AAE 666

Homework Four

Exercise 1 Determine whether or not the following function is positive definite.

$$V(x) = x_1^4 - x_1^2 x_2 + x_2^2$$

Exercise 2 Show that the following system is AS about zero

$$\dot{x} = -(1 + \sin x)x$$

Exercise 3 By appropriate choice of Lyapunov function, show that the origin is an asymptotically stable equilibrium state for

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 + x_1^3 - x_2 \end{aligned}$$

Exercise 4 (Stabilization of the Duffing system.) Consider the Duffing system with a scalar control input $u(t)$:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_1 - x_1^3 + u \end{aligned}$$

Obtain a linear controller of the form

$$u = -k_1 x_1 - k_2 x_2$$

which results in a closed loop system which is GAS about the origin. Numerically simulate the open loop system ($u = 0$) and the closed loop system for several initial conditions.

Exercise 5 Determine whether or not the the following function is radially unbounded.

$$V(x) = x_1 - x_1^3 + x_1^4 - x_2^2 + x_2^4$$

Exercise 6 (Forced Duffing's equation with damping.) Show that all solutions of the system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_1 - x_1^3 - cx_2 + 1 \qquad c > 0 \end{aligned}$$

are bounded.
Hint: Consider

$$V(x) = \frac{1}{2}\lambda c^2 x_1^2 + \lambda c x_1 x_2 + \frac{1}{2}x_2^2 - \frac{1}{2}x_1^2 + \frac{1}{4}x_1^4$$

where $0 < \lambda < 1$. Letting

$$P = \frac{1}{2}\begin{bmatrix} \lambda c^2 & \lambda c \\ \lambda c & 1 \end{bmatrix}$$

note that $P > 0$ and

$$\begin{aligned} V(x) &= x^T P x - \frac{1}{2}x_1^2 + \frac{1}{4}x_1^4 \\ &\geq x^T P x - \frac{1}{4} \end{aligned}$$

Exercise 7 Show that all solutions of

$$\dot{x} = \cos x - x^3 + 100$$

are bounded.

Exercise 8 Show that all solutions of

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \cos x_1 - x_1^3 + 100 \end{aligned}$$

are bounded.

Exercise 9 Show that the following system is GES about zero

$$\dot{x} = -(2 + \sin x)x$$

Give a rate of convergence.

Exercise 10 Show that the following system is GES about 1.

$$\dot{x} = -(2 + \sin x)(x - 1)$$

Give a rate of convergence.

Exercise 11 Show that the following system is GES about the zero state.

$$\begin{aligned} \dot{x}_1 &= -x_1 + (I_2 - I_3)x_2 x_3 \\ \dot{x}_2 &= -2x_2 + (I_3 - I_1)x_1 x_3 \\ \dot{x}_3 &= -3x_3 + (I_1 - I_2)x_1 x_2 \end{aligned}$$

where I_2, I_2, I_3 are arbitrary constants. Give a rate of convergence.