#1)

a) $A^T = A$

$A = \begin{pmatrix} a & d & e \\ d & b & f \\ e & f & c \end{pmatrix}$ $\dim(A_{3\times3}) = 6 = \frac{3^2+3}{2}$

$A = \begin{pmatrix} a & c \\ c & b \end{pmatrix}$ $\dim(A_{2\times2}) = 3 = \frac{2^2+2}{2}$

$$\boxed{\dim(A^T = A) = \frac{n^2+n}{2}}$$

b) $A^T = -A$

$A = \begin{pmatrix} 0 & -a & b \\ a & 0 & -c \\ -b & c & 0 \end{pmatrix}$ $\dim(A_{3\times3}) = 3 = \frac{3^2-3}{2}$

$A = \begin{pmatrix} 0 & -a \\ a & 0 \end{pmatrix}$ $\dim(A_{2\times2}) = 1 = \frac{2^2-2}{2}$

$$\boxed{\dim(A^T = A) = \frac{n^2-n}{2}}$$

c) $A = \begin{pmatrix} a & d & e \\ 0 & b & f \\ 0 & 0 & c \end{pmatrix}$ $\dim(A_{3\times3}) = 6$

$A = \begin{pmatrix} a & c \\ 0 & b \end{pmatrix}$ $\dim(A_{2\times2}) = 3$

$$\boxed{\dim(A_{ij} = 0 \quad i > j) = \frac{n^2+n}{2}}$$

#1)

d) $A = \begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix}$ $\dim(A_{3\times3}) = 3$

$A = \begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix}$ $\dim(A_{2\times2}) = 1$

$$\boxed{\dim(A_{ij} = 0 \ , \ i \geq j) = \frac{n^2 - n}{2}}$$

e) $A = \begin{pmatrix} a & b \\ c & -a \end{pmatrix}$ $\dim(A_{2\times2}) = 3 = 2^2 - 1$

$A = \begin{pmatrix} a & b & c \\ d & -a & g \\ e & f & b \end{pmatrix}$ $\dim(A_{3\times3}) = 8 = 3^2 - 1$

$$\boxed{\dim(A, \ \mathrm{trace}(A) = 0) = n^2 - 1}$$

#2)

$$A_1 = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 0 & -2 \\ 1 & 3 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 1 & -1 \\ 0 & -1 & -1 \\ 0 & 2 & 2 \end{pmatrix} \Rightarrow \begin{pmatrix} \boxed{1} & 1 & -1 \\ 0 & \boxed{-1} & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\boxed{\text{rank}(A_1) = 2}$$

$x_1 + x_2 - x_3 = 0$

$-x_2 - x_3 = 0$

x_3 is free, let $x_3 = 1 \therefore x_2 = -1$ & $x_1 = 2$

$$\boxed{N(A_1) = \text{span}\left\{ \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \right\}}$$ ← Null space A_1

$$\boxed{C(A_1) = \text{span}\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} \right\}}$$ ← Range A_1

rank $\neq 3$ & $N(A_1) \neq \vec{0}$ $\therefore$ A_1 is not onto or one to one.

$$\boxed{C(A_1^T) = \text{span}\left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} \right\}}$$ ← Range of A_1^T

$$A_1^T = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ -1 & -2 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & 2 \\ 0 & -1 & 2 \end{pmatrix} \Rightarrow \begin{pmatrix} \boxed{1} & 1 & 1 \\ 0 & \boxed{-1} & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\boxed{\text{rank}(A_1^T) = 2}$$

$x_1 + x_2 + x_3 = 0$

$-x_2 + 2x_3 = 0$ let $x_3 = 1$ $\therefore$ $x_2 = 2,\ x_1 = -3$

$$\boxed{N(A_1^T) = \text{span}\left\{ \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix} \right\}}$$

$$A_2 = \begin{pmatrix} 1 & 4 & 3 & 2 \\ 0 & -1 & -2 & 2 \\ 0 & 1 & 0 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} \boxed{1} & 4 & 3 & 2 \\ 0 & \boxed{-1} & -2 & 2 \\ 0 & 0 & \boxed{-2} & 3 \end{pmatrix}$$

$$\boxed{\text{rank}(A_2) = 3}$$

$$\boxed{C(A_2) = \text{span}\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix} \right\}}$$

$x_1 + 4x_2 + 3x_3 + 2x_4 = 0$

$-x_2 - 2x_3 + 2x_4 = 0$

$-2x_3 + 3x_4 = 0$

let $x_4 = 1$ $\therefore$ $x_3 = \frac{3}{2}$, $x_2 = -1$, $x_1 = -\frac{5}{2}$

$$\boxed{N(A_2) = \text{span}\left\{ \begin{pmatrix} -5/2 \\ -1 \\ 3/2 \\ 1 \end{pmatrix} \right\}}$$ ← Null space of A_2

$$\boxed{C(A_2^T) = \text{span}\left\{ \begin{pmatrix} 1 \\ 4 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ -2 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} \right\}}$$ ← Range of A_2^T

A_1^T is neither onto or one to one as $N(A_1^T) \neq 0$ & $\text{rank}(A_1^T) \neq 3$.

A_2 is onto as it's $\text{rank} = 3 = m$, but not one to one as $N(A_2) \neq 0$.

$$A_2^T = \begin{pmatrix} 1 & 0 & 0 \\ 4 & -1 & 1 \\ 3 & -2 & 0 \\ 2 & 2 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & -2 & 0 \\ 0 & 2 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -2 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \boxed{1} & 0 & 0 \\ 0 & \boxed{-1} & 1 \\ 0 & 0 & \boxed{-2} \\ 0 & 0 & 0 \end{pmatrix}$$

$$X_3 = 0$$
$$-X_2 + X_3 = 0$$
$$X_1 = 0$$

$$\therefore \boxed{N(A_2^T) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}}$$ ← Null space of A_2^T

$\text{rank}(A_2^T) = 3$

$\boxed{A_2^T \text{ is one to one but not onto as it's Null space} = \vec{0} \text{ but it's rank} \neq 4.}$

#3)

$$sI-A_c=\begin{pmatrix} s+a_1 & a_2 & & a_{n-1} & a_n \\ -1 & s & \cdots & 0 & 0 \\ 0 & -1 & s & 0 & 0 \\ \vdots & & & & \\ 0 & 0 & \cdots & -1 & s \end{pmatrix}$$

let $n=2$: $sI-A_{2\times2}=\begin{pmatrix} s+a_1 & a_2 \\ -1 & s \end{pmatrix}=(s+a_1)s+a_2$

$\det(sI-A_2)= s^2+a_1 s+a_2$

Let $n=3$: $sI-A_{3\times3}=\begin{pmatrix} s+a_1 & a_2 & a_3 \\ -1 & s & 0 \\ 0 & -1 & s \end{pmatrix}$

$(s+a_1)\begin{vmatrix} s & 0 \\ -1 & s \end{vmatrix} - a_2\begin{vmatrix} -1 & 0 \\ 0 & s \end{vmatrix} + a_3\begin{vmatrix} -1 & s \\ 0 & -1 \end{vmatrix}$

$(s+a_1)s^2 - a_2(-s) + a_3$

$\det = s^3+a_1 s^2+a_2 s+a_3$

For $n=n$: $(s+a_1)\underbrace{\begin{vmatrix} s & & \\ & \ddots & \\ & & s \end{vmatrix}}_{n-1} - a_2 \underbrace{\begin{vmatrix} -1 & & \\ & \ddots & \\ & & s \end{vmatrix}}_{n-2}\Big]^{n-2} + a_n$

$(s+a_1)s^{n-1} - a_2 - s^{n-2} + \ldots + a_n$

$$\boxed{\det(sI-A_c) = s^n+a_1 s^{n-1}+a_2 s^{n-2}+a_{n-1}s+a_n}$$

b) $A = \begin{pmatrix} 3 & -2 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$$\det(sI - A) = \begin{vmatrix} s-3 & 2 & 0 \\ -1 & s & 0 \\ 0 & -1 & s \end{vmatrix} = 0$$

$$= (s-3)\begin{vmatrix} s & 0 \\ -1 & s \end{vmatrix} - 2\begin{vmatrix} -1 & 0 \\ 0 & s \end{vmatrix} = (s-3)s^2 + 2s = 0$$

$s^3 - 3s^2 + 2s = 0 \qquad s(s^2 - 3s + 2) = 0$

$$\boxed{s_1 = 0, \quad s_2 = 2, \quad s_3 = 1}$$

c) $BK = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} (K_1 \ K_2 \ K_3) = \begin{pmatrix} K_1 & K_2 & K_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$$A - BK = \begin{pmatrix} 3-K_1 & -2-K_2 & -K_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\det(sI - (A - BK)) = 0$$

$$= \begin{vmatrix} s-3+K_1 & 2+K_2 & K_3 \\ -1 & s & 0 \\ 0 & -1 & s \end{vmatrix}$$

$$(s-3+K_1)\begin{vmatrix} s & 0 \\ -1 & s \end{vmatrix} - (2+K_2)\begin{vmatrix} -1 & 0 \\ 0 & s \end{vmatrix} + K_3\begin{vmatrix} -1 & s \\ 0 & -1 \end{vmatrix}$$

$$(s-3+K_1)s^2 + (s)(2+K_2) + K_3 = 0$$

$$s^3 - 3s^2 + K_1 s^2 + 2s + K_2 s + K_3 = 0$$

$$\lambda_1 = \lambda_2 = \lambda_3 = -1$$

$$(\lambda+1)^3 = (\lambda^2+2\lambda+1)(\lambda+1) = \lambda^3 + 2\lambda^2 + \lambda + \lambda^2 + 2\lambda + 1$$

$$(\lambda+1)^3 = \lambda^3 + 3\lambda^2 + 3\lambda + 1$$

$$(-3+K_1) = 3$$

$$(2+K_2) = 3$$

$$K_3 = 1$$

$$\boxed{K_1 = 6, \quad K_2 = 1, \quad K_3 = 1}$$

#4)

$$A = \begin{pmatrix} 0 & 1 & 3 \\ -2 & 1 & 1 \\ 2 & 1 & 1 \end{pmatrix}$$

a)

$$|\lambda I - A| = \chi_A = \begin{vmatrix} \lambda & -1 & -3 \\ 2 & \lambda-1 & -1 \\ -2 & -1 & \lambda-1 \end{vmatrix}$$

$$\chi_A = \lambda \begin{vmatrix} \lambda-1 & -1 \\ -1 & \lambda-1 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ -2 & \lambda-1 \end{vmatrix} - 3 \begin{vmatrix} 2 & \lambda-1 \\ -2 & -1 \end{vmatrix}$$

$$\chi_A = \lambda[(\lambda-1)^2 - 1] + 2\lambda - 4 - 3(-2 \cdot (-2\lambda + 2))$$

$$\chi_A = \lambda(\lambda-1)^2 - \lambda + 2\lambda - 4 + 6 - 6\lambda + 6$$

$$\chi_A = \lambda(\lambda^2 - 2\lambda + 1) - 5\lambda + 8$$

$$\boxed{\chi_A = \lambda^3 - 2\lambda^2 - 4\lambda + 8}$$

If $\lambda = 2$: $2^3 - 2(2)^2 - (4)(2) + 8 = 0$ ✓

$$\begin{array}{r|l} & \lambda^2 - 4 \\ \lambda - 2 & \lambda^3 - 2\lambda^2 - 4\lambda + 8 \\ & -\lambda^3 - 2\lambda^2 \\ & -4\lambda + 8 \\ & -4\lambda + 8 \\ & 0 \end{array}$$

$\chi_A = (\lambda-2)(\lambda^2 - 4) = \lambda^3 - 4\lambda - 2\lambda^2 + 8$.

$\lambda^2 = 4 \quad \lambda_{1,2} = \pm\sqrt{4}$

$$\boxed{\lambda_1 = 2, \lambda_2 = -2, \lambda_3 = 2}$$

$a.m(\lambda = 2) = 2$

$$(\lambda I - A)\Big|_{\lambda=2} = \begin{pmatrix} 2 & -1 & -3 \\ 2 & 1 & -1 \\ -2 & -1 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 2 & -1 & -3 \\ 0 & 2 & 2 \\ 0 & -2 & -2 \end{pmatrix} \Rightarrow \begin{pmatrix} 2 & -1 & -3 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$2x_1 - x_2 - 3x_3 = 0$$
$$2x_2 + 2x_3 = 0$$

x_3 is free, let $x_3 = 1$ $\therefore$ $x_2 = -1$ & $x_1 = 1$

$$V = \text{span}\left\{ \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right\}$$

$$\text{g.m.}(\lambda = 2) = 1 = \dim(N(\lambda I - A))$$

A isn't diagonlizable as the algebraic multiplicity of $\lambda=2$ doesn't equal the geometric multiplicity of $\lambda=2$. $(2 \neq 1)$

b) A^{1000} $\quad f(\lambda) = \lambda^{1000}$ $\quad f'(\lambda) = 1000\,\lambda^{999}$

$n = 3$ $\quad \therefore$ $h(\lambda) = a_2\lambda^2 + a_1\lambda + a_0$

$$h(2) = 4a_2 + 2a_1 + a_0 = f(2) = 2^{1000}$$
$$h(-2) = 4a_2 - 2a_1 + a_0 = f(-2) = (-2)^{1000}$$
$$h'(\lambda) = 2a_2\lambda + a_1$$
$$h'(2) = 4a_2 + a_1 = 1000(2)^{999}$$

$$\underbrace{\begin{pmatrix} 4 & 2 & 1 \\ 4 & -2 & 1 \\ 4 & 1 & 0 \end{pmatrix}}_{M} \underbrace{\begin{pmatrix} a_2 \\ a_1 \\ a_0 \end{pmatrix}}_{x} = \underbrace{\begin{pmatrix} 2^{1000} \\ (-2)^{1000} \\ 1000(2)^{999} \end{pmatrix}}_{b}$$

$$X = M^{-1}b = \begin{pmatrix} -\frac{1}{16} & \frac{1}{16} & \frac{4}{16} \\ \frac{4}{16} & -\frac{4}{16} & 0 \\ \frac{12}{16} & \frac{4}{16} & -\frac{16}{16} \end{pmatrix} \begin{pmatrix} 2^{1000} \\ (-2)^{1000} \\ 1000(2)^{999} \end{pmatrix} = \begin{pmatrix} a_2 \\ a_1 \\ a_0 \end{pmatrix}$$

$$a_2 = \cancel{\frac{-(2^{1000})}{16}} + \cancel{\frac{(-2)^{1000}}{16}} + \frac{4000(2)^{999}}{16}$$

$$= \frac{2000(2)^{1000}}{16} = 125(2)^{1000}$$

$$a_1 = \frac{4}{16} 2^{1000} - \frac{4}{16}(-2)^{1000} = 0$$

$$a_0 = \frac{12}{16}(2)^{1000} + \frac{4}{16}(-2)^{1000} - 1000(2)^{999}$$

$$a_0 = 2^{1000} - 500(2)^{1000}$$

$$a_0 = -499(2)^{1000}$$

$$h(\lambda) = 125(2)^{1000}\lambda^2 - 499(2)^{1000}$$

$$\boxed{A^{1000} = 125(2)^{1000}A^2 - 499(2)^{1000} I}$$

c) See MATLAB Code:

$$\boxed{J = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}}$$

$$T = \begin{bmatrix} -1/4 & 1 & 3/4 \\ -1/4 & -1 & 1/4 \\ 1/4 & 1 & 3/4 \end{bmatrix} \qquad T^{-1} = \begin{bmatrix} -2 & 0 & 2 \\ 1/2 & -3/4 & -1/4 \\ 0 & 1 & 1 \end{bmatrix}$$

$A = TJT^{-1}$ — JCF of A

e) $A^K = TJ^K T^{-1}$

$$A^K = \begin{pmatrix} -1/4 & 1 & 3/4 \\ -1/4 & -1 & 1/4 \\ 1/4 & 1 & 3/4 \end{pmatrix} \begin{pmatrix} J_1^K & \\ & J_2^K \end{pmatrix} \begin{pmatrix} -2 & 0 & 2 \\ 1/2 & -3/4 & -1/4 \\ 0 & 1 & 1 \end{pmatrix}$$

$J_1 = -2 \qquad f(J_1) = (-2)^K$

$J_2 = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} \qquad f(J_2) = 2^K \qquad f'(J_2) = K2^{K-1}$

$$J_2^K = \begin{pmatrix} 2^K & K2^{K-1} \\ 0 & 2^K \end{pmatrix}$$

$$\boxed{A^K = \begin{pmatrix} -1/4 & 1 & 3/4 \\ -1/4 & -1 & 1/4 \\ 1/4 & 1 & 3/4 \end{pmatrix} \begin{pmatrix} (-2)^K & 0 & 0 \\ 0 & 2^K & K(2)^{K-1} \\ 0 & 0 & 2^K \end{pmatrix} \begin{pmatrix} -2 & 0 & 2 \\ 1/2 & -3/4 & -1/4 \\ 0 & 1 & 1 \end{pmatrix}}$$

$e^{At} = Te^{Jt}T^{-1}$

$$e^{J_1 t} = e^{-2t}$$

$$e^{J_2 t} = \begin{pmatrix} e^{2t} & te^{2t} \\ 0 & e^{2t} \end{pmatrix}$$

$$\boxed{e^{At} = \begin{pmatrix} -1/4 & 1 & 3/4 \\ -1/4 & -1 & 1/4 \\ 1/4 & 1 & 3/4 \end{pmatrix} \begin{pmatrix} e^{-2t} & 0 & 0 \\ 0 & e^{2t} & te^{2t} \\ 0 & 0 & e^{2t} \end{pmatrix} \begin{pmatrix} -2 & 0 & 2 \\ 1/2 & -3/4 & -1/4 \\ 0 & 1 & 1 \end{pmatrix}}$$

#5) $A=\begin{pmatrix}1 & 0\\ 3 & -1\end{pmatrix}$

$\chi_A = \lambda^2 - 1 = 0 \qquad \lambda_1 = 1,\ \lambda_2 = -1$

$J = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$

$(A-\lambda_1 I)v_1 = \begin{pmatrix}0 & 0\\ 3 & -2\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} = \vec{0}$

let $x_2 = 1 \quad \therefore \quad v_1 = \begin{pmatrix}2/3\\ 1\end{pmatrix}$

$(A-\lambda_2 I)v_2 = \begin{pmatrix}2 & 0\\ 3 & -0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} \Rightarrow v_2 = \begin{pmatrix}0\\ 1\end{pmatrix}$

$T = \begin{pmatrix}2/3 & 0\\ 1 & 1\end{pmatrix} \qquad T^{-1} = \frac{3}{2}\begin{pmatrix}1 & 0\\ -1 & 2/3\end{pmatrix} = \begin{pmatrix}3/2 & 0\\ -3/2 & 1\end{pmatrix}$

$e^A = Te^JT^{-1} = \begin{pmatrix}2/3 & 0\\ 1 & 1\end{pmatrix}\begin{pmatrix}e & 0\\ 0 & e^{-1}\end{pmatrix}\begin{pmatrix}3/2 & 0\\ -3/2 & 1\end{pmatrix}$

$e^A = \begin{pmatrix}2/3\,e & 0\\ e & e^{-1}\end{pmatrix}\begin{pmatrix}3/2 & 0\\ -3/2 & 1\end{pmatrix}$

$e^A = \begin{pmatrix}e & 0\\ \frac{3}{2}e - \frac{3}{2}e^{-1} & e^{-1}\end{pmatrix} \Rightarrow \boxed{e^A = \begin{pmatrix}e & 0\\ \frac{3}{2}(e - e^{-1}) & e^{-1}\end{pmatrix}}$

$$sI - A = \begin{pmatrix} s-1 & 0 \\ -3 & s+1 \end{pmatrix} \qquad e^{At} = \mathcal{L}^{-1}\left((sI-A)^{-1}\right)$$

$$(sI-A)^{-1} = \frac{1}{(s-1)(s+1)} \begin{pmatrix} s+1 & 0 \\ 3 & s-1 \end{pmatrix} = \begin{pmatrix} \frac{1}{s-1} & 0 \\ \frac{3}{(s-1)(s+1)} & \frac{1}{s+1} \end{pmatrix}$$

$$\mathcal{L}^{-1}\left(\frac{1}{s-1}\right) = e^{t}$$

$$\mathcal{L}^{-1}\left(\frac{3}{(s-1)(s+1)}\right) : \frac{3}{(s-1)(s+1)} = \frac{A_1}{(s-1)} + \frac{A_2}{(s+1)}$$

$$A_1 = \frac{3}{s+1} + \frac{A_2(s-1)}{(s+1)} \Bigg|_{s=1} \qquad \therefore A_1 = \frac{3}{2}$$

$$A_2 = \frac{3}{s-1} + \frac{A_1(s+1)}{(s-1)} \Bigg|_{s=-1} \qquad \therefore A_2 = -3/2$$

$$\mathcal{L}^{-1}\left(\frac{3}{(s-1)(s+1)}\right) = \frac{3}{2}\mathcal{L}^{-1}\left(\frac{1}{s-1} - \frac{1}{s+1}\right) = \frac{3}{2}\left(e^{t} - e^{-t}\right)$$

$$\mathcal{L}^{-1}\left(\frac{1}{s+1}\right) = e^{-t}$$

$$\therefore e^{At} = \begin{pmatrix} e^{t} & 0 \\ \frac{3}{2}(e^{t} - e^{-t}) & e^{-t} \end{pmatrix}$$

$$\therefore \boxed{e^{A} = \begin{pmatrix} e & 0 \\ \frac{3}{2}(e - e^{-1}) & e^{-1} \end{pmatrix}}$$

Contents


```
clear
clc
```

Problem 4 Code

```
% Matrix of coeffcicients to solve for remainder polynomial
M       = [4 2 1; 4 -2 1; 4 1 0];
Minv    =  inv(M)

% Problem 4 A matrix
A       = [0 1 3; -2 1 1;2 1 1];
[T,J]   = jordan(A)

% Verify A = T*J*T^-1
A_check = T*J*inv(T)
```

```

Minv =

   -0.0625    0.0625    0.2500
    0.2500   -0.2500         0
    0.7500    0.2500   -1.0000


T =

   -0.2500    1.0000    0.7500
   -0.2500   -1.0000    0.2500
    0.2500    1.0000    0.7500


J =

    -2     0     0
     0     2     1
     0     0     2


A_check =

     0     1     3
    -2     1     1
     2     1     1

```

Problem 5 Code

```
% e^A from Laplace Inverse and Jordan Normal Form
eA_hand = [exp(1), 0; 3/2*(exp(1) - exp(-1)), exp(-1)]
```

```
% Problem 5 A Matrix
A       = [1 0; 3 -1];

% Matrix exponential
eA_mat  = expm(A)
```

```
eA_hand =

    2.7183         0
    3.5256    0.3679


eA_mat =

    2.7183         0
    3.5256    0.3679
```

Published with MATLAB® R2021b