ECE 602: LUMPED LINEAR SYSTEMS

Professor Jianghai Hu

Time-Invariant and Time-Varying Systems,
Causal and Non-Causal Systems

Time-Invariant vs. Time-Varying Systems

Continuos-time systems $\mathcal{N}$ are

- time-invariant if for all $u \in \mathcal{U}$ and all $\tau \in \mathcal{I}$, $\tau \geq 0$

$$y(\cdot) = \mathcal{N}(u(\cdot)) \quad \Rightarrow \quad y(\cdot - \tau) = \mathcal{N}(u(\cdot - \tau))$$

- time-varying if otherwise

Similar definitions for discrete-time systems

Examples

❶ $y(t) = [u(t)]^2$

❷ $y(t) = t^2 u(t)$

❸ $y(t) = \int_{t-1}^{t+2} u(s)\, ds$

❹ $y(t) = u(t) - u(t-1)$

❺ $y(t) = \begin{cases} t & \text{if } |u(t)| \leq 1 \\ 0 & \text{if } |u(t)| > 1 \end{cases}$

❻ $y[k] = \begin{cases} 3u[k-1] & \text{if } k = 0, 1, \ldots, \\ 0 & \text{if } k = -1, -2, \ldots \end{cases}$

Causal vs. Non-Causal Systems

Continuous-time systems $\mathcal{N}$ are

- causal if for all $t_0 \in \mathcal{I}$, $y(t_0)$ only depends on past input, $u(t)$ for $t \leq t_0$. Or more precisely,

$$\begin{cases} y_1 = \mathcal{N}(u_1),\ y_2 = \mathcal{N}(u_2) \\ u_1(t) = u_2(t),\ t \leq t_0 \end{cases} \qquad \Rightarrow \quad y_1(t_0) = y_2(t_0)$$

- non-causal if otherwise

Similar definitions for discrete-time systems

Examples

1. $y(t) = [u(t)]^2$
2. $y(t) = t^2 u(t)$
3. $y(t) = \int_{t-1}^{t+2} u(s)\, ds$
4. $y(t) = u(t) - u(t-1)$
5. $y(t) = \begin{cases} t & \text{if } |u(t)| \leq 1 \\ 0 & \text{if } |u(t)| > 1 \end{cases}$
6. $y[k] = \begin{cases} 3u[k-1] & \text{if } k = 0, 1, \ldots, \\ 0 & \text{if } k = -1, -2, \ldots \end{cases}$