#1) $2\ddot{q}_1 + \ddot{q}_2 + \sin(q_1) = 0$

$\ddot{q}_1 + 2\ddot{q}_2 + \sin(q_2) = 0$

$X_1 = q_1 \quad X_2 = \dot{q}_1 \quad X_3 = q_2 \quad X_4 = \dot{q}_2$

$2\dot{X}_2 + \dot{X}_4 + \sin(X_1) = 0$

$\dot{X}_2 + 2\dot{X}_4 + \sin(X_3) = 0$

$\dot{X}_2 = \frac{1}{2}(-\dot{X}_4 - \sin(X_1)) = -2\dot{X}_4 - \sin(X_3)$

$-\dot{X}_4 - \sin(X_1) = -4\dot{X}_4 - 2\sin(X_3)$

$3\dot{X}_4 = \sin(X_1) - 2\sin(X_3)$

$\dot{X}_4 = \frac{\sin(X_1)}{3} - \frac{2}{3}\sin(X_3)$

$\dot{X}_2 = -2\left(\frac{\sin(X_1)}{3} - \frac{2}{3}\sin(X_3)\right) - \sin(X_3)$

$\dot{X}_2 = -\frac{2}{3}\sin(X_1) + \frac{\sin(X_3)}{3}$

$$\boxed{\begin{aligned} \dot{X}_1 &= X_2 \\ \dot{X}_2 &= -\frac{2}{3}\sin(X_1) + \frac{\sin(X_3)}{3} \\ \dot{X}_3 &= X_4 \\ \dot{X}_4 &= \frac{\sin(X_1)}{3} - \frac{2}{3}\sin(X_3) \end{aligned}}$$

#2) $\ddot{q}_1 + \dot{q}_2 + q_1^3 = 0$

$\dot{q}_1 + \dot{q}_2 + q_2^3 = 0$

$X_1 = q_1 \quad X_2 = \dot{q}_1 \quad X_3 = q_2$

$\dot{X}_2 + \dot{X}_3 + X_1^3 = 0$

$X_2 + \dot{X}_3 + X_3^3 = 0$

$\dot{X}_3 = -\dot{X}_2 - X_1^3 = -X_2 - X_3^3$

$\dot{X}_2 = X_2 + X_3^3 - X_1^3$

$\dot{X}_3 = -X_2 - X_3^7$

$$\boxed{\begin{aligned} \dot{X}_1 &= X_2 \\ \dot{X}_2 &= X_2 + X_3^3 - X_1^3 \\ \dot{X}_3 &= -X_2 - X_3^3 \end{aligned}}$$

#3) $\ddot{q}_1 + q_1 + 2\dot{q}_2 = 0$

$\ddot{q}_1 + \dot{q}_2 + q_2 = 0$

$x_1 = q_1 \qquad x_2 = \dot{q}_1 \qquad x_3 = q_2$

$\dot{x}_2 + x_1 + 2\dot{x}_3 = 0$

$\dot{x}_2 + \dot{x}_3 + x_3 = 0$

$\dot{x}_2 = -x_1 - 2\dot{x}_3 = -\dot{x}_3 - x_3$

$\dot{x}_3 = -x_1 + x_3$

$\dot{x}_2 = x_1 - 2x_3$

$$\boxed{\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_1 - 2x_3 \\ \dot{x}_3 &= -x_1 + x_3 \end{aligned} \quad = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & -2 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}$$

#4) $q_1[k+2] + q_1[k] + 2q_2[k+1] = 0$

$$q_1[k+2] + q_1[k+1] + q_2[k] = 0$$

$X_1 = q_1[k]$ $\quad X_2 = q_1[k+1]$ $\quad X_3 = q_2[k]$

$$X_2[k+1] + X_1[k] + 2X_3[k+1] = 0$$

$$X_2[k+1] + X_2[k] + X_3[k] = 0$$

$$X_2[k+1] = -X_1[k] - 2X_3[k+1] = -X_2[k] - X_3[k]$$

$$X_3[k+1] = -\frac{X_1[k]}{2} + \frac{X_2[k]}{2} + \frac{X_3[k]}{2}$$

$$X_2[k+1] = -X_2[k] - X_3[k]$$

$$X_1[k+1] = X_2[k]$$

OR

$$\begin{pmatrix} X_1[k+1] \\ X_2[k+1] \\ X_3[k+1] \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -1 & -1 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} X_1[k] \\ X_2[k] \\ X_3[k] \end{pmatrix}$$

#5) $$x[k+1] = x[k] - \frac{g(x[k])}{g'(x[k])}$$

At equilibrium: $x[k+1] = x[k] = x_e$

$$x_e = x_e - \frac{g(x_e)}{g'(x_e)}$$

$$0 = -\frac{g(x_e)}{g'(x_e)}$$

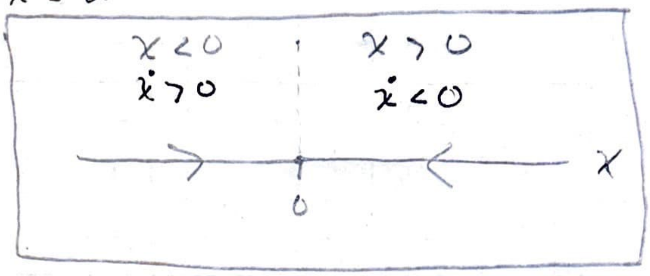
$$\boxed{\therefore \quad g(x_e) = 0}$$

#6) $\dot{x} = -\alpha \, \mathrm{sgm}(x)$ , $\alpha > 0$

$x_e = 0$ as $\mathrm{sgm}(0) = 0$

$x > 0$: $\dot{x} = -\alpha$

$x < 0$: $\dot{x} = \alpha$



#7) $\dot{x} = x^4 - x^2$

$0 = x^2(x^2-1)$

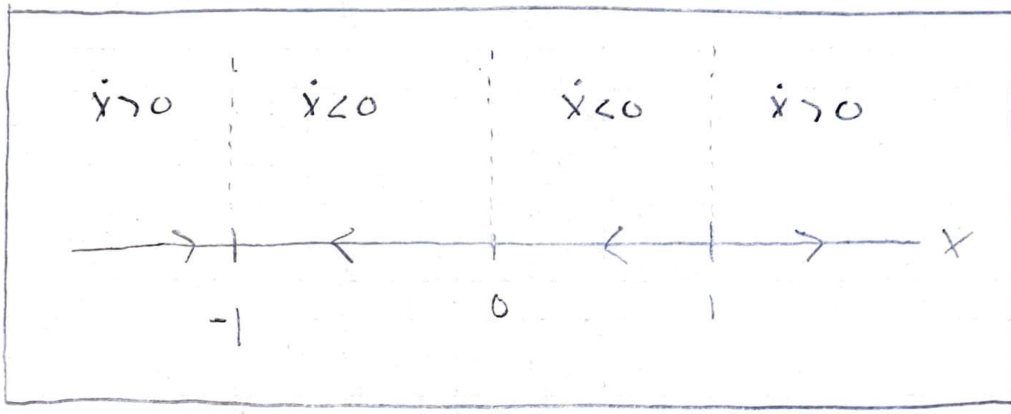
$\therefore\ x_e^{(1)} = 0\ ,\ x_e^{(2)} = 1\ ,\ x_e^{(3)} = -1$

$x>1:\ \dot{x}>0\ ,\quad 2^4 - 2^2 = 12$

$0<x<1:\ \dot{x}<0\ ,\quad (\tfrac{1}{2})^4 - (\tfrac{1}{2})^2 = -\tfrac{3}{16}$

$-1<x<0:\ \dot{x}<0\ ,\quad (-\tfrac{1}{2})^4 - (-\tfrac{1}{2})^2 = -\tfrac{3}{16}$

$x<-1:\ \dot{x}>0\ ,\quad (-2)^4 - (-2)^2 = 12$



#8) $\dot{x} = -x^3$

$$g(x) = \int_{x_0}^{x} \frac{1}{f(\eta)}\, d\eta = \int_{x_0}^{x} \frac{-1}{\eta^3}\, d\eta = \frac{\eta^{-2}}{2}\Bigg|_{x_0}^{x}$$

$$g(x) = \frac{1}{2x^2} - \frac{1}{2x_0^2}$$

$$g(x) = t = \frac{1}{2x^2} - \frac{1}{2x_0^2}$$

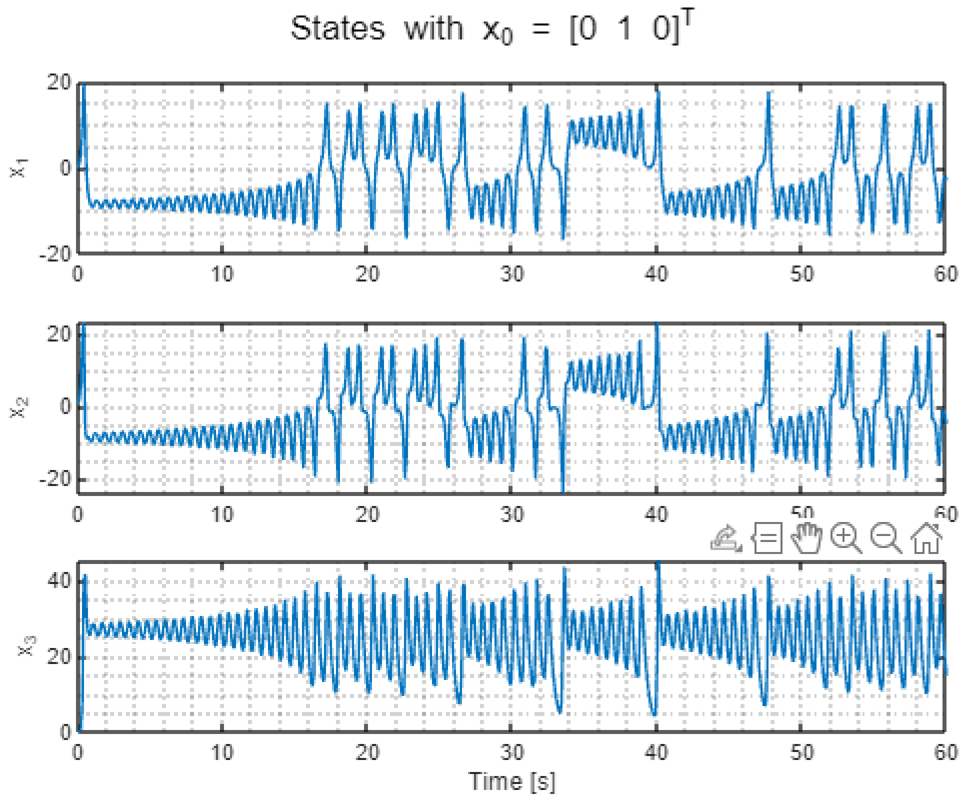
$$t + \frac{1}{2x_0^2} = \frac{1}{2x^2}$$

$$\frac{2x_0^2 t + 1}{2x_0^2} = \frac{1}{2x^2}$$

$$\boxed{x = \frac{x_0}{\sqrt{2x_0^2 t + 1}}}$$

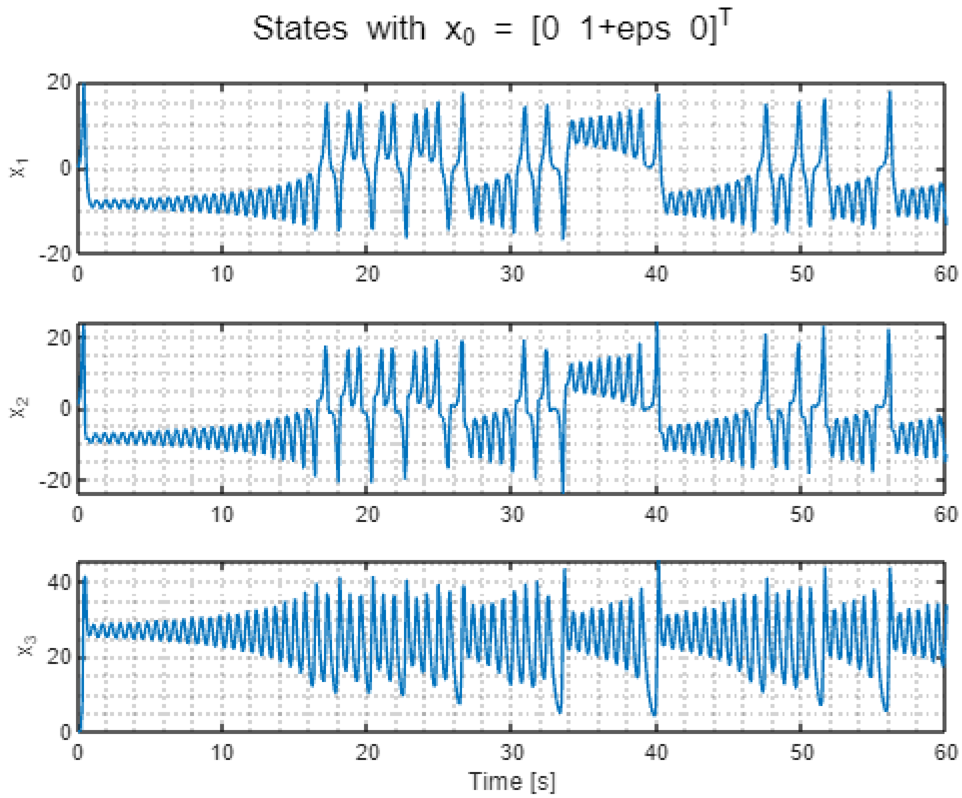
# Gabriel Colangelo HW1

```
clear
close all
clc

% Lorenz System Parameters
sigma       = 10;
b           = 8/3;
r           = 28;

% Time vector
t           = (0:.1:60);

% ODE45 solver options
opts        = odeset('AbsTol',1e-8,'RelTol',1e-8);

% Initial Condition 1
IC1         = [0 1 0]';

% Initial Condition 2
IC2         = [0 (1+eps) 0]';

% Sim for 1st IC set
[~, X1]     = ode45(@(t,x) Lorenz(t,x, sigma, r, b),t,IC1, opts);

% Sim for 2nd IC set
[~, X2]     = ode45(@(t,x) Lorenz(t,x, sigma, r, b),t,IC2, opts);

% Plots
figure
subplot(311)
plot(t,X1(:,1))
grid minor
ylabel('x_1')
subplot(312)
plot(t,X1(:,2))
grid minor
ylabel('x_2')
subplot(313)
plot(t,X1(:,3))
grid minor
ylabel('x_3')
xlabel('Time [s]')
sgtitle('States with x_0 = [0 1 0]^T')
```




**<u>This system is very osciallatory and chaotic over the simulation interval.</u>**

```
figure
subplot(311)
plot(t,X2(:,1))
grid minor
ylabel('x_1')
subplot(312)
plot(t,X2(:,2))
grid minor
ylabel('x_2')
subplot(313)
plot(t,X2(:,3))
grid minor
ylabel('x_3')
xlabel('Time [s]')
sgtitle('States with x_0 = [0 1+eps 0]^T')
```




**This system is also osciallatory and chaotic, appears to be similiar to the first simulation with x0 = [0 1 0].**

```
function xdot = Lorenz(t,x, sigma, r, b)
x1          = x(1,1);
x2          = x(2,1);
x3          = x(3,1);

xdot(1,1)   = sigma*(x2 - x1);
xdot(2,1)   = r*x1 - x2 - x1*x3;
xdot(3,1)   = -b*x3 + x1*x2;

end
```