Case Study

The objective of this case study is to construct a stabilizing controller for a fuzzy system and to investigate the performance of two different LMI solvers for finding fuzzy controller gains.

We consider a simple fuzzy system modeled as

$$\dot{x} = (w_1 A_1 + w_2 A_2)x + (w_1 B_1 + w_2 B_2)u,$$

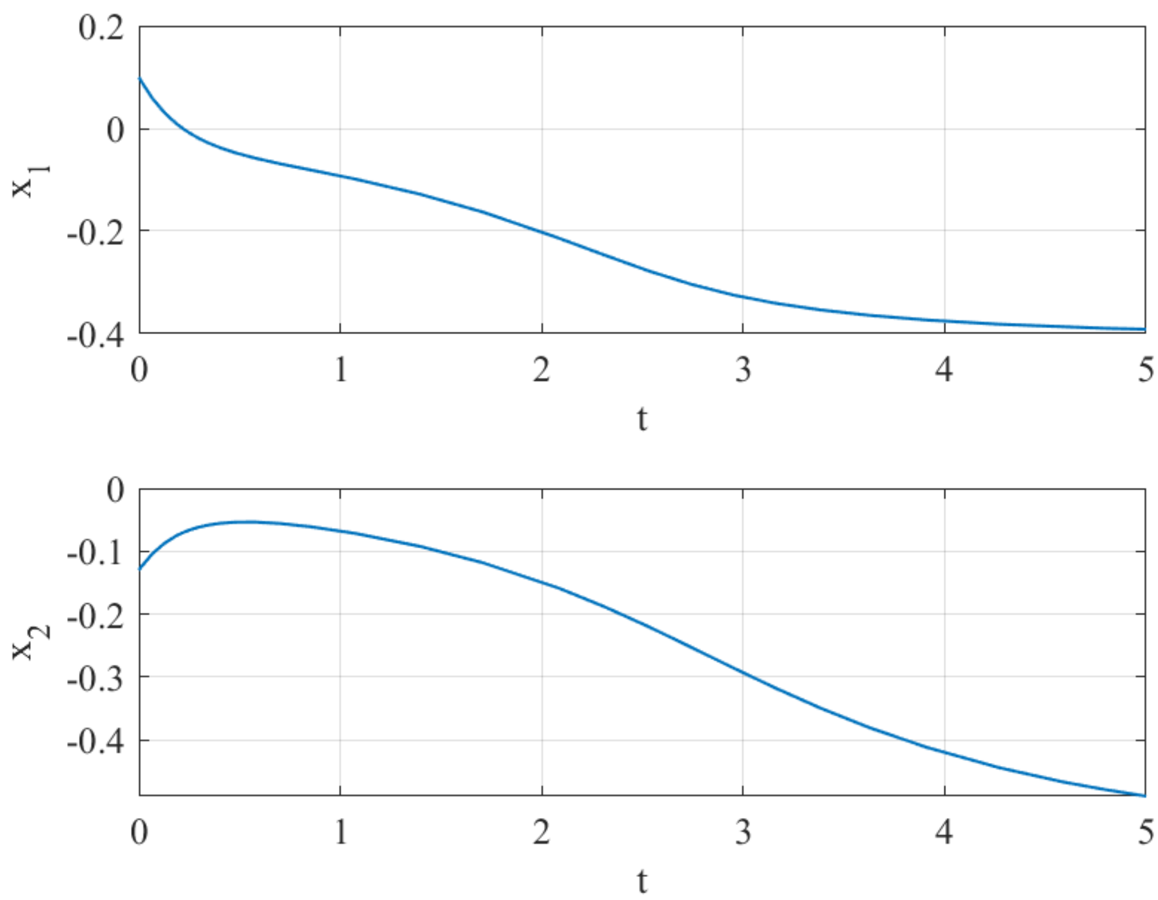
where

$$A_1 = \begin{bmatrix} -2 & 4 \\ 2 & -2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -3 & 0 \\ 2 & -1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

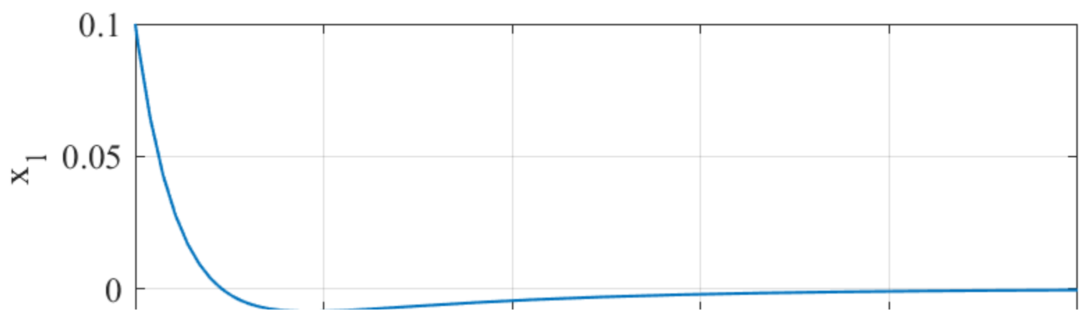
and

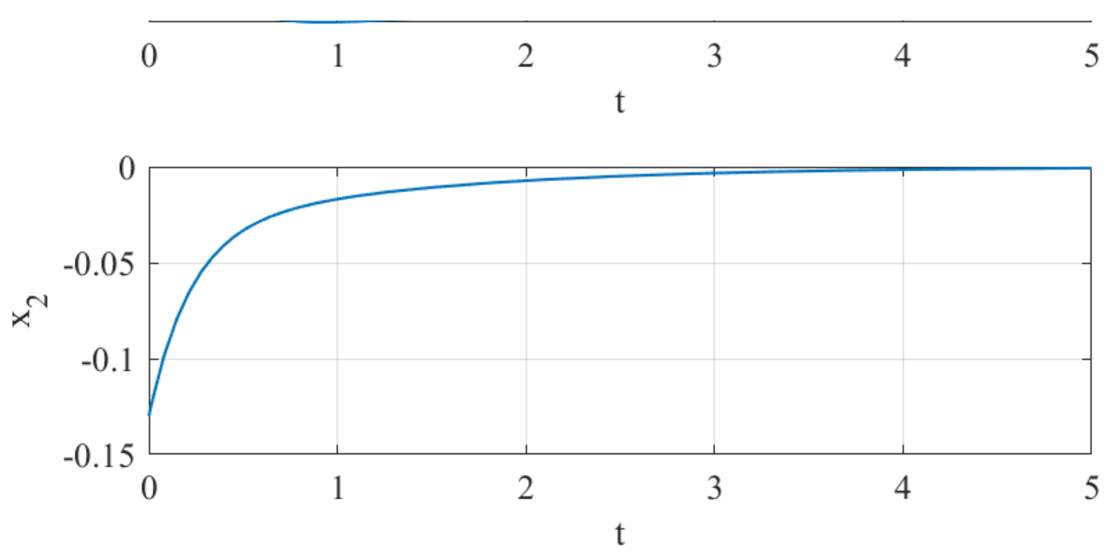
$$w_1(x_1) = \frac{1 - 1/\left(1 + \exp(-14(x_1 - \pi/8))\right)}{1 + \exp(-14(x_1 + \pi/8))} \quad \text{and} \quad w_2(x_1) = 1 - w_1(x_1).$$

We start with simulating the uncontrolled system behvaior. In the figure below, we show plots of the state variables of the uncontrolled fuzzy system.



We first design stabilizing controller using CVX. In the figure below, we show plots of the state variables of the controlled fuzzy system with the fuzzy controller's gains obtained using CVX.





The controller's gains are:

$$K_1 = \begin{bmatrix} 1.3588 & 2.4215 \end{bmatrix} \quad \text{and} \quad K_2 = \begin{bmatrix} -0.9378 & 2.4094 \end{bmatrix}.$$

The eigenvalues of $(A_1 - B_1K_1)$ are located at $\{-4.5817,\ -0.7771\}$.

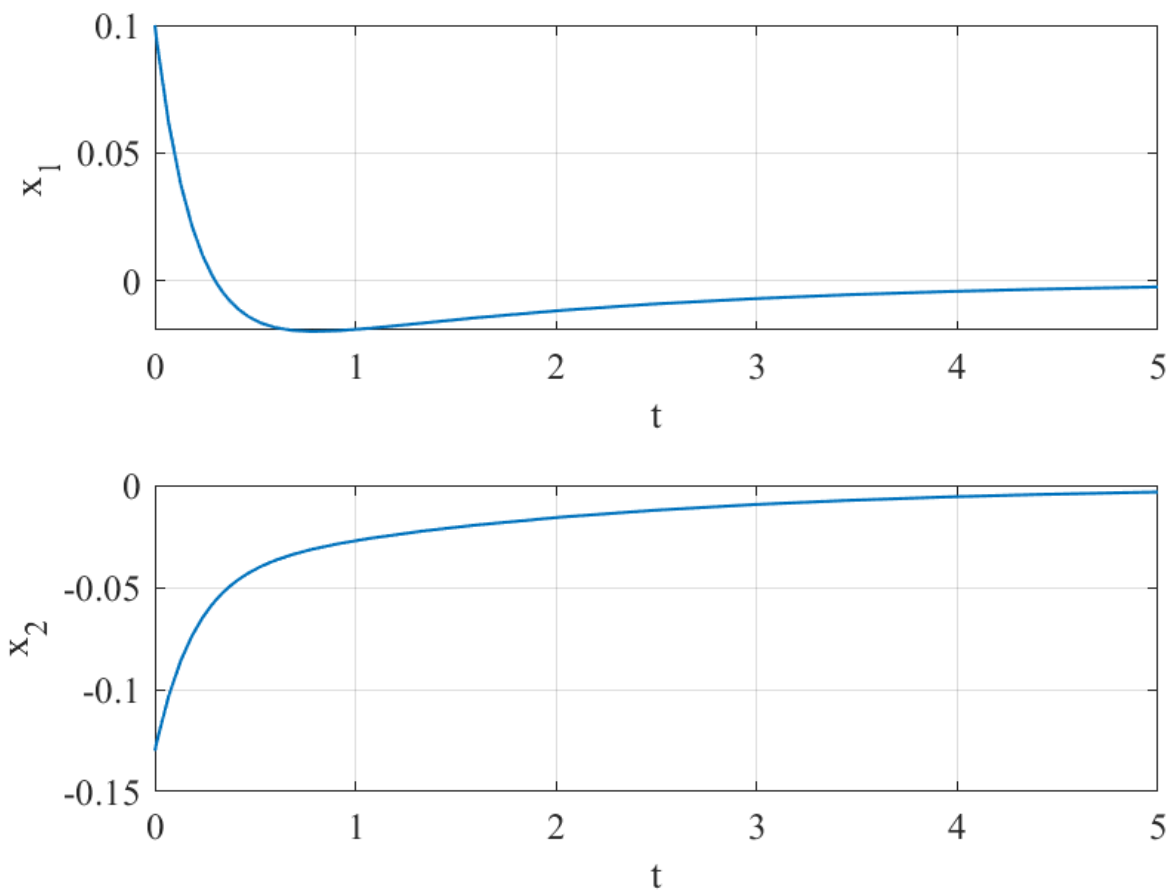
The eigenvalues of $(A_2 - B_2K_2)$ are located at $\{-3.4094,\ -3.0000\}$.

The common Lyapunov matrix P is

$$P = \begin{bmatrix} 0.4691 & -0.2038 \\ -0.2038 & 0.4103 \end{bmatrix}.$$

Its eigenvalues are located at $\{0.2338,\ 0.6456\}$.

We next design stabilizing controller using MATLAB's LMI toolbox. In the figure below, we show plots of the state variables of the controlled fuzzy system with controller gains obtained from CVX.



The controller's gains in this case are:

$$K_1 = \begin{bmatrix} 1.6804 & 1.6356 \end{bmatrix} \quad \text{and} \quad K_2 = \begin{bmatrix} -1.6220 & 3.1129 \end{bmatrix}.$$

The eigenvalues of $(A_1 - B_1K_1)$ are located at $\{-5.1714,\ -0.5090\}$.

The eigenvalues of $(A_2 - B_2K_2)$ are located at $\{-4.1129,\ -3.0000\}$.

The common Lyapunov matrix P is

$$P = \begin{bmatrix} 5.7685 & -2.6525 \\ -2.6525 & 5.5401 \end{bmatrix}.$$

Its eigenvalues are located at $\{2.9993,\ 8.3093\}$.

The above results were obtained using the following script:

```
function[]=Module48_CaseStudy()
clear all
clc
global A1 B1 A2 B2
% Example data
A1=[-2 4;2 -2];
B1=[1;0];
A2=[-3 0;2 -1];
B2=[0;1];
[n,~]=size(A1);
[~,m]=size(B1);
%----------------CVX-----------
 cvx_begin sdp
% Variable definition
variable S(n, n) symmetric
variable Z1(m, n)
variable Z2(m, n)

% LMIs
A1*S + S*A1' - B1*Z1 - Z1'*B1' <= 0
A2*S + S*A2' - B2*Z2 - Z2'*B2' <= 0
(A1+A2)*S+S*(A1+A2)'-Z2'*B1'-Z1'*B2'-B1*Z2-B2*Z1 <=0
S >= eps*eye(n)
 cvx_end
 %-------------MATLAB"s LMI Toolbox---------
 % Comment MATLAB"s LMI Toolbox commands; lines 28 through 45
 % to perform computations using CVX
setlmis([]);
S=lmivar(1,[n,1]);
Z1=lmivar(2,[m,n]);
Z2=lmivar(2,[m,n]);
lmiterm([1 1 1 S],A1,1,'s');
lmiterm([1 1 1 Z1],-B1,1,'s');
lmiterm([2 1 1 S],A2,1,'s');
lmiterm([2 1 1 Z2],-B2,1,'s');
lmiterm([3 1 1 S],A1+A2,1,'s');
lmiterm([3 1 1 Z1],-B2,1,'s');
lmiterm([3 1 1 Z2],-B1,1,'s');
lmiterm([-4 1 1 S],1,1);
lmiterm([4 1 1 0],0.01);
lmis=getlmis;
[tmin,xfeas]=feasp(lmis);
S=dec2mat(lmis,xfeas,S);
Z1=dec2mat(lmis,xfeas,Z1);
Z2=dec2mat(lmis,xfeas,Z2);

 global K1 K2
  K1 = Z1/S % compute K1 matrix
  K2 = Z2/S % compute K2 matrix
 P=inv(S)
 disp('eig(P)')
 eig(inv(S))
 disp('eig(A1-B1*K1)')
eig(A1-B1*K1)
disp('eig(A2-B2*K2)')
eig(A2-B2*K2)
G12=A1-B1*K2+A2-B2*K1;
disp('eig(A1-B1*K2+A2-B2*K1)')
eig(A1-B1*K2+A2-B2*K1)

%x_0 = 0.5*randn(n,1);
```

```
x_0=[.100 -.13]';
tspan = [0 5];
[t, x] = ode23(@model, tspan, x_0);

figure;
for i=1:n
    subplot(2,1,i);
    plot(t, x(:,i), 'linewidth', 1);
    xlabel('t','fontsize', 12);
    ylabel(['x_', num2str(i)],'fontsize',12);
    set(gca, 'fontsize', 12,'FontName','Times');
    grid
end

function xdot = model(t, x)
global A1 B1 A2 B2 K1 K2
% Control input
w1=(1-1/(1+exp(-14*(x(1)-pi/8))))/(1+exp(-14*(x(1)+pi/8)));
w2=1-w1;
u = -(w1*K1+w2*K2)*x;
% u=0 % Uncomment u=0 to simulate uncontrolled system
% System
xdot=(w1*A1+w2*A2)*x+(w1*B1+w2*B2)*u;
```