Frequency Response Method

Transfer function:

$$\frac{Y(s)}{R(s)} = G(s) = \frac{p(s)}{q(s)} = \frac{p(s)}{(s+s_1)(s+s_2)(s+s_3)\dots(s+s_n)}$$

For a stable system, the real parts of s_i lie in the left half of the complex plane.
The response of the system to a sinusoidal input of amplitude X, is: $R(s) = \frac{\omega X}{s^2+\omega^2}$

$$Y(s) = \frac{p(s)}{q(s)} \frac{\omega X}{s^2+\omega^2}$$

$$Y(s) = \frac{a}{s+j\omega} + \frac{\bar{a}}{s-j\omega} + \frac{b_1}{(s+s_1)} + \frac{b_2}{(s+s_2)} + \frac{b_3}{(s+s_3)} + \dots + \frac{b_n}{(s+s_n)}$$

Where a and $\bar{a}$ are complex conjugate constants and b_i are constants.

Frequency Response Method

The inverse Laplace transform leads to

$$y(t) = ae^{-j\omega t} + \bar{a}e^{j\omega t} + \underbrace{b_1e^{-s_1t} + b_2e^{-s_2t} + b_3e^{-s_3t} + \ldots + b_ne^{-s_nt}}_{=0 \text{ as } t \Rightarrow \infty}$$

Since the real parts of the poles of the system lie in the left half of the complex plane, all terms except the first two approach zero. The steady state response can be represented as:

$$y(t) = ae^{-j\omega t} + \bar{a}e^{j\omega t}$$

Where a *and* $\bar{a}$ *are:*

$$a = G(s)\frac{\omega X}{s^2+\omega^2}(s+j\omega)\bigg|_{s=-j\omega} = -\frac{XG(-j\omega)}{2j}$$

$$\bar{a} = G(s)\frac{\omega X}{s^2+\omega^2}(s-j\omega)\bigg|_{s=j\omega} = \frac{XG(j\omega)}{2j}$$

Frequency Response Method

The complex quantity G(jω) can be represented as:

$$G(j\omega) = |G(j\omega)|e^{j\phi}$$

where

$$\phi = tan^{-1}\left(\frac{Im(G(j\omega))}{Re(G(j\omega))}\right)$$

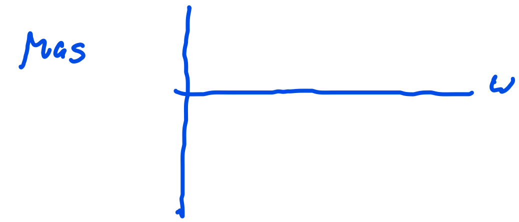
and:

$$G(-j\omega) = |G(-j\omega)|e^{-j\phi} = |G(j\omega)|e^{-j\phi}$$

The output at steady state can now be represented as:

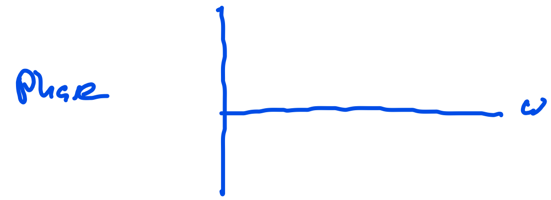
$$y(t) = X|G(j\omega)|\frac{e^{j(\omega t+\phi)} - e^{-j(\omega t+\phi)}}{2j}$$

$$y(t) = X|G(j\omega)|sin(\omega t + \phi) = Y sin(\omega t + \phi)$$



Bode Diagram

The complex quantity G(jω) can be represented as:



$$G(j\omega) = |G(j\omega)|e^{j\phi}$$

where

$$\phi = tan^{-1}\left(\frac{Im(G(j\omega))}{Re(G(j\omega))}\right)$$

Thus, the transfer function can be represented by two plots: one of the magnitude of G(jω) vs. frequency and φ vs frequency.

By plotting the logarithmic value of the magnitude of G(jω), multiplication operations are replaced by additions. This is the motivation for plotting

$20log(|G(j\omega)|)$ *(decibels) vs. frequency which helps is rapidly plotting asymptotic approximation of the magnitude plot.*

The basic factors which occur in a transfer function are: $s = j\omega$

-gain K (constant)

-Integral and derivative factor $(j\omega)^{-1}$ $\Rightarrow 1/s,\ s$

-First order factors $(1 + j\omega T)^{-1}$ $\Rightarrow (Ts+1),\ \frac{1}{Ts+1}$

-quadratic factors $(1 + 2\zeta\frac{j\omega}{\omega_n} + (\frac{j\omega}{\omega_n})^2)^{-1}$

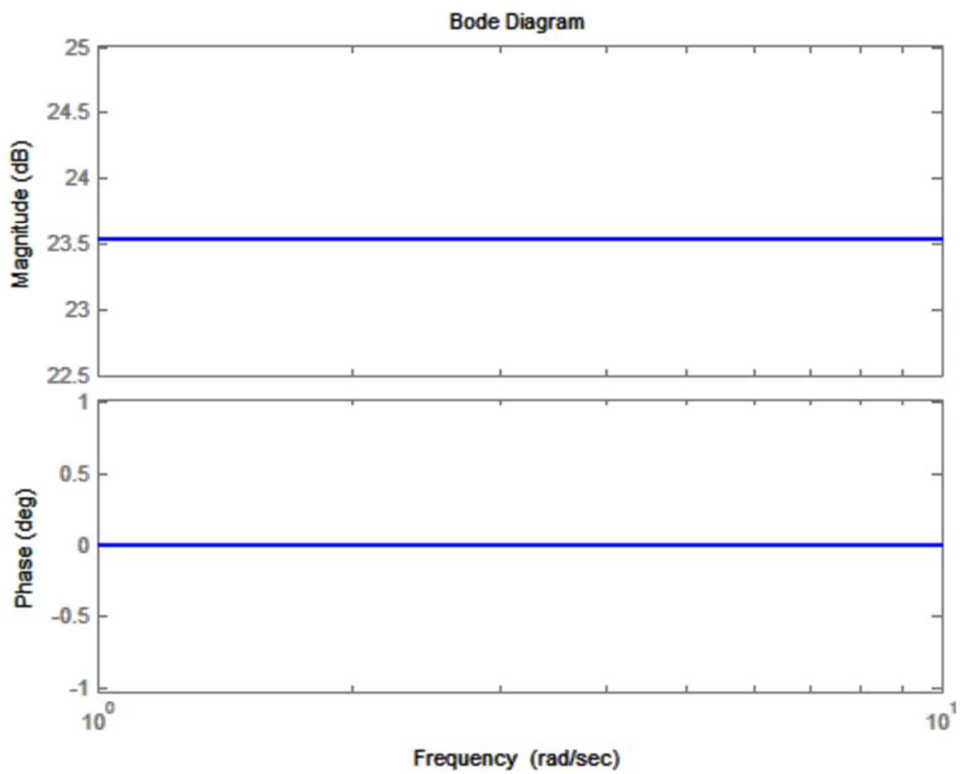
$$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} \Rightarrow \frac{1}{1+2\zeta\omega_n\frac{s}{\omega_n^2}+\frac{s^2}{\omega_n^2}}$$

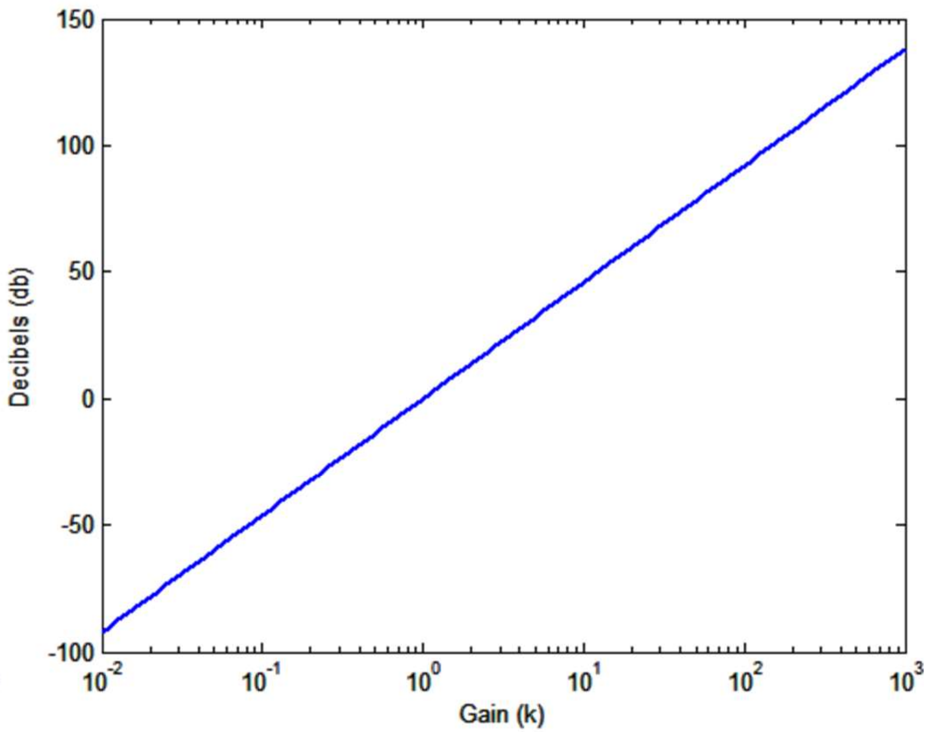
$G(i\omega) = k$ $|G| = K$ $\phi = \tan^{-1}\left(\frac{Im(K)}{Re(K)}\right) = 0$

Bode Diagram

Gain K: K is a real number and is not a function of frequency, so the magnitude plot does not change with frequency. Moreover, since the imaginary part of K is zero, the phase is zero for all frequency.

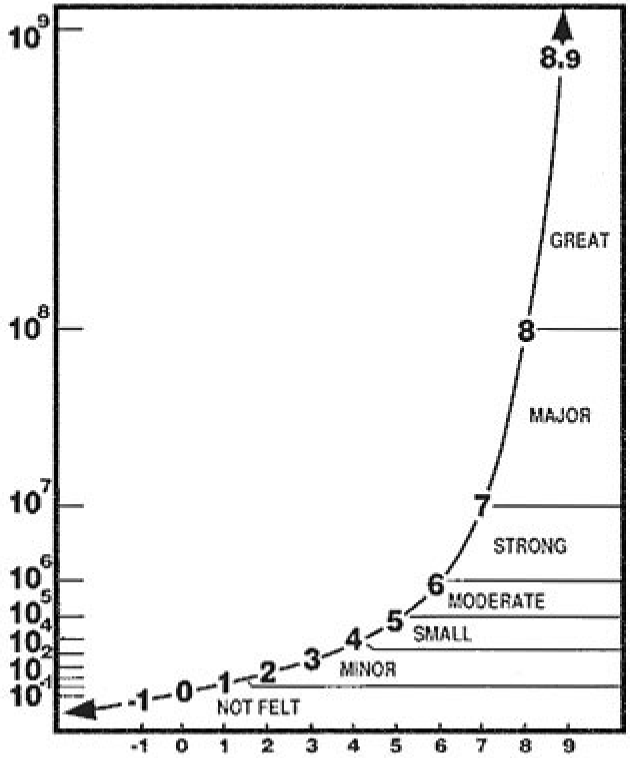
(NOTE: the matlab command to determine the db is 20*log10(K))





Why Decibels

Large range of values are conveniently expressed in logarithmic axis

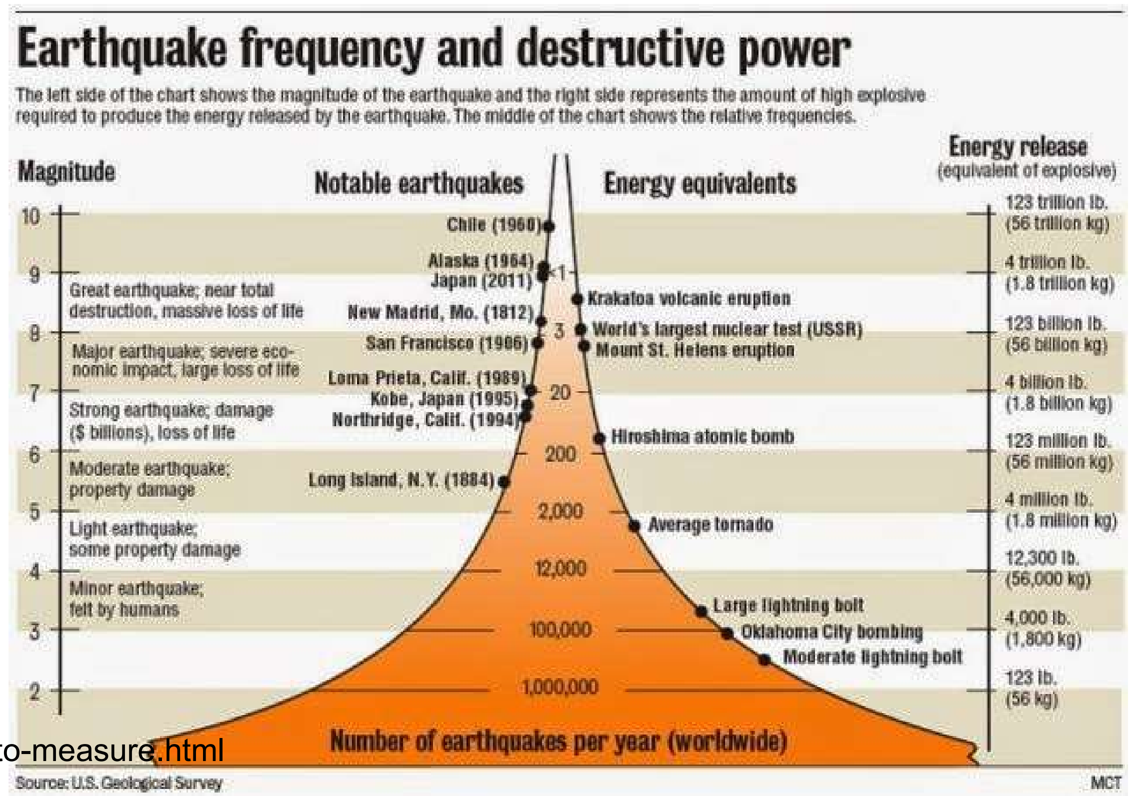


https://wiki.ubc.ca/

$$M = log_{10}(\frac{I}{I_0})$$

M – magnitude of the earthquake

$\frac{I}{I_0}$ – ratio of the earthquake intensity I to that of a zero-level earthquake that has intensity I_0 (seismic wave amplitude of 0.001 mm)



Products of gains reduces to addition on the log chart.

http://www.geologyin.com/2015/01/using-richter-scale-to-measure.html

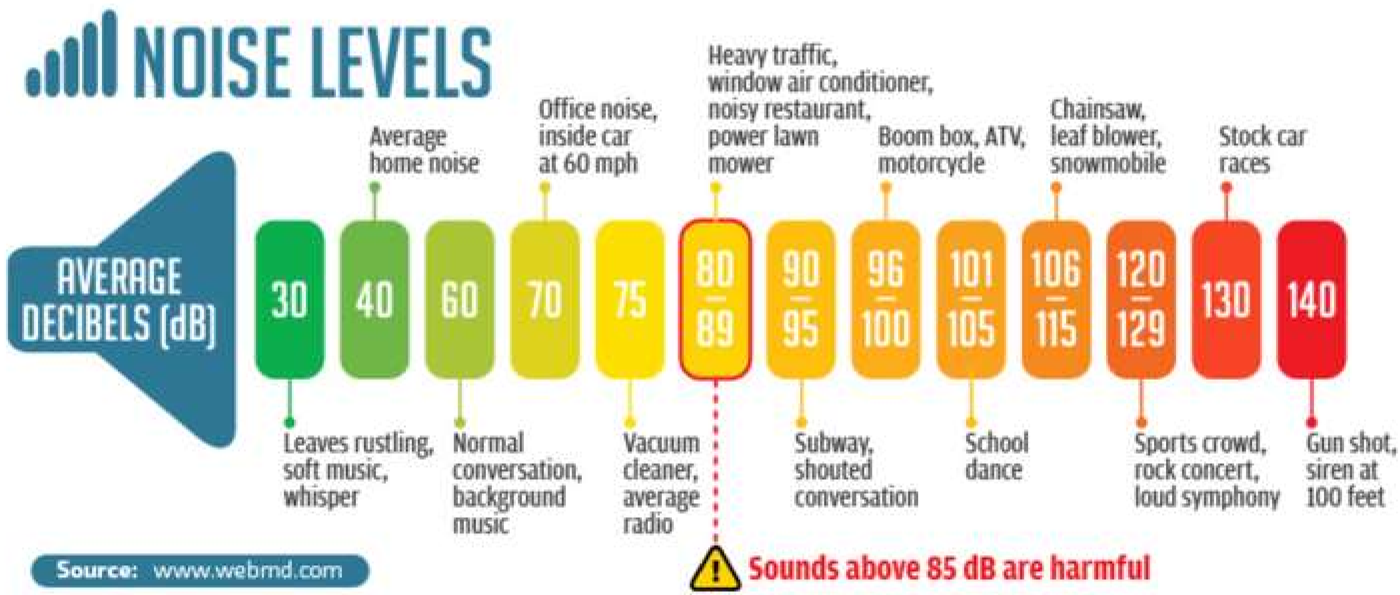
Why Decibels

Decibels is commonly used in acoustics to quantify sound levels relative to 0 db (defined as sound pressure level of 0.0002 microbar. The human ear has a large dynamic range. The ratio of sound intensity that causes permanent damage during short exposure to the quietest sound that the ear can hear is 1 trillion.

Original definition of Decibel (dB) is a measure of ratio of powers

$$X_{dB} = 10log_{10}\left(\frac{X}{X_0}\right)$$

Power is a quadratic quantity and absorbing the power (2) in the ratio leads to 20 being the coefficient of the definition of dB = $10log_{10}\left(\frac{V}{V_0}\right)$



https://blog.echobarrier.com/blog/the-decibel-scale-explained

$G = \frac{1}{s}$

Bode Diagram

Integral factor : The magnitude of an integrator transfer function in db is:

$$20log\left(\left|\frac{1}{j\omega}\right|\right) = -20log(\omega)$$ $\rightarrow$ $-20\ dB/dec$

The phase of an integrator transfer function is:

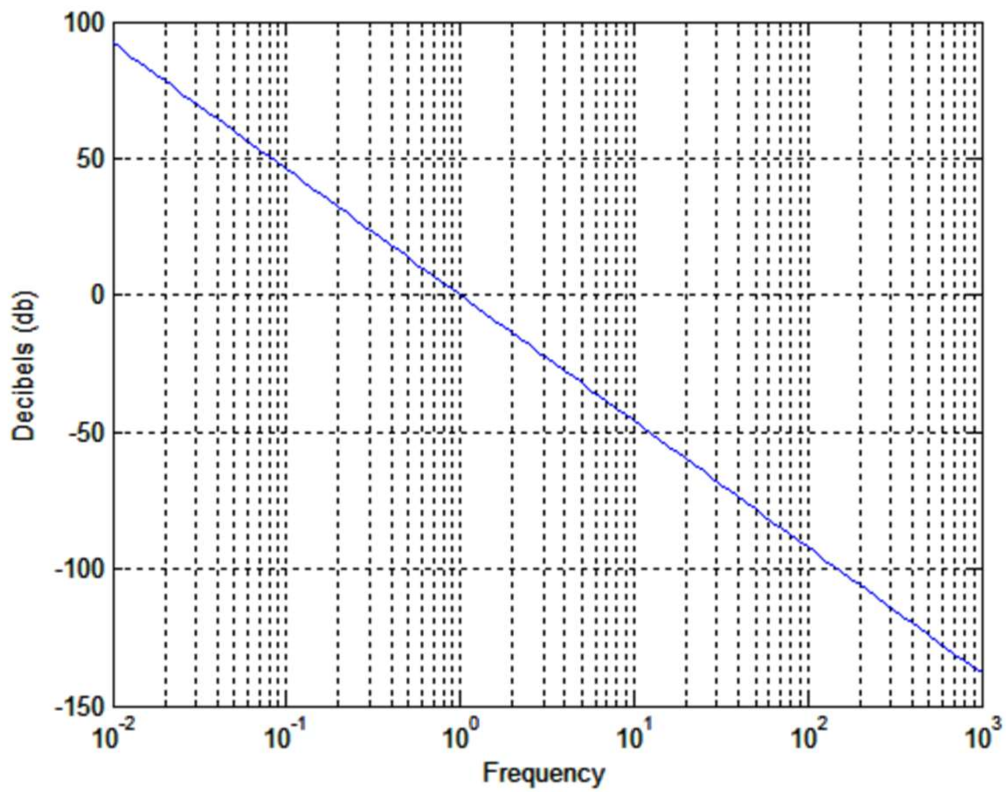
$$\phi = -tan^{-1}\left(\frac{\omega}{0}\right) = -\frac{\pi}{2}$$ $\rightarrow$ constant phase

The magnitude for ω = 1 is 0

The magnitude for ω = 10 is -20

The magnitude for ω = 100 is -40

It is clear if the magnitude if plotted on a semi-log graph, the magnitude plot is linear.



$G = s$

Bode Diagram

Derivative factor : The magnitude of an derivative transfer function in db is:

$$20log\left(|j\omega|\right) = 20log(\omega)$$

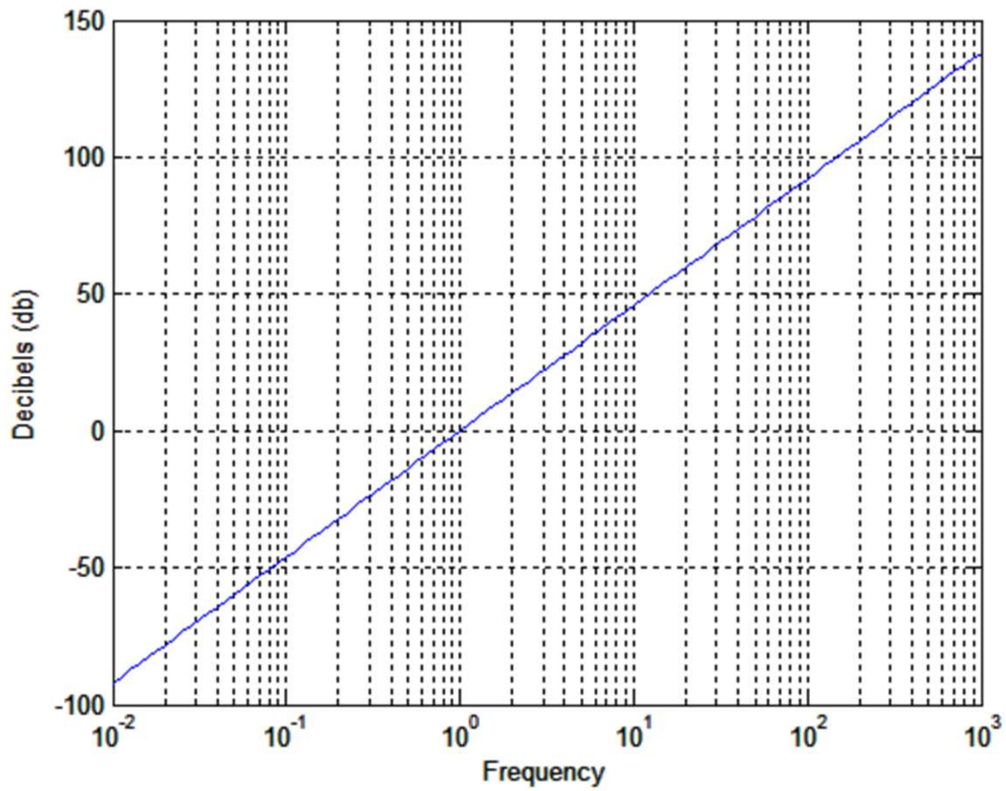
The phase of an differentiator transfer function is: $$\phi = tan^{-1}\left(\frac{\omega}{0}\right) = \frac{\pi}{2}$$

The magnitude for ω = 1 is 0

The magnitude for ω = 10 is 20

The magnitude for ω = 100 is 40

It is clear if the magnitude if plotted on a semi-log graph, the magnitude plot is linear.



$G = \frac{1}{Ts+1} = \frac{1}{Tj\omega+1}$

Bode Diagram

First order factor : The magnitude of a first order transfer function in db is:

$$20log\left(\left|\frac{1}{1+j\omega T}\right|\right) = -20log(\sqrt{1+\omega^2T^2})$$

For ωT << 1, the log magnitude can be approximated by $-20log(\sqrt{1}) = 0$

and the phase is zero

For ωT >> 1, the log magnitude can be approximated by

$$-20log(\sqrt{\omega^2T^2}) = -20log(\omega T)$$ $-20 \ dB/dec$

and the phase is -90°.

For ω = 1/T, called the ***Corner Frequency****, the log magnitude can be approximated by*

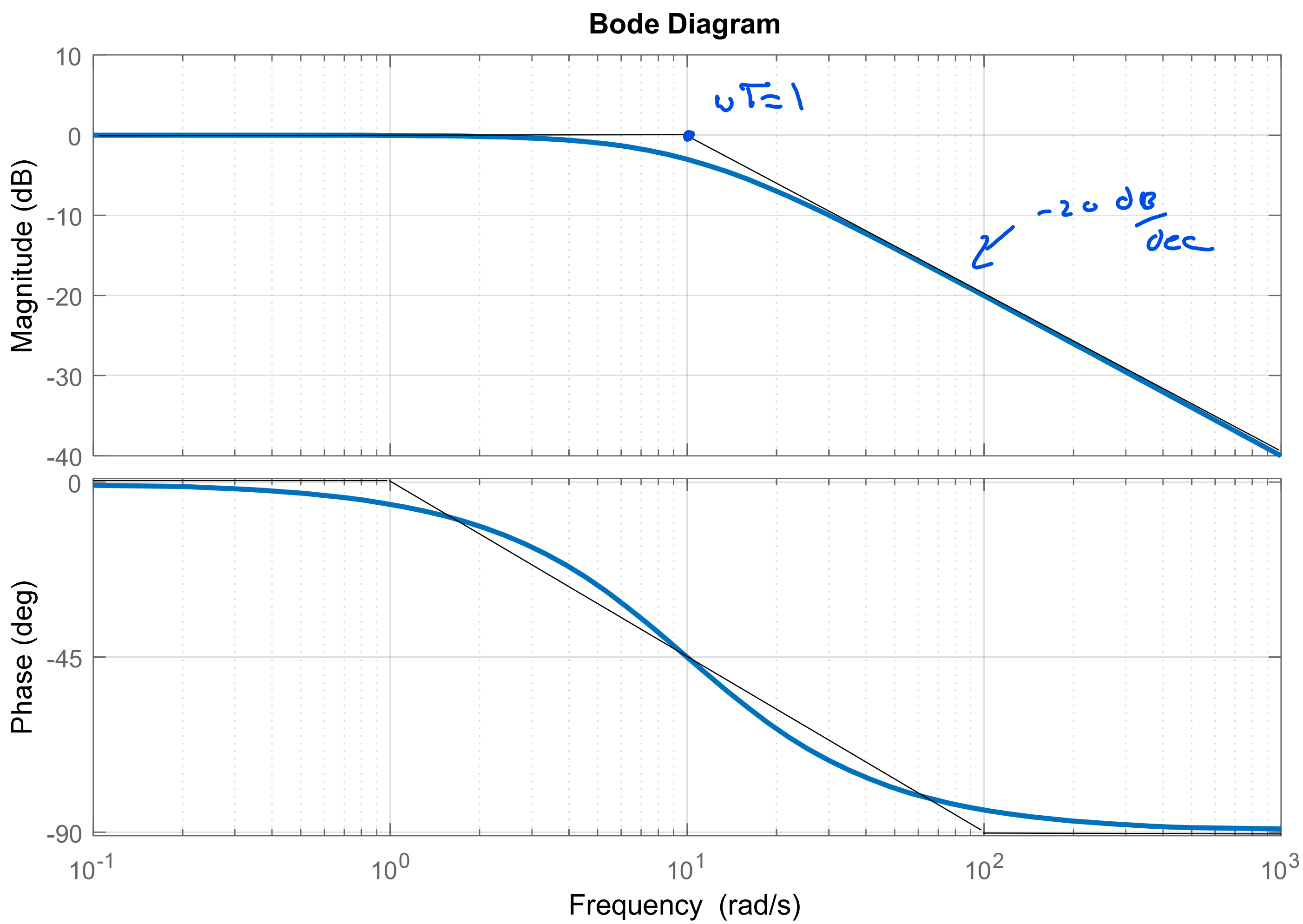
$$-20log(\sqrt{\omega^2T^2}) = -20log(1) = 0$$

and for large frequencies, the rest of the magnitude plot is the same as that of an integrator and the phase transitions from 0 to -90°.

Straight line approximation for the phase plot of a first order transfer function:

$$G(j\omega) \approx \begin{cases} 0 & \text{if } \omega T < \frac{1}{10}\omega T \\ -45 - 45(\log(\omega T)) & \text{if } \frac{1}{10}\omega T < \omega T < 10\omega T \\ -90 & \text{if } > 10\omega T \end{cases}$$

Bode Diagram



Bode Diagram

First order factor : The magnitude of a first order transfer function in db is:

$$20log\,(|1+j\omega T|) = 20log(\sqrt{1+\omega^2 T^2})$$

For ωT << 1, the log magnitude can be approximated by $20log(\sqrt{1}) = 0$

and the phase is zero

For ωT >> 1, the log magnitude can be approximated by

$$20log(\sqrt{\omega^2 T^2}) = 20log(\omega T)$$

and the phase is 90°.

*For ω = 1/T, called the **Corner Frequency**, the log magnitude can be approximated by*

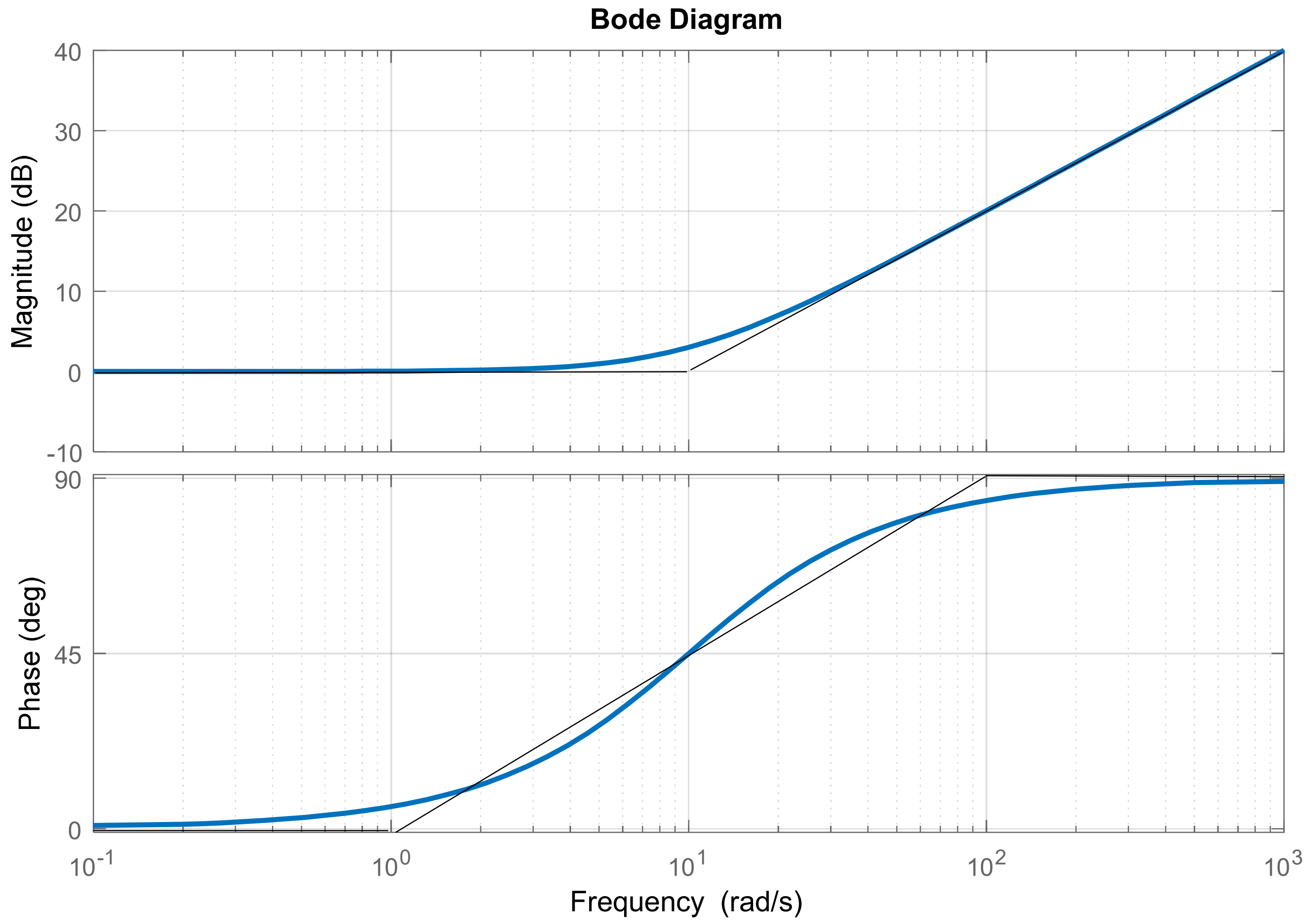
$$20log(\sqrt{\omega^2 T^2}) = 20log(1) = 0$$

and for large frequencies, the rest of the magnitude plot is the same as that of an differentiator and the phase transitions from 0 to 90°.

Straight line approximation for the phase plot of a first order transfer function:

$$G(j\omega) \approx \begin{cases} 0 & \text{if } \omega T < \frac{1}{10}\omega T \\ 45 + 45(\log(\omega T)) & \text{if } \frac{1}{10}\omega T < \omega T < 10\omega T \\ 90 & \text{if } > 10\omega T \end{cases}$$

Bode Diagram



Bode Diagram

Second order factor : Consider a second order transfer function:

$$G(j\omega) = \frac{1}{1 + 2\zeta(\frac{j\omega}{\omega_n}) + (\frac{j\omega}{\omega_n})^2}$$

The magnitude of the transfer function is:

$$20log\left|\frac{1}{1 + 2\zeta(\frac{j\omega}{\omega_n}) + (\frac{j\omega}{\omega_n})^2}\right| = -20log\sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}$$

For $\omega << \omega_n$, the log magnitude can be approximated by $-20log(\sqrt{1}) = 0$

and the phase is zero

For $\omega >> \omega_n$, the log magnitude can be approximated by

$$-20log\left(\frac{\omega^2}{\omega_n^2}\right) = -40log\left(\frac{\omega}{\omega_n}\right)$$

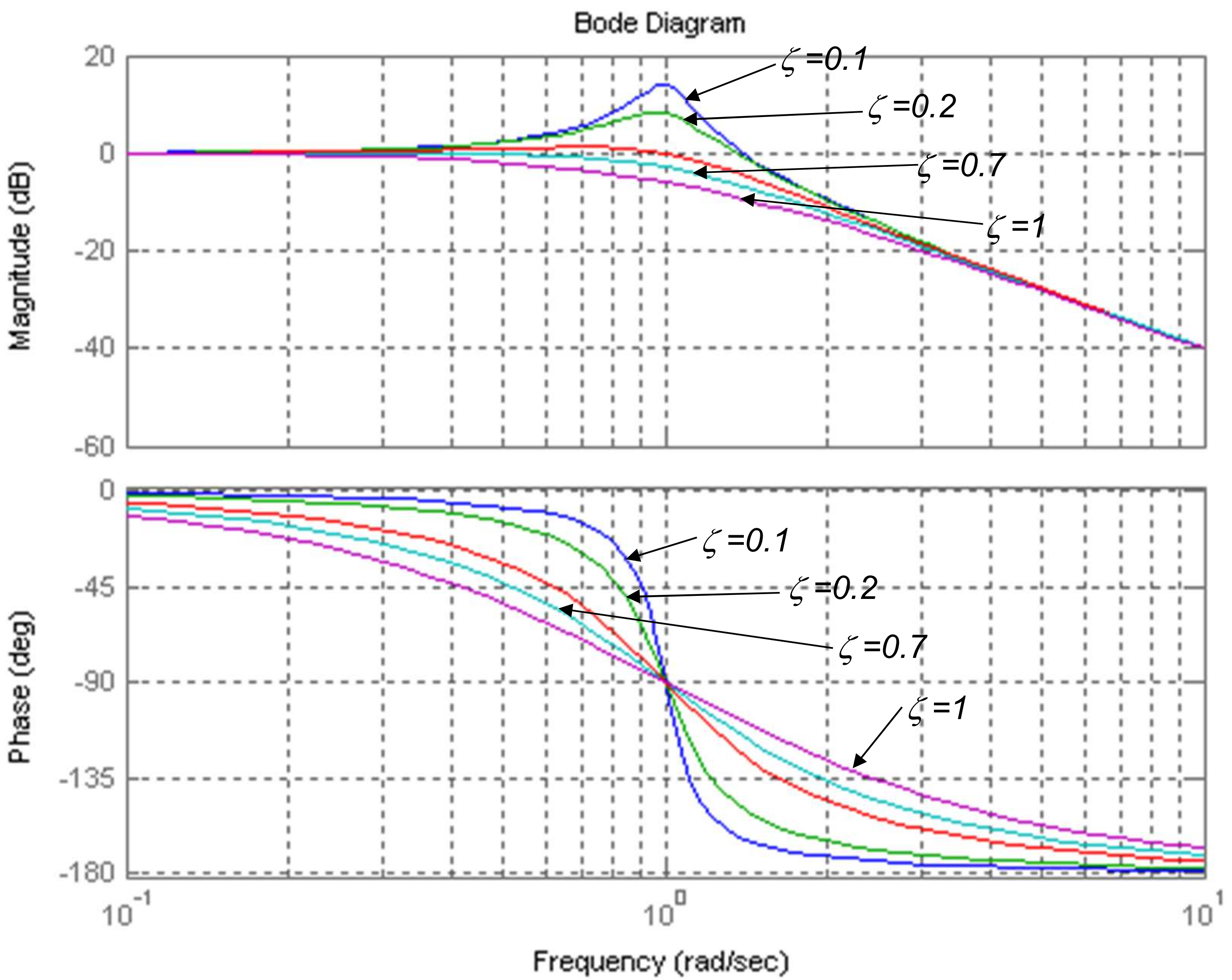
and the phase is 180^o.

For $\omega = \omega_n$, called the ***Corner Frequency****, the log magnitude can be approximated by*

$$-40log(\frac{\omega_n}{\omega_n}) = -40log(1) = 0$$

Bode Diagram

The two asymptotes determined are not function of ζ. Near the corner frequency a resonant peak occurs whose magnitude is a function of ζ.



Bode Diagram

The phase angle of a second order factor is:

$$\phi = -tan^{-1}\left(\frac{2\zeta\frac{\omega}{\omega_n}}{1-(\frac{\omega}{\omega_n})^2}\right)$$

The phase angle is a function of ω_n, and ζ.

For $\omega >> \omega_n$, the log phase can be approximated by 180°.

*For $\omega = \omega_n$, called the **Corner Frequency**, the phase is*

$$\phi = -tan^{-1}\left(\frac{2\zeta}{0}\right) = -tan^{-1}(\infty) = -90$$

Bode Diagram

Resonant Peak Value: The magnitude of the transfer function

$$G(j\omega) = \frac{1}{1 + 2\zeta(\frac{j\omega}{\omega_n}) + (\frac{j\omega}{\omega_n})^2}$$

$$|G(j\omega)| = \frac{1}{\sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}$$

Which has a peak value at some frequency which is called the resonant frequency. The occurs when the denominator of the magnitude equation is a minimum.

$$g = \left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2 = 1 + \frac{\omega^4}{\omega_n^4} - 2\frac{\omega^2}{\omega_n^2} + 4\zeta^2\frac{\omega^2}{\omega_n^2}$$

$$\frac{dg}{d\omega} = 0 \Rightarrow \omega = \omega_n\sqrt{1 - 2\zeta^2}$$

Thus, there is no peak for damping ratio > 0.707

Bode Diagram (non-minimum Phase Systems)

Transfer function which have no poles or zeros in the right-half plane are minimum-phase systems. Systems with poles or zeros in the right-half plane are called non-minimum phase systems.

For systems with the same magnitude plot, the range of the phase angle of the minimum-phase systems is minimum among all such systems, while the range of phase angle of any nonminimum-phase systems is greater than the minimum.

Consider the example of two systems:

$$G_1(j\omega) = \frac{1 + j\omega T}{1 + j\omega T_1}, \quad G_2(j\omega) = \frac{1 - j\omega T}{1 + j\omega T_1}, \quad 0 < T < T_1$$

The magnitudes of the two transfer function are:

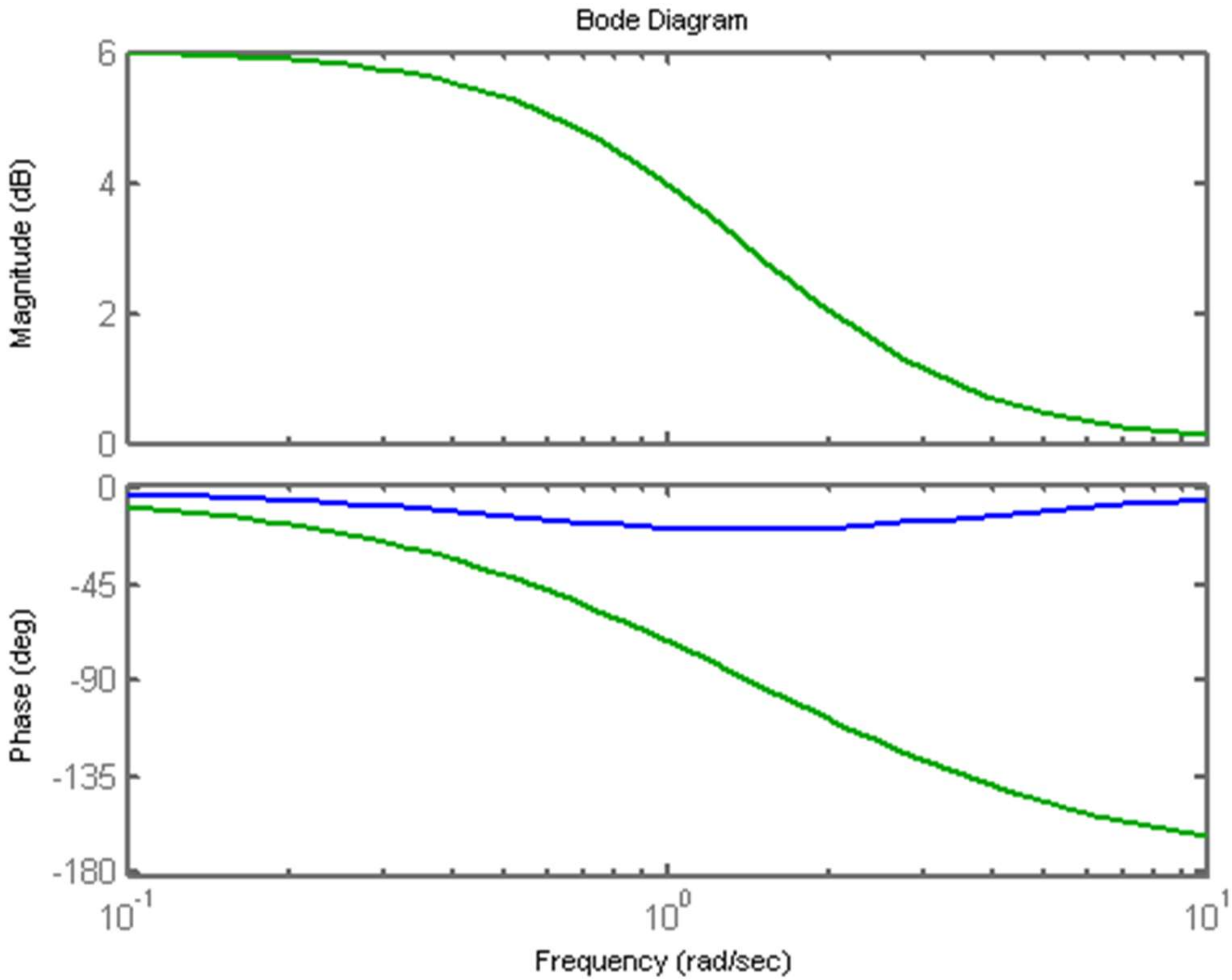
$$|G_1(j\omega)| = \frac{\sqrt{1 + \omega^2 T^2}}{\sqrt{1 + \omega^2 T_1^2}}, \quad |G_2(j\omega)| = \frac{\sqrt{1 + \omega^2 T^2}}{\sqrt{1 + \omega^2 T_1^2}},$$

The phase of the two transfer function are:

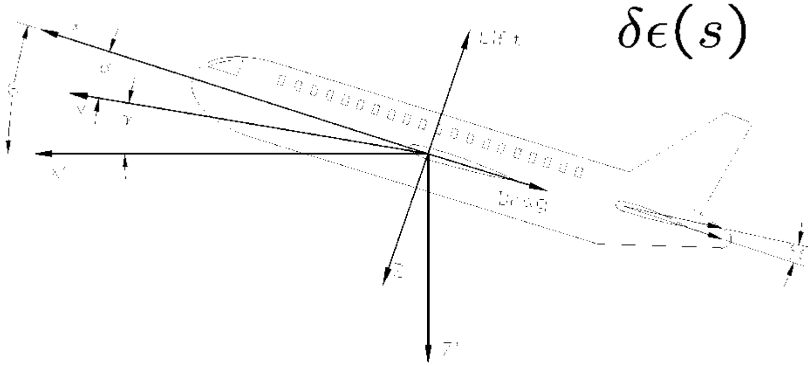
$$\angle G_1(j\omega) = tan^{-1}(\omega T) - tan^{-1}(\omega T_1)$$
$$\angle G_2(j\omega) = -tan^{-1}(\omega T) - tan^{-1}(\omega T_1)$$

Bode Diagram (non-minimum Phase Systems)

$$G_1(s) = \frac{s+2}{s+1}, \quad G_2(s) = \frac{-s+2}{s+1}.$$



Aircraft Altitude Dynamics



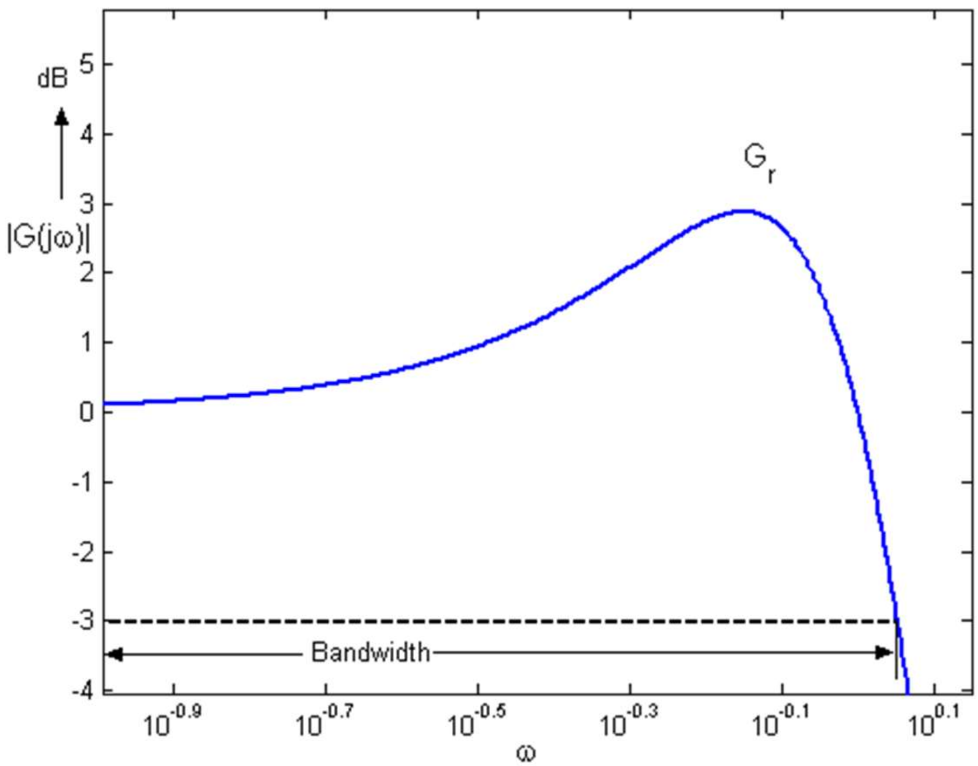
$$\frac{Z'(s)}{\delta\epsilon(s)} = G(s) = \frac{(s+6.24)(-s+2.24)}{s^2(s+2)^2}$$

$$\frac{\alpha(s)}{\delta\epsilon(s)} = \frac{3}{(s+2)^2}$$

When a step input ($\delta\varepsilon$) is applied to the elevator, it creates an instantaneous downward force resulting in an initial downward acceleration of the center of mass. The downward force also creates a torque which increases the angle α, resulting in an increase in the lift. Eventually the lift of the wings dominate the downward force of the elevator to move the airplane up. This is typical behavior of non-minimum phase systems.

Bode Diagram

<u>Resonant Peak:</u> *The resonant peak G_r is the maximum value of $|G(j\omega)|$*



- *G_r is indicative of the relative stability of a stable closed loop system.*
- *A large G_r corresponds to a large maximum overshoot for a step input.*
- *Generally a desirable value for G_r is between 1.1 and 1.5*

<u>Resonant Frequency:</u> *The resonant frequency ω_r is the frequency at which the peak resonance G_r occurs.*

Bode Diagram

<u>Bandwidth:</u> *The bandwidth BW is the frequency at which |G($j\omega$)| drops to 70.7% or 3bD down from its zero frequency value.* $20log10(\frac{1}{\sqrt{2}}) = -3db,$

•The bandwidth is indicative of the transient response properties in the time-domain

•A large bandwidth corresponds to a faster rise time, since higher frequencies are more easily passed through the system

•Bandwidth is indicative of noise-filtering characteristics and the robustness of the system.

<u>Second order system:</u>

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2},$$

Bandwidth is:

$$|G(j\omega)| = \frac{1}{\sqrt{(1 - \frac{\omega^2}{\omega_n^2})^2 + (2\zeta\frac{\omega}{\omega_n})^2}} = \frac{1}{\sqrt{2}} = 0.707$$

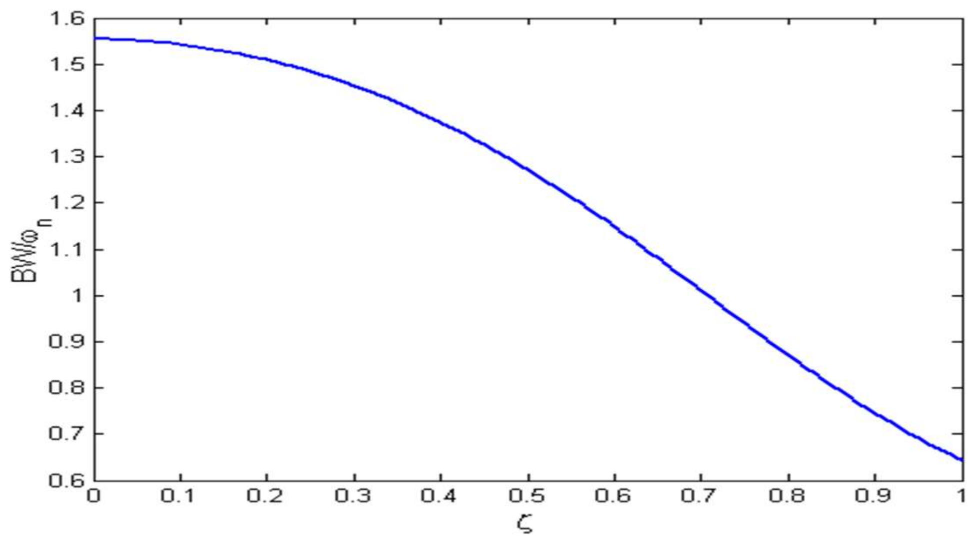
Bode Diagram

$$\sqrt{(1-\frac{\omega^2}{\omega_n^2})^2+(2\zeta\frac{\omega}{\omega_n})^2}=\sqrt{2}$$

$$\frac{\omega^2}{\omega_n^2}=(1-2\zeta^2)\pm\sqrt{4\zeta^4-4\zeta^2+2}$$

The positive sign should be chosen since the term on the left hand side is a positive real quantity for any ζ.

Bandwidth is: $$\omega=\omega_n\sqrt{(1-2\zeta^2)\pm\sqrt{4\zeta^4-4\zeta^2+2}}$$



BW/ω_n decreases monotonically with ζ

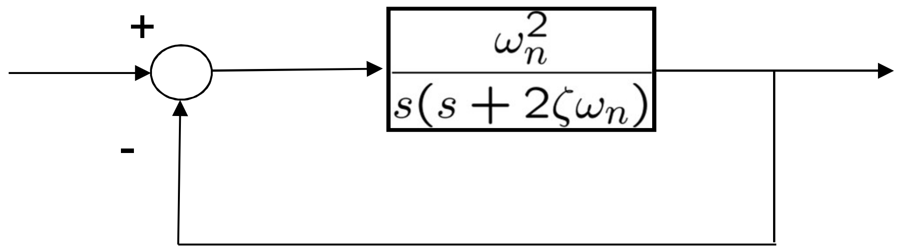
For a fixed ζ, BW increases with increasing ω_n

Effect of Adding a Zero to the Forward Path Transfer Function

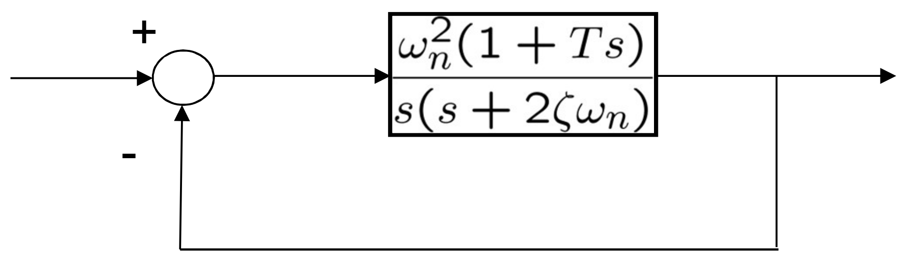
The system:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

can be represented in unity feedback form as:



Adding a zero at s = -1/T, results in



With a closed loop transfer function

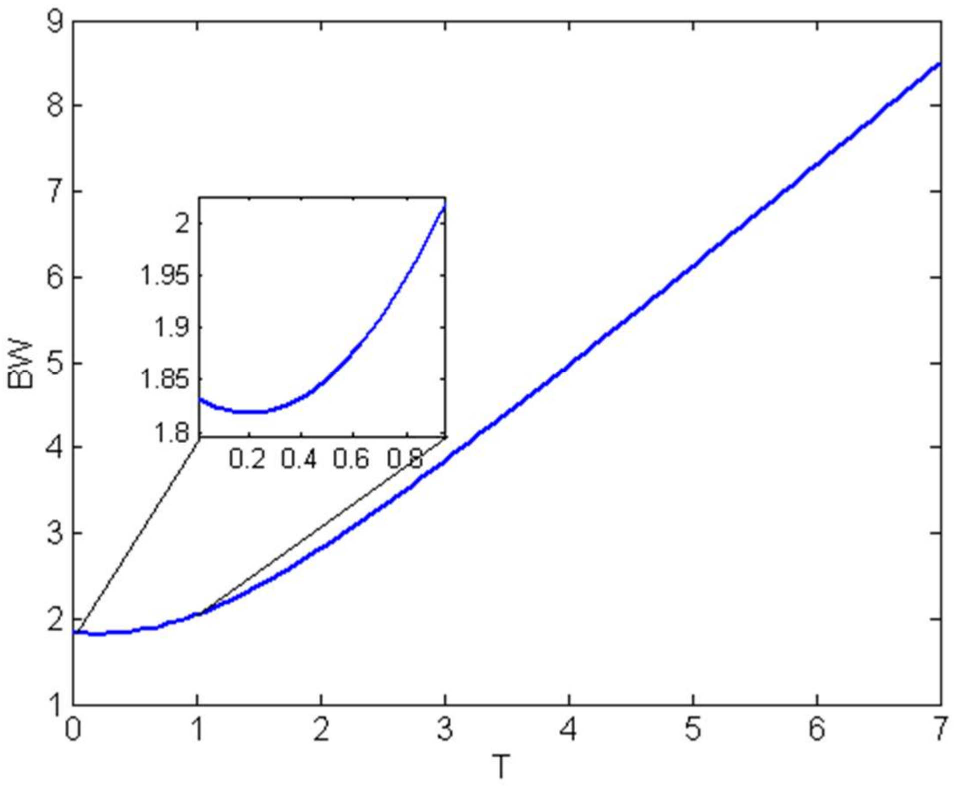
$$G(s) = \frac{\omega_n^2(1+Ts)}{s^2 + (2\zeta\omega_n + T\omega_n^2)s + \omega_n^2}$$

Effect of Adding a Zero to the Forward Path Transfer Function

In principle G_r, ω_r, and BW of the system can be derived as in the second order case. However since there are now three parameters ζ, ω_n, & T, the exact expressions for G_r, ω_r, & BW are difficult to derive analytically. After a lengthy derivation, it can be shown that:

$$BW = \left(-b \pm \frac{1}{2}\sqrt{b^2 + 4\omega_n^4}\right)^{0.5}$$

$$b = 4\zeta^2\omega_n^2 + 4\zeta\omega_n^3 T - 2\omega_n^2 - \omega_n^4 T^2$$

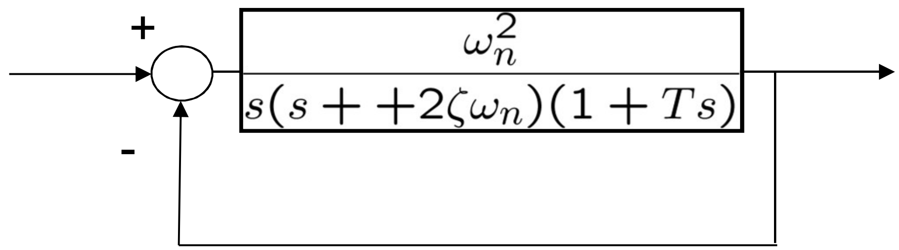


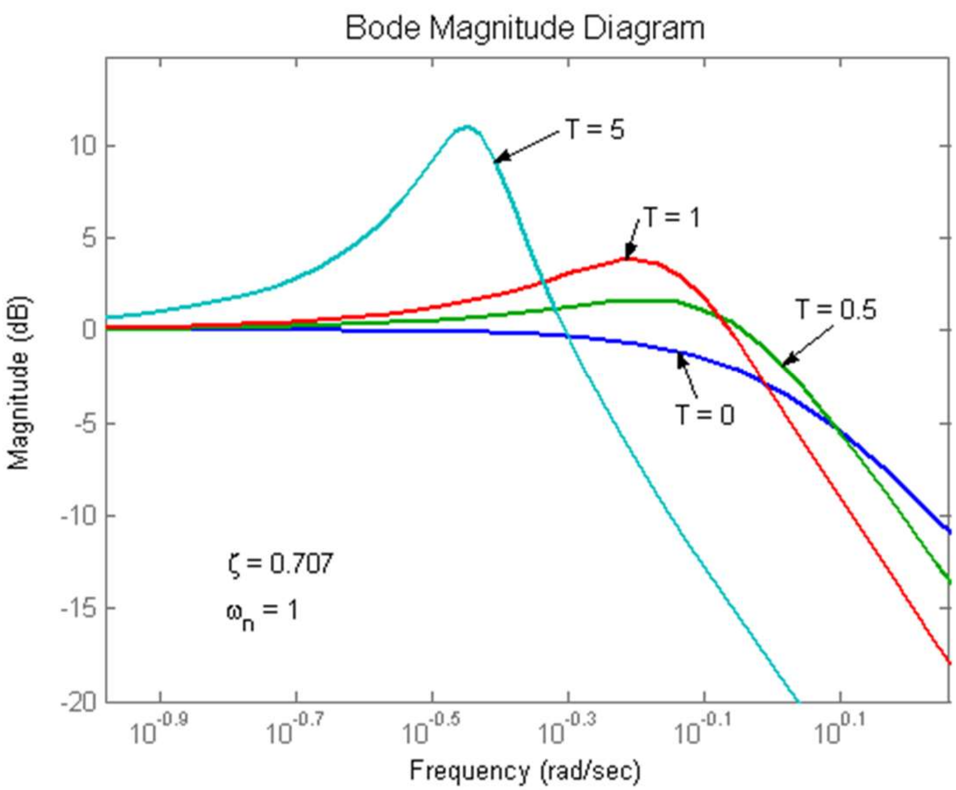
The general effect of adding a zero to the forward path transfer function is to increase the bandwidth of the closed-loop system.

However, over a small range of T, the bandwidth actually decreases as seen in the adjacent figure.

Effect of Adding a Pole to the Forward Path Transfer Function

Adding a pole at s = -1/T, results in



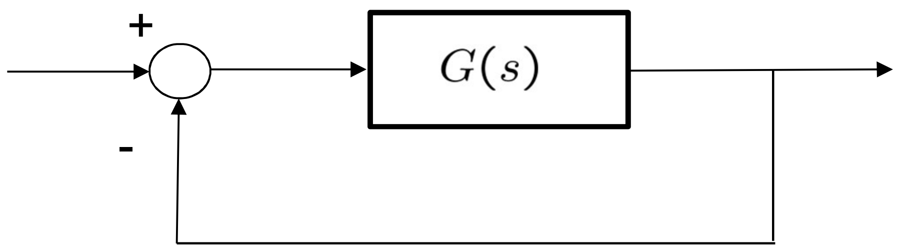


A qualitative feel can be acquired from the magnitude plot of the Bode diagram for the variation of the BW as a function of T.

The effect of adding a pole to the forward-path transfer function is to make the closed-loop system less stable, while decreasing the bandwidth.

Gain/Phase Margin

For the unity feedback system.



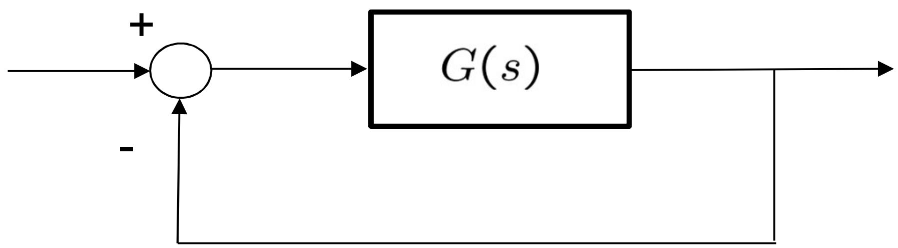
The characteristic equation is 1+G(s). When |G(s)|=1 with phase of -180^o, the characteristic equation has infinite gain which corresponds to the closed loop poles lying on the imaginary axis.

Gain margin is the amount of gain in decibels (dB) that can be added to the loop before the closed-loop system becomes unstable.

Phase margin is defined as the phase in degrees that can be added (i.e. from a transport delay), before the closed-loop system becomes unstable.

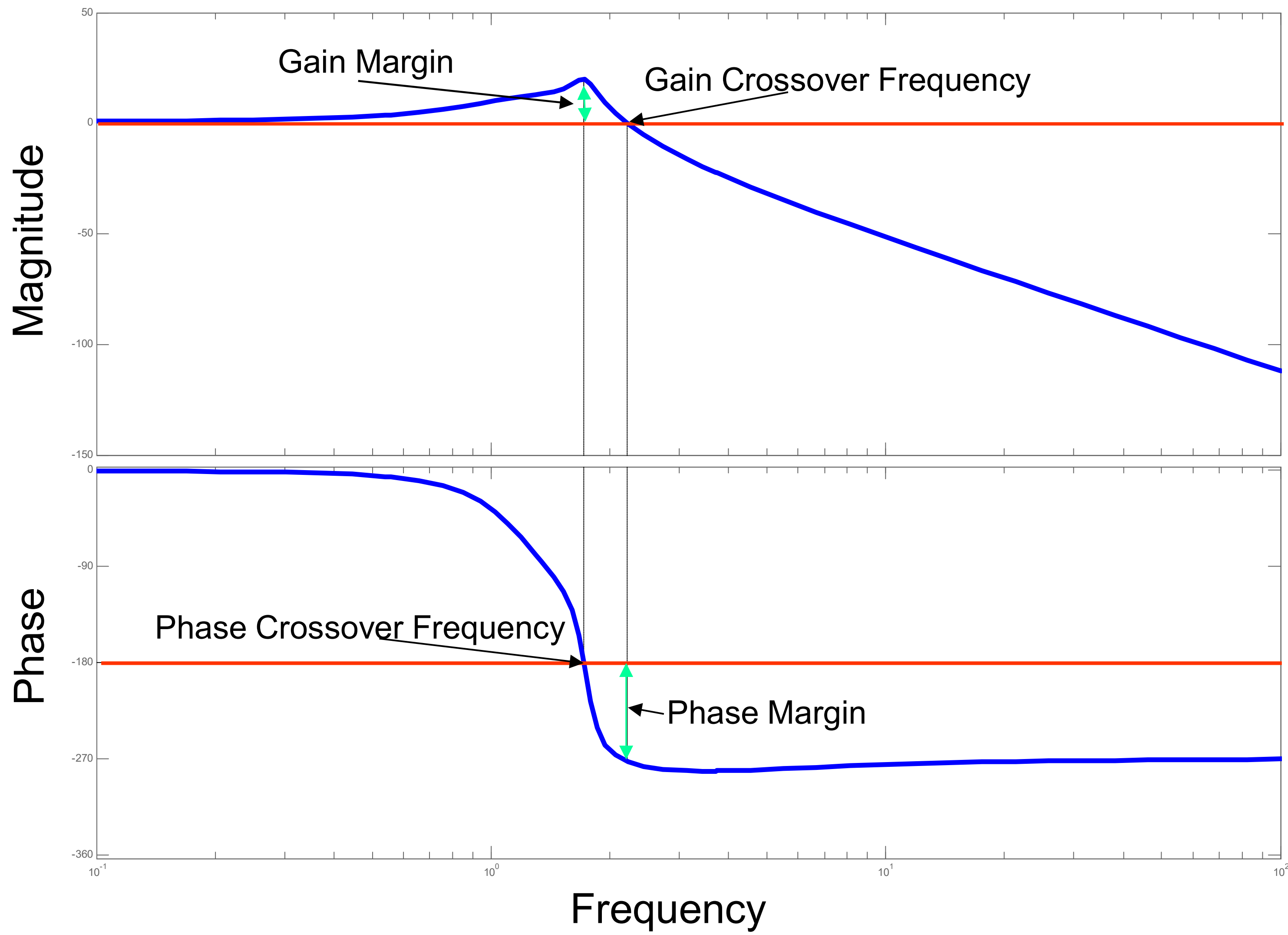
Bode Stability Criterion

For the unity feedback system.



Bode Stability Criterion: Consider an open-loop transfer function GOL=G(s) that is strictly proper (more poles than zeros) and has no poles located on or to the right of the imaginary axis, with the possible exception of a single pole at the origin. Assume that the open-loop frequency response has only a single critical frequency and a single gain crossover frequency . Then the closed-loop system is stable if magnitude(G(s)) is < 1. Otherwise it is unstable.

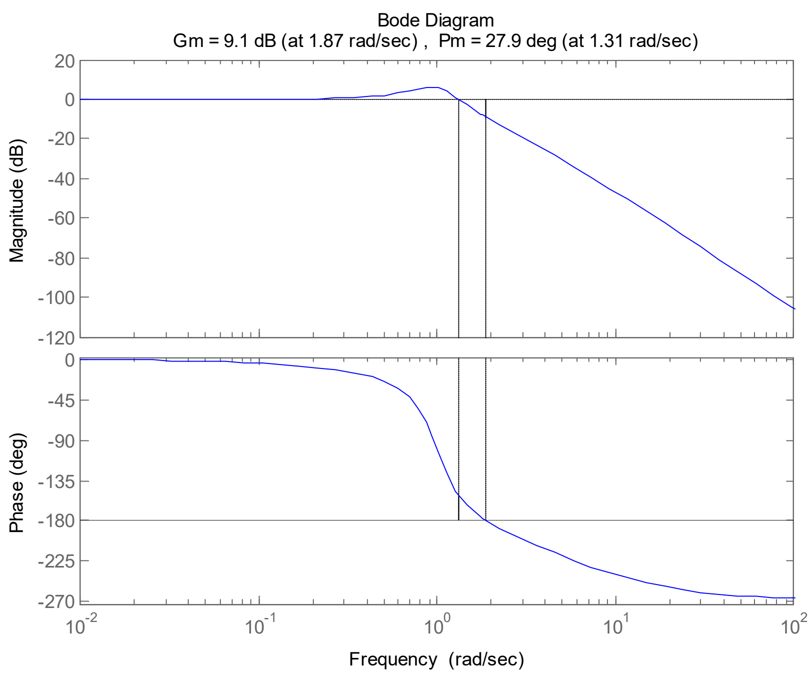
Gain and Phase Margin



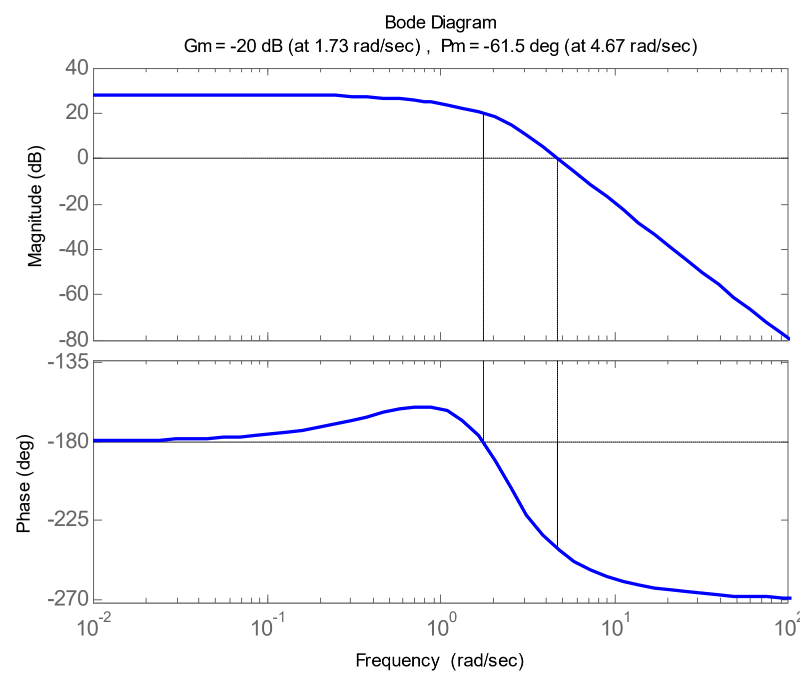
Gain Phase Margin

Gain Margin: *The gain margin is positive and the system is stable if the magnitude of G(jω) at the phase crossover is negative in db. That is, the gain margin is measured below the 0 db line. If the gain margin is measured above the 0 db line, the gain margin is negative and the system is unstable.*

Phase Margin: *The phase margin is positive and the system is stable if the phase of G(jω) at the gain crossover is greater than -180^o. That is, the phase margin is measured above the -180^o axis. If the phase margin is measured below the -180^o axis, the phase margin is negative and the system is unstable.*



Stable system



Unstable system

Lead Compensator Design

The primary function of the lead compensator is to reshape the frequency response curve to provide sufficient phase lead angle to offset the excessive phase lag associated with the components of the fixed system

- Determine the open-loop gain k to satisfy the requirements on the error coefficients
- Using the gain determined k, evaluate the phase margin of the uncompensated system
- Determine the necessary phase lead angle ϕ, to be added to the system
- Determine the attenuation factor α using the equation

$$sin(\phi_m) = \frac{1-\alpha}{1+\alpha}$$

- Determine the frequency where the magnitude of the uncompensated system is equal to -20 log(1/sqrt(α)). Select the frequency as the new gain crossover frequency. This frequency corresponds to ω_m and the maximum phase shift ϕ_m occurs at this frequency.
- Determine the corner frequency of the lead network from

$$\omega = \frac{1}{T}, \quad \omega = \frac{1}{\alpha T}$$

- Finally insert an amplifier with gain equal to $\frac{1}{\alpha}$

Lag Compensator Design

The primary function of the lag compensator is to provide attenuation in the high frequency range to give a system sufficient phase margin. The phase-lag characteristics are of no consequence in lag compensation.

- Select the form of the compensator

$$G_c(s) = k_c\beta\frac{Ts+1}{\beta Ts+1} = k_c\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}} \quad \beta > 1$$

- Define $k_c\beta = K \quad \Rightarrow G_c(s) = K\frac{Ts+1}{\beta Ts+1}$

- The open loop transfer function of the compensated system is:

$$G_c(s)G(s) = K\frac{Ts+1}{\beta Ts+1}G(s) = \frac{Ts+1}{\beta Ts+1}G_1(s)$$

where $G_1(s) = KG(s)$

- Determine K to satisfy the static velocity error constant
- If the system $G_1(s)$ does not satisfy the phase and gain margin requirements, ding the frequency point where the phase margin of the open-loop system is -180 + the required phase margin. Add 5-10 degrees to compensate for the phase-lag of the compensator

Lag Compensator Design (cont.)

- To prevent detrimental effects of the phase lag of the compensator, the pole and zero of the compensator must be located substantially lower than the new gain crossover frequency. Choose the corner frequency 1/T to be 1 octave or 1 decade below the new gain crossover frequency.

- Determine the attentuation necessary to bring the magnitude curve down to 0 db at the new gain crossover frequency. Since this attenuation is $-20\ log(\beta)$termine the value of β which leads to the other corner frequency of $1/\beta T$

- Using the value of K determined earlier, solve for k_c

$$k_c = \frac{K}{\beta}$$

Comparison of Lead and Lag Compensator

- Lead compensator achieves the desired result through the phase lead contribution of the compensator.

  Lag compensator achieves the desired results through the merits of its attenuation properties at high frequencies.

- Lead compensator yields a higher gain crossover frequency than is possible with lag compensation.

  The higher gain crossover frequency means larger bandwidth. If noise is present in the measurements, then a large bandwidth may not be desirable. It does result in faster response due to increased bandwidth.

- Lead compensator requires an additional increase in gain to offset the attentuation inherent in the lead network
- Lag compensation reduces the system gain at higher frequencies without reducing the system gain at lower frequencies.

  Since the system bandwidth is reduced, the system has a slower speed to respond. Also, high frequency noise can be attenuated.

- Lag compensation will introduce a pole-zero combination near the origin that will generate a long tail with small amplitude in the transient response.

Ziegler-Nichols (Z-N) Oscillation Method

Consider a PID controller parameterized as:

$$G_c(s) = K_p \left(1 + \frac{1}{T_r s} + \frac{T_d s}{\tau_D s + 1} \right)$$

Where T_r and T_d are known as the reset time and derivative time, respectively. The time constant τ_D is chosen as:

$$0.1 T_d \leq \tau_D \leq 0.2 T_d$$

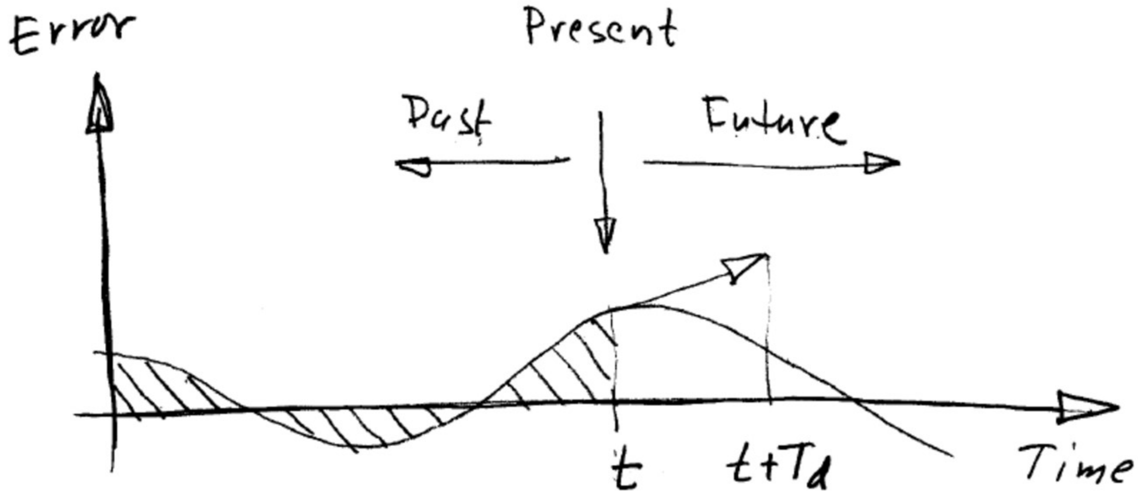
The classical argument to choose $\tau_D \neq 0$ was, apart from ensuing that the controller be proper, to attenuate high-frequency noise.

- The following procedure for selection the PID gains is only for open-loop stable plants.
  - Set the true plant under proportional control, with a very small gain
  - Increase the gain until the loop starts oscillating. Note that linear oscillation is required and that it should be detected at the controller output.
  - Record the controller critical gain $K_p = K_c$ and the oscillation period of the controller output P_c
  - Adjust the controller parameters according the table on the next viewgraph.

Control System Design, Goodwin, Graebe and Salgado, pp 162-165

Ziegler-Nichols (Z-N) Oscillation Method

- Consider a typical error evolution graph:

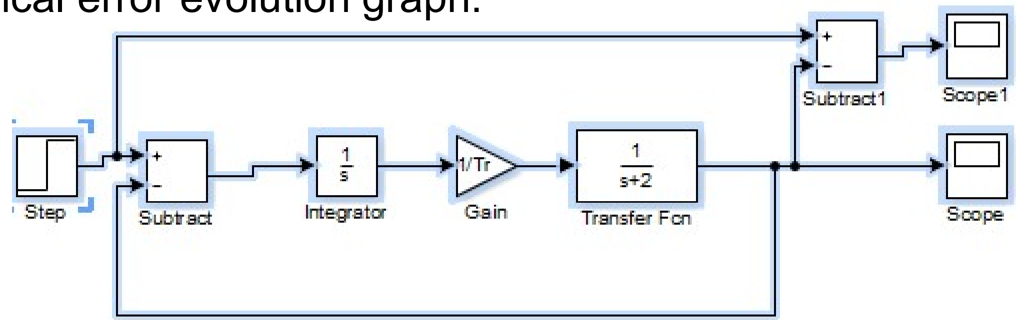


T_d, the derivative time is a measure of how far ahead into the future one can forecast the change in error based on the current slope of the error.

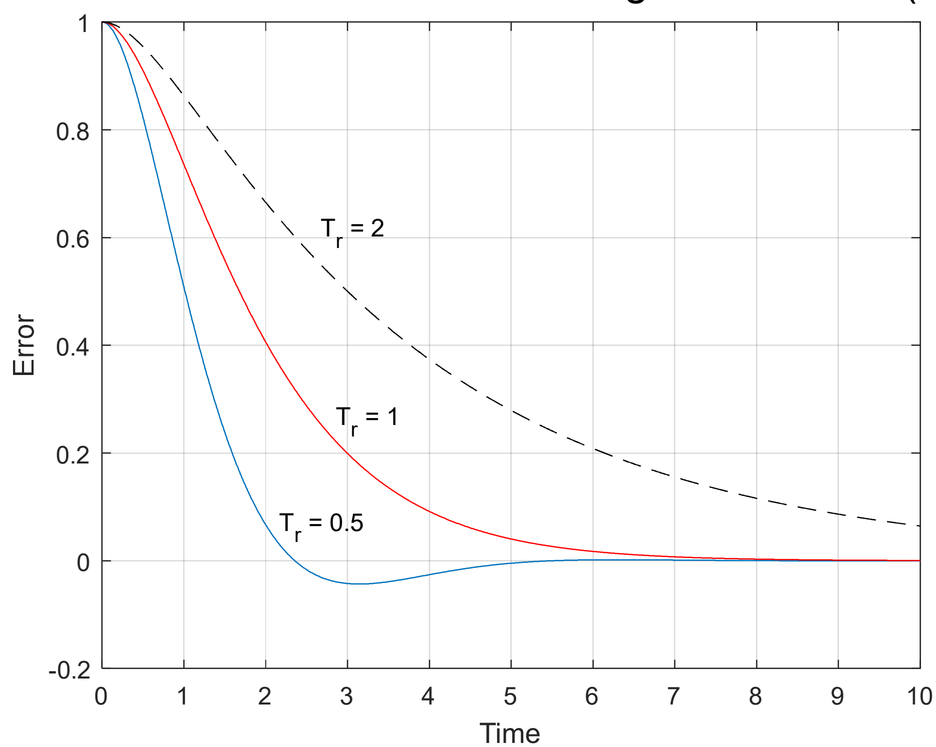
T_r, the reset time is approximately the time taken by the controller to overcome the steady state error.

Ziegler-Nichols (Z-N) Oscillation Method

- Consider a typical error evolution graph:



Varying the reset time T_r results into different time for the error to reach zero as shown below which reflect the time for the integrator to reset (start from zero).



Ziegler-Nichols (Z-N) Oscillation Method

	K_p	T_r	T_d
P	$0.50K_c$		
PI	$0.45K_c$	$\frac{P_c}{1.2}$	
PID	$0.60K_c$	$0.5P_c$	$\frac{P_c}{8}$

Ziegler-Nichols (Z-N) Oscillation Method (EXAMPLE)

Consider a plant with a transfer function:

$$G(s) = \frac{1}{(s+1)^3}$$

The critical gain K_c and the critical frequency ω_c can be determined by substituting s = $j\omega$ in the characteristic equation and solving for K_c and ω_c .

$$K_c + (j\omega_c + 1)^3 = 0$$

Equating the real and imaginary parts to zero, we have:

$$K_c - 3\omega_c^2 + 1 = 0$$
$$-\omega_c^3 + 3\omega = 0$$

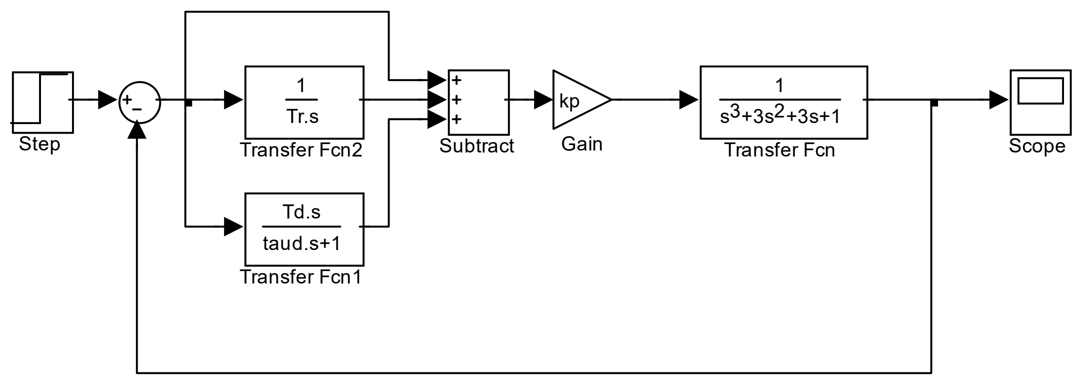
Which results in: $\omega_c = \sqrt{3} \rightarrow P_c = 3.6276$

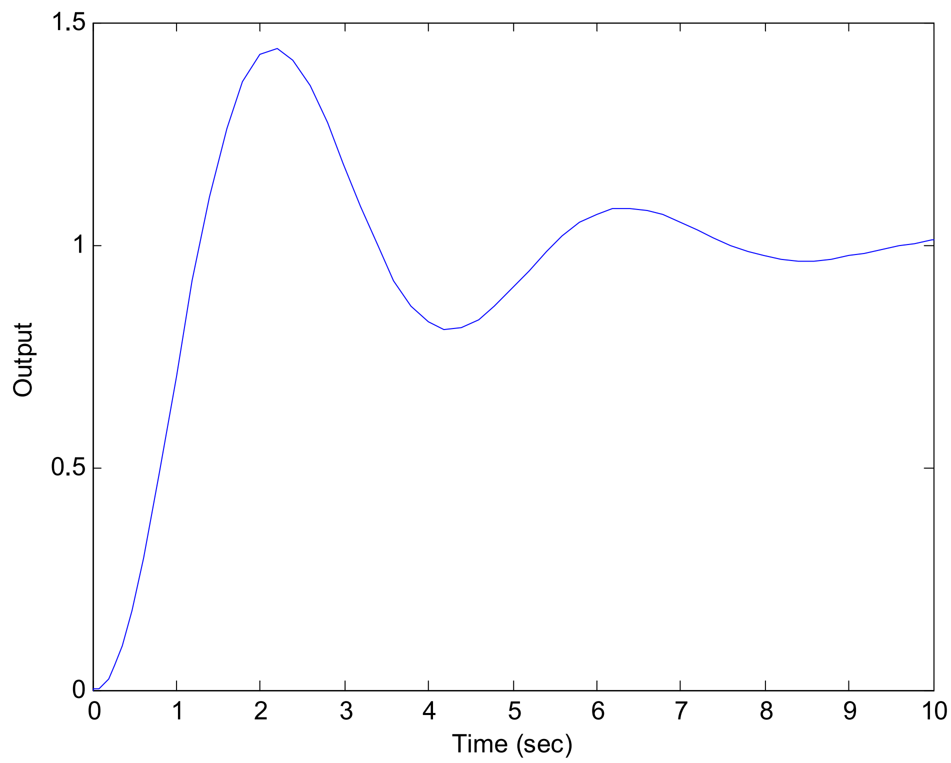
$$K_c = 8$$

Thus, the PID gains are: $K_p = 0.6K_c = 4.8$

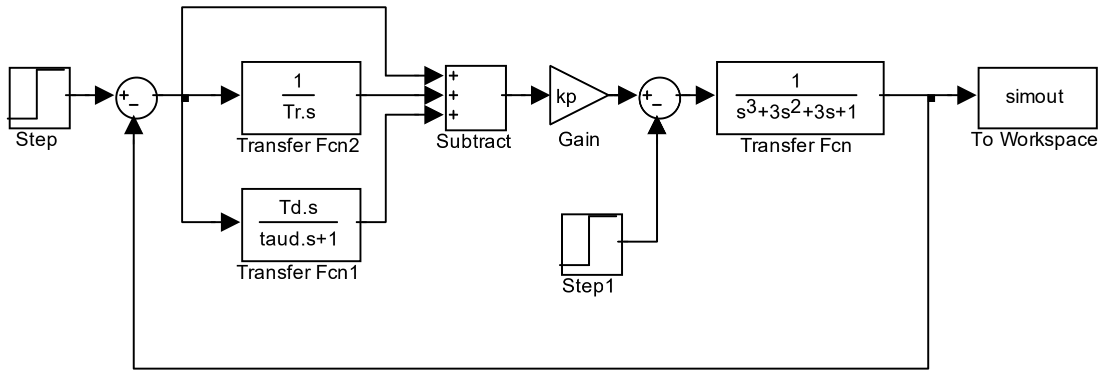
$$T_r = 0.5P_c = 1.81$$
$$T_d = 0.125P_c = 0.45$$

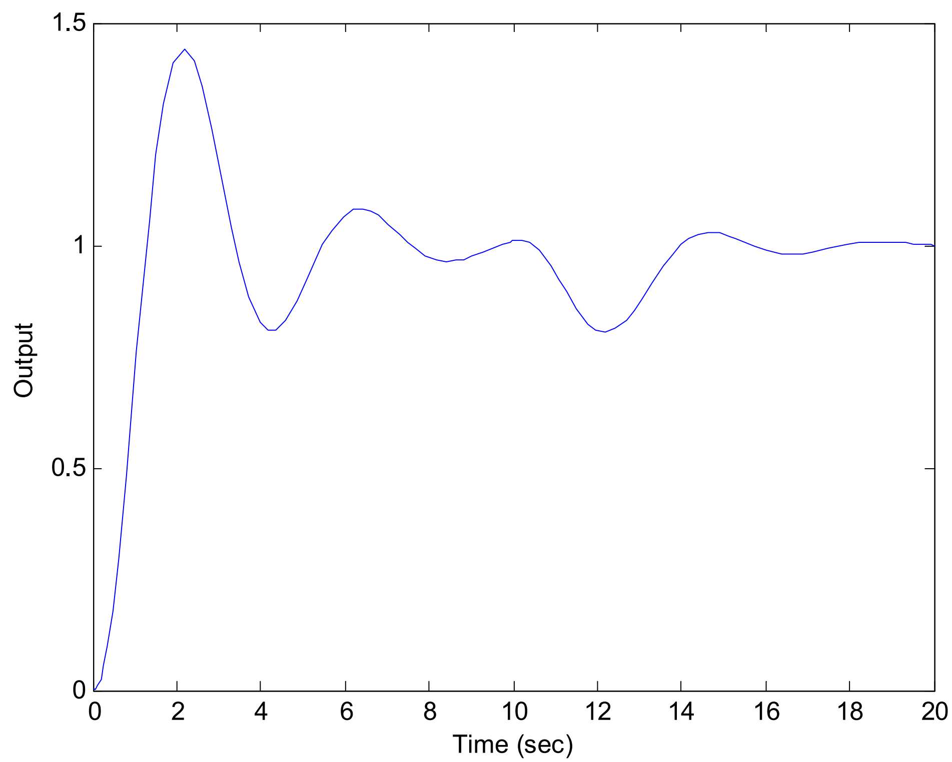
PID Controller





PID Controller (Step Input Disturbance applied at t = 10 sec)



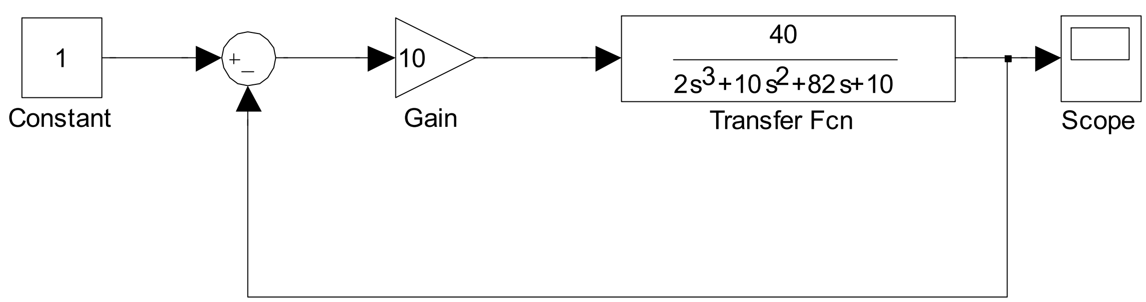


Ziegler-Nichols (Z-N) Oscillation Method (Cruise Control)

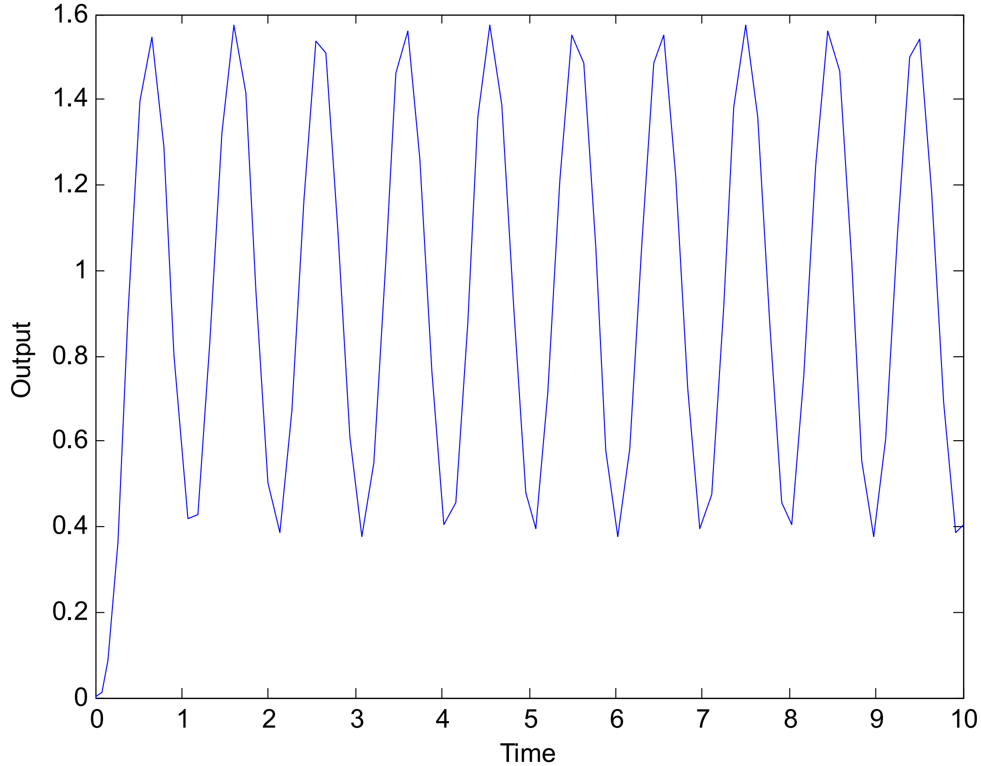
Consider a plant with a transfer function:

$$G(s) = \frac{40}{2s^3 + 10s^2 + 82s + 10}$$

Implement a proportional controller. Increase the gain till the system system response includes sustained oscillations. The critical gain K_c and the critical frequency ω_c can be observed from the graph.

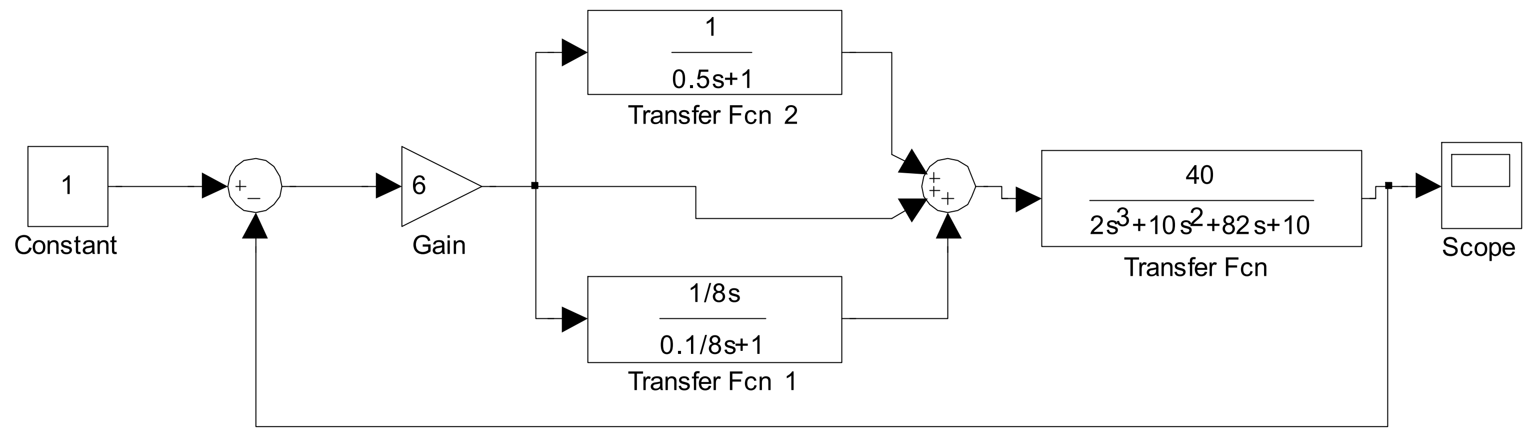


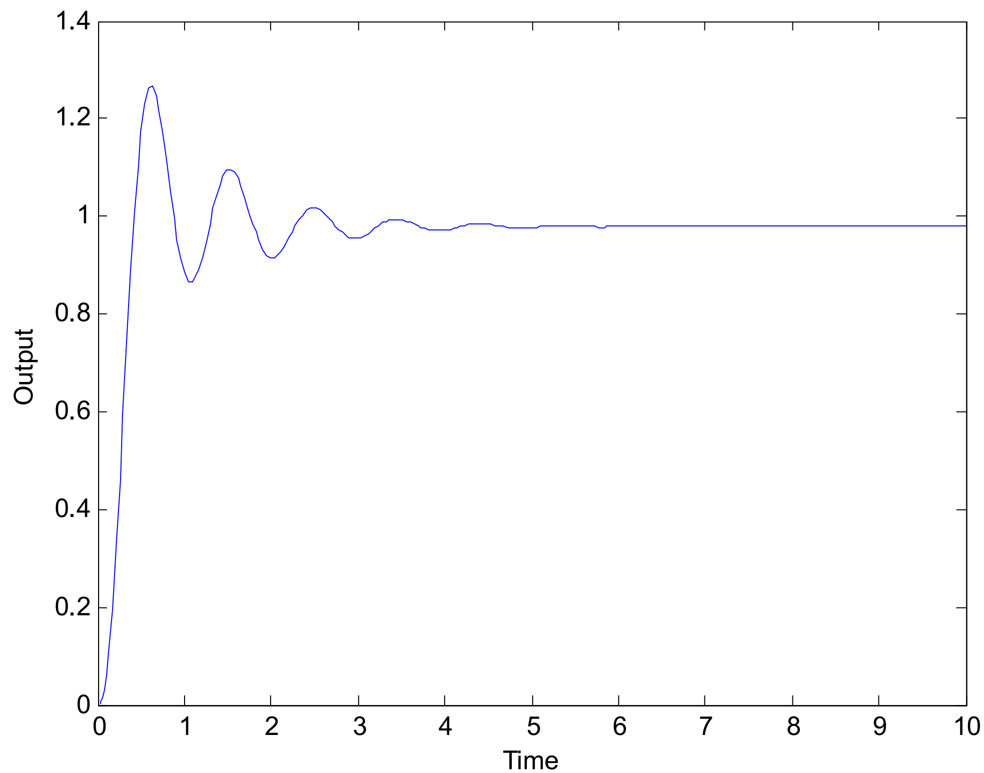
PID Controller



The critical gain Kc =10 and the critical frequency wc = 2pi rad/sec, or the period is 1 sec.

PID Controller





Integrator Antiwindup

- In any control system, the output of the actuator can saturate since the dynamic range of all actuators is limited.
- When the actuator saturates, the control signal stops changing and the feedback path is effectively broken.
- If the error signal continues to be applied to the integrator, the integrator output will grow (windup) until the sign of the error changes and the integration turns around.
- This can result in large overshoot, as the output must grow to produce the necessary unwinding error, resulting in poor transient response.

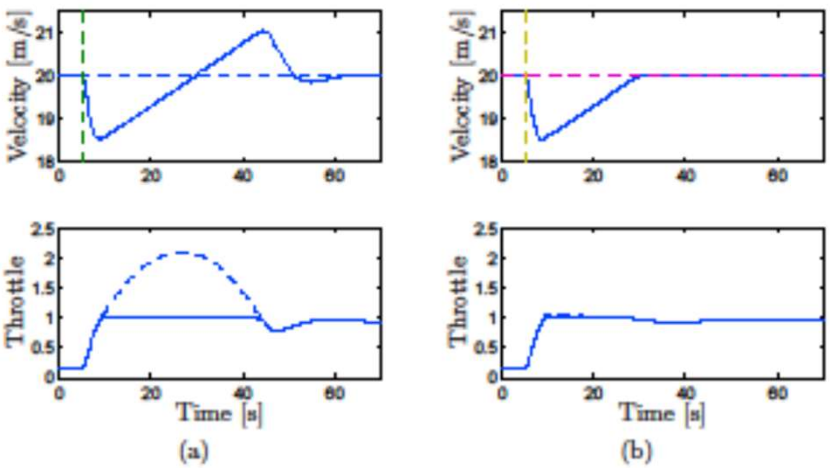
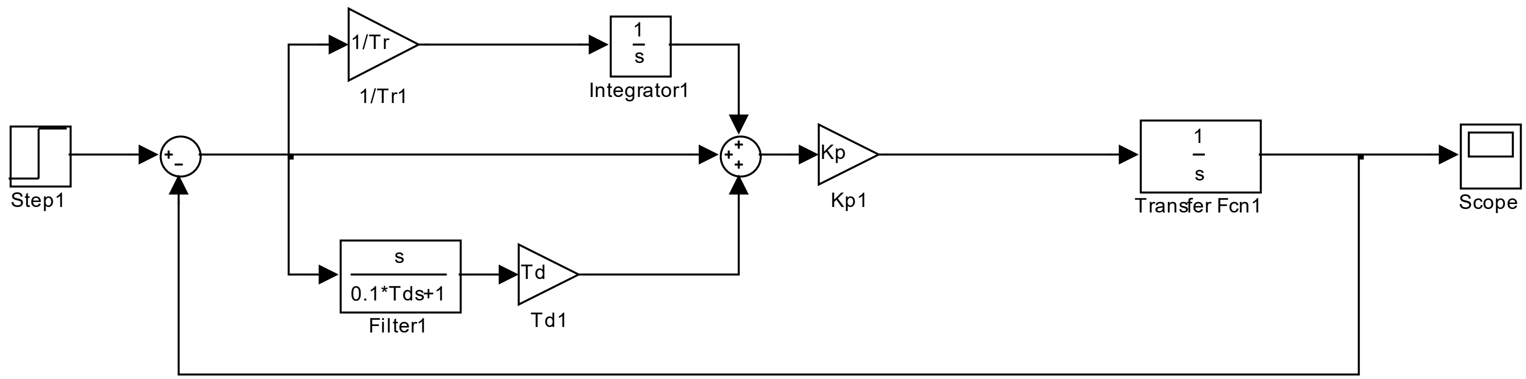
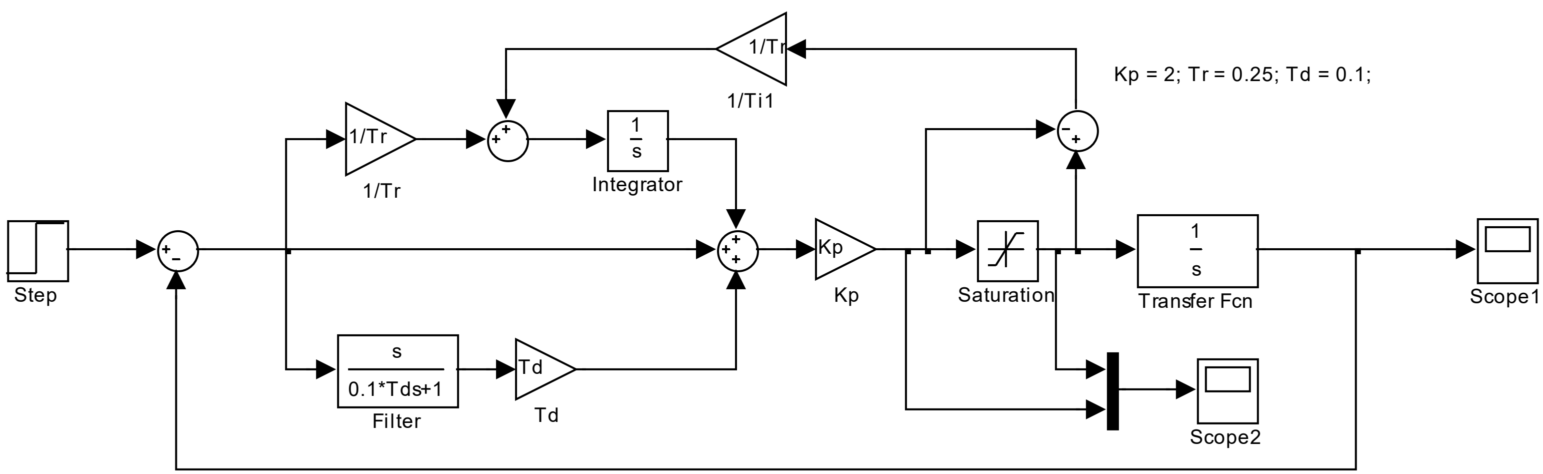


Figure 10.7: Simulation of windup (left) and anti-windup (right). The figure shows the speed v and the throttle u for a car that encounters a slope that is so steep that the throttle saturates. The controller output is dashed. The controller parameters are $k_p = 0.5$ and $k_i = 0.1$.

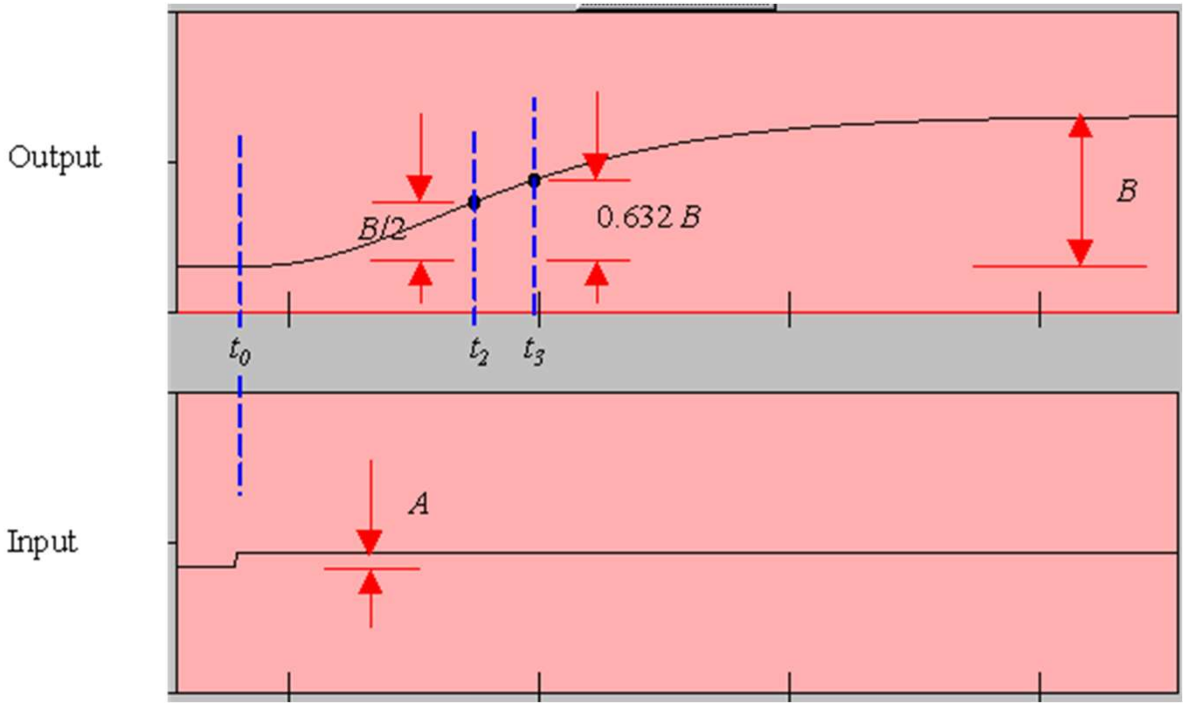
Integrator Antiwindup (example)





Cohen Coon Method

- The following procedure for selection the PID gains is only for open-loop stable plants.
  - Wait until process is at steady state
  - Introduce a step change in the input.
  - Based on the output, obtain an approximate first order process with a time constant τ delayed by τ_{DEL} units from when step input was applied.



  - From the measurements, evaluate:

$$t_1 = \frac{(t_2 - log(2)t_3)}{(1 - log(2))}, \quad \tau = t_3 - t_1, \ \tau_{DEL} = t_1 - t_0, \quad K = \frac{B}{A}$$

http://www.chem.mtu.edu/~tbco/cm416/cctune.html

Cohen Coon Tuning Method

$$G_c(s) = K_p\left(1 + \frac{1}{T_r s} + \frac{T_d s}{\tau_D s + 1}\right) \qquad r = \frac{\tau_{DEL}}{\tau}$$

	K_p	T_r	T_d
P	$\frac{1}{Kr}\left(1 + \frac{r}{3}\right)$		
PI	$\frac{1}{Kr}\left(0.9 + \frac{r}{12}\right)$	$\frac{30 + 3t}{9 + 20r}\tau_{DEL}$	
PID	$\frac{1}{Kr}\left(\frac{4}{3} + \frac{r}{4}\right)$	$\frac{32 + 6t}{13 + 8r}\tau_{DEL}$	$\frac{4}{11 + 2r}\tau_{DEL}$

Nyquist Stability Criterion

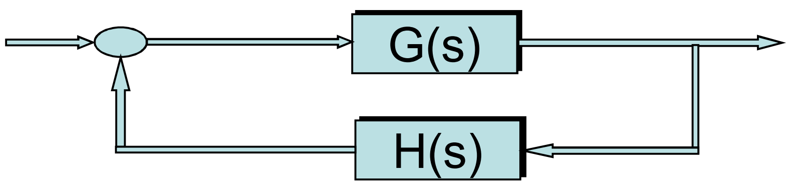
Thus far, stability has been analyzed by investigating the location of the roots of the characteristic equations by:

- Locus of the closed loop poles via the root-locus analysis
- Routh-Hurwitz criterion which provides information about the relative position of the closed loop poles with respect to the imaginary axis.

The Nyquist criterion is a frequency domain method which has the following desirable feature:

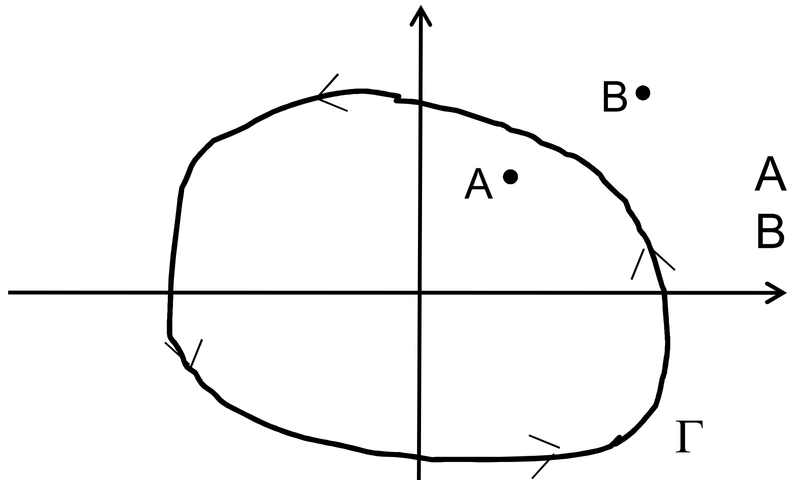
- It provides the same amount of information on the absolute stability as does the Routh-Hurwitz criterion
- The Nyquist criterion indicates the degree of stability of a stable system
- It gives information on the frequency-domain response of the system
- It can used to study system with time-delays
- It can be modified for use for nonlinear systems.

- The Nyquist criterion is a graphical method of determining the stabiltiy of a closed-loop system by investigating the properties of the frequency domain plots of the loop transfer function G(s)H(s)



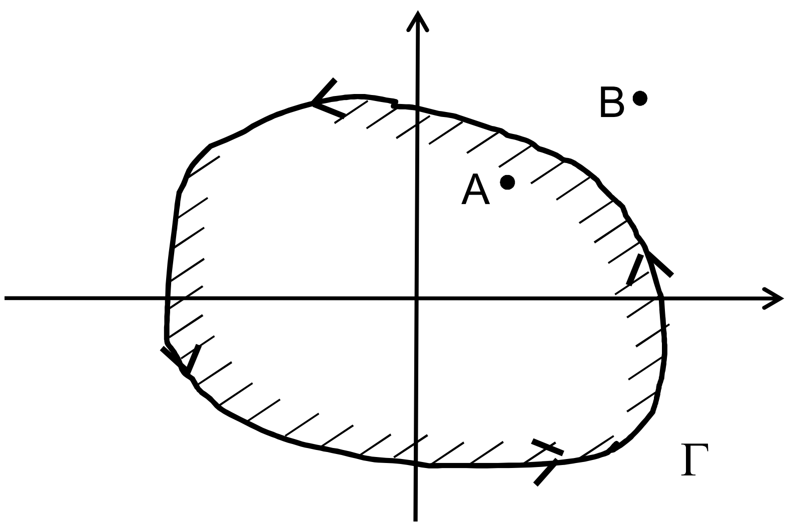
Nyquist Stability Criterion

- Encircled: A point is said to be encircled by a closed path if it is found inside the path.

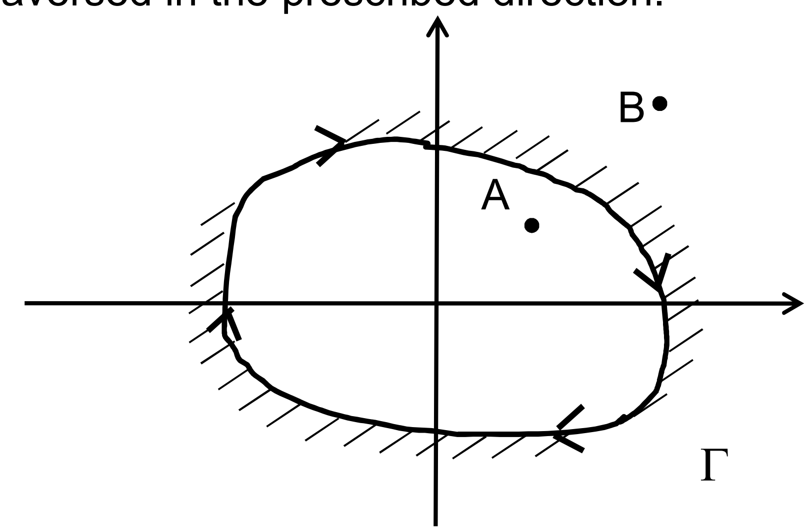


A is encircled by the path Γ
B is not encircled by the path Γ

- Enclosed: A point or a region is said to be enclosed by a closed path if it is found to lie to the left of the path when the path is traversed in the prescribed direction.



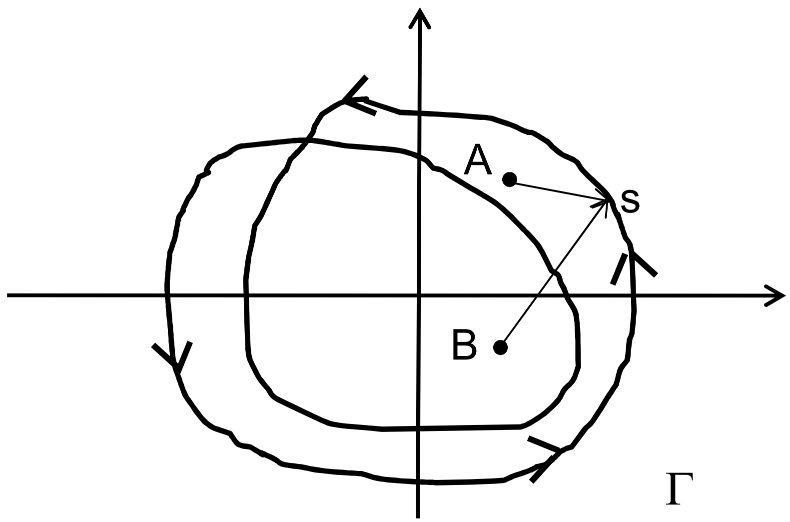
A is enclosed by the path Γ



A is not enclosed by the path Γ
B Is enclosed by the path Γ

Nyquist Stability Criterion

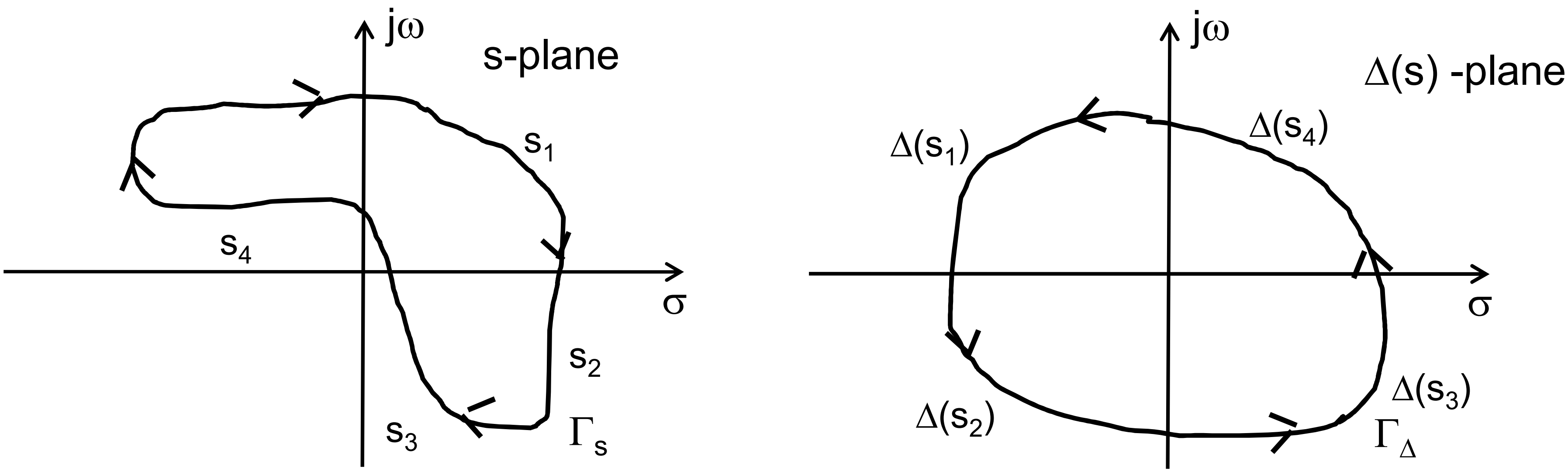
- Number of Encircled and Enclosure: When a point is said to be encircled or enclosed by a closed path, a number N may be assigned to the number of encirclement or enclosure, as the case may be. The value N may be determined by drawing a vector from A to any arbitrary point s on the closed path Γ and let s follow the path in the prescribed direction until it returns to the starting point. The total net number of revolutions traversed by this vector is N.



A is encircled once by the path Γ
B is encircled twice by the path Γ

Nyquist Stability Criterion

- Suppose that a continuous path Γ_s is arbitrarily chosen in the s-plane. If all points on Γ_s are in the specified region where $\Delta = 1+G(s)H(s)$ is analytic, then the curve Γ_Δ mapped by the function Δ into the $\Delta(s)$ plane is also a closed one.



Let $\Delta(s)$ be a single-valued rational function that is analytic in a given region in the s-plane except at a finite number of points. Suppose that an arbitrary closed path Γ_s is chosen in the s-plane so that $\Delta(s)$ is analytic at every point on Γ_s ; the corresponding $\Delta(s)$ locus mapped in the $\Delta(s)$-plane will encircle the origin as many times as the difference between the number of zeros and the number of poles of $\Delta(s)$ that are encircled by the s-plane locus Γ_s .

Nyquist Stability Criterion

- In equation form, this statement can be expressed as:

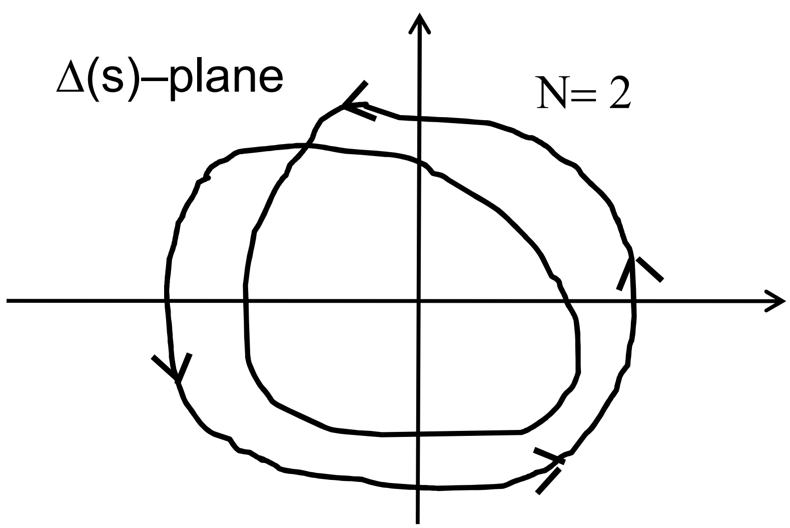
$$N = Z - P$$

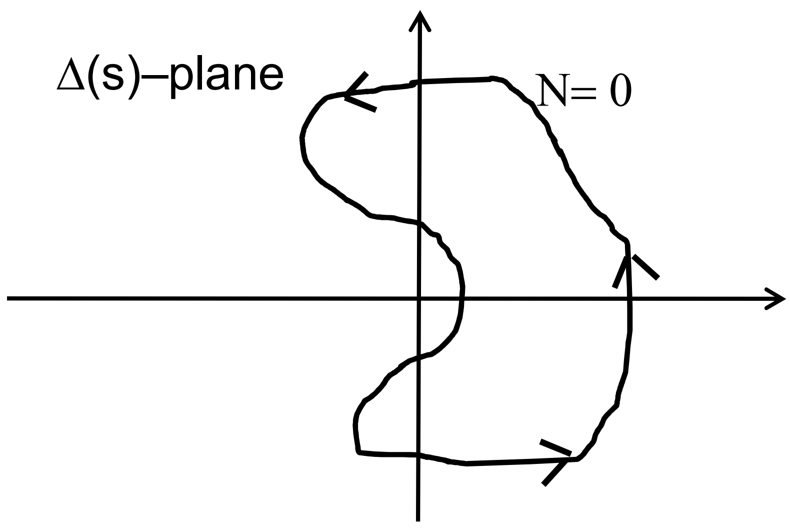
where

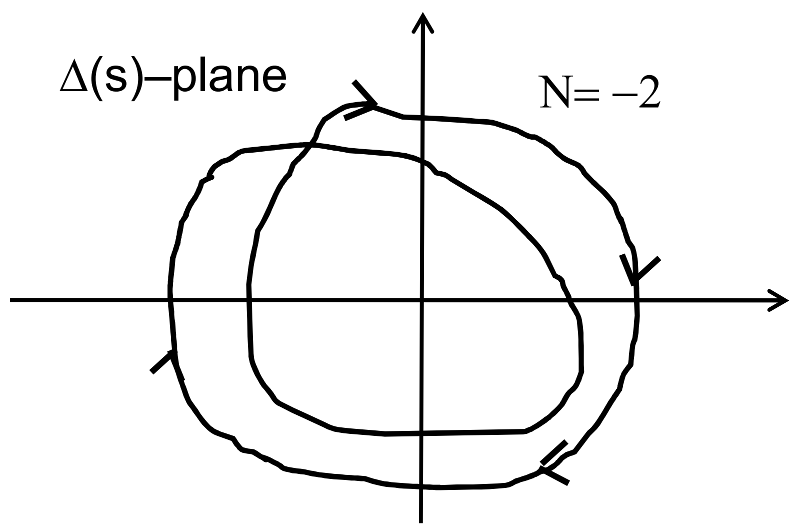
  - N = # of encirclement of the origin made by the Δ(s)–plane locus Γ_Δ
  - Z = # of zeros of Δ(s) encircled by the s-plane locus Γ_s in the s-plane.
  - P= # of poles of Δ(s) encircled by the s-plane locus Γ_s in the s-plane

- N can be positive, zero or negative
  1. N >0 (Z>P): If the s-plane locus encircles more zeros than poles of Δ(s)=1+G(s)H(s) in a certain prescribed direction (clockwise or counterclockwise), N is a positive integer. In this case the Δ(s)–plane locus will encircle the origin of the Δ(s)–plane N times in the same direction as that of Γ_s
  2. N =0 (Z=P): If the s-plane locus encircles as many zeros as poles, or no poles and zeros of Δ(s)=1+G(s)H(s), the Δ(s)–plane locus Γ_Δ will not encircle the origin of the Δ(s)–plane .
  3. N <0 (Z<P): If the s-plane locus encircles more poles than zeros of Δ(s)=1+G(s)H(s) in a certain prescribed direction (clockwise or counterclockwise), N is a negative integer. In this case the Δ(s)–plane locus will encircle the origin of the Δ(s)–plane N times in the opposite direction as that of Γ_s

Nyquist Stability Criterion





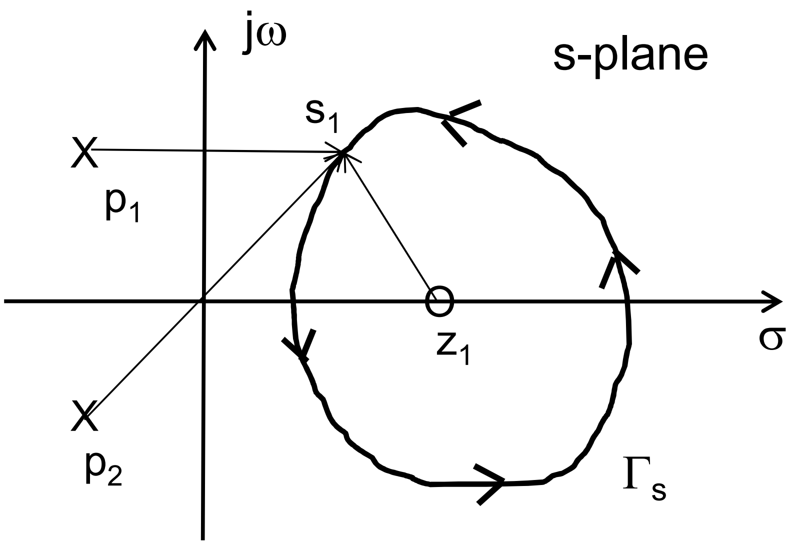


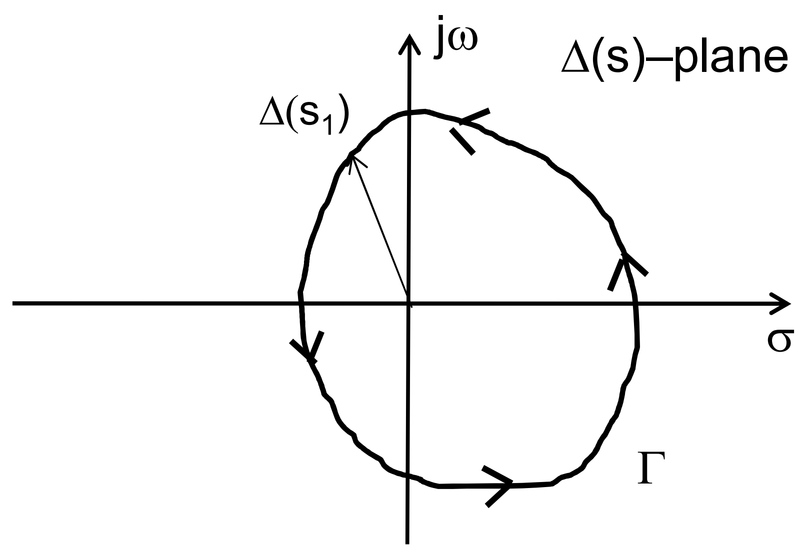
Nyquist Stability Criterion

- Consider the system with a positive K:

$$\Delta(s) = \frac{K(s+z_1)}{(s+p_1)(s+p_2)}$$

The poles and zeros are assumed to be located as shown below:





The function Δ(s) can be written as:

$$\Delta(s) = \frac{K|s+z_1|}{|s+p_1||s+p_2|}(\angle s+z_1 - \angle s+p_1 - \angle s+p_2)$$

Nyquist Stability Criterion

- The factors $(s+z_1)(s+p_1)(s+p_2)$ can be represented by the arrows.

- Now if the points s_1 moves along the locus of Γ_s in the prescribed counterclockwise direction until it returns to the starting point, the angle generated by the vectors drawn from the poles that are not encircled by Γ_s when s_1 completes one round trip are zero, whereas the vector drawn from z_1 which is encircled by Γ_s , generates a positive angle (counterclockwise sense) of 360 degrees. This is why only the poles and zeros of Δ(s) which are inside the Γ_s path in the s-plane, would contribute to the value of N.

- Since poles of Δ(s) correspond to negative phase angle and zeros correspond to positive phase angle, the value of N depends only on the difference between Z and P.

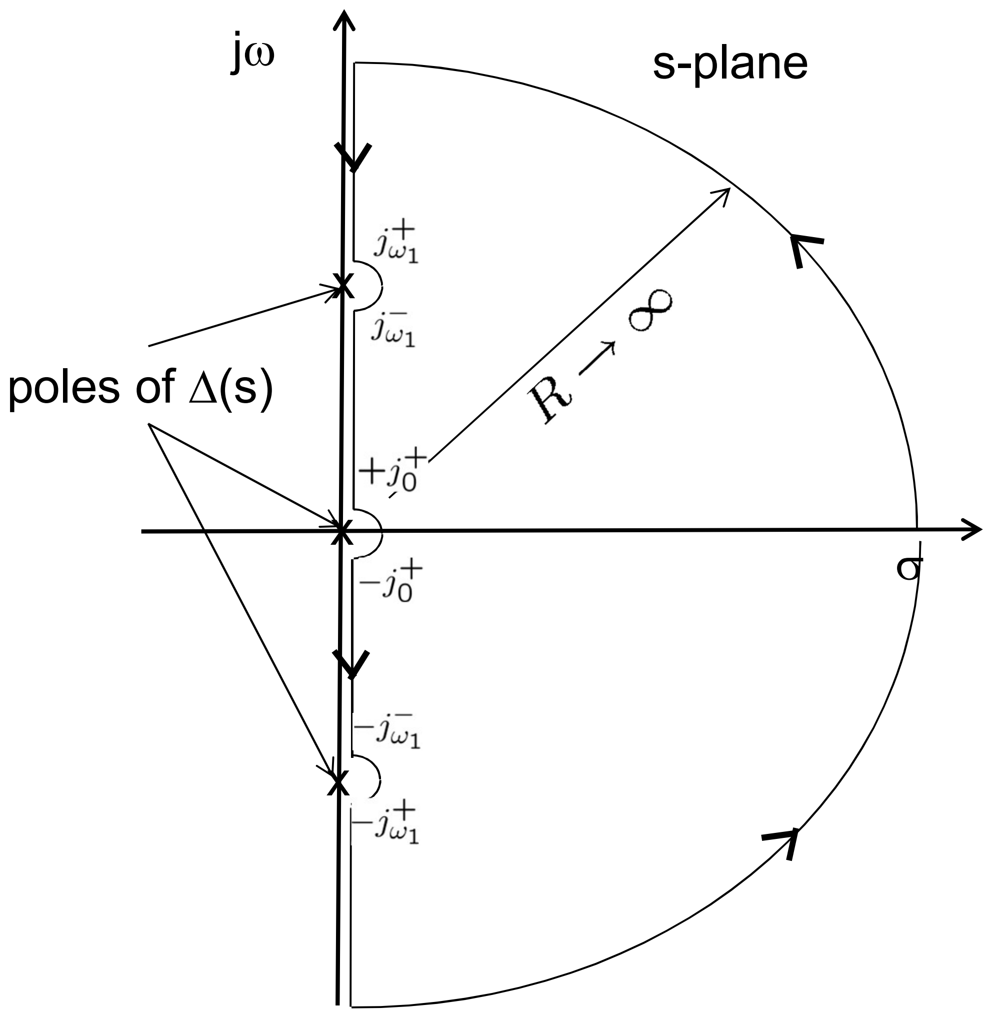
- In the present case Z = 1, P = 0

- Thus N = Z-P = 1

  Which means that the Δ(s)-plane locus should encircle the origin one in the same direction as the s-plane locus.

Nyquist Path

- If the Γ_s path in the s-plane is taken to be one that encircles the entire right-half of the s-plane , the mapping of the Γ_s path in the Δ(s)-plane can be used to determine if any poles of the closed loop system lie in the right-half of the complex plane.



Nyquist Path

The Nyquist path must not pass through any singularity of Δ(s), the small semi-circles shown along the y-axis, are used to indicate that the path should go around these singular points. It is apparent that if any pole or zero of Δ(s) lies inside the right half of the s-plane, it will be encircled by this Nyquist path.

Nyquist Stability Criterion

- Once the Nyquist path is specified, the stability of the closed-loop system can be determined by plotting the Δ = 1+G(s)H(s) locus when s takes on values along the Nyquist path and investigating the behavior of Δ(s) plot with respect to the origin in the Δ(s)-plane.
- However, since G(s)H(s) are generally known functions, it is simpler to construct the Nyquist plot of G(s)H(s), and the same conclusion of stability of the closed-loop system can be determined from the plot of G(s)H(s) with respect to the (-1,0jω) point in the G(s)H(s)—plane. This because the origin of the Δ(s) corresponds to the (-1,0jω) point on the G(s)H(s)—plane.
- Closed loop stability implies that Δ = 1+G(s)H(s) has zeros only in the left half of the s-plane. Open-loop stability implies that G(s)H(s) has poles only in the left-half of the s-plane.

- N_0=# of encirclements of the origin made by G(s)H(s)
- Z_0=#of zeros of G(s)H(s) in the right-half of the s-plane
- P_0=#of poles of G(s)H(s) in the right-half of the s-plane
- N_{-1}=# of encirclements of (-1,0jω) made by G(s)H(s)
- Z_{-1}=#of zeros of 1+G(s)H(s) in the right-half of the s-plane
- P_{-1}=#of poles of 1+G(s)H(s) in the right-half of the s-plane

Nyquist Stability Criterion

- Note: $P_{0}= P_{-1,}$ since G(s)H(s) and (1+G(s)H(s)) always have the same poles.
- Closed loop stability implies that $Z_{-1} = 0$.
- Open-loop stability requires that $P_0 = 0$.
- The procedure of determining stability using the Nyquist plot is:
  - Define the Nyquist path according to the pole-zero properties of G(s)H(s)
  - The Nyquist plot of G(s)H(s) is constructed
  - The values of N_0 and N_{-1} are determined by observing the behavior of the Nyquist plot of G(s)H(s) with respect to the origin and the (-1,0jω) point
  - Once N_0 and N_{-1} are determined, the value of P_0 is determined from $N_0= Z_0-P_0$.
  - Once P_0 is determined, : $P_{0}= P_{-1,}$ and Z_{-1} is determined from

$$N_{-1} = Z_{-1} - P_{-1}$$

since it has been established that for a stable closed-loop system Z_{-1} must be zero, gives

$$N_{-1} = -P_{-1}$$

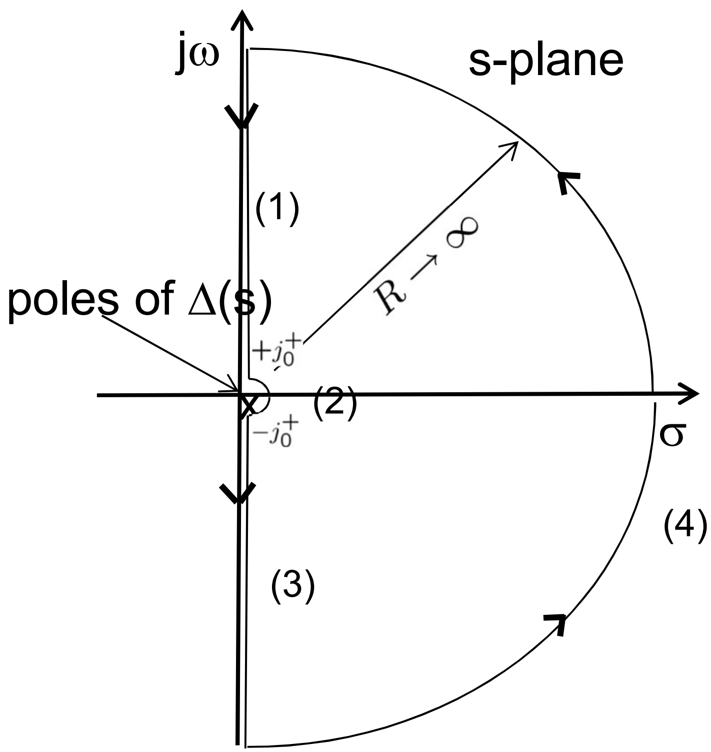
Therefore, the Nyquist criterion may be formally stated as: For a closed-loop system to be stable, the Nyquist plot of 1+G(s)H(s) must encircle the (-1,0jω) point as many times as the number of poles of 1+G(s)H(s) that are in the right half of the s-plane, and the encirclements if any, must be made in the clockwise direction.

Nyquist Stability Criterion (Example)

- Consider a system with a transfer function (K and a are positive numbers):

$$G(s)H(s) = \frac{K}{s(s+a)}$$

- It is clear that G(s)H(s) does not have any pole in the right-half of the s-plane, thus $P_{0}=P_{-1}=0$. The Nyquist path for the system is show below.



Since G(s)H(s) has a pole at the origin, it is necessary for the Nyquist path include a small semicircle around s=0.The entire Nyquist path is divided into four sections. section (2) of the Nyquist path can be represented by a phasor as:

$$s = \epsilon e^{j\theta}$$

Where $\epsilon \longrightarrow 0$ and θ denote the magnitude and phase of the phasor, respectively. as the Nyquist path is traversed from $+j0^+$ To $-j0^+$, along section (2), the phasor rotates through 180 degrees starting at +90 and reaching -90 degrees.

Nyquist Stability Criterion (Example)

- The Nyquist plot G(s)H(s) can be determined as:

$$G(s)H(s)|_{s=\epsilon e^{j\theta}} = \frac{K}{\epsilon e^{j\theta}(\epsilon e^{j\theta} + a)}$$

- Which can be approximated as:

$$G(s)H(s)|_{s=\epsilon e^{j\theta}} \approx \frac{K}{a\epsilon e^{j\theta}} = \infty e^{-j\theta}$$

- Which indicates that all points on the Nyquist plot G(s)H(s) that correspond to section (2) on the Nyquist path have an infinite magnitude and the corresponding phases is opposite that of the s-plane locus. Since the phase varies from +90 to -90 degrees in the clockwise direction, the G(s)H(s) plot should have a phase that varies from -90 to +90 degrees in the counter-clockwise direction.
- The same technique can be used to determine the behavior of the G(s)H(s) plot, which corresponds to the semi-circle with infinite radius on the Nyquist path (section(4)). The points on this section can be represented by the phasor:

$$s = Re^{j\theta} \text{ where } R \to \infty$$

- Thus G(s)H(s) can be represented as:

$$G(s)H(s)|_{s=Re^{j\theta}} = \frac{K}{R^2 e^{2j\theta}} = 0e^{-2j\theta}$$

- Which corresponds to a phasor of infinitesimally small magnitude which rotates around the origin 2x180 degrees in the clockwise direction.

Nyquist Stability Criterion (Example)

- The Nyquist plot G(s)H(s) for section (1) and (3) can now be determined. For section (3) substituting s = jω

$$G(s)H(s)|_{s=j\omega} = \frac{K}{j\omega(j\omega + a)}$$

- Which can be rewritten as:

$$G(s)H(s)|_{s=j\omega} = \frac{K(-\omega^2 - ja\omega)}{\omega^4 + a^2\omega^2}$$

- The intersect of G(s)H(s) with the real axis is determined by equating the imaginary part of G(s)H(s) to zero, which leads to:

$$IM\left(G(j\omega)H(j\omega)\right) = \frac{-Ka\omega}{\omega^4 + a^2\omega^2} = 0$$

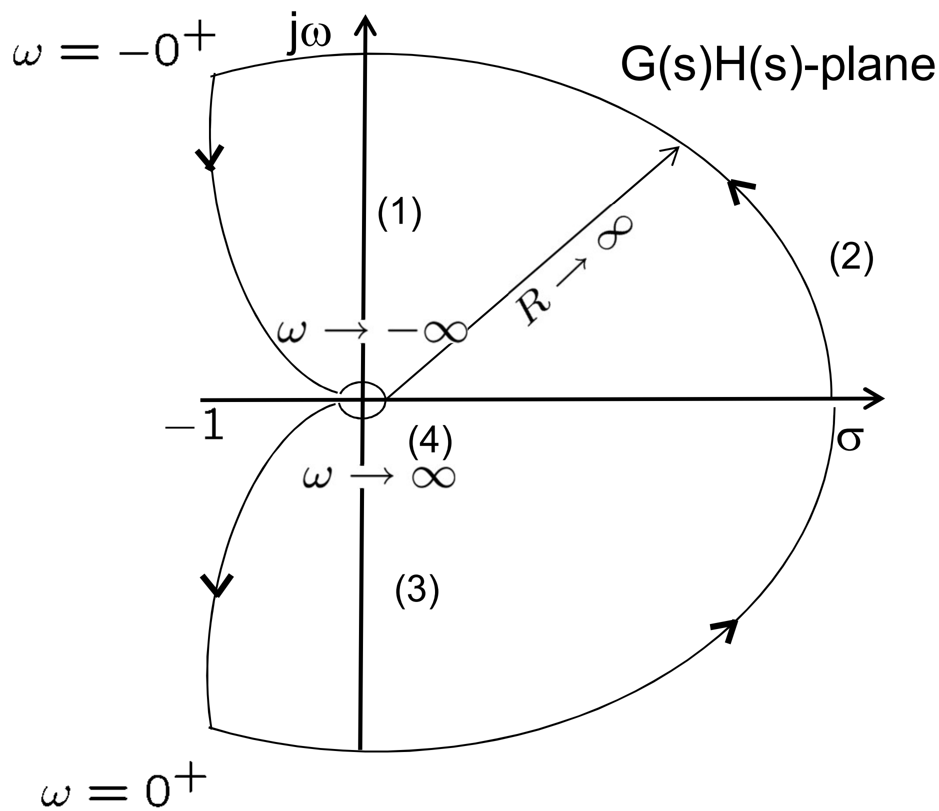
- Which gives $\omega = \infty$. This means that the only intersect on the real-axis in the G(s)H(s) plane is at the origin with $\omega = \infty$

Nyquist Stability Criterion (Example)

- Since $Z_0 = P_0 = 0$ from G(s)H(s) and from the figure below $N_0 = N_{-1} = 0$, we have

$$Z_{-1} = N_{-1} + P_{-1} = 0$$

- since $P_0 = P_{-1}$. Therefore, the close-loop system is stable.

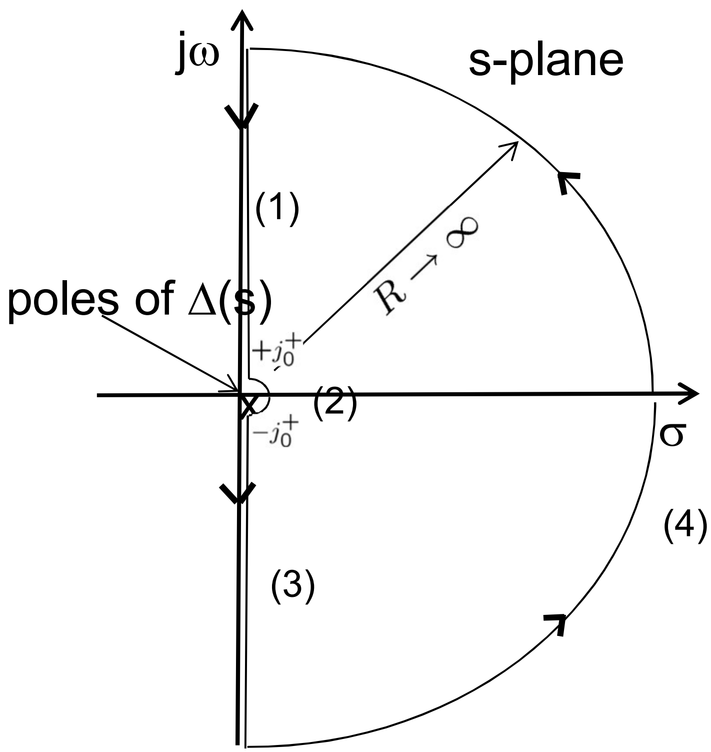


Nyquist Stability Criterion (Example 2)

- Consider a system with a transfer function (K and a are positive numbers):

$$G(s)H(s) = \frac{K(s-1)}{s(s+1)}$$

- It is clear that G(s)H(s) does not have any pole in the right-half of the s-plane, thus $P_{0=}$ $P_{-1,}$ =0. The Nyquist path for the system is show below.



Since G(s)H(s) has a pole at the origin, it is necessary for the Nyquist path include a small semicircle around s=0.The entire Nyquist path is divided into four sections. section (2) of the Nyquist path can be represented by a phasor as:

$$s = \epsilon e^{j\theta}$$

Where $\epsilon \longrightarrow 0$ and θ denote the magnitude and phase of the phasor, respectively. as the Nyquist path is traversed from $+j0^+$ To $-j0^+$, along section (2), the phasor rotates through 180 degrees starting at +90 and reaching -90 degrees.

Nyquist Stability Criterion (Example 2)

- The Nyquist plot G(s)H(s) can be determined as:

$$G(s)H(s)|_{s=\epsilon e^{j\theta}} = \frac{K(\epsilon e^{j\theta} - 1)}{\epsilon e^{j\theta}(\epsilon e^{j\theta} + 1)}$$

- Which can be approximated as:

$$G(s)H(s)|_{s=\epsilon e^{j\theta}} \approx \frac{-K}{\epsilon e^{j\theta}} = \infty e^{-j(\theta+\pi)}$$

- Which indicates that all points on the Nyquist plot G(s)H(s) that correspond to section (2) on the Nyquist path have an infinite magnitude and the corresponding phases starts at an angle of +90 and ends at -90 degrees and goes around the origin in the counter-clockwise direction.
- The same technique can be used to determine the behavior of the G(s)H(s) plot, which corresponds to the semi-circle with infinite radius on the Nyquist path (section(4)). The points on this section can be represented by the phasor:

$$s = Re^{j\theta} \text{ where } R \to \infty$$

- Thus G(s)H(s) can be represented as:

$$G(s)H(s)|_{s=Re^{j\theta}} = \frac{K}{Re^{j\theta}} = 0e^{-j\theta}$$

- Which corresponds to a phasor of infinitesimally small magnitude which rotates around the origin 180 degrees in the clockwise direction.

Nyquist Stability Criterion (Example)

- The Nyquist plot G(s)H(s) for section (1) and (3) can now be determined. For section (3) substituting s = jω

$$G(s)H(s)|_{s=j\omega} = \frac{K(j\omega - 1)}{j\omega(j\omega + 1)}$$

- Which can be rewritten as:

$$G(s)H(s)|_{s=j\omega} = \frac{K(2\omega + j(1 - \omega^2))}{\omega^3 + \omega}$$

- The intersect of G(s)H(s) with the real axis is determined by equating the imaginary part of G(s)H(s) to zero, which leads to:

$$IM\left(G(j\omega)H(j\omega)\right) = \frac{K(1 - \omega^2)}{\omega^3 + \omega} = 0$$

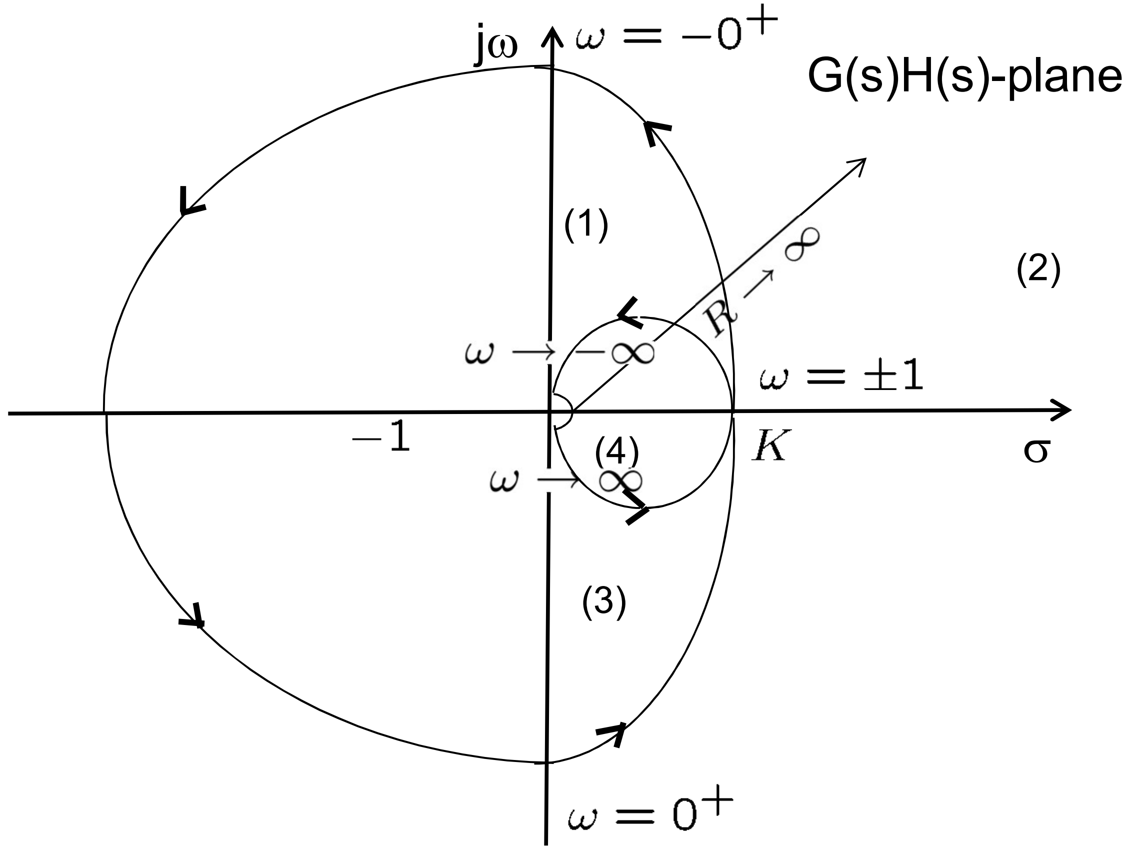
- Which gives $\omega = \pm 1$, which are the frequencies at which the G(s)H(s) curve crosses the real axis.

Nyquist Stability Criterion (Example)

- Since Z_0= 1 from G(s)H(s) and from the figure below N_{-1}=1, we have

$$Z_{-1} = N_{-1} + P_{-1} = 1$$

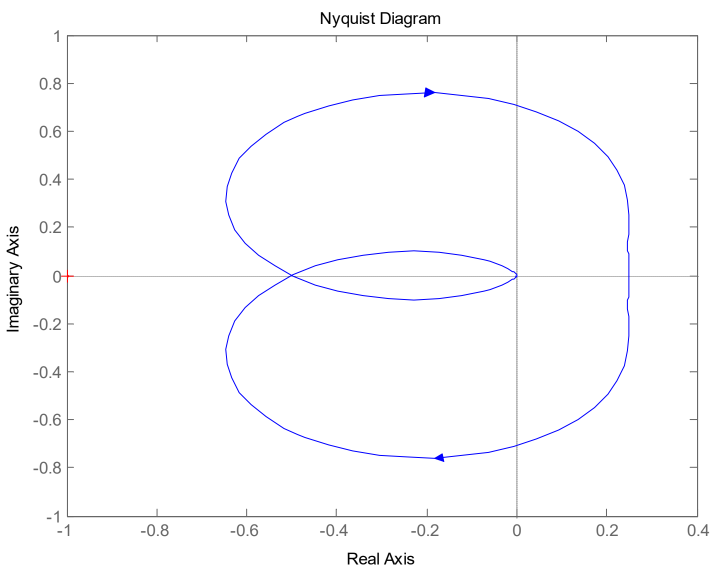
- since P_0= P_{-1}.Therefore, the close-loop system is unstable.

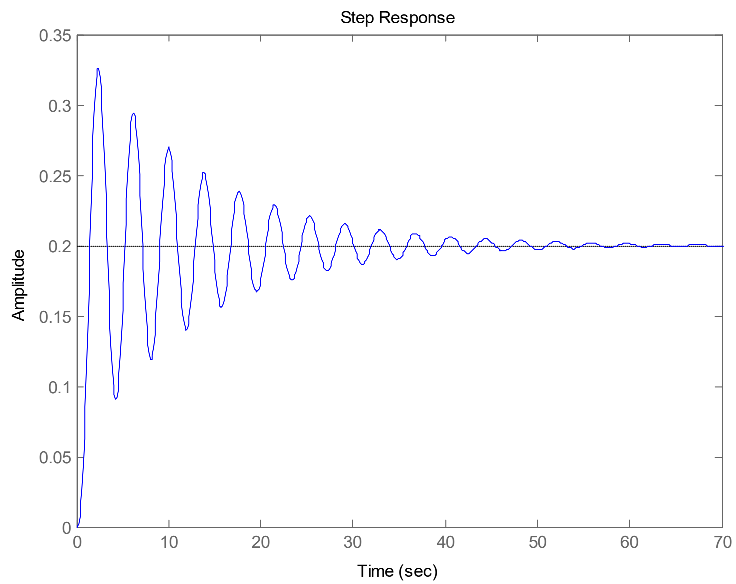


Relative Stability—Gain Margin, Phase Margin

- To demostrate the concept of relative stability, the Nyquist plot and the step response of a typical third-order system is shown below.

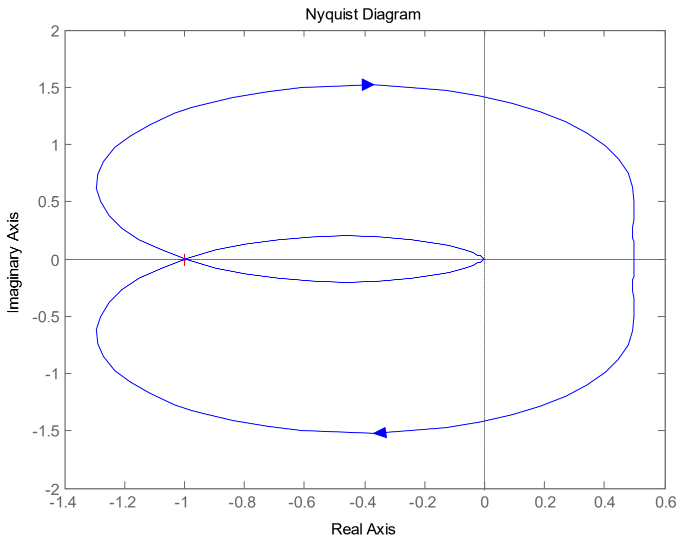
$$G(s)H(s) = \frac{K}{s^3 + 2s^3 + 3s + 4}$$

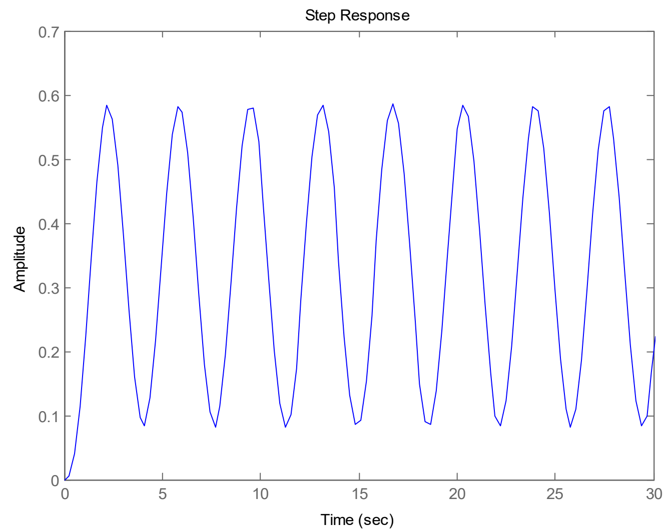




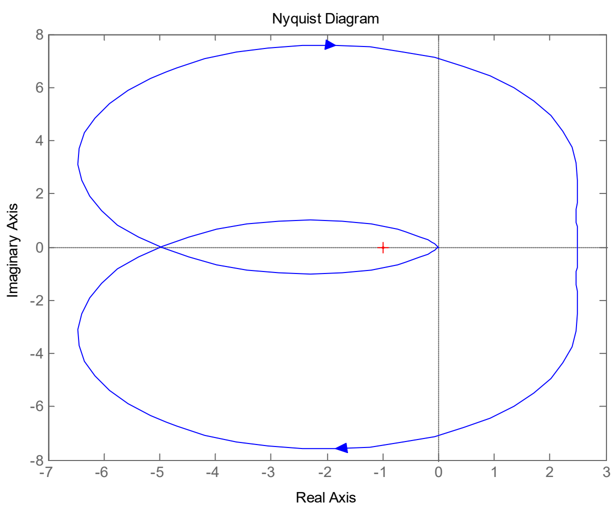
$$K = 1$$

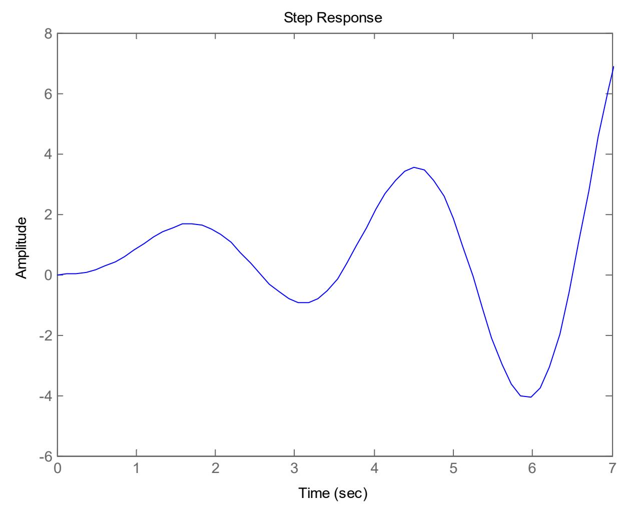
Relative Stability—Gain Margin, Phase Margin





$$K = 2$$



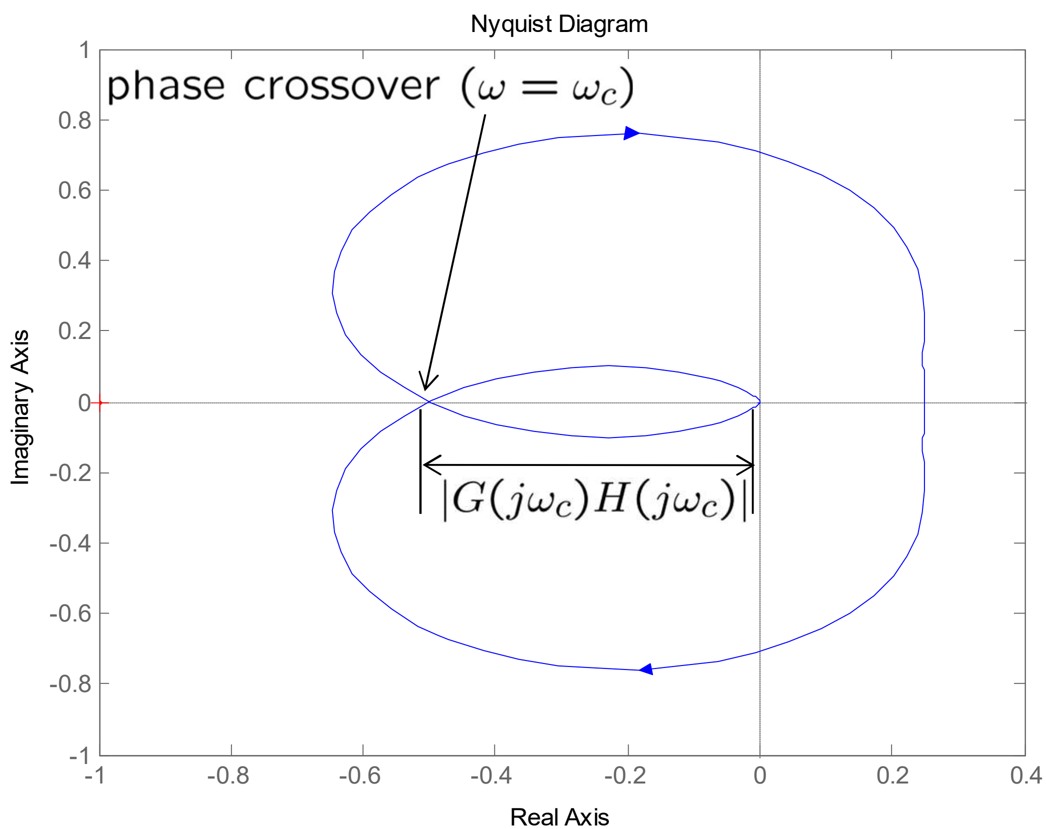


$$K = 10$$

Relative Stability—Gain Margin, Phase Margin

- Note that as K is increased, the point at which the Nyquist plot crosses the real axis (phase crossover point) moves closer to the (-1,0jω) point and once it crosses it, the system is unstable.
- GAIN MARGIN: The gain margin is a measure of the closeness of the phase-crossover point to the (-1,0jω) point. The gain margin of the closed-loop system that has G(s)H(s) as its loop transfer function is:

$$\mathrm{GM} = 20\log_{10}\left(\frac{1}{|G(j\omega_c)H(j\omega_c)|}\right)$$

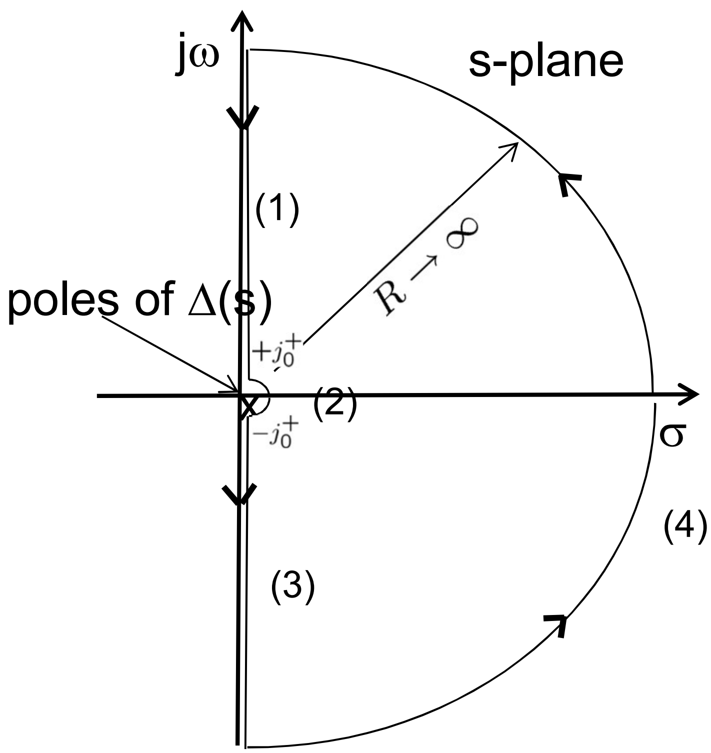


Relative Stability—Gain Margin, Phase Margin

- GAIN MARGIN: is the amount of gain in decibels that can be allowed to increase in the loop before the closed-loop system reaches instability.
- When the G(s)H(s) plot goes through the (-1,0jω) point, the gain margin is 0 db, which implies that the loop gain can no longer be increased as the system is already on the margin of stability.
- When the G(s)H(s) plot does not intersect the negative real axis at any finite nonzero frequency, and the Nyquist stability criterion indicates that the (-1,0jω) point must not be enclosed for system stability, the gain margin is infinite in decibels, which means that theoretically, the value of the loop gain can be increased to infinity before any instability occurs.
- When the (-1,0jω) point is to the right of the phase-crossover point, the magnitude of G(s)H(s) is greater than unity and the gain margin is negative in decibels, which implies that the system is unstable.
- NOTE: If G(s)H(s) has poles or zeros in the right-half of the s-plane, (-1,0jω) point must be encircled by the G(s)H(s) plot for stability. Under this condition, a stable system yields a negative gain margin.

Nyquist Example

- Consider the system: G(s)H(s) = 5(s+3)/(s(s-1))



Since G(s)H(s) has a pole at the origin, it is necessary for the Nyquist path include a small semicircle around s=0.The entire Nyquist path is divided into four sections. section (2) of the Nyquist path can be represented by a phasor as:

$$s = \epsilon e^{j\theta}$$

Where $\epsilon \longrightarrow 0$ and θ denote the magnitude and phase of the phasor, respectively. as the Nyquist path is traversed from $+j0^+$ To $-j0^+$, along section (2), the phasor rotates through 180 degrees starting at +90 and reaching -90 degrees.

Nyquist Stability Criterion (Example 2)

- The Nyquist plot G(s)H(s) can be determined as:

$$G(s)H(s)|_{s=\epsilon e^{j\theta}} = \frac{5(\epsilon e^{j\theta}+3)}{\epsilon e^{j\theta}(\epsilon e^{j\theta}-1)}$$

- Which can be approximated as:

$$G(s)H(s)|_{s=\epsilon e^{j\theta}} \approx \frac{-15}{\epsilon e^{j\theta}} = \infty e^{-j(\theta+\pi)}$$

- Which indicates that all points on the Nyquist plot G(s)H(s) that correspond to section (2) on the Nyquist path have an infinite magnitude and the corresponding phases starts at an angle of +90 and ends at -90 degrees and goes around the origin in the counter-clockwise direction.
- The same technique can be used to determine the behavior of the G(s)H(s) plot, which corresponds to the semi-circle with infinite radius on the Nyquist path (section(4)). The points on this section can be represented by the phasor:

$$s = Re^{j\theta} \text{ where } R \to \infty$$

- Thus G(s)H(s) can be represented as:

$$G(s)H(s)|_{s=Re^{j\theta}} = \frac{K}{Re^{j\theta}} = 0e^{-j\theta}$$

- Which corresponds to a phasor of infinitesimally small magnitude which rotates around the origin 180 degrees in the clockwise direction starting at +90 degrees.

Nyquist Stability Criterion (Example)

- The Nyquist plot G(s)H(s) for section (1) and (3) can now be determined. For section (3) substituting s = jω

$$G(s)H(s)|_{s=j\omega} = \frac{5(j\omega+3)}{j\omega(j\omega-1)}$$

- Which can be rewritten as:

$$G(s)H(s)|_{s=j\omega} = -\frac{5(4\omega^2 + j(\omega^3 - 3\omega))}{\omega^4+\omega^2}$$

- The intersect of G(s)H(s) with the real axis is determined by equating the imaginary part of G(s)H(s) to zero, which leads to:

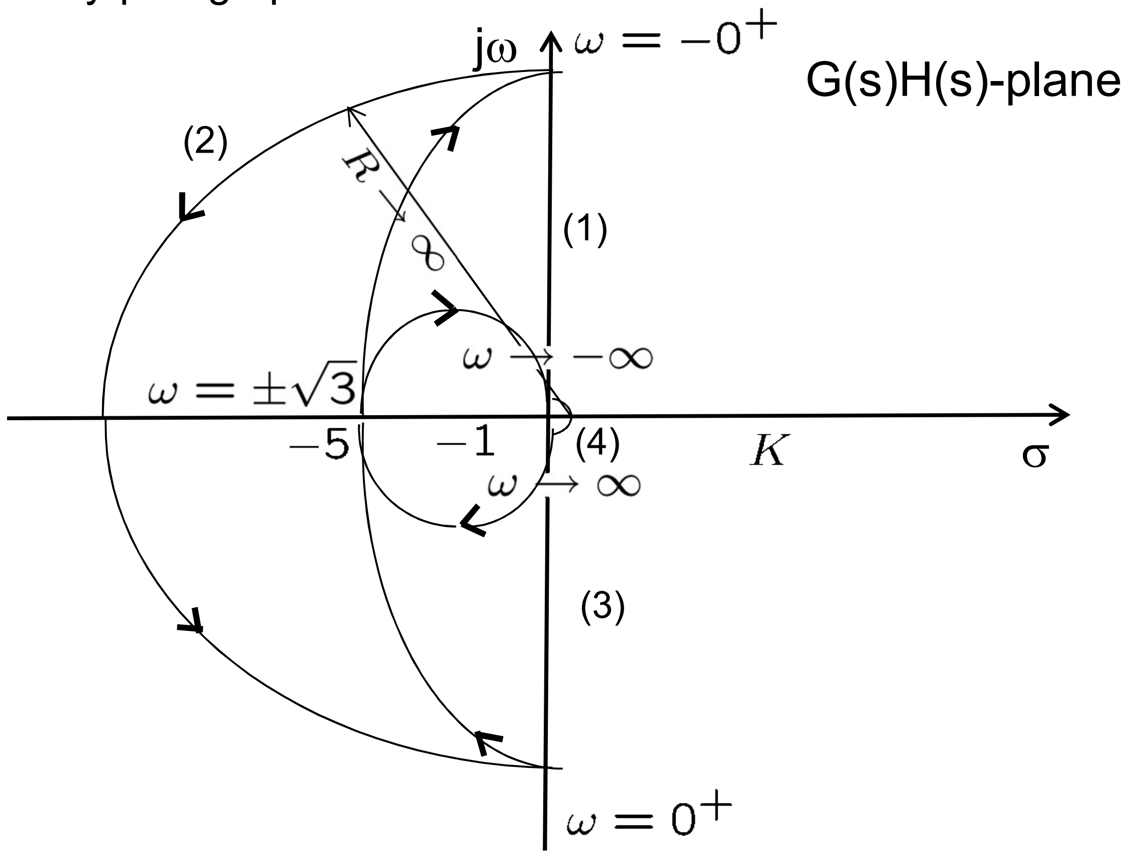
$$IM\,(G(j\omega)H(j\omega)) = \frac{5(3-\omega^2)}{\omega^3+\omega} = 0$$

- Which gives $\omega = \pm\sqrt{3}$ which are the frequencies at which the G(s)H(s) curve crosses the real axis.
- The corresponding real part is:

$$IM\,(G(j\omega)H(j\omega)) = \frac{20\omega}{\omega^3+\omega} = -5$$

Nyquist Stability Criterion (Example)

- The final Nyquist graph is:



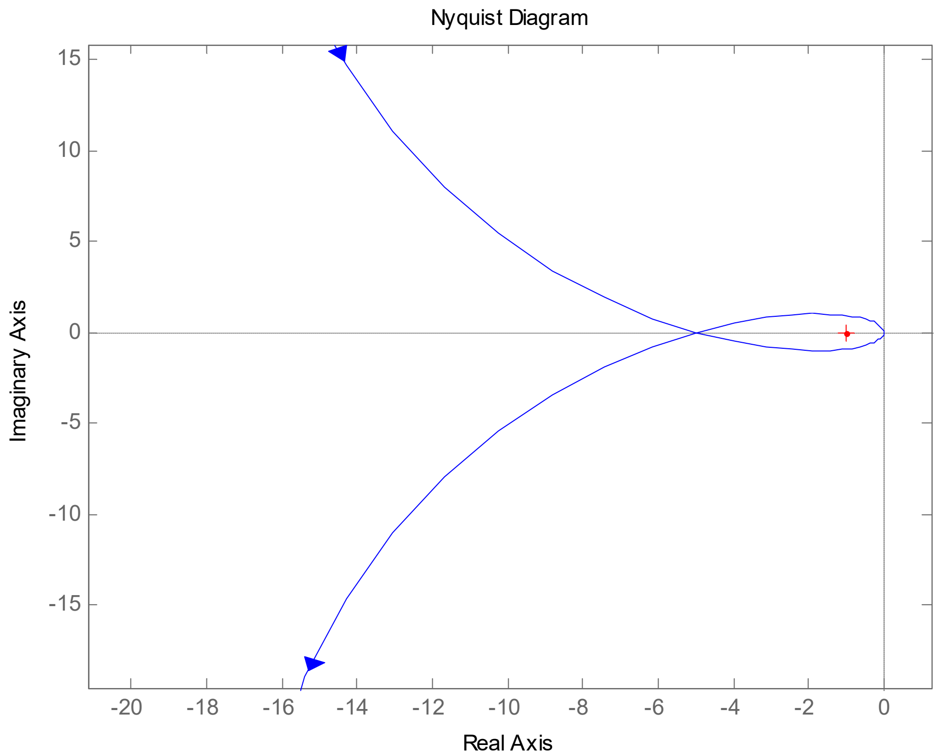
- Which shows that the (-1,j0) point is encircled in a clockwise manner: $N_{-1} = -1$
- $P_{-1} = 1$, which corresponds to a right half plane zero. Therefore

$$Z_{-1} = N_{-1} + P_{-1} = 0$$

- Which indicates that the closed loop system is stable.

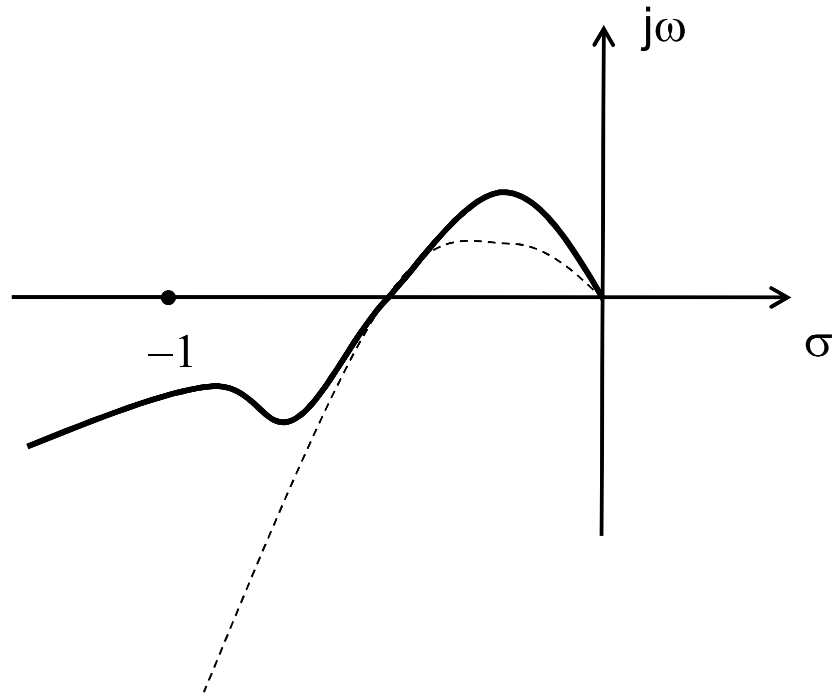
Relative Stability—Gain Margin, Phase Margin

- Consider the system: $G(s) = 5(s+3)/(s(s-1))$
- Gain Margin = -14 db (Note system is stable, even though the gain margin is negative)



Relative Stability—Gain Margin, Phase Margin

- Gain margin is one way of representing the relative stability of a feedback control system. In general a system has a large gain margin, should be relatively more stable than one with a smaller gain margin.
- The gain margin of the two system shown below by the solid and the dashed lines, have the same gain margin. However, the system represented by the dashed line is more stable than the system represented by the solid line. This is due to the fact that with any change in the system parameter, other than the loop gain, it is easier for the system represented by the solid line to pass through the (-1,0jω) point.



Relative Stability—Gain Margin, Phase Margin

- PHASE MARGIN: is defined as the angle in degrees through which G(s)H(s) plot must be rotated about the origin in order that the gain-crossover point on the locus passes through the (-1,0jω) point .
- The phase margin indicates the effect on stability of changes in system parameters which alter the phase of G(s)H(s).

$$\text{PM} = \angle G(j\omega_g)H(j\omega_g) - 180$$