# ECE 602: LUMPED LINEAR SYSTEMS

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Stability of Continuous-Time LTV Systems

# Stability of CT LTV Systems

LTV system $\dot{x}(t) = A(t)x(t)$ has solution $x(t) = \Phi(t, t_0)x(t_0)$, $t \geq t_0$

**Theorem**

- *LTV system is asymptotically stable if* $\lim_{t\to\infty} \Phi(t) = 0$ *for all* $x(t_0)$
- *LTV system is exponentially stable if there exist* $C, r > 0$ *such that*

$$\|\Phi(t)\| \leq Ce^{-r(t-t_0)}, \quad \forall t \geq t_0, \forall x(t_0)$$

- In the above, $\|\cdot\|$ denotes (any) matrix norm (more on this later)
- Stability property does not depend on initial time $t_0 \neq 0$

# Stability of CT LTV Systems (cont.)

Stability of LTV systems much more difficult to characterize than LTI systems

- Asymptotic stability does **not** imply exponential stability

  **Example:** LTV system $\dot{x}(t) = -\frac{1}{t+1}x(t)$ has a solution $x(t) = \frac{x(0)}{t+1}$

- Stability cannot be determined based on eigenvalues of $A(t)$

  **Example:** LTV system

$$\dot{x}(t) = \begin{bmatrix} -e^{-t} & \frac{1}{t+1} \\ 0 & -e^{-t} \end{bmatrix} x(t)$$

  has fundamental matrix $\Phi(t) = e^{(e^{-t}-1)} \begin{bmatrix} 1 & \ln(t+1) \\ 0 & 1 \end{bmatrix}$