AAE 532 - Orbit Mechanics

Purdue University

PS1 Solutions

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Problem 1

Problem Statement

The Dawn spacecraft was launched in September 2007 as part of NASA's Discovery Program. The spacecraft successfully visited the protoplanet Vesta and the dwarf planet Ceres, delivering detailed images and other data back to Earth. The goal for the mission was to characterize the conditions and processes in the early solar system by investigating two objects that have remained intact since their formation. Both reside in the asteroid belt with many other objects.

(a) Vesta is labelled a protoplanet. Determine the definition of a protoplanet and explain why Vesta is denoted as such; be sure to cite your sources. Find 5 facts about Vesta that you did not know previously. (You must use vetted primary sources!)

(b) View Vesta's orbit relative to the solar system. Go to the website below the figure on the next page to visit a 3D Solar System Simulator. Along the bottom of the screen is a string of ption in 'gold'. The 'home' button will offer lots of details about Vesta and other objects. Experiment with the various options to view the motion of the objects as well as change dates, etc.
Is Vesta's orbit in the same plane as the Earth orbit? Estimate the angular difference (note that you can find the inclination of Vesta's orbit in the data!) What is Vesta's 'magnitude'? One criterion for viewing an object is its 'magnitude'. Typically, an object is visible if the magnitude is < 20 (approximately). Where does Vesta fit on this scale?

(c) Dawn was launched 9/27/2007. Take a screenshot of the solar system on that date. Note the locations of Earth and Vesta at launch. Check the line-of-sight (LOS). Given the magnitude as well as the LOS, the launch date was selected, in part, to ensure that Vesta is viewable. Is that accomplished? How do you know?

(d) Change the date and note the orientation of Vesta as well as the Earth on the date. Take three more screenshots:

  (i) 7/15/11 when the spacecraft arrived at Vesta and was inserted into an orbit in the Vesta vicinity

  (ii) 9/4/12 when the spacecraft departed Vesta

  (iii) In a top-down view, the date when Earth/Sun/Vesta all appear to be 'aligned'. What is the date?

  At each date, observe the line-of-sight (LOS) from Earth to the spacecraft (or Earth to Vesta). When the solar system objects are aligned, is the Sun in the LOS from Earth to Vesta? Anyother planets likely to interfere? Is the alignment a problem on this date? Why or why not? (Offer any additional views that support your conclusions.)

(e) Animate to the date of YOUR birthday 10 years from this year. Capture the orientation on this date. In the bottom of the image, if you click on the three lines, a box will appear with 'Asteroid 4 Vesta'; click on the term Vesta. From lots of information, 'Distance to Earth' is also available. You can find the distance on all the dates of interest in (d) as well at your birthday in 10 years! Make a list. Add two screenshots on your birthday in 10 years: one that is top-down; one that demonstrates the out-of-plane characteristics. Will there be a LOS to Vesta on your future birthday? Will Vesta be closer or further from the Earth than on the date when Dawn departed Vesta?

(f) Scrolling further down the Vesta page to orbital elements, the inclination, i, is also available, that is, the orientation of Vesta's orbital plane relative to the ecliptic. What is the value of i? Compare it to your earlier estimate. How does orbital inclination affect LOS when the objects are all aligned? The period is also available—what is the orbital period of Vesta? How many orbital periods will Vesta have completed between Dawn's departure from Vesta and your future birthday?

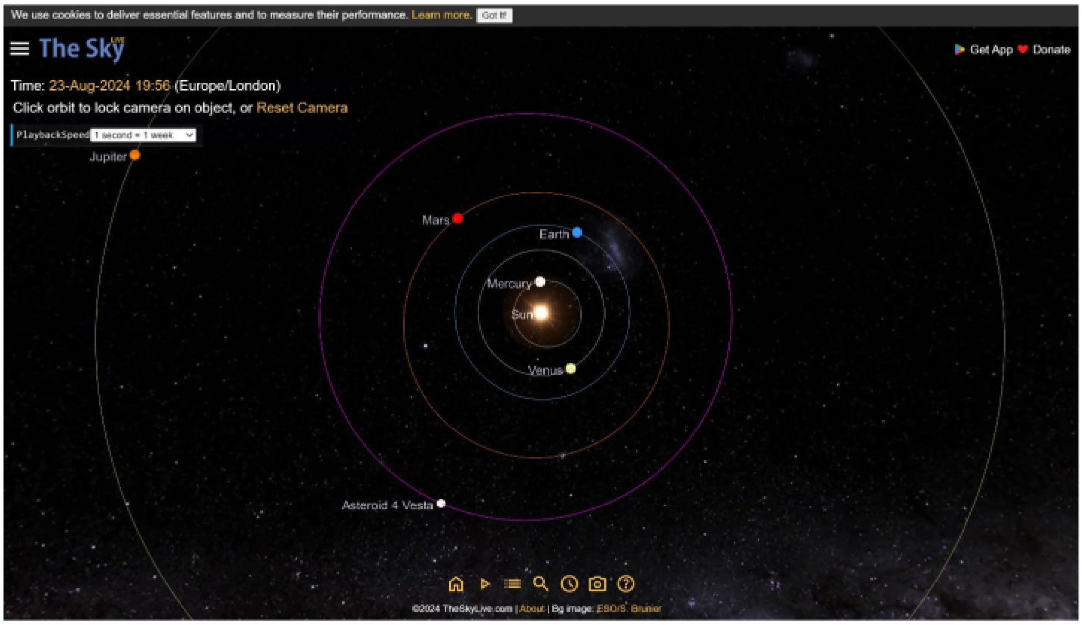

Figure 1: 3D Solar System Simulator

Part (a)

According to the Oxford Dictionary of Astronomy [1], *protoplanet* is defined as "A body presumed to have formed in the early history of the Solar System, from which the major planets grew." In [2], this idea is expanded and talks about short-lived radioactive nuclides which were present in the first 2 million years of our solar system's formation. Bodies which contain this radioactive material were early-forming and of these some went on to form our planets. The composition of Vesta was found to place it in this early formation regime. This period coincides with the formation and early stages of evolution of the giant planets [3]. Additionally, Vesta is differentiated and contains a large iron core capable of sustaining a magnetic dynamo like that seen here on Earth [2]. The early formation, iron core, geological history, and size all support Vesta's classification as a protoplanet.

Five more facts about Vesta:

1. Vesta was the fourth asteroid ever to be discovered (March 29, 1807) [4]
2. Vesta was named after the Roman goddess of the hearth and household by Carl Friedrich Gauss [4]
3. Vesta has one of the largest brightness ranges observed by any rocky solar system body; the bright materials likely originate from Vesta, whereas the darker materials are believed to have been left other asteroids that collided with Vesta [4]
4. The large peak in the middle of Rheasilvia, a giant crater on Vesta, is taller than Mauna Kea on Hawaii (relative to the ocean floor) [2]
5. Vesta accounts for almost 9% of the total mass of all asteroids and is second in mass only to the dwarf planet Ceres within the asteroid belt [4]

References:

[1] Oxford Dictionary of Astronomy

[2] Russell, et al. "Dawn at Vesta: Testing the Protoplanetary Paradigm," *Science*, May 2012

[3] Consolmagno, et al. "Is Vesta an intact and pristine protoplanet?" *Icarus*, July 2015.

[4] NASA Science website

Part (b)

Vesta's orbit is not in the same plane as Earth's. Vesta's inclination with respect to Earth's orbital plane looks to be around 7 degrees. From the data table, Vesta's inclination with respect to the ecliptic is 7.139 degrees. The magnitude of Vesta is 8.0, which is smaller than 20, so Vesta is visible.

Part (c)

When Dawn is launched, the Earth has a clear line-of-sight (LOS) to Vesta. As the magnitude of Vesta is $8.0 < 20$, and there is an unobstructed LOS between Earth and Vesta, Vesta is visible from Earth. Therefore, Dawn's launch date was selected appropriately.

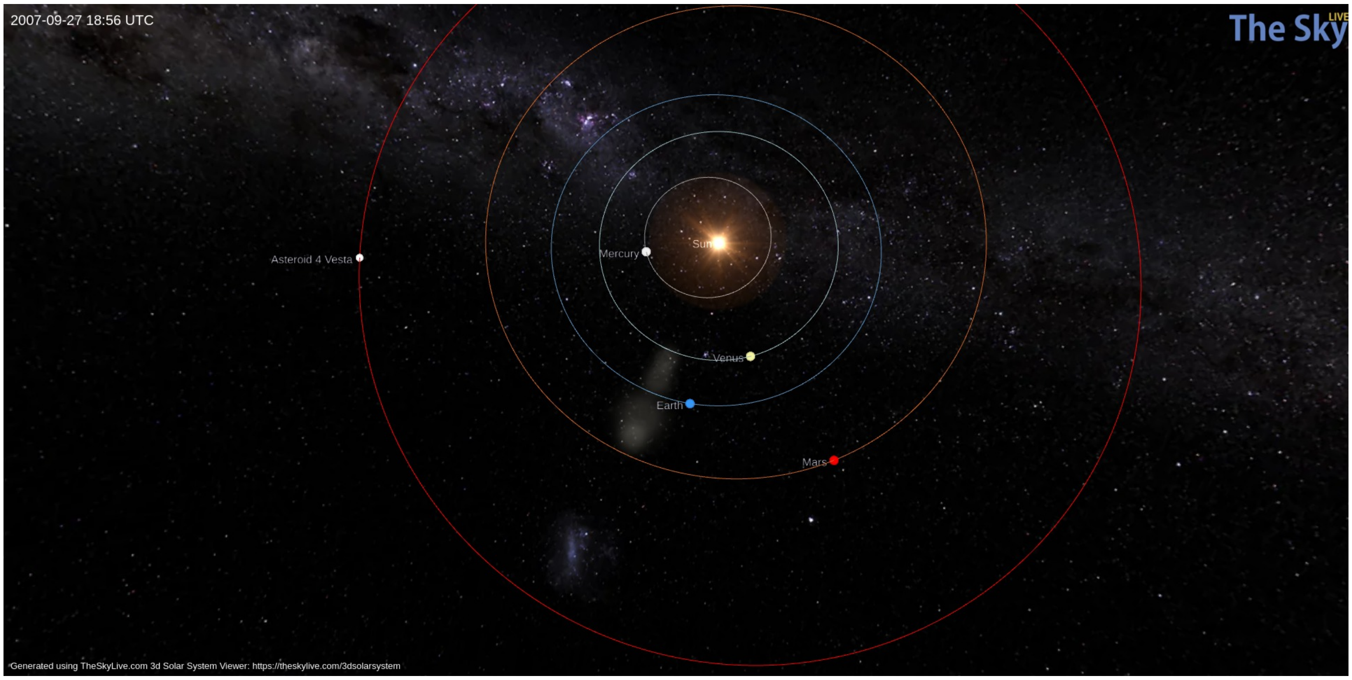

Figure 2: Solar system state on 09/27/2007, when Dawn was launched

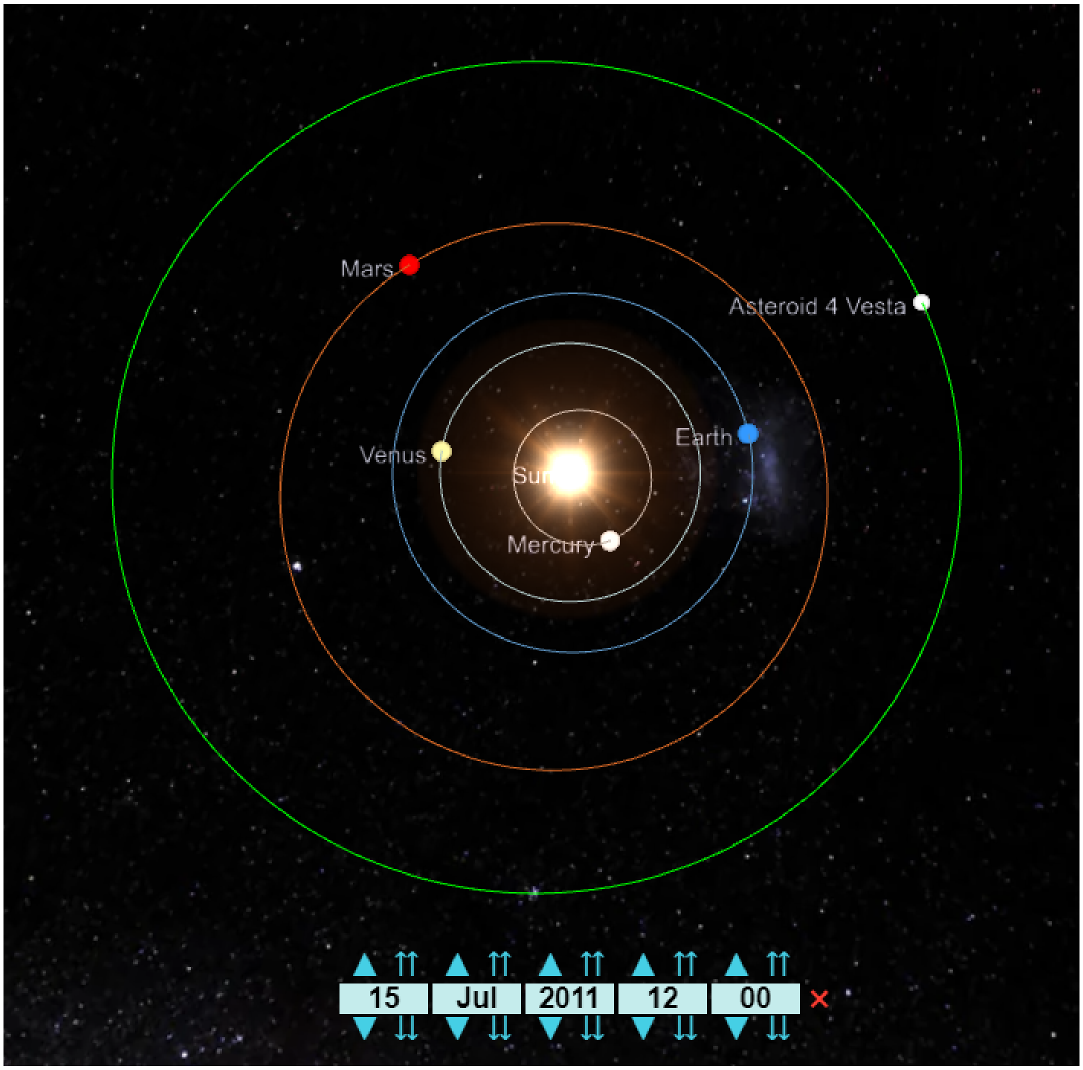

Figure 3: Orientation of Orbits of the Inner Planets and Vesta (green) on 7/15/2011 at 12:00

Part (d)

(i) Figure 3 above depicts the orientation of the orbits of the inner planets and Vesta when the spacecraft Dawn inserts into an orbit near Vesta. While figure 3 only provides a view looking down onto the ecliptic plane, one can clearly see that the line-of-sight (LOS) from Earth to the spacecraft near Vesta would not be obscured (unless the spacecraft's orbit is eclipsed by Vesta itself).

The Sun does not directly pass between the Earth and Vesta, eclipsing LOS. However, one must be careful as the Sun is close to back-lighting Earth, and since the Sun is such a large emitter of electromagnetic radiation, it can indirectly interfere with satellite communications between Earth and the spacecraft. Thus, if solar activity is high on this day, the alignment may prove to be slightly problematic. However, no other planets are likely to interfere with the LOS on this date.

(ii) Figure 4 below depicts the orientation of the orbits of the inner planets and Vesta when the spacecraft Dawn departs from its orbit near Vesta. While figure 4 only provides a view looking down onto the ecliptic plane, one can clearly see that the line-of-sight (LOS) from Earth to the spacecraft near Vesta would not be obscured (unless, again, the spacecraft's orbit is eclipsed by Vesta itself).

The Sun does not directly pass between the Earth and Vesta, nor is it close to back-lighting the Earth. In addition, there are no other planets are likely to interfere with the LOS on this date.

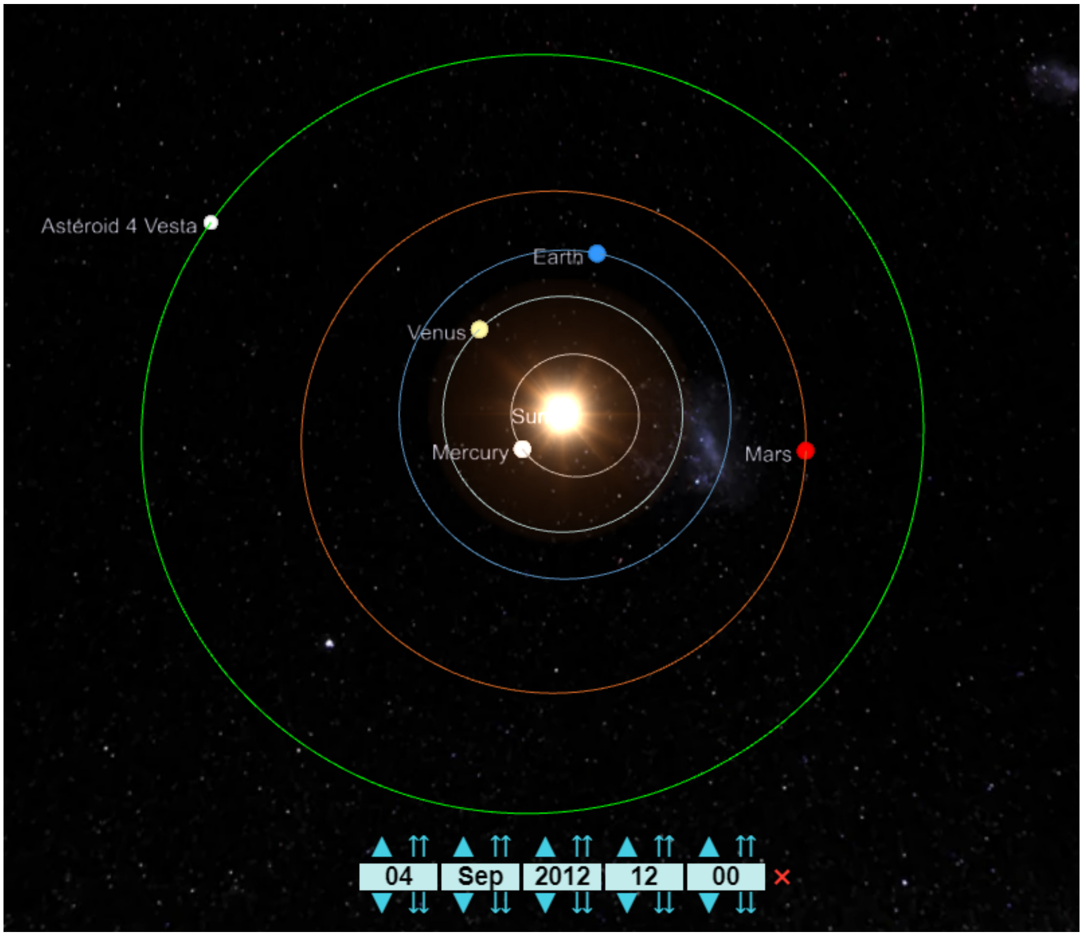

Figure 4: Orientation of Orbits of the Inner Planets and Vesta (green) on 9/4/2012 at 12:00

(iii) Figure 5 below depicts the orientation of the orbits of the inner planets and Vesta when the Sun, Earth, and Vesta are aligned. The Sun does, essentially, eclipse the Earth on April 12, 2012, denying LOS between Earth and the spacecraft at Vesta. This can specifically be seen in the edge-on view in figure 6. Without LOS, all communications with the spacecraft are, momentarily, lost. Nevertheless, it should be noted that no other planets are likely to interfere with the LOS on this date.

Recognize that this orientation is just one way that communications can be interfered. As suggested earlier, the Sun can also interfere with telecommunications even in the orientation of Sun-Earth-Vesta, such as on August 5, 2011 (see below).

On August 5, 2011, the Sun is not in between Earth and Vesta in this aligned configuration, so a LOS theoretically exists. However, because the Sun sits right behind the Earth when viewed from Dawn (see the edge-on view), any signals sent from the ground at Earth will be drowned in the noise generated from the Sun's solar radiation when picked up by Dawn. This orientation, therefore, poses a serious problem for telecommunications. Given that this orientation occurs during the period that Dawn was at Vesta, this hurdle had to be dealt with by the mission operations team.

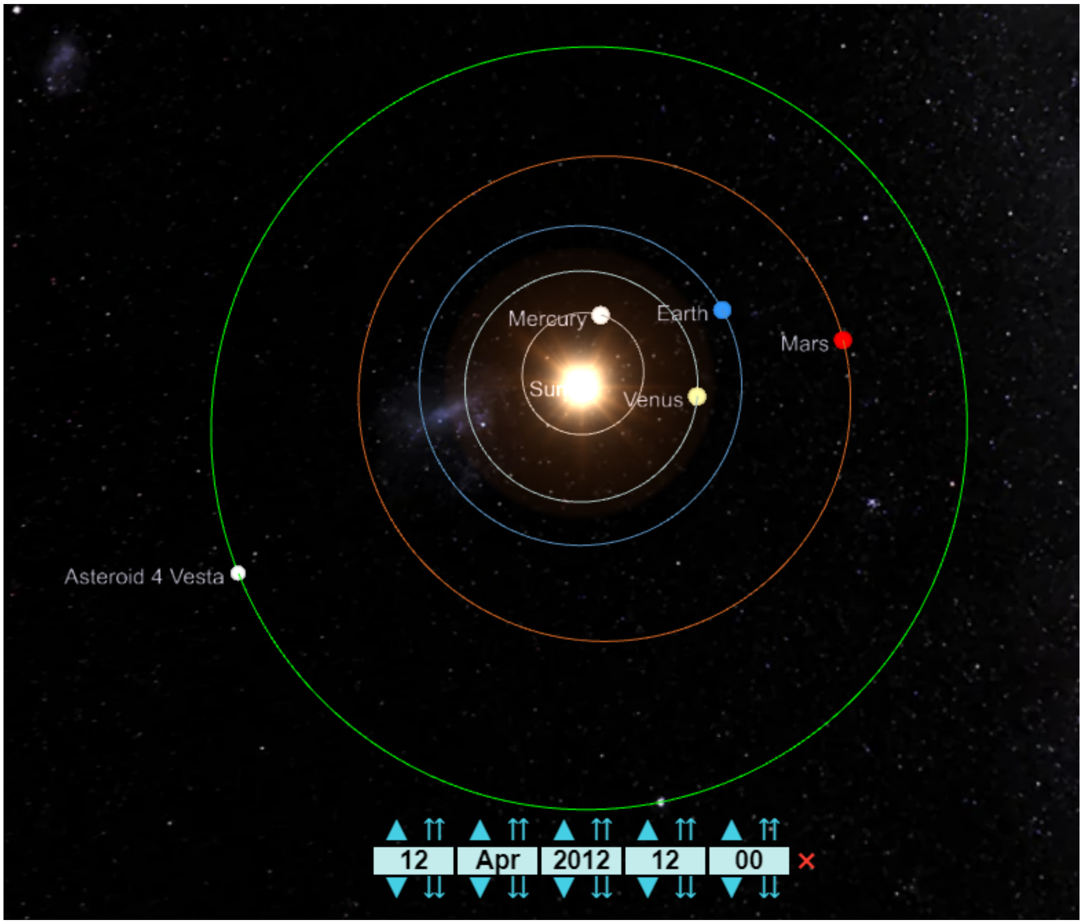

Figure 5: Orientation of Orbits of the Inner Planets and Vesta (green) on 4/12/2012 at 12:00

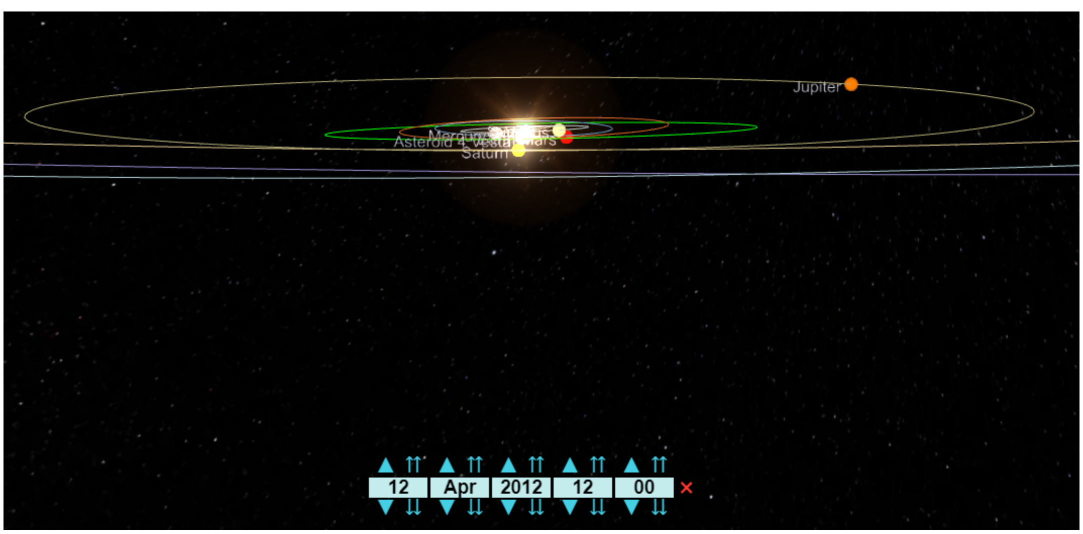

Figure 6: Edge-on view of the Inner Planets and Vesta (green) on 4/12/2012 at 12:00

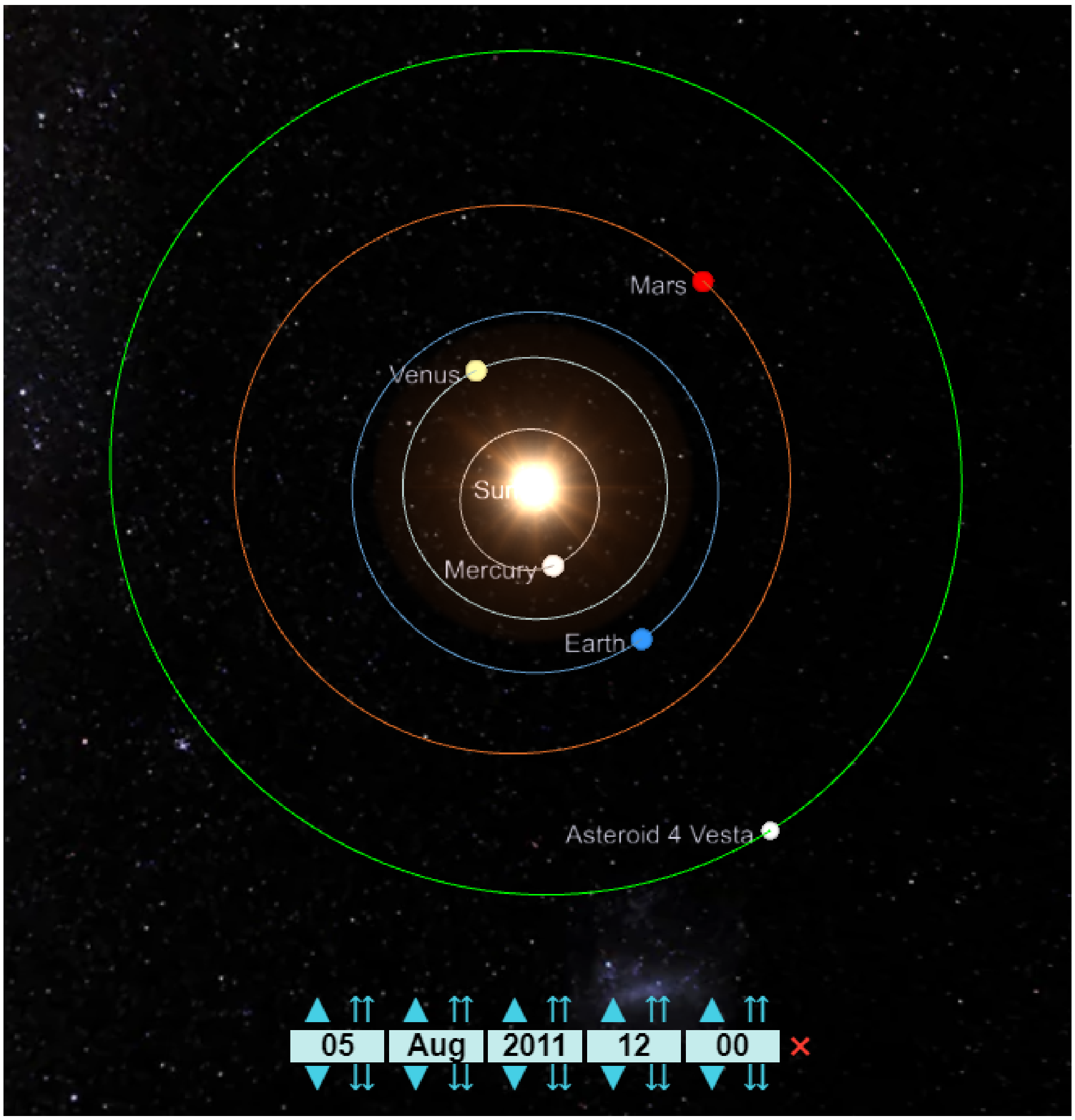

Figure 7: Orientation of Orbits of the Inner Planets and Vesta (green) on 8/5/2011 at 12:00

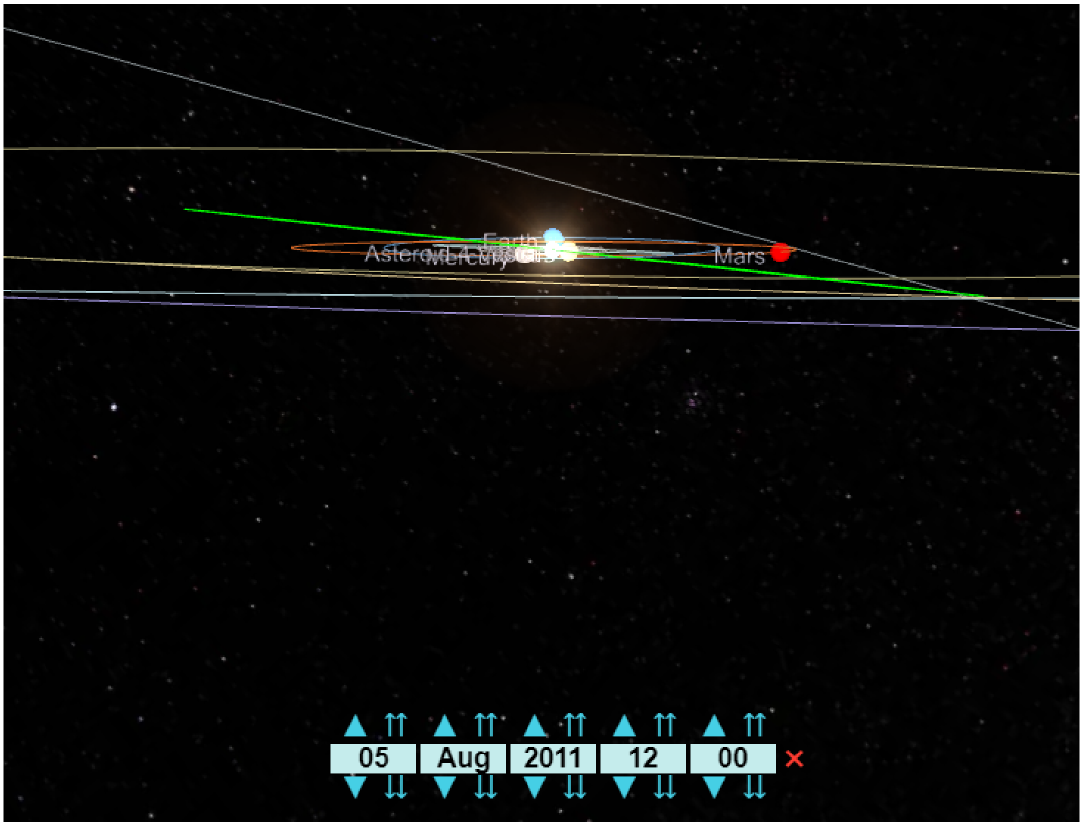

Figure 8: Edge-on view of the Inner Planets and Vesta (green) on 8/5/2011 at 12:00

Part (e)

Let's create a list of the "Distance to Earth" from the dates in part (d) above and for my birthday in ten years:

(i) 7/15/2011: Distance to Earth $= 188.67 \cdot 10^6$ km

(ii) 9/4/2012: Distance to Earth $= 369.25 \cdot 10^6$ km

(iii) 4/12/2012: Distance to Earth $= 519.33 \cdot 10^6$ km

(iv) 8/5/2011: Distance to Earth $= 183.93 \cdot 10^6$ km

(v) 12/25/2034: Distance to Earth $= 236.01 \cdot 10^6$ km

Like on 7/15/2011, as one can see in figure 9 while one can clearly see that the LOS from Earth to Vesta would not be obscured (unless the spacecraft's orbit is eclipsed by Vesta itself), the Sun is close to back-lighting Earth, and since the Sun is such a large emitter of electromagnetic radiation, it can indirectly interfere with satellite communications between Earth and the spacecraft. Vesta will be closer to Earth on my birthday, 12/25/2034, than it was when Dawn departed Vesta on 9/4/2012.

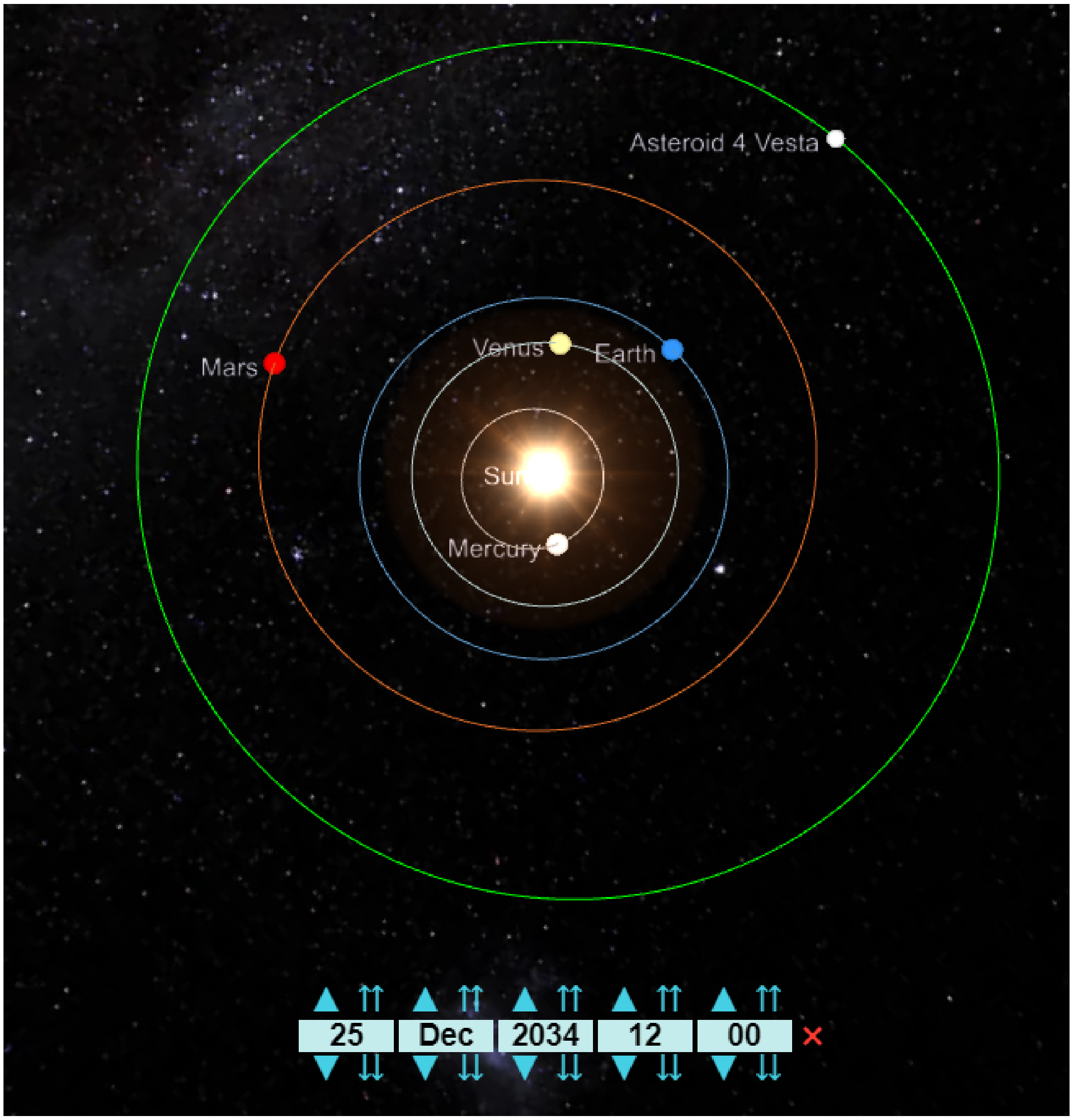

Figure 9: Orientation of Orbits of the Inner Planets and Vesta (green) on 12/25/2034 at 12:00

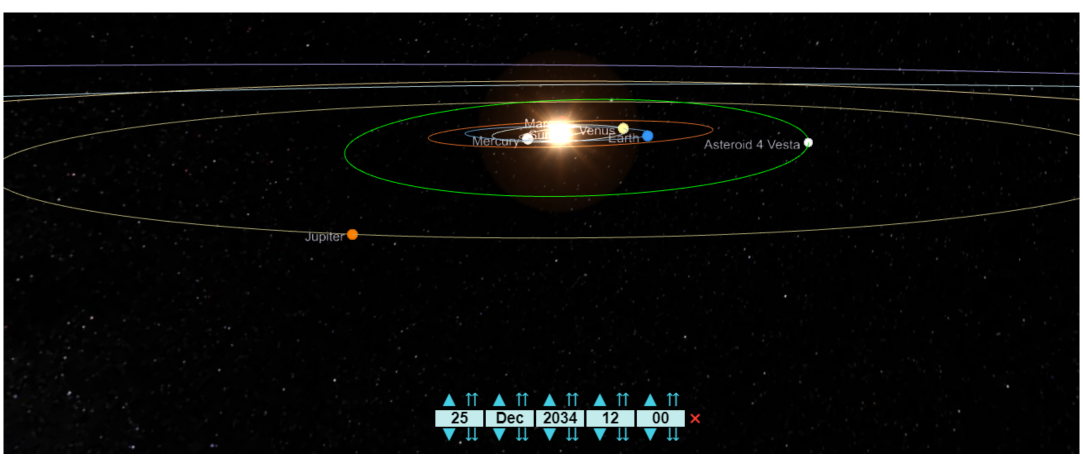

Figure 10: Side view of the Inner Planets and Vesta (green) on 12/25/2034 at 12:00

Part (f)

The inclination of Vesta relative to the ecliptic is 7.14 degree. This agrees well with the estimate made earlier. When the bodies are aligned, the inclination contributes an out-of-plane component. This means that the bodies may appear to lie on a straight line from a top down view, but when looked at edge-on, they may not be as aligned. With an inclination this low, the effects are minimal (especially given the size of the Sun). However, in some cases this out-of-plane motion can play a significant role, i.e. when looking at whether a smaller body is blocking LOS between Vesta and Earth.

The orbital period of Vesta is 3.63 years. Using an online calendar calculator, there are 8147 days between Dawn's arrival at Vesta and my birthday in 2034. So:

$$8147 \text{ days} \cdot \frac{1 \text{ year}}{365.25 \text{ days}} \cdot \frac{1 \text{ period}}{3.63 \text{ years}} \approx 6.1447 \text{ orbits completed} \tag{1}$$

Problem 2

Problem Statement

The solar system consists of the Sun and those celestial objects bound to it by gravity. These objects are the eight planets, over 150 known moons, dwarf planets, and billions of small bodies. There are likely thousands of dwarf planets waiting to be discovered beyond Neptune! However, as of June 2024, the IAU has recognized **five** dwarf planets: Pluto, Eris, Makemake, Haumea, and Ceres. However, there are four more planetary objects, namely Orcus, Sedna, Gonggong and Quaoar, that the majority of the scientific community recognizes as dwarf planets as of 2021. (Note that Vesta is not among them.) Check out the site from problem #1 and investigate the orbits of the nine generally accepted dwarf planets in the Orbit Visualizations.

(a) Submit an image for the orbit of each the 9 dwarf planets and observe their orientation relative to the rest of the solar system. Also, your figures should be dated on your birthday in the year 2024. The International Astronomical Union (IAU), the official scientific body for astronomical nomenclature, defines a **dwarf planet** as a celestial body within the solar system that satisfies these four conditions:

  (i) is in orbit around the Sun

  (ii) has sufficient mass for its self-gravity to overcome rigid body forces so that it assumes a hydrostatic equilibrium (near-spherical) shape

  (iii) has not cleared the neighborhood around its orbit

  (iv) is not a satellite/moon

  Do the 5 IAU dwarf planets appear to meet criteria (i) and (iv)? How about the additional 4 dwarf planets generally accepted by the community? Justify your statement.

(b) Create a table with the following information on the nine dwarf planets: distance from the Earth on your birthday in 2024, period of the orbit, the semi-major axis and the inclination. Note that the Sun-Earth distance is 1 AU (where AU = Astronomical Unit = 1.496×10^8 km) where the average distance from the Sun is the semi-major axis in the same units. Compare the semi-major axis distance to the distance from Earth on your birthday; is the dwarf planet above or below the ecliptic plane on this date? Consider which dwarf planet is closest to Earth on your birthday for a potential mission? Spacecraft delivery is easiest when the object is in the ecliptic plane at arrival. To arrive and be located in the ecliptic plane, is it likely to arrive on your birthday in any year? Why or why not? Given your birthday in 2024, what is the next earliest date when the dwarf planet crosses the ecliptic plane; how long do we have to wait? (Add an image).

(c) For each of the additional four generally accepted dwarf planets, answer the following questions:

  (a) Name, year of discovery, and size?

  (b) Moons (if any) and their names and sizes?

  (c) Source of your information? (Website or source; Wikipedia is not a primary source)

  (d) One interesting fact that is not common knowledge about the new dwarf planet.

Part (a)

Selecting the date January 3, 2024, we can observe the orbits of the 9 dwarf planets in figures 11 - 19. Each of the bodies orbit the Sun, often in highly elliptical and inclined orbits. Since they all orbit the Sun directly (they do not orbit a larger planet which orbits the Sun), they meet both conditions (i) and (iv) defined by the International Astronomical Union (IAU) to distinguish dwarf planets within the Solar System.

Sources for orbit viewing:

- https://theskylive.com/asteroids-and-dwarf-planets
- https://ssd.jpl.nasa.gov/tools/orbit_viewer.html

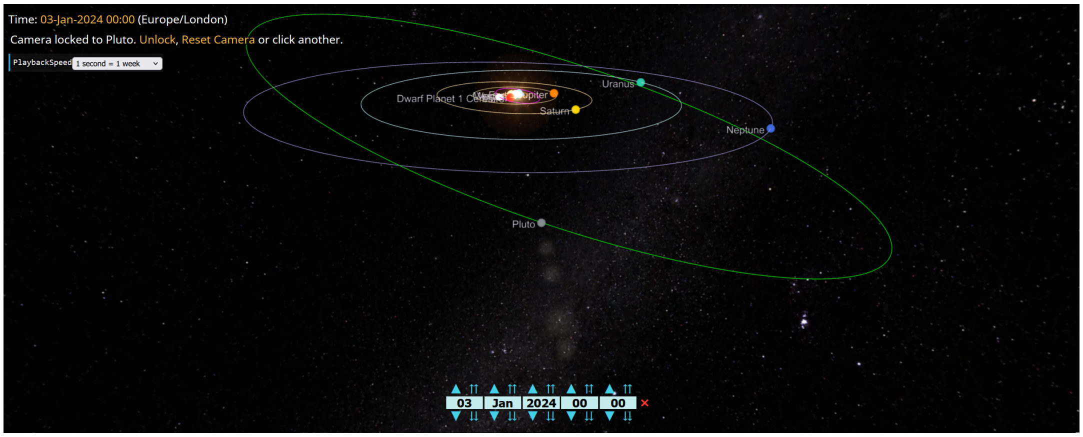

Figure 11: Orbit of Pluto on January 3, 2024.

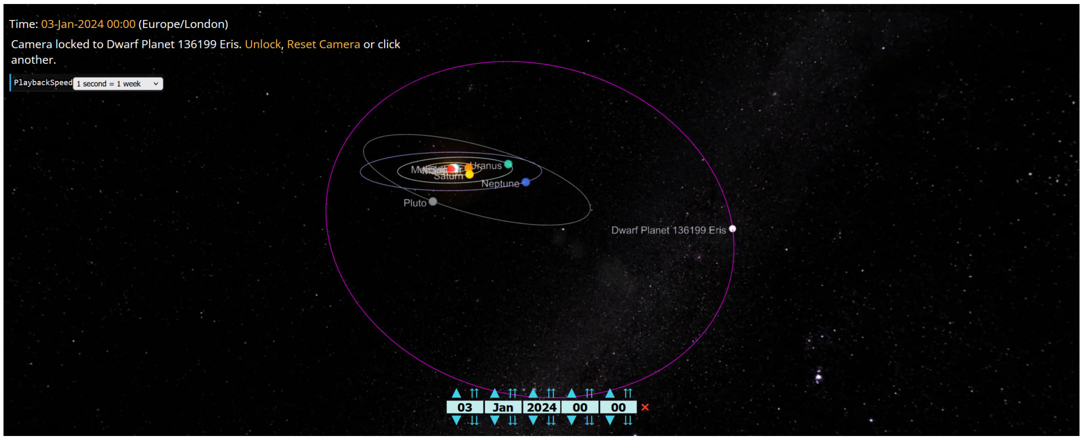

Figure 12: Orbit of Eris on January 3, 2024.

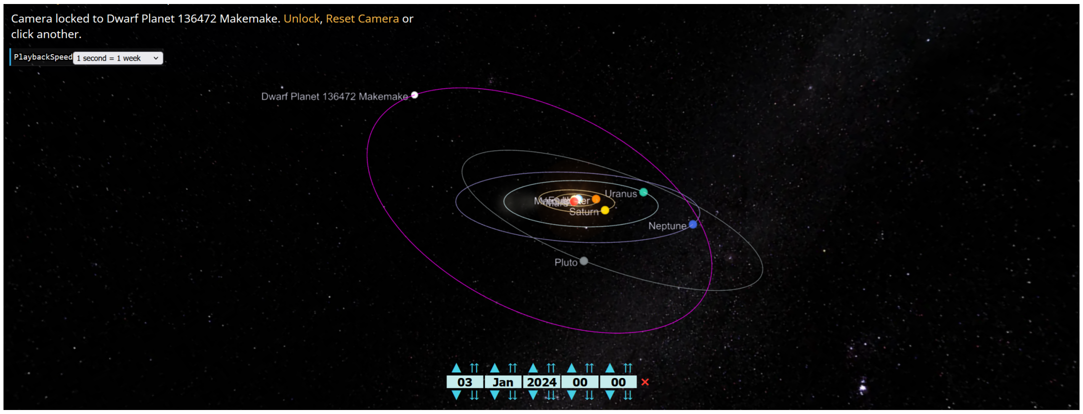

Figure 13: Orbit of Makemake on January 3, 2024.

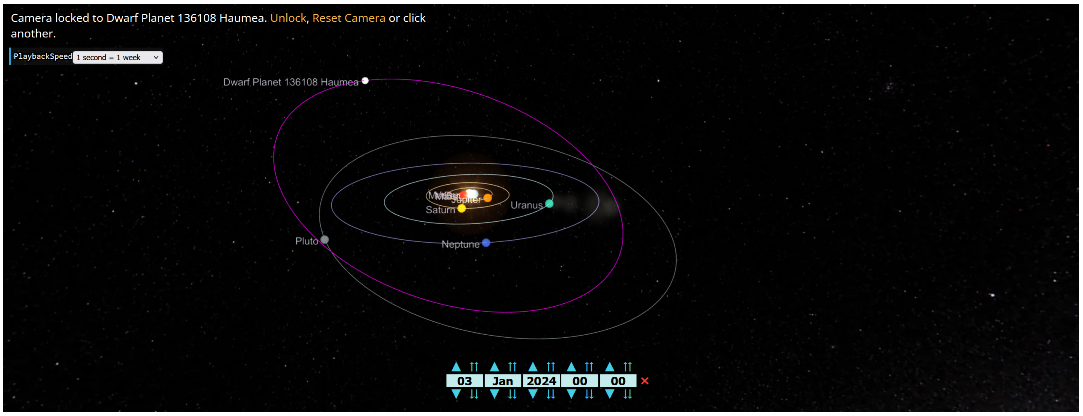

Figure 14: Orbit of Haumea on January 3, 2024.

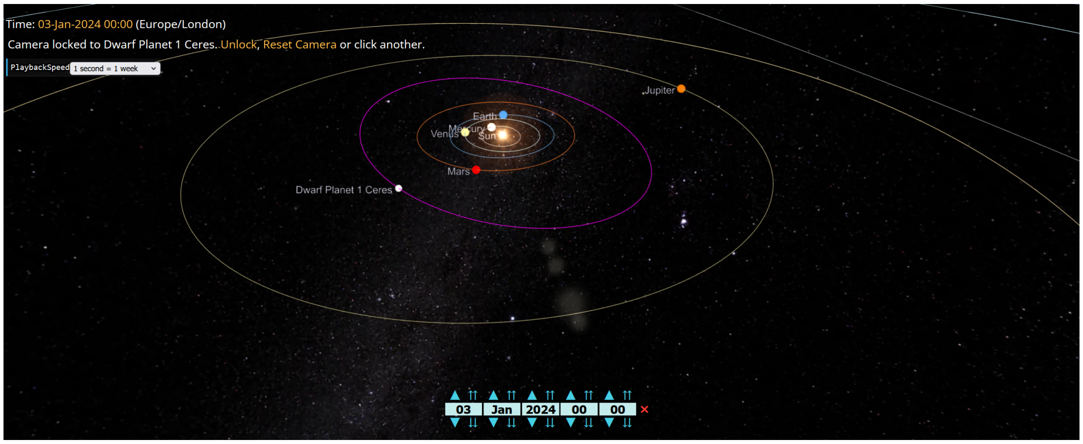

Figure 15: Orbit of Ceres on January 3, 2024.

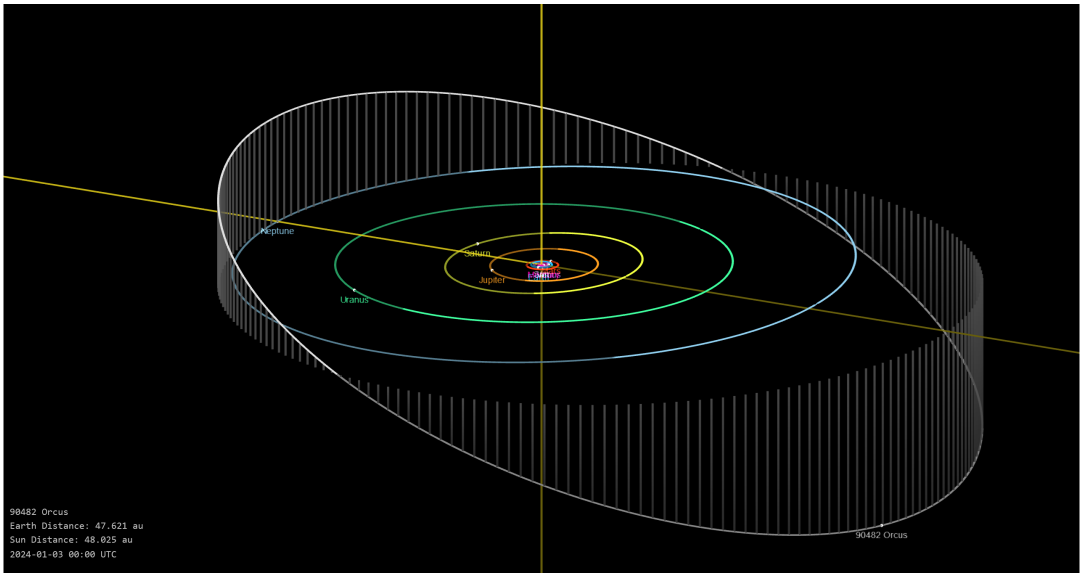

Figure 16: Orbit of Orcus on January 3, 2024.

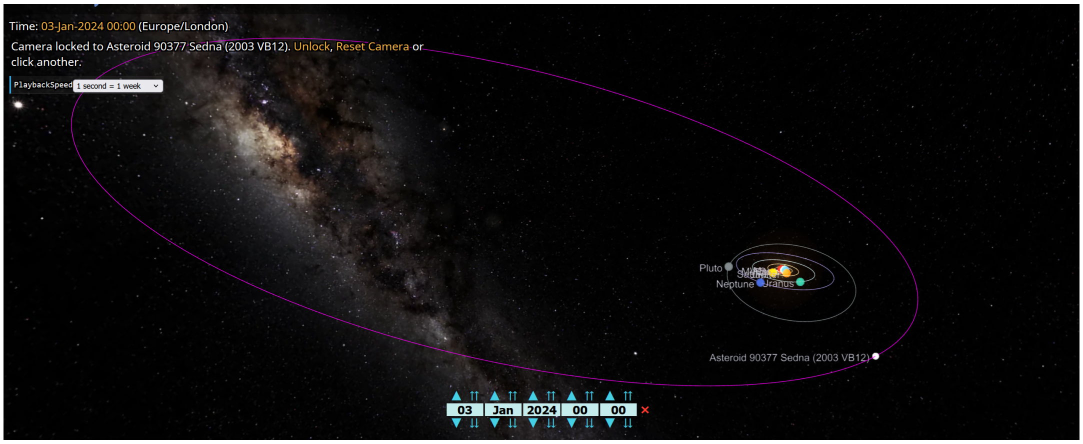

Figure 17: Orbit of Sedna on January 3, 2024.

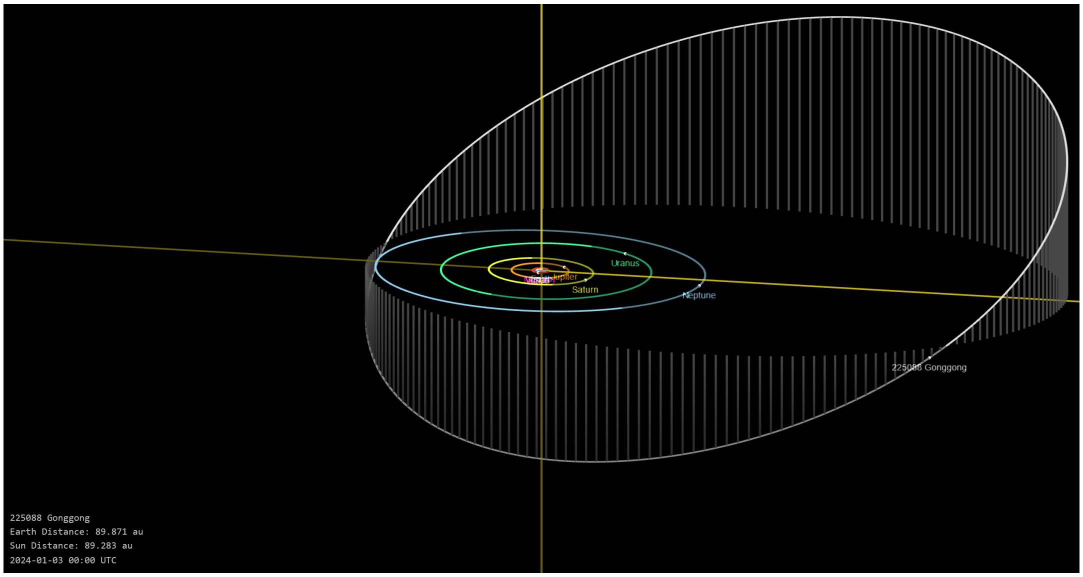

Figure 18: Orbit of Gonggong on January 3, 2024.

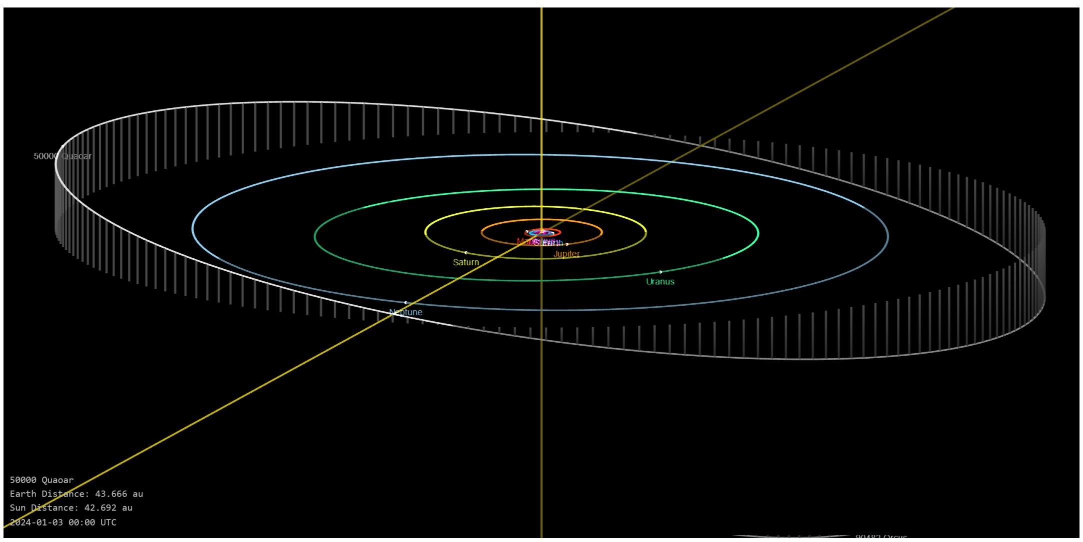

Figure 19: Orbit of Quaoar on January 3, 2024.

Part (b)

A table comparing some of the characteristic values for each of the dwarf planets is presented below along with the location of each relative to the ecliptic as observed on January 3 2024:

Name	Earth Dis. (AU)	P (Earth years)	SMA (AU)	i (deg)	Above/Below Ecliptic
Pluto	35.88	247.7407	39.4551	17.0890	below
Eris	95.492	562.2081	68.1176	43.7937	below
Makemake	52.646	304.4884	45.2594	29.0101	above
Haumea	50.297	281.3993	42.9414	28.2115	above
Ceres	3.613	4.6018	2.7666	10.5868	above
Orcus	47.621	244.4703	39.0972	20.5734	below
Sedna	82.839	11900.2080	521.2318	11.9309	below
Gonggong	89.871	552.9649	67.3689	30.6148	below
Quaoar	43.666	286.6307	43.4720	7.9912	above

Looking at the second column of the table, Ceres is by far the closest dwarf planet to Earth due to its location within the asteroid belt as opposed to the other bodies which are all transneptunian objects, meaning they orbit the Sun beyond the orbit of Neptune. Ceres will cross the ecliptic twice over the course of its orbital period of 4.6 Earth years. With only two opportunities for this to occur over such a long range of time, it is unlikely (but not impossible) that it will cross the ecliptic on the specified date. If we step forward in time, we can see that the next time Ceres crosses the ecliptic is around March 7 2024 as seen below:

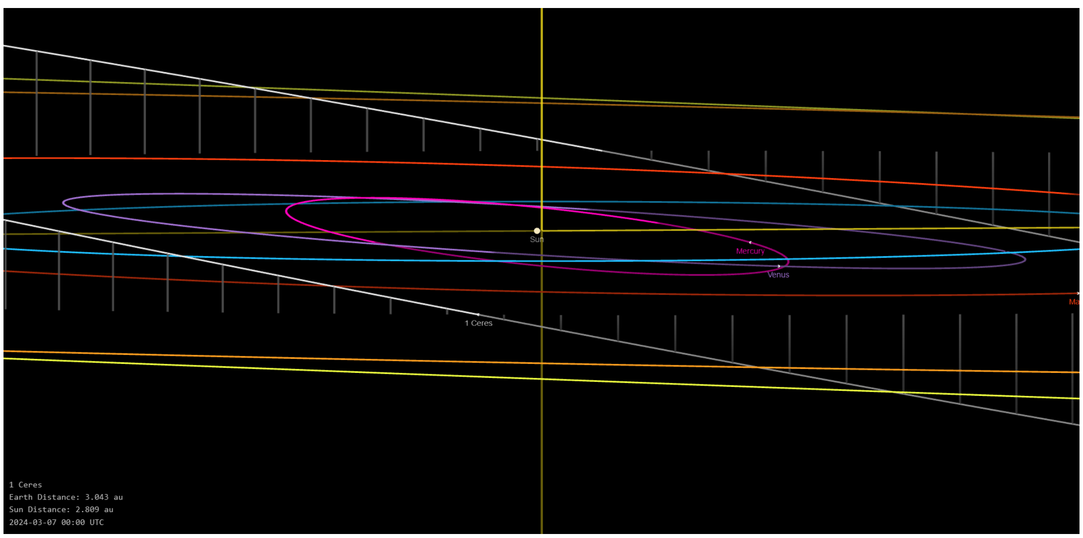

Figure 20: Location of Ceres on March 7, 2024.

Part (c)

Name	Discovery Year	Diameter (km)	Moon(s)	Moon Diameter(km)
Orcus	2005	910	Vanth	475
Sedna	2003	995	-	-
Gonggong	2007	1230	Xiangliu	<100
Quaoar	2002	1110	Weywot	170

Some fun facts:

- Orcus and Vanth seem to be composed of different materials. While there is evidence of water ice on the surface of Orcus, Vanth's more reddish coloring indicates that it has a more similar composition to other Kuiper Belt objects. [1]
- If you were to stand on the surface of Sedna, you could block the entire Sun with the head of a pen held at arms length due to its extreme distance from the Sun. [2]
- Gonggong was the first dwarf planet in the Solar System to be named after a Chinese mythical figure.[5]
- Quaoar is another body whose surface is comprised of water ice and also retains other volatiles including methane. Additionally, studies show that it may have at one point maintained an atmosphere.[7]

Sources:

[1] https://iopscience.iop.org/article/10.3847/1538-3881/aad9f2

[2] https://web.gps.caltech.edu/ mbrown/sedna/

[3] https://www.spacereference.org/asteroid/225088-gonggong-2007-or10

[4] https://www.sciencedirect.com/science/article/pii/S0019103518304354?via

[5] https://dbpedia.org/page/225088_Gonggong

[6] https://iopscience.iop.org/article/10.1088/0004-637X/773/1/26

[7] https://ui.adsabs.harvard.edu/abs/2020JOA....10a..24K/abstract