U = mgl(\cos(x_1) - 1) + \frac{K_p x_1^2}{2}$$

Candidate Lyapunov function: $V = KE + PE$

$$V(x) = \frac{1}{2} I x_2^2 + mgl(\cos(x_1) - 1) + \frac{K_p x_1^2}{2}$$

$V(0) = 0 + \frac{mgl}{I}(1-1) + 0 = 0$ ✓

$$\frac{\partial V}{\partial x} = [-mgl\sin(x_1) + K_p x_1 \quad I x_2]$$

$\frac{\partial V(0)}{\partial x} = [0 \quad 0]$ ✓

$$\frac{\partial^2 V}{\partial x^2} = \begin{bmatrix} -mgl\cos(x_1) + K_p & 0 \\ 0 & I \end{bmatrix} \qquad (m, g, l > 0)$$

$$\frac{\partial^2 V}{\partial x^2} \geq 0 \quad \text{if} \quad -mgl\cos(x_1) + K_p > 0$$

$\therefore$ $K_p > mgl\cos(x_1)$, $\cos(x_1)$ has max value of 1 $\therefore$

$\underline{K_p > mgl}$ for $\frac{\partial^2 V}{\partial x^2} \geq 0$ for all x.

$\|x\| = \infty$ if $x_1 = \infty$ & $x_2 = 0$ or $x_1 = 0$ & $x_2 = \infty$

$$\lim_{x \to (\infty, 0)} V(x) = \infty \qquad \lim_{x \to (0, \infty)} V(x) = \infty$$

$\therefore \lim_{\|x\| \to \infty} V(x) = \infty$, $\underline{V(x) \text{ is radially unbounded.}}$

$V(0) = 0$, $V(x) > 0$ for all x if $K_p > mgl$, & $V(x)$ is radially unbounded $\therefore$ $\underline{V(x) \text{ is a positive definite function}}$.

<u>For GAS about state for $q \equiv 0$</u> : V is P.D. , $\dot{V} \leq 0$, & $\dot{V} \equiv 0$ only for $q \equiv 0$

$$\dot{V} = \frac{\partial V}{\partial x}\dot{x} = [-mgl\sin(x_1) + K_p x_1 \quad I x_2] \begin{bmatrix} x_2 \\ \frac{mgl}{I}\sin(x_1) - \frac{K_p}{I}x_1 - \frac{K_d x_2}{I} \end{bmatrix}$$

$$\dot{V} = -\cancel{mgl\sin(x_1)x_2} + \cancel{K_p x_1 x_2} + \cancel{mgl x_2 \sin(x_1)} - \cancel{K_p x_1 x_2} - K_d x_2^2$$

$\dot{V} = -K_d x_2^2 \leq 0$ for all x only if $\underline{K_d > 0}$

$\therefore \dot{V} \equiv 0 \Rightarrow X_2 \equiv 0 \quad \therefore \dot{X}_2 = 0$

$\dot{x}_2 = 0 = \frac{mgl}{I} \sin(x_1) - \frac{1}{I} K_p x_1 - \frac{K_d}{I}(0) \Rightarrow$

$0 = \frac{mgl}{I} \sin(x_1) - \frac{K_p}{I} x_1$

$\frac{mgl}{I}$ & $\frac{K_p}{I} \neq 0 \quad \therefore$

$\underline{\sin(x_1) \text{ \& } x_1 \text{ must } = 0}$

$\therefore \underline{x_1 \equiv 0}$

$$\boxed{\begin{array}{l}\text{If } K_p > mgl \text{ \& } K_d > 0, \text{ then } V(x) \text{ is Positive Definite} \\ \text{\& } \dot{V} \leq 0 \text{ for all } x, \text{ where } \dot{V} \equiv 0 \text{ only for } q = \dot{q} = 0 \\ \text{Thus the closed loop System is G.A.S about the State} \\ \text{Corresponding to } q \equiv 0, \ \begin{pmatrix} q \\ \dot{q} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.\end{array}}$$

#6)

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_1 - x_2 + \theta \sin(x_1) + u \qquad \theta - \text{unknown constant}$$

Nominal linear system: $\dot{x}_1 = x_2$
$\dot{x}_2 = -x_1 - x_2$

Nominal Candidate Lyapunov function: $V(x) = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2 + \lambda x_2 x_1 + \frac{\lambda}{2}x_1^2$

where $0 < \lambda < 1$. $P = \frac{1}{2}\begin{pmatrix} 1+\lambda & \lambda \\ \lambda & 1 \end{pmatrix}$

$\therefore \underline{V(x) = x^T P x}$

$P(1,1) = 1+\lambda > 0$
$\det(P) = \frac{1}{4}(1+\lambda - \lambda^2) > 0$ if $1+\lambda > \lambda^2$

$0 < \lambda < 1 \therefore 1+\lambda > 1$ & $\lambda^2 < 1 \therefore \det(P) > 0$

Leading principal minors of P are > 0 $\therefore$ $\underline{P = P^T \succ 0}$ via Sylvester criteria.

P is P.D. $\therefore$ <u>$V(x)$ is positive definite function.</u>

$V(x) = x^T P x = \frac{1}{2}(1+\lambda)x_1^2 + \frac{x_2^2}{2} + \lambda x_2 x_1$

Choose $\underline{u(t) = -\hat{\theta}\sin(x_1)}$ to eliminate $\theta \sin(x_1)$ & get nominal linear system.

$\underline{\delta\theta \triangleq \hat{\theta} - \theta}$ $\therefore$ $\dot{x}_2 = -x_1 - x_2 + \theta\sin(x_1) - \hat{\theta}\sin(x_1) = -x_1 - x_2 - \delta\theta \sin(x_1)$

Candidate Lyapunov function for compensated system: $U(\delta\theta) = \frac{1}{2}\delta\theta^2$

Total Candidate Lyapunov function: $W(x,\delta\theta) = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2 + \frac{\lambda x_1^2}{2} + \lambda x_2 x_1 + \frac{1}{2}\delta\theta^2$

For G.A.S. about origin: $\dot{V} \leq 0$ for all x, $\dot{V} \equiv 0$ only if $x \equiv 0$ &
V is P.D.

$W = V(x) + U(\delta\theta)$ $\therefore$ $W(x,\delta\theta)$ is P.D. as both $V(x)$ & $U(\delta\theta)$ are P.D.

$$\dot{W} = \begin{bmatrix} \frac{\partial W}{\partial x_1} & \frac{\partial W}{\partial x_2} & \frac{\partial W}{\partial \delta\theta} \end{bmatrix} \begin{bmatrix} x_2 \\ -x_1 - x_2 - \delta\theta\sin(x_1) \\ \delta\dot{\theta} \end{bmatrix}$$

$$\dot{W} = \begin{bmatrix} x_1 + \lambda x_1 + \lambda x_2 & x_2 + \lambda x_1 & \delta\theta \end{bmatrix} \begin{bmatrix} x_2 \\ -x_1 - x_2 - \delta\theta\sin(x_1) \\ \delta\dot{\theta} \end{bmatrix}$$

$$\dot{W} = \cancel{x_2 x_1} + \cancel{\lambda x_1 x_2} + \lambda x_2^2 - \cancel{x_2 x_1} - x_2^2 - \delta\theta\sin(x_1)x_2 - \lambda x_1^2 - \cancel{\lambda x_1 x_2} - \lambda x_1\delta\theta\sin(x_1) + \delta\theta\delta\dot{\theta}$$

$$\dot{W} = -x_2^2(1-\lambda) - \lambda x_1^2 - \delta\theta\left(\sin(x_1)x_2 + \lambda x_1\sin(x_1)\right) + \delta\theta\delta\dot{\theta}$$

If $\delta\dot{\theta} = \sin(x_1)x_2 + \lambda x_1\sin(x_1)$ then $\dot{W} = -x_2^2(1-\lambda) - \lambda x_1^2$

$0<\lambda<1$ $\therefore$ $-x_2^2(1-\lambda) < 0$ for $x_2 \neq 0$ & λx_1^2 for $x_1 \neq 0$

$\therefore$ $\dot{W} \leq 0$ for all x

$\dot{W} = -x_2^2(1-\lambda) - \lambda x_1^2 \leq 0$ for all x

$\dot{W} \equiv 0 \Rightarrow x_1 = x_2 = 0 \Rightarrow \dot{x}_1 = 0 = \dot{x}_2 = -\tilde{\theta}\sin(0) = 0$

$\therefore$ $\dot{W} \equiv$ only if $x_1 = x_2 = 0$

$$\dot{\tilde{\theta}} = \dot{\hat{\theta}} - \cancel{\dot{\theta}}^{\,0\ (\text{Constant})} = \dot{\hat{\theta}} = \underline{\sin(x_1)x_2 + \lambda x_1 \sin(x_1)}$$

If $u = -\hat{\theta}\sin(x_1)$ with $\dot{\hat{\theta}} = \sin(x_1)x_2 + \lambda x_1 \sin(x_1)$, (where $0 < \lambda < 1$) then $W(x, \tilde{\theta})$ is Positive definite with $\dot{W} \leq 0$ for all x. $\dot{W} \equiv 0$ only for $x \equiv 0$ $\therefore$ the closed loop system is G.A.S. about $x \equiv 0$. Because W is P.D (& radially unbounded), then $\tilde{\theta}$ & x must be bounded. Since θ is a constant, $\hat{\theta}$ must also be bounded, $\therefore$ $u(t)$ is also bounded.

Adaptive feedback controller: $u(t) = -\hat{\theta}\sin(x_1)$

$$\dot{\hat{\theta}} = \sin(x_1)x_2 + \lambda x_1 \sin(x_1)$$

Contents

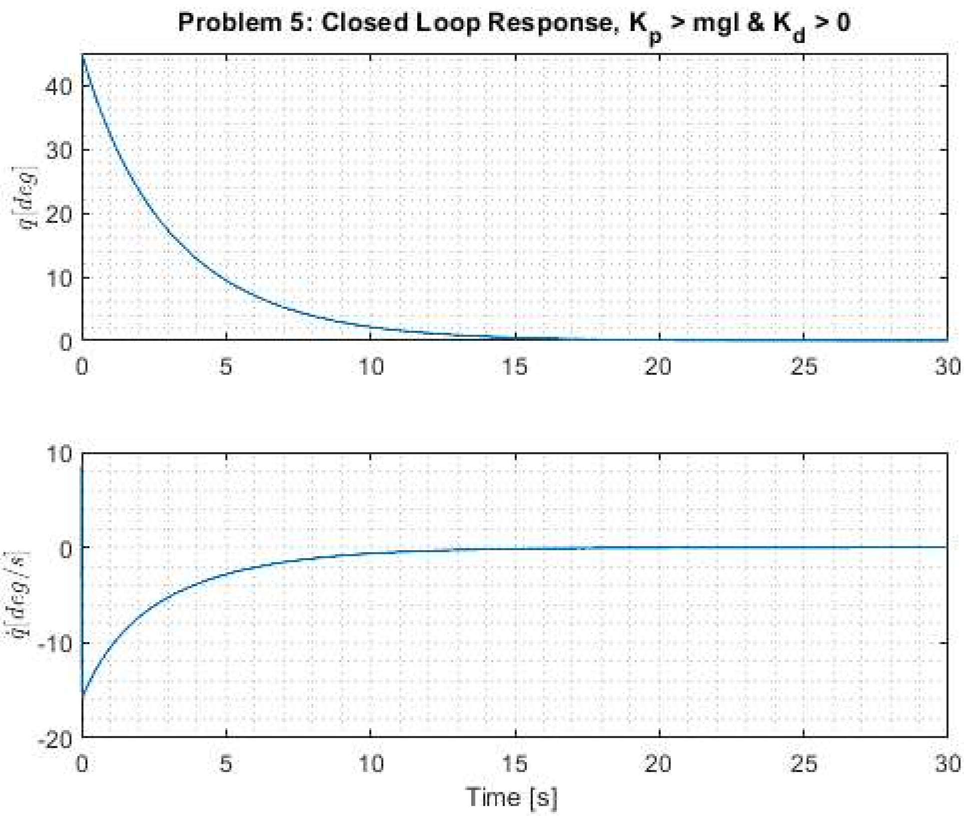
Gabriel Colangelo HW 8

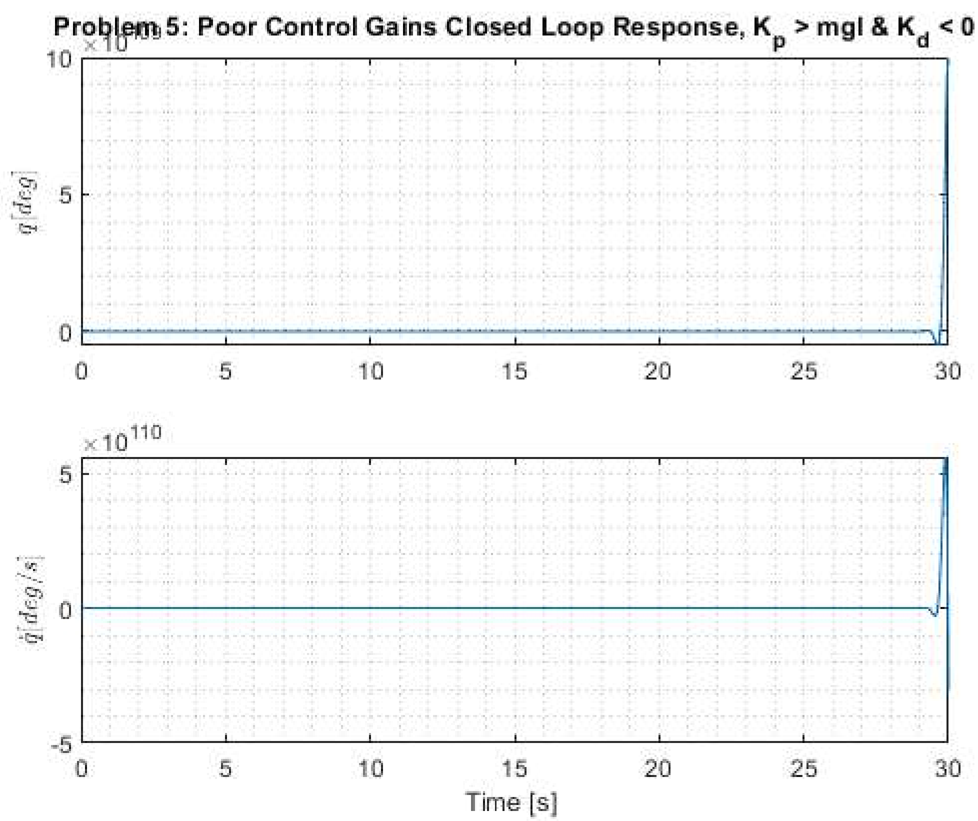
```
clear
close all
clc
```

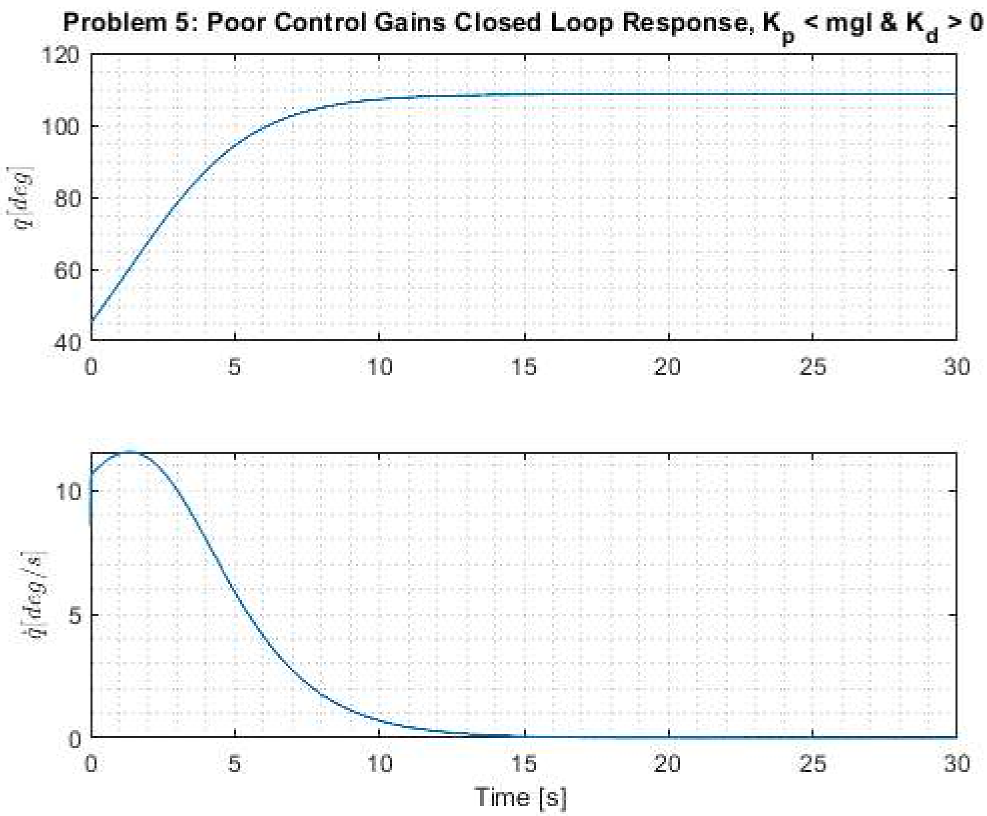
Problem 5

```
% Numerical parameters
m           = 0.2;          % [kg] Pendulum mass
l           = 0.3;          % [m] Distance to center of mass
I           = .006;         % [kg-m^2]
g           = 9.81;         % [m/s^2]
a           = m*g*l/I;
b           = 1/I;

% sim time
time        = (0:.005:30)';

% ODE45 solver options
opts        = odeset('AbsTol',1e-8,'RelTol',1e-8);

% Initial Conditions
x0          = [45*pi/180; 0.15];

% Choose control gains
Kp          = 1.5*m*g*l;    % Kp > m*g*l
Kd          = 1;            % Kd > 0
K           = [Kp Kd];

% Choose bad control gains
Kd_bad      = [Kp -0.1];
Kp_bad      = [0.5*m*g*l 1];

% ODE45 Function calls
[T, X]      = ode45(@(t,x) InvertedPendulumPD(t, x, a, b, K),...
                time, x0, opts);
[~, X_kd]   = ode45(@(t,x) InvertedPendulumPD(t, x, a, b, Kd_bad),...
                time, x0, opts);
[~, X_kp]   = ode45(@(t,x) InvertedPendulumPD(t, x, a, b, Kp_bad),...
                time, x0, opts);

figure
subplot(211)
plot(T,X(:,1)*180/pi)
grid minor
ylabel('$q [deg]$','Interpreter','latex')
title('Problem 5: Closed Loop Response, K_p > mgl & K_d > 0')
subplot(212)
```

```
plot(T,X(:,2)*180/pi)
grid minor
ylabel('$\dot{q} [deg/s]$','Interpreter','latex')
xlabel('Time [s] ')

figure
subplot(211)
plot(T,X_kd(:,1)*180/pi)
grid minor
ylabel('$q [deg]$','Interpreter','latex')
title('Problem 5: Poor Control Gains Closed Loop Response, K_p > mgl & K_d < 0')
subplot(212)
plot(T,X_kd(:,2)*180/pi)
grid minor
ylabel('$\dot{q} [deg/s]$','Interpreter','latex')
xlabel('Time [s] ')

figure
subplot(211)
plot(T,X_kp(:,1)*180/pi)
grid minor
ylabel('$q [deg]$','Interpreter','latex')
title('Problem 5: Poor Control Gains Closed Loop Response, K_p < mgl & K_d > 0')
subplot(212)
plot(T,X_kp(:,2)*180/pi)
grid minor
ylabel('$\dot{q} [deg/s]$','Interpreter','latex')
xlabel('Time [s] ')
```


Problem 5: Closed Loop Response, K_p > mgl & K_d > 0
q [deg]
q̇ [deg/s]
Time [s]


Problem 5: Poor Control Gains Closed Loop Response, K_p > mgl & K_d < 0
q [deg]
×10^110
q̇ [deg/s]
Time [s]

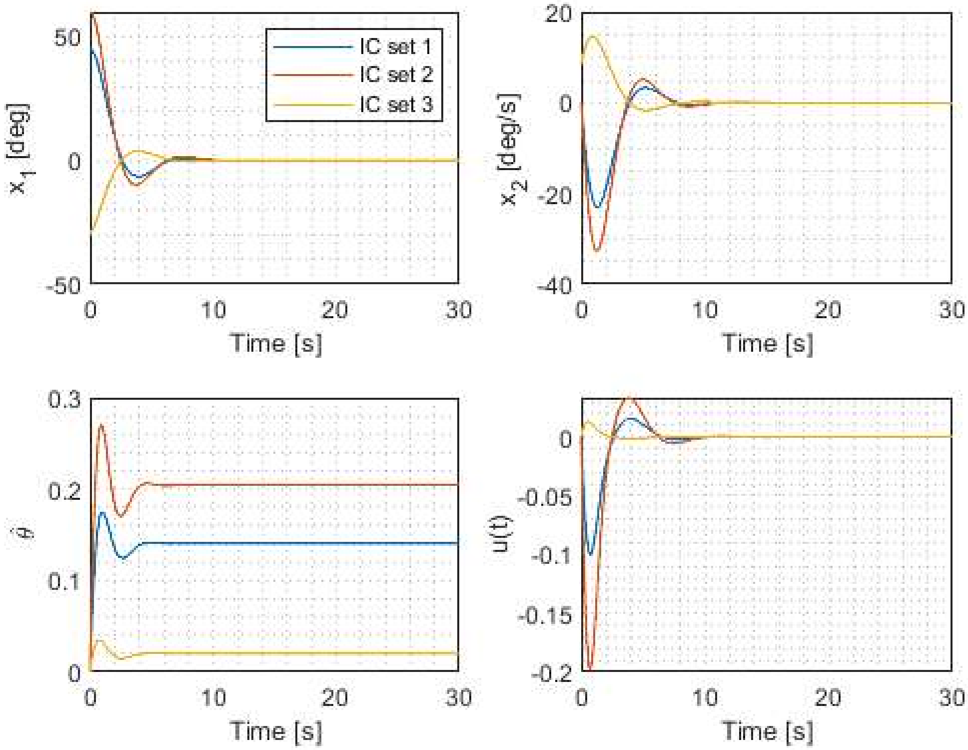


Problem 6

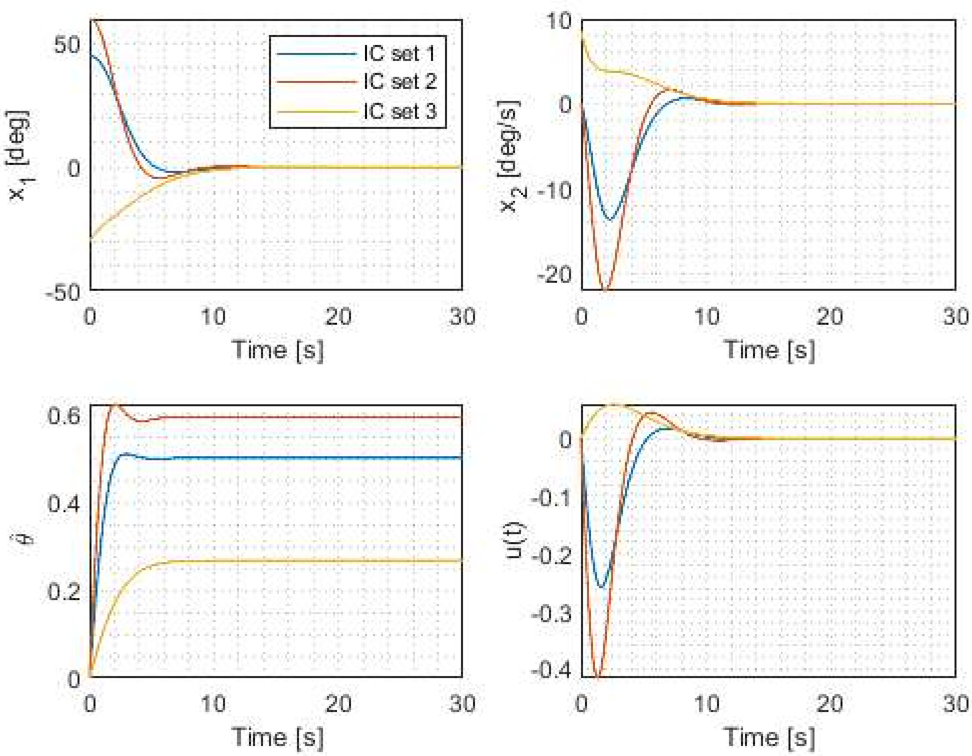
```
% Unknown parameter vector
theta       = [0.25, 1, 4];

% Constant tuning parameter (0 < lambda < 1)
lambda      = 0.7;

% Matrix of Initial Conditions: [x1;x2;theta_hat]
IC          = [45*pi/180 60*pi/180 -30*pi/180; 0 0 0.15; 0 0 0];

% Loop through various unknown parameters
for j = 1:length(theta)

    % Initialize vectors
    x1          = zeros(length(time),length(IC));
    x2          = x1;
    theta_hat   = x1;
    u           = x1;

    for i = 1:length(IC)
        % ODE45 Function call
        [T, Y]          = ode45(@(t,x) AdaptiveController(t,x, theta(j), lambda),...
                            time, IC(:,i), opts);
        % Extract and Store States
        x1(:,i)         = Y(:,1)*180/pi;
        x2(:,i)         = Y(:,2)*180/pi;
        theta_hat(:,i)  = Y(:,3);

        % Calculate control input
        u(:,i)          = -theta_hat(:,i).*sind(x1(:,i));
    end
```

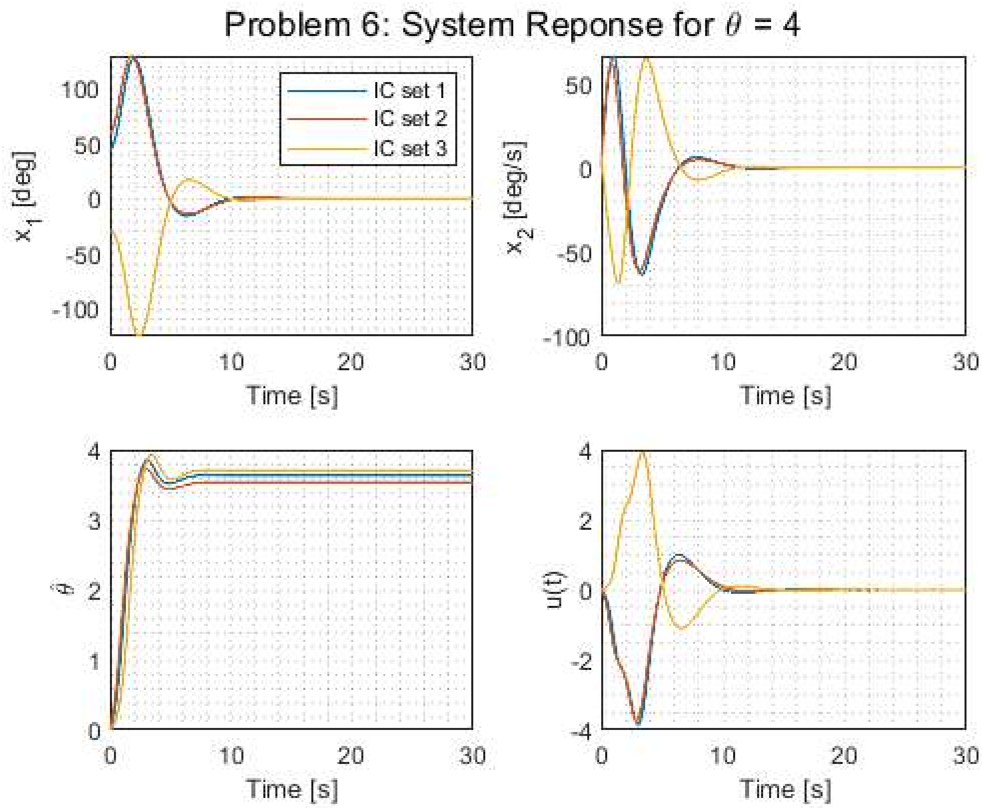
```
    figure
    subplot(221)
    plot(T,x1)
    grid minor
    legend('IC set 1','IC set 2', 'IC set 3')
    xlabel('Time [s]')
    ylabel('x_1 [deg]')
    subplot(222)
    plot(T,x2)
    xlabel('Time [s]')
    grid minor
    ylabel('x_2 [deg/s]')
    subplot(223)
    plot(T,theta_hat)
    xlabel('Time [s]')
    grid minor
    ylabel('$$\hat{\theta}$$','Interpreter','latex')
    subplot(224)
    plot(T,u)
    grid minor
    ylabel('u(t)')
    xlabel('Time [s]')
    sgtitle(['Problem 6: System Reponse for \theta = ' num2str(theta(j))])
end
```

Problem 6: System Reponse for θ = 0.25



Problem 6: System Reponse for θ = 1





Functions

```
function xdot = InvertedPendulumPD(t, x, a, b, K)
    % K = [Kp Kd]
    u       = -K*x;   % PD Controller

    % Plant: x2dot - asin(x1) = bu
    xdot    = [x(2,1); a*sin(x(1,1)) + b*u];
end

function xdot = AdaptiveController(t, x, theta, lambda)
    % States: [x1, x2, theta_hat]
    x1              = x(1,1);
    x2              = x(2,1);
    theta_hat       = x(3,1);

    % Adaptive Controller
    u               = -theta_hat*sin(x1);
    theta_hat_dot   = x2*sin(x1) + lambda*x1*sin(x1);

    % Plant
    x1dot           = x2;
    x2dot           = -x1 -x2 + theta*sin(x1) + u;

    % State dynamics
    xdot            = [x1dot;x2dot;theta_hat_dot];
end
```

Published with MATLAB® R2021b