ECE 68000: MODERN AUTOMATIC CONTROL

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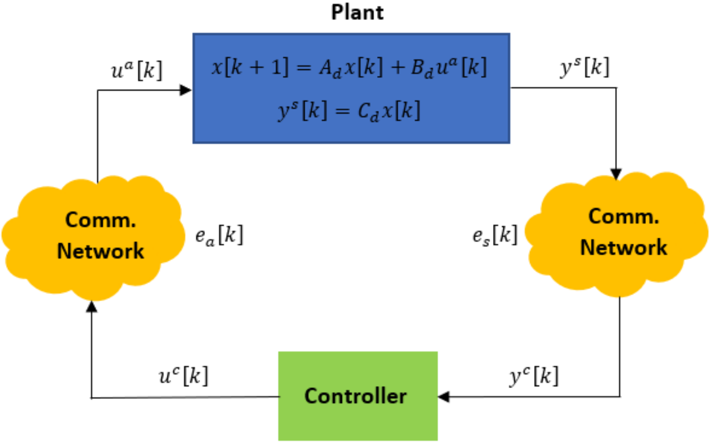
Modeling Networked Control Systems Corrupted by Unknown Input and Output Sparse Errors

Networked Control System Corrupted by Unknown Input and Output Sparse Errors



Plant Design Model

$$\left.\begin{array}{rcl}\boldsymbol{x}[k+1] & = & \boldsymbol{A}\boldsymbol{x}[k]+\boldsymbol{B}\boldsymbol{u}^a[k] \\ \boldsymbol{y}^s[k] & = & \boldsymbol{C}\boldsymbol{x}[k]\end{array}\right\}$$

where

- $\boldsymbol{A} \in \mathbb{R}^{n\times n}$, $\boldsymbol{B} \in \mathbb{R}^{n\times m}$, $\boldsymbol{C} \in \mathbb{R}^{p\times n}$
- $\boldsymbol{B}$ full column rank, that is, $\operatorname{rank} \boldsymbol{B} = m$
- $\boldsymbol{u}^a[k] \in \mathbb{R}^m$—input received by actuators
- $\boldsymbol{y}^s[k] \in \mathbb{R}^p$—output measured by sensors

Modeling Malicious Attacks on Sensors

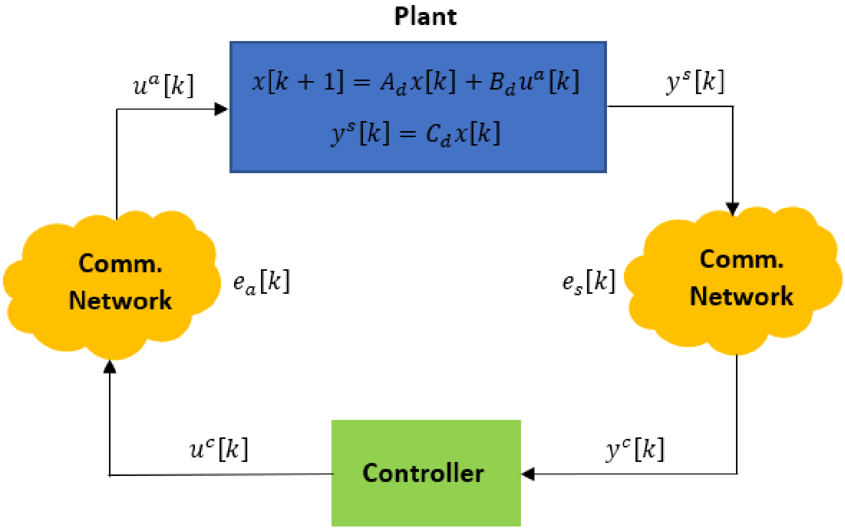
- Sensor measurements, $\boldsymbol{y}^s[k]$, are being sent to the controller through a communication network
- Malicious attacks cause packet drops in the communication network
- Malicious packet drops model:

$$\boldsymbol{\Gamma}(k) = \mathrm{diag}\{\gamma_1(k), \gamma_2(k), \cdots, \gamma_p(k)\}$$

  where $\gamma_i(k), i = 1, \ldots, p$ are Boolean variables, $\gamma_i(k) = 1$ if the packet is correctly received; $\gamma_i(k) = 0$ if the packet is dropped
- Signal received by the controller:

$$\boldsymbol{y}^c[k] = \boldsymbol{\Gamma}(k)\boldsymbol{y}^s[k]$$

NCS


Plant
$x[k+1] = A_d x[k] + B_d u^a[k]$
$y^s[k] = C_d x[k]$
$u^a[k]$
$y^s[k]$
Comm. Network
$e_a[k]$
$e_s[k]$
Comm. Network
$u^c[k]$
Controller
$y^c[k]$

Modeling Malicious Attacks on Actuators

- The control signal is being sent to the plant through a communication network
- Malicious packet drops model:

$$\mathbf{\Lambda}(k) = \mathrm{diag}\{\lambda_1(k), \lambda_2(k), \cdots, \lambda_m(k)\}$$

  where $\lambda_i(k), i = 1, \ldots, m$ are Boolean variables, $\lambda_i(k) = 1$ if the packet is correctly received; $\lambda_i(k) = 0$ if the packet is dropped by the actuator
- Signal received by the actuator:

$$\boldsymbol{u}^a[k] = \mathbf{\Lambda}(k)\boldsymbol{u}^c[k]$$

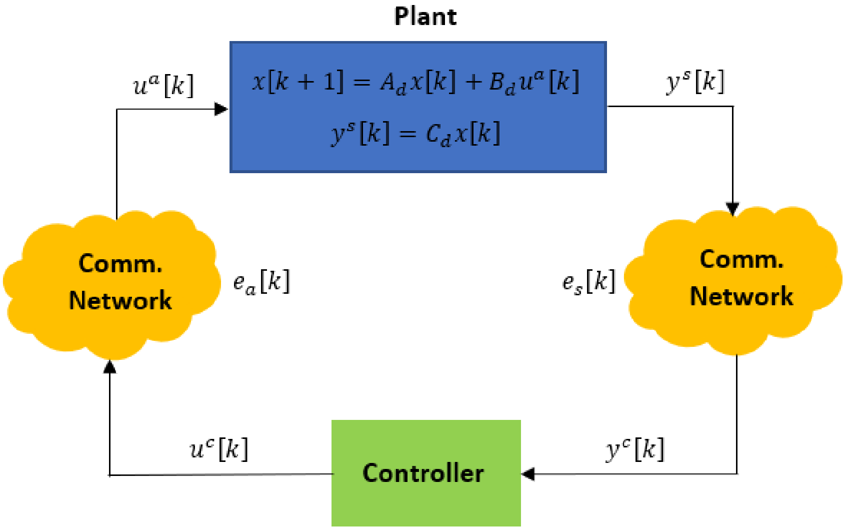
Errors in communication between sensors and the controller

- Network communication errors in the communication flow from the sensor to the controller—$\boldsymbol{e}_s[k]$
- Hence,
$$\boldsymbol{e}_s[k] = \boldsymbol{y}^c[k] - \boldsymbol{y}^s[k] \in \mathbb{R}^p$$

Errors in communication between the controller and actuators

- Errors in the communication between the controller to the actuator—$\boldsymbol{e}_a[k]$
- Hence,
$$\boldsymbol{e}_a[k] = \boldsymbol{u}^a[k] - \boldsymbol{u}^c[k] \in \mathbb{R}^m$$

NCS considered



NCS model

- Let $\overline{\mathbf{\Gamma}}(k) = \mathbf{\Gamma}(k) - \boldsymbol{I}_p \in \mathbb{R}^{p \times p}$ and $\overline{\mathbf{\Lambda}}(k) = \mathbf{\Lambda}(k) - \boldsymbol{I}_m \in \mathbb{R}^{m \times m}$
- Then
$$\boldsymbol{e}_s[k] = \overline{\mathbf{\Gamma}}(k)\boldsymbol{y}^s[k] \text{ and } \boldsymbol{e}_a[k] = \overline{\mathbf{\Lambda}}(k)\boldsymbol{u}^c[k]$$
- We analyze the case when malicious packet drops are sparse
- The system model under consideration
$$\left.\begin{array}{rcl} \boldsymbol{x}[k+1] & = & \boldsymbol{A}\boldsymbol{x}[k] + \boldsymbol{B}(\boldsymbol{u}^c[k] + \boldsymbol{e}_a[k]) \\ \boldsymbol{y}^c[k] & = & \boldsymbol{C}\boldsymbol{x}[k] + \boldsymbol{e}_s[k] \end{array}\right\}$$
- **Objective**: obtain an estimate of the state $\boldsymbol{x}[k]$ of the NCS in the presence of malicious packet drops $\boldsymbol{e}_s[k]$ and $\boldsymbol{e}_a[k]$

An alternative approach to the problem

- Plant linear model

$$\left.\begin{array}{rcl} \boldsymbol{x}[k+1] & = & \boldsymbol{A}\boldsymbol{x}[k] + \boldsymbol{B}(\boldsymbol{u}^c[k] + \boldsymbol{e}_a[k]) \\ \boldsymbol{y}^c[k] & = & \boldsymbol{C}\boldsymbol{x}[k] + \boldsymbol{e}_s[k] \end{array}\right\}$$

- Communication links subject to attacks
  - $\boldsymbol{e}_a[k]$—sparse attacks injected in the actuators
  - $\boldsymbol{e}_s[k]$—sparse attacks injected in the sensors
- **Objective**: correctly estimate the initial state

H. Fawzi, P. Tabuada, S. Diggavi, *Secure estimation and control for cyber-physical systems under adversarial attacks*, IEEE TAC, Vol. 59, No. 6, pp. 1454–1467, June 2014

Our Approach—Use State Observer