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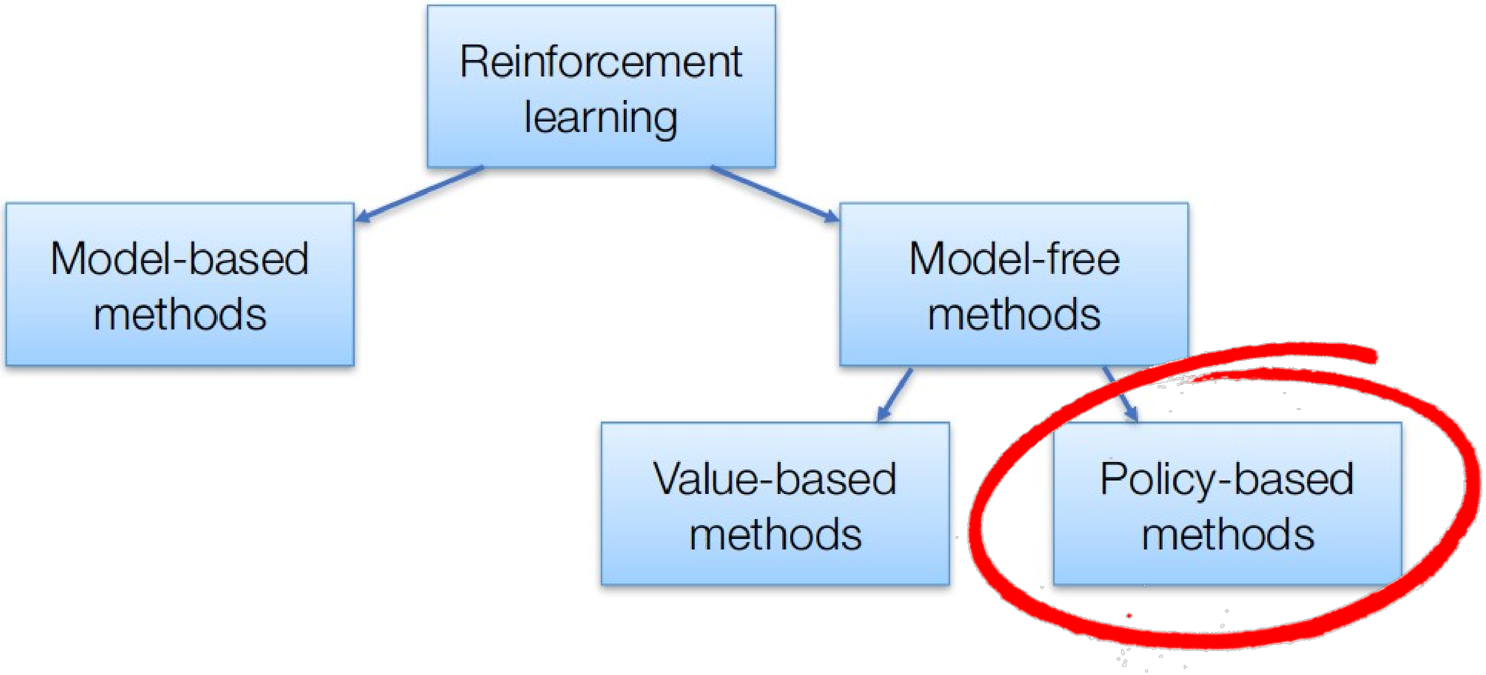

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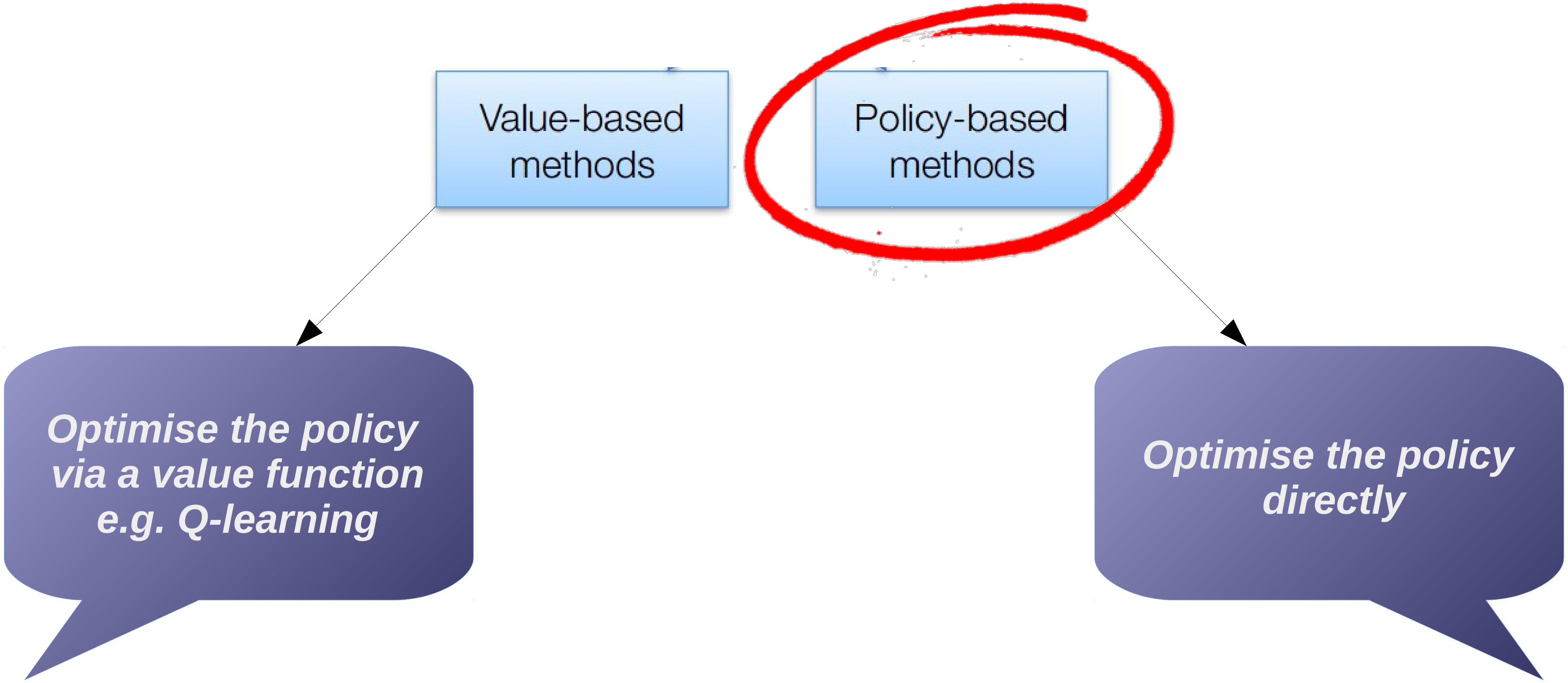

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Pros & cons of policy gradient methods (vs Q-learning)

- ➕ Learns policy directly (good when value function is very complex)
- ➕ Can learn stochastic policies
- ➕ Works with continuous action spaces
- ➕ (Often) faster convergence

- ➖ Can converge to local optima
- ➖ High variance of rewards leads to low sample efficiency
- ➖ Less stable performance

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Definitions

$\tau = (s_0, a_0, s_1, a_1, ..., s_{H-1}, a_{H-1}, s_H)$ is a trajectory

$\pi_\theta : \mathcal{S} \times \mathcal{A} \to [0, 1]$ is a stochastic policy with parameters $\theta$

$R(s_t, a_t)$ is the expected reward for taking action $a_t$ in state $s_t$

$\mathcal{R}(\tau) = \sum_{t=0}^{T-1} R(s_t, a_t)$ is the expected reward over a trajectory $\tau$

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**Note**

$\mathcal{R}(\tau)$ isn't in the sum…

…but $Q(s_t, a_t)$ is

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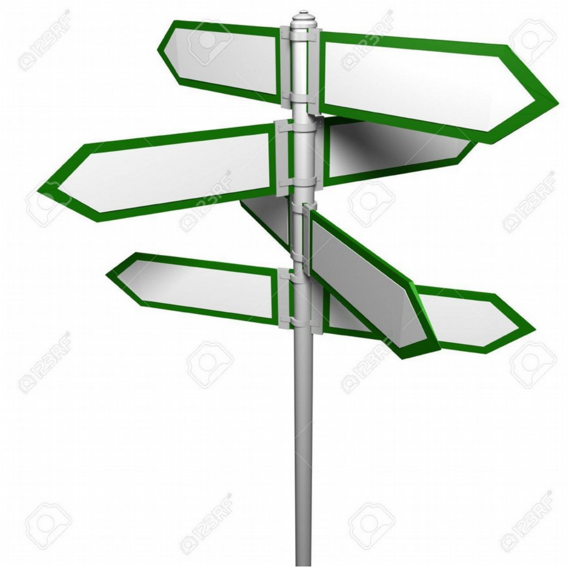

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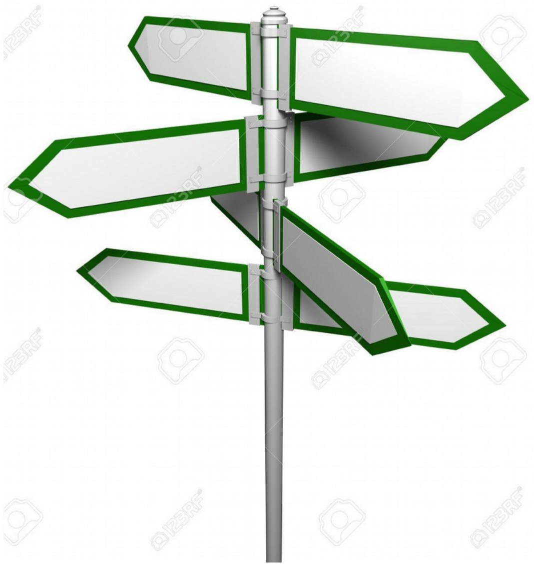

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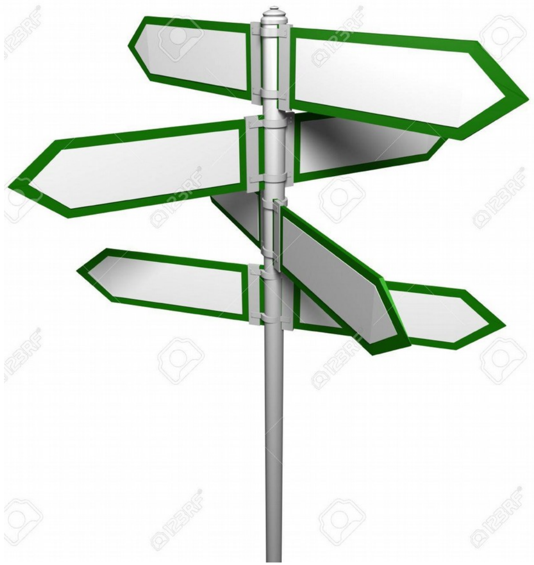

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$$\nabla_\theta U(\theta_n) = \frac{1}{m}\sum_{i=1}^{m}\left[\sum_{t=1}^{T-1} \left(\nabla_\theta \left[\log \pi_{\theta_n}\left(a_t^{(i)}|s_t^{(i)}\right)\right]\right)\mathcal{R}(\tau^{(i)})\right]$$

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