#1) $A = \underbrace{\begin{pmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{T} \underbrace{\begin{pmatrix} -3 & & & \\ & -3 & 1 & \\ & & -3 & \\ & & & 0 \end{pmatrix}}_{J} \begin{pmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1}$

$J_1 = -3$, $J_2 = \begin{pmatrix} -3 & 1 \\ 0 & -3 \end{pmatrix}$, $J_3 = 0$

a)

$$T_1 e^{J_1 t} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{-3t}$$

$$\boxed{x^{(1)}(t) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{-3t}}$$

$$T_2 e^{J_2 t} = \begin{bmatrix} 0 & -2 \\ 1 & 2 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} e^{J_2 t}$$

$$= \begin{bmatrix} 0 & -2 \\ 1 & 2 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} e^{-3t} & te^{-3t} \\ 0 & e^{-3t} \end{pmatrix} = \begin{pmatrix} 0 & -2e^{-3t} \\ e^{-3t} & te^{-3t} + 2e^{-3t} \\ 0 & e^{-3t} \\ 0 & 0 \end{pmatrix}$$

$$\boxed{x^{(2)}(t) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} e^{-3t}}$$

$$\boxed{x^{(3)}(t) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} te^{-3t} + \begin{pmatrix} -2 \\ 2 \\ 1 \\ 0 \end{pmatrix} e^{-3t}}$$

$$X^{(4)}t = T_3 e^{J_3 t} = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 1 \end{pmatrix} e^{0t}$$

$$\boxed{X^{(4)}(t) = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 1 \end{pmatrix}}$$

$$Te^{Jt} = \underbrace{\begin{pmatrix} e^{-3t} & 0 & -2e^{-3t} & -1 \\ 0 & e^{-3t} & (t+2)e^{-3t} & 0 \\ 0 & 0 & e^{-3t} & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{[X^{(1)}(t),\, X^{(2)}(t),\, X^{(3)}(t),\, X^{(4)}(t)]}$$

b) $X(0) = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}$

From Matlab: $T^{-1} = \begin{pmatrix} 1 & 0 & 2 & -1 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$$S_1^T x(0) = (1\ 0\ 2\ -1)\begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix} = 0$$

$$S_2^T x(0) = (0\ 1\ -2\ 2)\begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix} = 1$$

$$S_3^T x(0) = (0\ 0\ 1\ -1)\begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix} = -1$$

$$S_4^T x(0) = (0\ 0\ 0\ 1)\begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix} = 1$$

$$x(t) = \sum_{i=1}^{3} T_i e^{J_i t}(S_i^T x(0)) = X^{(2)}(t) - X^{(3)}(t) + X^{(4)}(t)$$

$$\boxed{x(t) = \underbrace{\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} e^{-3t}}_{X^{(2)}} \underbrace{- \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} te^{-3t} - \begin{pmatrix} -2 \\ 2 \\ 1 \\ 0 \end{pmatrix} e^{-3t}}_{-X^{(3)}} + \underbrace{\begin{pmatrix} -1 \\ 0 \\ 1 \\ 1 \end{pmatrix}}_{X^{(4)}}}$$

c)

$$T_1 J_1^k = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} (-3)^k$$

$$\boxed{x^{(1)}[k] = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} (-3)^k}$$

$$T_2 J_2^k = \begin{bmatrix} 0 & -2 \\ 1 & 2 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} J_2^k = \begin{bmatrix} 0 & -2 \\ 1 & 2 \\ 8 & 1 \\ & 0 \end{bmatrix} \begin{bmatrix} (-3)^k & k(-3)^{k-1} \\ 0 & (-3)^k \end{bmatrix}$$

$$= \begin{pmatrix} 0 & -2(-3)^k \\ (-3)^k & k(-3)^{k-1} + 2(-3)^k \\ 0 & (-3)^k \\ 0 & 0 \end{pmatrix}$$

$$\boxed{x^{(2)}[k] = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} (-3)^k}$$

$$\boxed{x^{(3)}[k] = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} k(-3)^{k-1} + \begin{pmatrix} -2 \\ 2 \\ 1 \\ 0 \end{pmatrix} (-3)^k}$$

$$T_3 J_3^k = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 1 \end{pmatrix} 0^k$$

$$\boxed{x^{(4)}[k] = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 1 \end{pmatrix} 0^k}$$

$$TJ^k = \begin{bmatrix} (-3)^k & 0 & (-2)(-3)^k & -0^k \\ 0 & (-3)^k & k(-3)^{k-1} + (2)(-3)^k & 0 \\ 0 & 0 & (-3)^k & 0^k \\ 0 & 0 & 0 & 0^k \end{bmatrix} = \left(x^{(1)}[k], x^{(2)}[k], x^{(3)}[k], x^{(4)}[k]\right)$$

d) $x(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

$s_1^T x(0) = -1$

$s_2^T x(0) = 2$

$s_3^T x(0) = -1$

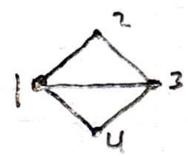
$s_4^T x(0) = 1$

$$x[k] = -x^{(1)}[k] + 2x^{(2)}[k] - x^{(3)}[k] + x^{(4)}[k]$$

$$\boxed{x[k] = \underbrace{-\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}(-3)^k}_{-x^{(1)}} + \underbrace{2\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}(-3)^k}_{2x^{(2)}} \underbrace{- \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} k(-3)^{k-1} - \begin{pmatrix} -2 \\ 2 \\ 1 \\ 0 \end{pmatrix}(-3)^k}_{-x^{(3)}} + \underbrace{\begin{pmatrix} -1 \\ 0 \\ 1 \\ 1 \end{pmatrix} 0^k}_{x^{(4)}}}$$

#2)

ia)



$$X_1[k+1] = \frac{X_1[k] + X_2[k] + X_3[k] + X_4[k]}{4}$$

$$X_2[k+1] = \frac{X_1[k] + X_3[k] + X_2[k]}{3}$$

$$X_3[k+1] = \frac{X_1[k] + X_2[k] + X_3[k] + X_4[k]}{4}$$

$$X_4[k+1] = \frac{X_1[k] + X_3[k] + X_4[k]}{3}$$

$$X[k+1] = A\, X[k]$$

$$\boxed{A = \begin{pmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ 1/3 & 1/3 & 1/3 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/3 & 0 & 1/3 & 1/3 \end{pmatrix}}$$

iia) $$V_1 = \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{pmatrix}, \underbrace{V_2 = \begin{pmatrix} .4243 \\ -.5657 \\ .4243 \\ -.5657 \end{pmatrix}, V_3 = \begin{pmatrix} -.7071 \\ 0 \\ .7071 \\ 0 \end{pmatrix}, V_4 = \begin{pmatrix} 0 \\ -.7071 \\ 0 \\ .7071 \end{pmatrix}}_{\text{Right e-vecs}}$$

$$w_1 = \begin{pmatrix} .5714 \\ .4286 \\ .5714 \\ .4286 \end{pmatrix},\quad w_2 = \begin{pmatrix} .5051 \\ -.5051 \\ .5051 \\ -.5051 \end{pmatrix},\quad w_3 = \begin{pmatrix} -.7071 \\ 0 \\ .7071 \\ 0 \end{pmatrix},\quad w_4 = \begin{pmatrix} 0 \\ -.7071 \\ 0 \\ .7071 \end{pmatrix}$$

left e-vecs

$$\lambda_1 = 1,\quad \lambda_2 = -.1667 = -\tfrac{1}{6},\quad \lambda_3 = 0,\quad \lambda_4 = .333 = \tfrac{1}{3}$$

A is diagonlizable as all λ_i are distinct $\therefore$

$$X[k] = \sum_{i}^{4} w_i^T X(0)\, \lambda_i^k\, V_i = T \Lambda^k T^{-1} X(0)$$

$$\boxed{\begin{aligned} X[k] = {} & \underbrace{(.5714\ \ .4286\ \ .5714\ \ .4286)\, X(0)\, (1)^k \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{pmatrix}}_{w_1^T X(0) \lambda_1^k V_1} \\ & + \underbrace{(.5051\ \ -.5051\ \ .5051\ \ -.5051)\, X(0)\, (-1/6)^k \begin{pmatrix} .4243 \\ -.5657 \\ .4243 \\ -.5657 \end{pmatrix}}_{w_2^T X(0) \lambda_2^k V_2} \\ & + \underbrace{(-.7071\ \ 0\ \ .7071\ \ 0)\, X(0)\, (0)^k \begin{pmatrix} -.7071 \\ 0 \\ .7071 \\ 0 \end{pmatrix}}_{w_3^T X(0) \lambda_3^k V_3} \\ & + \underbrace{(0\ \ -.7071\ \ 0\ \ .7071)\, X(0)\, (1/3)^k \begin{pmatrix} 0 \\ -.7071 \\ 0 \\ .7071 \end{pmatrix}}_{w_4^T X(0) \lambda_4^k V_4} \end{aligned}}$$

$$X[k] = \begin{pmatrix} 1/2 & .4243 & -.7071 & 0 \\ 1/2 & -.5657 & 0 & -.7071 \\ 1/2 & .4243 & .7071 & 0 \\ 1/2 & -.5657 & 0 & .7071 \end{pmatrix} \begin{pmatrix} 1^k & 0 & 0 & 0 \\ 0 & (-1/6)^k & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (1/3)^k \end{pmatrix} \begin{pmatrix} .5714 & .4286 & .5714 & .4286 \\ .5051 & -.5051 & .5051 & -.5051 \\ -.7071 & 0 & .7071 & 0 \\ 0 & -.7071 & 0 & .7071 \end{pmatrix} X(0)$$

iii a)

$$\lim_{K\to\infty}(X[K]) = \lim_{K\to\infty}\left(\omega_1^T X_0 \cancel{(1)^K}^{1} V_1 + \omega_2^T X_0 \cancel{(-\tfrac{1}{6})^K}^{0} V_2 + \omega_3^T X_0 \cancel{(0)^K}^{0} V_3 + \omega_4^T X_0 \cancel{(\tfrac{1}{3})^K}^{0} V_4\right)$$

$$\lim_{K\to\infty}(X[K]) = \omega_1^T X(0) V_1 = (.5714\, X_1(0) + .4286\, X_2(0) + .5714\, X_3(0) + .4286\, X_4(0)) \begin{pmatrix} \tfrac{1}{2} \\ \tfrac{1}{2} \\ \tfrac{1}{2} \\ \tfrac{1}{2} \end{pmatrix} = \begin{pmatrix} \bar{X} \\ \bar{X} \\ \bar{X} \\ \bar{X} \end{pmatrix}$$

$$\boxed{\bar{X} = .2857\, X_1(0) + .2143\, X_2(0) + .2857\, X_3(0) + .2143\, X_4(0)}$$

Indivuals 1 & 3 opinions matter more

iv a) $e[K] = X[K] - \begin{bmatrix} \bar{X} \\ \bar{X} \\ \bar{X} \\ \bar{X} \end{bmatrix}$

$\bar{X} = \omega_1^T X(0)(1)^K V_1$

$e[K] = \omega_2^T X(0)(-\tfrac{1}{6})^K V_2 + \omega_3^T X(0)(0)^K V_3 + \omega_4^T X(0)(\tfrac{1}{3})^K V_4$

The dominant rate is $(\tfrac{1}{3})^K$. Consenus will then be acheived when $(\tfrac{1}{3})^K$ reaches 0 as $(-\tfrac{1}{6})^K$ and $(0)^K$ will have reached 0 already.

ib)

$$X_1[k+1] = \frac{X_1[k] + X_2[k] + X_4[k]}{3}$$

$$X_2[k+1] = \frac{X_2[k] + X_1[k] + X_3[k]}{3}$$

$$X_3[k+1] = \frac{X_3[k] + X_2[k] + X_4[k]}{3}$$

$$X_4[k+1] = \frac{X_4[k] + X_3[k] + X_1[k]}{3}$$

$$\boxed{A = \begin{pmatrix} 1/3 & 1/3 & 0 & 1/3 \\ 1/3 & 1/3 & 1/3 & 0 \\ 0 & 1/3 & 1/3 & 1/3 \\ 1/3 & 0 & 1/3 & 1/3 \end{pmatrix}}$$

iib) From MATLAB: $X[k] = \overbrace{[V_1\ V_2\ V_3\ V_4]}^{T} \overbrace{\begin{pmatrix} \lambda_1^k & 0 & 0 & 0 \\ 0 & \lambda_2^k & 0 & 0 \\ 0 & 0 & \lambda_3^k & 0 \\ 0 & 0 & 0 & \lambda_4^k \end{pmatrix}}^{\lambda^k} \overbrace{\begin{bmatrix} w_1^T \\ w_2^T \\ w_3^T \\ w_4^T \end{bmatrix}}^{T^{-1}} X(0)$

$$V_1 = \begin{pmatrix} 1/2 \\ -1/2 \\ 1/2 \\ -1/2 \end{pmatrix}, V_2 = \begin{pmatrix} 0.3627 \\ -0.607 \\ -0.3627 \\ 0.607 \end{pmatrix}, V_3 = \begin{pmatrix} 0.607 \\ 0.3627 \\ -0.607 \\ -0.3627 \end{pmatrix}, V_4 = \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{pmatrix}$$

Right e-vecs

$$\lambda_1 = -1/3,\ \lambda_2 = 1/3,\ \lambda_3 = 1/3,\ \lambda_4 = 1$$

$$w_1 = \begin{pmatrix} 1/2 \\ -1/2 \\ 1/2 \\ -1/2 \end{pmatrix}, w_2 = \begin{pmatrix} .3627 \\ -.6070 \\ -.3627 \\ .6070 \end{pmatrix}, w_3 = \begin{pmatrix} .6070 \\ .3627 \\ -.6070 \\ -.3627 \end{pmatrix}, w_4 = \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{pmatrix}$$

left evecs

Diagonalizable as $am(\lambda_i) = gm(\lambda_i)$

$$x[k] = \left(\tfrac{1}{2} \; -\tfrac{1}{2} \; \tfrac{1}{2} \; -\tfrac{1}{2}\right) x(0) \left(-\tfrac{1}{3}\right)^k \begin{pmatrix} \tfrac{1}{2} \\ -\tfrac{1}{2} \\ \tfrac{1}{2} \\ -\tfrac{1}{2} \end{pmatrix} \Big\} \, w_1^T x_0 \lambda_1^k v_1$$

$$+ \left(.3627 \; -.6070 \; -.3627 \; .6070\right) x(0) \left(\tfrac{1}{3}\right)^k \begin{pmatrix} .3627 \\ -.6070 \\ -.3627 \\ .6070 \end{pmatrix} \Big\} \, w_2^T x_0 \lambda_2^k v_2$$

$$+ \left(.6070 \; .3627 \; -.6070 \; -.3627\right) x(0) \left(\tfrac{1}{3}\right)^k \begin{pmatrix} .6070 \\ .3627 \\ -.6070 \\ -.3627 \end{pmatrix} \Big\} \, w_3^T x_0 \lambda_3^k v_3$$

$$+ \left(\tfrac{1}{2} \; \tfrac{1}{2} \; \tfrac{1}{2} \; \tfrac{1}{2}\right) x(0) (1)^k \begin{pmatrix} \tfrac{1}{2} \\ \tfrac{1}{2} \\ \tfrac{1}{2} \\ \tfrac{1}{2} \end{pmatrix} \Big\} \, w_4^T x_0 \lambda_4^k v_4$$

iiib) $$\lim_{k \to \infty} (x[k]) = \left(\tfrac{1}{2} \; \tfrac{1}{2} \; \tfrac{1}{2} \; \tfrac{1}{2}\right) x(0) (1) \begin{pmatrix} \tfrac{1}{2} \\ \tfrac{1}{2} \\ \tfrac{1}{2} \\ \tfrac{1}{2} \end{pmatrix} = \begin{pmatrix} \bar{x} \\ \bar{x} \\ \bar{x} \\ \bar{x} \end{pmatrix}$$

$$= \tfrac{1}{2}\left(x_1(0) + x_2(0) + x_3(0) + x_4(0)\right) \begin{pmatrix} \tfrac{1}{2} \\ \tfrac{1}{2} \\ \tfrac{1}{2} \\ \tfrac{1}{2} \end{pmatrix} = \begin{pmatrix} \bar{x} \\ \bar{x} \\ \bar{x} \\ \bar{x} \end{pmatrix} = w_4^T x_0 \lambda_4^k v_4$$

$$\bar{x} = \frac{x_1(0) + x_2(0) + x_3(0) + x_4(0)}{4}$$

All Indivuals matter equally

ivb) $e = x - \bar{x} = \omega_1^T x_0 \lambda_1^k v_1 + \omega_2^T x_0 \lambda_2^k v_2 + \omega_3^T x_0 \lambda_3^k v_3$

Dominant rate is $(1/3)^k$. Consensus will be reached when all three modes of consensus error die out simultaneously

ic)

$$x_1[k+1] = \frac{x_1[k] + x_2[k] + x_4[k]}{3}$$

$$x_2[k+1] = \frac{x_2[k] + x_3[k] + x_1[k]}{3}$$

$$x_3[k+1] = \frac{x_3[k] + x_2[k]}{2}$$

$$x_4[k+1] = \frac{x_4[k] + x_1[k]}{2}$$

$$A = \begin{pmatrix} 1/3 & 1/3 & 0 & 1/3 \\ 1/3 & 1/3 & 1/3 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 & 1/2 \end{pmatrix}$$

iic) $\lambda_1 = -0.2287$, $\lambda_2 = 0.1667$, $\lambda_3 = 1$, $\lambda_4 = 0.7287$

Four distinct λ_i, $\therefore$ A is diagonalizable

Right e-vecs:

$$V_1 = \begin{pmatrix} .5831 \\ -.5831 \\ .4001 \\ -.4001 \end{pmatrix}, \quad V_2 = \begin{pmatrix} -.3922 \\ -.3922 \\ .5883 \\ .5883 \end{pmatrix}, \quad V_3 = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}, \quad V_4 = \begin{pmatrix} .2941 \\ -.2941 \\ -.6340 \\ .6340 \end{pmatrix}$$

Left e-vecs:

$$\omega_1 = \begin{pmatrix} .6527 \\ -.6527 \\ .2986 \\ -.2986 \end{pmatrix}, \quad \omega_2 = \begin{pmatrix} -.5099 \\ -.5099 \\ .5099 \\ .5099 \end{pmatrix}, \quad \omega_3 = \begin{pmatrix} .6 \\ .6 \\ .4 \\ .4 \end{pmatrix}, \quad \omega_4 = \begin{pmatrix} .4061 \\ -.4061 \\ -.5918 \\ .5918 \end{pmatrix}$$

$$X[k] = T \Lambda^k T^{-1} x(0) = \sum \omega_i^T x(0)\, \lambda_i^k\, v_i \qquad \text{(Diagonalizable)}$$

$$X[k] = \begin{pmatrix} .5831 & -.3922 & \frac{1}{2} & .2941 \\ -.5831 & -.3922 & \frac{1}{2} & -.2941 \\ .4001 & .5883 & \frac{1}{2} & -.6340 \\ -.4001 & .5883 & \frac{1}{2} & .6340 \end{pmatrix} \begin{pmatrix} (-.2287)^k & 0 & 0 & 0 \\ 0 & (\frac{1}{2})^k & 0 & 0 \\ 0 & 0 & 1^k & 0 \\ 0 & 0 & 0 & (.7287)^k \end{pmatrix} \begin{pmatrix} .6527 & -.6527 & .2986 & -.2986 \\ -.5099 & -.5099 & .5099 & .5099 \\ .6 & .6 & .4 & .4 \\ .4061 & -.4061 & -.5918 & .5918 \end{pmatrix} x(0)$$

$$\boxed{\begin{aligned} X[k] &= (.6527 \ -.6527 \ .2986 \ -.2986)\, x(0)\, (-.2287)^k \begin{pmatrix} .5831 \\ -.5831 \\ .4001 \\ -.4001 \end{pmatrix} + \\ &(-.5099 \ -.5099 \ .5099 \ .5099)\, x(0)\, (\tfrac{1}{2})^k \begin{pmatrix} -.3922 \\ -.3922 \\ .5833 \\ .5833 \end{pmatrix} + \\ &(.6 \ .6 \ .4 \ .4)\, x(0)\, (1)^k \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} + (.4061 \ -.4061 \ -.5918 \ .5918)\, x(0)\, (.7287)^k \begin{pmatrix} .2941 \\ -.2941 \\ -.6340 \\ .6340 \end{pmatrix} \end{aligned}}$$

iii c) $$\lim_{k \to \infty} (X[k]) = (.6 \ .6 \ .4 \ .4)\, x(0)\, 1 \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \bar{x} \\ \bar{x} \\ \bar{x} \\ \bar{x} \end{pmatrix}$$

$$\boxed{\bar{X} = .3\, X_1(0) + .3\, X_2(0) + .2\, X_1(0) + .2\, X_1(0)}$$

Indivuals 1 & 2 opinions matter more

iv c) $e[k] = x[k] - \begin{bmatrix} \bar{x} \\ \bar{x} \\ \bar{x} \\ \bar{x} \end{bmatrix}$

$$e[k] = w_1^T x(0)\, \lambda_1^k v_1 + w_2^T x(0)\, \lambda_2^k v_2 + w_4^T x(0)\, \lambda_4^k v_4$$

$\lambda_4 = .7287$ which has largest magnitude $\therefore$

The decay rate is $(.7287)^k$ and consenous will be acheived when the fourth mode goes to 0.

i d) 

$X_1[k+1] = \dfrac{X_1[k] + X_4[k]}{2}$ $\quad$ $X_2[k+1] = \dfrac{X_2[k] + X_3[k]}{2}$

$X_4[k+1] = \dfrac{X_1[k] + X_4[k]}{2}$ $\quad$ $X_3[k+1] = \dfrac{X_2[k] + X_3[k]}{2}$

$$A = \begin{pmatrix} 1/2 & 0 & 0 & 1/2 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ 1/2 & 0 & 0 & 1/2 \end{pmatrix}$$

iid) $\lambda_1 = 0, \lambda_2 = 0, \lambda_3 = 1, \lambda_4 = 1$

A is diagonalizable as $a.m.(\lambda=0) = g.m.(\lambda=0) = 2$ & $a.m.(\lambda=1) = g.m.(\lambda=1) = 2$

Right e-vecs:

$$V_1 = \begin{pmatrix} 0 \\ -.7071 \\ .7071 \\ 0 \end{pmatrix}, \quad V_2 = \begin{pmatrix} .7071 \\ 0 \\ 0 \\ -.7071 \end{pmatrix}, \quad V_3 = \begin{pmatrix} .7071 \\ 0 \\ 0 \\ .7071 \end{pmatrix}, \quad V_4 = \begin{pmatrix} 0 \\ .7071 \\ .7071 \\ 0 \end{pmatrix}$$

Left e-vecs:

$$w_1 = \begin{pmatrix} 0 \\ -.7071 \\ .7071 \\ 0 \end{pmatrix}, \quad w_2 = \begin{pmatrix} .7071 \\ 0 \\ 0 \\ -.7071 \end{pmatrix}, \quad w_3 = \begin{pmatrix} .7071 \\ 0 \\ 0 \\ .7071 \end{pmatrix}, \quad w_4 = \begin{pmatrix} 0 \\ .7071 \\ .7071 \\ 0 \end{pmatrix}$$

$$X[k] = T\Lambda^k T^{-1} X(0) = \begin{pmatrix} 0 & .7071 & .7071 & 0 \\ -.7071 & 0 & 0 & .7071 \\ .7071 & 0 & 0 & .7071 \\ 0 & -.7071 & .7071 & 0 \end{pmatrix} \begin{pmatrix} 0^k & 0 & 0 & 0 \\ 0 & 0^k & 0 & 0 \\ 0 & 0 & 1^k & 0 \\ 0 & 0 & 0 & 1^k \end{pmatrix} \begin{pmatrix} 0 & -.7071 & .7071 & 0 \\ .7071 & 0 & 0 & -.7071 \\ .7071 & 0 & 0 & .7071 \\ 0 & .7071 & .7071 & 0 \end{pmatrix} X(0)$$

$$\boxed{\begin{aligned} X[k] = {} & (0\ \ -.7071\ \ .7071\ \ 0)\, X(0)\, (0)^k \begin{pmatrix} 0 \\ -.7071 \\ .7071 \\ 0 \end{pmatrix} + (.7071\ \ 0\ \ 0\ \ -.7071)\, X(0)\, (0)^k \begin{pmatrix} .7071 \\ 0 \\ 0 \\ -.7071 \end{pmatrix} \\ & + (.7071\ \ 0\ \ 0\ \ .7071)\, X(0)\, (1)^k \begin{pmatrix} .7071 \\ 0 \\ 0 \\ .7071 \end{pmatrix} + (0\ \ .7071\ \ .7071\ \ 0)\, X(0)\, (1)^k \begin{pmatrix} 0 \\ .7071 \\ .7071 \\ 0 \end{pmatrix} \end{aligned}}$$

iii d) $\lim_{k \to \infty} (x[k]) = \begin{pmatrix} \bar{x} \\ \bar{x} \\ \bar{x} \\ \bar{x} \end{pmatrix} = (w_3^T x(0))\, v_3 + w_4^T x(0)\, v_4$

$= (.7071\, x_1[0] + .7071\, x_4[0]) \begin{pmatrix} .7071 \\ 0 \\ 0 \\ .7071 \end{pmatrix} + (.7071\, x_3[0] + .7071\, x_2[0]) \begin{pmatrix} 0 \\ .7071 \\ .7071 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} \frac{x_1[0] + x_4[0]}{2} \\ 0 \\ 0 \\ \frac{x_1[0] + x_4[0]}{2} \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{x_3[0] + x_2[0]}{2} \\ \frac{x_3[0] + x_2[0]}{2} \\ 0 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} x_1[0] + x_4[0] \\ x_2[0] + x_3[0] \\ x_3[0] + x_2[0] \\ x_1[0] + x_4[0] \end{pmatrix}$

$$\boxed{\begin{aligned} \bar{x}_1 = \bar{x}_4 &= \frac{x_1[0] + x_4[0]}{2} \\ \bar{x}_2 = \bar{x}_3 &= \frac{x_2[0] + x_3[0]}{2} \end{aligned}}$$

All indiwals opinions matter equaly in the two disconnected groups, consenous is obtained in both groups seperately.

iv d) Consensous can't be reached asumptocally due to the divided nature of the social network.

#3)

$$\dot{x}_1 = x_2 - x_1$$

$$\dot{x}_2 = \frac{x_1 + x_3}{2} - x_2$$

$$\dot{x}_3 = x_2 - x_3$$

$$A = \begin{pmatrix} -1 & 1 & 0 \\ \frac{1}{2} & -1 & \frac{1}{2} \\ 0 & 1 & -1 \end{pmatrix}$$

$$|\lambda I - A| = \begin{vmatrix} \lambda+1 & -1 & 0 \\ -\frac{1}{2} & \lambda+1 & -\frac{1}{2} \\ 0 & -1 & \lambda+1 \end{vmatrix} = (\lambda+1)\begin{vmatrix} \lambda+1 & -\frac{1}{2} \\ -1 & \lambda+1 \end{vmatrix} + \begin{vmatrix} -\frac{1}{2} & -\frac{1}{2} \\ 0 & \lambda+1 \end{vmatrix}$$

$$= (\lambda+1)\left[(\lambda+1)^2 - \tfrac{1}{2}\right] + \tfrac{1}{2}(\lambda+1) = (\lambda+1)^3 - (\lambda+1) = 0$$

$$= (\lambda+1)\left((\lambda+1)^2 - 1\right) = 0 \qquad \underline{\lambda_1 = -1}$$

$$\lambda^2 + 2\lambda + 1 - 1 = \lambda^2 + 2\lambda = 0 = \lambda(\lambda+2) = 0 \qquad \underline{\lambda_2 = -2} \quad \underline{\lambda_3 = 0}$$

$$(A - \lambda_1 I)v_1 = 0$$

$$\begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} \boxed{\frac{1}{2}} & 0 & \frac{1}{2} \\ 0 & \boxed{1} & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \begin{matrix} x_1 + x_3 = 0 \\ x_2 = 0 \end{matrix}$$

x_3 is free, let $x_3 = 1 \therefore x_2 = 0, \; x_1 = -1$

$$V_1 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \underset{\frac{1}{\sqrt{2}}}{\Rightarrow} \begin{pmatrix} -.7071 \\ 0 \\ .7071 \end{pmatrix}$$

$(A-\lambda_2 I)v_2 = 0$

$$\begin{pmatrix} 1 & 1 & 0 \\ \frac{1}{2} & 1 & \frac{1}{2} \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} \boxed{1} & 1 & 0 \\ 0 & \boxed{\frac{1}{2}} & \frac{1}{2} \\ 0 & 0 & 0 \end{pmatrix}$$

$x_1 + x_2 = 0$

$x_2 + x_3 = 0$

$x_3 = 1 \quad \therefore \quad x_2 = -1 \;\&\; x_1 = 1$

$$V_2 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \underset{1/\sqrt{3}}{\Rightarrow} \begin{pmatrix} .5774 \\ -.5774 \\ .5774 \end{pmatrix}$$

$$(A-\lambda_3 I)V_3 = AV_3 = \begin{pmatrix} -1 & 1 & 0 \\ \frac{1}{2} & -1 & \frac{1}{2} \\ 0 & 1 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} \boxed{-1} & 1 & 0 \\ 0 & \boxed{-\frac{1}{2}} & \frac{1}{2} \\ 0 & 0 & 0 \end{pmatrix}$$

$x_3 = 1 \quad \therefore \quad x_2 = 1 \;\&\; x_1 = 1$

$$V_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \overset{1/\sqrt{3}}{\Rightarrow} \begin{pmatrix} .5774 \\ -.5774 \\ .5774 \end{pmatrix}$$

$$T = [V_1 \; V_2 \; V_3] \qquad T^{-1} = \begin{bmatrix} w_1^T \\ w_2^T \\ w_3^T \end{bmatrix}$$

From MATLAB: $T^{-1} = \begin{pmatrix} -.7071 & 0 & .7071 \\ .4330 & -.866 & .4330 \\ .4330 & .866 & .4330 \end{pmatrix}$

$$w_1 = \begin{pmatrix} -.7071 \\ 0 \\ .7071 \end{pmatrix}, \quad w_2 = \begin{pmatrix} .4330 \\ -.8660 \\ .4330 \end{pmatrix}, \quad w_3 = \begin{pmatrix} .4330 \\ .8660 \\ .4330 \end{pmatrix}$$

A is diagonlitable $\therefore$ $x(t) = e^{At}x(0) = Te^{\Lambda t}T^{-1}x(0)$

$$= \begin{pmatrix} -.7071 & .5774 & .5774 \\ 0 & -.5774 & .5774 \\ .7071 & .5774 & .5774 \end{pmatrix} \begin{pmatrix} e^{-t} & 0 & 0 \\ 0 & e^{-2t} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -.7071 & 0 & .7071 \\ .4330 & -.866 & .433 \\ .433 & .866 & .433 \end{pmatrix} x(0)$$

$$\lim_{t \to \infty} (x(t)) = T \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} T^{-1} x(0) =$$

$$\begin{pmatrix} 0 & 0 & .5774 \\ 0 & 0 & .5774 \\ 0 & 0 & .5774 \end{pmatrix} \begin{pmatrix} -.7071 & 0 & .7071 \\ .4330 & -.866 & .433 \\ .433 & .866 & .433 \end{pmatrix} \begin{pmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{pmatrix}$$

$$= \begin{pmatrix} 1/4 & 1/2 & 1/4 \\ 1/4 & 1/2 & 1/4 \\ 1/4 & 1/2 & 1/4 \end{pmatrix} \begin{pmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{pmatrix} = \begin{pmatrix} \bar{x} \\ \bar{x} \\ \bar{x} \end{pmatrix}$$

$$\boxed{\bar{x} = \frac{x_1(0)}{4} + \frac{x_2(0)}{2} + \frac{x_3(0)}{4}} = \omega_3^T x(0) \cancel{e^{\lambda_3 t}} V_3$$

b) The 3 cars will meet at the same location, as described by the position $\bar{x}$.

c) $e = x - \bar{x}$

$$e(t) = (\omega_1^T x(0) e^{\lambda_1 t} V_1 + \omega_2^T x(0) e^{\lambda_2 t} V_2 + \omega_3^T x(0) e^{\lambda_3 t} V_3) - \omega_3^T x(0) e^{\lambda_3 t} V_3$$

$$e(t) = \omega_1^T x(0) e^{\lambda_1 t} V_1 + \omega_2^T x(0) e^{\lambda_2 t} V_2$$

$$\boxed{\text{The dominate exponential rate is given by the } \lambda \text{ closest to } 0, \therefore \text{ the dominant } \alpha = -1, \ e^{-t}.}$$

#4)

a) $\boxed{\text{False}}$, the eigenvalues of $-A$ are $(-1)(\lambda_i)$ of A. Therefore if A is stable all λ_i are negative & the eigenvalues of $-A$ are positive and then $\dot{X} = -AX$ is unstable.

b) $\boxed{\text{True}}$, the eigenvalues of A^T = the eigenvalues of A.

c) $\boxed{\text{True}}$, the $\det(A^{-1}) = \frac{1}{\det(A)}$ $\therefore$ the eigenvalues of A are the reciprocal of the e-vals of A^{-1}. Sign is the same

d) $\boxed{\text{False}}$ If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $A^T = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$

$$A + A^T = \begin{pmatrix} 2a & b+c \\ b+c & 2d \end{pmatrix}$$

$\det(A) = ad - bc$ $\det(A+A^T) = 4ad - (b+c)^2$

$\therefore$ the e-vals of A can have a different sign then that of A.

e) $\boxed{\text{False}}$, the eigenvalues of A^2 are the square of the e-vals of A. Therefore if A is stable, the e-vals of A^2 are positive and $\dot{x} = Ax$ is unstable.

f)

$\boxed{\text{True}}$, the eigenvalues magnitude remain unchanged $\therefore$ the e-vals of $-A$ remain < 1.

g) $\boxed{\text{True}}$, e-vals remain unchanged

h) $\boxed{\text{False}}$, if A is stable, $\lambda_i < 1$. The e-vals of A^{-1} are the reciercal of the e-vals of A. Therefore the e-vals of A^{-1} are > 1.

I) $\boxed{\text{False}}$ $A = \begin{pmatrix} .3 & .3 \\ .6 & .7 \end{pmatrix} \Rightarrow \lambda_{1,2} = .031, .969$ (stable)

$A + A^T = \begin{pmatrix} .6 & .6 \\ .9 & 1.4 \end{pmatrix} \Rightarrow \lambda_{1,2} = .0151, 1.9849$ (unstable)

J) $\boxed{\text{True}}$, If λ's of A are < 1, then the λ of A^2 are also < 1 due to the e-vals of A^2 being equal to the square of the e-vals of A.

#5)

$$A = \begin{pmatrix} -3\cos t & 4\cos t \\ -2\cos t & 3\cos t \end{pmatrix}$$

$$A(t)\,A(\tau) = \begin{pmatrix} -3\cos t & 4\cos t \\ -2\cos t & 3\cos t \end{pmatrix}\begin{pmatrix} -3\cos\tau & 4\cos\tau \\ -2\cos\tau & 3\cos\tau \end{pmatrix}$$

$$A(t)\,A(\tau) = \begin{pmatrix} \cos t\cos\tau & 0 \\ 0 & \cos t\cos\tau \end{pmatrix}$$

$$A(\tau)\,A(t) = \begin{pmatrix} \cos t\cos\tau & 0 \\ 0 & \cos t\cos\tau \end{pmatrix}$$

$\therefore$ $\underline{A(\tau)}$ & $\underline{A(t)}$ $\underline{\text{commute}}$ $\therefore$

$\dot{x}(t) = A(t)\,x(t)$ has solution $x(t) = e^{\int_0^t A(\tau)\,d\tau}\,x(0)$

$$\phi(t,\tau) = e^{\int_\tau^t A(s)\,ds}$$

$$\int_\tau^t A(s)\,ds = \begin{pmatrix} \int_\tau^t -3\cos(s)\,ds & \int_\tau^t 4\cos(s)\,ds \\ \int_\tau^t -2\cos(s)\,ds & \int_\tau^t 3\cos(s)\,ds \end{pmatrix}$$

$$= \begin{pmatrix} -3(\sin(t) - \sin(\tau)) & 4(\sin(t) - \sin(\tau)) \\ -2(\sin(t) - \sin(\tau)) & 3(\sin(t) - \sin(\tau)) \end{pmatrix}$$

$$= \sin(t) - \sin(\tau)\begin{pmatrix} -3 & 4 \\ -2 & 3 \end{pmatrix}$$

$$e^{\overline{\sin t - \sin \tau}\begin{pmatrix} -3 & 4 \\ -2 & 3 \end{pmatrix}} = e^{\alpha M} \qquad |\lambda I - M| = 0$$

$$\begin{vmatrix} \lambda+3 & -4 \\ 2 & \lambda-3 \end{vmatrix} = \lambda^2 - 9 + 8 = \lambda^2 - 1 = 1 = 0$$

$$\lambda_1 = -1$$
$$\lambda_2 = 1$$

$\lambda = -1$

$$\begin{pmatrix} 2 & -4 \\ 2 & -4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \qquad X_2 = 1 \ \therefore X_1 = 2$$

$$T_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$\lambda = 1$

$$\begin{pmatrix} 4 & -4 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \qquad X_2 = 1, \ X_1 = 1$$

$$T_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$T = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \qquad T^{-1} = \frac{1}{2-1}\begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

$$T^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

M is diagonalizable $\qquad e^{M} = T e^{J} T^{-1} = T e^{\Lambda} T^{-1}$

$$\therefore e^{\alpha M} = T e^{\alpha \Lambda} T^{-1}$$

$$\Lambda = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \alpha\Lambda = \begin{pmatrix} -\sin(t) + \sin(\tau) & 0 \\ 0 & \sin(t) - \sin(\tau) \end{pmatrix}$$

$$e^{\alpha\Lambda} = \begin{pmatrix} e^{-\sin(t)+\sin(\tau)} & 0 \\ 0 & e^{\sin(t)-\sin(\tau)} \end{pmatrix} = \begin{pmatrix} K_1 & 0 \\ 0 & K_2 \end{pmatrix}$$

$$\Phi(t,\tau) = e^{\int_\tau^t A(s)\,ds} = T e^{\alpha\Lambda} T^{-1}$$

$$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} K_1 & 0 \\ 0 & K_2 \end{pmatrix} = \begin{pmatrix} 2K_1 & K_2 \\ K_1 & K_2 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} =$$

$$= \begin{pmatrix} 2K_1 - K_2 & -2K_1 + 2K_2 \\ K_1 - K_2 & -K_1 + 2K_2 \end{pmatrix}$$

state transition Matrix

$$\boxed{\Phi(t,\tau) = \left(\begin{array}{c|c} 2e^{-\sin(t)+\sin(\tau)} - e^{\sin(t)-\sin(\tau)} & 2(-e^{-\sin(t)+\sin(\tau)} + e^{\sin(t)-\sin(\tau)}) \\ \hline e^{-\sin(t)+\sin(\tau)} - e^{\sin(t)-\sin(\tau)} & -e^{-\sin(t)+\sin(\tau)} + 2e^{\sin(t)-\sin(\tau)} \end{array}\right)}$$

$$\boxed{\Phi(t,0) = \left(\begin{array}{c|c} 2e^{-\sin(t)} - e^{\sin(t)} & -2e^{-\sin(t)} + 2e^{\sin(t)} \\ \hline e^{-\sin(t)} - e^{\sin(t)} & -e^{-\sin(t)} + 2e^{\sin(t)} \end{array}\right)}$$

Fundamental Matrix

Contents


```
clear
clc
```

Problem 1

```
disp('--Problem 1 ---------------------------------------------------------')
disp(' ')

T           =  [1 0 -2 -1;...
                0 1 2 0; ....
                0 0 1 1;...
                0 0 0 1];
Tinv        = inv(T)
```

```
--Problem 1 ---------------------------------------------------------


Tinv =

     1     0     2    -1
     0     1    -2     2
     0     0     1    -1
     0     0     0     1
```

Problem 2

Social Network A---

```
Aa          =  [1/4 1/4 1/4 1/4;...
                1/3 1/3 1/3 0;
                1/4 1/4 1/4 1/4;
                1/3 0 1/3 1/3];
% Eigenvalues
lambda_a     = eig(Aa);

disp('--Problem 2a ------------------------------------------------------')
disp(' ')

% Check Algebraic and Geometric Multiplcities
AlgMulta     = GetAlgebraicMultiplicity(lambda_a);
```

```
GeoMulta     = GetGeometricMultiplicity(lambda_a, Aa);
[Ta,Ja]      = CheckDiagonalizable(AlgMulta, GeoMulta, lambda_a, Aa)

% left evecs are rows of T^-1
Tainv        = inv(Ta)
```

Social Network B---

```
Ab           =  [1/3 1/3 0 1/3;...
                 1/3 1/3 1/3 0;
                 0 1/3 1/3 1/3;
                 1/3 0 1/3 1/3];
% Eigenvalues
lambda_b     = eig(Ab);

disp('--Problem 2b ------------------------------------------------------')
disp(' ')

% Check Algebraic and Geometric Multiplcities
AlgMultb     = GetAlgebraicMultiplicity(lambda_b);
GeoMultb     = GetGeometricMultiplicity(lambda_b, Ab);
[Tb,Jb]      = CheckDiagonalizable(AlgMultb, GeoMultb, lambda_b, Ab)

% left evecs are rows of T^-1
Tbinv        = inv(Tb)
```

Social Network C---

```
Ac           =  [1/3 1/3 0 1/3;...
                 1/3 1/3 1/3 0;
                 0 1/2 1/2 0;
                 1/2 0 0 1/2];
% Eigenvalues
lambda_c     = eig(Ac);

disp('--Problem 2c ------------------------------------------------------')
disp(' ')

% Check Algebraic and Geometric Multiplcities
AlgMultc     = GetAlgebraicMultiplicity(lambda_c);
GeoMultc     = GetGeometricMultiplicity(lambda_c, Ac);
[Tc,Jc]      = CheckDiagonalizable(AlgMultc, GeoMultc, lambda_c, Ac)

% left evecs are rows of T^-1
Tcinv        = inv(Tc)
```

Social Network D---

```
Ad           =  [1/2 0 0 1/2;...
                 0 1/2 1/2 0;
                 0 1/2 1/2 0;
                 1/2 0 0 1/2];
% Eigenvalues
lambda_d     = eig(Ad);

disp('--Problem 2d ------------------------------------------------------')
disp(' ')
```

```
% Check Algebraic and Geometric Multiplcities
AlgMultd     = GetAlgebraicMultiplicity(lambda_d);
GeoMultd     = GetGeometricMultiplicity(lambda_d, Ad);
[Td,Jd]      = CheckDiagonalizable(AlgMultd, GeoMultd, lambda_d, Ad)

% left evecs are rows of T^-1
Tdinv        = inv(Td)
```

```
--Problem 2a ------------------------------------------------------

Eigenvalue of -0.1667 has Alg Mult of 1
Eigenvalue of -0.0000 has Alg Mult of 1
Eigenvalue of 0.3333 has Alg Mult of 1
Eigenvalue of 1.0000 has Alg Mult of 1
Matrix is diagonalizable

Ta =

    0.5000    0.4243   -0.7071    0.0000
    0.5000   -0.5657   -0.0000   -0.7071
    0.5000    0.4243    0.7071   -0.0000
    0.5000   -0.5657   -0.0000    0.7071


Ja =

    1.0000         0         0         0
         0   -0.1667         0         0
         0         0   -0.0000         0
         0         0         0    0.3333


Tainv =

    0.5714    0.4286    0.5714    0.4286
    0.5051   -0.5051    0.5051   -0.5051
   -0.7071    0.0000    0.7071    0.0000
   -0.0000   -0.7071    0.0000    0.7071

--Problem 2b ------------------------------------------------------

Eigenvalue of -0.3333 has Alg Mult of 1
Eigenvalue of 0.3333 has Alg Mult of 2
Eigenvalue of 1.0000 has Alg Mult of 1
Matrix is diagonalizable

Tb =

    0.5000    0.3627    0.6070    0.5000
   -0.5000   -0.6070    0.3627    0.5000
    0.5000   -0.3627   -0.6070    0.5000
   -0.5000    0.6070   -0.3627    0.5000


Jb =

   -0.3333         0         0         0
         0    0.3333         0         0
```

```
         0         0    0.3333         0
         0         0         0    1.0000


Tbinv =

    0.5000   -0.5000    0.5000   -0.5000
    0.3627   -0.6070   -0.3627    0.6070
    0.6070    0.3627   -0.6070   -0.3627
    0.5000    0.5000    0.5000    0.5000

--Problem 2c ------------------------------------------------------

Eigenvalue of -0.2287 has Alg Mult of 1
Eigenvalue of 0.1667 has Alg Mult of 1
Eigenvalue of 0.7287 has Alg Mult of 1
Eigenvalue of 1.0000 has Alg Mult of 1
Matrix is diagonalizable

Tc =

    0.5831   -0.3922    0.5000    0.2941
   -0.5831   -0.3922    0.5000   -0.2941
    0.4001    0.5883    0.5000   -0.6430
   -0.4001    0.5883    0.5000    0.6430


Jc =

   -0.2287         0         0         0
         0    0.1667         0         0
         0         0    1.0000         0
         0         0         0    0.7287


Tcinv =

    0.6527   -0.6527    0.2986   -0.2986
   -0.5099   -0.5099    0.5099    0.5099
    0.6000    0.6000    0.4000    0.4000
    0.4061   -0.4061   -0.5918    0.5918

--Problem 2d ------------------------------------------------------

Eigenvalue of 0.0000 has Alg Mult of 2
Eigenvalue of 1.0000 has Alg Mult of 2
Matrix is diagonalizable

Td =

         0    0.7071    0.7071         0
   -0.7071         0         0    0.7071
    0.7071         0         0    0.7071
         0   -0.7071    0.7071         0


Jd =

     0     0     0     0
     0     0     0     0
     0     0     1     0
```

```
     0     0     0     1


Tdinv =

         0   -0.7071    0.7071         0
    0.7071         0         0   -0.7071
    0.7071         0         0    0.7071
         0    0.7071    0.7071         0
```

Problem 3

```
disp('--Problem 3 --------------------------------------------------------')
disp(' ')

% Eigenvectors - from hand written
V1  = [-1; 0; 1]*1/sqrt(2);
V2  = [1;-1;1]*1/sqrt(3);
V3  = [1;1;1]*1/sqrt(3);

T   = [V1,V2,V3];
Tinv= inv(T)
```

Functions

```
% Function to find Alg Mult of each Eigenvalue
function am = GetAlgebraicMultiplicity(evals)
    % Initialize
    lam_uni = uniquetol(evals,1e-8); % unique evals
    am      = zeros(size(lam_uni));

    % Check to see how many times each eigenvalue repeats
    for i = 1:length(am)
        ind     = find(abs(lam_uni(i) - evals) <= 1e-8);
        am(i)   = length(ind);
        fprintf('Eigenvalue of %.4f has Alg Mult of %i \n',lam_uni(i), am(i))
    end
end

% Function to find Geo Mult of each Eigenvalue
function gm = GetGeometricMultiplicity(evals, A)
    % Initialize
    lam_uni = uniquetol(evals,1e-8); % unique evals
    gm      = zeros(size(lam_uni));

    % Check dimension of null space of lamda*I - A
    for i = 1:length(gm)
        gm(i) = size(null(lam_uni(i)*eye(size(A)) - A),2);
    end
end

function [T,J] = CheckDiagonalizable(AlgMult, GeoMult, lambda, A)
    % Check if Matrix is diagnolizable (n distinct evals or AlgMult = GeoMult)
    if (AlgMult == GeoMult)
        disp('Matrix is diagonalizable')
        [T,J] = eig(A);
    elseif ((length(uniquetol(lambda)) == length(lambda)) == 1)
```

```
        disp('Matrix is diagonalizable')
        [T,J] = eig(A);
    else
        disp('Matrix is not diagonalizable')
        [T,J] = jordan(A);
    end
end
```

```
--Problem 3 --------------------------------------------------------


Tinv =

   -0.7071    0.0000    0.7071
    0.4330   -0.8660    0.4330
    0.4330    0.8660    0.4330
```

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