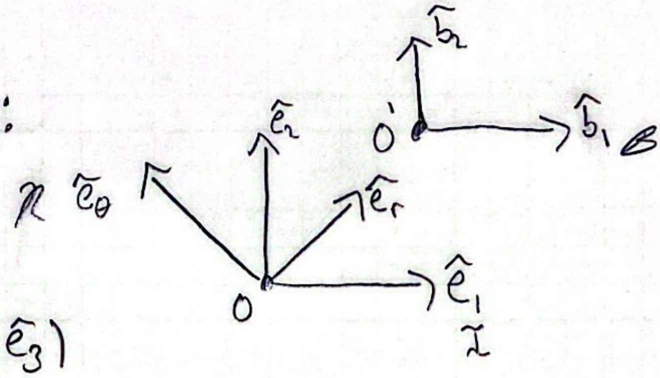
MAE 562	HW5	50223306

8.7)

Reference Frames:

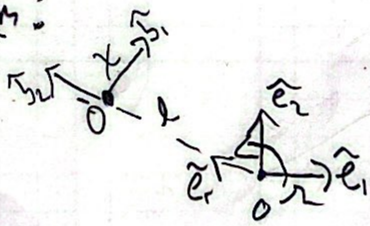


$\mathcal{I} = (O, \hat e_1, \hat e_2, \hat e_3)$

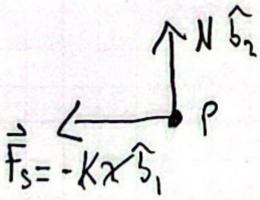
$\mathcal{R} = (O, \hat e_r, \hat e_\theta, \hat e_3)$

$\mathcal{B} = (O', \hat b_1, \hat b_2, \hat b_3)$ $\qquad \hat b_3 = \hat e_3, \quad \hat b_2 = \hat e_r, \quad \hat b_1 = -\hat e_\theta$

Coordinate System:



a) **FBD:**



Kinematics: $\vec r_{P/O} = \vec r_{O'/O} + \vec r_{P/O'} = l\hat e_r + x\hat b_1$

$${}^I\vec v_{P/O} = \left[\frac{{}^R d}{dt}(\vec r_{O'/O}) + {}^I\vec\omega^R \times \vec r_{O'/O}\right] + \left[\frac{{}^B d}{dt}(\vec r_{P/O'}) + {}^I\vec\omega^B \times \vec r_{P/O'}\right]$$

$${}^I\vec v_{P/O} = \cancelto{0}{\dot l}\hat e_r + (\Omega\hat e_3 \times l\hat e_r) + (\dot x\hat b_1 + \Omega\hat b_3 \times x\hat b_1)$$

$${}^I\vec v_{P/O} = \Omega l\hat e_\theta + \dot x\hat b_1 + x\Omega\hat b_2$$

$${}^I\vec a_{P/O} = \frac{{}^R d}{dt}(\Omega l\hat e_\theta) + {}^I\vec\omega^R \times (\Omega l\hat e_\theta) + \frac{{}^B d}{dt}(\dot x\hat b_1) + {}^I\vec\omega^B \times \dot x\hat b_1$$
$$+ \frac{{}^B d}{dt}(x\Omega\hat b_2) + {}^I\vec\omega^B \times x\Omega\hat b_2$$

$$^{I}\vec{a}_{P/O} = \Omega\hat{e}_3 \times \Omega l \hat{e}_\theta + \ddot{x}\hat{b}_1 + \Omega\hat{b}_3 \times \dot{x}\hat{b}_1 + \dot{x}\Omega\hat{b}_2 + \Omega\hat{b}_3 \times x\Omega\hat{b}_2$$

$$^{I}\vec{a}_{P/O} = -\Omega^2 l \hat{e}_r + \ddot{x}\hat{b}_1 + \Omega\dot{x}\hat{b}_2 + \dot{x}\Omega\hat{b}_2 - x\Omega^2\hat{b}_1$$

Note $\hat{e}_r = \hat{b}_2$ & $\hat{b}_1 = -\hat{e}_\theta$ $\therefore$

$$^{I}\vec{a}_{P/O} = -\Omega^2 l \hat{b}_2 + \ddot{x}\hat{b}_1 + 2\Omega\dot{x}\hat{b}_2 - x\Omega^2\hat{b}_1$$

$$\vec{F}_P = M_P \, {}^{I}\vec{a}_{P/O} = -Kx\hat{b}_1 + N\hat{b}_2 = (\ddot{x} - x\Omega^2)\hat{b}_1 + (2\Omega\dot{x} - \Omega^2 l)\hat{b}_2$$

$$\boxed{\ddot{x} = -\frac{Kx}{m} + x\Omega^2}$$ - From N2L in $\hat{b}_1$

b) $\ddot{x} = 0 = -\frac{Kx}{m} + x\Omega^2$

$$\Rightarrow \boxed{\Omega = \sqrt{\frac{K}{m}}}$$

- The angular velocity of disk is equivalent to the natural frequency of the spring. $\omega_n = \sqrt{\frac{K}{M}}$

c) N2L $\hat{b}_2$: $\boxed{N = m_P(2\Omega\dot{x} - \Omega^2 l)}$

D) Total Energy: $E_{P/O}(t_2) = E_{P/O}(t_1) + W_P^{(nc)}(\vec{r}_{P/O}; \gamma_P)$

$\therefore$ For Energy to be conserved, $W_P^{(nc)}(\vec{r}_{P/O}; \gamma_P)$ must $= 0$.

Normal Force does No work, & the spring applies a conservative force. Therefore no work is done by non-conservative forces & Energy is conserved.

e) ${}^{\mathcal{I}}\vec{h}_{P/O} = \vec{r}_{P/O} \times m_P \, {}^{\mathcal{I}}\vec{v}_{P/O}$

$${}^{\mathcal{I}}\vec{h}_{P/O} = (l\hat{e}_r + x\hat{b}_1) \times m_P(\Omega l \hat{e}_\theta + \dot{x}\hat{b}_1 + x\Omega\hat{b}_2)$$

$${}^{\mathcal{I}}\vec{h}_{P/O} = (l\hat{b}_2 + x\hat{b}_1) \times m_P(-\Omega l\hat{b}_1 + \dot{x}\hat{b}_1 + x\Omega\hat{b}_2)$$

$${}^{\mathcal{I}}\vec{h}_{P/O} = -(-\Omega l^2 + \dot{x} l) m_P \hat{b}_3 + m_P x^2\Omega \hat{b}_3$$

$$\boxed{{}^{\mathcal{I}}\vec{h}_{P/O} = m_P(x^2\Omega - \dot{x}l + \Omega l^2)\,\hat{b}_3}$$ Note: $\hat{b}_3 = \hat{e}_3$

$${}^{\mathcal{I}}\frac{d}{dt}\left({}^{\mathcal{I}}\vec{h}_{P/O}\right) = \vec{M}_{P/O} = \vec{r}_{P/O} \times \vec{F}_P$$

For ${}^{\mathcal{I}}\vec{h}_{P/O}$ to be conserved, $\vec{M}_{P/O} = 0$.

$$\vec{M}_{P/O} = (l\hat{b}_2 + x\hat{b}_1) \times (N\hat{b}_2 - Kx\hat{b}_1)$$

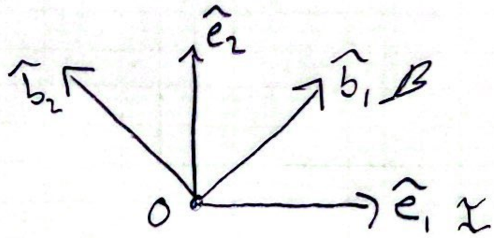
$$\vec{M}_{P/O} = Kxl\,\hat{b}_3 + xN\hat{b}_3 \qquad N = m_P(2\Omega\dot{x} - l\Omega^2)$$

$$\vec{M}_{P/O} = Kxl + m_P x(2\Omega\dot{x} - l\Omega^2) \neq 0$$

$\therefore$ angular momentum is not conserved.

8.10)

<u>Reference Frames:</u>

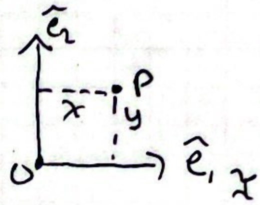


$\mathcal{I} = (O, \hat{e}_1, \hat{e}_2, \hat{e}_3)$

$\mathcal{B} = (O, \hat{b}_1, \hat{b}_2, \hat{b}_3)$

$\hat{e}_3 = \hat{b}_3$

a) <u>Coordinate System:</u>



$(x, y)_{\mathcal{I}}$

<u>Kinematics:</u> $\vec{r}_{P/O} = x\hat{e}_1 + y\hat{e}_2$

$^{\mathcal{I}}\vec{v}_{P/O} = \dot{x}\hat{e}_1 + \dot{y}\hat{e}_2$

$^{\mathcal{I}}\vec{a}_{P/O} = \ddot{x}\hat{e}_1 + \ddot{y}\hat{e}_2$

<u>N2L:</u> $\vec{F}_P = m_P\, {^{\mathcal{I}}\vec{a}_{P/O}} = 0 \quad = m_P(\ddot{x}\hat{e}_1 + \ddot{y}\hat{e}_2)$

<u>Equations of motion:</u>

$$\boxed{\begin{aligned} \hat{e}_1:&\quad \ddot{x} = 0 \\ \hat{e}_2:&\quad \ddot{y}_1 = 0 \end{aligned}}$$

Solution of $\ddot{x} = 0$: $\quad \frac{d\dot{x}}{dt} = 0 \Rightarrow \int d\dot{x} = \int_0^t 0\,dt$

$= \dot{x} = \frac{dx}{dt} + C_1 = 0 \quad \Rightarrow \int dx + \int_0^t C_1 dt = \int_0^t 0\,dt$

$\Rightarrow x(t) + C_1 t + C_2 = 0$

Similarily it can be shown the solution of $\ddot{y} = 0$ is:

$y(t) + C_1 t + C_2 = 0.$

Assuming $\mathcal{B}$ is aligned with $\mathcal{I}$ at $t=0$ & ${}^{\mathcal{B}}\vec{v}_{P/O} = -1\hat{b}_1$ m/s and P starts at $R = 1$ m, gives IC's as follows:

$x(0) = 1\,\hat{e}_1$ m $\quad \dot{x}(0) = -1\,\hat{e}_1$ m/s

$\dot{y}(0) = 0\,\hat{e}_2$ m/s $\quad y(0) = 0\,\hat{e}_2$ m

$x(0) + C_1(0) + C_2 = 0 \quad \Rightarrow \quad C_2 = -1 \qquad y(0) + C_1(0) + C_2 = 0 \Rightarrow C_2 = 0$

$\dot{x}(0) + C_1 = 0 \quad \Rightarrow \quad C_1 = 1 \qquad \dot{y}(0) + C_1 \Rightarrow C_1 = 0$

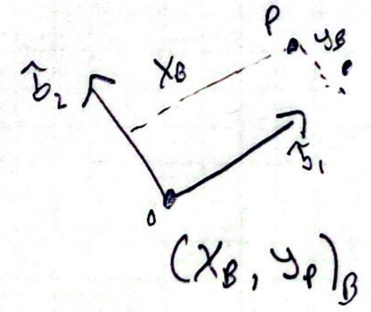
$x(t) = -t + 1, \quad y(t) = 0$

$\therefore \quad \vec{r}_{P/O} = x\,\hat{e}_1 + y\,\hat{e}_2 = (-t+1)\,\hat{e}_1$

See Matlab for simulation:

c)

Coordinate System:



Kinematics: $\vec{r}_{P/O} = x_B \hat{b}_1 + y_B \hat{b}_2$

$${}^{I}\vec{v}_{P/O} = \frac{{}^{B}d}{dt}(\vec{r}_{P/O}) + {}^{I}\vec{\omega}^{B} \times \vec{r}_{P/O}$$

$$= (\dot{x}_B \hat{b}_1 + \dot{y}_B \hat{b}_2) + (\Omega \hat{b}_3 \times [x_B \hat{b}_1 + y_B \hat{b}_2])$$

$${}^{I}\vec{v}_{P/O} = (\dot{y}_B + x_B \Omega)\hat{b}_2 + (\dot{x}_B - \Omega y_B)\hat{b}_1$$

$${}^{I}\vec{a}_{P/O} = \frac{{}^{B}d}{dt}({}^{I}\vec{v}_{P/O}) + {}^{I}\vec{\omega}^{B} \times {}^{I}\vec{v}_{P/O}$$

$$= (\ddot{y}_B + \dot{x}_B \Omega)\hat{b}_2 + (\ddot{x}_B - \dot{y}_B \Omega)\hat{b}_1$$
$$+ \Omega \hat{b}_3 \times ([\dot{y}_B + x_B \Omega]\hat{b}_2 + [\dot{x}_B - \Omega y_B]\hat{b}_1)$$

$$= (\ddot{y}_B + \dot{x}_B \Omega)\hat{b}_2 + (\ddot{x}_B - \dot{y}_B \Omega)\hat{b}_1 - (\dot{y}_B \Omega + x_B \Omega^2)\hat{b}_1$$
$$+ (\dot{x}_B \Omega - \Omega^2 y_B)\hat{b}_2$$

$${}^{I}\vec{a}_{P/O} = (\ddot{x}_B - 2\dot{y}_B \Omega - x_B \Omega^2)\hat{b}_1 + (\ddot{y}_B + 2\dot{x}_B \Omega - y_B \Omega^2)\hat{b}_2$$

Equations of motion: $\vec{F}_P = 0 = m_P \, {}^{I}\vec{a}_{P/O} \Rightarrow {}^{I}\vec{a}_{P/O} = 0$ (N2L)

$$\boxed{\begin{aligned} \hat{b}_1 &: \ddot{x}_B = 2\dot{y}_B \Omega + x_B \Omega^2 \\ \hat{b}_2 &: \ddot{y}_B = -2\dot{x}_B \Omega + y_B \Omega^2 \end{aligned}}$$

D) Known Initial Conditions: $x_B(0) = R$, $y_B(0) = 0$

$\dot{x}_B(0) = 1\ m/s$, $\dot{y}_B(0) = 0$

Let $\vec{z} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} = \begin{pmatrix} x_B \\ y_B \\ \dot{x}_B \\ \dot{y}_B \end{pmatrix}$

$\therefore \quad \dot{\vec{z}} = \begin{pmatrix} z_3 \\ z_4 \\ 2z_4\Omega + z_1\Omega^2 \\ -2z_3\Omega + z_2\Omega^2 \end{pmatrix}$

See MATLAB for simulation

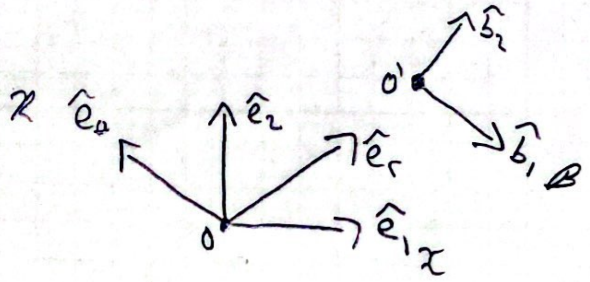
9.6)

Reference Frame:

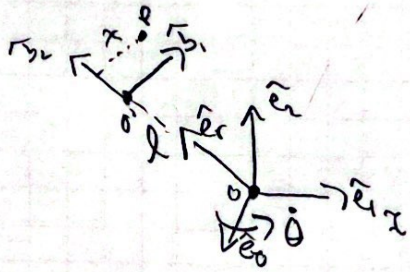
$\mathcal{I} = (O, \hat{e}_1, \hat{e}_2, \hat{e}_3)$

$\mathcal{R} = (O, \hat{e}_r, \hat{e}_\theta, e_3)$

$\mathcal{B} = (O', \hat{b}_1, \hat{b}_2, \hat{b}_3)$



Coordinate System:



${}^{\mathcal{I}}\vec{\omega}^{\mathcal{B}} = \dot{\theta}\hat{b}_3$

Kinematics: $\vec{r}_{P/O} = l\hat{e}_r + x\hat{b}_1 = x\hat{b}_1 + l\hat{b}_2$

$${}^{\mathcal{I}}\vec{v}_{P/O} = {}^{\mathcal{B}}\frac{d}{dt}(\vec{r}_{P/O}) + {}^{\mathcal{I}}\vec{\omega}^{\mathcal{B}} \times \vec{r}_{P/O}$$

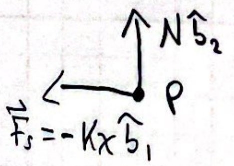
$${}^{\mathcal{I}}\vec{v}_{P/O} = \dot{x}\hat{b}_1 + \dot{\theta}x\hat{b}_2 - \dot{\theta}l\hat{b}_1$$

$${}^{\mathcal{I}}\vec{a}_{P/O} = {}^{\mathcal{B}}\frac{d}{dt}((\dot{x} - \dot{\theta}l)\hat{b}_1 + \dot{\theta}x\hat{b}_2) + \dot{\theta}\hat{b}_3 \times [(\dot{x} - \dot{\theta}l)\hat{b}_1 + \dot{\theta}x\hat{b}_2]$$

$${}^{\mathcal{I}}\vec{a}_{P/O} = (\ddot{x} - \ddot{\theta}l)\hat{b}_1 + (\ddot{\theta}x + \dot{\theta}\dot{x})\hat{b}_2 + (\dot{x}\dot{\theta} - \dot{\theta}^2 l)\hat{b}_2 - (\dot{\theta}^2 x)\hat{b}_1$$

$${}^{\mathcal{I}}\vec{a}_{P/O} = (\ddot{x} - \ddot{\theta}l - \dot{\theta}^2 x)\hat{b}_1 + (\ddot{\theta}x + 2\dot{\theta}\dot{x} - \dot{\theta}^2 l)\hat{b}_2$$

FBD:



a)

N2L: (Scalar form) $\vec{F}_P = m_P \, {}^{\mathcal{I}}\vec{a}_{P/O} = N\hat{b}_2 - Kx\hat{b}_1$

$\hat{b}_1$: $-Kx = (\ddot{x} - \ddot{\theta}l - \dot{\theta}^2 x)m$

$\hat{b}_2$: $-N = (\ddot{\theta}x + 2\dot{\theta}\dot{x} - \dot{\theta}^2 l)m$ (Constraint force)

$\Rightarrow$

$$\boxed{\ddot{x} = \ddot{\theta}l + \dot{\theta}^2 x - \frac{Kx}{m}}$$

Total Moment about O: $M_O = \sum r_{i/O} \times F_i^{(ext)}$

$M_O = \vec{r}_{P/O} \times \vec{F}_P = (x\hat{b}_1 + l\hat{b}_2) \times (-|\vec{F}_s|\hat{b}_1 + N\hat{b}_2)$

$= xN\hat{b}_3 + l|\vec{F}_s|\hat{b}_3$

$= [xm(\ddot{\theta}x + 2\dot{\theta}\dot{x} - \dot{\theta}^2 l) + lKx]\hat{b}_3$

$M_O = I_G \, {}^{\mathcal{I}}\alpha^B = -I\ddot{\theta}\hat{b}_3$

$-I\ddot{\theta} = mx^2\ddot{\theta} + 2\dot{\theta}\dot{x}xm - \dot{\theta}^2 lmx + lKx$ $\quad I + mx^2$ (Parallel Axis theorem)

$\ddot{\theta}(-I - mx^2) = 2\dot{\theta}\dot{x}xm - \dot{\theta}^2 lmx + lKx$

$I - mx^2$

$$\ddot{\theta} = \frac{2\dot{\theta}\dot{x}xm - \dot{\theta}^2 lmx + lKx}{-I - mx^2} = \frac{-(-2\dot{\theta}\dot{x}xm + \dot{\theta}^2 lmx - lKx)}{-(I + mx^2)}$$

$$\boxed{\ddot{\theta} = \frac{-2\dot{\theta}\dot{x}xm + \dot{\theta}^2 lmx - lKx}{I + mx^2}}$$

Scanned with CamScanner

b) Energy is conserved as there are no non-conservative forces acting.

c) $^{I}h_o = m\vec{r}_{P/O} \times {}^{I}\vec{v}_{P/O} + \vec{h}_B = m\vec{r}_{P/O} \times {}^{I}\vec{v}_{P/O} + I_B\, {}^{I}\vec{\omega}^{B}$

$^{I}h_o = m(x\hat{b}_1 + l\hat{b}_2) \times [(\dot{x} - \dot{\theta}l)\hat{b}_1 + \dot{\theta}x\hat{b}_2] + I\dot{\theta}\hat{b}_3$

$$\boxed{{}^{I}h_o = mx^2\dot{\theta} - ml\dot{x} + ml^2\dot{\theta} + I\dot{\theta}\ \hat{b}_3} \quad \hat{e}_3 = \hat{b}_3$$

$$^{I}\frac{d}{dt}({}^{I}h_o) = m\ddot{\theta}x^2 + m2x\dot{x}\dot{\theta} - ml\ddot{x} + ml^2\ddot{\theta} + I\ddot{\theta}$$

$$= mx^2\left(\frac{-2mx\dot{x}\dot{\theta} + mlx\dot{\theta}^2 - Klx}{I + mx^2}\right) + 2mx\dot{x}\dot{\theta} - ml[l\left(\frac{-2mx\dot{x}\dot{\theta} + mlx\dot{\theta}^2 - Klx}{I + mx^2}\right)$$

$$+ x\dot{\theta}^2 - \frac{Kx}{m}] + ml^2\left(\frac{-2mx\dot{x}\dot{\theta} + mlx\dot{\theta}^2 - Klx}{I + mx^2}\right) + I\ddot{\theta}$$

$$= mx^2\left(\frac{-2mx\dot{x}\dot{\theta} + mlx\dot{\theta}^2 - Klx}{I + mx^2}\right) + 2mx\dot{x}\dot{\theta} - mlx\dot{\theta}^2 - lKx + I\ddot{\theta}$$

$$= mx^2\left(\frac{-2m\dot{x}x\dot{\theta} + mlx\dot{\theta}^2 - Klx}{I + mx^2}\right) + I + mx^2\left(\frac{2mx\dot{x}\dot{\theta} - mlx\dot{\theta}^2 - lKx}{I + mx^2}\right) + I\ddot{\theta}$$

$$= mx^2\ddot{\theta} + (I + mx^2)(-\left(\frac{-2mx\dot{x}\dot{\theta} + mlx\dot{\theta}^2 + lKx}{I + mx^2}\right) + I\ddot{\theta}$$

$$= mx^2\ddot{\theta} - (I + mx^2)\ddot{\theta} + I\ddot{\theta} = mx^2\ddot{\theta} - I\ddot{\theta} - mx^2\ddot{\theta} + I\ddot{\theta}$$

$$^{I}\frac{d}{dt}({}^{I}h_o) = 0$$

$\therefore$ $\boxed{\text{Angular momentum is conserved}}$

$$\vec{Z} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} = \begin{pmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{pmatrix}$$

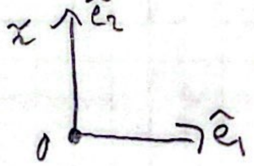
$$\dot{\vec{Z}} = \begin{pmatrix} z_2 \\ -2 z_2 z_3 z_4 m + z_2^2 lm z_3 - lK z_3 / (I + m z_3^2) \\ z_4 \\ \dot{z}_2 l + z_2^2 z_3 - \frac{K z_3}{m} \end{pmatrix}$$

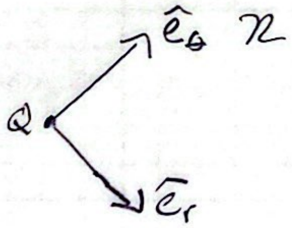
9.8)

Reference Frames:

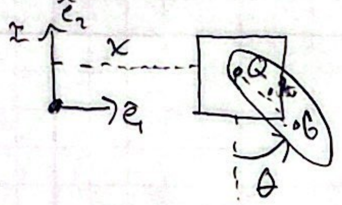
$\mathcal{I} = (O, \hat{e}_1, \hat{e}_2, \hat{e}_3)$

$\mathcal{R} = (Q, \hat{e}_r, \hat{e}_\theta, \hat{e}_3)$





Coordinates:



$\dot{\theta}\hat{e}_3 = {}^{\mathcal{I}}\vec{\omega}^{\mathcal{R}}$

Kinematics:

$\vec{r}_{Q/O} = x\hat{e}_1$ $\qquad$ $\vec{r}_{G/Q} = 0.25\hat{e}_r$

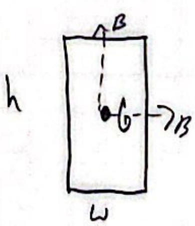
${}^{\mathcal{I}}\vec{v}_{Q/O} = \dot{x}\hat{e}_1$ $\qquad$ ${}^{\mathcal{I}}\vec{v}_{G/Q} = \frac{{}^{\mathcal{R}}d}{dt}(0.25\hat{e}_r) + \dot{\theta}\hat{e}_3 \times .25\hat{e}_r$

${}^{\mathcal{I}}\vec{a}_{Q/O} = \ddot{x}\hat{e}_1$ $\qquad$ ${}^{\mathcal{I}}\vec{v}_{G/Q} = .25\dot{\theta}\hat{e}_\theta$

${}^{\mathcal{I}}\vec{a}_{G/Q} = .25\ddot{\theta}\hat{e}_\theta - .25\dot{\theta}^2\hat{e}_r$

${}^{\mathcal{I}}\vec{a}_{G/O} = -.25\dot{\theta}^2\hat{e}_r + .25\ddot{\theta}\hat{e}_\theta + \ddot{x}\hat{e}_1$

Moment of Inertia:



h = .6m

w = .3m

m = 2Kg

$I_G = \rho\int_{-h/2}^{h/2}\int_{-w/2}^{w/2}(x^2+y^2)\,dx\,dy$

$= \rho\int_{-h/2}^{h/2}\left[\frac{x^3}{3} + y^2x\Big|_{-w/2}^{w/2} dy\right]$

$= \rho\int_{-h/2}^{h/2}\left[\frac{(\frac{w}{2})^3}{3} + y^2(\frac{w}{2}) + \frac{(\frac{w}{2})^3}{3} + y^2(\frac{w}{2})\right]dy$

$$= \rho \int_{-h/2}^{h/2} \frac{2w^3}{24} + y^2 w \, dy$$

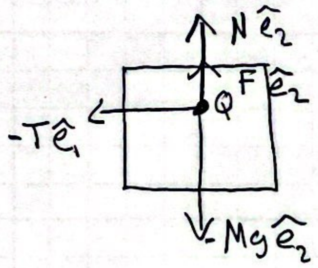
$$= \rho \left[\frac{w^3}{12} y + \frac{y^3}{3} w \right] \Big|_{-h/2}^{h/2} = \rho \left[\frac{w^3 \frac{h}{2}}{12} + \frac{(\frac{h}{2})^3 w}{3} + \frac{w^3 \frac{h}{2}}{12} + \frac{(\frac{h}{2})^3 w}{3} \right]$$

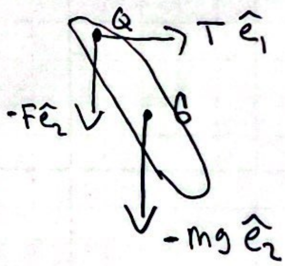
$$= \rho \left[\frac{w^3 h}{12} + \frac{h^3 w}{12} \right]$$

$$\rho = \frac{m}{wh}$$

$$\therefore \quad I_G = \frac{mw^2}{12} + \frac{mh^2}{12}$$

FBD:





Eulers First law: Mass, M $\quad \vec{F}_Q = M \, {}^{\mathcal{I}}\vec{a}_{Q/O}$

$$N\hat{e}_2 + F\hat{e}_2 - T\hat{e}_1 - Mg\hat{e}_2 = m\ddot{x}\,\hat{e}_1$$

$$\hat{e}_1 : m\ddot{x} = -T\,\hat{e}_1 \quad (1)$$

$$\hat{e}_2 : \quad N + F - Mg = 0 \quad (2)$$

Eulers First law: Mass, m $\quad \vec{F}_G = m \, {}^{\mathcal{I}}\vec{a}_{G/O}$

$$T\hat{e}_1 - F\hat{e}_2 - mg\hat{e}_2 = (-0.25\dot{\theta}^2 \hat{e}_r + .25\ddot{\theta}\hat{e}_\theta + \ddot{x}\hat{e}_1)m$$

$$\begin{array}{c|cc} & \hat{e}_r & \hat{e}_\theta \\ \hline \hat{e}_1 & S\theta & C\theta \\ \hat{e}_2 & -C\theta & S\theta \end{array}$$

Scanned with CamScanner

$\hat{e}_1$: $T = (-0.25\dot{\theta}^2 \sin\theta + .25\ddot{\theta}\cos\theta + \ddot{x})\, m$ (3)

$\hat{e}_2$: $-F - mg = (0.25\dot{\theta}^2\cos\theta + .25\ddot{\theta}\sin\theta)\, m$ (4)

Eulers 2^{nd} law about arbitrary point Q:

$$I_Q \,{}^{I}\alpha^{R} = \vec{M}_Q + \vec{r}_{Q/G} \times m_G \,{}^{I}\vec{a}_{Q/O}$$

$$\vec{r}_{Q/G} = -0.25\,\hat{e}_r$$

$$I_Q = I_G + m_G \|\vec{r}_{Q/G}\|^2 = I_G + m_G\left(\sqrt{(-.25)^2}\right)^2$$

$$I_Q = \frac{mw^2}{12} + \frac{mh^2}{12} + m_G(.25)^2$$

$${}^{I}\alpha^{R} = \ddot{\theta}\,\hat{e}_3$$

$$M_Q = \Sigma\, \vec{r}_{i/Q} \times F_i = 0.25\,\hat{e}_r \times -mg\,\hat{e}_2 = 0.25\,\hat{e}_r \times -mg(-\cos\theta\,\hat{e}_r + \sin\theta\,\hat{e}_\theta)$$

$$\vec{M}_Q = -0.25\,mg\sin\theta\,\hat{e}_3$$

$$\vec{r}_{Q/G} \times m_G \,{}^{I}\vec{a}_{Q/O} = -0.25\,\hat{e}_r \times m_G\,\ddot{x}\,\hat{e}_1 = -0.25\,\hat{e}_r \times m_G\ddot{x}(\sin\theta\,\hat{e}_r + \cos\theta\,\hat{e}_\theta)$$

$$= -0.25\, m\ddot{x}\cos\theta\,\hat{e}_3$$

$$\ddot{\theta}\, I_Q = -0.25\, mg\sin\theta - 0.25\, m\ddot{x}\cos\theta$$

$$\ddot{\theta} = \frac{-0.25m(g\sin\theta + \ddot{x}\cos\theta)}{I_Q} \quad (5)$$

Combine (1) & (3):

$$m\ddot{x} = -(-0.25\dot{\theta}^2\sin\theta + .25\ddot{\theta}\cos\theta + \ddot{x})m$$

$$2\ddot{x} = 0.25\dot{\theta}^2\sin\theta - .25\ddot{\theta}\cos\theta$$

Insert (5):

$$2\ddot{x} = 0.25\dot{\theta}^2\sin\theta - .25\cos\theta\left[\frac{-0.25mg\sin\theta - 0.25m\cos\theta\,\ddot{x}}{I_Q}\right]$$

$$2\ddot{x} - \frac{(0.25)^2\cos^2\theta\, m}{I_Q}\ddot{x} = 0.25\dot{\theta}^2\sin\theta + \frac{(0.25)^2 mg\sin\theta\cos\theta}{I_Q}$$

$$\boxed{\ddot{x}\left(2 - \frac{(0.25)^2\cos^2\theta\, m}{I_Q}\right) = 0.25\dot{\theta}^2\sin\theta + \frac{(0.25)^2 mg\sin\theta\cos\theta}{I_Q}}$$

$$\ddot{\theta} = \frac{-0.25mg\sin\theta}{I_Q} + \frac{\cos\theta}{I_Q}\left(\frac{.25\dot{\theta}^2\sin\theta + \frac{(.25)^2 mg\sin\theta\cos\theta}{I_Q}}{2 - \frac{(0.25)^2\cos^2\theta\, m}{I_Q}}\right)$$

$$\ddot{\theta} = \frac{-0.25mg\sin\theta}{I_Q} + \frac{.25\dot{\theta}^2\sin\theta + \left(\frac{(.25)^2 mg\sin\theta\cos\theta}{I_Q}\right)}{2 - (0.25)^2\cos\theta\, m}$$

$$\boxed{\ddot{\theta} = \frac{-0.25mg\sin\theta}{I_Q} + \frac{0.25\dot{\theta}^2\sin\theta}{2-(0.25)^2\cos\theta m} + \frac{(0.25)^2 mg\sin\theta\cos\theta}{I_Q[2-(0.25)^2\cos\theta m]}}$$

$$\boxed{\ddot{x} = \frac{\left[0.25\,\dot{\theta}^2 \sin\theta + \dfrac{(0.25)^2 mg \sin\theta \cos\theta}{I_Q} \right]}{\left(2 - \dfrac{(0.25)^2 \cos^2\theta \; m}{I_Q} \right)}}$$

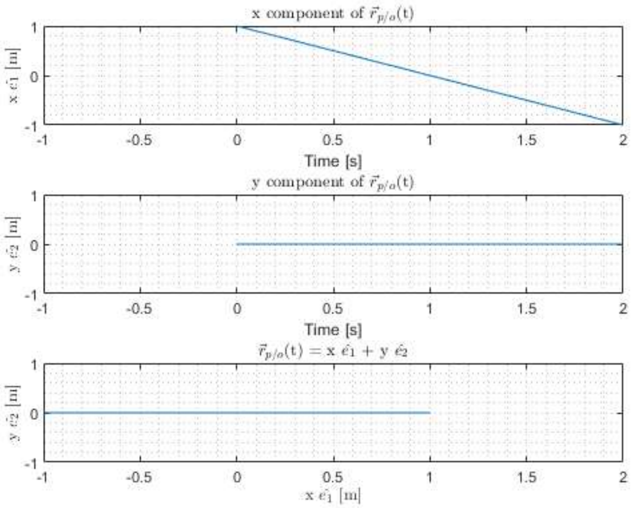
Contents

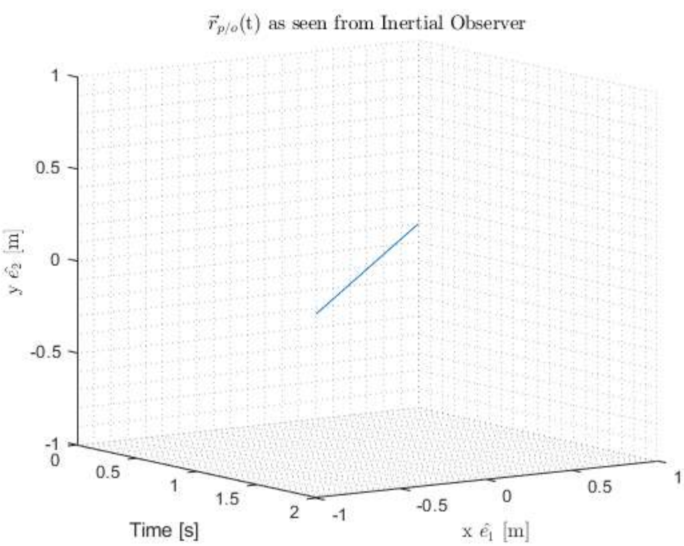
MAE 562 HW5 Gabriel Colangelo 50223306

```
clear
close all
clc
```

Problem 8.10.B

```
t           = (0:0.01:2)';          % Time [s]
x           = -t + 1;               % component of r_p/o in e1 [m], derived from xdotdot = 0
y           = zeros(length(t),1);   % component of r_p/o in e2 [m], derived from ydotdot = 0

figure()
ax1         = subplot(3,1,1);
plot(t,x)
ylabel('x $\hat{e_1}$ [m]','Interpreter','latex')
xlabel('Time [s]')
title('x component of $\vec{r}_{p/o}$(t)','Interpreter','latex')
grid minor

ax2         = subplot(3,1,2);
plot(t,y)
ylabel('y $\hat{e_2}$ [m]','Interpreter','latex')
xlabel('Time [s]')
title('y component of $\vec{r}_{p/o}$(t)','Interpreter','latex')
grid minor

ax3         = subplot(3,1,3);
plot(x,y)
ylabel('y $\hat{e_2}$ [m]','Interpreter','latex')
xlabel('x $\hat{e_1}$ [m]','Interpreter','latex')
title('$\vec{r}_{p/o}$(t) = x $\hat{e_1}$ + y $\hat{e_2}$','Interpreter','latex')
grid minor

linkaxes([ax1 ax2 ax3],'x')

figure()
plot3(t,x,y)
grid minor
xlabel('Time [s]')
ylabel('x $\hat{e_1}$ [m]','Interpreter','latex')
zlabel('y $\hat{e_2}$ [m]','Interpreter','latex')
title('$\vec{r}_{p/o}$(t) as seen from Inertial Observer','Interpreter','latex')
view(55,10);
```

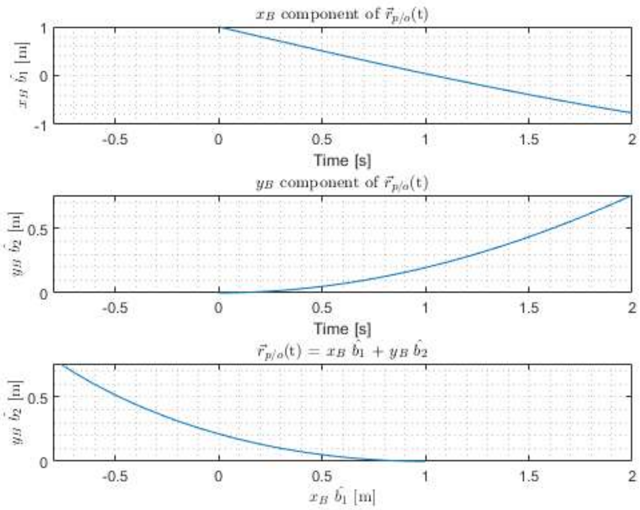


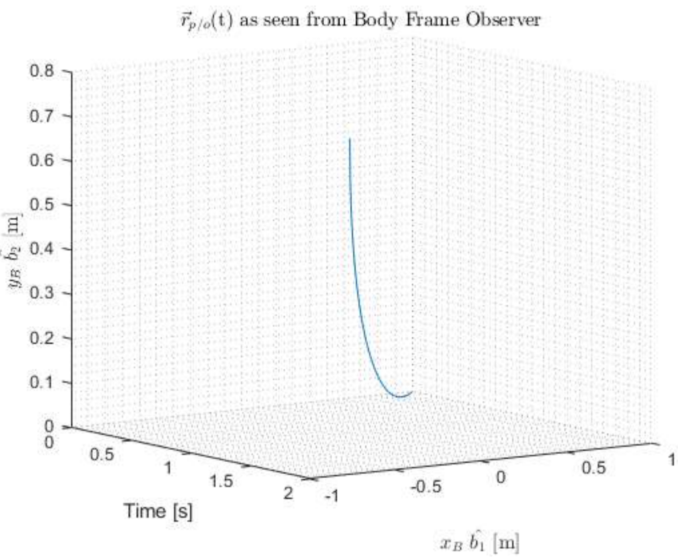


Problem 8.10.D

```
Omega   = 0.2                                                      ; % Angular velocity IwB [rad/s] about b3 = e3

IC1      = [1 0 -1 0]'                                             ; % Initial conditions of xb = R = 1 [m] and v0 = -1 [m/s] b1
options = odeset('AbsTol',1e-8,'RelTol',1e-8)                      ; % ODE45 solver options

[T,Z1] = ode45(@(t,z) BodyParticle(t,z,Omega),t,IC1,options)       ; % Simulation

figure()
ax1         = subplot(3,1,1);
plot(t,Z1(:,1))
ylabel('$x_B$ $\hat{b_1}$ [m]','Interpreter','latex')
xlabel('Time [s]')
title('$x_B$ component of $\vec{r}_{p/o}$(t)','Interpreter','latex')
grid minor

ax2         = subplot(3,1,2);
plot(t,Z1(:,2))
ylabel('$y_B$ $\hat{b_2}$ [m]','Interpreter','latex')
xlabel('Time [s]')
title('$y_B$ component of $\vec{r}_{p/o}$(t)','Interpreter','latex')
grid minor

ax3         = subplot(3,1,3);
plot(Z1(:,1),Z1(:,2))
ylabel('$y_B$ $\hat{b_2}$ [m]','Interpreter','latex')
xlabel('$x_B$ $\hat{b_1}$ [m]','Interpreter','latex')
title('$\vec{r}_{p/o}$(t) = $x_B$ $\hat{b_1}$ + $y_B$ $\hat{b_2}$','Interpreter','latex')
grid minor

linkaxes([ax1 ax2 ax3],'x')

figure()
plot3(t,Z1(:,1),Z1(:,2))
grid minor
```

```
xlabel('Time [s]')
ylabel('$x_B$ $\hat{b_1}$ [m]','Interpreter','latex')
zlabel('$y_B$ $\hat{b_2}$ [m]','Interpreter','latex')
title('$\vec{r}_{p/o}$(t) as seen from Body Frame Observer','Interpreter','latex')
view(55,10);
```

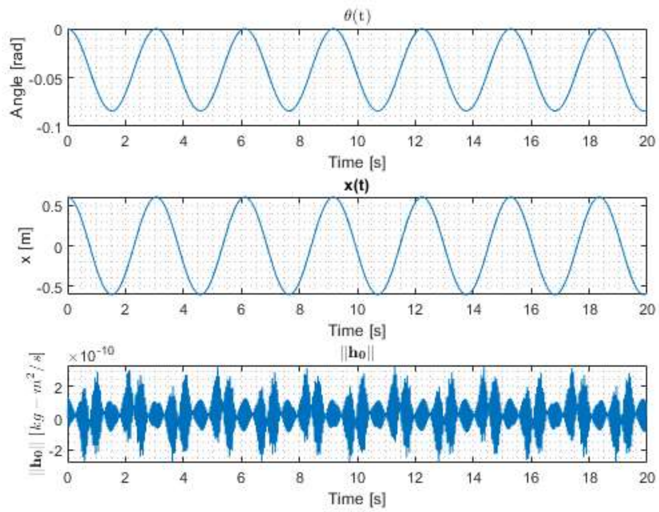




Problem 9.6

```
r           = 1             ; % radius of disk [m]
I           = 2.5           ; % Moment of Inertia [kg-m^2]
l           = .75           ; % offset [m]
m           = 0.25          ; % mass of particle m [kg]
k           = 1             ; % spring constant [N/m]
time        = (0:.01:20)'   ; % time [s]
IC1         = [0 0 .6 0]    ; % IC of x = 60 cm

[T,Z2]      = ode45(@(t,z) SlottedDisk(t,z,m,l,k,I),time,IC1,options);

x           = Z2(:,3);
theta       = Z2(:,1);
xdot        = Z2(:,4);
thetadot    = Z2(:,2);

h0          = m.*thetadot.*x.^2 - m*l.*xdot + m*l^2.*thetadot + I.*thetadot;

figure()
ax1         = subplot(3,1,1);
plot(time,Z2(:,1))
xlabel('Time [s]')
ylabel('Angle [rad]')
grid minor
title('$\theta$(t)','Interpreter','latex')

ax2         = subplot(3,1,2);
plot(time,Z2(:,3))
```

```
xlabel('Time [s]')
ylabel('x [m]')
grid minor
title('x(t)')

ax3         = subplot(3,1,3);
plot(time,h0)
xlabel('Time [s]')
ylabel('$\| \mathbf{h_0} \|$ $[kg-m^2/s]$','Interpreter','latex')
grid minor
title('$\| \mathbf{h_0} \|$','Interpreter','latex')
```



Functions

```
function zdot  = BodyParticle(t,z,Omega)
z1          = z(1,1); % z1 = xB
z2          = z(2,1); % z2 = yB
z3          = z(3,1); % z3 = xBdot
z4          = z(4,1); % z4 = yBdot

% Equations of motion is first order form
zdot(1,1)   = z3;
zdot(2,1)   = z4;
zdot(3,1)   = 2*z4*Omega + z1*Omega^2;
zdot(4,1)   = -2*z3*Omega + z2*Omega^2;
end

function zdot  = SlottedDisk(t,z,m,l,k,I)
z1          = z(1,1); % z1 = theta
z2          = z(2,1); % z2 = thetadot
z3          = z(3,1); % z3 = x
z4          = z(4,1); % z4 = xdot

% Equations of motion is first order form
zdot(1,1)   = z2;
zdot(2,1)   = (-2*m*z3*z4*z2 + m*l*z3*z2^2 - k*l*z3)/(I + m*z3^2);
zdot(3,1)   = z4;
zdot(4,1)   = l*zdot(2,1) + z3*z2^2 - (k/m)*z3;
end
```

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