(HW8)

Sec 12.6

#7 $k = \dfrac{\text{thermal conductivity}}{(\text{specific heat})(\text{density})} = \dfrac{1.04}{0.056 \cdot 10.6}$

$\cong 1.752$

(IBVP)
$$\begin{cases} u_t - 1.752\, u_{xx} = 0, & 0 < x < 10,\ t > 0 \\ u(0,t) = 0, \quad u(10,t) = 0 & \\ u(x,0) = \underbrace{x(10-x)}_{= f(x)} & ; \ L = 10 \end{cases}$$

$$u(x,t) = \sum_{n=1}^{\infty} B_n e^{-1.752\left(\frac{n\pi}{10}\right)^2 t} \sin\left(\frac{n\pi x}{10}\right)$$

$$u(x,0) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{10}\right) = x(10-x).$$

$$B_n = \frac{2}{10}\int_0^{10} x(10-x)\sin\left(\frac{n\pi x}{10}\right) dx$$

$$= \frac{400\left(1-(-1)^n\right)}{(n\pi)^3}$$

$$u(x,t) = \sum_{n=1}^{\infty}\left[\frac{400\left(1-(-1)^n\right)}{(n\pi)^3}\right] e^{-1.752\left(\frac{n\pi}{10}\right)^2 t} \sin\left(\frac{n\pi x}{10}\right)$$

#11 $k = c^2$:

$$\begin{cases} u_t - c^2 u_{xx} = 0, & 0 < x < L, \ t > 0 \\ u_x(0,t) = 0, \quad u_x(L,t) = 0, & t > 0 \\ u(x,0) = f(x) \end{cases}$$

(Separation of variables). Set $u(x,t) = F(x)G(t)$

$u_t - c^2 u_{xx} = 0$: (1) $F'' = mF$

(2) $G' = mc^2 G$

BC: $\begin{cases} u_x(0,t) = F'(0)G(t) = 0 & \text{for any } t > 0 \\ u_x(L,t) = F'(L)G(t) = 0 & \text{''} \end{cases}$:

$F'(0) = 0, \quad F'(L) = 0$

(1) $F''(x) - mF(x) = 0, \quad F'(0) = 0, \quad F'(L) = 0$

$r^2 - m = 0$: $r = \pm\sqrt{m}$ ① $m > 0$: No eigenfunction

② $m = 0$: $F_0(x) = 1$ ③ $m < 0$: $F_n(x) = \cos\left(\frac{n\pi x}{L}\right)$

$m_n = -\left(\frac{n\pi}{L}\right)^2, \ n = 1, 2, \cdots$

(2) $G' + c^2\left(\frac{n\pi}{L}\right)^2 G = 0$: $G_n(t) = A_n e^{-\left(\frac{cn\pi}{L}\right)^2 t}$

$$u(x,t) = A_0 + \sum_{n=1}^{\infty} A_n e^{-\left(\frac{cn\pi}{L}\right)^2 t} \cos\left(\frac{n\pi x}{L}\right),$$

$$A_n = \frac{2}{L}\int_0^L f(x) \cos\left(\frac{n\pi x}{L}\right) dx, \quad n = 0, 1, 2, \cdots$$