Case Study

1. Determine if the polynomial

$$P(s) = s^7 + 2s^6 + 5s^5 + 7s^4 + 2s^3 - 5s^2 - 8s - 4$$

   is Hurwitz. Use the Routh-Hurwitz criterion to find the number of the polynomial right-half plane (RHP) zeros and determine the $j\omega$-axis zeros.

2. Consider a negative unity feedback system with the forward loop transfer function

$$G(s) = \frac{K}{s(s+3)(s+10)}.$$

   Use the Routh-Hurwitz criterion to find the interval of K for which the closed-loop system is asymptotically stable.

Please take a moment to work through this problem on your own. When you are ready, scroll down the page to view the solution.

Explanation:

1. The polynomial $P(s)$ is not Hurwitz because its coefficients have different signs. The Routh-Hurwitz array for the polynomial $P(s)$ is:

$$\begin{array}{c|cccc} s^7 & 1 & 5 & 2 & -8 \\ s^6 & 2 & 7 & -5 & -4 \\ s^5 & \frac{3}{2} & \frac{9}{2} & -\frac{12}{2} & \end{array}$$

   To simplify further calculations, we multiply the last row of the above array by $1/2$ and proceed to obtain

$$\begin{array}{c|cccc} s^7 & 1 & 5 & 2 & -8 \\ s^6 & 2 & 7 & -5 & -4 \\ s^5 & 3 & 9 & -12 & \\ s^4 & 1 & 3 & -4 & \\ s^3 & 0 & 0 & 0 & \end{array}$$

   We next form the auxiliary polynomial,

$$A(s) = s^4 + 3s^2 - 4.$$

   Its derivative with respect to s is

$$\frac{d}{ds}A(s) = 4s^3 + 6s.$$

We then proceed with the subsequent rows of the Routh-Hurwitz array to get

$$\begin{array}{c|cccc} s^7 & 1 & 5 & 2 & -8 \\ s^6 & 2 & 7 & -5 & -4 \\ s^5 & 3 & 9 & -12 & \\ s^4 & 1 & 3 & -4 & \\ s^3 & 4 & 6 & & \\ s^2 & \frac{3}{2} & -4 & & \\ s^1 & \frac{50}{3} & & & \\ s^0 & -4 & & & \end{array}$$

There is one sign change in the first column of the Routh-Hurwitz array. This implies that there is one RHP-zero of $P(s)$. There are two $j\omega$-axis zeros at $s = j2$ and $s = -j2$. We obtain the $j\omega$-axis zeros by factoring the auxiliary polynomial.

2. The closed-loop characteristic polynomial $P(s)$ is

$$P(s) = s^3 + 13s^2 + 30s + K.$$

The Routh-Hurwitz array corresponding to the polynomial $P(s)$ is

$$\begin{array}{c|cc} s^3 & 1 & 30 \\ s^2 & 13 & K \\ s^1 & \frac{390-K}{13} & \\ s^0 & K & \end{array}$$

From the Routh-Hurwitz array, it follows that the closed-loop system will be stable if and only if, $0 < K < 390$.