Examples – 3D Representations

Example 1:

Given : $\bar{r}_1 = 1.6772\,R_\oplus\,\hat{x} - 1.6772\,R_\oplus\,\hat{y} + 2.3719\,R_\oplus\,\hat{z}$
$\bar{v}_1 = 3.1574\,\hat{x} + 2.4987\,\hat{y} + 0.4658\,\hat{z}$ km/s

UNITS!!

Find : $a, e, i, \Omega, \omega, \theta^*$

Analysis :

Shape → $r_1 = |\bar{r}_1| \qquad v_1 = |\bar{v}_1|$

$$\underbrace{\varepsilon = -\frac{\mu}{2a}} = \frac{v_1^2}{2} - \frac{\mu}{r_1} \qquad \Rightarrow \qquad \boxed{a = 3\,R_\oplus}$$

$$\bar{h} = \bar{r}_1 \times \bar{v}_1 \rightarrow h = |\bar{h}| = \sqrt{\mu p} = \sqrt{\mu a(1-e_2)}$$

$$\rightarrow \boxed{e = 0.2}$$

From conic equation →

$$\theta_1^* = \pm\cos^{-1}\left\{\frac{1}{e}\left(\frac{p}{r_1} - 1\right)\right\}$$

orbit orientations →

$$\hat{h} = \underbrace{\frac{\bar{r}\times\bar{v}}{\left|\bar{r}\times\bar{v}\right|}} = -.5\,\hat{x} + .5\,\hat{y} + .7071\,\hat{z}$$

${}^{I}C^{R}$	$\hat{r}_1$	$\hat{\theta}_1$	$\hat{h}_1$
$\hat{x}$	....	....	$s_\Omega s_i$
$\hat{y}$	....	....	$-c_\Omega s_i$
$\hat{z}$	$s_i s_\theta$	$s_i c_\theta$	c_i

$$c_i = .7071 \Rightarrow$$

$$s_\Omega s_i = -.5$$

$$-c_\Omega s_i = +.5$$

Numerical values

Note that we can also obtain the remaining elements of the direction cosine matrix

$$\hat{r}_1 = \frac{\bar{r}_1}{\left|\bar{r}_1\right|} = .5\,\hat{x} - .5\,\hat{y} + .7071\hat{z}$$

$$\hat{\theta}_1 = \hat{h}\times\hat{r}_1 = .7071\hat{x} + .7071\hat{y}$$

$$\left.\begin{aligned} s_i s_{\theta_1} &= .7071 \\ s_i c_{\theta_1} &= 0 \end{aligned}\right\} \qquad \boxed{\theta_1 = 90^\circ}$$

Back to θ_1^* $\longrightarrow$ Recall

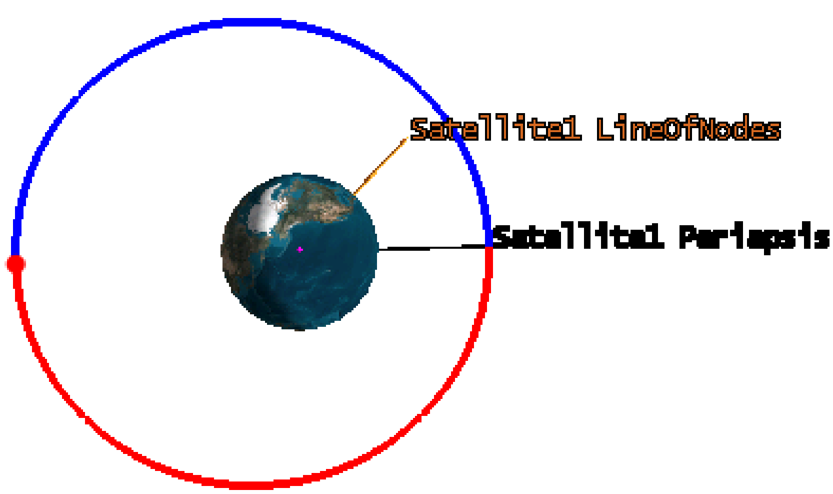
$$\bar{v}_1 = \underbrace{\left(\bar{v}_1 \bullet \hat{r}_1\right)}\,\hat{r}_1 + \left(\bar{v}_1 \bullet \hat{\theta}_1\right)\hat{\theta}_1$$

$$\bar{v}_1 \bullet \hat{r}_1 \;\rightarrow$$

in this case $\dot{r}_1 =$

$$\Rightarrow \boxed{\theta_1^* = +135^\circ}$$

$$\omega = \theta_1 - \theta_1^*$$



Example 2:

<u>Given</u> :
$$\bar{r}_1 = 14450.6\,\hat{x} - 1529.9\,\hat{y} - 6524.0\,\hat{z} \text{ km}$$
$$\bar{r}_2 = -6199.5\,\hat{x} + 14699.2\,\hat{y} + 8531.9\,\hat{z} \text{ km}$$

UNITS!!

$$p = 2.88\,R_{\oplus}$$

<u>Find</u>: $a, e, i, \Omega, \omega, \theta_1^*, \theta_2^*, \bar{v}_1, \bar{v}_2$

<u>Analysis</u>

Available $\quad r_1 = |\bar{r}_1| \qquad r_2 = |\bar{r}_2|$

$$\hat{h} = \frac{\bar{r}_1 \times \bar{r}_2}{|\bar{r}_1 \times \bar{r}|}$$ why? Are $\hat{h}_1, \hat{h}_2$ both necessary?

$$\hat{r}_1 = \frac{\bar{r}_1}{|\bar{r}_1|}$$

${}^I C^R$	$\hat{r}_1$	$\hat{\theta}_1$	$\hat{h}$
$\hat{x}$	.9072	.2280	.3536
$\hat{y}$	-.0960	.9305	-.3536
$\hat{z}$	-.4096	.2868	.8660

Can I check ?

What conditions must be satisfied so there is a chance that this DC matrix is correct?

Compare to

${}^{I}C^{R}$	$\hat{r}_1$	$\hat{\theta}_1$	$\hat{h}$
$\hat{x}$			$\sin\Omega\sin i$
$\hat{y}$			$-\cos\Omega\sin i$
$\hat{z}$	$\sin i\sin\theta_1$	$\sin i\cos\theta_1$	$\cos i$

$$\cos i = .866$$

$$\left.\begin{aligned} \sin\Omega\sin i &= .3536 \\ -\cos\Omega\sin i &= -.3536 \end{aligned}\right\}$$

$$\left.\begin{aligned} \sin\theta_1\sin i &= -.4096 \\ \cos\theta_1\sin i &= 2868 \end{aligned}\right\}$$

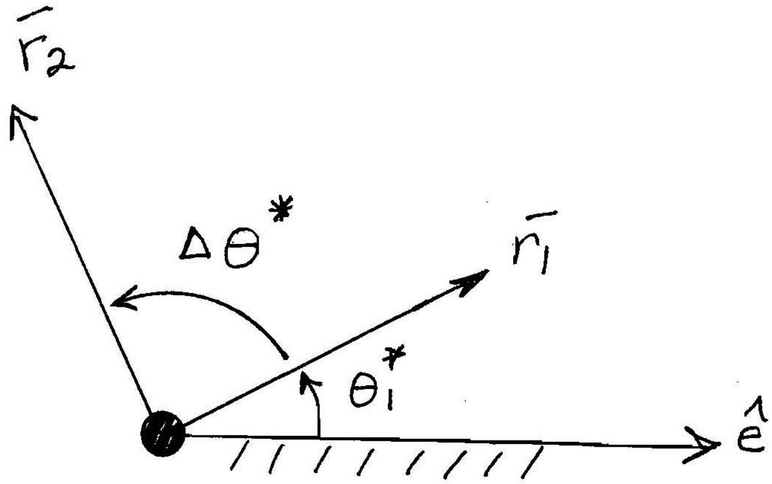
$$\bar{r}_2 = f\;\bar{r}_1 + g\;\bar{v}_1 \Longrightarrow$$

Use f and g in terms of $\Delta\theta^*$!!

$$f, g = f\left(r_1, r_2, \Delta\theta^*, p\right); \quad g\left(r_1, r_2, \Delta\theta^*, p\right)$$

How to find $\Delta\theta^*$?

Any assumptions required?



$$\bar{r}_1 \bullet \bar{r}_2 = r_1 r_2 \cos\Delta\theta^*$$

$$\boxed{\Delta\theta^* = \pm 125.6^\circ}$$

$$\bar{v}_1 = 0.6814\,\hat{x} + 5.0560\,\hat{y} + 1.7859\,\hat{z} \text{ km/s}$$

θ_1^*? Is the vehicle ascending or descending?

Recall $\bar{v}_1 = \underbrace{\left(\bar{v}_1 \bullet \hat{r}_1\right)}_{\dot{r}_1}\hat{r}_1 + \underbrace{\left(\bar{v}_1 \bullet \hat{\theta}_1\right)}_{r_1\dot{\theta}_1}\hat{\theta}_1$

$$\dot{r}_1 = \frac{\bar{v}_1 \bullet \bar{r}_1}{r_1} =$$

Can γ_1 now be determined? How? Would it be + or - ?

Now $v_1 = |\overline{v}_1|$ $\qquad r_1 = |\overline{r}_1|$

$$\mathcal{E} = \frac{v_1^2}{2} - \frac{\mu}{r_1} \quad \Rightarrow \quad \boxed{a = 3R_\oplus}$$

$$p = a\left(1 - e^2\right) \quad \Rightarrow \quad \boxed{e = .2}$$

conic equation

$$\theta_1^* = \pm\cos^{-1}\left\{\frac{1}{e}\left(\frac{p}{r_1} - 1\right)\right\} \quad \Rightarrow$$

$$\theta_2^* = \theta_1^* + \Delta\theta^* \quad \rightarrow \quad \boxed{\theta_2^* = 85.6^\circ}$$

$$\omega = \theta_1 - \theta_1^* \quad \rightarrow$$

θ_2 ?

$\theta_2 = \omega + \theta_2^*$?

$$\boxed{\overline{v}_2 = \dot{f}\,\overline{r}_1 + \dot{g}\,\overline{v}_1}$$